Fundamental Constants and Their Derivation from Natural Units

Johann Pascher

03.25.2025

Contents

1 Extrapolation of Physics Beyond Known Boundaries

1.1 Physics Beyond the Speed of Light

The speed of light c is considered an absolute speed limit for matter and signal transmission in standard physics. This is a direct consequence of the Lorentz transformation and relativity theory. Within this framework, all fundamental constants and the Planck scale have been defined.

It is important to understand, however, that this limit may only apply within our current theoretical framework. Beyond the speed of light, completely new physical laws might apply:

- **Tachyonic Fields:** Hypothetical particles (tachyons) with velocities $v > c$ would exhibit an imaginary rest mass. In a system with absolute time (T_0 model), this could be interpreted as a real mass variation.
- **Modified Transformation Laws:** Instead of the Lorentz transformation, extended transformations might apply that allow superluminal velocities without causing causality violations.
- **Extended Constants:** Known constants such as α, $\hbar$, and G would need to be redefined or replaced by extended versions that are also valid in the superluminal range.

A possible approach would be to modify the energy-momentum relationship:

$$E^2 = (mc^2)^2 + (pc)^2 + \alpha_c p^4 c^2 / E_P^2 \tag{1}$$

where α_c is a dimensionless parameter and E_P is the Planck energy. At low energies, this reduces to the known relation, while at high energies near the Planck scale, deviations could occur.

1.2 Consequences for Causality and Information

The possibility of physics beyond the speed of light would have profound consequences for our understanding of causality and information transmission:

- **Reformulation of Cause and Effect:** In a T_0 model with absolute time, causality could be defined through energy states and not through event sequences.
- **Non-local Information Transfer:** While in the standard model seemingly non-local quantum correlations do not allow information transfer, an extended model could provide mechanisms for superluminal information transfer without violating fundamental causality.
- **Extended Light Cones:** The classical light cone as a boundary of causal connections would need to be replaced by an extended concept that possibly includes mass-dependent or energy-dependent causality boundaries.

1.3 Possible Experimental Indications

Although physics beyond the speed of light is speculative, subtle experimental indications might point to a necessary extension of our physical framework:

- **Anomalies in High-Energy Experiments:** Deviations from expected dispersion relations at very high energies.
- **Cosmological Observations:** Unexpected correlations in the cosmic microwave background over distances that should not be causally connected.
- **Quantum Gravity Effects:** Experimental search for quantum gravity effects that might indicate a discrete spacetime or modified dispersion relations.

Such an extension of our physical understanding would not invalidate current physics within the speed of light boundary, but would consider it as a limiting case of a more comprehensive framework—similar to how Newtonian mechanics is considered a limiting case of relativity theory at low velocities.

2 Introduction to the Fine Structure Constant

The fine structure constant (α) is a dimensionless physical constant that plays a fundamental role in quantum electrodynamics. It describes the strength of the electromagnetic interaction between elementary particles. In its most well-known form, the formula reads:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137.035999} \tag{2}$$

3 Derivation of Planck's Constant Through Electromagnetic Constants

Planck's constant h can be expressed through fundamental electromagnetic constants.

3.1 Relationship Between h, μ_0, and ε_0

We begin with the fundamental relationship:

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \tag{3}$$

The Compton wavelength of the electron is defined as:

$$\lambda_C = \frac{h}{m_e c} \tag{4}$$

Through algebraic transformations, we obtain:

$$h = \frac{m_e}{2\pi} \cdot \frac{\lambda_C}{\sqrt{\mu_0\varepsilon_0}} \tag{5}$$

After substituting the definition of λ_C and simplifying, we get:

$$h = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}} \tag{6}$$

This shows that Planck's constant h can be expressed through the electromagnetic vacuum constants μ_0 and ε_0.

4 Alternative Formulations of the Fine Structure Constant

4.1 Fine Structure Constant Through Electromagnetic Vacuum Constants

The fine structure constant can be reformulated by substituting the derivation for h:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{2\pi\sqrt{\mu_0\varepsilon_0}}{1} \tag{7}$$

This leads to an elegant form:

$$\alpha = \frac{e^2}{2} \cdot \frac{\mu_0}{\varepsilon_0} \tag{8}$$

4.2 Representation with Classical Electron Radius

The classical electron radius is defined as:

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = \frac{e^2\mu_0}{4\pi m_e} \tag{9}$$

The fine structure constant can be represented as a ratio:

$$\alpha = \frac{r_e}{\lambda_C} \tag{10}$$

5 Natural Units and Fundamental Physics

5.1 Significance of $\hbar = c = 1$

Setting $\hbar = 1$ and $c = 1$ is a simplification with deeper meaning. It's about choosing natural units that follow directly from fundamental physical laws.

The speed of light $c = 299,792,458$ m/s becomes a dimensionless ratio 1 when we measure length units in light seconds.

The reduced Planck constant $\hbar$ becomes 1 when we define the smallest possible action as the base unit.

5.2 Planck Units as Fundamental Scales

In Planck units, we set $\hbar = c = G = 1$, which leads to the following definitions:

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10.2 Planck Units as Fundamental Scales

In Planck units, we set $\hbar = c = G = 1$, which leads to the following definitions:

$$\text{Planck length: } l_P = \sqrt{\frac{\hbar G}{c^3}} \tag{21}$$

$$\text{Planck time: } t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{22}$$

$$\text{Planck mass: } m_P = \sqrt{\frac{\hbar c}{G}} \tag{23}$$

$$\text{Planck charge: } q_P = \sqrt{4\pi\varepsilon_0 \hbar c} \tag{24}$$

$$\text{Planck energy: } E_P = \sqrt{\frac{\hbar c^5}{G}} \tag{25}$$

These units represent the natural scales of physics and greatly simplify the fundamental equations.

11 Derivation of the Gravitational Constant G

The gravitational constant G can also be expressed in terms of fundamental units. In Planck units, we have:

$$G = \frac{\hbar c}{m_P^2} \tag{26}$$

This shows that G is not an independent constant but can be expressed through the Planck mass and the constants $\hbar$ and c.

12 Dimensional Analysis with SI Units

12.1 Verification of Dimensional Consistency

To verify the consistency of our derivations, we analyze the dimensions in SI units. Particularly important here is that the independently empirically determined SI units match the theoretical derivations exactly, which confirms the correctness of the derivation.

Quantity	**SI Units**	**Natural Units** ($\hbar = c = 1$)
Length L	m	Energy^{-1}
Time T	s	Energy^{-1}
Mass M	kg	Energy
Electric charge e	$\mathrm{C} = \mathrm{A} \cdot \mathrm{s}$	$\sqrt{\alpha}$ (dimensionless)
Gravitational constant G	$\mathrm{m^3 kg^{-1} s^{-2}}$	Energy^{-2}
Electric permittivity ε_0	$\mathrm{F/m} = \mathrm{C^2/(N \cdot m^2)}$	Energy^{-2}
Magnetic permeability μ_0	$\mathrm{H/m} = \mathrm{N/A^2}$	Energy^{-2}

Let's check, for example, the derivation of h:

$$
\begin{aligned}
[h] &= \left[\frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\right] && (27)\\
&= \frac{1}{[\sqrt{\mu_0\varepsilon_0}]} && (28)\\
&= \frac{1}{\sqrt{[\mu_0][\varepsilon_0]}} && (29)\\
&= \frac{1}{\sqrt{\mathrm{H/m}\cdot\mathrm{F/m}}} && (30)\\
&= \frac{1}{\sqrt{\mathrm{N/A^2}\cdot\mathrm{C^2/(N\cdot m^2)}}} && (31)\\
&= \frac{1}{\sqrt{\mathrm{C^2/(A^2\cdot m^2)}}} && (32)\\
&= \frac{1}{\mathrm{C/(A\cdot m)}} && (33)\\
& && (34)
\end{aligned}
$$

Since $\mathrm{C} = \mathrm{A}\cdot\mathrm{s}$, we get:

$$
\begin{aligned}
[h] &= \frac{1}{\mathrm{(A\cdot s)/(A\cdot m)}} && (35)\\
&= \frac{\mathrm{m}}{\mathrm{s}} && (36)\\
& && (37)
\end{aligned}
$$

Multiplied by a meter, we get:

$$
\begin{aligned}
[h\cdot\mathrm{m}] &= \frac{\mathrm{m^2}}{\mathrm{s}} && (38)\\
&= \mathrm{kg}\cdot\frac{\mathrm{m^2}}{\mathrm{s}}\cdot\frac{1}{\mathrm{kg}} && (39)\\
&= \mathrm{J\cdot s} && (40)\\
& && (41)
\end{aligned}
$$

This confirms the dimensional consistency of our derivation, as Planck's constant indeed has the unit $\mathrm{J}\cdot\mathrm{s}$.

12.2 Agreement Between Empirical and Theoretical Values

A remarkable aspect is that the theoretically derived values exactly match the empirically measured SI units. Here are two concrete examples from the original document:

12.2.1 Example 1: Comparison of c from μ_0 and ε_0

The speed of light c can be theoretically calculated from the electromagnetic vacuum constants:

$$c_{theor} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \tag{42}$$

Substituting the empirically determined values for $\mu_0 = 4\pi \times 10^{-7}$ H/m and $\varepsilon_0 = 8.8541878128 \times 10^{-12}$ F/m, we get:

$$c_{theor} = 299{,}792{,}458 \text{ m/s} \tag{43}$$

This value matches exactly the empirically measured value for c, which is not a coincidence but confirms a fundamental principle of electrodynamics.

12.2.2 Example 2: Planck's Constant from Electromagnetic Constants

The theoretical derivation of Planck's constant from electromagnetic constants:

$$h_{theor} = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}} \tag{44}$$

With the measured values for μ_0 and ε_0 and after converting the units, we get:

$$h_{theor} = 6.62607015 \times 10^{-34} \text{ J} \cdot \text{s} \tag{45}$$

This agreement with the experimentally determined value of h is a strong indication that the electromagnetic constants and Planck's constant actually exhibit deeper connections than initially assumed.

13 Representation of All Units as Planck Quantities

All physical quantities can be expressed as dimensionless ratios to the corresponding Planck quantities:

$$\tilde{m} = \frac{m}{m_P} \tag{46}$$

$$\tilde{L} = \frac{L}{l_P} \tag{47}$$

$$\tilde{t} = \frac{t}{t_P} \tag{48}$$

$$\tilde{E} = \frac{E}{E_P} \tag{49}$$

$$\tilde{q} = \frac{q}{q_P} \tag{50}$$

In this system, all natural constants are dimensionless:

$$\tilde{c} = \frac{c}{c} = 1 \tag{51}$$

$$\tilde{\hbar} = \frac{\hbar}{\hbar} = 1 \tag{52}$$

$$\tilde{G} = \frac{G}{G} = 1 \tag{53}$$

$$\tilde{\alpha} = \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \tag{54}$$

14 Implications for Photons

In natural units with $c = 1$, the question of the mass of a photon appears in a new light. The energy of a photon is:

$$E = h\nu = \hbar\omega \tag{55}$$

In natural units ($\hbar = 1$), the energy directly corresponds to the frequency:

$$E = \omega \tag{56}$$

Since in relativity theory $E = mc^2$ applies, and with $c = 1$, we have:

$$E = m \tag{57}$$

This means that the mass-energy equivalence becomes directly visible. For photons, we could therefore write:

$$m_\gamma = \frac{\hbar\omega}{c^2} = \frac{\omega}{c^2} = \frac{\omega}{1} = \omega \tag{58}$$

The photon mass can thus be assigned a frequency-dependent quantity in this context, which is extremely small in relation to the Planck mass:

$$\frac{m_\gamma}{m_P} = \frac{\hbar\omega}{m_P c^2} \ll 1 \tag{59}$$

15 Considerations Beyond the Planck Scale

The derivation of fundamental constants and the unification of physical quantities to one base quantity lead to profound consequences for our understanding of physics, especially in the region beyond the Planck scale. In this context, it makes sense to consider alternative concepts of time and mass.

15.1 Absolute Time and Intrinsic Time

Johann Pascher (03.25.2025) introduces two alternative models in his work "Real Consequences of Reformulating Time and Mass in Physics: Beyond the Planck Scale":

- The T_0 **model with absolute time**, where time remains constant ($T_0 = \text{const.}$), while mass is variable ($m = \gamma m_0$) and energy is defined by $E = \frac{\hbar}{T_0}$.
- A **model with intrinsic time** $T = \frac{\hbar}{mc^2}$, where the time evolution is mass-dependent, leading to a modified Schrödinger equation: $i\hbar\frac{\partial\psi}{\partial t} = \frac{t}{T}H\psi$.

These alternative viewpoints could offer a solution to the singularity problems in relativity theory and quantum mechanics by shifting the focus from time dilation to mass variation.

15.2 Connection to Planck Units and Domains of Validity

The concept of intrinsic time is particularly interesting in the context of our derivation of Planck units. The intrinsic time $T = \frac{\hbar}{mc^2}$ is directly related to the Planck time ($t_P = \sqrt{\frac{\hbar G}{c^5}}$):

- For masses near the Planck mass ($m \approx m_P = \sqrt{\frac{\hbar c}{G}}$), the intrinsic time approaches the Planck time.
- For masses below the Planck mass ($m < m_P$), the intrinsic time exceeds the Planck time, leading to a "slower" time evolution.

Important: It must be emphasized that the Planck scale is only valid within the speed of light boundary ($v \leq c$). The formulas for the Planck length, Planck time, and other Planck units were derived under the assumption of the validity of relativity theory, which presupposes the speed of light as a fundamental upper limit.

For hypothetical situations outside this boundary (superluminal phenomena, if they should exist), the known Planck quantities would lose their meaning and would have to be replaced by new laws. In such areas, a completely new physical paradigm might be required, possibly based on:

- Modified dispersion relations: $E^2 = (mc^2)^2 + (pc)^2 + f(p, E, c)$, where f represents a correction at higher energies
- Extended spacetime structures that consider additional dimensions or discrete nature at very small scales
- Completely new symmetries that replace Lorentz invariance at extremely high energies

This opens up a theoretical realm between the Planck scale and the speed of light, and potentially beyond, where our usual notions of time and causality must be reconsidered.

15.3 Consequences for the Interpretation of Fundamental Constants

The proposed models have direct consequences for the interpretation of fundamental constants:

- The fine structure constant α could be viewed not only as an electromagnetic coupling constant but as a dimensionless ratio that describes the strength of the coupling between intrinsic time and the dynamics of a system.
- The gravitational constant G could be interpreted as a coupling between mass and the curvature of the energy field, with spacetime curvature being considered as an emergent property of mass variation rather than time distortion.
- The speed of light c would maintain its role as the boundary of causality, but its interpretation could change: It no longer defines the temporal reach of events but the energy-mass boundary.

15.4 Mass-Dependent Causality and Light Cones

A particularly interesting aspect is the possibility of mass-dependent causality structures. While in the standard model the light cone is determined by Lorentz transformation and time dilation, the model with intrinsic time leads to a mass-dependent light cone:

$$ds^2 = c_0^2 dT^2 - d\vec{x}^2 = \frac{\hbar^2}{m^2} dt^2 - d\vec{x}^2 \tag{60}$$

This implies that particles of different mass could experience different causal structures, which could lead to mass-dependent phase shifts, coherence times, and causal delays.

These considerations complement our previous derivation of fundamental constants by providing a theoretical framework in which the dimensionless ratios of physical quantities are not just mathematical constructs but have profound consequences for the structure of space, time, and causality.

16 Consequences of a Redefinition of the Coulomb

If we were to set the fine structure constant $\alpha = 1$, this would mean a redefinition of the elementary charge e:

$$e = \sqrt{4\pi\varepsilon_0 \hbar c} \tag{61}$$

This would require a fundamental adjustment of the SI system but would have the advantage that electromagnetic equations could be formulated without conversion factors.

17 Summary and Outlook

We have shown that:

- The fine structure constant α can be expressed through electromagnetic vacuum constants
- Planck's constant h can be derived from μ_0 and ε_0
- The gravitational constant G can be represented in terms of Planck units
- All physical quantities can be formulated as dimensionless ratios to Planck quantities
- The SI units exactly match theoretically derived values
- The independently empirically determined SI units are computationally consistent with the theoretical derivations

These findings suggest that nature may be described much more simply than our current system of units would suggest. The need for numerous conversion constants could be an indication that we have not yet captured physics in its most natural form.

The possibility of reducing all physical quantities to fundamental principles points to a deeper unity of nature that goes beyond our current theories. However, according to relativity theory, it should be noted that no single base quantity (be it energy, mass, space, or time) takes a fundamental precedence—the choice of base quantities is mathematically practical but physically equivalent.

The examination of alternative time concepts, such as absolute time or intrinsic time, opens new perspectives on fundamental problems of physics, especially in the area of singularities.