An Introduction To Information Theory Solutions

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1 Introduction

1. An alphabet consists of four letters *A, B, C, D* with respective probabilities of transmission $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{6}$. Find the average amount of information associated with the transmission of a letter.

Solution:
To find the average amount of information associated with the transmission of a letter we need to find the entropy of the system.

$$\text{Entropy} = -\sum_{i=1}^{n} P(x_i)\log_2 P(x_i)$$
$$\text{Entropy} = -(\tfrac{1}{3}\log_2\tfrac{1}{3} + \tfrac{1}{4}\log_2\tfrac{1}{4} + \tfrac{1}{4}\log_2\tfrac{1}{4} + \tfrac{1}{6}\log_2\tfrac{1}{6})$$
$$\text{Entropy} = 1.9591 \text{ bits}$$

2. An independent, discrete source transmits letters selected from an alphabet consisting of three letters A, B, and C, with respective probabilities

$$p_A = 0.7 \quad p_B = 0.2 \quad p_C = 0.1$$

a) Find the entropy per letter.
b) If consecutive letters are statistically independent and two-symbol words are transmitted, find all the pertinent probabilities for all two-letter words and the entropy of the system of such words.

Solution:
a) To find the entropy per letter multiply the information of each letter by the probability the letter occurs and sum the results.

$$\text{Information} = I_x = -\log_2 p_x$$
$$\text{Entropy per letter} = H_x = I_x * p_x$$
$$H_A = 0.36 \text{ bits} \quad H_B = 0.46 \text{ bits} \quad H_C = 0.33 \text{ bits}$$

$$\Sigma H_x = H_A + H_B + H_C = 1.15 \text{ bits}$$

b) First we need to find the probability of each two-letter word occurring. Assuming each letter is statistically independent of the other, we multiply together the probabilities of each possible two-letter combination to get the probability of each two-letter word.

$$p_{AA} = 0.49 \quad p_{AB} = 0.14 \quad p_{AC} = 0.07$$

$$p_{BA} = 0.14 \quad p_{BB} = 0.04 \quad p_{BC} = 0.02$$

$$p_{CA} = 0.07 \quad p_{CB} = 0.02 \quad p_{CC} = 0.01$$

Then we can simply use the Shannon entropy formula to get the entropy of the system.

$$H(X) = -\sum_{i=1}^{n} P(x_i) \log_2 P(x_i)$$

$$H(X) = 2.31 \text{ bits}$$