Assignment 2

Q1

a) Prog1 has 4 paths

① 1, 2, 3, 4, 9, 10, 11, 15

② 1, 2, 5, 6, 7, 9, 10, 11, 15

③ 1, 2, 3, 4, 9, 12, 13, 14, 15

④ 1, 2, 5, 6, 7, 9, 12, 13, 14, 15

b)

Edge	Symbolic State	Path Condition
① $2 \to 3,4$	$x \to X_0+1$ $y \to Y_0-2$	$X_0 - Y_0 > 30$
$3,4 \to 9 \to 10,11$	$x \to 3X_0+3$ $y \to 2Y_0-4$	$3X_0 - 3Y_0 < 43$
		PC: $X_0 - Y_0 > 30 \wedge 3X_0 - 3Y_0 < 43$
③ $2 \to 3,4$	$x \to X_0+1$ $y \to Y_0-2$	$X_0 - Y_0 > 30$
$3,4 \to 9 \to 13,14$	$x \to 4X_0+4$ $y \to 7Y_0-14$	$3X_0 - 3Y_0 \geq 43$
		PC: $X_0 - Y_0 > 30 \wedge 3X_0 - 3Y_0 \geq 43$
② $2 \to 5,6$	$x \to X_0-3$ $y \to Y_0+5$	$X_0 - Y_0 \leq 30$
$5,6 \to 9 \to 10,11$	$x \to 3X_0-9$ $y \to 2Y_0+10$	$3X_0 - 3Y_0 < 64$
		PC: $X_0 - Y_0 \leq 30 \wedge 3X_0 - 3Y_0 < 64$
④ $2 \to 5,6$	$x \to X_0-3$ $y \to Y_0+5$	$X_0 - Y_0 \leq 30$
$5,6 \to 9 \to 13,14$	$x \to 4X_0-12$ $y \to 7Y_0+35$	$3X_0 - 3Y_0 \geq 64$
		PC: $X_0 - Y_0 \leq 30 \wedge 3X_0 - 3Y_0 \geq 64$

c) ① Infeasible $(30 < X_0 - Y_0 < 14)$

② Feasible $(30 < X_0 - Y_0)$ $X_0 = 100$

$Y_0 = 50$

③ Feasible $(X_0 - Y_0 < 21)$ $X_0 = 52$

$Y_0 = 50$

④ Feasible $(22 \leq X_0 - Y_0 \leq 30)$ $X_0 = 50$

$Y_0 = 25$

Q2

a) $\bigwedge_{1\leq i\leq 4} \neg a_i \vee \bigvee_{1\leq i\leq 4}\left(\bigwedge_{\substack{1\leq j\leq 4\\ j\neq i}} \neg a_j \wedge a_i\right)$

no-true

outer loop a_i should be true

inner loop a_j is false

b) $\bigvee\left(v_u \wedge \left[\bigwedge_{u<x<v} v_x\right] \wedge v_w\right)$ $(v_{init}, u) \in E$, $(w, v_{end}) \in E$

meaning x is directed nodes further from u and directly nodes closer to v.

c) $\bigwedge_{1\leq i\leq n} \neg a_i \vee \bigvee_{1\leq i\leq n}\left(\bigwedge_{\substack{1\leq j\leq n\\ j\neq i}} \neg a_j \wedge a_i\right)$

Q3

a) Let A be the first string such that A_i represents ith character
B be the second " B_i "
C be the third " C_i "
S be any string.
then the puzzle logic is:

$$IntVal(C_1) = (IntVal(A_1) + IntVal(B_1)) \bmod 10$$

$$a = len(A) \wedge b = len(B) \wedge c = len(C) \wedge n = max(a,b,c)$$
$$\wedge\ 0 < i < n \rightarrow IntVal(C_i) = [IntVal(A_i) + IntVal(B_i) + (IntVal(A_{i-1}) + IntVal(B_{i-1}))//10] \bmod 10$$

$$a = len(A) \wedge b = len(B) \wedge c = len(C) \wedge n = min(a,b,c)$$
$$\wedge\ i \geq n \rightarrow IntVal(S_i) = 0$$

$$S_i \neq S_j \rightarrow IntVal(S_i) \neq IntVal(S_j)$$

$$0 \leq IntVal(S_i) \leq 9$$

Q5

a) True (Proof by contradiction)
(Valid)

① $\Rightarrow$ side: Assume RHS is false such that

$$(\forall x \cdot \exists y \cdot P(x) \vee Q(y)) \rightarrow \neg((\forall x \cdot P(x)) \vee (\exists y \cdot Q(y)))$$

$$\Leftrightarrow (\forall x \cdot \exists y \cdot P(x) \vee Q(y)) \rightarrow (\exists x \cdot \neg P(x)) \wedge (\forall y \neg Q(y))$$

This is a contradiction since LHS is not satisfied where RHS is there exists x s.t. $\neg P(x)$ and for all y $\neg Q(y)$.

② $\Leftarrow$ side: Assume LHS is false such that

$$(\forall x \cdot P(x)) \vee (\exists y \cdot Q(y)) \rightarrow \neg(\forall x \cdot \exists y \cdot P(x) \vee Q(y))$$

$$\Leftrightarrow (\forall x \cdot P(x)) \vee (\exists y \cdot Q(y)) \rightarrow \exists x \forall y \neg (P(x) \vee Q(y))$$

This is a contradiction since LHS is not satisfied where RHS is there exists x such that for all y $P(x) \vee Q(y)$ is NOT true.

$\boxtimes$

b) False

Counter Example:

$X = \{a, b, c\}$ $\quad$ $Y = \{d, e, f\}$

$M(a) = a$, $M(b) = b$, $M(c) = c$, $M(d) = d$, $M(e) = e$, $M(f) = f$

$M(P) = \{(a, d), (c, f)\}$ $\quad$ $M(Q) = \{(b, e)\}$

then $\forall x \cdot \exists y\ P(x, y) \vee Q(x, y)$ is true since $P(a, d) \wedge P(c, f) \wedge P(b, e)$

but the RHS is false:

$\forall x \cdot \exists y\ P(x, y) = \text{false} \vee \forall x \exists y\ Q(x, y) = \text{false}$

$\therefore$ The sentence is false.

c) $(\exists x, y \cdot x \neq y) \wedge (\forall a_0, a_1 \bigvee_{0 \leq i < j \leq 1} P(a_i) = P(a_j))$

d) Array(A)

$\exists i \forall j, \forall k \cdot 0 \leq i < len(A) \wedge 0 \leq j < i \wedge i < k < len(A) \rightarrow A[j] < A[k]$

e) Array(A)

$\forall i \forall j \cdot 0 \leq i < j \wedge i < j < len(A) \rightarrow A[i] \neq A[j]$