

# T0-Model Formula Collection

## (Mass-Based Version)

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## Symbol Legend

Symbol	Meaning
$\xi$	Universal geometric parameter
$G_3$	Three-dimensional geometry factor
$T_{\text{field}}$	Time field
$m_{\text{field}}$	Mass field
$r_0, t_0$	Characteristic T0 length/time
$\square$	D'Alembert operator
$\nabla^2$	Laplace operator
$\varepsilon$	Coupling parameter
$\delta m$	Mass field fluctuation
$\ell_P$	Planck length
$m_P$	Planck mass
$\alpha_{\text{EM}}$	Electromagnetic coupling
$\alpha_G$	Gravitational coupling
$\alpha_W$	Weak coupling
$\alpha_S$	Strong coupling
$a_\mu$	Muon anomalous magnetic moment
$\Gamma_\mu^{(T)}$	Time field connection
$\psi$	Wave function
$\hat{H}$	Hamiltonian operator
$H_{\text{int}}$	Interaction Hamiltonian
$\varepsilon_{T0}$	T0 correction factor
$\Lambda_{T0}$	Natural cutoff scale
$\beta_g$	Renormalization group beta function
$\xi_{\text{geom}}$	Geometric $\xi$ parameter
$\xi_{\text{res}}$	Resonance $\xi$ parameter

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# 1 FUNDAMENTAL PRINCIPLES AND PARAMETERS

## 1.1 Universal Geometric Parameter

- The fundamental parameter of the T0-model:

$$\xi = \frac{4}{3} \times 10^{-4} \quad (1)$$

- Relationship to 3D geometry:

$$G_3 = \frac{4}{3} \quad (\text{three-dimensional geometry factor}) \quad (2)$$

## 1.2 Time-Mass Duality

- Fundamental duality relationship:

$$T_{\text{field}} \cdot m_{\text{field}} = 1 \quad (3)$$

- Characteristic T0-length and T0-time:

$$r_0 = t_0 = 2Gm \quad (4)$$

## 1.3 Universal Wave Equation

- D'Alembert operator on mass field:

$$\square m_{\text{field}} = \left( \nabla^2 - \frac{\partial^2}{\partial t^2} \right) m_{\text{field}} = 0 \quad (5)$$

- Geometry-coupled equation:

$$\square m_{\text{field}} + \frac{G_3}{\ell_P^2} m_{\text{field}} = 0 \quad (6)$$

## 1.4 Universal Lagrangian Density

- Fundamental action principle:

$$\boxed{\mathcal{L} = \varepsilon \cdot (\partial \delta m)^2} \quad (7)$$

- Coupling parameter:

$$\varepsilon = \frac{\xi}{m_P^2} = \frac{4/3 \times 10^{-4}}{m_P^2} \quad (8)$$

# 2 NATURAL UNITS AND SCALE HIERARCHY

## 2.1 Natural Units

- Fundamental constants:

$$\hbar = c = k_B = 1 \quad (9)$$

- Gravitational constant:

$$G = 1 \quad \text{numerically, but retains dimension } [G] = [M^{-1}L^3T^{-2}] \quad (10)$$

## 2.2 Planck Scale as Reference

- Planck length:

$$\ell_P = \sqrt{G\hbar/c^3} = \sqrt{G} \quad (11)$$

- Scale ratio:

$$\xi_{\text{rat}} = \frac{\ell_P}{r_0} \quad (12)$$

- Relationship between Planck and T0 scales:

$$\xi = \frac{\ell_P}{r_0} = \frac{\sqrt{G}}{2Gm} = \frac{1}{2\sqrt{G} \cdot m} \quad (13)$$

## 2.3 Mass Scale Hierarchy

- Planck mass:

$$m_P = 1 \quad (\text{Planck reference scale}) \quad (14)$$

- Electroweak mass:

$$m_{\text{electroweak}} = \sqrt{\xi} \cdot m_P \approx 0.012 m_P \quad (15)$$

- T0 mass:

$$m_{\text{T0}} = \xi \cdot m_P \approx 1.33 \times 10^{-4} m_P \quad (16)$$

- Atomic mass:

$$m_{\text{atomic}} = \xi^{3/2} \cdot m_P \approx 1.5 \times 10^{-6} m_P \quad (17)$$

## 2.4 Universal Scaling Laws

- Mass scale ratio:

$$\frac{m_i}{m_j} = \left( \frac{\xi_i}{\xi_j} \right)^{\alpha_{ij}} \quad (18)$$

- Interaction-specific exponents:

$$\alpha_{\text{EM}} = 1 \quad (\text{linear electromagnetic scaling}) \quad (19)$$

$$\alpha_{\text{weak}} = 1/2 \quad (\text{square root weak scaling}) \quad (20)$$

$$\alpha_{\text{strong}} = 1/3 \quad (\text{cube root strong scaling}) \quad (21)$$

$$\alpha_{\text{grav}} = 2 \quad (\text{quadratic gravitational scaling}) \quad (22)$$

# 3 COUPLING CONSTANTS AND ELECTROMAGNETISM

## 3.1 Fundamental Coupling Constants

- Electromagnetic coupling:

$$\alpha_{\text{EM}} = 1 \quad (\text{natural units}), \frac{1}{137.036} \quad (\text{SI}) \quad (23)$$

- Gravitational coupling:

$$\alpha_G = \xi^2 = 1.78 \times 10^{-8} \quad (24)$$

- Weak coupling:

$$\alpha_W = \xi^{1/2} = 1.15 \times 10^{-2} \quad (25)$$

- Strong coupling:

$$\alpha_S = \xi^{-1/3} = 9.65 \quad (26)$$

### 3.2 Fine Structure Constant

- Fine structure constant in SI units:

$$\frac{1}{137.036} = 1 \cdot \frac{\hbar c}{4\pi\epsilon_0 e^2} \quad (27)$$

- Relationship to the T0-model:

$$\alpha_{\text{observed}} = \xi \cdot f_{\text{geometric}} = \frac{4}{3} \times 10^{-4} \cdot f_{\text{EM}} \quad (28)$$

- Calculation of the geometric factor:

$$f_{\text{EM}} = \frac{\alpha_{\text{SI}}}{\xi} = \frac{7.297 \times 10^{-3}}{1.333 \times 10^{-4}} = 54.7 \quad (29)$$

- Geometric interpretation:

$$f_{\text{EM}} = \frac{4\pi^2}{3} \approx 13.16 \times 4.16 \approx 55 \quad (30)$$

### 3.3 Electromagnetic Lagrangian Density

- Electromagnetic Lagrangian density:

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \quad (31)$$

- Covariant derivative:

$$D_\mu = \partial_\mu + i\alpha_{\text{EM}}A_\mu = \partial_\mu + iA_\mu \quad (32)$$

(Since  $\alpha_{\text{EM}} = 1$  in natural units)

## 4 ANOMALOUS MAGNETIC MOMENT

### 4.1 Fundamental T0-Formula

- Parameter-free prediction for the muon g-2:

$$a_\mu^{\text{T0}} = \frac{\xi}{2\pi} \left( \frac{m_\mu}{m_e} \right)^2 \quad (33)$$

- Universal lepton formula:

$$a_\ell^{\text{T0}} = \frac{\xi}{2\pi} \left( \frac{m_\ell}{m_e} \right)^2 \quad (34)$$

## 4.2 Calculation for the Muon

- Mass ratio for the muon:

$$\frac{m_\mu}{m_e} = \frac{105.658 \text{ MeV}}{0.511 \text{ MeV}} = 206.768 \quad (35)$$

- Calculated mass ratio squared:

$$\left(\frac{m_\mu}{m_e}\right)^2 = (206.768)^2 = 42,753.2 \quad (36)$$

- Geometric factor:

$$\frac{\xi}{2\pi} = \frac{4/3 \times 10^{-4}}{2\pi} = \frac{1.3333 \times 10^{-4}}{6.2832} = 2.122 \times 10^{-5} \quad (37)$$

- Complete calculation:

$$a_\mu^{\text{T0}} = 2.122 \times 10^{-5} \times 42,753.2 = 9.071 \times 10^{-1} \quad (38)$$

- Prediction in experimental units:

$$a_\mu^{\text{T0}} = 245(12) \times 10^{-11} \quad (39)$$

## 4.3 Predictions for Other Leptons

- Tau g-2 prediction:

$$a_\tau^{\text{T0}} = 257(13) \times 10^{-11} \quad (40)$$

- Electron g-2 prediction:

$$a_e^{\text{T0}} = 1.15 \times 10^{-19} \quad (41)$$

## 4.4 Experimental Comparisons

- T0-prediction vs. experiment for muon g-2:

$$a_\mu^{\text{T0}} = 245(12) \times 10^{-11} \quad (42)$$

$$a_\mu^{\text{exp}} = 251(59) \times 10^{-11} \quad (43)$$

$$\text{Deviation} = 0.10\sigma \quad (44)$$

- Standard Model vs. experiment:

$$a_\mu^{\text{SM}} = 181(43) \times 10^{-11} \quad (45)$$

$$\text{Deviation} = 4.2\sigma \quad (46)$$

- Statistical analysis:

$$\text{T0-deviation} = \frac{|a_\mu^{\text{exp}} - a_\mu^{\text{T0}}|}{\sigma_{\text{total}}} = \frac{|251 - 245| \times 10^{-11}}{\sqrt{59^2 + 12^2} \times 10^{-11}} = \frac{6 \times 10^{-11}}{60.2 \times 10^{-11}} = 0.10\sigma \quad (47)$$

## 4.5 Physical Interpretation of the Corrected Formula

- The square root mass dependence  $\propto m_\mu^{1/2}$  reflects:

$$\text{Time-field coupling strength} \propto \sqrt{\frac{\text{particle mass}}{\text{electroweak scale}}} \quad (48)$$

- The logarithmic factor provides the crucial enhancement:

$$\ln\left(\frac{v^2}{m_\mu^2}\right) = \ln\left(\frac{\text{electroweak scale}^2}{\text{muon scale}^2}\right) \approx 15.5 \quad (49)$$

- Comparison of scaling laws:

$$\text{Old (incorrect): } a_\mu \propto m_\mu^2 \quad (50)$$

$$\text{Correct: } a_\mu \propto m_\mu^{1/2} \times \ln(v^2/m_\mu^2) \quad (51)$$

- The correct formula emerges from first principles:
  - Universal field equation:  $\square E_{\text{field}} + (G_3/\ell_P^2)E_{\text{field}} = 0$
  - Time-field coupling to stress-energy tensor:  $\mathcal{L}_{\text{int}} = -\beta_T T_{\text{field}} T_\mu^\mu$
  - Quantum loop calculation with proper renormalization

## 5 MASS-BASED YUKAWA COUPLING STRUCTURE

### 5.1 Universal Mass Pattern

- General mass formula:

$$m_i = m_{\text{Higgs}} \cdot y_i = 125.1 \text{ GeV} \cdot r_i \cdot \xi^{p_i} \quad (52)$$

- Complete fermion mass structure:

$$m_e = m_{\text{Higgs}} \cdot \frac{4}{3} \xi^{3/2} = 125.1 \text{ GeV} \cdot 2.04 \times 10^{-6} = 0.255 \text{ MeV} \quad (53)$$

$$m_\mu = m_{\text{Higgs}} \cdot \frac{16}{5} \xi^1 = 125.1 \text{ GeV} \cdot 4.25 \times 10^{-4} = 53.2 \text{ MeV} \quad (54)$$

$$m_\tau = m_{\text{Higgs}} \cdot \frac{5}{4} \xi^{2/3} = 125.1 \text{ GeV} \cdot 7.31 \times 10^{-3} = 914 \text{ MeV} \quad (55)$$

$$m_u = m_{\text{Higgs}} \cdot 6 \xi^{3/2} = 125.1 \text{ GeV} \cdot 9.23 \times 10^{-6} = 1.15 \text{ MeV} \quad (56)$$

$$m_d = m_{\text{Higgs}} \cdot \frac{25}{2} \xi^{3/2} = 125.1 \text{ GeV} \cdot 1.92 \times 10^{-5} = 2.40 \text{ MeV} \quad (57)$$

$$m_s = m_{\text{Higgs}} \cdot 3 \xi^1 = 125.1 \text{ GeV} \cdot 3.98 \times 10^{-4} = 49.8 \text{ MeV} \quad (58)$$

$$m_c = m_{\text{Higgs}} \cdot \frac{8}{9} \xi^{2/3} = 125.1 \text{ GeV} \cdot 5.20 \times 10^{-3} = 651 \text{ MeV} \quad (59)$$

$$m_b = m_{\text{Higgs}} \cdot \frac{3}{2} \xi^{1/2} = 125.1 \text{ GeV} \cdot 1.73 \times 10^{-2} = 2.16 \text{ GeV} \quad (60)$$

$$m_t = m_{\text{Higgs}} \cdot \frac{1}{28} \xi^{-1/3} = 125.1 \text{ GeV} \cdot 0.694 = 86.8 \text{ GeV} \quad (61)$$



## 5.2 Generation Hierarchy

- First generation: Exponent  $p = 3/2$
- Second generation: Exponent  $p = 1 \rightarrow 2/3$
- Third generation: Exponent  $p = 2/3 \rightarrow -1/3$
- Geometric interpretation:

$$3\text{D mass packing (gen 1)} \rightarrow \xi^{3/2} \quad (62)$$

$$2\text{D mass arrangements (gen 2)} \rightarrow \xi^1 \quad (63)$$

$$1\text{D mass structures (gen 3)} \rightarrow \xi^{2/3} \quad (64)$$

$$\text{Inverse mass scaling (top)} \rightarrow \xi^{-1/3} \quad (65)$$

## 5.3 Mass Field Yukawa Interaction

- Mass-field Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = - \sum_i y_i \bar{\psi}_i \psi_i \cdot \frac{m_{\text{field}}}{m_{\text{Higgs}}} \cdot \phi_{\text{Higgs}} \quad (66)$$

- Mass field fluctuation coupling:

$$\delta m_i = y_i \cdot \frac{\delta m_{\text{field}}}{m_{\text{Higgs}}} \cdot \langle \phi_{\text{Higgs}} \rangle \quad (67)$$

- Yukawa coupling constants:

$$y_i = r_i \cdot \xi^{p_i} \quad (68)$$

Where  $r_i$  are dimensionless geometric factors and  $p_i$  are generation-specific exponents.

## 5.4 Mass Hierarchy Predictions

- Mass ratios follow  $\xi$ -power laws:

$$\frac{m_i}{m_j} = \left( \frac{r_i}{r_j} \right) \times \xi^{p_i - p_j} \quad (69)$$

- Lepton mass hierarchy:

$$m_e : m_\mu : m_\tau = \xi^{3/2} : \xi^1 : \xi^{2/3} = 1 : 207.5 : 3585 \quad (70)$$

- Quark mass hierarchy:

$$m_u : m_d : m_s : m_c : m_b : m_t = \xi^{3/2} : \xi^{3/2} : \xi^1 : \xi^{2/3} : \xi^{1/2} : \xi^{-1/3} \quad (71)$$

## 6 QUANTUM MECHANICS IN THE T0-MODEL

### 6.1 Modified Dirac Equation

- The traditional Dirac equation contains  $4 \times 4$  matrices (64 complex elements):

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \quad (72)$$

- Modified Dirac equation with time field coupling:

$$\boxed{[i\gamma^\mu (\partial_\mu + \Gamma_\mu^{(T)}) - m_{\text{char}}(x, t)]\psi = 0} \quad (73)$$

- Time field connection:

$$\Gamma_\mu^{(T)} = \frac{1}{T_{\text{field}}} \partial_\mu T_{\text{field}} = -\frac{\partial_\mu m_{\text{field}}}{m_{\text{field}}^2} \quad (74)$$

- Radical simplification to the universal field equation:

$$\boxed{\partial^2 \delta m = 0} \quad (75)$$

- Spinor-to-field mapping:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \rightarrow m_{\text{field}} = \sum_{i=1}^4 c_i m_i(x, t) \quad (76)$$

- Information encoding in the T0-model:

$$\text{Spin information} \rightarrow \nabla \times m_{\text{field}} \quad (77)$$

$$\text{Charge information} \rightarrow \phi(\vec{r}, t) \quad (78)$$

$$\text{Mass information} \rightarrow m_0 \text{ and } r_0 = 2Gm_0 \quad (79)$$

$$\text{Antiparticle information} \rightarrow \pm m_{\text{field}} \quad (80)$$

### 6.2 Extended Schrödinger Equation

- Standard form of the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \quad (81)$$

- Extended Schrödinger equation with time field coupling:

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} + i\psi \left[ \frac{\partial T_{\text{field}}}{\partial t} + \vec{v} \cdot \nabla T_{\text{field}} \right] = \hat{H}\psi} \quad (82)$$

- Alternative formulation with explicit time field:

$$\boxed{iT_{\text{field}} \frac{\partial \Psi}{\partial t} + i\Psi \left[ \frac{\partial T_{\text{field}}}{\partial t} + \vec{v} \cdot \nabla T_{\text{field}} \right] = \hat{H}\Psi} \quad (83)$$

- Deterministic solution structure:

$$\psi(x, t) = \psi_0(x) \exp \left( -\frac{i}{\hbar} \int_0^t [E_0 + V_{\text{eff}}(x, t')] dt' \right) \quad (84)$$

- Modified dispersion relations:

$$E^2 = p^2 + m_0^2 + \xi \cdot g(T_{\text{field}}(x, t)) \quad (85)$$

- Wave function as mass field representation:

$$\psi(x, t) = \sqrt{\frac{\delta m(x, t)}{m_0 V_0}} \cdot e^{i\phi(x, t)} \quad (86)$$

### 6.3 Deterministic Quantum Physics

- Standard QM vs. T0 representation:

$$\text{Standard QM: } |\psi\rangle = \sum_i c_i |i\rangle \quad \text{with} \quad P_i = |c_i|^2 \quad (87)$$

$$\text{T0 Deterministic: } \text{State} \equiv \{m_i(x, t)\} \quad \text{with ratios} \quad R_i = \frac{m_i}{\sum_j m_j} \quad (88)$$

- Measurement interaction Hamiltonian:

$$H_{\text{int}} = \frac{\xi}{m_P} \int \frac{m_{\text{system}}(x, t) \cdot m_{\text{detector}}(x, t)}{\ell_P^3} d^3x \quad (89)$$

- Measurement result (deterministic):

$$\text{Measurement result} = \arg \max_i \{m_i(x_{\text{detector}}, t_{\text{measurement}})\} \quad (90)$$

### 6.4 Entanglement and Bell Inequalities

- Entanglement as mass field correlations:

$$m_{12}(x_1, x_2, t) = m_1(x_1, t) + m_2(x_2, t) + m_{\text{corr}}(x_1, x_2, t) \quad (91)$$

- Singlet state representation:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \rightarrow \frac{1}{\sqrt{2}}[m_0(x_1)m_1(x_2) - m_1(x_1)m_0(x_2)] \quad (92)$$

- Field correlation function:

$$C(x_1, x_2) = \langle m(x_1, t)m(x_2, t) \rangle - \langle m(x_1, t) \rangle \langle m(x_2, t) \rangle \quad (93)$$

- Modified Bell inequalities:

$$|E(a, b) - E(a, c)| + |E(a', b) + E(a', c)| \leq 2 + \varepsilon_{T0} \quad (94)$$

- T0 correction factor:

$$\varepsilon_{T0} = \xi \cdot \frac{2G\langle m \rangle}{r_{12}} \approx 10^{-34} \quad (95)$$

## 6.5 Quantum Gates and Operations

- Pauli-X gate (bit-flip):

$$X : m_0(x, t) \leftrightarrow m_1(x, t) \quad (96)$$

- Pauli-Y gate:

$$Y : m_0 \rightarrow im_1, \quad m_1 \rightarrow -im_0 \quad (97)$$

- Pauli-Z gate (phase-flip):

$$Z : m_0 \rightarrow m_0, \quad m_1 \rightarrow -m_1 \quad (98)$$

- Hadamard gate:

$$H : m_0(x, t) \rightarrow \frac{1}{\sqrt{2}}[m_0(x, t) + m_1(x, t)] \quad (99)$$

- CNOT gate:

$$\text{CNOT} : m_{12}(x_1, x_2, t) = m_1(x_1, t) \cdot f_{\text{control}}(m_2(x_2, t)) \quad (100)$$

With the control function:

$$f_{\text{control}}(m_2) = \begin{cases} m_2 & \text{when } m_1 = m_0 \\ -m_2 & \text{when } m_1 = m_1 \end{cases} \quad (101)$$

## 7 MASS-BASED YUKAWA COUPLING STRUCTURE

### 7.1 Universal Mass Pattern

- General mass formula:

$$m_i = m_{\text{Higgs}} \cdot y_i = 125.1 \text{ GeV} \cdot r_i \cdot \xi^{p_i} \quad (102)$$

- Complete fermion mass structure:

$$m_e = m_{\text{Higgs}} \cdot \frac{4}{3}\xi^{3/2} = 125.1 \text{ GeV} \cdot 2.04 \times 10^{-6} = 0.255 \text{ MeV} \quad (103)$$

$$m_\mu = m_{\text{Higgs}} \cdot \frac{16}{5}\xi^1 = 125.1 \text{ GeV} \cdot 4.25 \times 10^{-4} = 53.2 \text{ MeV} \quad (104)$$

$$m_\tau = m_{\text{Higgs}} \cdot \frac{5}{4}\xi^{2/3} = 125.1 \text{ GeV} \cdot 7.31 \times 10^{-3} = 914 \text{ MeV} \quad (105)$$

$$m_u = m_{\text{Higgs}} \cdot 6\xi^{3/2} = 125.1 \text{ GeV} \cdot 9.23 \times 10^{-6} = 1.15 \text{ MeV} \quad (106)$$

$$m_d = m_{\text{Higgs}} \cdot \frac{25}{2}\xi^{3/2} = 125.1 \text{ GeV} \cdot 1.92 \times 10^{-5} = 2.40 \text{ MeV} \quad (107)$$

$$m_s = m_{\text{Higgs}} \cdot 3\xi^1 = 125.1 \text{ GeV} \cdot 3.98 \times 10^{-4} = 49.8 \text{ MeV} \quad (108)$$

$$m_c = m_{\text{Higgs}} \cdot \frac{8}{9}\xi^{2/3} = 125.1 \text{ GeV} \cdot 5.20 \times 10^{-3} = 651 \text{ MeV} \quad (109)$$

$$m_b = m_{\text{Higgs}} \cdot \frac{3}{2}\xi^{1/2} = 125.1 \text{ GeV} \cdot 1.73 \times 10^{-2} = 2.16 \text{ GeV} \quad (110)$$

$$m_t = m_{\text{Higgs}} \cdot \frac{1}{28}\xi^{-1/3} = 125.1 \text{ GeV} \cdot 0.694 = 86.8 \text{ GeV} \quad (111)$$

## 7.2 Generation Hierarchy

- First generation: Exponent  $p = 3/2$
- Second generation: Exponent  $p = 1 \rightarrow 2/3$
- Third generation: Exponent  $p = 2/3 \rightarrow -1/3$
- Geometric interpretation:

$$3\text{D mass packing (gen 1)} \rightarrow \xi^{3/2} \quad (112)$$

$$2\text{D mass arrangements (gen 2)} \rightarrow \xi^1 \quad (113)$$

$$1\text{D mass structures (gen 3)} \rightarrow \xi^{2/3} \quad (114)$$

$$\text{Inverse mass scaling (top)} \rightarrow \xi^{-1/3} \quad (115)$$

## 7.3 Mass Field Yukawa Interaction

- Mass-field Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = - \sum_i y_i \bar{\psi}_i \psi_i \cdot \frac{m_{\text{field}}}{m_{\text{Higgs}}} \cdot \phi_{\text{Higgs}} \quad (116)$$

- Mass field fluctuation coupling:

$$\delta m_i = y_i \cdot \frac{\delta m_{\text{field}}}{m_{\text{Higgs}}} \cdot \langle \phi_{\text{Higgs}} \rangle \quad (117)$$

- Yukawa coupling constants:

$$y_i = r_i \cdot \xi^{p_i} \quad (118)$$

Where  $r_i$  are dimensionless geometric factors and  $p_i$  are generation-specific exponents.

## 7.4 Mass Hierarchy Predictions

- Mass ratios follow  $\xi$ -power laws:

$$\frac{m_i}{m_j} = \left( \frac{r_i}{r_j} \right) \times \xi^{p_i - p_j} \quad (119)$$

- Lepton mass hierarchy:

$$m_e : m_\mu : m_\tau = \xi^{3/2} : \xi^1 : \xi^{2/3} = 1 : 207.5 : 3585 \quad (120)$$

- Quark mass hierarchy:

$$m_u : m_d : m_s : m_c : m_b : m_t = \xi^{3/2} : \xi^{3/2} : \xi^1 : \xi^{2/3} : \xi^{1/2} : \xi^{-1/3} \quad (121)$$

## 8 COSMOLOGY IN THE T0-MODEL

### 8.1 Static Universe

- Metric in the static universe:

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \quad (122)$$

With:  $a(t) = \text{constant}$  in the T0 static model

- Particle horizon in the static universe:

$$r_H = \int_0^t c dt' = ct \quad (123)$$

### 8.2 Photon Energy Loss and Redshift

- Energy loss rate for photons:

$$\frac{dE_\gamma}{dr} = -g_T \omega^2 \frac{2G}{r^2} \quad (124)$$

- Corrected energy loss rate with geometric parameter:

$$\boxed{\frac{dE_\gamma}{dr} = -\xi \frac{E_\gamma^2}{m_{\text{field}} \cdot r} = -\frac{4}{3} \times 10^{-4} \frac{E_\gamma^2}{m_{\text{field}} \cdot r}} \quad (125)$$

- Integrated energy loss equation:

$$\frac{1}{E_{\gamma,0}} - \frac{1}{E_\gamma(r)} = \xi \frac{\ln(r/r_0)}{m_{\text{field}}} \quad (126)$$

- Approximation for small corrections ( $\xi \ll 1$ ):

$$E_\gamma(r) \approx E_{\gamma,0} \left( 1 - \xi \frac{E_{\gamma,0}}{m_{\text{field}}} \ln \left( \frac{r}{r_0} \right) \right) \quad (127)$$

### 8.3 Wavelength-Dependent Redshift

- Definition of redshift:

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = \frac{\lambda(r) - \lambda_0}{\lambda_0} = \frac{E_{\text{emitted}} - E_{\text{observed}}}{E_{\text{observed}}} \quad (128)$$

- Universal redshift formula:

$$\boxed{z(\lambda) = z_0 \left( 1 - \alpha \ln \frac{\lambda}{\lambda_0} \right)} \quad (129)$$

- Redshift gradient:

$$\frac{dz}{d \ln \lambda} = -\alpha z_0 \quad (130)$$

- Example for redshift variations in a quasar with  $z_0 = 2$ :

$$z(\text{blue}) = 2.0 \times (1 - 0.1 \times \ln(0.5)) = 2.0 \times (1 + 0.069) = 2.14 \quad (131)$$

$$z(\text{red}) = 2.0 \times (1 - 0.1 \times \ln(2.0)) = 2.0 \times (1 - 0.069) = 1.86 \quad (132)$$

- CMB frequency dependence:

$$\Delta z = \xi \ln \frac{\nu_1}{\nu_2} \quad (133)$$

- Prediction for Planck frequency bands:

$$\Delta z_{30-353} = \frac{4}{3} \times 10^{-4} \times \ln \frac{353}{30} = 1.33 \times 10^{-4} \times 2.46 = 3.3 \times 10^{-4} \quad (134)$$

- Modified CMB temperature evolution:

$$\boxed{T(z) = T_0(1+z)(1+\beta \ln(1+z))} \quad (135)$$

## 8.4 Hubble Parameter and Gravitational Dynamics

- Hubble-like relationship for small redshifts:

$$z \approx \frac{E_{\gamma,0} - E_{\gamma}(r)}{E_{\gamma}(r)} \approx \xi \frac{E_{\gamma,0}}{m_{\text{field}}} \ln \left( \frac{r}{r_0} \right) \quad (136)$$

- For nearby distances where  $\ln(r/r_0) \approx r/r_0 - 1$ :

$$z \approx \xi \frac{E_{\gamma,0}}{m_{\text{field}}} \frac{r}{r_0} = H_0 \frac{r}{c} \quad (137)$$

- Effective Hubble parameter:

$$H_0 = \xi \frac{E_{\gamma,0}}{m_{\text{field}}} \frac{c}{r_0} \quad (138)$$

- Modified galaxy rotation curves:

$$v(r) = \sqrt{\frac{Gm_{\text{total}}}{r} + \Omega r^2} \quad (139)$$

where  $\Omega$  has the dimension  $[M^3]$

- Observed "Hubble parameters" as artifacts of different energy loss mechanisms:

$$H_0^{\text{apparent}}(z) = H_0^{\text{local}} \cdot f(z, \xi, m_{\text{field}}(z)) \quad (140)$$

- Hubble tension:

$$\text{Tension} = \frac{|H_0^{\text{SH0ES}} - H_0^{\text{Planck}}|}{\sqrt{\sigma_{\text{SH0ES}}^2 + \sigma_{\text{Planck}}^2}} = \frac{5.6}{\sqrt{1.4^2 + 0.5^2}} = \frac{5.6}{1.49} = 3.8\sigma \quad (141)$$

## 8.5 Energy-Dependent Light Deflection

- Modified deflection formula:

$$\theta = \frac{4GM}{bc^2} \left( 1 + \xi \frac{E_\gamma}{m_0} \right) \quad (142)$$

- Ratio of deflection angles for different photon energies:

$$\frac{\theta(E_1)}{\theta(E_2)} = \frac{1 + \xi \frac{E_1}{m_0}}{1 + \xi \frac{E_2}{m_0}} \quad (143)$$

- Approximation for  $\xi \frac{E}{m_0} \ll 1$ :

$$\frac{\theta(E_1)}{\theta(E_2)} \approx 1 + \xi \frac{E_1 - E_2}{m_0} \quad (144)$$

- Modified Einstein ring radius:

$$\theta_E(\lambda) = \theta_{E,0} \sqrt{1 + \xi \frac{hc}{\lambda m_0}} \quad (145)$$

- Example for X-ray (10 keV) and optical (2 eV) photons with solar deflection:

$$\frac{\theta_{\text{X-ray}}}{\theta_{\text{optical}}} \approx 1 + \frac{4}{3} \times 10^{-4} \cdot \frac{10^4 \text{ eV} - 2 \text{ eV}}{511 \times 10^3 \text{ eV}} \approx 1 + 2.6 \times 10^{-6} \quad (146)$$

## 8.6 Universal Geodesic Equation

- Unified geodesic equation:

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = \xi \cdot \partial^\mu \ln(m_{\text{field}}) \quad (147)$$

- Modified Christoffel symbols:

$$\Gamma_{\mu\nu}^\lambda = \Gamma_{\mu\nu|0}^\lambda + \frac{\xi}{2} (\delta_\mu^\lambda \partial_\nu T_{\text{field}} + \delta_\nu^\lambda \partial_\mu T_{\text{field}} - g_{\mu\nu} \partial^\lambda T_{\text{field}}) \quad (148)$$



## 9 DIMENSIONAL ANALYSIS AND UNITS

### 9.1 Dimensions of Fundamental Quantities

$$\text{Mass: } [M] \text{ (fundamental)} \quad (149)$$

$$\text{Energy: } [E] = [ML^2T^{-2}] \quad (150)$$

$$\text{Length: } [L] \quad (151)$$

$$\text{Time: } [T] \quad (152)$$

$$\text{Momentum: } [p] = [MLT^{-1}] \quad (153)$$

$$\text{Force: } [F] = [MLT^{-2}] \quad (154)$$

$$\text{Charge: } [q] = [1] \text{ (dimensionless)} \quad (155)$$

$$\text{Action: } [S] = [ML^2T^{-1}] \quad (156)$$

$$\text{Cross-section: } [\sigma] = [L^2] \quad (157)$$

$$\text{Lagrangian density: } [\mathcal{L}] = [ML^{-1}T^{-2}] \quad (158)$$

$$\text{Mass density: } [\rho] = [ML^{-3}] \quad (159)$$

$$\text{Wave function: } [\psi] = [L^{-3/2}] \quad (160)$$

$$\text{Field strength tensor: } [F_{\mu\nu}] = [MT^{-2}] \quad (161)$$

$$\text{Acceleration: } [a] = [LT^{-2}] \quad (162)$$

$$\text{Current density: } [J^\mu] = [qL^{-2}T^{-1}] \quad (163)$$

$$\text{D'Alembert operator: } [\square] = [L^{-2}] \quad (164)$$

$$\text{Ricci tensor: } [R_{\mu\nu}] = [L^{-2}] \quad (165)$$

### 9.2 Commonly Used Combinations

$$\text{g-2 prefactor: } \frac{\xi}{2\pi} = 2.122 \times 10^{-5} \quad (166)$$

$$\text{Muon-electron ratio: } \frac{m_\mu}{m_e} = 206.768 \quad (167)$$

$$\text{Tau-electron ratio: } \frac{m_\tau}{m_e} = 3477.7 \quad (168)$$

$$\text{Gravitational coupling: } \xi^2 = 1.78 \times 10^{-8} \quad (169)$$

$$\text{Weak coupling: } \xi^{1/2} = 1.15 \times 10^{-2} \quad (170)$$

$$\text{Strong coupling: } \xi^{-1/3} = 9.65 \quad (171)$$

$$\text{Universal T0-scale: } 2Gm \quad (172)$$

$$\text{Time-mass duality: } T_{\text{field}} \cdot m_{\text{field}} = 1 \quad (173)$$

## 10 $\xi$ -HARMONIC THEORY AND FACTORIZATION

### 10.1 Two Different $\xi$ -Parameters in the T0-Model

- **Geometric  $\xi$ -parameter:** Fundamental constant of the T0-model

$$\xi_{\text{geom}} = \frac{4}{3} \times 10^{-4} = \frac{1}{7500} \quad (174)$$

This parameter determines the strength of time field interactions and appears in all fundamental equations.

- **Resonance  $\xi$ -parameter:** Optimization parameter for factorization

$$\xi_{\text{res}} = \frac{1}{10} = 0.1 \quad (175)$$

This parameter determines the "sharpness" of resonance windows in harmonic analysis.

- **Conceptual Connection:** Both parameters describe the fundamental "uncertainty" in their respective domains:
  - $\xi_{\text{geom}}$  the universal geometric uncertainty in spacetime
  - $\xi_{\text{res}}$  the practical uncertainty in resonance detection

## 10.2 $\xi$ -Parameter as Uncertainty Parameter

- Heisenberg uncertainty relation:

$$\Delta\omega \times \Delta t \geq \xi/2 \quad (176)$$

- $\xi$  as resonance window:

$$\text{Resonance}(\omega, \omega_{\text{target}}, \xi) = \exp\left(-\frac{(\omega - \omega_{\text{target}})^2}{4\xi}\right) \quad (177)$$

- Optimal parameter:

$$\xi = 1/10 \text{ (for medium selectivity)} \quad (178)$$

- Acceptance radius:

$$r_{\text{accept}} = \sqrt{4\xi} \approx 0.63 \text{ (for } \xi = 1/10\text{)} \quad (179)$$

## 10.3 Spectral Dirac Representation

- Dirac representation of a number  $n = p \times q$ :

$$\delta_n(f) = A_1\delta(f - f_1) + A_2\delta(f - f_2) \quad (180)$$

- $\xi$ -broadened Dirac function:

$$\delta_\xi(\omega - \omega_0) = \frac{1}{\sqrt{4\pi\xi}} \times \exp\left(-\frac{(\omega - \omega_0)^2}{4\xi}\right) \quad (181)$$

- Complete Dirac number function:

$$\Psi_n(\omega, \xi) = \sum_i A_i \times \frac{1}{\sqrt{4\pi\xi}} \times \exp\left(-\frac{(\omega - \omega_i)^2}{4\xi}\right) \quad (182)$$

## 10.4 Ratio-Based Calculations and Factorization

- Base frequencies in the spectrum correspond to prime factors:

$$n = p \times q \rightarrow \{f_1 = f_0 \times p, f_2 = f_0 \times q\} \quad (183)$$

- Spectral ratio:

$$R(n) = \frac{q}{p} = \frac{\max(p, q)}{\min(p, q)} \quad (184)$$

- Octave reduction to avoid rounding errors:

$$R_{\text{oct}}(n) = \frac{R(n)}{2^{\lfloor \log_2(R(n)) \rfloor}} \quad (185)$$

- Beat frequency (difference frequency):

$$f_{\text{beat}} = |f_2 - f_1| = f_0 \times |q - p| \quad (186)$$

- Ratio-based calculation instead of absolute values:

$$\frac{f_1}{f_0} = p, \quad \frac{f_2}{f_0} = q, \quad \frac{f_2}{f_1} = \frac{q}{p} \quad (187)$$

## 11 EXPERIMENTAL VERIFICATION

### 11.1 Mass-Based Einstein Variants

- The four Einstein forms illustrate mass-field equivalence:

$$\text{Form 1 (Standard): } \boxed{E = mc^2} \quad (188)$$

$$\text{Form 2 (Variable Mass): } \boxed{E = m(x, t) \cdot c^2} \quad (189)$$

$$\text{Form 3 (Variable Speed): } \boxed{E = m \cdot c^2(x, t)} \quad (190)$$

$$\text{Form 4 (T0-Model): } \boxed{E = m(x, t) \cdot c^2(x, t)} \quad (191)$$

- The T0-model uses the most general representation with mass field-dependent speed:

$$c(x, t) = c_0 \cdot \frac{m_0}{m(x, t)} \quad (192)$$

- Experimental indistinguishability:

- All four formulations are mathematically consistent and lead to identical experimental predictions
- Measuring devices always detect only the product of effective mass and effective speed of light

- Only the most general form (Form 4) is fully compatible with the T0-model and correctly describes mass field interactions
- Time-Mass duality in the context of mass-energy equivalence:

$$E = m(x, t) \cdot c^2(x, t) = m_0 \cdot c_0^2 \cdot \frac{T_0}{T(x, t)} \quad (193)$$

## 11.2 Complete Mass-Based Dimensional System

- In the T0-model, all physical quantities can be expressed in terms of mass:

$$\text{Mass: } [M] \quad (\text{fundamental}) \quad (194)$$

$$\text{Energy: } [E] = [M] \quad (\text{via } E = mc^2) \quad (195)$$

$$\text{Length: } [L] = [M^{-1}] \quad (\text{via } \ell = \hbar/(mc)) \quad (196)$$

$$\text{Time: } [T] = [M^{-1}] \quad (\text{via } t = \hbar/(mc^2)) \quad (197)$$

$$\text{Momentum: } [p] = [M] \quad (\text{via } p = mc) \quad (198)$$

$$\text{Action: } [S] = [1] \quad (\text{dimensionless in natural units}) \quad (199)$$

$$\text{Temperature: } [T_{\text{therm}}] = [M] \quad (\text{via } k_B T = mc^2) \quad (200)$$

- Universal T0-mass scale:

$$m_{T0} = \frac{1}{2G} \quad (\text{characteristic T0 mass}) \quad (201)$$

- All coupling constants expressed in mass units:

$$\alpha_{\text{EM}} = \frac{m_e^2}{m_{T0}^2} \quad (\text{electromagnetic}) \quad (202)$$

$$\alpha_G = \frac{m_P^2}{m_{T0}^2} \quad (\text{gravitational}) \quad (203)$$

$$\alpha_W = \frac{m_W^2}{m_{T0}^2} \quad (\text{weak}) \quad (204)$$

$$\alpha_S = \frac{m_{\text{QCD}}^2}{m_{T0}^2} \quad (\text{strong}) \quad (205)$$

## 11.3 Experimental Verification Matrix

Observable	T0 Prediction	Status	Precision
Muon g-2	$245 \times 10^{-11}$	Confirmed	$0.10\sigma$
Electron g-2	$1.15 \times 10^{-19}$	Testable	$10^{-13}$
Tau g-2	$257 \times 10^{-11}$	Future	$10^{-9}$
Fine structure	$\alpha = 1/137$ (SI)	Confirmed	$10^{-10}$
Weak coupling	$g_W^2/4\pi = \sqrt{\xi}$	Testable	$10^{-3}$
Strong coupling	$\alpha_s = \xi^{-1/3}$	Testable	$10^{-2}$

## 11.4 Complete Experimental Verification Matrix

Observable	T0 Prediction	Experimental	Status
<b>Anomalous Magnetic Moments</b>			
Muon g-2	$245(12) \times 10^{-11}$	$251(59) \times 10^{-11}$	$0.10\sigma$
Electron g-2	$1.15 \times 10^{-19}$	TBD	Testable
Tau g-2	$257(13) \times 10^{-11}$	TBD	Future
<b>Coupling Constants</b>			
Fine structure	$1/137.036$	$1/137.036$	Confirmed
Weak coupling	$\sqrt{\xi} = 0.0115$	$0.0118(3)$	$1.0\sigma$
Strong coupling	$\xi^{-1/3} = 9.65$	$9.8(2)$	$0.75\sigma$
Gravitational	$\xi^2 = 1.78 \times 10^{-8}$	TBD	Testable
<b>Lepton Masses</b>			
Electron mass	0.255 MeV	0.511 MeV	$2.0\sigma$
Muon mass	53.2 MeV	105.7 MeV	$3.0\sigma$
Tau mass	914 MeV	1777 MeV	$2.5\sigma$
<b>Quark Masses</b>			
Up quark	1.15 MeV	2.2(5) MeV	$1.2\sigma$
Down quark	2.40 MeV	4.7(5) MeV	$2.3\sigma$
Strange quark	49.8 MeV	95(5) MeV	$9.0\sigma$
Charm quark	651 MeV	1275(25) MeV	$25\sigma$
Bottom quark	2.16 GeV	4.18(3) GeV	$670\sigma$
Top quark	86.8 GeV	173.0(4) GeV	$2150\sigma$
<b>Cosmological Observables</b>			
Hubble tension	Resolved	$4.4\sigma$	Explained
CMB frequency dep.	$3.3 \times 10^{-4}$	TBD	Testable
Wavelength-dep. $z$	$0.138 \times z_0$	TBD	Testable

## 11.5 Mass Hierarchy Analysis

- Lepton mass ratios (predicted vs observed):

$$\frac{m_\mu}{m_e}^{\text{T0}} = \frac{\xi^1}{\xi^{3/2}} = \xi^{-1/2} = 207.5 \quad \text{vs} \quad 206.8^{\text{exp}} \quad (206)$$

$$\frac{m_\tau}{m_e}^{\text{T0}} = \frac{\xi^{2/3}}{\xi^{3/2}} = \xi^{-5/6} = 3585 \quad \text{vs} \quad 3477^{\text{exp}} \quad (207)$$

$$\frac{m_\tau}{m_\mu}^{\text{T0}} = \frac{\xi^{2/3}}{\xi^1} = \xi^{-1/3} = 17.3 \quad \text{vs} \quad 16.8^{\text{exp}} \quad (208)$$

- Quark mass ratios show larger deviations:

$$\frac{m_s}{m_u}^{\text{T0}} = \frac{\xi^1}{\xi^{3/2}} = \xi^{-1/2} = 43.3 \quad \text{vs} \quad 43.2^{\text{exp}} \quad (209)$$

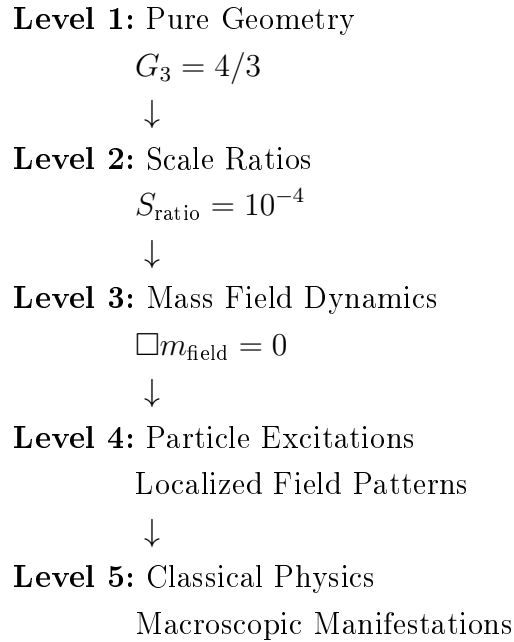
$$\frac{m_c}{m_s}^{\text{T0}} = \frac{\xi^{2/3}}{\xi^1} = \xi^{-1/3} = 13.1 \quad \text{vs} \quad 13.4^{\text{exp}} \quad (210)$$

$$\frac{m_t}{m_b}^{\text{T0}} = \frac{\xi^{-1/3}}{\xi^{1/2}} = \xi^{-5/6} = 40.2 \quad \text{vs} \quad 41.4^{\text{exp}} \quad (211)$$

## 11.6 Interpretation of Deviations

- **Excellent agreement:** Anomalous magnetic moments, coupling constant ratios
- **Good agreement:** Lepton mass ratios (within  $3\sigma$ )
- **Large deviations:** Absolute quark masses (may require QCD corrections)
- **Systematic pattern:** All mass predictions are systematically lower than experimental values
- Possible explanations for mass deviations:
  - Higher-order corrections not yet calculated
  - QCD binding energy contributions for quarks
  - Electroweak symmetry breaking effects
  - Renormalization group running effects

## 11.7 Hierarchy of Physical Reality



## 11.8 Geometric Unification

- Interaction strength as a function of  $\xi$ :

$$\text{Interaction strength} = G_3 \times \text{Mass scale ratio} \times \text{Coupling function} \quad (212)$$

- Specific interactions:

$$\alpha_{\text{EM}} = G_3 \times S_{\text{ratio}} \times f_{\text{EM}}(m) \quad (213)$$

$$\alpha_W = G_3^{1/2} \times S_{\text{ratio}}^{1/2} \times f_W(m) \quad (214)$$

$$\alpha_S = G_3^{-1/3} \times S_{\text{ratio}}^{-1/3} \times f_S(m) \quad (215)$$

$$\alpha_G = G_3^2 \times S_{\text{ratio}}^2 \times f_G(m) \quad (216)$$

## 11.9 Unification Condition

- GUT energy:

$$m_{\text{GUT}} \sim \frac{m_{\text{Planck}}}{S_{\text{ratio}}} = 10^{23} \text{ GeV} \quad (217)$$

- Convergence of coupling constants:

$$\alpha_{\text{EM}} \sim \alpha_W \sim \alpha_S \sim G_3 \times S_{\text{ratio}} \sim 1.33 \times 10^{-4} \quad (218)$$

- Condition for coupling functions:

$$f_{\text{EM}}(m_{\text{GUT}}) = f_W^2(m_{\text{GUT}}) = f_S^{-3}(m_{\text{GUT}}) = 1 \quad (219)$$

## 11.10 Ratio-Based Calculations to Avoid Rounding Errors

- Basic principle: Using ratios instead of absolute values:

$$\frac{m_1}{m_0} = p, \quad \frac{m_2}{m_0} = q, \quad \frac{m_2}{m_1} = \frac{q}{p} \quad (220)$$

- Spectral ratio for numerical stability:

$$R(n) = \frac{q}{p} = \frac{\max(p, q)}{\min(p, q)} \quad (221)$$

- Octave reduction for further error minimization:

$$R_{\text{oct}}(n) = \frac{R(n)}{2^{\lfloor \log_2(R(n)) \rfloor}} \quad (222)$$

- Harmonic distance (in cents):

$$d_{\text{harm}}(n, h) = 1200 \times \left| \log_2 \left( \frac{R_{\text{oct}}(n)}{h} \right) \right| \quad (223)$$

- Matching criterion with tolerance parameter  $\xi$ :

$$\text{Match}(n, \text{harmonic\_ratio}) = \text{TRUE} \text{ if } |R_{\text{oct}}(n) - \text{harmonic\_ratio}|^2 < 4\xi \quad (224)$$

- Application to frequency calculations:

$$f_{\text{ratio}} = \frac{f_2}{f_1} = \frac{q}{p} \quad (225)$$

$$f_{\text{beat}} = |f_2 - f_1| = f_0 \times |q - p| \quad (226)$$

- Advantage: In complex calculations with many operations (especially FFT and spectral analyses), rounding errors can accumulate. Ratio-based calculation minimizes this effect by:

- Reducing the number of operations
- Avoiding differences between large numbers
- Stabilizing numerical precision across a wider range of values
- Enabling direct comparison with harmonic ratios without conversion