



Fully-Online Suffix Tree and Directed Acyclic Word Graph Construction for Multiple Texts

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Received: 8 March 2018 / Accepted: 22 October 2019 / Published online: 31 October 2019
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Abstract

We consider the construction of the *suffix tree* and the *directed acyclic word graph* (DAWG) indexing data structures for a collection \mathcal{T} of texts, where a new symbol may be appended to any text in $\mathcal{T} = \{T_1, \dots, T_K\}$, at any time. This *fully-online* scenario, which arises in dynamically indexing multi-sensor data, is a natural generalization of the long solved *semi-online* text indexing problem, where texts T_1, \dots, T_k are permanently fixed before the next text T_{k+1} is processed for each k ($1 \leq k < K$). We first show that a direct application of Weiner’s right-to-left online construction for the suffix tree of a single text to fully-online multiple texts requires superlinear time. This also means that Blumer et al.’s left-to-right online construction for the DAWG of a single text requires superlinear time in the fully-online setting. We then present our fully-online versions of these algorithms that run in $O(N \log \sigma)$ time and $O(N)$ space, where N is the total length of the texts in \mathcal{T} and σ is their alphabet size. We then show how to extend Ukkonen’s left-to-right online suffix tree construction to fully-online multiple strings, with the aid of Weiner’s suffix tree for the reversed texts.

Keywords String algorithms · Suffix trees · DAWGs · Multiple texts · Online algorithms

1 Introduction

Text indexing is a fundamental problem in computer science, which arises in many applications including text retrieval, molecular biology, signal processing, and sensor

Dany Breslauer—deceased.

A preliminary conference version of this paper [14] contained an error in the analysis of the fully-online versions of Weiner’s algorithm and of Blumer et al.’s algorithm, where the claimed $O(N \log \sigma)$ time bounds neglected to account for the redirections of the soft Weiner links or DAWG edges. We show that in some cases $\Theta(N \min(K, \sqrt{N}))$ or $\Theta(N^{1.5})$ DAWG edge re-directions may be structurally required, and that the correct time bounds for the algorithms in the conference paper [14] are $O(N \min(K, \sqrt{N}) \log \sigma)$ (Lemma 2).

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data analysis. In this paper, we focus on indexing a collection of multiple texts, so that subsequent pattern matching queries can be answered quickly. In particular, we study online indexing for a collection \mathcal{T} of multiple texts, where a new character can be appended to each text at *any* time. Such fully-online indexing for multiple growing texts has potential applications to continuous processing of data streams, where a number of symbolic events or data items are produced from multiple, rapid, time-varying, and unbounded data streams [1,12]. For example, motif mining system tries to discover characteristic or interesting collective behaviors, such as frequent path or anomalies, from data streams generated by a collection of moving objects or sensors [12,16].

In this paper we consider two fundamental text indexing data structures, the *suffix tree* [17] and the *directed acyclic word graph (DAWG)* [2] (a.k.a. *suffix automaton* [6]). The suffix tree of a string T is an edge-labeled rooted tree that represents all the suffixes of T in space linear in the length of T , while the DAWG of T is the smallest (partial) DFA that recognizes all suffixes of T that also occupies linear space. The suffix tree can be constructed in $O(n \log \sigma)$ time and $O(n)$ space, in a right-to-left online manner by Weiner’s algorithm [17], and in a left-to-right online manner by Ukkonen’s algorithm [15], where n is the length of T and σ is the alphabet size. The DAWG of a given text T can also be built in $O(n \log \sigma)$ time and $O(n)$ space, in a left-to-right online manner by Blumer et al.’s algorithm [2]. The “duality” of the DAWG of T and the suffix tree of the reversal \bar{T} of T is known. More specifically, the tree of the *suffix links* of the DAWG of T coincides with the suffix tree of \bar{T} [2,7].

This paper deals with online constructions of indexing structures for multiple texts. The *generalized suffix tree* [10] for multiple texts is the suffix tree that represents all suffixes of the texts. Throughout this paper, we will simply call generalized suffix trees as suffix trees. The DAWG for multiple texts were proposed in [3]. We remark that the aforementioned duality property also holds for multiple texts.

Let N be the final total length of the growing texts in a fully-online text collection $\mathcal{T} = \{T_1, \dots, T_K\}$. The above existing suffix tree and DAWG construction algorithms for a single text also work within the same $O(N \log \sigma)$ time and $O(N)$ space bounds for a collection of growing texts in the *semi-online* setting, where only the last inserted text can be extended [3,10]. However, special attention is needed for the construction of the suffix tree and of the DAWG in our *fully-online* setting. For the fully-online *right-to-left* setting where new characters are prepended to the multiple texts, we show that a matching upper and lower bound $\Theta(N \min(K, \sqrt{N}))$ or $\Theta(N^{1.5})$ holds for a direct extension of Weiner’s original algorithm, where K is the number of texts in the collection. We prove these bounds by showing that the total number of suffix tree nodes in the paths that Weiner’s algorithm walks up is $\Theta(N \min(K, \sqrt{N}))$. It is known that the number of edge-redirections on DAWGs for the reverse strings is proportional to this number of suffix tree nodes [2]. Hence, $\Theta(N \min(K, \sqrt{N}))$ or $\Theta(N^{1.5})$ DAWG edge re-directions can be required during the DAWG construction in the fully-online *left-to-right* setting. Also, we show that during the construction of the suffix tree for a fully-online left-to-right text collection, the open-ended suffix tree leaf edge label representation, the cornerstone of Ukkonen’s [15] on-line suffix tree algorithm, may have to update the association between the numerous suffix tree

leaf edge labels and the various texts $\Omega(\frac{N^2}{K})$ times, which turns $\Omega(N^2)$ when the collection contains only a constant number of texts. Thus, if we wish to stay within the $O(N \log \sigma)$ time bounds in the *fully-online* setting, the DAWG edges and the suffix tree leaf edge labels in the fully-online left-to-right setting cannot be directly explicitly maintained. We call this as the *leaf ownership* problem.

To overcome the above difficulties, we first show how to extend Weiner's algorithm in the fully-online right-to-left setting, with the aid of the nearest marked ancestor (NMA) data structures [18]. The resulting algorithm runs in $O(N \log \sigma)$ time and takes $O(N)$ total space for a general ordered alphabet of size σ . We then show that how an $O(N)$ -space representation of the DAWG can be incrementally maintained for a left-to-right online text collection, in overall $O(N \log \sigma)$ time and $O(N)$ space. Hence, at any moment during the fully-online growth of the texts, we can find all *occ* occurrences of a given pattern of length M in the current text collection in $O(M \log \sigma + occ)$ time, using any of these two text indexing structures.

Our next goal is to extend Ukkonen's construction [15] to fully-online left-to-right construction of suffix trees for multiple texts in $O(N \log \sigma)$ time and $O(N)$ space bounds. As was already mentioned above, however, it is not possible to explicitly maintain the owners of the leaf edges in the suffix tree here. To overcome this difficulty, we present a new novel technique that *swaps* the active points among the texts that are involved in the update of the suffix tree, together with a query algorithm that efficiently answers the owners of the particular leaf edges involved in the update. As a result, we obtain a natural extension of Ukkonen's construction where suffixes are inserted to the current tree in decreasing order of their lengths (called the *forward approach*). We also present an alternative method that inserts suffixes in increasing order of their lengths each time a new character is appended to one of the texts (called the *backward approach*). Both methods work in $O(N \log \sigma)$ time and $O(N)$ space, with the aid of the extended Weiner algorithm for right-to-left text collection.

Related Work

We note that we can obtain fully-online text indexing data structure for multiple texts by using existing more general dynamic text indexing data structures as follows. To use the indexing data structure of Ferragina and Grossi [8] that permits character-wise updates, we build a text $\$1 \cdots \K that initially consists only of K delimiters. Then, appending a character a to the k th text in the collection reduces to prepending a to the k th delimiter $\$k$. Using this approach, the indexing structure of Ferragina and Grossi [8] takes $O(N \log N)$ total time to be constructed, requires $O(N \log N)$ space, and allows pattern matching in $O(M + \log N + N \log M + occ)$ time.¹ Using the compressed indexing data structure for a dynamic text collection of Chan et al. [5], we can append a new character a to the k th text T_k by removing T_k and then adding $T_k a$ in $O(|T_k|)$ time. This yields a fully-online text indexing structure with $O(N^2 \log N)$ construction

¹ The original claimed bound in [8] is $O(M + \log N + upd \log M + occ)$, where *upd* denotes the number of string edit operations performed so far. Since we append N total characters in our online setting, *upd* = $O(N)$ in this case.

time and $O(N)$ bits of space (or $O(N/\log N)$ words of space assuming $\Theta(\log N)$ -bit machine word), supporting pattern matching in $O(M \log N + occ \log^2 N)$ time.

2 Preliminaries

2.1 String Notations

Let Σ be a general ordered alphabet. Any element of Σ^* is called a *string*. For any string T , let $|T|$ denote its length. Let ε be the empty string, namely, $|\varepsilon| = 0$. If $T = XYZ$, then X , Y , and Z are called a *prefix*, a *substring*, and a *suffix* of T , respectively. For any $1 \leq i \leq j \leq |T|$, let $T[i..j]$ denote the substring of T that begins at position i and ends at position j in T . For convenience, let $T[i..j] = \varepsilon$ if $i > j$. For any $1 \leq i \leq |T|$, let $T[i]$ denote the i th character of T . For any string T , let $\text{Suffix}(T)$ denote the set of suffixes of T , and for any set \mathcal{T} of strings, let $\text{Suffix}(\mathcal{T})$ denote the set of suffixes of all strings in \mathcal{T} . Namely, $\text{Suffix}(\mathcal{T}) = \bigcup_{T \in \mathcal{T}} \text{Suffix}(T)$. For any string T , let \bar{T} denote the reversed string of T , i.e., $\bar{T} = T[|T|] \cdots T[1]$.

Let $\mathcal{T} = \{T_1, \dots, T_K\}$ be a collection of K texts. For each text T_k , the integer k is called its *id*. For any $1 \leq k \leq K$, let $\text{lrs}_{\mathcal{T}}(T_k)$ be the longest repeating suffix of T_k that occurs at least twice in the texts of \mathcal{T} .

2.2 Suffix Trees and DAWGs for Multiple Texts

2.2.1 Suffix Tries

The *suffix trie* for a text collection $\mathcal{T} = \{T_1, \dots, T_K\}$, denoted by $\text{STrie}(\mathcal{T})$, is a trie which represents $\text{Suffix}(\mathcal{T})$. The size of $\text{STrie}(\mathcal{T})$ is $O(N^2)$, where N is the total length of texts in \mathcal{T} . We identify each node v of $\text{STrie}(\mathcal{T})$ with the string that v represents. A substring x of a text in \mathcal{T} is said to be *branching* in \mathcal{T} , if there exist two distinct characters $a, b \in \Sigma$ such that both xa and xb are substrings of some texts in \mathcal{T} . Clearly, node x of $\text{STrie}(\mathcal{T})$ is branching iff x is branching in \mathcal{T} .

For each node av of $\text{STrie}(\mathcal{T})$ with $a \in \Sigma$ and $v \in \Sigma^*$, let $\text{slink}(av) = v$. This auxiliary edge $\text{slink}(av) = v$ from av to v is called a *suffix link*. We define the *reversed suffix link* $\mathcal{W}_a(v) = av$ iff $\text{slink}(av) = v$. For any node v and $a \in \Sigma$, if av is not a substring of the texts in \mathcal{T} , then $\mathcal{W}_a(v)$ is undefined. By definition, the reversed suffix links on $\text{STrie}(\mathcal{T})$ form a rooted tree which coincides with $\text{STrie}(\bar{\mathcal{T}})$, the suffix trie for the collection $\bar{\mathcal{T}} = \{\bar{T}_1, \dots, \bar{T}_K\}$ of the reversed texts.

2.2.2 Suffix Trees

A compacted trie is a rooted tree such that (1) each edge is labeled by a non-empty string, (2) each internal node is branching, and (3) the string labels of the out-going edges of each node begin with mutually distinct characters. The *suffix tree* [17] for a text collection \mathcal{T} , denoted by $\text{STree}(\mathcal{T})$, is a compacted trie which represents $\text{Suffix}(\mathcal{T})$. In other words, $\text{STree}(\mathcal{T})$ is obtained by compacting every maximal path of $\text{STrie}(\mathcal{T})$

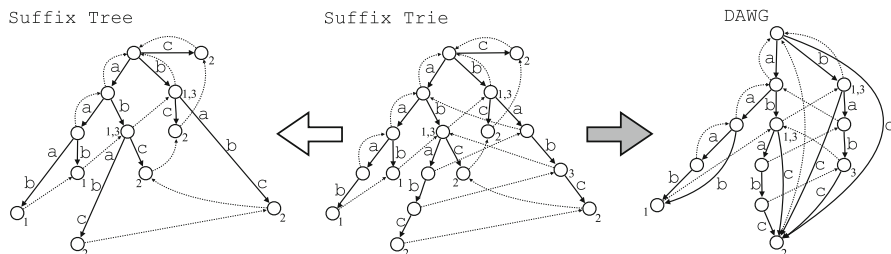


Fig. 1 Illustration for $STrie(\mathcal{T})$, $STree(\mathcal{T})$, and $DAWG(\mathcal{T})$ with $\mathcal{T} = \{T_1 = aaab, T_2 = ababc, T_3 = bab\}$. The solid arrows and broken arrows represent the edges and the suffix links of each data structure, respectively. The number k ($k = 1, 2, 3$) beside each node indicates that the node represents a suffix of T_k . The nodes $[ab]_{\mathcal{T}}$ and $[b]_{\mathcal{T}}$ are separated in $DAWG(\mathcal{T})$ since the node bab in $STrie(\mathcal{T})$ represents a suffix of T_3 , while the node $abab$ does not (see also the subtrees rooted at nodes ab and b in $STrie(\mathcal{T})$)

which consists of non-branching internal nodes (see Fig. 1). Since every internal node of $STree(\mathcal{T})$ is branching, and since there are at most N leaves in $STree(\mathcal{T})$, the numbers of edges and nodes are $O(N)$. The edge labels of $STree(\mathcal{T})$ are non-empty substrings of some text in \mathcal{T} . By representing each edge label x with a triple $\langle k, i, j \rangle$ of integers s.t. $x = T_k[i..j]$, $STree(\mathcal{T})$ can be stored with $O(N)$ space. We say that any branching (resp. non-branching) substring of \mathcal{T} is an *explicit node* (resp. *implicit node*) of $STree(\mathcal{T})$. An implicit node x is represented by a triple (v, a, ℓ) , called a *reference* to x , such that v is an explicit ancestor of x , a is the first character of the path from v to x , and ℓ is the length of the path from v to x . A reference (v, a, ℓ) to node x is called *canonical* if v is the lowest explicit ancestor of x .

For each explicit node av of $STree(\mathcal{T})$ with $a \in \Sigma$ and $v \in \Sigma^*$, let $slink(av) = v$. For each explicit node v and $a \in \Sigma$, we also define the reversed suffix link $\mathcal{W}_a(v) = avx$ where $x \in \Sigma^*$ is the shortest string such that avx is an explicit node of $STree(\mathcal{T})$. $\mathcal{W}_a(v)$ is undefined if av is not a substring of texts in \mathcal{T} . These reversed suffix links are also called as *Weiner links* (or *W-link* in short) in the literature [4]. A W-link $\mathcal{W}_a(v) = avx$ is said to be *hard* if $x = \varepsilon$, and *soft* if $x \in \Sigma^+$. Let w be a Boolean function such that for any explicit node v and $a \in \Sigma$, $w_a(v) = 1$ iff (soft or hard) W-link $\mathcal{W}_a(v)$ exists. Notice that if $w_a(v) = 1$ for a node v and $a \in \Sigma$, then $w_a(u) = 1$ for every ancestor u of v .

2.2.3 Directed Acyclic Word Graphs (DAWGs)

The *directed acyclic word graph* (DAWG in short) [2,3] of a text collection \mathcal{T} , denoted by $DAWG(\mathcal{T})$, is a small DAG which represents $Suffix(\mathcal{T})$. $DAWG(\mathcal{T})$ is obtained by merging identical subtrees of $STrie(\mathcal{T})$ connected by the suffix links (see Fig. 1). Hence, the label of every edge of $DAWG(\mathcal{T})$ is a single character. The leaves of $STrie(\mathcal{T})$ that correspond to different texts are not connected by the suffix links. On the other hand, it is possible that \mathcal{T} contains some identical texts, and also it is possible that one text is a prefix of other texts. Thus, $DAWG(\mathcal{T})$ contains at most K sink nodes, where K is the number of texts in \mathcal{T} . The numbers of nodes and edges of $DAWG(\mathcal{T})$ are $O(N)$ [3], and hence $DAWG(\mathcal{T})$ can be stored with $O(N)$ space. $DAWG(\mathcal{T})$ can be defined formally as follows: For any string x , let $Epos_{\mathcal{T}}(x)$ be the set of ending

positions of x in the texts in \mathcal{T} , i.e.,

$$Epos_{\mathcal{T}}(x) = \{(k, j) \mid x = T_k[j - |x| + 1..j], 1 \leq j \leq |T_k|, 1 \leq k \leq K\}.$$

Consider an equivalence relation $\equiv_{\mathcal{T}}$ on substrings x, y of texts in \mathcal{T} such that $x \equiv_{\mathcal{T}} y$ iff $Epos_{\mathcal{T}}(x) = Epos_{\mathcal{T}}(y)$. For any substring x of texts of \mathcal{T} , let $[x]_{\mathcal{T}}$ denote the equivalence class w.r.t. $\equiv_{\mathcal{T}}$. There is a one-to-one correspondence between each node v of $DAWG(\mathcal{T})$ and each equivalence class $[x]_{\mathcal{T}}$, and hence we will identify each node v of $DAWG(\mathcal{T})$ with its corresponding equivalence class $[x]_{\mathcal{T}}$. Let $long([x]_{\mathcal{T}})$ denote the longest member of $[x]_{\mathcal{T}}$. By the definition of equivalence classes, $long([x]_{\mathcal{T}})$ is unique for each $[x]_{\mathcal{T}}$ and every member of $[x]_{\mathcal{T}}$ is a suffix of $long([x]_{\mathcal{T}})$. If x, xa are substrings of some text in \mathcal{T} with $x \in \Sigma^*$ and $a \in \Sigma$, then there exists an edge labeled with character $a \in \Sigma$ from node $[x]_{\mathcal{T}}$ to node $[xa]_{\mathcal{T}}$. This edge is called *primary* if $|long([x]_{\mathcal{T}})| + 1 = |long([xa]_{\mathcal{T}})|$, and is called *secondary* otherwise. For each node $[x]_{\mathcal{T}}$ of $DAWG(\mathcal{T})$ with $|x| \geq 1$, let $slink([x]_{\mathcal{T}}) = y$, where y is the longest suffix of $long([x]_{\mathcal{T}})$ which does not belong to $[x]_{\mathcal{T}}$. In the example of Fig. 1, $[aaab]_{\mathcal{T}} = \{aaab, aab\}$. The edge labeled with b from node $[aaa]_{\mathcal{T}}$ to node $[aaab]_{\mathcal{T}}$ is primary, while the edge labeled with b from $[aa]_{\mathcal{T}}$ to node $[aaab]_{\mathcal{T}}$ is secondary. $slink([aaab]_{\mathcal{T}}) = [ab]_{\mathcal{T}}$.

2.2.4 Duality Between Suffix Trees and DAWGs

There exists a nice duality between suffix trees and DAWGs. To observe this, it is convenient to consider the collection $\overline{\mathcal{T}}$ of the reversed texts each of which begins with a special marker $\$,$ i.e., $\overline{\mathcal{T}} = \{\$1\overline{T}_1, \dots, \$K\overline{T}_K\}$. For ease of notation, let $S_k = \overline{T}_k$ for $1 \leq k \leq K$ and $\mathcal{S} = \{\$1S_1, \dots, \$KS_K\} = \overline{\mathcal{T}}$. Then, it is known (c.f. [2,3,7]) that the reversed suffix links of $DAWG(\mathcal{S})$ coincide with the suffix tree $STree(\mathcal{T})$ for the original text collection \mathcal{T} . This fact can also be observed from the other direction. Namely, the hard (resp. soft) W-links of $STree(\mathcal{T})$ coincide with the primary (resp. secondary) edges of $DAWG(\mathcal{S})$.

Intuitively, this duality holds because

- (1) The reversed suffix links of $STree(\mathcal{S})$ form $STree(\mathcal{T})$ (and vice versa), and
- (2) When we construct $DAWG(\mathcal{S})$ from $STree(\mathcal{S})$, we merge isomorphic subtrees that are connected by suffix links. During this merging process, the reversed suffix links get compacted and the resulting compacted links form the edges of $STree(\mathcal{T})$.

Using this duality, we can immediately show that the total number of hard and soft W-links is linear in the total text length N , since the number of edges of the DAWG is linear in N . This also means that we can easily maintain the Boolean indicator w with $O(N)$ space, so that $w_a(v)$ for a given node v and $a \in \Sigma$ can be answered in $O(\log \sigma)$ time (e.g., at each node v we can maintain a binary search tree dictionary storing only the characters c s.t. $w_c(v) = 1$.)

2.3 Fully-Online Text Collection

We consider a collection $\{T_1, \dots, T_K\}$ of K growing texts, where each text T_k ($1 \leq k \leq K$) is initially the empty string ε . Given a pair (k, a) of a text id k and a character $a \in \Sigma$ which we call an *update operator*, the character a is appended to the k -th text of the collection. For a sequence U of update operators, let $U[1..i]$ denote the sequence of the first i update operators in U with $0 \leq i \leq |U|$. Also, for $0 \leq i \leq |U|$ let $\mathcal{T}_{U[1..i]}$ denote the collection of texts which have been updated according to the first i update operators of U . For instance, consider a text collection of three texts which grow according to the following sequence $U = (1, a), (2, b), (2, a), (3, a), (1, a), (3, c), (3, b), (2, b), (1, a), (1, b), (3, c), (3, b), (1, c), (3, b), (2, c)$ of 15 update operators. Then,

$$\mathcal{T}_{U[1..0]} = \left\{ \begin{array}{c} \varepsilon \\ \varepsilon \\ \varepsilon \end{array} \right\}, \dots, \mathcal{T}_{U[1..14]} = \left\{ \begin{array}{c} 1 \ 5 \ 9 \ 10 \ 13 \\ a \ a \ a \ b \ c \\ 2 \ 3 \ 8 \\ b \ a \ b \\ 4 \ 6 \ 7 \ 11 \ 12 \ 14 \\ a \ c \ b \ c \ b \ b \end{array} \right\},$$

$$\mathcal{T}_{U[1..15]} = \left\{ \begin{array}{c} 1 \ 5 \ 9 \ 10 \ 13 \\ a \ a \ a \ b \ c \\ 2 \ 3 \ 8 \ 15 \\ b \ a \ b \ c \\ 4 \ 6 \ 7 \ 11 \ 12 \ 14 \\ a \ c \ b \ c \ b \ b \end{array} \right\}$$

where the superscript i over each character a in the k -th text implies that $U[i] = (k, a)$. For instance, $U[15] = (2, c)$ and hence c was appended to the 2nd text $T_2 = bab$ in $\mathcal{T}_{U[1..14]}$, yielding $T_2 = babc$ in $\mathcal{T}_{U[1..15]}$.

If there is no restriction on U like the one in the example above, then U is called *fully-online*. If there is a restriction on U such that once a new character is appended to the k -th text, then no characters will be appended to its previous $k - 1$ texts, then U is called *semi-online*. Hence, any semi-online sequence of update operators is of form

$$(1, T_1[1]), \dots, (1, T_1[|T_1|]), \dots, (K, T_K[1]), \dots, (K, T_K[|T_K|]).$$

When we talk about the duality of suffix trees and DAWGs in our fully-online scenario, $\mathcal{S}_{U[1..i]}$ represents the set of the reversed texts from $\mathcal{T}_{U[1..i]}$.

2.4 Nearest Marked Ancestors

Consider a rooted tree in which each node is either marked or unmarked. The *nearest marked ancestor (NMA)* query is, given a node u in the tree, return the lowest ancestor of u that is marked. We will use the following NMA data structure as a building block of our algorithms.

Lemma 1 ([18]) *There exists an NMA data structure for a growing rooted tree, which supports the following operations in amortized $O(1)$ time each: (1) find the NMA of a given node; (2) insert an unmarked node; (3) mark an unmarked node. This NMA data structure requires linear space in the size of the tree.*

3 Fully-Online Version of DAWG and Weiner’s Suffix Tree Algorithm

Blumer et al. [2,3] and Crochemore [6] introduced the DAWG, also called suffix automaton, and gave a DAWG construction algorithm for a collection of semi-online texts. Their DAWG construction algorithm is very closely related to Weiner’s reverse right-to-left suffix tree construction algorithm [7,10,13,17]. In fact, both algorithms build dual structures and each exposes different parts of these structures, where the collection of semi-online left-to-right text inputs to the DAWG algorithm can be perceived as the same texts reversed right-to-left inputs to Weiner’s suffix tree algorithm. Blumer et al.’s algorithm does not require a terminating $\$$ symbol and it was noted that the set of nodes of the DAWG and the reverse string’s suffix tree coincide if the terminator symbols are present in both sets of inputs.

3.1 Semi-online Construction of Weiner’s Suffix Trees and DAWGs

We briefly explain how the suffix tree of a collection of semi-online right-to-left texts can be built by using Weiner’s algorithm. For convenience, we assume that there is an auxiliary node \perp that is the parent of the root r . We also assume that the edge from \perp to r is labeled with *any* character c from Σ , $\mathcal{W}_c(\perp) = r$, and $slink(r) = \perp$. Assume that we have constructed $STree(\{T_1\$, \dots, T_{k-1}\$_{k-1}\})$ in which all the hard W-links have been constructed and the Boolean indicator w have been appropriately maintained. Now we process the k -th text $T_k\$$. Since the end-marker $\$$ is a unique character, a new leaf representing $\$$ is created first and initially $T_k = \varepsilon$. Now T_k is going to grow from right to left. Suppose that we have inserted the leaves for the suffixes of $T_k\$$ with $T_k \in \Sigma^*$. The leaf that represents the k -th text $T_k\$$ is called the *handle leaf* for $T_k\$$. Now we are to prepend a new character a and insert the extended text $aT_k\$$ to the tree. See Fig. 2 for illustration. We begin with the handle leaf ℓ for $T_k\$$. We walk up from the handle leaf ℓ until finding the lowest explicit ancestor u' of ℓ that has hard W-link $\mathcal{W}_a(u')$ defined for the added character a . Also, let u be the lowest explicit ancestor of ℓ such that $w_a(u) = 1$. Note that u is a descendant of u' . Let b be the first character of the path label from u' to ℓ . We move to the node $v' = au'$ using the hard W-link $\mathcal{W}_a(u')$, and let $v'' = au'by$ be the child of v' below the edge whose label begins with b , where $y \in \Sigma^*$. There are two cases: (1) If $|v''| - |v'| = |au'by| - |au'| = |by| > |u| - |u'|$, then we create a new explicit node $v = v''[1..|u| + 1]$ and set $\mathcal{W}_a(u) = v$. (2) Otherwise ($|by| = |u| - |u'|$), then $|v''| = |u| + 1$ and thus let $v = v''$. In both cases, we insert a new leaf ℓ' representing $aT_k\$$ as a child of v , and create a new hard W-link $\mathcal{W}_a(\ell) = \ell'$. This insertion point v for ℓ' represents the longest prefix of $aT_k\$$ that appears at least twice in the updated text collection, and hence, v is sometimes called as the *longest repeating prefix* of

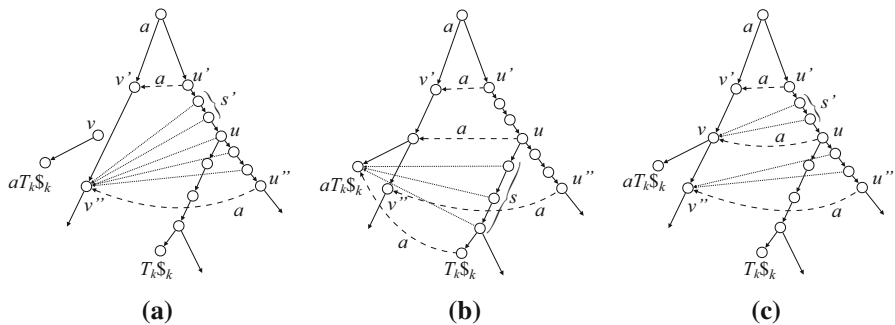


Fig. 2 Extending $T_k\$$ to $aT_k\$$. Soft W-links are shown short-dashed and hard W-links are shown long-dashed. For simplicity, the labels (character a) of the soft W-links are omitted. **a** Relevant existing W-links before the extension. **b** New W-links pointing to $aT_k\$$ are created from all nodes on the path between $T_k\$$ up to u . **c** Existing W-links pointing to v'' from all nodes on the path between u up to u' are redirected to point to v instead of v'' . The new hard W-link $\mathcal{W}_a(T_k\$) = aT_k\$$ and redirected hard W-link $\mathcal{W}_a(u) = v$ have corresponding nodes on the path to $aT_k\$$, while all the other new soft W-links involved point to $aT_k\$$ and the redirected soft W-links involved point to v . The new node v also adopts all the outgoing W-links from v'' (not shown)

$aT_k\$$. Let s be any node in the path from u to ℓ such that $s \neq u$ (if any). In the suffix tree before the text $T_k\$$ was extended with a , we had $w_a(s) = 0$. Now in the updated suffix tree, we update $w_a(s) = 1$ due to the insertion of the new handle leaf ℓ' that represents $aT_k\$$. Also, node s gets a new soft W-link $\mathcal{W}_a(s) = \ell'$. These updates are common to both of Cases (1) and (2). There can be further updates in Case (1): Let s' be any node in the path from u' to u such that $s' \neq u'$ and $s' \neq u$ (if any). In the suffix tree before the text $T_k\$$ was extended with a , node s' had a soft W-link $\mathcal{W}_a(s') = v''$. Now in the updated suffix tree, this soft W-link is redirected as $\mathcal{W}_a(s') = v$. Also, the soft W-link $\mathcal{W}_a(u) = v''$ in the previous suffix tree gets redirected and becomes the hard W-link $\mathcal{W}_a(u) = v$ in the updated suffix tree.

Weiner's original algorithm is designed for a single right-to-left text, and for each prepended character a to the text $T\$$, the number of internal explicit nodes from the leaf for T to its lowest ancestor u' for which hard W-link $\mathcal{W}_a(u')$ exists can be amortized to a constant. This amortization argument is based on the fact that the depth of the path from the root to the handle leaf ℓ' representing the extended text $aT\$$ is at most one unit larger than that of the path from the root to the handle leaf ℓ representing $T\$$. This property holds also in the semi-online setting, since while the k th text $T_k\$$ is being extended from right to left, other texts remain static and thus do not change the topology of the suffix tree. Hence, we can build $STree(T)$ for a collection of semi-online left-to-right texts in $O(N \log \sigma)$ time and $O(N)$ space. A pseudo-code of Weiner's original algorithm for a single text, as well as a more detailed analysis, can be found e.g. in [7].

Blumer et al. [2] showed how the DAWG for a collection \mathcal{S} of semi-online left-to-right texts can be built in $O(N \log \sigma)$ time. Recall that each DAWG node represents an equivalence class of substrings that have the same ending positions in the texts. Appending a new character a to the currently processed text $\$_k\$$ can affect some equivalence class under the current text collection. This can cause splitting an existing

node into two nodes. Let w be the node that gets split and w' be the copy of this node w . The original node w will contain longer substrings than the copy w' . The longest element belonging to w' is the *longest repeating suffix* X of $S_k S_k a$ in the updated text collection, and any element of w that is shorter than X will belong to w' . Eventually, any element of w that is longer than X remains in w . This node split operation can be done by redirecting corresponding in-coming edges from w to w' . The key argument in the time analysis of Blumer et al.'s algorithm is that this cost of redirecting in-coming edges can also be amortized constant per added character a . Observe that this update is exactly the same as the aforementioned update of the suffix tree for the corresponding right-to-left text collection. For instance, the longest repeating suffix of $S_k S_k a$ for the current left-to-right text collection is the reverse of the longest repeating prefix of $a T_k S_k$ for the corresponding right-to-left text collection. Also, redirecting those in-coming edges in the DAWG are exactly the same as updating a soft W-link to a hard one and redirecting soft W-links, in the suffix tree of the corresponding right-to-left texts (recall Case (1) above). Consequently, we can build the DAWG for a collection of semi-online left-to-right texts in $O(N \log \sigma)$ time and $O(N)$ space as well.

3.2 Fully-Online Construction of Weiner's Suffix Trees and DAWGs

In this subsection, we consider how to maintain the suffix tree for a collection of K texts that grow from right to left in a fully-online manner. This means that we will have to maintain K handle leaves for the K texts simultaneously. We also consider how to maintain the DAWG for a collection of K texts that grow from left to right in a fully-online manner.

Unfortunately, the identical amortization argument in both algorithms does not carry over in the fully-online setting. However, we will show next that Weiner's algorithm can be modified to work within the desired $O(N \log \sigma)$ time and $O(N)$ space bounds with the aid of σ nearest marked ancestor (NMA) data structures [18] of total size $O(N)$, where σ denotes the number of all distinct characters appearing in the texts in the collection. Moreover, the same data structures can provide access to the DAWG edges, which cannot be maintained explicitly within our bounds, in $O(\log \sigma)$ time per edge query.

Suppose that we have $STree(\mathcal{T}_{U[1..i-1]})$ for a fully-online right-to-left text collection $\mathcal{T}_{U[1..i-1]}$ and assume $U[i] = (k, a)$, i.e., the k th text $T_k S_k$ gets extended with a new character a being prepended to it. As in the case with the semi-online texts, some new soft and hard W-links are created in the updated $STree(\mathcal{T}_{U[1..i]})$. Fortunately, the number of such newly created W-links are bounded by the size of the resulting suffix tree, which is $O(N)$. However, the number of *redirected* soft W-links, which are the same as the number of DAWG edges to be redirected, can be too large to be treated within our desired bounds as the next lemma shows.

Lemma 2 *Weiner's suffix tree algorithm takes $\Theta(N \min(K, \sqrt{N}))$ time in the fully-online setting, where N is the total length of the K texts. Hence, for $K = \Omega(\sqrt{N})$ it also takes $\Theta(N\sqrt{N})$ time to explicitly maintain the soft W-links (equivalently, the DAWG secondary edges) in the fully-online setting. The lower bound holds for a constant-size alphabet.*

Proof To show that these bounds hold for constant-size alphabets, we here assume that each text in the collection terminates with the same end-marker $\$$. However, in our collection of texts each text will be distinct, so that each $T_k\$$ will be represented by a unique handle leaf.

First, we consider a lower bound. Consider the following K right-to-left texts $\mathcal{T} = \{T_k = a^k\$ \mid 1 \leq k \leq K\}$ where $a \in \Sigma$ and each text terminates with a common end-marker $\$$. Suppose we have constructed the suffix tree of \mathcal{T} in any order. Then, we prepend a new character $c \in \Sigma$, such that $c \neq a$, to each text $T_k = a^k\$$ in decreasing order of their length, $k = K, \dots, 1$. Since we process each text in decreasing order of k , there are $\Omega(k)$ explicit nodes in the path from the handle leaf for $T_k = a^k\$$ to its lowest ancestor $r = \varepsilon$ (the root) for which hard W-link $\mathcal{W}_c(r)$ is defined. Hence, it takes $\Omega(k)$ time to naïvely walk up this path. Also, with the exception of the first longest text T_K that introduces $\Omega(k)$ new soft W-links, for all other $k < K$, there are $\Omega(k)$ soft W-links to be redirected along the way. Thus, there are $\Omega(K^2)$ explicit nodes to walk up and $\Omega(K^2)$ edge re-directions in total for all k 's. We then repeat the above procedure several times. At each repetition i ($i > 1$), for each k in decreasing order it again takes $\Omega(k)$ time to walk up from the handle leaf for $c^{i-1}a^k\$$ until reaching its lowest ancestor r for which hard W-link $\mathcal{W}_c(r)$ is defined. Also, there are $\Omega(k)$ soft W-links to be redirected along the way. Thus, at each repetition i , it takes a total of $\Omega(K^2)$ time for all k 's, too.

Let N be the total length of the texts in the collection after performing the above procedure several times. The initial total length of the text collection $\mathcal{T} = \{a^k\$ \mid 1 \leq k \leq K\}$ is $\frac{K(K+3)}{2}$. We then append c 's to each of the K texts, and the text collection of total length finally becomes N . Hence, the number of iterations is $(N - \frac{K(K+3)}{2})/K = \Theta(N/K - K)$, which is $\Theta(N/K)$ in the case where $K < \alpha\sqrt{N}$ with some constant α . Since each iteration requires re-directions of $\Omega(K^2)$ soft W-links, it takes a total of $\Omega(NK)$ time in this case. Now consider the case where $K > \alpha\sqrt{N}$. In this case, we can apply the same procedure as above only to $\alpha\sqrt{N}$ texts in the collection, and the other $K - \alpha\sqrt{N}$ texts remain empty. This leads to $\Omega(N\sqrt{N})$ total work for re-directing soft W-links. Combining these two, we obtain an $\Omega(N \min(K, \sqrt{N}))$ lower bound.

To see that this lower bound actually gives rise to the worse case in Weiner's algorithm, we can focus only on the time required for soft W-link re-direction, since new edge insertions and node insertions are always accounted globally to be the total size of the the suffix tree, which is $O(N)$.

Recall that the number of soft W-link re-directions when appending a symbol a to text $T_k\$$ is no larger than the suffix tree depth of the handle leaf representing $T_k\$$, which is in turn smaller than the length of $T_k\$$. Also, the depth of the new leaf $aT_k\$$ is at most one more than the depth of leaf $T_k\$$ minus the number of edge re-directions that reduce the depth of the current handle leaf associated with each of the K texts, while the depth of all current handle leaves $T_i\$$, $i \neq k$, may also increase by at most one while updating $T_k\$$, by the insertion of the internal node from which the leaf $T_k\$$ is hanging above the handle leaf of T_i . Thus, each of the $O(N)$ symbols may increase by at most one the depth of all the K handle leaves. This depth increase was not an issue in the semi-online setting since previous $T_k\$$ are no longer updated and their

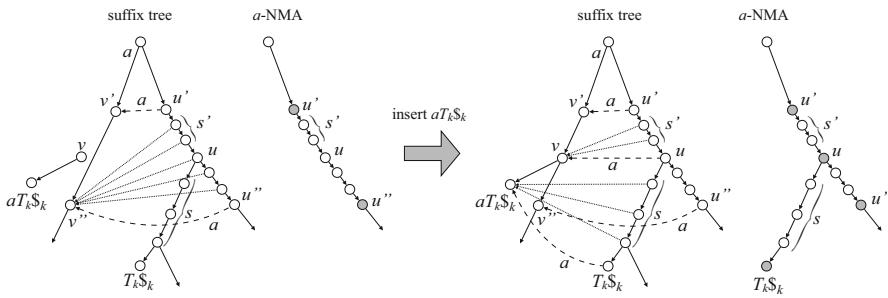


Fig. 3 Extending text $T_k\$$ to $aT_k\$$ by our fully-online version of Weiner's algorithm, with the aid of the NMA structures. Soft W-links are shown short-dashed and hard W-links are shown long-dashed. We remark that soft W-links are only imaginary and are not explicitly maintained (Lemma 3), and their character labels (with a) are omitted for simplicity. The gray nodes of the a -NMA data structures are the marked nodes. After the insertion of a new leaf representing $aT_k\$$, node u gets marked in the a -NMA data structure since u now has a new hard W-link with a in the suffix tree. Then, nodes s with new soft W-links with a are newly added to the a -NMA data structure as unmarked nodes, in a top-down manner from u . Finally, leaf $T_k\$$ with a new hard W-link with label a is also newly added to the a -NMA data structure

handle leaves were no longer used. In the fully-online setting, this depth increase is problematic. The depth reduction argument gives an obvious $O(N)$ upper bound on the soft W-link re-directions while updating each of the K texts, which adds up to $O(KN)$ overall upper bound.

The analysis will separate those short texts $T_k\$$, such that $|T_k\$| \leq \sqrt{N}$ from the longer texts. For the short texts, each time a symbol is prepended to a text $T_k\$$, the number of soft W-link edge re-directions is bounded by the length of each short text, which is at most \sqrt{N} , totaling at most $O(N\sqrt{N})$ such re-directions. For the long texts, we observe that there are at most $O(\sqrt{N})$ such long texts, and for each specific text, the total number of soft W-link edge re-directions is at most $O(N)$, totaling at most $O(KN) \subseteq O(N\sqrt{N})$. Combining these bounds, we get the desired $\Theta(N \min(K, \sqrt{N}))$ tight bound. \square

Remark 1 To show that the bounds hold for a constant alphabet, we used the same end-marker $\$$ for all the texts in the proof of Lemma 2. We remark that the same arguments hold for the case where each text T_k is terminated with a unique end-marker $\$k$, as we assume elsewhere in this paper, since also in this case each text $T_k\$k$ is represented by a unique handle text. We then use $K + 2$ characters in the lower bound example (the alphabet is $\{a, b, \$_1, \dots, \$K\}$).

To avoid the above-stated super-linear cost in Lemma 2, we shall only maintain hard W-links and will not explicitly maintain soft W-links. Instead of soft W-links we will maintain only the Boolean indicator $w_a(v)$ that tells us whether a (soft or hard) W-link $\mathcal{W}_a(v)$ is defined or not. Once $w_a(v)$ is set to 1, it remains 1 and does not need to be updated even when the corresponding soft W-link would have to be redirected.

Like in the semi-online setting, we here also go up from the leaf ℓ representing $T_k\$k$ to its lowest ancestor u' for which $\mathcal{W}_a(u')$ is defined. The cost for walking up to the lowest ancestor u of ℓ for which $w_a(u) = 1$ can be charged to the cost for creating

new soft W-links (or equivalently, that for creating new corresponding DAWG edges), which is amortized constant per added character a . See also Fig. 3 for illustration.

One problem remains: We would like to skip all explicit nodes s' in the path from node u to u' , since naïvely walking up this path can be as costly as redirecting W-links $\mathcal{W}_a(s')$ for all such nodes s' . In so doing, we shall also maintain for each character σ an NMA data structure of Lemma 1 on the subtree of the suffix tree that consists of the two following disjoint sets of nodes: (1) the set of unmarked nodes v such that $w_a(v) = 1$ and $\mathcal{W}_a(v)$ is a soft W-link, and (2) the set of marked nodes v such that $\mathcal{W}_a(v)$ is a hard W-link. Our version of Weiner's algorithm will naïvely walk up the suffix tree from the leaf ℓ representing $T_k\$_k$ until the lowest node u such that $w_a(u) = 1$, and from there it will jump to u' using the NMA data structure for the prepended character a . In what follows, we will denote this as the a -NMA data structure.

Theorem 1 *Given a fully-online sequence U of N update operators for a collection of K right-to-left texts \mathcal{T} , our version of Weiner's algorithm can update the suffix tree in a total of $O(N \log \sigma)$ time and $O(N)$ space.*

Proof The correctness of our algorithm should be clear from the above discussion.

Let us analyze the time complexity. The algorithm will now still climb up the suffix tree from the currently focused leaf ℓ up to its lowest ancestor u with $w_a(u) = 1$. From there, it would jump to its nearest ancestor u' of u having hard W-link $\mathcal{W}_a(u')$ defined in constant amortized time using an NMA query on the a -NMA data structure. Now we update the a -NMA data structure. If the insertion point v for the new leaf ℓ' representing $aT_k\$_k$ is newly created (see Case (1) in the previous sub-section), then the soft W-link $\mathcal{W}_a(u)$ becomes hard. Hence, we mark node u in the a -NMA data structure. Otherwise, the W-link $\mathcal{W}_a(u)$ is already hard and hence u is already marked in the a -NMA data structure. Recall that each node s between the leaf ℓ and u obtain new soft W-links and hence $w_a(s)$ is now set to 1. Hence, we insert an unmarked node for each s in the a -NMA data structure. Since the NMA data structure allows us to insert a new leaf in amortized constant time, we insert these unmarked nodes in increasing order of depth, from the child of u to the parent of ℓ contained in the path. We also spend $O(\log \sigma)$ time at each visited node for searching the corresponding NMA data structure. Overall, it takes a total of $O(N \log \sigma)$ time to construct the suffix tree for a fully-online right-to-left text collection of total length N .

Let us now analyze the space complexity. For each character $c \in \Sigma$, each marked node u in the c -NMA data structure corresponds to a unique hard W-link $\mathcal{W}_c(u)$. Also, each unmarked node s in the c -NMA data structure corresponds to a unique soft W-link $\mathcal{W}_c(s)$. Since the total number of hard and soft W-links for all characters $c \in \Sigma$ is $O(N)$, the total size of the c -NMA data structures for all characters $c \in \Sigma$ is $O(N)$. \square

Now we turn our attention to the construction of the DAWG for a fully-online left-to-right text collection \mathcal{S} . Since our version of Weiner's algorithm does not explicitly maintain soft W-links, we do not have an explicit representation of the secondary edges of the DAWG for the left-to-right texts. However, Weiner's suffix tree augmented with the NMA data structures indeed is an implicit representation of the DAWG secondary edges:

Lemma 3 *Using Weiner’s suffix tree augmented with the NMA data structures, we can simulate each soft W-link in amortized $O(\log \sigma)$ time per query.*

Proof As a query input, we are given a node u and character a . Then, node u has soft W-link $\mathcal{W}_a(u)$ w.r.t. a iff $w_a u = 1$ and $\mathcal{W}_a(u)$ is not a hard W-link. Suppose u has soft W-link $\mathcal{W}_a(u)$. We query the NMA u' of u in the a -NMA data structure. Let b be the first character of the path label from u' to u . We follow the hard W-link $\mathcal{W}_a(u') = v'$, and find the out-going edge of v' whose edge label begins with b . Then, the child of v' below this edge is the destination of the soft W-link $\mathcal{W}_a(u)$. See also Fig. 3 for illustration. The time for each NMA query is amortized to $O(1)$ and finding the appropriate a -NMA data structure and the appropriate out-going edge of v' takes $O(\log \sigma)$ time each. \square

It should be noted that we can simulate secondary edge redirections using the approach of Lemma 3. See Fig. 3. The soft W-links of nodes s' before the insertion of $aT_k\$k$ point to node v'' (the left diagrams). These can be simulated by first finding the NMA u' of s' in the a -NMA data structure, following the hard W-link of u' , and then selecting the child v'' of v below the edge whose label begins with b . After the insertion of $aT_k\$k$ (in the right diagrams), the soft W-links of nodes s' are redirected to node v . Here, the child of v' below the edge whose label begins with b is v , and thus we can find this v using the same approach as above. The new soft W-links of nodes s can also be simulated similarly, since u is now marked in the a -NMA data structure.

The next corollary immediately follows from Theorem 1 and Lemma 3.

Corollary 1 *Given a fully-online sequence U of N update operators for a collection of K left-to-right texts, the DAWG can be maintained in a total of $O(N \log \sigma)$ time and $O(N)$ space with $O(\log \sigma)$ query time for an out-going DAWG edge.*

4 Fully-Online Version of Ukkonen’s Suffix Tree Algorithm

Ukkonen’s algorithm [15] constructs the suffix tree of a given text in an online manner, from left to right. In this section, we show how Ukkonen’s algorithm can be extended to maintain the suffix tree for a fully-online left-to-right text collection. We will do so by first explaining that Ukkonen’s algorithm can readily be extended to the semi-online setting. Then, we will describe some difficulties in extending Ukkonen’s algorithm to our fully-online setting, and finally we will present how to overcome these difficulties achieving $O(N \log \sigma)$ -time algorithm.

4.1 Semi-online Left-to-Right Suffix Tree Construction

Ukkonen’s algorithm [15] can easily be extended to incrementally construct the suffix tree for multiple texts in the semi-online setting.

Let U be a semi-online sequence of N update operators such that the last update operator for each k ($1 \leq k \leq K$) is $(k, \#_k)$, where $\#_k$ is a special end-marker for the k th text in the collection. Also, assume that we have already constructed $STree(\mathcal{S}_{U[1..i-1]})$

and that the next update operator is $U[i] = (k, a)$. Thus a new character a is appended to the text S_k and it becomes S_ka .

In updating $STree(S_{U[1..i-1]})$ to $STree(S_{U[1..i]})$, we have to assure that all suffixes of the extended text S_ka will be represented by $STree(S_{U[1..i]})$. These suffixes are categorized to three different types:

- Type-1 The suffixes of S_ka that are longer than $lrs_{S_{U[1..i-1]}}(S_k)a$.
- Type-2 The suffixes of S_ka that are not longer than $lrs_{S_{U[1..i-1]}}(S_k)a$ but are longer than $lrs_{S_{U[1..i]}}(S_ka)$.
- Type-3 The suffixes of S_ka that are not longer than $lrs_{S_{U[1..i]}}(S_ka)$.

The suffixes of S_ka are inserted in decreasing order of length.

The Type-1 suffixes are maintained as follows. Let s be any suffix of S_k that is represented by a leaf of $STree(S_{U[1..i-1]})$. Since s is a non-repeating suffix of S_k in $S_{U[1..i-1]}$, sa is a non-repeating suffix of S_ka in $S_{U[1..i]}$, which implies that sa will also be a leaf of $STree(S_{U[1..i]})$. Based on this observation, the label of the incoming edge of the leaf is represented by a pair $\langle k, b \rangle$ called an *open edge*, where b is the beginning position of the label of the in-coming edge in the k th text. We can retrieve the ending position of the edge label in constant time by looking at the current length of the k th text. This way, every existing leaf will then be “automatically” extended.

Hence, updating $STree(S_{U[1..i-1]})$ to $STree(S_{U[1..i]})$ reduces to inserting the Type-2 suffixes of S_ka (note that the Type-3 suffixes of S_ka already exist in the suffix tree, since they are suffixes of $lrs_{S_{U[1..i]}}(S_ka)$ occurring at least twice in S_ka). For this sake, the algorithm maintains an invariant that indicates the locus of $lrs_{S_{U[1..i]}}(S_k)$ on $STree(S_{U[1..i-1]})$ called the *active point*. This active point will be maintained so that new leaves will be inserted from its locus. The algorithm also maintains an invariant j that indicates the beginning position of the suffix of S_ka that is to be added to the tree. Thus, currently j is the beginning position of $lrs_{S_{U[1..i-1]}}(S_k)$ in S_k , and let x_j denote the locus of the active point. Later, j will be incremented one by one as a new suffix is inserted into the tree. Since x_j can be an implicit node, the algorithm maintains the canonical reference to x_j , which is hereafter denoted by (v_i, c_i, ℓ_i) . For convenience, if x_j is an explicit node, then let its canonical reference be $(x_j, \varepsilon, 0)$. The update starts from active point x_j represented by its canonical reference pair, and the Type-2 suffixes of S_ka are inserted in decreasing order of length, by using the chain of (virtual) suffix links. There are two cases:

- I. If it is possible to go down from x_j with character a , then the suffix of S_ka that begins at position j is the longest repeating suffix of S_ka , namely $x_ja = lrs_{S_{U[1..i]}}(S_ka)$. Thus, the value of j does not change and no updates to the tree topology are needed. The locus of the active point is updated as $x_j \leftarrow x_ja$, and the reference to x_j is made canonical if necessary. The update ends.
- II. If it is impossible to go down from x_j with character a , then we create a new leaf. The following procedure is repeated until Case I happens.
 - (a) If active point x_j is on an explicit node, then a new leaf node is created as a new child of x_j , with its incoming edge labeled by $\langle k, |S_ka| \rangle$. The locus of the active point is updated to $slink(x_j)$, and then we update $j \leftarrow j + 1$.

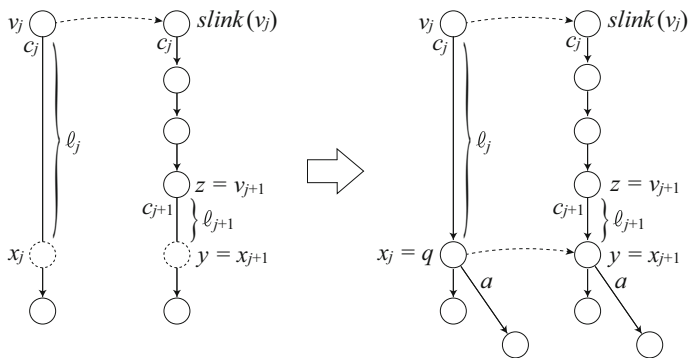


Fig. 4 Illustration for Case II-b-(ii)

(b) If active point x_j is on an implicit node, then this implicit node becomes explicit in this step. A new leaf is created as a new child of the new explicit node q_j with its incoming edge labeled by $\langle k, |S_k a| \rangle$. Since the suffix link of the new explicit node does not yet exist, we simulate the suffix link traversal as follows: Recall that (v_j, c_j, ℓ_j) denotes the canonical reference to the current locus of active point x_j . First, we follow the suffix link $slink(v_j)$ of v_j , and then go down along the path of length ℓ_j from $slink(v_j)$ starting with character c_j . Let this locus be y , and let z be the deepest explicit node in this path. By definition, the string represented by z is a prefix of that represented by y .

- (i) If $z = y$, then we firstly create the new suffix link $slink(q_j) = z$ for the new explicit node q_j . Now we update $j \leftarrow j + 1$, and the locus of active point x_i is updated to z , which is now represented by canonical reference $(v_j, c_j, \ell_j) = (z, \varepsilon, 0)$.
- (ii) If $|z| < |y|$, then the next locus of the active point is implicit. For the next position $j + 1$ in text $S_k a$, we denote by $(v_{j+1}, c_{j+1}, \ell_{j+1})$ the canonical reference to active point x_{j+1} . Since $x_{j+1} = y$, $(v_{j+1}, c_{j+1}, \ell_{j+1}) = (z, c, |y| - |z|)$, where c is the first character of the edge label from z to y . The suffix link of the new created explicit node q will be set to the current locus of the active point x_{j+1} when it becomes explicit in the next step. See Fig. 4 for illustration of this case.

We update $j \leftarrow j + 1$.

The most expensive case is II-b-(ii). Since the path from $slink(v_j)$ to $v_{j+1} = z$ contains at most $\ell_j - \ell_{j+1}$ explicit nodes, it takes $O((\ell_j - \ell_{j+1} + 1) \log \sigma)$ time to locate the next active point x' (note $\ell_j - \ell_{j+1} \geq 0$ holds). All the other operations take $O(\log \sigma)$ time. Hence, the total cost to insert all leaves (suffixes) for the k th text is $O(\sum_{j=1}^{N_k} (\ell_j - \ell_{j+1} + 1) \log \sigma) = O(N_k \log \sigma)$, where N_k is the final length of the k th text. Thus the amortized time cost for each leaf (suffix) for the k th text is $O(\log \sigma)$. Overall, it takes a total of $O(N \log \sigma)$ time to construct $STree(S_U)$ for a semi-online sequence U of update operators. The space requirement is $O(N)$.

4.2 Difficulties in Fully-Online Left-to-Right Suffix Tree Construction

The following observations suggest that it does not seem easy to extend Ukkonen's algorithm to our left-to-right fully-online setting. See also Fig. 5 for concrete examples.

- A. Keeping Track of Active Points** Let $U[i] = (k, a)$, which updates the current k th text S_k to $S_k a$, and assume that we have just constructed $STree(S_{U[1..i]})$. Recall that we defined the initial locus of the active point for $S_k a$ on $STree(S_{U[1..i]})$ to be the longest repeating suffix of $S_k a$ in $S_{U[1..i]}$. However, since U is fully-online, any other text S_h ($h \neq k$) in the collection would be updated by following update operators $U[r]$ with $r > i$. Then, the longest repeating suffix of $S_k a$ in $S_{U[1..r]}$ can be much longer than that of $S_k a$ in $S_{U[1..i]}$. In the example of Fig. 5, the active point of the fixed string S_1 is initially on the root, but it moved to the locus for bab in the last two phases. In addition, some Type-1 suffixes of $S_k a$ in $S_{U[1..i]}$ can become of Type-2 in $S_{U[1..r]}$. Again in the example of Fig. 5, ab is initially a Type-1 suffix of S_1 and thus it was a leaf in the beginning. However, after S_2 is updated to aba, ab becomes a Type-2 suffix of S_1 as it occurs twice in the collection $\{S_1 = \text{bab}, S_2 = \text{aba}\}$. What is worse, updating S_h can affect the longest repeating suffix of any other text in the collection as well. This can also be seen in the example of Fig. 5, where the active point (the grey rectangle) of fixed string S_1 often changes its locus. A naïve way to maintain these active points would be to check the active points of all the K texts whenever a text is updated. However, this takes at least $O(K)$ time per new character, which leads to a prohibitive $O(KN)$ total bound.
- B. Canonization of Active Points** As was discussed in Difficulty A, the active point for a text S_h may be forced to move deeper in the suffix tree due to the updates of some other texts. Then, when a new character is appended to S_h , we need to canonize the active point of S_h . In the example of Fig. 5, the active point of S_1 in the last phase needs to be canonized. In the worse case, this can take time proportional to the difference of the (string) depths of the current and previous loci of the active point for S_h . Note that this cost was originally introduced by the updates of other strings than S_h , and hence it is not trivial how one can amortize this cost for canonization.
- C. Maintaining Leaf Ownerships** The phenomenon mentioned in Difficulty A also causes a problem of how to represent the labels of the in-coming edges to the leaves. Assume that we created a new leaf w.r.t. an update operator (k, a) , and let $\langle k, b_k \rangle$ be the pair representing the label of the in-coming edge to the leaf, where b_k is the beginning position of the edge label in the k th text. We say that the k th text S_k is the *owner* of the leaf. It corresponds to a Type-1 suffix of the k th text, but the leaf can later be extended by another growing text S_h . Namely, S_h can overtake the ownership of the leaf from S_k . After this happens, then the pair $\langle k, b_k \rangle$ has to be updated to $\langle h, b_h \rangle$, where b_h is the beginning position of the edge label in the h th text. In the example of Fig. 5, the owner of the leaf ab is initially $S_1 = \text{bab}$. Hence, the label of its incoming edge is represented by pair $\langle 1, 2 \rangle$, which indicates that $\text{ab} = S_1[2..|S_1|]$. However, in the fourth phase, the leaf is extended to aba which is no more a suffix of $S_1 = \text{bab}$. Thus, the new

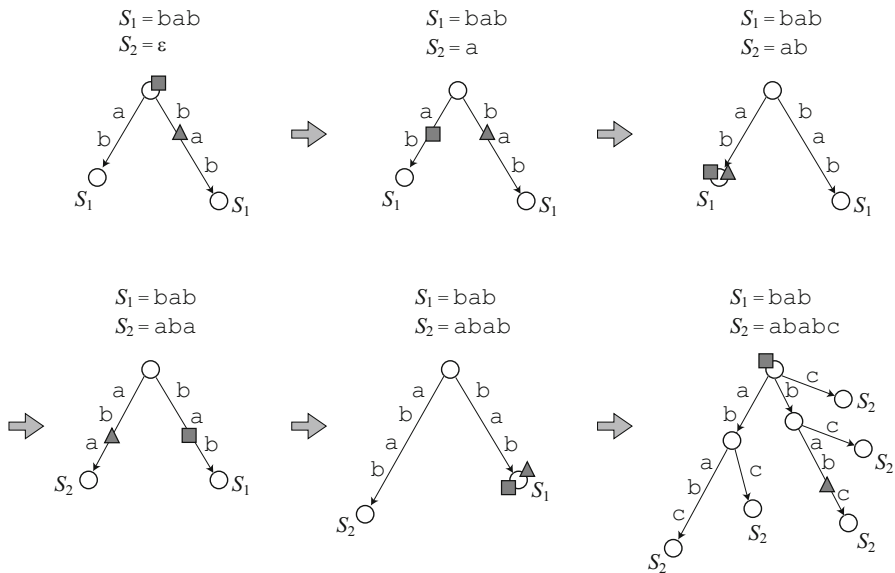


Fig. 5 A snapshot of fully-online left-to-right construction of the suffix tree for two strings S_1 and S_2 , where S_1 is fixed to be bab and S_2 grows from the empty string ε to ababc . The active points of S_1 and S_2 are indicated by the grey triangle and square, respectively. As can be seen in this example, the locus of the active point of one string is affected by the other string. This is Difficulty A. Difficulty B occurs in the last step, where the parent of the active point bab for S_1 is changed from the root to node b . Also, in this figure the owner of each leaf is explicitly shown (e.g., the owner of the leaf representing ab at the top-left diagram is S_1 , and so on). In the beginning, the owners of all the leaves are S_1 . Then, as S_2 grows, S_2 takes over the ownership of the existing leaves. This is Difficulty C

owner of this leaf is $S_2 = \text{aba}$, and its incoming edge label needs to be updated as $(2, 1)$, which indicates that $\text{aba} = S_2[1..|S_2|]$. Notice that this update may happen repeatedly. For instance, if we update S_1 to bababcc after the last phase of Fig. 5, then S_1 will take over the ownership of all the existing leaves.

4.3 Fully-Online Left-to-Right Suffix Tree Algorithms

Let us now consider how to construct the suffix tree for a fully-online left-to-right text collection. Our fully-online version of Ukkonen's algorithm works with the aid of the fully-online version of Weiner's algorithm proposed in Sect. 3. Namely, for a fully-online left-to-right text collection \mathcal{S} with K texts, we build $\text{STree}(\mathcal{S})$ in tandem with $\text{STree}(\mathcal{T})$, where \mathcal{T} is the set of reversed texts from \mathcal{S} (i.e., $\mathcal{T} = \overline{\mathcal{S}}$). These two constructions are synchronized since appending a new character to a text in \mathcal{S} is equivalent to prepending the new character to the reversed text in \mathcal{T} . Since we use the fully-online version of Weiner's algorithm, as in Sect. 3, we assume that each text in \mathcal{T} terminates with a special symbol $\$k$, namely, $\mathcal{T} = \{T_1\$1, \dots, T_K\$K\}$. This in turn implies that each text in \mathcal{S} begins with $\$k$, namely, $\mathcal{S} = \{\$1S_1, \dots, \$KS_K\}$, where $S_i = \overline{T_i}$ for $1 \leq i \leq K$.

4.3.1 Proposed Algorithm

Now we present our fully-online left-to-right suffix tree construction algorithm, which is a generalization to Ukkonen's algorithm for a single online text. The key notions in our algorithm are *swapping active points* and *tight connections between active points and leaf ownerships*. In what follows we will explain these notions in full details.

Let us first consider maintaining active points (Point A). This is indeed closely related to maintaining leaf ownerships (Point C). We will for now put it aside the cost for maintaining leaf ownerships, and will focus on describing how active points can affect ownerships of leaves.

For a single right-to-left online text, the suffix links of the leaves form a single path from the longest leaf to the shortest one. On top of them we also consider a virtual suffix link from the shortest leaf to the active point.

We generalize the above notion to our fully-online text collection S . Unlike the single text case, a leaf can represent a suffix of multiple texts in our fully-online setting. This implies that the suffix links of $STree(S)$ form a forest. Let F_S denote this forest. This forest is only conceptual, namely, in our algorithms to follow we will *not* explicitly maintain it. However, the forest gives us more insights into Points A and C. Formally, the forest F_S is a set of maximal trees such that each maximal tree SLT in F_S satisfies:

- the root of SLT is the locus (an implicit or an explicit internal node) of the active point of a text,
- the other nodes of SLT are leaves of $STree(S)$, and
- the (reversed) edges of SLT are suffix links of $STree(S)$ (if the root of SLT is an implicit node, then the (reversed) edges from the root to its children are virtual suffix links from the children).

Since a leaf of $STree(S)$ can be a suffix of multiple texts, there are multiple choices for the owner of each leaf. Our choice of the owner of a leaf is either

- (R1) the text that created the leaf, or
- (R2) the last text whose active point has extended the leaf.

Regarding Rule (R2) above, we will soon describe in more details how the active point of a text can extend an existing leaf.

Suppose that we have constructed $STree(S_{U[1..i-1]})$ and that we are given an update operator $U[i] = (k, a)$ that appends new character a to text $S_k S_k$.

If the active point of $S_k S_k$ is not on a leaf of $STree(S_{U[1..i-1]})$, then the suffix tree is updated as in the semi-online setting and there are no changes on the ownerships of the leaves. Hence, in what follows we consider the case where the active point of $S_k S_k$ is on a leaf of $STree(S_{U[1..i-1]})$.

Let s be the leaf of $STree(S_{U[1..i-1]})$ where the active point of the text $S_k S_k$ lies. Let SLT denote the suffix link tree in $F_{S_{U[1..i-1]}}$ that contains this node s , and let P_i be the path from s to the root of SLT . Also, let \mathcal{O}_i be the set of texts that are the owners of the suffix tree leaves in P_i . Finally, let L_i be the list of all nodes u in the path from the *parent* of s to the root of SLT such that the active point of some text in \mathcal{O}_i lies on u . For each x ($1 \leq x \leq m = |L_i|$), let $u_x = L_i[x]$. For convenience, let $u_0 = s$.

For each x ($1 \leq x \leq m$), let k_x denote the text id of the owner of u_x . Then, due to the way how the ownerships of leaves are defined by Rules (R1) and (R2) above, for every j ($1 \leq j < m$) the owner of every leaf between u_{j-1} and u_j is the k_j th text in the collection. See also the left diagram of Fig. 6 for illustration.

Now we describe how the ownerships of leaves and the active points of texts can change when a new character is appended to a text in the fully-online setting. We begin with the first node $u_1 = s$ in the list L_i whose current owner is text $S_{k_1} S_{k_1}$. See also the left diagram of Fig. 6. Since $S_{k_1} S_{k_1}$ now gets extended to $S_{k_1} S_{k_1} a$, the active point of this text *extends* the suffix tree leaf u_1 . Then, the extended leaf u_1 no more represents a suffix of its original owner $S_{k_1} S_{k_1}$. This implies that the new owner of this suffix tree leaf u_1 is $S_{k_1} S_{k_1} a$. The same happens to all leaves in the path up to u_1 . Then, we *swap* the active points of texts $S_{k_1} S_{k_1}$ and $S_{k_1} S_{k_1} a$. We continue the same procedure recursively for the other nodes u_2, \dots, u_m in the list L_i , and finally the new owner of each leaf in the path P_i becomes the updated k th text $S_{k_1} S_{k_1} a$. After reaching the root of SLT , we possibly create new edges labeled with a following virtual suffix links, and finally arrive at the new locus of the active point for the updated k th text $S_{k_1} S_{k_1} a$. This operation may split the original suffix link tree SLT into some smaller suffix link trees (see also Fig. 6).

Lemma 4 *The above procedure correctly maintains the active points of texts and the leaf ownerships under Rules (R1) and (R2).*

Proof It is clear that the above procedure correctly maintains the leaf ownerships under Rules (R1) and (R2).

Let $S_{k_j} S_{k_j}$ be any text in \mathcal{O}_i . After swapping the active points of $S_{k_1} S_{k_1} a$ and those texts in \mathcal{O}_i , the locus of the active point of $S_{k_j} S_{k_j}$ is one character above the suffix tree leaf (say u) that has just been extended by $S_{k_1} S_{k_1} a$. By the definition of list L_i , this leaf u *before extension* was the longest leaf whose previous owner was $S_{k_j} S_{k_j}$. Hence, the string depth of the new active point of $S_{k_j} S_{k_j}$ is at least $|u| - 1$. Also, it cannot be larger than $|u| - 1$, since otherwise it contradicts with the definitions of \mathcal{O}_i and L_i (see also Fig. 6). Hence, the above procedure of swapping active points correctly maintains the active points of the texts in the collection. \square

Now we wish to maintain leaf ownerships as described above. However, the next lemma shows that it requires super-linear cost to explicitly maintain leaf ownerships.

Lemma 5 *There is a left-to-right fully-online collection of K texts of total length N for which explicitly maintaining leaf ownerships requires $\Omega(\frac{N^2}{K})$ time.*

Proof Consider an initial text collection $\mathcal{S} = \{S_1, \dots, S_K\}$. We will update this text collection in i rounds so that in each j th round the same character a_j is appended to each text. The order of the texts to which a_j is appended is arbitrary in each round. Thus, after the j th round, the text collection becomes of form $\{S_1 a_1 \cdots a_j, \dots, S_K a_1 \cdots a_j\}$. We also assume that $a_j \neq a_h$ for any $1 \leq j \neq h \leq i$. This implies that in each j th round, we will have j leaves representing common suffixes $a_1 \cdots a_j, a_2 \cdots a_j, \dots, a_j$.

Notice that during the j th round, the ownership of each such leaf has to be updated K times since each such leaf is shared by the K texts. Therefore, the total number of

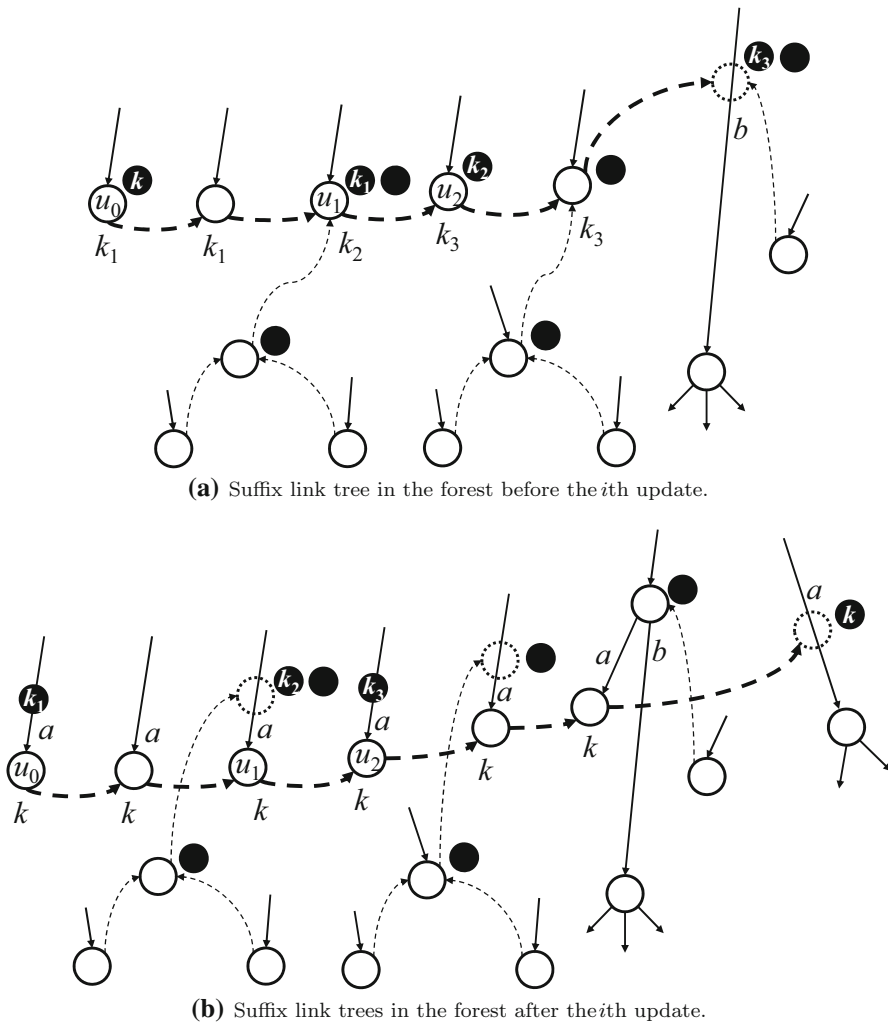


Fig. 6 Diagram (a) depicts a suffix link tree in the forest before the i th update with update operator $U[i] = (k, a)$, where the solid and broken arrows respectively represent suffix tree edges and suffix links, and the white and black circles respectively represent suffix tree leaves and active points. The dotted circle represents the root of the suffix link tree (which is an implicit node in this case). The suffix link path P_i of interest is shown with bold broken arrows, where the starting node is $s = u_0$. The integers k_j below each leaf shows the current owner of the leaf, and hence $\mathcal{O}_i = \{S_k S_k, S_{k_1} S_{k_1}, S_{k_2} S_{k_2}, S_{k_3} S_{k_3}\}$. The integer k_j in each black circle implies that it is the active point for text $S_{k_j} S_{k_j}$. The black circles without text id's are the active points of texts that are not in \mathcal{O}_i . Diagram (b) shows how it looks after the text $S_k S_k$ has been extended to $S_k S_k a$ with a new character a . Since its active point has extended the leaf u_0 with a , text $S_k S_k$ becomes the new owner of every leaf in the path P_i . In the meantime, we swap the active point for text $S_k S_k a$ with the active points of texts in \mathcal{O}_i , in the order they appear in the path P_i . After the active point of text $S_k S_k a$ and that of the last text in the path (which in this figure is $S_{k_3} S_{k_3}$) have been swapped, we possibly create new leaves (in this figure we create just one new leaf), and eventually we find the new locus for the active point for the updated text $S_k S_k a$. Since all the leaves in the path P_i have been extended by the new character a , this path breaks away from the original suffix link tree. As a result, we obtain several smaller suffix link trees

updates for the leaf ownership after the final i th round is at least

$$K(1 + 2 + \cdots + i) = \frac{Ki(i+1)}{2}. \quad (1)$$

Since N is the total length of the resulting text collection after the i th round, we get $N = K(i+1)$. Hence, $i = \Theta(\frac{N}{K})$. Plugging this into Eq. 1, we obtain the desired lower bound $\Omega(\frac{N^2}{K})$. \square

The above $\Omega(\frac{N^2}{K})$ lower bound requires us a super-linear cost for explicit leaf ownership maintenance when $K = o(N)$. Indeed, $K = o(N)$ is the only meaningful case in our fully-online problem: If $K = \Theta(N)$, then each of the K texts is of constant size and hence a naïve algorithm would update the suffix tree in constant time per each text no matter how they are updated, resulting in an $O(N)$ -time construction anyway. Hence, in what follows, we will only consider the case where $K = o(N)$.

Due to Lemma 5, we shall not explicitly maintain leaf ownerships in our fully-online algorithm. However, when swapping the active point of the k th text with those of the texts in the set \mathcal{O}_i , we need to know the owner of the leaf that has just been extended by the active point of the k th text. We also need to know the set \mathcal{O}_i of texts that are the owners of the leaves in the path P_i , and need to know the list L_i of leaves where those active points currently lie. For this sake we use the aid of our version of Weiner's algorithm for fully-online right-to-left construction. Namely, we build $STree(S_{U[1..i]})$ in tandem with $STree(\mathcal{T}_{U[1..i]})$ for each increasing $i = 1, \dots, N$. For simplicity, we will call the left-to-right fully-online suffix tree $STree(S_{U[1..i]})$ as the *Ukkonen tree* and the right-to-left fully-online suffix tree $STree(\mathcal{T}_{U[1..i]})$ as the *Weiner tree*.

Below we show key observations that connect our versions of Weiner's algorithm and Ukkonen's algorithm in the fully-online setting. For each node v of the Weiner tree, let $w_deg(v)$ denote the number of (soft or hard) W-links from v , namely, $w_deg(v) = |\{c \in \Sigma \mid w_c(v) = 1\}|$.

Lemma 6 *Let u be any leaf in the list L_i of Ukkonen tree $STree(S_{U[1..i-1]})$. Then, there exists an explicit node v of Weiner tree $STree(\mathcal{T}_{U[1..i-1]})$ such that (1) $v = \bar{u}$, (2) v is in the path from the root to the leaf representing $T_k\$k$, and (3) $w_deg(v) = 0$.*

Proof Since u is a leaf of Ukkonen tree $STree(S_{U[1..i-1]})$, it is a suffix of the text $\$k\k to which a new character a will be appended. Hence $v = \bar{u}$ is a prefix of the reversed text $T_k\$k$, and is located on the path from the root to the leaf $T_k\$k$ in Weiner tree $STree(\mathcal{T}_{U[1..i-1]})$. By the definition of the list L_i , the active point of some other text (say $\$h\h , with $h \neq k$) lies on the leaf u in the Ukkonen tree, which implies that u is the longest suffix of $\$h\h that occurs at least twice in the left-to-right collection. Since each left-to-right text begins with a distinct $\$$ symbol, there must be at least two distinct characters that immediately precede occurrences of u . This in turn implies that there are at least two distinct characters that immediately follow occurrences of $v = \bar{u}$ in the right-to-left text collection, and hence $v = \bar{u}$ is an explicit node in the Weiner tree. To prove (3) assume on the contrary that $w_deg(v) > 0$, and let c be any character such that $w_c(v) = 1$. Since $cv = c\bar{u}$ is a substring of some text in the right-to-left collection $\mathcal{T}_{U[1..i-1]}$, uc is a substring of some text in the left-to-right collection

$\mathcal{S}_{U[1..i-1]}$. However, this contradicts that u is a leaf of Ukkonen tree $STree(\mathcal{S}_{U[1..i-1]})$. Hence $w_deg(v) = 0$. \square

As was shown in Sect. 3, when we update Weiner tree $STree(\mathcal{T}_{U[1..i-1]})$ to $STree(\mathcal{T}_{U[1..i]})$ with update operator $U[i] = (k, a)$ that prepends character a to text $T_k\$k$, we walk up from the leaf $T_k\$k$ until finding the first node with a (soft or hard) W-link w.r.t. a defined. Since the total cost of walking up these paths for all characters prepended to the right-to-left texts is linear in the final total length N of all texts, the number of nodes in the list L_i for $1 \leq i \leq N$ is also linear in N .

Notice that not every explicit node v with $w_deg(v) = 0$ in the path from the leaf $T_k\$k$ to the root of the Weiner tree corresponds to a leaf in the list L_i on the Ukkonen tree. However, as was shown above, we can afford to check each such explicit node v in total linear time.

The next lemma shows how to maintain correspondence between these nodes in the Weiner tree and the Ukkonen tree.

Lemma 7 *We can maintain correspondence between each node v of the Weiner tree with $w_deg(v) = 0$ and its corresponding leaf u in the Ukkonen tree in $O(N \log \sigma)$ total time.*

Proof Let v be any node of Weiner tree $STree(\mathcal{T}_{U[1..i-1]})$ with $w_deg(v) = 0$. Suppose we have maintained correspondence between v and its corresponding leaf u in Ukkonen tree $STree(\mathcal{S}_{U[1..i-1]})$. This correspondence is maintained by bidirectional links between the two trees.

Now suppose we are given an update operator $U[i] = (k, a)$ that appends a new character a to $\$k\k and prepends a to $T_k\$k$. There are three cases to consider.

- (a) If the active point of the k th left-to-right text extends a leaf of the Ukkonen tree: In this case, as was described previously and was illustrated in Fig. 6, the leaves in the path P_i get extended by the new character a that was appended to the k th left-to-right text $\$k\k . This implies that v in the updated Weiner tree $STree(\mathcal{T}_{U[1..i]})$ does not correspond to a leaf in the updated Ukkonen tree $STree(\mathcal{S}_{U[1..i]})$. Thus, we remove the bidirectional link that connects v and the corresponding leaf in the Ukkonen tree.
- (b) If the active point of the k th text catches up a leaf u of the Ukkonen tree: Since u is a leaf whose current owner is another text $\$h\h with $h \neq k$, u is a suffix of at least two distinct left-to-right texts in the updated collection $\mathcal{S}_{U[1..i]}$. Hence, \bar{u} is a prefix of at least two distinct right-to-left texts in the updated collection $\mathcal{T}_{U[1..i]}$, and hence is represented by an explicit node in the updated Weiner tree $STree(\mathcal{T}_{U[1..i]})$. Let v be this explicit node. Moreover, since u is the locus of the active point of $\$k\$k\$a$, u is the longest repeating suffix of $\$k\$k\$a$ and hence $v = \bar{u}$ is the longest repeating prefix of $aT_k\$k$. This node v is exactly the insertion point of the new leaf $aT_k\$k$ in the Weiner tree. Hence, we can find the locus of $v = \bar{u}$ during the updates of the Weiner tree and can easily create a bidirectional link between v and u .
- (c) Otherwise, there are no changes in the correspondence and hence no maintenance of bidirectional links is needed.

In both cases (a) and (b), the costs can be charged to the construction of the Weiner tree that takes total $O(N \log \sigma)$ time. \square

In Lemmas 6 and 7 we have shown how to efficiently find those suffix tree leaves in the list L_i of the Ukkonen tree with the aid of the Weiner tree. What remains is how to find each text in the set \mathcal{O}_i of owners of the leaves in the list L_i . The next lemma shows yet another application of the Weiner tree for this purpose.

Lemma 8 *With the aid of the Weiner tree, we can find the owner of each leaf in the list L_i in total $O(N \log \sigma)$ time for all i ($1 \leq i \leq N$).*

Proof In each internal explicit node of the Weiner tree, we store the id of the text that created the oldest leaf in the subtree rooted at this internal explicit node. This can be easily maintained in $O(1)$ time per node: When we split an edge and create a new internal node, then we simply copy the text id stored in its unique child.

Consider any update operator $U[i] = (k, a)$. Let u be any leaf in the list L_i of the Ukkonen tree and let v be its corresponding node in the Weiner tree (hence $v = \bar{u}$ and it is an explicit node due to Lemma 6). Then, if the text id stored in v is h , then the h th text is the current owner of the leaf u in the Ukkonen tree. This is true in either case where the leaf u was created by the h th text and has never been extended by an active point, or the leaf u was last extended by the h th text. In both cases, the subtree rooted at $v = \bar{u}$ in the Weiner tree may contain leaves that correspond to suffixes of some other texts than the h th text, but in the Ukkonen tree the active points of these texts only caught up with the leaf u . Hence none of these texts is the one that created the leaf u , or the last one that has extended u . Therefore, the h th text is the current owner of u .

A careful consideration is required when the leaf u gets extended by the active point of text S_k . Now the extended leaf represents the extended string ua and its new owner is the k th text $S_k a$. As was shown in the proof for Lemma 7, in the Weiner tree the reversed extended string $a\bar{u}$ is represented by a new, different locus than the locus for \bar{u} . It is also possible that $a\bar{u}$ is on an implicit node in the Weiner tree at this stage, but it will become explicit when the active point of another text catches up the leaf ua in the Ukkonen tree. Thus, we will be able to return the text id k as the correct answer for a leaf ownership query when the active point of another text extends the leaf ua in future. \square

In the above arguments we have shown that Difficulties A and C can be efficiently resolved by swapping active points and by neglecting explicit maintenance of leaf ownerships.

Meanwhile, this lazy maintenance of leaf ownership causes two more issues; Suppose that the active point of some text S_i lies on an edge that leads to a leaf u , and that a new character a has been appended to this text. Let x be the string represented by the active point.

- The first question is how we can determine whether the active point can step forward along this edge by character a , or a new explicit node must be created at the locus of x together with a new edge labeled with a . Since we do not know the owner of the leaf u , we are not able to answer the above question by a simple

character comparison. However, this can be answered again by the aid of the Weiner tree. Recall that there is an explicit node representing the reversed string \bar{x} in the Weiner tree and we know its locus through the updates of the Weiner tree. Now, the active point can step forward with character a if and only if the node \bar{x} has a (soft or hard) W-link for character a . Hence, we can answer the above question in $O(\log \sigma)$ time. In case where we cannot step forward with character a , then we need to create a new edge leading to a new leaf. Instead of explicitly maintaining the owner of the leaf, we only maintain the first character a of this edge label. If the locus of the active point is on an edge, then we create a new explicit node u representing x in the Ukkonen tree. Now u has two out-going edges both leading to leaves, one of which is labeled with a as was described above. Since x was on an edge, there was a unique character, say b , such that $b \neq a$ and the W-link of node \bar{x} for character b is defined in the Weiner tree. Thus the other out-going edge of u is labeled with b in the Ukkonen tree. Also, by storing the string depth in each active point, the whole label of the edge from the parent of u to u can be easily determined in constant time. Thus, we are able to eagerly maintain the whole label of every edge leading to an internal explicit node.

- The second question is how we can know that the active point catches up the leaf. In the preceding discussions, we only proved that we can find the owner of the leaf *after* we know that the active point has caught up the leaf. We observe that the active point catches up the leaf if and only if the Weiner tree node v representing $a\bar{x}$ is of Weiner degree zero, namely, the W-link of node v is undefined for any character. Hence, this question can also be answered by the aid of the Weiner tree in constant time.

The final issue in this method is how to overcome Difficulty B on the cost for canonizing active points. The next lemma implies that the cost in our fully-online setting can indeed be amortized by a simple modification to the original amortization arguments in the semi-online setting.

Lemma 9 *The total cost for canonizing the active points for all the K texts in a left-to-right collection \mathcal{S} is $O(N \log \sigma)$.*

Proof Since we swap active points, the owner of each active point can change during the construction of the Ukkonen tree. However, our analysis below does not consider which text is the owner of each active point and hence it will lead us to simple arguments.

Let A denote any active point and let (u_A, c_A, ℓ_A) denote the reference pair of A . We remark that in our fully-online setting, this reference pair may not be canonical, since some other text can split the out-going edge of node u_A whose label begins with c_A . The *potential* of the active point A is ℓ_A of the string that hangs off from the explicit node u_A .

Suppose we have constructed $STree(S_{U[1..i-1]})$, and that we are given the i th update operator $U[i] = (k, a)$ that appends new character a to the k th text $\$_k S_k$. Also, suppose that A is the active point for $\$_k S_k$ at this stage. Now the algorithm finds the new locus for the active point A for the updated text $\$_k S_k a$, while possibly swapping several active points and inserting new leaves. In this event the algorithm traverses a chain of

(virtual) suffix links. When a canonization is conducted after tracing a virtual suffix link, then the potential ℓ_A decreases at least one. Also, when the new locus of the active point A is found on the updated suffix tree $S\text{Tree}(\mathcal{S}_{U[1..i]})$, then the potential increases exactly by one with the new character a . Hence, the total number of canonizations performed for all the N added characters is at most N .

Each canonization operation requires $O(\log \sigma)$ time to find the out-going edge whose label begins with the corresponding character. Hence, the total cost for canonizations for all the N characters is $O(N \log \sigma)$. \square

Putting the above arguments all together, we have proven the following theorem.

Theorem 2 *Given a fully-online sequence U of N update operators for a collection of K left-to-right texts \mathcal{S} , our version of Ukkonen’s algorithm can update the suffix tree in a total of $O(N \log \sigma)$ time and $O(N)$ space.*

A snapshot of left-to-right fully-online suffix tree construction is shown in Fig. 7, where the $\$i$ symbols are omitted for simplicity.

4.3.2 Alternative Approach

The algorithm proposed in the previous subsection is a direct extension of Ukkonen’s original algorithm, which inserts the new leaves of the updated text $\$_k S_k a$ in *decreasing* order of the lengths of the suffixes of $\$_k S_k a$, by following the chain of (virtual) suffix links *forward*. An alternative approach would be to insert the new leaves in *increasing* order of the lengths of the suffixes of $\$_k S_k a$, by following the chain of (virtual) suffix links *backward*. This backward approach can be seen as an extension of Breslauer and Italiano’s algorithm [4] that was originally proposed for real-time suffix tree construction for a single left-to-right text.

The basic idea of this alternative backward approach is given in the previous version of this paper [14], but as was noted previously in this paper, the original left-to-right algorithm in [14] takes superlinear time. This is because the original algorithm used the explicit representation of the DAWG for a left-to-right text collection. Still, if we use our version of Weiner’s algorithm proposed in Sect. 3 in place of the DAWG, then the left-to-right suffix tree construction algorithm can be fixed so that it runs in $O(N \log \sigma)$ time with $O(N)$ space. Interested readers are referred to “Appendix” which shows the details of this modified backward approach.

5 Conclusions and Open Problems

In this paper, we considered construction of the suffix tree and the DAWG of the fully-online multiple texts, where new characters can be added to any of the texts. Our contribution is twofold:

First, we proposed the fully-online version of Weiner’s suffix tree construction algorithm for a collection of K right-to-left growing texts. This algorithm runs in $O(N \log \sigma)$ time with $O(N)$ space, where N is the total length of the texts in the collection and σ is the alphabet size. We showed that the direct application of Weiner’s

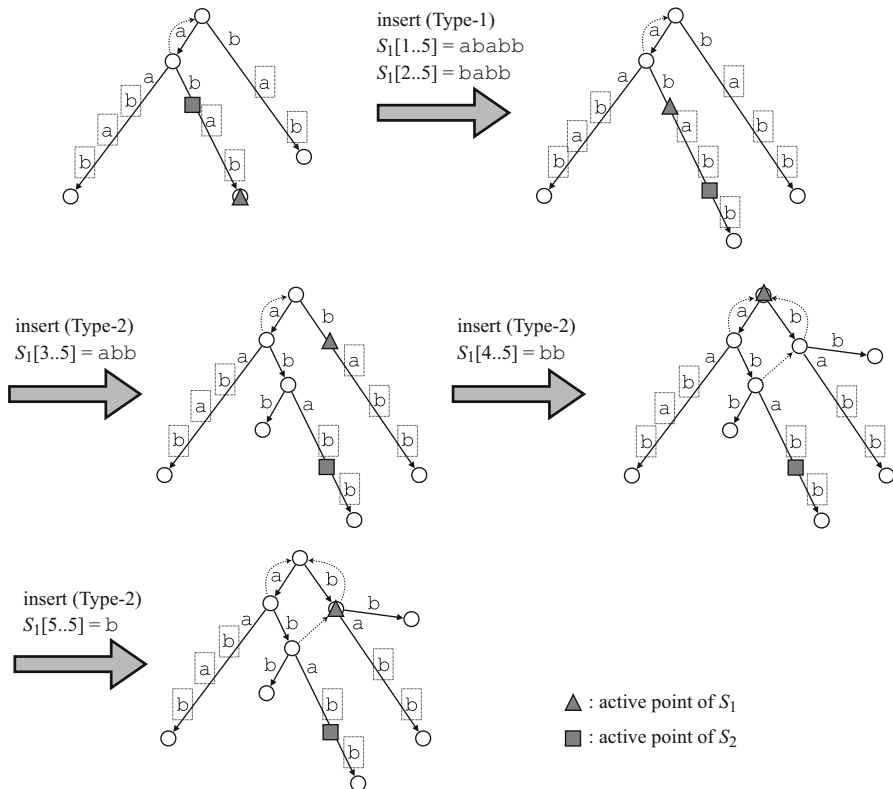


Fig. 7 A snapshot of left-to-right fully-online suffix tree construction, where we update $STree(\mathcal{S})$ to $STree(\mathcal{S}')$ with $\mathcal{S} = \{S_1 = abab, S_2 = aabab\}$ and $\mathcal{S}' = \{S_1b, S_2\}$ (here the terminate symbols S_1 and S_2 are omitted for simplicity). Recall that we employ lazy maintenance of leaf ownership, and hence each character within a box is only imaginary and is not computed during the updates. Due to lazy representation of leaves, we do nothing to insert the Type-1 suffixes of S_1b . The active point of S_1 was on a leaf whose owner was S_2 , and then it has extended the leaf. Hence, we swap the active points of S_1 and S_2 . To start inserting the Type-2 suffixes in decreasing order of length, we first insert the longest Type-2 suffix abb at the locus of the active point of S_1 . With the aid of the Weiner tree, we determine whether the active point can step forward along this edge by character b . In this case, the active point cannot step forward, and hence create a new internal node in the middle of this edge. After creating a new leaf from the new internal node and its in-coming edge with the first character label b , we determine the label of the in-coming edge of the new internal node using Weiner tree. Then the active point traces the virtual suffix link from the new internal node ab to node b . This virtual link can be computed by using the suffix link of node a . The next Type-2 suffix is bb , and the active point cannot step forward with b . Therefore we create a new internal node in the middle of this edge. After creating a new leaf from the new internal node and its in-coming edge with the first character label b , we determine the label of the in-coming edge of the new internal node using Weiner tree. The reversed suffix link is set from this new internal node b to node ab . Then the active point traces the virtual suffix link from the new internal node b to the root. The next shorter suffix b is Type-3, since we can step forward with character b from the root. Therefore, we move the active point from the root to node b that represents the longest repeating suffix of S_1b , and the reversed suffix link is set from root to the node b . Since we have inserted all the Type-2 suffixes, the update finishes

algorithm to our fully-online setting takes $\Theta(N \min(K, \sqrt{N}))$ time (Lemma 2), and showed that how it can be modified to run in $O(N \log \sigma)$ time with a novel use of NMA data structures that occupy $O(N)$ total space (Theorem 1). We also showed an algorithm that simulates soft W-links with hard W-links and these NMA data structures in $O(\log \sigma)$ time per query, which immediately gives us an $O(N \log \sigma)$ -time construction algorithm for an $O(N)$ -space representation of the DAWG for a fully-online left-to-right text collection (Corollary 1).

Second, we proposed two variants of the fully-online version of Ukkonen’s suffix tree construction algorithm for a collection of K left-to-right growing texts. We showed that explicit maintenance of the owners of leaves requires us super-linear cost (Lemma 5) in the worst case. Then, we proposed the first variant called the forward approach, which runs in $O(N \log \sigma)$ time with $O(N)$ space. The key to this forward approach is the notion of swapping active points and the efficient algorithm for answering leaf ownerships in a spacial case that happens during the construction of the suffix tree. The second variant called the backward approach traces a virtual suffix link chain in the reversed direction to the forward approach, and also works in $O(N \log \sigma)$ time with $O(N)$ space (see “Appendix”).

There are many intriguing open problems for the left-to-right fully-online suffix tree construction. Examples are the following:

- (1) Is it possible to maintain the Ukkonen’s tree for a left-to-right text collection *without* the aid of the Weiner tree for the corresponding right-to-left text collection?
- (2) Is there a data structure that maintains leaf ownerships in an *implicit* manner, so that the ownership of an *arbitrary* leaf can be efficiently answered upon query, at any stage of the construction algorithm?
- (3) Our bound is amortized, namely, for each new character our algorithm takes $O(\log \sigma)$ amortized time. Is it possible to de-amortize it, e.g. by using techniques in [4,9]?
- (4) Is it possible to extend our approach for a *bidirectional* fully-online text collection, where each text can grow both directions? There is a $O(n \log \sigma)$ -time $O(n)$ -space algorithm for constructing the suffix tree for a single bidirectional text of length n [11]. We note that for a bidirectional fully-online text collection, we cannot use terminal $\$_k$ symbols either ends of each text during the updates.

Appendix: Backward Approach

In this appendix, we propose the backward approach that traces a chain of (virtual) suffix links in the reversed order and inserts new leaves in increasing order of their string lengths.

Suppose we have constructed $STree(S_{U[1..i-1]})$ and we are now given an update operator $U[i] = (k, a)$. Consider the locus of the insertion point of the shortest Type-2 suffix of the updated text $\$_k S_k a$ in the Ukkonen tree $STree(S_{U[1..i-1]})$. This locus corresponds to the suffix of $\$_k S_k a$ that is exactly one character longer than the longest Type-3 suffix $lrs_{S_{U[1..i-1]}}(\$_k S_k a)$ of $\$_k S_k a$ in the text collection $S_{U[1..i-1]}$ before update. In the backward approach we first find this locus, and insert the Type-2 suffixes of the

updated text $S_k S_k a$ in increasing order of lengths. Since we trace the chain of suffix links backward, we use the reversed suffix links with character labels. In other words, we maintain *the hard W-links on the Ukkonen tree*.

We also remark that we do not need to swap active points in this backward approach, since we begin with the *shortest* Type-2 suffix. This somewhat simplifies the concept of the algorithm and might be an advantage over the forward approach.

To find the canonical reference to the locus of the insertion point of the shortest Type-2 suffix of $S_k S_k a$, we use the spanning tree of $DAWG(S_{U[1..i]})$ that consists only of the primary edges. This tree consists of the longest paths from the source of the DAWG to its nodes, and hence, it coincides with the tree of *the reversed hard W-links of the Weiner tree* for the collection $\mathcal{T}_{U[1..i]}$ of the reversed texts (this should not be confused with the hard W-links on the Ukkonen tree for backward suffix link traversals). For each i ($1 \leq i \leq N$), let $LPT(S_{U[1..i]})$ denote this tree. By the property of DAWGs (and hence that of the equivalence relation), the following fact holds (see also Fig. 8).

Fact 1 *For any i ($2 \leq i \leq N$), if an edge e is a primary edge of $DAWG(S_{U[1..i-1]})$, then e is a primary edge of $DAWG(S_{U[1..i]})$.*

We also use the following fact in our algorithm.

Fact 2 *For any substring x of texts in a left-to-right text collection S , node x is branching (explicit) in $STree(S)$ iff node $[x]_S$ is branching in $DAWG(S)$.*

Based on Fact 2, for each i ($1 \leq i \leq N$), we will maintain the NMA data structure for $LPT(S_{U[1..i]})$ and mark its nodes iff they correspond to the branching nodes of $STree(S_{U[1..i-1]})$. Note that, due to Fact 1, no edges of $LPT(S_{U[1..i-1]})$ will be deleted in $LPT(S_{U[1..i]})$ and only new edges will be added. Hence we can use the NMA data structure on top of this tree.

The next lemma shows how we can efficiently find the new locus of the active point for the updated text $S_k S_k a$ in the Ukkonen tree.

Lemma 10 *We can compute, in amortized $O(\log \sigma)$ time, the canonical reference to the locus of the active point of $S_k S_k a$ on the Ukkonen tree, using a data structure that requires $O(N)$ space.*

Proof Suppose we have constructed the Ukkonen tree $STree(S_{U[1..i-1]})$ in tandem with the Weiner tree $STree(\mathcal{T}_{U[1..i-1]})$ and $LPT(S_{U[1..i-1]})$. A node v of $LPT(S_{U[1..i-1]})$ is marked iff its corresponding node \bar{v} in the Weiner tree $STree(\mathcal{T}_{U[1..i-1]})$ has at least two W-links defined, namely, $w_c(\bar{v}) = w_{c'}(\bar{v}) = 1$ with at least two distinct characters $c \neq c'$. This in turn implies that the corresponding node of the (implicitly maintained) DAWG is branching. Every marked node of $LPT(S_{U[1..i-1]})$ is linked to its corresponding node of the Ukkonen tree $STree(S_{U[1..i-1]})$ that is also branching by Fact 2 (see also Fig. 8). We also maintain an NMA data structure on $LPT(S_{U[1..i-1]})$.

Given an update operator $U[i] = (k, a)$, we first update the Weiner tree to $STree(\mathcal{T}_{U[1..i]})$. This introduces at most two new hard W-links, one for the new leaf and one for its parent. This means that these edges are also inserted to $LPT(S_{U[1..i-1]})$ and we then obtain $LPT(S_{U[1..i]})$. Because of these new edges, at most two DAWG

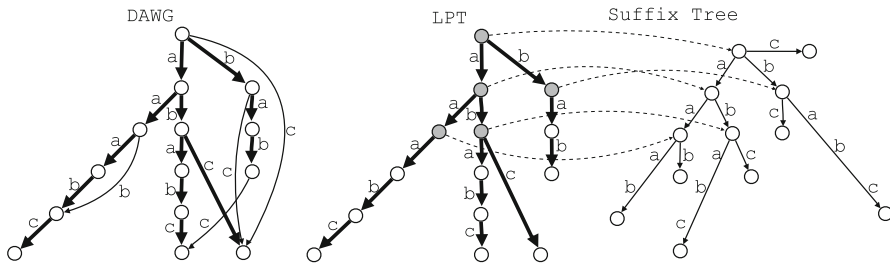


Fig. 8 Illustration for $DAWG(S_{U[1..14]})$, $LPT(S_{U[1..14]})$, and the Ukkonen tree $STree(S_{U[1..13]})$ before update, where $S_{U[1..13]} = \{S_1 = aaab, S_2 = ababc, S_3 = bab\}$ and $S_{U[1..14]} = \{S_1c, S_2, S_3\}$. For simplicity, we here omit the terminate symbols $\$1$, $\$2$, and $\$3$. The bold solid arrows represent the primary edges of $DAWG(S_{U[1..14]})$, the gray nodes are the marked nodes of $LPT(S_{U[1..14]})$, and the dashed arrows represent the links between the marked nodes of $LPT(S_{U[1..14]})$ and the corresponding branching nodes of $STree(S_{U[1..13]})$. The longest repeating suffix of S_1c in $S_{U[1..14]}$ is abc , and hence we perform an NMA query from node abc on $LPT(S_{U[1..14]})$, obtaining node ab . We then access the suffix tree node ab using the link from $LPT(S_{U[1..14]})$, and obtain the canonical reference $(ab, c, 1)$ to abc on the Ukkonen tree $STree(S_{U[1..13]})$ before update

non-branching nodes can become branching. We mark their corresponding nodes in $LPT(S_{U[1..i-1]})$, and link them to the corresponding Ukkonen tree nodes *only after* we have built the updated Ukkonen tree $STree(S_{U[1..i-1]})$. This is because the corresponding nodes of $STree(S_{U[1..i-1]})$ before the update are still non-branching (see Fact 2).

Let \bar{y} be the insertion point of the leaf $aT_k\$k$ in the Weiner tree that is the longest repeating prefix of $aT_k\$k$ in the right-to-left text collection $\mathcal{T}_{U[1..i]}$. By the definition of $LPT(S_{U[1..i]})$, there is a node in $LPT(S_{U[1..i]})$ that represents y . We conduct an NMA query from y on $LPT(S_{U[1..i]})$, and let v be the NMA of y . Let $\ell = |y| - |v|$, and let c be the label of the first edge in the path from v to y . We move from v to its corresponding node x in the Ukkonen tree $STree(S_{U[1..i-1]})$. Then, (x, c, ℓ) is a reference to the insertion point of the shortest Type-2 suffix of $\$k\ka . Since v is the NMA of y in $LPT(S_{U[1..i]})$, and since updating $\$k\k to $\$k\ka does not explicitly insert any suffix of $\$k\ka that is shorter than the longest repeating suffix of $\$k\ka in $S_{U[1..i]}$, this reference is canonical by Fact 2.

Clearly the total size of the above data structures is linear in the total length N of the texts in the final text collection \mathcal{S} . We analyze the time complexity. We can find the insertion point y of the new leaf in the Weiner tree in amortized $O(\log \sigma)$ time due to Theorem 1. Using the link from the node y in $LPT(S_{U[1..i]})$, the corresponding node in the Ukkonen tree $STree(S_{U[1..i-1]})$ can be found in $O(1)$ time. Updating $LPT(S_{U[1..i-1]})$ to $LPT(S_{U[1..i]})$ takes $O(\log \sigma)$ amortized time. Inserting a new node and querying an NMA from a given node takes amortized $O(1)$ time. We can link a new marked node of $LPT(S_{U[1..i]})$ to the corresponding new branching node of $STree(S_{U[1..i]})$ in $O(1)$ time, since it is easy to remember this new branching node when updating $STree(S_{U[1..i-1]})$ to $STree(S_{U[1..i]})$. Hence, the total amortized bound is $O(\log \sigma)$. \square

Let w and w' denote the strings that are represented by the loci of the insertion points of the shortest and longest new leaves w.r.t. the update operator $U[i] = (k, a)$.

Let $q = |w'| - |w| + 1$ be the number of new leaves to be inserted in the Ukkonen tree. Our backward approach terminates the i th update after inserting the q th new leaf. How do we compute this value q ? If (x, c, ℓ) is the canonical reference to the locus for w , then $|w| = |x| + \ell$, and hence what remains is how to compute $|w'|$. We note that w' is the longest suffix of $S_k S_k$ that has at least one more occurrence in $S_{U[1..i]}$ immediately followed by another character $b \neq a$. This is because any longer suffix of $S_k S_k$ is immediately followed only by a , and will thus correspond to existing leaves in the updated Ukkonen tree. These two occurrences of w' must be immediately preceded by distinct characters, say c and d , in the left-to-right text collection $S_{U[1..i]}$ since otherwise there will be a longer suffix of $S_k S_k$ that has at least one more occurrence in $S_{U[1..i]}$, a contradiction. Also, $\overline{w'}c$ and $\overline{w'}d$ occur in the right-to-left text collection $\mathcal{T}_{U[1..i-1]}$ before the i th update. Thus, $\overline{w'}$ is represented by an explicit node in the Weiner tree $STree(\mathcal{T}_{U[1..i-1]})$. Since this node is on the path from the leaf for $T_k S_k$ to the root of the Weiner tree, and since it is the deepest node with the hard W-link for character a , we visit this node during the update of the Weiner tree. Hence, we can compute $|w'|$ in $O(\log \sigma)$ amortized time by the aid of the Weiner tree.

The cost to trace the suffix link chains in this backward approach is exactly the same as that in the forward approach. Hence, the total cost is for suffix link chain traversals is $O(N \log \sigma)$ for all i ($1 \leq i \leq N$) by Lemma 9.²

The lower bound of Lemma 5 also applies to this backward approach. Hence, we do not maintain the leaf ownerships, and we label the edges leading to the leaves only with their first characters.

We have shown the following:

Theorem 3 *Given a fully-online sequence U of N update operators for a collection of K left-to-right texts S , our backward version of Ukkonen's algorithm can update the suffix tree in a total of $O(N \log \sigma)$ time and $O(N)$ space.*

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² In the preliminary version [14] of this paper, a simplified version of the *suffix tree oracle* [9] was used to obtain the same bound. However, we do not need it any more due to our amortization argument of Lemma 9.

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