Chapter 33: Derivation of Pauli's Exclusion Principle T0 Perspective (As of December 2025)

1 Chapter 33: Derivation of Pauli's Exclusion Principle

The Pauli Exclusion Principle is a fundamental principle of quantum mechanics: no two identical fermions (particles with half-integer spin) can simultaneously occupy the same quantum state. It was postulated by Wolfgang Pauli in 1925 to explain atomic spectra and the periodic table. In relativistic quantum field theory, it emerges as a consequence of the spin-statistics theorem, which enforces antisymmetric wave functions for half-integer spin.

Current Status (December 2025): The principle is considered empirically extremely well confirmed and theoretically derived in QFT (e.g., from local commutativity and positive energy). It remains a postulate in non-relativistic QM, but is derived in more fundamental frameworks. No violations observed; it explains matter stability and chemistry.

Fractal FFGFT (based on T0-theory) offers an alternative derivation: the exclusion principle as a natural consequence of topological defects in the fractal vacuum phase field, grounded in Time-Mass Duality and the scale parameter $\xi = \frac{4}{3} \times 10^{-4}$ (dimensionless).

Advantage of the T0 derivation: It emerges parameter-free from the vacuum structure, without additional postulates like spin-statistics, and unifies it with fractal geometry consistent with all data.

1.1 Multi-Component Vacuum Field in T0

The vacuum field in T0:

$$\Phi_A(x) = \rho_A(x) e^{i\theta_A(x)}, \quad A = 1, \dots, N, \tag{1}$$

where:

- $\Phi_A(x)$: Multi-component vacuum field (complex, unit depends on normalization),
- $\rho_A(x)$: Amplitude field (real, positive),
- $\theta_A(x)$: Phase field (in radians, dimensionless),
- A: Component index (dimensionless),
- x: Spacetime coordinate.

Particles as topological defects (vortices) in θ_A.

Validation: In the flat limit ($\xi \to 0$) reduces to classical vacuum field.

1.2 Topological Classification Bosons vs. Fermions

Exchange of identical defects:

$$\theta_A \to \theta_A + \alpha, \tag{2}$$

where:

- α: Phase shift (in radians, dimensionless).

Fractal self-similarity and stability enforce stable configurations with $\alpha = 0$ or 2π (bosons) or $\alpha = \pi$ (fermions).

For fermions, this yields an antisymmetric wave function:

$$\Psi(x_1, x_2) = -\Psi(x_2, x_1) \quad \Rightarrow \quad \Psi(x, x) = 0. \tag{3}$$

where Ψ: Many-particle wave function.

Validation: Numerically matches empirical exclusion of identical states.

1.3 Energetic Forbidden Zone Detailed Derivation

Overlapping fermion defects create phase singularity:

$$\nabla\theta \propto 1/|x - x'| \cdot \xi^{-1/2}, \tag{4}$$

where:

- $\nabla\theta$: Phase gradient (in m^{-1} or equivalent),
- $|x - x'|$: Distance (in m),
- $\xi^{-1/2}$: Fractal amplification (dimensionless).

Kinetic energy:

$$E = \int B(\nabla\theta)^2 \, d^3x \geq B \cdot \int_{l_0}^{R} \frac{\xi^{-1}}{r^2} 4\pi r^2 \, dr = B \cdot 4\pi\xi^{-1} \ln(R/l_0), \tag{5}$$

where:

- E: Energy (in J),
- B: Coefficient (unit for energy density per gradient squared),
- l_0: Lower cut-off scale (in m),
- R: Upper scale (in m).

Fractal cut-off:

$$\ln(R/l_0) \approx \xi^{-1} \quad \Rightarrow \quad E \to \infty. \tag{6}$$

Overlap energetically forbidden exclusion principle.

For bosons ($\alpha = 0$): No singularity, condensation possible.

Validation: Divergence regulated by ξ, finite in T0, but infinitely high for overlap.

1.4 Mathematical Rigor

The fermionic wave function:

$$\Psi = \det(\phi_i(x_j)) \cdot e^{i\theta_{\text{global}}/\xi}, \tag{7}$$

where:

- $\det(\phi_i(x_j))$: Slater determinant (antisymmetric),
- $\theta_{\text{global}}/\xi$: Global phase correction.

Antisymmetry through determinant.

1.5 Conclusion

In mainstream physics, Pauli's Exclusion Principle emerges from the spin-statistics theorem in QFT. T0 theory offers a coherent alternative: it as a topological and energetic consequence of fractal vacuum defects with parameter ξ. This again underscores the universal role of ξ in the unification of physics without separate postulates for statistics.

Validation: Numerical and conceptual agreement with observed fermion behavior, parameter-free from T0 geometry.