Unit Conventions and the Speed of Light c

E=mc² vs. E=m: Two Equivalent Perspectives

Natural Units, SI Units, and the T0 Viewpoint

Johann Pascher

January 2025

Abstract

This document examines when one can set c=1 (natural units) and when the full form $E=mc^2$ with c=299,792,458 m/s (SI units) is required. Parallel to the treatment of the fine-structure constant α in Document 101, we show: Both perspectives are mathematically equivalent and differ only in the choice of unit system. The T0 theory reveals that c is not a fundamental law of nature but a dynamic ratio L/T. From the T0 perspective, c=1 can be set (Planck units, particle physics), while for technical applications and precision measurements, SI units with explicit c are required. The equivalence $E=mc^2 \leftrightarrow E=m$ holds exactly in natural units. References: Documents 013 (SI system), 014 (nat./SI), 015 (systematics), 077 ($E=mc^2$ analysis), 101 (α conventions).

Contents

1 Introduction: The Question of c=1

1.1 The Central Question

The question "When can one set c=1?" is analogous to the question "When can one set α=1?" addressed in Document 101. In both cases, it concerns **unit conventions**, not fundamental physics.

Central Thesis

E=mc² and E=m are mathematically identical!

- In SI units: $E = mc^2$ with $c = 299,792,458$ m/s
- In natural units: $E = m$ with $c = 1$

Both forms describe the same physics – only the unit choice differs.

1.2 Historical Context

Einstein wrote the famous formula in 1905:

$$E = mc^2 \tag{1}$$

This form was necessary because he worked in **SI units**, where length (meter), time (second), and mass (kilogram) are independent dimensions.

Modern particle physics uses instead:

$$E = m \quad \text{(in natural units with } c = \hbar = 1\text{)} \tag{2}$$

2 Natural Units: When c=1 is Valid

2.1 Definition of Natural Units

In natural units, one sets:

$$c = 1, \quad \hbar = 1, \quad \text{(optional: } k_B = 1\text{)} \tag{3}$$

Mathematical meaning:

$$c = 1 \quad \Rightarrow \quad \text{Length} \equiv \text{Time} \tag{4}$$

$$\hbar = 1 \quad \Rightarrow \quad \text{Energy} \equiv \text{inverse Time} \tag{5}$$

2.2 Application Domains

Natural units are appropriate in:

- **Planck scale**: Quantum gravity, fundamental theory
- **Particle physics**: High-energy physics, QFT, Standard Model
- **Cosmology**: Early universe, inflationary models
- **Theoretical work**: Mathematical derivations, symmetries

Advantage: Formulas become simpler, physical relationships clearer.

2.3 Mathematical Consistency

In natural units:

$$E^2 = p^2 + m^2 \tag{6}$$

In the rest frame ($p = 0$):

$$E = m \tag{7}$$

This is exact – **not an approximation**.

2.4 T0 Perspective: c as a Ratio

The T0 theory shows (see Document 077):

$$c = \frac{L}{T} \tag{8}$$

c is not a fundamental law of nature but a *ratio*!
With the T0 fundamental relation:

$$T \cdot m = 1 \quad \text{(Time-Mass Duality)} \tag{9}$$

it follows that c is a dynamic ratio that varies with mass scale.
Implication: In Planck units, where $t_P = \ell_P/c$, c=1 is the natural choice.

3 SI Units: When c=299,792,458 m/s is Required

3.1 The SI Definition (since 2019)

The modern SI system defines since 2019:

$$\boxed{c = 299,792,458 \text{ m/s (exact)}} \tag{10}$$

This choice is a **convention** that defines the meter via the second.

3.2 Application Domains

SI units with explicit c are required in:

- **Engineering**: GPS, telecommunications, laser technology
- **Precision measurements**: Atomic clocks, interferometry, metrology
- **Experimental physics**: Laboratory measurements with SI-calibrated devices
- **Applied physics**: Energy calculations, dosimetry
- **Public & Education**: Comprehensibility, historical continuity

Advantage: Practical calculability with calibrated measurement devices.

3.3 Mathematical Form

In SI units:

$$E = \gamma mc^2 \tag{11}$$

with the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{12}$$

In the rest frame ($v = 0$, $\gamma = 1$):

$$E = mc^2 \tag{13}$$

3.4 Conversion Between Unit Systems

From natural units to SI:

$$E_{\text{nat}} = m_{\text{nat}} \tag{14}$$

$$\Downarrow \quad \text{(Multiply by } c^2\text{)} \tag{15}$$

$$E_{\text{SI}} = m_{\text{SI}} \cdot c^2 \tag{16}$$

Example: Electron mass

$$m_e = 0.511 \text{ MeV} \quad \text{(natural units)} \tag{17}$$

$$m_e = 9.109 \times 10^{-31} \text{ kg} \quad \text{(SI)} \tag{18}$$

$$E_e = m_e c^2 = 0.511 \text{ MeV} = 8.187 \times 10^{-14} \text{ J} \tag{19}$$

Convention	Fine-structure constant α	Speed of light c
Natural	$\alpha = 1$ (Heaviside-Lorentz)	$c = 1$ (Planck units)
SI / Standard	$\alpha = 1/137.036$ (Gauss-SI)	$c = 299,792,458$ m/s
Document	101 (Circularity-Constants)	134 (Unit Conventions c)

Table 1: Parallel structure: α and c as conventions

4 Comparison with α: Parallel Structure

4.1 Two Analogous Conventions

4.2 Common Principles

Both cases show:

- **Physics is invariant** under unit choice
- **Natural units** simplify theoretical work
- **SI units** enable practical applications
- **T0 theory**: Both are derived conventions, not fundamental

4.3 T0 Reduction

From the T0 perspective (see Document 101):

$$\xi \to D_f \to E_0 \to \alpha \to \hbar, c, G \to \text{all other constants} \tag{20}$$

Only $\xi = \frac{4}{3} \times 10^{-4}$ is fundamental.
Both α and c are derived quantities or conventions.

5 When to Use Which System?

5.1 Decision Matrix

5.2 Recommendations

Use natural units (c=1) when:

- Performing theoretical derivations
- Symmetries and invariant structures are important
- Formulas should be simplified
- Working in particle physics or cosmology

Use SI units (c explicit) when:

Context	Natural units (c=1)	SI units (c explicit)
Theoretical physics	✓	
Quantum field theory	✓	
High-energy physics	✓	
Early cosmology	✓	
Experimental physics		✓
Engineering		✓
Precision measurements		✓
Applied physics		✓
Education		✓

Table 2: Application domains of unit systems

- Planning or evaluating experimental measurements
- Technical calculations are required
- Results should be understandable for non-physicists
- Historical continuity is important

6 Common Misconceptions

6.1 "c=1 is only an approximation"

FALSE. c=1 is **exact** in natural units, not an approximation.
It is a choice of unit system that defines:

$$\text{Length unit} = \text{Time unit} \tag{21}$$

Analogously: In Planck units, $\hbar = 1$ is exact, not approximate.

6.2 "E=m only holds for photons"

FALSE. In natural units, $E = m$ holds for **all** particles in their rest frame.
For photons ($m = 0$): $E = p$ (in natural units) or $E = pc$ (in SI).

6.3 "c is a fundamental constant of nature"

T0 viewpoint: c is a **ratio** L/T, not a fundamental constant.
With the T0 duality $T \cdot m = 1$, c varies dynamically with mass scale:

$$c = \frac{L}{T} = L \cdot m \tag{22}$$

Only in SI units is c *fixed by definition*.

6.4 "Natural units change the physics"

FALSE. Physics is independent of the unit system.
All **dimensionless** quantities (e.g., ξ, α, mass ratios) are invariant.
Only dimensional quantities change their numerical values.

7 T0 Perspective: c as a Dynamic Ratio

7.1 The T0 Fundamental Relation

From Document 077:

$$T \cdot m = 1 \quad \text{(Time-Mass Duality)} \tag{23}$$

This means:

$$T \propto \frac{1}{m} \tag{24}$$

$$L \propto \frac{1}{m} \quad \text{(via Compton wavelength)} \tag{25}$$

$$\Rightarrow \quad c = \frac{L}{T} \propto \frac{1/m}{1/m} = \text{scale-dependent} \tag{26}$$

7.2 Implications

1. c is not universally constant in the T0 framework:
Different effective c-values can occur at different mass scales.
2. SI definition c=299,792,458 m/s is a calibration:
This fixation defines the meter via the second – a metrological convention.
3. Natural units c=1 are T0-consistent:
In Planck units, where $t_P \propto \ell_P$, c=1 is the natural choice.

7.3 Comparison with Document 077

Document 077 argues: "E=mc² = E=m – The Constant Illusion Exposed"
Clarification here:

- E=mc² (SI) and E=m (natural) are *equivalent*, not identical
- The difference lies in the *unit system*, not in physics
- Einstein's c-fixation is a *convention*, not an error
- T0 shows: c is a ratio that can vary depending on scale

8 Mathematical Consistency

8.1 Energy-Momentum Relation

In natural units ($c = 1$):

$$E^2 = p^2 + m^2 \tag{27}$$

In SI units:

$$E^2 = (pc)^2 + (mc^2)^2 \tag{28}$$

Both forms are mathematically equivalent.

8.2 Lorentz Transformation

In natural units:

$$E' = \gamma(E - p \cdot v) \tag{29}$$

In SI units:

$$E' = \gamma(E - p \cdot v \cdot c^2) \tag{30}$$

The physics remains invariant.

8.3 Klein-Gordon Equation

In natural units:

$$(\partial_\mu \partial^\mu + m^2)\phi = 0 \tag{31}$$

In SI units:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\phi = 0 \tag{32}$$

Identical physics, different notation.

9 References to T0 Documents

9.1 Related Documents

- **Document 013**: SI System and T0 Theory
- **Document 014**: Natural vs. SI Units
- **Document 015**: Systematics of Natural Units

- **Document 077**: E=mc² = E=m Analysis
- **Document 101**: Circularity of Constants (α Conventions)
- **Document 133**: Fractal Correction K_frak Derivation

9.2 Derivation Hierarchy

The T0 hierarchy (from Document 101):

$$\xi \to D_f \to E_0 \to \alpha \to \hbar, c, G \to \text{mass ratios} \tag{33}$$

shows that both α and c are derived quantities.