Homework 3

① Write expression for loss w/ single datapoint

$$\boxed{\text{Loss}(x, y, \vec{\omega}) = (\sigma(\vec{\omega}\phi(x)) - y)^2}$$

② $$\nabla_{\vec{\omega}} \text{Loss} = 2(\sigma(\vec{\omega}\phi(x)) - y)\,\sigma'(\vec{\omega}\phi(x))\,\phi(x)$$

$$= 2\phi(x)(\sigma(\vec{\omega}\phi(x)) - y)\,\sigma'(\vec{\omega}\phi(x))$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\sigma'(z) = -1(1+e^{-z})^{-2}(-e^{-z})$$

$$= \frac{e^{-z}}{(1+e^{-z})^2} = \left(\frac{1}{1+e^{-z}}\right)\left(\frac{1}{1+e^{-z}}\right)e^{-z}$$

$$= \sigma^2(z)e^{-z}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$1 + e^{-z} = \sigma^{-1}(z)$$

$$e^{-z} = \sigma^{-1}(z) - 1$$

$$= \sigma^2(z)(\sigma^{-1}(z) - 1)$$

$$\sigma'(z) = \sigma(z) - \sigma^2(z)$$

$$= 2\phi(x)(\sigma(\vec{\omega}\phi(x)) - y)(\sigma(\vec{\omega}\phi(x)) - \sigma^2(\vec{\omega}\phi(x)))$$

$$= 2\phi(x)(P - y)(P - P2)$$

$$= \boxed{-2\phi(x)P(P-y)(P-1)}$$

③ to make $\nabla_{\vec{\omega}}$ Loss $-2\phi(x)P(P-1)^2$ arbitrarily small, and given $\sigma(z) = (1+e^{-z})^{-1}$, the value of $\vec{\omega}$ would have to approach $-\infty$. The ∇ magnitude can never be 0.

④ Same datapoint: $\nabla_{\vec{w}} \text{loss} = -2\phi(x)\, p(p-1)^2$

Find largest magnitude.

$= -2\phi(x)\, p(p^2-2p+1)$

$= -2\phi(x)(p^3-2p^2+p)$

$$\frac{d}{dp}\left[-2\phi(x)(p^3-2p^2+p)\right] = -2\phi(x)\left[3p^2-4p+1\right]$$

$= -2\phi(x)(3p-1)(p-1)$

largest magnitude can occur @ $p = \frac{1}{3}$ or $p = 1$

$\hookrightarrow \|2\phi(x)(\frac{1}{3})(\frac{2}{3})^2\|$ or $\|2\phi(x)(1)(0)\|$

$= \phi(x)(\frac{8}{27})$ $\qquad = 0$

largest magnitude $= (\frac{8}{27})\,\phi(x)$

⑤ $\text{loss}_D = (\sigma(\vec{w}\phi(x)) - y)^2 = 0$

$\text{loss}_{D'} = (\vec{w}'\phi(x) - y')^2 = 0$

$\sigma(\vec{w}\phi(x)) - y = 0$

$\vec{w}'\phi(x) - y' = 0$

$y = \sigma(\vec{w}\phi(x))$

$y' = \vec{w}'\phi(x)$

$\sigma^{-1}(\sigma(\vec{w}\phi(x))) = \sigma^{-1}(y)$

$\sigma^{-1}(\vec{w}'\phi(x)) = \sigma^{-1}(y')$

$w'\phi(x) = y' = \sigma^{-1}(y)$