

# Unit Conventions and the Speed of Light $c$

$E=mc^2$  vs.  $E=m$ : Two Equivalent Perspectives

Natural Units, SI Units, and the T0 Viewpoint

Johann Pascher

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## Abstract

This document examines when one can set  $c=1$  (natural units) and when the full form  $E=mc^2$  with  $c=299,792,458$  m/s (SI units) is required. Parallel to the treatment of the fine-structure constant  $\alpha$  in Document 101, we show: Both perspectives are mathematically equivalent and differ only in the choice of unit system. The T0 theory reveals that  $c$  is not a fundamental law of nature but a dynamic ratio  $L/T$ . From the T0 perspective,  $c=1$  can be set (Planck units, particle physics), while for technical applications and precision measurements, SI units with explicit  $c$  are required. The equivalence  $E=mc^2 \leftrightarrow E=m$  holds exactly in natural units. References: Documents 013 (SI system), 014 (nat./SI), 015 (systematics), 077 ( $E=mc^2$  analysis), 101 ( $\alpha$  conventions).

## Contents

# 1 Introduction: The Question of $c=1$

## The Central Question

The question "When can one set  $c=1$ ?" is analogous to the question "When can one set  $\alpha=1$ ?" addressed in Document 101. In both cases, it concerns **unit conventions**, not fundamental physics.

### Central Thesis

**$E=mc^2$  and  $E=m$  are mathematically identical!**

- In SI units:  $E = mc^2$  with  $c = 299,792,458$  m/s
- In natural units:  $E = m$  with  $c = 1$

Both forms describe the same physics – only the unit choice differs.

## Historical Context

Einstein wrote the famous formula in 1905:

$$E = mc^2 \quad (1)$$

This form was necessary because he worked in **SI units**, where length (meter), time (second), and mass (kilogram) are independent dimensions.

**Modern particle physics** uses instead:

$$E = m \quad (\text{in natural units with } c = \hbar = 1) \quad (2)$$

# 2 Natural Units: When $c=1$ is Valid

## Definition of Natural Units

In natural units, one sets:

$$c = 1, \quad \hbar = 1, \quad (\text{optional: } k_B = 1) \quad (3)$$

**Mathematical meaning:**

$$c = 1 \quad \Rightarrow \quad \text{Length} \equiv \text{Time} \quad (4)$$

$$\hbar = 1 \quad \Rightarrow \quad \text{Energy} \equiv \text{inverse Time} \quad (5)$$

## Application Domains

Natural units are appropriate in:

- **Planck scale:** Quantum gravity, fundamental theory
  - **Particle physics:** High-energy physics, QFT, Standard Model
  - **Cosmology:** Early universe, inflationary models
  - **Theoretical work:** Mathematical derivations, symmetries
- Advantage:** Formulas become simpler, physical relationships clearer.

## Mathematical Consistency

In natural units:

$$E^2 = p^2 + m^2 \quad (6)$$

In the rest frame ( $p = 0$ ):

$$E = m \quad (7)$$

This is exact – **not an approximation**.

## T0 Perspective: c as a Ratio

The T0 theory shows (see Document 077):

$$c = \frac{L}{T} \quad (8)$$

**c is not a fundamental law of nature but a *ratio*!**

With the T0 fundamental relation:

$$T \cdot m = 1 \quad (\text{Time-Mass Duality}) \quad (9)$$

it follows that c is a dynamic ratio that varies with mass scale.

**Implication:** In Planck units, where  $t_P = \ell_P/c$ ,  $c=1$  is the natural choice.

## 3 SI Units: When c=299,792,458 m/s is Required

### The SI Definition (since 2019)

The modern SI system defines since 2019:

$$c = 299,792,458 \text{ m/s (exact)} \quad (10)$$

This choice is a **convention** that defines the meter via the second.

## Application Domains

SI units with explicit  $c$  are required in:

- **Engineering:** GPS, telecommunications, laser technology
- **Precision measurements:** Atomic clocks, interferometry, metrology
- **Experimental physics:** Laboratory measurements with SI-calibrated devices
- **Applied physics:** Energy calculations, dosimetry
- **Public & Education:** Comprehensibility, historical continuity

**Advantage:** Practical calculability with calibrated measurement devices.

## Mathematical Form

In SI units:

$$E = \gamma mc^2 \quad (11)$$

with the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad (12)$$

In the rest frame ( $v = 0, \gamma = 1$ ):

$$E = mc^2 \quad (13)$$

## Conversion Between Unit Systems

From natural units to SI:

$$E_{\text{nat}} = m_{\text{nat}} \quad (14)$$

$$\Downarrow \quad (\text{Multiply by } c^2) \quad (15)$$

$$E_{\text{SI}} = m_{\text{SI}} \cdot c^2 \quad (16)$$

**Example:** Electron mass

$$m_e = 0.511 \text{ MeV} \quad (\text{natural units}) \quad (17)$$

$$m_e = 9.109 \times 10^{-31} \text{ kg} \quad (\text{SI}) \quad (18)$$

$$E_e = m_e c^2 = 0.511 \text{ MeV} = 8.187 \times 10^{-14} \text{ J} \quad (19)$$

Convention	Fine-structure constant $\alpha$	Speed of light $c$
<b>Natural</b>	$\alpha = 1$ (Heaviside-Lorentz)	$c = 1$ (Planck units)
<b>SI / Standard</b>	$\alpha = 1/137.036$ (Gauss-SI)	$c = 299,792,458$ m/s
<b>Document</b>	101 (Circularity-Constants)	134 (Unit Conventions c)

**Table 1:** Parallel structure:  $\alpha$  and  $c$  as conventions

## 4 Comparison with $\alpha$ : Parallel Structure

### Two Analogous Conventions

#### Common Principles

Both cases show:

- **Physics is invariant** under unit choice
- **Natural units** simplify theoretical work
- **SI units** enable practical applications
- **T0 theory:** Both are derived conventions, not fundamental

#### T0 Reduction

From the T0 perspective (see Document 101):

$$\xi \rightarrow D_f \rightarrow E_0 \rightarrow \alpha \rightarrow \hbar, c, G \rightarrow \text{all other constants} \quad (20)$$

**Only  $\xi = \frac{4}{3} \times 10^{-4}$  is fundamental.**

Both  $\alpha$  and  $c$  are derived quantities or conventions.

## 5 When to Use Which System?

### Decision Matrix

#### Recommendations

**Use natural units ( $c=1$ ) when:**

- Performing theoretical derivations
- Symmetries and invariant structures are important
- Formulas should be simplified
- Working in particle physics or cosmology

**Use SI units ( $c$  explicit) when:**

Context	Natural units ( $c=1$ )	SI units ( $c$ explicit)
Theoretical physics	✓	
Quantum field theory	✓	
High-energy physics	✓	
Early cosmology	✓	
Experimental physics		✓
Engineering		✓
Precision measurements		✓
Applied physics		✓
Education		✓

**Table 2:** Application domains of unit systems

- Planning or evaluating experimental measurements
- Technical calculations are required
- Results should be understandable for non-physicists
- Historical continuity is important

## 6 Common Misconceptions

### “ $c=1$ is only an approximation”

**FALSE.**  $c=1$  is **exact** in natural units, not an approximation.

It is a choice of unit system that defines:

$$\text{Length unit} = \text{Time unit} \quad (21)$$

Analogously: In Planck units,  $\hbar = 1$  is exact, not approximate.

### “ $E=m$ only holds for photons”

**FALSE.** In natural units,  $E = m$  holds for **all** particles in their rest frame.

For photons ( $m = 0$ ):  $E = p$  (in natural units) or  $E = pc$  (in SI).

### “ $c$ is a fundamental constant of nature”

**T0 viewpoint:**  $c$  is a **ratio**  $L/T$ , not a fundamental constant.

With the T0 duality  $T \cdot m = 1$ ,  $c$  varies dynamically with mass scale:

$$c = \frac{L}{T} = L \cdot m \quad (22)$$

Only in SI units is  $c$  *fixed by definition*.

## “Natural units change the physics”

**FALSE.** Physics is independent of the unit system.

All **dimensionless** quantities (e.g.,  $\xi$ ,  $\alpha$ , mass ratios) are invariant.

Only dimensional quantities change their numerical values.

## 7 T0 Perspective: c as a Dynamic Ratio

### The T0 Fundamental Relation

From Document 077:

$$T \cdot m = 1 \quad (\text{Time-Mass Duality}) \quad (23)$$

This means:

$$T \propto \frac{1}{m} \quad (24)$$

$$L \propto \frac{1}{m} \quad (\text{via Compton wavelength}) \quad (25)$$

$$\Rightarrow c = \frac{L}{T} \propto \frac{1/m}{1/m} = \text{scale-dependent} \quad (26)$$

### Implications

#### 1. c is not universally constant in the T0 framework:

Different effective c-values can occur at different mass scales.

#### 2. SI definition c=299,792,458 m/s is a calibration:

This fixation defines the meter via the second – a metrological convention.

#### 3. Natural units c=1 are T0-consistent:

In Planck units, where  $t_P \propto \ell_P$ , c=1 is the natural choice.

### Comparison with Document 077

Document 077 argues: “E=mc<sup>2</sup> = E=m – The Constant Illusion Exposed”

**Clarification here:**

- E=mc<sup>2</sup> (SI) and E=m (natural) are *equivalent*, not identical
- The difference lies in the *unit system*, not in physics
- Einstein’s c-fixation is a *convention*, not an error
- T0 shows: c is a ratio that can vary depending on scale

## 8 Mathematical Consistency

### Energy-Momentum Relation

In natural units ( $c = 1$ ):

$$E^2 = p^2 + m^2 \quad (27)$$

In SI units:

$$E^2 = (pc)^2 + (mc^2)^2 \quad (28)$$

Both forms are mathematically equivalent.

### Lorentz Transformation

In natural units:

$$E' = \gamma(E - p \cdot v) \quad (29)$$

In SI units:

$$E' = \gamma(E - p \cdot v \cdot c^2) \quad (30)$$

The physics remains invariant.

### Klein-Gordon Equation

In natural units:

$$(\partial_\mu \partial^\mu + m^2)\phi = 0 \quad (31)$$

In SI units:

$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2 c^2}{\hbar^2} \right) \phi = 0 \quad (32)$$

Identical physics, different notation.

## 9 References to T0 Documents

### Related Documents

- **Document 013:** SI System and T0 Theory
- **Document 014:** Natural vs. SI Units
- **Document 015:** Systematics of Natural Units

- **Document 077:**  $E=mc^2 = E=m$  Analysis
- **Document 101:** Circularity of Constants ( $\alpha$  Conventions)
- **Document 133:** Fractal Correction  $K_{\text{frak}}$  Derivation

## Derivation Hierarchy

The T0 hierarchy (from Document 101):

$$\xi \rightarrow D_f \rightarrow E_0 \rightarrow \alpha \rightarrow \hbar, c, G \rightarrow \text{mass ratios} \quad (33)$$

shows that both  $\alpha$  and  $c$  are derived quantities.