

Unit Conventions and the Speed of Light c

$E=mc^2$ vs. $E=m$: Two Equivalent Perspectives

Natural Units, SI Units, and the T0 Viewpoint

Johann Pascher

December 22, 2025

Abstract

This document examines when one can set $c=1$ (natural units) and when the full form $E=mc^2$ with $c=299,792,458$ m/s (SI units) is required. Parallel to the treatment of the fine-structure constant α in Document 101, we show: Both perspectives are mathematically equivalent and differ only in the choice of unit system. The T0 theory reveals that c is not a fundamental law of nature but a dynamic ratio L/T . From the T0 perspective, $c=1$ can be set (Planck units, particle physics), while for technical applications and precision measurements, SI units with explicit c are required. The equivalence $E=mc^2 \leftrightarrow E=m$ holds exactly in natural units. References: Documents 013 (SI system), 014 (nat./SI), 015 (systematics), 077 ($E=mc^2$ analysis), 101 (α conventions).

Contents

1 Introduction: The Question of $c=1$

1.1 The Central Question

The question "When can one set $c=1$?" is analogous to the question "When can one set $\alpha=1$?" addressed in Document 101. In both cases, it concerns **unit conventions**, not fundamental physics.

Central Thesis

$E=mc^2$ and $E=m$ are mathematically identical!

- In SI units: $E = mc^2$ with $c = 299,792,458$ m/s
- In natural units: $E = m$ with $c = 1$

Both forms describe the same physics – only the unit choice differs.

1.2 Historical Context

Einstein wrote the famous formula in 1905:

$$E = mc^2 \tag{1}$$

This form was necessary because he worked in **SI units**, where length (meter), time (second), and mass (kilogram) are independent dimensions.

Modern particle physics uses instead:

$$E = m \quad (\text{in natural units with } c = \hbar = 1) \tag{2}$$

2 Natural Units: When $c=1$ is Valid

2.1 Definition of Natural Units

In natural units, one sets:

$$c = 1, \quad \hbar = 1, \quad (\text{optional: } k_B = 1) \tag{3}$$

Mathematical meaning:

$$c = 1 \quad \Rightarrow \quad \text{Length} \equiv \text{Time} \tag{4}$$

$$\hbar = 1 \quad \Rightarrow \quad \text{Energy} \equiv \text{inverse Time} \tag{5}$$

2.2 Application Domains

Natural units are appropriate in:

- **Planck scale:** Quantum gravity, fundamental theory

- **Particle physics:** High-energy physics, QFT, Standard Model
- **Cosmology:** Early universe, inflationary models
- **Theoretical work:** Mathematical derivations, symmetries

Advantage: Formulas become simpler, physical relationships clearer.

2.3 Mathematical Consistency

In natural units:

$$E^2 = p^2 + m^2 \quad (6)$$

In the rest frame ($p = 0$):

$$E = m \quad (7)$$

This is exact – **not an approximation**.

2.4 T0 Perspective: c as a Ratio

The T0 theory shows (see Document 077):

$$c = \frac{L}{T} \quad (8)$$

c is not a fundamental law of nature but a *ratio*!

With the T0 fundamental relation:

$$T \cdot m = 1 \quad (\text{Time-Mass Duality}) \quad (9)$$

it follows that c is a dynamic ratio that varies with mass scale.

Implication: In Planck units, where $t_P = \ell_P/c$, $c=1$ is the natural choice.

3 SI Units: When c=299,792,458 m/s is Required

3.1 The SI Definition (since 2019)

The modern SI system defines since 2019:

$$\boxed{c = 299,792,458 \text{ m/s (exact)}} \quad (10)$$

This choice is a **convention** that defines the meter via the second.

3.2 Application Domains

SI units with explicit c are required in:

- **Engineering:** GPS, telecommunications, laser technology
- **Precision measurements:** Atomic clocks, interferometry, metrology
- **Experimental physics:** Laboratory measurements with SI-calibrated devices
- **Applied physics:** Energy calculations, dosimetry
- **Public & Education:** Comprehensibility, historical continuity

Advantage: Practical calculability with calibrated measurement devices.

3.3 Mathematical Form

In SI units:

$$E = \gamma mc^2 \quad (11)$$

with the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad (12)$$

In the rest frame ($v = 0$, $\gamma = 1$):

$$E = mc^2 \quad (13)$$

3.4 Conversion Between Unit Systems

From natural units to SI:

$$E_{\text{nat}} = m_{\text{nat}} \quad (14)$$

$$\Downarrow \quad (\text{Multiply by } c^2) \quad (15)$$

$$E_{\text{SI}} = m_{\text{SI}} \cdot c^2 \quad (16)$$

Example: Electron mass

$$m_e = 0.511 \text{ MeV} \quad (\text{natural units}) \quad (17)$$

$$m_e = 9.109 \times 10^{-31} \text{ kg} \quad (\text{SI}) \quad (18)$$

$$E_e = m_e c^2 = 0.511 \text{ MeV} = 8.187 \times 10^{-14} \text{ J} \quad (19)$$

Convention	Fine-structure constant α	Speed of light c
Natural	$\alpha = 1$ (Heaviside-Lorentz)	$c = 1$ (Planck units)
SI / Standard	$\alpha = 1/137.036$ (Gauss-SI)	$c = 299,792,458$ m/s
Document	101 (Circularity-Constants)	134 (Unit Conventions c)

Table 1: Parallel structure: α and c as conventions

4 Comparison with α : Parallel Structure

4.1 Two Analogous Conventions

4.2 Common Principles

Both cases show:

- **Physics is invariant** under unit choice
- **Natural units** simplify theoretical work
- **SI units** enable practical applications
- **T0 theory**: Both are derived conventions, not fundamental

4.3 T0 Reduction

From the T0 perspective (see Document 101):

$$\xi \rightarrow D_f \rightarrow E_0 \rightarrow \alpha \rightarrow \hbar, c, G \rightarrow \text{all other constants} \quad (20)$$

Only $\xi = \frac{4}{3} \times 10^{-4}$ is fundamental.

Both α and c are derived quantities or conventions.

5 When to Use Which System?

5.1 Decision Matrix

5.2 Recommendations

Use **natural units** ($c=1$) when:

- Performing theoretical derivations
- Symmetries and invariant structures are important
- Formulas should be simplified
- Working in particle physics or cosmology

Use **SI units** (c explicit) when:

- Planning or evaluating experimental measurements

Context	Natural units ($c=1$)	SI units (c explicit)
Theoretical physics	✓	
Quantum field theory	✓	
High-energy physics	✓	
Early cosmology	✓	
Experimental physics		✓
Engineering		✓
Precision measurements		✓
Applied physics		✓
Education		✓

Table 2: Application domains of unit systems

- Technical calculations are required
- Results should be understandable for non-physicists
- Historical continuity is important

6 Common Misconceptions

6.1 “ $c=1$ is only an approximation”

FALSE. $c=1$ is **exact** in natural units, not an approximation.

It is a choice of unit system that defines:

$$\text{Length unit} = \text{Time unit} \quad (21)$$

Analogously: In Planck units, $\hbar = 1$ is exact, not approximate.

6.2 “ $E=m$ only holds for photons”

FALSE. In natural units, $E = m$ holds for **all** particles in their rest frame.

For photons ($m = 0$): $E = p$ (in natural units) or $E = pc$ (in SI).

6.3 “ c is a fundamental constant of nature”

T0 viewpoint: c is a **ratio** L/T , not a fundamental constant.

With the T0 duality $T \cdot m = 1$, c varies dynamically with mass scale:

$$c = \frac{L}{T} = L \cdot m \quad (22)$$

Only in SI units is c *fixed by definition*.

6.4 “Natural units change the physics”

FALSE. Physics is independent of the unit system.

All **dimensionless** quantities (e.g., ξ , α , mass ratios) are invariant.

Only dimensional quantities change their numerical values.

7 T0 Perspective: c as a Dynamic Ratio

7.1 The T0 Fundamental Relation

From Document 077:

$$T \cdot m = 1 \quad (\text{Time-Mass Duality}) \quad (23)$$

This means:

$$T \propto \frac{1}{m} \quad (24)$$

$$L \propto \frac{1}{m} \quad (\text{via Compton wavelength}) \quad (25)$$

$$\Rightarrow \quad c = \frac{L}{T} \propto \frac{1/m}{1/m} = \text{scale-dependent} \quad (26)$$

7.2 Implications

1. c is not universally constant in the T0 framework:

Different effective c-values can occur at different mass scales.

2. SI definition c=299,792,458 m/s is a calibration:

This fixation defines the meter via the second – a metrological convention.

3. Natural units c=1 are T0-consistent:

In Planck units, where $t_P \propto \ell_P$, c=1 is the natural choice.

7.3 Comparison with Document 077

Document 077 argues: “E=mc² = E=m – The Constant Illusion Exposed”

Clarification here:

- E=mc² (SI) and E=m (natural) are *equivalent*, not identical
- The difference lies in the *unit system*, not in physics
- Einstein’s c-fixation is a *convention*, not an error
- T0 shows: c is a ratio that can vary depending on scale

8 Mathematical Consistency

8.1 Energy-Momentum Relation

In natural units ($c = 1$):

$$E^2 = p^2 + m^2 \quad (27)$$

In SI units:

$$E^2 = (pc)^2 + (mc^2)^2 \quad (28)$$

Both forms are mathematically equivalent.

8.2 Lorentz Transformation

In natural units:

$$E' = \gamma(E - p \cdot v) \quad (29)$$

In SI units:

$$E' = \gamma(E - p \cdot v \cdot c^2) \quad (30)$$

The physics remains invariant.

8.3 Klein-Gordon Equation

In natural units:

$$(\partial_\mu \partial^\mu + m^2)\phi = 0 \quad (31)$$

In SI units:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2 c^2}{\hbar^2} \right) \phi = 0 \quad (32)$$

Identical physics, different notation.

9 References to T0 Documents

9.1 Related Documents

- **Document 013:** SI System and T0 Theory
https://github.com/jpascher/T0-Time-Mass-Duality/blob/main/2/pdf/013_T0_SI_En.pdf
- **Document 014:** Natural vs. SI Units
https://github.com/jpascher/T0-Time-Mass-Duality/blob/main/2/pdf/014_T0_nat-si_En.pdf
- **Document 015:** Systematics of Natural Units
https://github.com/jpascher/T0-Time-Mass-Duality/blob/main/2/pdf/015_NatEinheitenSystematik_En.pdf
- **Document 077:** E=mc² = E=m Analysis
https://github.com/jpascher/T0-Time-Mass-Duality/blob/main/2/pdf/077_E-mc2_En.pdf
- **Document 101:** Circularity of Constants (α Conventions)
https://github.com/jpascher/T0-Time-Mass-Duality/blob/main/2/pdf/101_zirkularitaet-Konstanten_En.pdf
- **Document 133:** Fractal Correction K_frak Derivation
https://github.com/jpascher/T0-Time-Mass-Duality/blob/main/2/pdf/133_Fraktale_Korrektur_Herleitung_En.pdf

9.2 Derivation Hierarchy

The T0 hierarchy (from Document 101):

$$\xi \rightarrow D_f \rightarrow E_0 \rightarrow \alpha \rightarrow \hbar, c, G \rightarrow \text{mass ratios} \quad (33)$$

shows that both α and c are derived quantities.

10 Summary and Conclusion

10.1 Key Insights

Summary

1. **$E=mc^2$ and $E=m$ are mathematically equivalent** – different only in unit system
2. **$c=1$ (natural units)** is exact, not approximate, in particle physics/theory
3. **$c=299,792,458$ m/s (SI)** is required for experiment, engineering, education
4. **T0 perspective:** c is a ratio L/T , not a fundamental law of nature
5. **Unit choice does not change physics** – only notation
6. **Parallel to α :** Both are conventions/derived quantities, not fundamental

10.2 Practical Recommendation

For theoretical work: Use $c=1$ (natural units)

For experimental/applied work: Use c explicitly (SI units)

Important: Be consistent within a calculation and document the unit choice.

10.3 T0 Deeper Insight

The T0 theory shows that the debate “ $c=1$ or $c=299,792,458$?” can ultimately be reduced to **unit conventions** – similar to the debate about $\alpha=1$ vs. $\alpha=1/137$ in Document 101.

The fundamental physics resides in **dimensionless quantities** like $\xi = \frac{4}{3} \times 10^{-4}$, from which all other constants emerge.

c is a **derived ratio**, not a fundamental constant.

10.4 Outlook

Future work could investigate:

- Experimental tests of the T0 prediction of variable c -values at different scales
- Precision measurements to verify the Time-Mass Duality $T \cdot m = 1$
- Didactic approaches to convey the equivalence of natural/SI units

References

- [1] Johann Pascher, *The Complete Closure of FFGFT: From ξ to the 2019 SI Reform*, Document 013, T0 Theory Series (2024).
- [2] Johann Pascher, *Natural vs. SI Units*, Document 014, T0 Theory Series (2024).
- [3] Johann Pascher, *Systematics of Natural Units*, Document 015, T0 Theory Series (2024).
- [4] Johann Pascher, *$E=mc^2 = E=m$: The Constant Illusion Exposed*, Document 077, T0 Theory Series (2024).
- [5] Johann Pascher, *The Circularity in the Debate over Fundamental Constants*, Document 101, T0 Theory Series (2025).
- [6] Johann Pascher, *Fractal Correction K_{frak} – Complete Mathematical Proof*, Document 133, T0 Theory Series (2025).
- [7] Max Planck, *Über irreversible Strahlungsvorgänge*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (1899).
- [8] George Johnstone Stoney, *On the Physical Units of Nature*, Philosophical Magazine, Series 5, Vol. 11 (1881).
- [9] BIPM, *The International System of Units (SI)*, 9th edition (2019).