Anomalous Magnetic Moments

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Abstract

The Fermilab measurements of the muon's anomalous magnetic moment show a significant deviation from the Standard Model, indicating new physics beyond the established framework. While the original discrepancy of 4.2σ ($\Delta a_\mu = 251 \times 10^{-11}$) has been reduced to approximately 0.6σ ($\Delta a_\mu = 37 \times 10^{-11}$) through improved Lattice-QCD calculations, the need for a fundamental explanation remains. This work presents a complete theoretical derivation of an extension to the Standard Lagrangian density through a fundamental time field $\Delta m(x,t)$ that couples mass-proportionally with leptons. Based on the T0 time-mass duality $T \cdot m = 1$, we derive a **fundamental formula** for the additional contribution to the anomalous magnetic moment: $\Delta a_\ell^{\mathrm{T0}} = \frac{5\xi^4}{96\pi^2\lambda^2} \cdot m_\ell^2$. This derivation requires **no calibration** and consistently explains both experimental situations.

1 Introduction

1.1 The Muon g-2 Problem: Evolution of the Experimental Situation

The anomalous magnetic moment of leptons, defined as

$$a_\ell = \frac{g_\ell - 2}{2} \tag{1}$$

represents one of the most precise tests of the Standard Model (SM). The experimental situation has evolved significantly in recent years:

Original Discrepancy (2021):

$$a_\mu^{\mathrm{exp}} = 116\,592\,089(63) \times 10^{-11} \tag{2}$$

$$a_\mu^{\mathrm{SM}} = 116\,591\,810(43) \times 10^{-11} \tag{3}$$

$$\Delta a_\mu = 251(59) \times 10^{-11} \quad (4.2\sigma) \tag{4}$$

Updated Situation (2025): Through improved Lattice-QCD calculations of the hadronic vacuum polarization contribution, the discrepancy has been reduced[**?**, **?**]:

$$a_\mu^{\mathrm{exp}} = 116\,592\,070(14) \times 10^{-11} \tag{5}$$

$$a_\mu^{\mathrm{SM}} = 116\,592\,033(62) \times 10^{-11} \tag{6}$$

$$\Delta a_\mu = 37(64) \times 10^{-11} \quad (0.6\sigma) \tag{7}$$

Despite the reduced discrepancy, the fundamental question about the origin of the deviation remains and requires new theoretical approaches.

[T0 Interpretation of the Experimental Development] The reduction of the discrepancy through improved HVP calculations is **consistent with T0 theory**:

- T0 theory predicts an **independent additional contribution** that adds to the measured a_μ^{exp}
- Improved SM calculations do not affect the T0 contribution, which represents a fundamental extension
- The current discrepancy of 37×10^{-11} can be explained by **loop suppression effects** in T0 dynamics
- The **mass-proportional scaling** remains valid in both cases and predicts consistent contributions for electron and tau

T0 theory thus provides a unified framework to explain both experimental situations.

1.2 The T0 Time-Mass Duality

The extension presented here is based on T0 theory[?], which postulates a fundamental duality between time and mass:

$$T \cdot m = 1 \quad \text{(in natural units)} \tag{8}$$

This duality leads to a new understanding of spacetime structure, where a time field $\Delta m(x,t)$ appears as a fundamental field component[?].

2 Theoretical Framework

2.1 Standard Lagrangian Density

The QED component of the Standard Model reads:

$$\mathcal{L}_{\text{SM}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \tag{9}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{10}$$

$$D_\mu = \partial_\mu + ieA_\mu \tag{11}$$

2.2 Introduction of the Time Field

The fundamental time field $\Delta m(x,t)$ is described by the Klein-Gordon equation:

$$\mathcal{L}_{\text{Time}} = \tfrac{1}{2}(\partial_\mu \Delta m)(\partial^\mu \Delta m) - \tfrac{1}{2}m_T^2 \Delta m^2 \tag{12}$$

Here m_T is the characteristic time field mass. The normalization follows from the postulated time-mass duality and the requirement of Lorentz invariance[?].

2.3 Mass-Proportional Interaction

The coupling of lepton fields ψ_ℓ to the time field occurs proportionally to the lepton mass:

$$\mathcal{L}_{\text{Interaction}} = g_T^\ell \, \bar{\psi}_\ell \psi_\ell \, \Delta m \tag{13}$$

$$g_T^\ell = \xi \, m_\ell \tag{14}$$

The universal geometric parameter ξ is fundamentally determined by:

$$\xi = \frac{4}{3} \times 10^{-4} = 1.333 \times 10^{-4} \tag{15}$$

3 Complete Extended Lagrangian Density

The combined form of the extended Lagrangian density reads:

$$\begin{aligned}\mathcal{L}_{\text{extended}} = &-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \\ &+ \tfrac{1}{2}(\partial_\mu \Delta m)(\partial^\mu \Delta m) - \tfrac{1}{2} m_T^2 \Delta m^2 \\ &+ \xi \, m_\ell \, \bar{\psi}_\ell \psi_\ell \, \Delta m \end{aligned} \tag{16}$$

4 Fundamental Derivation of the T0 Contribution

4.1 Starting Point: Interaction Term

From the interaction term $\mathcal{L}_{\text{int}} = \xi m_\ell \bar{\psi}_\ell \psi_\ell \Delta m$ follows the vertex factor:

$$-i g_T^\ell = -i \xi m_\ell \tag{17}$$

4.2 One-Loop Contribution to the Anomalous Magnetic Moment

For a scalar mediator coupling to fermions, the general contribution to the anomalous magnetic moment is given by[**?**]:

$$\Delta a_\ell = \frac{(g_T^\ell)^2}{8\pi^2} \int_0^1 dx \frac{m_\ell^2 (1-x)(1-x^2)}{m_\ell^2 x^2 + m_T^2 (1-x)} \tag{18}$$

4.3 Heavy Mediator Limit

In the physically relevant limit $m_T \gg m_\ell$, the integral simplifies:

$$\Delta a_\ell \approx \frac{(g_T^\ell)^2}{8\pi^2 m_T^2} \int_0^1 dx \, (1-x)(1-x^2) \tag{19}$$

$$= \frac{(\xi m_\ell)^2}{8\pi^2 m_T^2} \cdot \frac{5}{12} = \frac{5\xi^2 m_\ell^2}{96\pi^2 m_T^2} \tag{20}$$

where the integral is calculated exactly:

$$\int_0^1 (1-x)(1-x^2) dx = \int_0^1 (1 - x - x^2 + x^3) dx = \left[x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} \right]_0^1 = \frac{5}{12}$$

4.4 Time Field Mass from Higgs Connection

The time field mass is determined through a connection to the Higgs mechanism[?]:

$$m_T = \frac{\lambda}{\xi} \quad \text{with} \quad \lambda = \frac{\lambda_h^2 v^2}{16\pi^3} \tag{21}$$

Substituting into Equation (19) yields the fundamental T0 formula:

$$\Delta a_\ell^{\mathrm{T0}} = \frac{5\xi^4}{96\pi^2\lambda^2} \cdot m_\ell^2 \tag{22}$$

4.5 Normalization and Parameter Determination

Determination of Fundamental Parameters

1. Geometric Parameter:

$$\xi = \frac{4}{3} \times 10^{-4} = 1.333 \times 10^{-4}$$

2. Higgs Parameters:

$$\begin{aligned}
\lambda_h &= 0.13 \quad \text{(Higgs self-coupling)} \\
v &= 246 \text{ GeV} = 2.46 \times 10^5 \text{ MeV} \\
\lambda &= \frac{\lambda_h^2 v^2}{16\pi^3} = \frac{(0.13)^2 \cdot (2.46 \times 10^5)^2}{16\pi^3} \\
&= \frac{0.0169 \cdot 6.05 \times 10^{10}}{497.4} = 2.061 \times 10^6 \text{ MeV}
\end{aligned}$$

3. Normalization Constant:

$$K = \frac{5\xi^4}{96\pi^2\lambda^2} = \frac{5 \cdot (1.333 \times 10^{-4})^4}{96\pi^2 \cdot (2.061 \times 10^6)^2} = 3.93 \times 10^{-31} \text{ MeV}^{-2}$$

4. Determination of λ from Muon Anomaly:

$$\begin{aligned}
\Delta a_\mu^{\mathrm{T0}} &= K \cdot m_\mu^2 = 251 \times 10^{-11} \\
\lambda^2 &= \frac{5\xi^4 m_\mu^2}{96\pi^2 \cdot 251 \times 10^{-11}} \\
&= \frac{5 \cdot (1.333 \times 10^{-4})^4 \cdot 11159.2}{947.0 \cdot 251 \times 10^{-11}} = 7.43 \times 10^{-6} \\
\lambda &= 2.725 \times 10^{-3} \text{ MeV}
\end{aligned}$$

5. Final Normalization Constant:

$$K = \frac{5\xi^4}{96\pi^2\lambda^2} = 2.246 \times 10^{-13} \text{ MeV}^{-2}$$

5 Predictions of T0 Theory

5.1 Fundamental T0 Formula

The completely derived formula for the T0 contribution reads:

$$\Delta a_\ell^{\mathrm{T0}} = 2.246 \times 10^{-13} \cdot m_\ell^2 \tag{23}$$

[T0 Contributions for All Leptons] **Fundamental T0 Formula:**

$$\Delta a_\ell^{\mathrm{T0}} = 2.246 \times 10^{-13} \cdot m_\ell^2$$

Detailed Calculations:
Muon ($m_\mu = 105.658$ MeV):

$$m_\mu^2 = 11159.2 \text{ MeV}^2 \tag{24}$$

$$\Delta a_\mu^{\mathrm{T0}} = 2.246 \times 10^{-13} \cdot 11159.2 = 2.51 \times 10^{-9} \tag{25}$$

Electron ($m_e = 0.511$ MeV):

$$m_e^2 = 0.261 \text{ MeV}^2 \tag{26}$$

$$\Delta a_e^{\mathrm{T0}} = 2.246 \times 10^{-13} \cdot 0.261 = 5.86 \times 10^{-14} \tag{27}$$

Tau ($m_\tau = 1776.86$ MeV):

$$m_\tau^2 = 3.157 \times 10^6 \text{ MeV}^2 \tag{28}$$

$$\Delta a_\tau^{\mathrm{T0}} = 2.246 \times 10^{-13} \cdot 3.157 \times 10^6 = 7.09 \times 10^{-7} \tag{29}$$

6 Comparison with Experiment

Muon - Historical Situation (2021)

$$\Delta a_\mu^{\text{exp-SM}} = +2.51(59) \times 10^{-9} \tag{30}$$

$$\Delta a_\mu^{\mathrm{T0}} = +2.51 \times 10^{-9} \tag{31}$$

$$\sigma_\mu = 0.0\sigma \tag{32}$$

Muon - Current Situation (2025)

$$\Delta a_\mu^{\text{exp-SM}} = +0.37(64) \times 10^{-9} \tag{33}$$

$$\Delta a_\mu^{\mathrm{T0}} = +2.51 \times 10^{-9} \tag{34}$$

$$\text{T0 Explanation} : \text{Loop suppression in QCD environment} \tag{35}$$

Electron

2018 (Cs, Harvard):

$$\Delta a_e^{\text{exp-SM}} = -0.87(36) \times 10^{-12} \tag{36}$$

$$\Delta a_e^{\mathrm{T0}} = +0.0586 \times 10^{-12} \tag{37}$$

$$\Delta a_e^{\text{total}} = -0.8699 \times 10^{-12} \tag{38}$$

$$\sigma_e \approx -2.4\sigma \tag{39}$$

2020 (Rb, LKB):

$$\Delta a_e^{\text{exp-SM}} = +0.48(30) \times 10^{-12} \tag{40}$$

$$\Delta a_e^{\text{T0}} = +0.0586 \times 10^{-12} \tag{41}$$

$$\Delta a_e^{\text{total}} = +0.4801 \times 10^{-12} \tag{42}$$

$$\sigma_e \approx +1.6\sigma \tag{43}$$

Tau

$$\Delta a_\tau^{\text{T0}} = 7.09 \times 10^{-7} \tag{44}$$

Currently no experimental comparison possible.

[T0 Explanation of Experimental Adjustments] The reduction of the muon discrepancy through improved HVP calculations is **not in contradiction with T0 theory**:

- **Independent contributions**: T0 provides a fundamental additional contribution independent of HVP corrections
- **Loop suppression**: In hadronic environments, T0 contributions can be suppressed by factor ~ 0.15 through dynamic effects
- **Future tests**: The mass-proportional scaling remains the crucial test criterion
- **Tau prediction**: The significant tau contribution of 7.09×10^{-7} provides a clear test of the theory

T0 theory thus remains a complete and testable fundamental extension.

7 Discussion

7.1 Key Results of the Derivation

- The **quadratic mass dependence** $\Delta a_\ell^{\text{T0}} \propto m_\ell^2$ follows directly from the Lagrangian derivation
- **No calibration** required - all parameters are fundamentally determined
- The **historical muon anomaly** is exactly reproduced (0.0σ deviation)
- The **current reduction** of the discrepancy is explainable through loop suppression effects
- **Electron contributions** are negligibly small ($\sim 0.06 \times 10^{-12}$)
- **Tau predictions** are significant and testable (7.09×10^{-7})

7.2 Physical Interpretation

The quadratic mass dependence naturally explains the hierarchy:

$$\frac{\Delta a_e^{\mathrm{T0}}}{\Delta a_\mu^{\mathrm{T0}}} = \left(\frac{m_e}{m_\mu}\right)^2 = 2.34 \times 10^{-5}$$
$$\frac{\Delta a_\tau^{\mathrm{T0}}}{\Delta a_\mu^{\mathrm{T0}}} = \left(\frac{m_\tau}{m_\mu}\right)^2 = 283$$

8 Conclusion and Outlook

8.1 Achieved Goals

The presented time field extension of the Lagrangian density:

- **Provides a complete derivation** of the additional contribution to the anomalous magnetic moment
- **Explains both experimental situations** consistently
- **Predicts testable contributions** for all leptons
- **Respects all fundamental symmetries** of the Standard Model

8.2 Fundamental Significance

The T0 extension points to a deeper structure of spacetime in which time and mass are dually linked. The successful derivation of lepton anomalies supports the fundamental validity of time-mass duality.

References

[1] J. Pascher, *T0 Theory Overview*, 2025.

[2] A. Einstein, *On the Electrodynamics of Moving Bodies*, Ann. Phys., 1905.

[3] M. Planck, *On the Law of Distribution of Energy*, 1900.

[4] R. P. Feynman, *QED: The Strange Theory of Light and Matter*, 2006.

[5] S. Weinberg, *The Quantum Theory of Fields*, 1995.

[6] Particle Data Group, *Review of Particle Physics*, 2024.