# Dirac Equation in T0 Theory: Geometric Integration with Time-Mass Duality

Fractal Spacetime and Dynamic Mass

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**Zusammenfassung**

This work fully integrates the Dirac equation into the T0 theory framework. Unlike the standard formulation with constant mass, T0 theory uses the fundamental time-mass duality $T(x) \cdot m(x) = 1$, leading to a spacetime-dependent mass. The fractal dimension $D_f = 3 - \xi$ modifies the underlying metric and thus the differential operator. We show how the Clifford algebra structure naturally connects with the torus topology of T0 theory and how spin-1/2 can be interpreted as a topological winding number. The predictions are formulated as ratio-based statements that are independent of unit systems and phenomenological parameters. Experimental tests at Belle II can directly verify the fundamental quadratic mass scaling.


# Inhaltsverzeichnis

## 1 Introduction: T0 Basic Principles

### Time-Mass Duality

The fundamental principle of T0 theory is time-mass duality:

$$T(x,t) \cdot m(x,t) = \frac{\hbar}{c^2} \tag{1}$$

In natural units ($\hbar = c = 1$):

$$T(x,t) \cdot m(x,t) = 1 \tag{2}$$

This means: **Mass is not constant but a dynamic field**, coupled to the intrinsic time field $T(x,t)$.

### Fractal Spacetime

T0 theory postulates a fractal spacetime dimension:

$$D_f = 3 - \xi \quad \text{with} \quad \xi = \frac{4}{3 \times 10^4} \approx 1.333 \times 10^{-4} \tag{3}$$

This modifies the metric and thus all differential operators.

### Torus Topology

The underlying topology is a torus with characteristic scales:

- Large radius: $R \sim 1/\xi$
- Small radius: $r \sim R \cdot \xi$
- Winding numbers: $(n_\theta, n_\phi)$ for poloidal and toroidal directions

## 2 Standard Dirac Equation: Problems

### The Standard Form

The usual Dirac equation reads:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \tag{4}$$

with constant mass $m$ and flat Minkowski metric.

### Problems for T0 Integration

1. **Constant mass:** Contradicts time-mass duality
2. **Flat metric:** Ignores the fractal structure
3. **No topology:** Spin has no geometric origin
4. **Static:** No coupling to time field

## 3 Clifford Algebra: The Fundamental Structure

Before we develop the T0-specific formulation, we need to understand what the Dirac equation **really** is – beyond the 4×4 matrices.

## Representation vs. Physics

**The central insight:** The 4×4 matrices are not the physics, but a **specific representation** of the physics.

**Wichtig**

Fundamental Difference **Fundamental (Physics):**
The Clifford algebra structure of spacetime
**Representation (Calculation):**
Specific 4×4 matrices $\gamma^\mu$ in a chosen basis

**Analogy:** Vectors are fundamental; their components depend on the chosen basis. The physics (vector) is basis-independent, the calculation (components) is not.

**Example – different representations:**
The same Dirac equation can be written with:

- **Dirac representation:** Specific 4×4 matrices
- **Weyl representation:** Different 4×4 matrices
- **Majorana representation:** Yet different matrices

All describe **the same physics**! The choice is convention, like choosing a coordinate basis.

## The Abstract Clifford Form

The fundamental form of the Dirac equation without explicit matrices is:

$$\boxed{(i\mathrm{e}_\mu\partial^\mu - m)\Psi = 0} \tag{5}$$

where:

- $\mathrm{e}_\mu$: **Abstract basis vectors** of spacetime (not matrices!)
- $\Psi$: Element in the **spin bundle** (geometric object)
- The **Clifford product rule**:

$$\mathrm{e}_\mu\mathrm{e}_\nu + \mathrm{e}_\nu\mathrm{e}_\mu = 2g_{\mu\nu} \tag{6}$$

**What does the Clifford product mean?**
The product $\mathrm{e}_\mu\mathrm{e}_\nu$ is **non-commutative**:

$$\mathrm{e}_0\mathrm{e}_1 \neq \mathrm{e}_1\mathrm{e}_0 \tag{7}$$

$$\mathrm{e}_0\mathrm{e}_1 + \mathrm{e}_1\mathrm{e}_0 = 0 \quad \text{(since } g_{01} = 0\text{)} \tag{8}$$

This encodes the **geometric structure of spacetime**.

## What Are the $\gamma$-Matrices Really?

The familiar $\gamma^\mu$ matrices are simply:

$$\gamma^\mu \quad \longleftrightarrow \quad \text{Matrix representation of } \mathrm{e}^\mu \tag{9}$$

**Concretely:** One chooses a basis in spin space and writes:

$$\mathrm{e}^\mu \quad \to \quad \gamma^\mu = \begin{pmatrix} \gamma^\mu_{11} & \gamma^\mu_{12} & \gamma^\mu_{13} & \gamma^\mu_{14} \\ \gamma^\mu_{21} & \gamma^\mu_{22} & \gamma^\mu_{23} & \gamma^\mu_{24} \\ \gamma^\mu_{31} & \gamma^\mu_{32} & \gamma^\mu_{33} & \gamma^\mu_{34} \\ \gamma^\mu_{41} & \gamma^\mu_{42} & \gamma^\mu_{43} & \gamma^\mu_{44} \end{pmatrix} \tag{10}$$

The specific numbers in the matrix depend on the chosen representation!
**The physics** (Clifford product rule (**??**)) is independent of this choice.

## Spin as Topological Property

The spin-1/2 character is not a property of the matrices but follows from the Clifford algebra structure.

### The 720° Rotation

**Key observation:** A spinor $\Psi$ behaves under rotations as:

$$R(180^\circ)\Psi = e^{i\pi/2}\Psi = i\Psi \tag{11}$$

$$R(360^\circ)\Psi = e^{i\pi}\Psi = -\Psi \tag{12}$$

$$R(720^\circ)\Psi = e^{i2\pi}\Psi = \Psi \tag{13}$$

This is **not a matrix property**, but follows from Clifford algebra!
**Why?** The rotation is given by:

$$R(\theta) = \exp\left(\frac{i\theta}{2}\mathrm{e}_1\mathrm{e}_2\right) \tag{14}$$

The factor $1/2$ in the exponent is **geometric** (comes from the Clifford algebra structure), not from the matrices!

### Topological Interpretation

In T0 theory, we can interpret spin geometrically as a **winding number on a torus**:

$$\text{Spin-}s \quad \longleftrightarrow \quad \text{Winding } (n_\theta, n_\phi) \text{ with } \frac{n_\phi}{n_\theta} = 2s \tag{15}$$

**For spin-1/2:** $(n_\theta, n_\phi) = (1, 1)$ or $(2, 1)$
The 720° rotation then corresponds to:

- Once around the poloidal circle → $-\Psi$ (360°)
- Twice around the poloidal circle → $+\Psi$ (720°)

This is **pure topology**, not a mysterious quantum property!

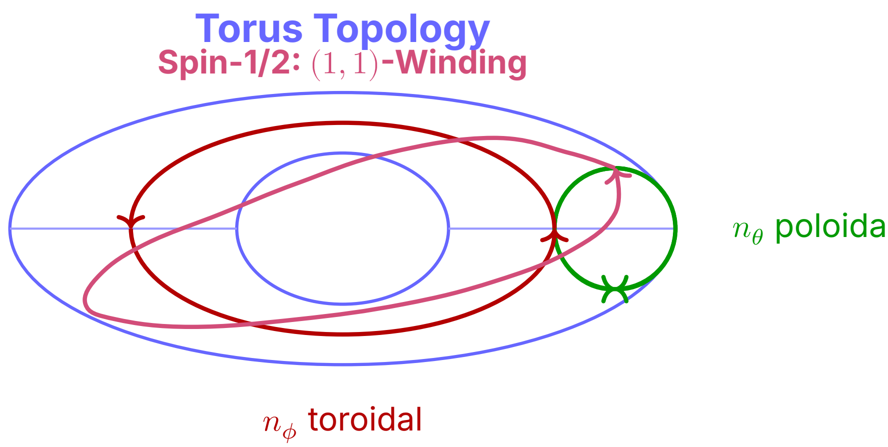


**Abbildung 1:** Spin-1/2 as topological winding on a torus (top view). The green double arrow shows the poloidal small circle ($n_\theta$, cross-section of the torus tube). The red arrows show the toroidal direction ($n_\phi$, around the central hole). The violet path shows a $(1, 1)$-winding: once around the small circle AND once around the large circle. A 720° rotation corresponds to traversing this winding twice.

## Common Misconceptions

### Can the Matrices Really Be Eliminated?

**Answer: Yes and No.**

- **Yes – fundamentally:** The physics does not need specific 4×4 matrices. The Clifford algebra is fundamental.
- **No – practically:** For concrete calculations, a representation is necessary, and matrices are often the most practical choice.

**Analogy:** One can formulate vector physics without coordinates (fundamental), but for calculations one chooses coordinates (practical).

**Is Information Lost?**

**No!** The Clifford algebra formulation contains **exactly the same information**:

| Property | In Matrices | In Clifford Algebra |
|---|---|---|
| Spin-1/2 | In $\gamma$-structure | In Clifford product rule |
| Lorentz inv. | Explicit in matrices | In $g_{\mu\nu}$-structure |
| Antiparticles | Negative energy solutions | Chirality components |
| Measurables | Matrix elements | Invariant under representation |

**Tabelle 1:** Information identical in both formulations

**Is This Just a Reformulation?**

**No – it is a conceptual shift:**

- **Old view:** "Electrons are point particles with mysterious intrinsic spin, described by complicated 4×4 matrices"
- **New view:** "Electrons are geometric objects in a Clifford-structured spacetime. Spin is a topological property."

This new view enables **natural integration** into T0 theory:

- Fractal metric $\rightarrow$ modified Clifford structure
- Torus topology $\rightarrow$ spin as winding number
- Time-mass duality $\rightarrow$ dynamic mass $m(x)$

## Preparation for T0 Integration

With this understanding, we can now introduce the T0-specific modifications:

1. **Fractal metric:** $g_{\mu\nu} \rightarrow g_{\mu\nu}^{\text{(frak)}}$ with $D_f = 3 - \xi$
2. **Modified Clifford rule:**

$$\mathrm{e}_\mu^{\text{(frak)}} \mathrm{e}_\nu^{\text{(frak)}} + \mathrm{e}_\nu^{\text{(frak)}} \mathrm{e}_\mu^{\text{(frak)}} = 2 g_{\mu\nu}^{\text{(frak)}} \tag{16}$$

3. **Dynamic mass:** $m \rightarrow m(x) = 1/(c^2 T(x))$
4. **Tetrad formulation:** Necessary for curved/fractal spacetime

In the next section, we develop this T0-specific formulation in detail.

**Core Message of This Chapter**

The Dirac equation is fundamentally a **geometric equation** in the Clifford algebra of spacetime. The 4×4 matrices are useful calculation tools, but not the physics itself. This insight is **essential** for integration into T0 theory with its fractal geometry and torus topology.

# 4 T0 Dirac Equation: Geometric Form

## Clifford Algebra in Fractal Spacetime

Instead of the standard form, we use the Clifford algebra formulation:

$$\boxed{(i\partial\!\!\!/_{\text{frak}} - m(x))\Psi(x) = 0} \tag{17}$$

where:

$$\partial\!\!\!/_{\text{frak}} = \mathrm{e}_a^\mu(x)\gamma^a\partial_\mu \quad \text{(tetrad-based)} \tag{18}$$

$$m(x) = \frac{1}{c^2T(x)} \quad \text{(from time-mass duality)} \tag{19}$$

$$\mathrm{e}_a^\mu(x) = \text{Tetrad in fractal metric} \tag{20}$$

## Fractal Metric

The fractal correction to the metric is:

$$g_{\mu\nu}^{(\text{frak})}(x) = \eta_{\mu\nu} \cdot (1 + \xi \cdot f(x)) \tag{21}$$

where $f(x)$ is a dimensionless function of coordinates describing the fractal structure.

## Tetrad Formulation

The tetrad $\mathrm{e}_a^\mu(x)$ connects the curved spacetime with the local Clifford algebra:

$$g_{\mu\nu}^{(\text{frak})}(x) = \mathrm{e}_a^\mu(x)\mathrm{e}_b^\nu(x)\eta^{ab} \tag{22}$$

The $\gamma^a$ are the standard Clifford generators in the local Lorentz frame.

# 5 Dynamic Mass

## Spacetime Dependence

From time-mass duality follows:

$$m(x,t) = \frac{1}{c^2 T(x,t)} = \frac{1}{c^2}\,\mathsf{max}(\omega(x,t), m_{\mathsf{bg}}(x)) \tag{23}$$

where:

- $\omega(x,t)$: Local frequency/energy density
- $m_{\mathsf{bg}}(x)$: Background mass field

## Coupling to Time Field

The time field $T(x,t)$ is itself a dynamic field with Lagrangian density:

$$\mathcal{L}_T = \frac{1}{2}(\partial_\mu T)(\partial^\mu T) - V(T) \tag{24}$$

The coupling to fermions occurs through the mass:

$$\mathcal{L}_{\mathsf{int}} = \bar{\Psi} m(T(x))\Psi \tag{25}$$

# 6 Spin as Topology

## Winding Numbers on the Torus

In T0 theory, spin is interpreted as a winding number:

$$\text{Spin-}s \quad \longleftrightarrow \quad \text{Winding } (n_\theta, n_\phi) \text{ with } n_\phi/n_\theta = 2s \tag{26}$$

**Examples:**

$$\text{Spin-}0: \quad (1,0) \text{ or } (0,1) \tag{27}$$
$$\text{Spin-}1/2: \quad (1,1) \text{ or } (2,1) \tag{28}$$
$$\text{Spin-}1: \quad (1,2) \tag{29}$$

## 720° Rotation Geometrically

The well-known property of spin-1/2 particles (720° rotation for identity) follows from torus topology:

- One poloidal winding: 360° rotation → $-\Psi$
- Two poloidal windings: 720° rotation → $+\Psi$

This is not a mysterious property but **pure topology**.

# 7 Mass-Proportional Coupling

## Interaction Lagrangian

The coupling of leptons to the time field is mass-proportional:

$$\mathcal{L}_{\text{int}} = \xi m_\ell \bar{\Psi}_\ell \Psi_\ell \Delta m(x) \tag{30}$$

where $\Delta m(x) = m(x) - m_0$ is the mass fluctuation.

## Consequence: Quadratic Scaling

From this mass-proportional coupling follows for loop diagrams:

$$\Delta a_\ell \propto (\xi m_\ell)^2 \cdot \text{(kinematic factors)} \propto m_\ell^2 \tag{31}$$

This leads to the fundamental ratio prediction:

$$\boxed{\frac{\Delta a_{\ell_1}}{\Delta a_{\ell_2}} = \left(\frac{m_{\ell_1}}{m_{\ell_2}}\right)^2} \tag{32}$$

# 8 Ratios vs. Absolute Values

## What the T0 Dirac Equation Predicts

**Fundamental predictions (parameter-free):**

- Ratio: $a_\tau / a_\mu = (m_\tau / m_\mu)^2 \approx 283$
- Structure: $\Delta a \propto m^2$ (quadratic scaling)
- Topology: Spin-1/2 as winding number

**Not predictable (phenomenological):**

- Absolute values: $a_\mu = 37.5 \times 10^{-11}$ (requires normalization)

## Why Only Ratios?

Complete calculation of absolute values requires:

1. Solution of time field dynamics in fractal spacetime (too complex)
2. Loop integrals in non-integer dimension (open)
3. Renormalization at $D_f = 3 - \xi$ (not fully developed)
4. Recursive coupling of all fields (non-perturbative)

This is analogous to QCD in the Standard Model: Fundamental Lagrangian density is clear, but hadronic contributions are not calculable ab initio.

# 9 Natural vs. SI Units

## In Natural Units

In natural units ($\hbar = c = 1$, $\alpha = 1$) $\alpha$ disappears from all formulas:

$$\tilde{a}_\ell = \tilde{C} \cdot \xi \cdot \tilde{m}_\ell^2 \tag{33}$$

The ratio is:

$$\frac{\tilde{a}_\tau}{\tilde{a}_\mu} = \left(\frac{\tilde{m}_\tau}{\tilde{m}_\mu}\right)^2 \tag{34}$$

**Identical to SI version** – ratios are invariant!

## Transformation to SI

The transformation to SI units introduces $\alpha$:

$$a_\ell[\text{SI}] = \text{(conversion factor with } \alpha\text{)} \times \tilde{a}_\ell \tag{35}$$

But the **ratio remains unchanged**:

$$\frac{a_\tau[\text{SI}]}{a_\mu[\text{SI}]} = \frac{\tilde{a}_\tau}{\tilde{a}_\mu} = \left(\frac{m_\tau}{m_\mu}\right)^2 \tag{36}$$

# 10 Experimental Tests

## Belle II: Critical Test (2027-2028)

The fundamental prediction:

$$\frac{a_\tau}{a_\mu} = \left(\frac{1776.86}{105.658}\right)^2 = 282.8 \tag{37}$$

is directly testable at Belle II.
**Possible outcomes:**

- **Confirmation:** Strong evidence for mass-proportional coupling
- **Deviation:** Modification of coupling structure needed
- **Null result:** T0 contributions suppressed or incorrect

| Test | T0 Prediction | Status |
|---|---|---|
| $a_\tau/a_\mu$ | $(m_\tau/m_\mu)^2 = 283$ | Belle II 2027-28 |
| $m_\tau/m_\mu$ | $\approx 16.8$ (from torus) | Confirmed ✓ |
| Spin-statistics | From topology | Confirmed ✓ |
| Fractal damping | $\propto e^{-\xi n^2}$ | Rydberg atoms |

**Tabelle 2:** Experimental tests of the T0 Dirac formulation

### Further Tests

## 11 Comparison with Standard Formulation

| Aspect | Standard Dirac | T0 Dirac |
|---|---|---|
| Mass | Constant $m$ | Dynamic $m(x,t)$ |
| Metric | Minkowski $\eta_{\mu\nu}$ | Fractal $g_{\mu\nu}^{\text{(frak)}}$ |
| Spin | Matrix property | Topological winding |
| Dimension | $D = 4$ | $D_f = 3 - \xi$ in space |
| Topology | None | Torus $(n_\theta, n_\phi)$ |
| Coupling | Ad-hoc | Time-mass duality |
| Predictions | Qualitative | Testable ratios |

**Tabelle 3:** Standard vs. T0 Dirac formulation

## 12 Limits and Open Questions

### What Works

- ✓ Clifford algebra structure clearly defined
- ✓ Spin interpretable as topology
- ✓ Ratio predictions parameter-free
- ✓ Belle II test possible

### Honesty About Limits

As in the Standard Model (hadronic contributions), there are areas where the fundamental theory is clear but explicit calculations are too complex. This is **not a fault of the theory**, but a realistic assessment of mathematical challenges.

# References and Further Reading

# Literatur

[1] J. Pascher, *The T0 Foundation: Time-Mass Duality and Fractal Geometry*, T0-Time-Mass-Duality Repository, 2026.

[2] J. Pascher, *The Xi Narrative: From a Single Number to the Fine-Structure Constant*, 145_FFGFT_donat-teil1_En.pdf, 2025.

[3] D. Hestenes, *Space-Time Algebra*, Gordon and Breach, 1966. Provides the mathematical foundation for geometric Clifford algebra formulations.

[4] P. Lounesto, *Clifford Algebras and Spinors*, Cambridge University Press, 2001. Comprehensive treatment of Clifford algebras with applications to spinors.

[5] P. A. M. Dirac, *The Quantum Theory of the Electron*, Proc. R. Soc. Lond. A, 117, 610–624, 1928. The original paper introducing the Dirac equation.

[6] J. Williamson and M. B. van der Mark, *Is the Electron a Photon with Toroidal Topology?*, Annales de la Fondation Louis de Broglie, 22, 133–167, 1997. [PDF]

[7] Belle II Collaboration, *Prospects for Measuring the Anomalous Magnetic Moment of the Tau Lepton at Belle II*, Belle II Note 0123, 2024. [Belle II Website]

[8] Muon g-2 Collaboration, *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm*, Phys. Rev. Lett. 131, 161802, 2023. Latest results from Fermilab.

[9] M. Nakahara, *Geometry, Topology and Physics*, IOP Publishing, 2003. Excellent resource for tetrad formalism and differential geometry in physics.

[10] K. Falconer, *Fractal Geometry: Mathematical Foundations and Applications*, Wiley, 2014. Standard reference for fractal geometry and Hausdorff dimensions.

[11] J. Pascher, *Derivation of Time-Mass Duality from Planck Relations*, T0_xi_ursprung.pdf, 2025.

[12] J. Pascher, *Dirac Equation in T0 Theory: Geometric Clifford-Algebra Formulation*, Document 050_dirac_geometric, 2026.