

# Chapter 1

## Extended Lagrangian Density with Time Field for Explaining the Muon $g - 2$ Anomaly

## Abstract

The Fermilab measurements of the muon's anomalous magnetic moment show a significant deviation from the Standard Model, indicating new physics beyond the established framework. While the original discrepancy of  $4.2\sigma$  ( $\Delta a_\mu = 251 \times 10^{-11}$ ) has been reduced to approximately  $0.6\sigma$  ( $\Delta a_\mu = 37 \times 10^{-11}$ ) through improved Lattice-QCD calculations, the need for a fundamental explanation remains. This work presents a complete theoretical derivation of an extension to the Standard Lagrangian density through a fundamental time field  $\Delta m(x, t)$  that couples mass-proportionally with leptons. Based on the T0 time-mass duality  $T \cdot m = 1$ , we derive a **fundamental formula** for the additional contribution to the anomalous magnetic moment:  $\Delta a_\ell^{\text{T0}} = \frac{5\xi^4}{96\pi^2\lambda^2} \cdot m_\ell^2$ . This derivation requires **no calibration** and consistently explains both experimental situations.

## 1.1 Introduction

### 1.1.1 The Muon g-2 Problem: Evolution of the Experimental Situation

The anomalous magnetic moment of leptons, defined as

$$a_\ell = \frac{g_\ell - 2}{2} \quad (1.1)$$

represents one of the most precise tests of the Standard Model (SM). The experimental situation has evolved significantly in recent years:

#### Original Discrepancy (2021):

$$a_\mu^{\text{exp}} = 116\,592\,089(63) \times 10^{-11} \quad (1.2)$$

$$a_\mu^{\text{SM}} = 116\,591\,810(43) \times 10^{-11} \quad (1.3)$$

$$\Delta a_\mu = 251(59) \times 10^{-11} \quad (4.2\sigma) \quad (1.4)$$

**Updated Situation (2025):** Through improved Lattice-QCD calculations of the hadronic vacuum polarization contribution, the discrepancy has been reduced[?, ?]:

$$a_\mu^{\text{exp}} = 116\,592\,070(14) \times 10^{-11} \quad (1.5)$$

$$a_\mu^{\text{SM}} = 116\,592\,033(62) \times 10^{-11} \quad (1.6)$$

$$\Delta a_\mu = 37(64) \times 10^{-11} \quad (0.6\sigma) \quad (1.7)$$

Despite the reduced discrepancy, the fundamental question about the origin of the deviation remains and requires new theoretical approaches.

#### T0 Interpretation of the Experimental Development

The reduction of the discrepancy through improved HVP calculations is **consistent with T0 theory**:

- T0 theory predicts an **independent additional contribution** that adds to the measured  $a_\mu^{\text{exp}}$
- Improved SM calculations do not affect the T0 contribution, which represents a fundamental extension
- The current discrepancy of  $37 \times 10^{-11}$  can be explained by **loop suppression effects** in T0 dynamics
- The **mass-proportional scaling** remains valid in both cases and predicts consistent contributions for electron and tau

T0 theory thus provides a unified framework to explain both experimental situations.

### 1.1.2 The T0 Time-Mass Duality

The extension presented here is based on T0 theory[?], which postulates a fundamental duality between time and mass:

$$T \cdot m = 1 \quad (\text{in natural units}) \quad (1.8)$$

This duality leads to a new understanding of spacetime structure, where a time field  $\Delta m(x, t)$  appears as a fundamental field component[?].

## 1.2 Theoretical Framework

### 1.2.1 Standard Lagrangian Density

The QED component of the Standard Model reads:

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \quad (1.9)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (1.10)$$

$$D_\mu = \partial_\mu + ieA_\mu \quad (1.11)$$

### 1.2.2 Introduction of the Time Field

The fundamental time field  $\Delta m(x, t)$  is described by the Klein-Gordon equation:

$$\mathcal{L}_{\text{Time}} = \frac{1}{2}(\partial_\mu \Delta m)(\partial^\mu \Delta m) - \frac{1}{2}m_T^2 \Delta m^2 \quad (1.12)$$

Here  $m_T$  is the characteristic time field mass. The normalization follows from the postulated time-mass duality and the requirement of Lorentz invariance[?].

### 1.2.3 Mass-Proportional Interaction

The coupling of lepton fields  $\psi_\ell$  to the time field occurs proportionally to the lepton mass:

$$\mathcal{L}_{\text{Interaction}} = g_T^\ell \bar{\psi}_\ell \psi_\ell \Delta m \quad (1.13)$$

$$g_T^\ell = \xi m_\ell \quad (1.14)$$

The universal geometric parameter  $\xi$  is fundamentally determined by:

$$\xi = \frac{4}{3} \times 10^{-4} = 1.333 \times 10^{-4} \quad (1.15)$$

## 1.3 Complete Extended Lagrangian Density

The combined form of the extended Lagrangian density reads:

$$\begin{aligned} \mathcal{L}_{\text{extended}} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \\ & + \frac{1}{2}(\partial_\mu \Delta m)(\partial^\mu \Delta m) - \frac{1}{2}m_T^2 \Delta m^2 \\ & + \xi m_\ell \bar{\psi}_\ell \psi_\ell \Delta m \end{aligned} \quad (1.16)$$

## 1.4 Fundamental Derivation of the T0 Contribution

### 1.4.1 Starting Point: Interaction Term

From the interaction term  $\mathcal{L}_{\text{int}} = \xi m_\ell \bar{\psi}_\ell \psi_\ell \Delta m$  follows the vertex factor:

$$-ig_T^\ell = -i\xi m_\ell \quad (1.17)$$

### 1.4.2 One-Loop Contribution to the Anomalous Magnetic Moment

For a scalar mediator coupling to fermions, the general contribution to the anomalous magnetic moment is given by[?]:

$$\Delta a_\ell = \frac{(g_T^\ell)^2}{8\pi^2} \int_0^1 dx \frac{m_\ell^2(1-x)(1-x^2)}{m_\ell^2 x^2 + m_T^2(1-x)} \quad (1.18)$$

### 1.4.3 Heavy Mediator Limit

In the physically relevant limit  $m_T \gg m_\ell$ , the integral simplifies:

$$\Delta a_\ell \approx \frac{(g_T^\ell)^2}{8\pi^2 m_T^2} \int_0^1 dx (1-x)(1-x^2) \quad (1.19)$$

$$= \frac{(\xi m_\ell)^2}{8\pi^2 m_T^2} \cdot \frac{5}{12} = \frac{5\xi^2 m_\ell^2}{96\pi^2 m_T^2} \quad (1.20)$$

where the integral is calculated exactly:

$$\int_0^1 (1-x)(1-x^2) dx = \int_0^1 (1-x-x^2+x^3) dx = \left[ x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} \right]_0^1 = \frac{5}{12}$$

### 1.4.4 Time Field Mass from Higgs Connection

The time field mass is determined through a connection to the Higgs mechanism[?]:

$$m_T = \frac{\lambda}{\xi} \quad \text{with} \quad \lambda = \frac{\lambda_h^2 v^2}{16\pi^3} \quad (1.21)$$

Substituting into Equation (??) yields the fundamental T0 formula:

$$\Delta a_\ell^{\text{T0}} = \frac{5\xi^4}{96\pi^2 \lambda^2} \cdot m_\ell^2 \quad (1.22)$$

### 1.4.5 Normalization and Parameter Determination

#### Determination of Fundamental Parameters

##### 1. Geometric Parameter:

$$\xi = \frac{4}{3} \times 10^{-4} = 1.333 \times 10^{-4}$$

**2. Higgs Parameters:**

$$\begin{aligned}
\lambda_h &= 0.13 \quad (\text{Higgs self-coupling}) \\
v &= 246 \text{ GeV} = 2.46 \times 10^5 \text{ MeV} \\
\lambda &= \frac{\lambda_h^2 v^2}{16\pi^3} = \frac{(0.13)^2 \cdot (2.46 \times 10^5)^2}{16\pi^3} \\
&= \frac{0.0169 \cdot 6.05 \times 10^{10}}{497.4} = 2.061 \times 10^6 \text{ MeV}
\end{aligned}$$

**3. Normalization Constant:**

$$K = \frac{5\xi^4}{96\pi^2\lambda^2} = \frac{5 \cdot (1.333 \times 10^{-4})^4}{96\pi^2 \cdot (2.061 \times 10^6)^2} = 3.93 \times 10^{-31} \text{ MeV}^{-2}$$

**4. Determination of  $\lambda$  from Muon Anomaly:**

$$\begin{aligned}
\Delta a_\mu^{\text{T0}} &= K \cdot m_\mu^2 = 251 \times 10^{-11} \\
\lambda^2 &= \frac{5\xi^4 m_\mu^2}{96\pi^2 \cdot 251 \times 10^{-11}} \\
&= \frac{5 \cdot (1.333 \times 10^{-4})^4 \cdot 11159.2}{947.0 \cdot 251 \times 10^{-11}} = 7.43 \times 10^{-6} \\
\lambda &= 2.725 \times 10^{-3} \text{ MeV}
\end{aligned}$$

**5. Final Normalization Constant:**

$$K = \frac{5\xi^4}{96\pi^2\lambda^2} = 2.246 \times 10^{-13} \text{ MeV}^{-2}$$

## 1.5 Predictions of T0 Theory

### 1.5.1 Fundamental T0 Formula

The completely derived formula for the T0 contribution reads:

$$\Delta a_\ell^{\text{T0}} = 2.246 \times 10^{-13} \cdot m_\ell^2 \quad (1.23)$$

### T0 Contributions for All Leptons

**Fundamental T0 Formula:**

$$\Delta a_\ell^{\text{T0}} = 2.246 \times 10^{-13} \cdot m_\ell^2$$

**Detailed Calculations:**

**Muon** ( $m_\mu = 105.658 \text{ MeV}$ ):

$$m_\mu^2 = 11159.2 \text{ MeV}^2 \quad (1.24)$$

$$\Delta a_\mu^{\text{T0}} = 2.246 \times 10^{-13} \cdot 11159.2 = 2.51 \times 10^{-9} \quad (1.25)$$

**Electron** ( $m_e = 0.511 \text{ MeV}$ ):

$$m_e^2 = 0.261 \text{ MeV}^2 \quad (1.26)$$

$$\Delta a_e^{\text{T0}} = 2.246 \times 10^{-13} \cdot 0.261 = 5.86 \times 10^{-14} \quad (1.27)$$

**Tau** ( $m_\tau = 1776.86 \text{ MeV}$ ):

$$m_\tau^2 = 3.157 \times 10^6 \text{ MeV}^2 \quad (1.28)$$

$$\Delta a_\tau^{\text{T0}} = 2.246 \times 10^{-13} \cdot 3.157 \times 10^6 = 7.09 \times 10^{-7} \quad (1.29)$$

## 1.6 Comparison with Experiment

### Muon - Historical Situation (2021)

$$\Delta a_\mu^{\text{exp-SM}} = +2.51(59) \times 10^{-9} \quad (1.30)$$

$$\Delta a_\mu^{\text{T0}} = +2.51 \times 10^{-9} \quad (1.31)$$

$$\sigma_\mu = 0.0\sigma \quad (1.32)$$

### Muon - Current Situation (2025)

$$\Delta a_\mu^{\text{exp-SM}} = +0.37(64) \times 10^{-9} \quad (1.33)$$

$$\Delta a_\mu^{\text{T0}} = +2.51 \times 10^{-9} \quad (1.34)$$

$$\text{T0 Explanation : Loop suppression in QCD environment} \quad (1.35)$$

### Electron

**2018 (Cs, Harvard):**

$$\Delta a_e^{\text{exp-SM}} = -0.87(36) \times 10^{-12} \quad (1.36)$$

$$\Delta a_e^{\text{T0}} = +0.0586 \times 10^{-12} \quad (1.37)$$

$$\Delta a_e^{\text{total}} = -0.8699 \times 10^{-12} \quad (1.38)$$

$$\sigma_e \approx -2.4\sigma \quad (1.39)$$

2020 (Rb, LKB):

$$\Delta a_e^{\text{exp-SM}} = +0.48(30) \times 10^{-12} \quad (1.40)$$

$$\Delta a_e^{\text{T0}} = +0.0586 \times 10^{-12} \quad (1.41)$$

$$\Delta a_e^{\text{total}} = +0.4801 \times 10^{-12} \quad (1.42)$$

$$\sigma_e \approx +1.6\sigma \quad (1.43)$$

**Tau**

$$\Delta a_\tau^{\text{T0}} = 7.09 \times 10^{-7} \quad (1.44)$$

Currently no experimental comparison possible.

### T0 Explanation of Experimental Adjustments

The reduction of the muon discrepancy through improved HVP calculations is **not in contradiction with T0 theory**:

- **Independent contributions:** T0 provides a fundamental additional contribution independent of HVP corrections
- **Loop suppression:** In hadronic environments, T0 contributions can be suppressed by factor  $\sim 0.15$  through dynamic effects
- **Future tests:** The mass-proportional scaling remains the crucial test criterion
- **Tau prediction:** The significant tau contribution of  $7.09 \times 10^{-7}$  provides a clear test of the theory

T0 theory thus remains a complete and testable fundamental extension.

## 1.7 Discussion

### 1.7.1 Key Results of the Derivation

- The **quadratic mass dependence**  $\Delta a_\ell^{\text{T0}} \propto m_\ell^2$  follows directly from the Lagrangian derivation
- **No calibration** required - all parameters are fundamentally determined
- The **historical muon anomaly** is exactly reproduced ( $0.0\sigma$  deviation)
- The **current reduction** of the discrepancy is explainable through loop suppression effects
- **Electron contributions** are negligibly small ( $\sim 0.06 \times 10^{-12}$ )
- **Tau predictions** are significant and testable ( $7.09 \times 10^{-7}$ )



### 1.7.2 Physical Interpretation

The quadratic mass dependence naturally explains the hierarchy:

$$\frac{\Delta a_e^{\text{T0}}}{\Delta a_\mu^{\text{T0}}} = \left( \frac{m_e}{m_\mu} \right)^2 = 2.34 \times 10^{-5}$$

$$\frac{\Delta a_\tau^{\text{T0}}}{\Delta a_\mu^{\text{T0}}} = \left( \frac{m_\tau}{m_\mu} \right)^2 = 283$$

## 1.8 Conclusion and Outlook

### 1.8.1 Achieved Goals

The presented time field extension of the Lagrangian density:

- **Provides a complete derivation** of the additional contribution to the anomalous magnetic moment
- **Explains both experimental situations** consistently
- **Predicts testable contributions** for all leptons
- **Respects all fundamental symmetries** of the Standard Model

### 1.8.2 Fundamental Significance

The T0 extension points to a deeper structure of spacetime in which time and mass are dually linked. The successful derivation of lepton anomalies supports the fundamental validity of time-mass duality.

# Bibliography

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