Project Proposal

Project Mittens

Kate Brown, Dan Hayes, James Parkington

We're in the middle of a massive surge in popularity surrounding the game of chess. What once was considered a niche hobby relegated to clubs of intellectuals has now becomes a prominent thread in the social fabric. This sudden rise in interest, no doubt magnified by the number of us at home, can be attributed to several factors, including:

- the increase in options for accessible chess-playing platforms
- the emergence of popular chess streamers garnering millions of views alongside the top gamers in the country
- the portrayal of chess in mainstream television, particularly *The Queen's Gambit*

With this increase in attention comes a significant increase in players of all backgrounds. The added attention to the game has been amplified by the rapid development of advanced chess bots, evolving the game into a playing ground for machine intelligence, and thereby inviting an entirely new form of competition for eager new users.

This fever pitch for bot play broke the sound barrier with the emergence and sudden decommissioning of **Mittens** in January of 2022. Mittens was, at the time, the most competitive bot available for players to challenge on **chess.com.** Like every player on **chess.com** (*and the other platforms in the space*), each bot is given a rating that informs you of its general strength as a player. These ratings are generally based on ELO, a scoring algorithm devised by Arpad Elo in the 1960s. Players who are adept at finding winning combinations tend to have higher ELO values approaching and sometimes surpassing a value of 2,000.

However, **chess.com** upon releasing Mittens bestowed it with an ELO of **1**! About 100 times worse than the poorest chess players on its platform. Naturally, this sparked curiosity and attention, followed by thousands of Twitch streams and YouTube videos of grandmasters playing Mittens and attempting to guess its ELO, even pitting Mittens against other well-known bots like Stockfish and AlphaZero to get a stronger sense of its abilities.

This raised some major questions for the 3 of us chess fans of different backgrounds:

- How could someone develop a sense of a competitor's ELO without a value for reference?
- How was Mittens designed to be "better" than other prominent bots?

- **Can bots actually make human chess-players more competitive as a whole?**
- **What's the natural growth of a chess player improving via learning, rather than relying on bots? Can we tell when players cheat by the growth of their ELO?**
- **And most importantly, did Arpad Elo have any idea what was in store for the future when he devised this algorithm?**

1. Hypotheses

Our goal is to substantiate these 4 premises with available math and research to conclude that **bots are responsible for a measurable growth in the average chess player's ability to play chess to such a degree that it is tempting for top players to cheat.**

1. ELO as a mathematical construct is **positioned to scale** with the introduction of a new "population" of players, bots, that compute depths of permutations too large for humans to comprehend.
2. Experienced players have an **intrinsic understanding of ELO** and its variations from platform to platform, and therefore can make educated guesses about a competitor's strength based on their playing style, opening choices, and overall strategic acumen.
3. The algorithms chess bots use to play at previously unreached levels of precision and accuracy have become **increasingly efficient** over time, resulting in their accessibility for a wider audience.
4. Humans have shifted their perception of "good" chess and their ability to recreate it in large part to the increased accessibility of said bots, to such a degree that **cheating has become more rampant** at all levels of chess.

2. Proposed Sources

1. ***(1978) "The Rating of Chessplayers, Past and Present" by Arpad Elo***
   - This is the foundational book for the scoring algorithm that was later translated to other scoring-based systems in Go, player rankings, and potential political outcomes.
   - The ELO rating system is defined as a logistic function that maps the probability of winning a game between two players to their relative ratings.
   - The formula for his function involves logarithms and summation.
2. ***(2005) " A Psychometric Analysis of Chess Expertise" by Han L. J. Van Der Maas, Eric-Jan Wagenmakers***
   - This study introduces the Amsterdam Chess Test (ACT) and explains the math behind the 5 tests within, a lot of which uses sets and series to understand large ranges of possible permutations of moves
   - Their results show that the ACT has high predictive validity, suggesting that strong players have an inherent human understanding of chess complexity and when it's created by other humans
   - ELO is their foundational measurement, which they use as a dependency for much of their reporting

3. ***(2005) "Chess: A Cover Up" by Eric K. Henderson et al.***

- The goal of this team's mathematical approach was to supply a more "modern" formula set for how possible moves could be "pruned" down to a sequence of logical next steps for a player to make
- This study does an excellent job of breaking down some of the possible permutations and piece movements on the board into digestible mathematical formulas
- Much of this seems like it would be foundational to the programming of an early-stage chess bot, in that its pruning mechanism suggests our current application of "depth"

4. ***(2022) "Chess AI: Competing Paradigms for Machine Intelligence" by Shiva Maharaj et al.***

- This paper explores some of the formulas and methodologies used for AlphaZero, Stockfish 14, and Maia, among some of the other bots
- It touches on how the methodologies have developed to become more sophisticated over time, and specifically highlights the progeession from some of the early computing ideas to more modern mathematical approaches, with lots of examples of formulas

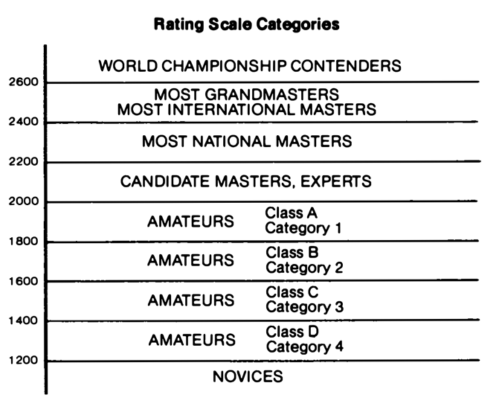
3. Supporting Evidence & Theories

The History of the ELO Formula

The ELO rating formula is a player rating formula that can be traced back to the 1960s. Arpad Elo, a Hungarian American physics professor and avid chess player, developed a rating system to replace then current system, the Harkness system.

In the 1978 publication of *The Rating of Chessplayers Past & Present* Elo defines the rating system, the formulas, and the mathematics used to create the performance of players and to properly rate them. The system is built on relative inference based on other player's scores.

Providing a breakdown of the inputs considered



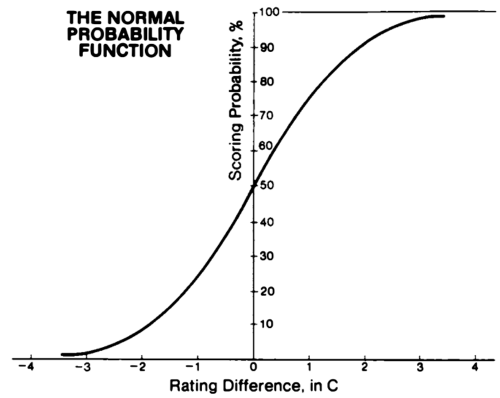
The class subdivision into 200 points and choice of 2000 points as reference was preserved when Elo created his calculations.

Since we have an established point system, we can consider the variability in player performance. With the number of players contributing to the system there is a normal distribution curve that can explain the range of expected scores. The statistical probability theory, however, needs to be combined with the competition, Elo suggested. So enters the normal probability function.

The rating system is defined as a logistic function that maps the probability of winning a game between two players to their relative ratings. The formula for his function involves logarithms and summation.

The Elo Rating System can be broken down to the following two equations

These equations calculate the expected score for player A and player B.



$$E_A = \frac{1}{1 + 10^{(R_B - R_A)/400}} \qquad E_B = \frac{1}{1 + 10^{(R_A - R_B)/400}}$$

- E is the expected result of the game
- R_A and R_B are the ratings of player 1 and player 2, respectively.

The formula for his function involves logarithms and summation. We can see that in the ***Current Rating Formula – Continuous Measurement*** [1.61] which is recognized as

$$\Delta R = K \cdot (S - E)$$

- ΔR is the change in ratings
- K is the constant, set at $K = 16$ for masters and $K = 32$ for weaker players
- S is the actual result of the game (1 for a win, 0 for a loss, 0.5 for a draw)

In this formula, the base-10 logarithm represents the odds of one player winning, given the difference in their ratings. The expected result E is then used to calculate the change in ratings ΔR using that same logistic function.

In understanding these formulas, we'd like to take these equations from Arpad Elo's book on use them to trace the likelihood of different-level players winning matches.

Explore How Its Calculation Has Changed Over Time

In its first publication Elo remarked that ratings were subject to the current population. Change in population causes the equations to fluctuate. Hence inevitable change over time.

The change in K-factor for different level of players. The USCF "uses a three-level K-factor for players below 2100, between 2100–2400 and above 2400 of 32, 24 and 16, respectively. FIDE uses similar factors (40, 20 and 10) albeit with somewhat different rules."

Comparison to current ELO divisions using the rating of chess players past & present

Here, we'll would like to supplement some data on current ratings of top players as well as the agreed upon range of different player levels, and compare them to historical figures.

What was a good ELO when it was first introduced?

- Elo: 5.4 | The Crosstables of 120 Years*
- Elo: 5.5 | The 500 Best Five-Year Averages*

Implementation of the Equation

Start with how ELO works generally, as well as for experienced players using data from chess.com / lichess.org. Then implement the equation and take a look at summations of the equation to see what growth would look like long term.

Getting a look at how long it would take for a strong chess player to reach their accurate strength, and perhaps a bot that never loses and see what would eventually happen there.

Elo and how it represents chess knowledge and understanding

Using the rating of chess players past and present / chess.com and lichess.org.

From there we'll look more in depth at how the Elo represents the strength of chess players past and present. How over time with the assistance of computers, we have increased the upper end of chess knowledge. How this increased knowledge has increased the range of Elo and also how important it is that in the system of Elo everyone must have the same starting value. Example of chess.com and lichess.org and how one starts at 1500 and the other 1200, which then skews the inherit strengths of their players.

Known Bot Algorithms

LCZero uses Q-matrices, denoted by $Q(s, a)$, which determine optimal sequential decisions.

The values of $Q(s, a)$ can be found through Q-learning, and describe the value of taking action a (a chess move) in the current state s (the chess board position), followed by optimal play.

In the context of chess, $V(s)$ represents the probability of winning, which can be converted from a pawn-advantage metric that takes into account the arrangement of s.

$$V(s) = max_a Q(s, a) = Q(s, a^*(s))$$

The Bellman equation for the Q-values (assuming instantaneous utility $u(s, a)$ and a time-inhomogeneous Q matrix) is given by:

$$Q(s,a) = u(s,a) + \sum_{s^* \in S} P(s^* | s,a) max_a Q(s^*, a)$$

$P(s^*||s,a)$ denotes the transition matrix of states and represents the probability of moving to a new state s^* given the current state s and action a.

s^* is the board position after the current player has taken action a and the opponent has responded

AlphaZero uses the Monte Carlo tree search (MCTS) algorithm to determine the best moves in a game by repeatedly sampling different variations. The algorithm starts with one node, which represents the current position in the game, and performs simulations to trace paths through the game tree. The tree is expanded by adding new nodes at the end of each simulation, and the quality of each node is evaluated using the evaluation function. The algorithm uses the polynomial upper confidence tree (PUCT) algorithm to optimistically select nodes during simulations.

The PUCT algorithm is defined as:

- $a_t = \text{error}_a \left(Q(s_t, a) + U(s_t, a)\right)$
- $U(s,a) = C(s)P(s,a)\dfrac{\sqrt{\sum_b N(s,b)}}{1 + N(s,a)}$
- $C(s) = \log \dfrac{1 + N(s) + c_{base}}{c_{base}} + c_{init}$

In the above formulas:

- $Q(s,a)$ is the mean action value of the node
- $P(s,a)$ is the prior probability of the node according to the policy network
- $N(s,a)$ is the number of times that action a has been taken from state s
- The value of $C(s)$ controls the amount of exploration, and increases as the search progresses
- The constants C_{base} and C_{init} determine the rate of exploration during the search

Quantifying Hans Neimann's Rise to GM

1. Rate of Change

One way to represent the speed of a player like Neimann's progression in mathematical terms is by calculating the rate of change of their ELO rating over time. This can be done by fitting a mathematical function to the player's ELO ratings and finding its **derivative**, which represents the **rate of change** of the function at any given time.

Let $ELO(t)$ be the ELO rating of a player at time t. The rate of change of the player's ELO rating with respect to time t can be represented as:

$$\frac{dELO}{dt} = \frac{d}{dt}ELO(t)$$

This would offer a measure of how quickly the player's ELO rating was changing at that point in time, which can be used to compare the player's progress to others.

Growth rate is also measureable to some degree by $\mathcal{O}$, which we use to compare if some algorithm is more efficient than another. A player's ability to improve their chess performance is **inherently algorithmic**, since we're evaluating the player's efficiency in finding the most competitive moves. Therefore, the application of **big-O** applies, since it measures the growth rate of that change.

2. Magnitude

The magnitude of the rate of change can be calculated as the absolute value of that same function. Magnitude tells us how different some function size is to some other function size, in this case the progressions of ELO from player to player:

$$\left|\frac{dELO}{dt}\right|$$

This gives us the absolute rate at which the player's ELO rating is changing. If the player's ELO rating is increasing rapidly, this value will be large. If the player's ELO rating is changing slowly, it will be small.

3. Acceleration

To measure acceleration, we can take the derivative of the rate of change:

$$\frac{d^2ELO}{dt^2} = \frac{d}{dt}\left(\frac{dELO}{dt}\right)$$

This formula represents the **second derivative** of the function $ELO(t)$ with respect to time t. If $\frac{d^2ELO}{dt^2}$ is positive, the player's ELO rating is increasing at an increasing rate. If $\frac{d^2ELO}{dt^2}$ is negative, the player's ELO rating is increasing at a decreasing rate (or more slowly over time). We could also calculate a magnitude of this second derivative function to see how quickly a rating is speeding up or slowing down, compared to others.

In the case of Hans Niemann, calculating the rate of change of his ELO ratings over time would give us a quantitative measure of the speed of his progression compared to other players. This could provide a useful mathematical representation of the remarkable nature of his rise to Grandmaster status at such a young age.