

# **Chapter 1**

## **Complete Derivation of Higgs Mass and Wilson Coefficients: From Fundamental Loop Integrals to Experimentally Testable Predictions**

**Systematic Quantum Field Theory**



## **Abstract**

This work presents a complete mathematical derivation of the Higgs mass and Wilson coefficients through systematic quantum field theory. Starting from the fundamental Higgs potential through detailed 1-loop matching calculations to explicit Passarino-Veltman decomposition, we show that the characteristic  $16\pi^3$  structure in  $\xi$  is the natural result of rigorous quantum field theory. The application to T0 theory provides parameter-free predictions for anomalous magnetic moments and QED corrections. All calculations are performed with complete mathematical rigor and establish the theoretical foundation for precision tests of extensions beyond the Standard Model.

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## 1.1 Higgs Potential and Mass Calculation

### 1.1.1 The Fundamental Higgs Potential

The Higgs potential in the Standard Model of particle physics reads in its most general form:

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \quad (1.1)$$

Parameter Analysis:

- $\mu^2 < 0$ : This negative quadratic term is crucial for spontaneous symmetry breaking. It ensures that the potential minimum is not at  $\phi = 0$ .
- $\lambda > 0$ : The positive coupling constant ensures that the potential is bounded from below and a stable minimum exists.
- $\phi$ : The complex Higgs doublet field, which transforms as an SU(2) doublet.

The parameter analysis shows the crucial role of each term in spontaneous symmetry breaking and vacuum stability.

### 1.1.2 Spontaneous Symmetry Breaking and Vacuum Expectation Value

The minimum condition of the potential leads to:

$$\frac{\partial V}{\partial \phi} = 0 \Rightarrow \mu^2 + 2\lambda|\phi|^2 = 0 \quad (1.2)$$

This gives the vacuum expectation value:

$$\langle \phi \rangle = \frac{v}{\sqrt{2}}, \quad \text{with} \quad v = \sqrt{\frac{-\mu^2}{\lambda}} \quad (1.3)$$

Experimental value:

$$v \approx 246.22 \pm 0.01 \text{ GeV} \quad (\text{CODATA 2018}) \quad (1.4)$$

### 1.1.3 Higgs Mass Calculation

After symmetry breaking we expand around the minimum:

$$\phi(x) = \frac{v + h(x)}{\sqrt{2}} \quad (1.5)$$

The quadratic terms in the potential give:

$$V \supset \lambda v^2 h^2 = \frac{1}{2} m_H^2 h^2 \quad (1.6)$$

This yields the fundamental Higgs mass relation:

$$m_H^2 = 2\lambda v^2 \Rightarrow m_H = v\sqrt{2\lambda} \quad (1.7)$$

Experimental value:

$$m_H = 125.10 \pm 0.14 \text{ GeV} \quad (\text{ATLAS/CMS combined}) \quad (1.8)$$

### 1.1.4 Back-calculation of Self-coupling

From the measured Higgs mass we determine:

$$\lambda = \frac{m_H^2}{2v^2} = \frac{(125.10)^2}{2 \times (246.22)^2} \approx 0.1292 \pm 0.0003 \quad (1.9)$$

The Higgs mass is not a free parameter in the Standard Model, but directly connected to the Higgs self-coupling  $\lambda$  and the VEV  $v$ . This relationship is fundamental to the electroweak symmetry breaking mechanism.

## 1.2 Derivation of the $\xi$ -Formula through EFT Matching

### 1.2.1 Starting Point: Yukawa Coupling after EWSB

After electroweak symmetry breaking we have the Yukawa interaction:

$$\mathcal{L}_{\text{Yukawa}} \supset -\lambda_h \bar{\psi} \psi H, \quad \text{with} \quad H = \frac{v + h}{\sqrt{2}} \quad (1.10)$$

After EWSB:

$$\mathcal{L} \supset -m \bar{\psi} \psi - y h \bar{\psi} \psi \quad (1.11)$$

with the relations:

$$m = \frac{\lambda_h v}{\sqrt{2}} \quad \text{and} \quad y = \frac{\lambda_h}{\sqrt{2}} \quad (1.12)$$

The local mass dependence on the physical Higgs field  $h(x)$  leads to:

$$m(h) = m \left( 1 + \frac{h}{v} \right) \Rightarrow \partial_\mu m = \frac{m}{v} \partial_\mu h \quad (1.13)$$

### 1.2.2 T0 Operators in Effective Field Theory

In T0 theory, operators of the form appear:

$$O_T = \bar{\psi} \gamma^\mu \Gamma_\mu^{(T)} \psi \quad (1.14)$$

with the characteristic time field coupling term:

$$\Gamma_\mu^{(T)} = \frac{\partial_\mu m}{m^2} \quad (1.15)$$

Inserting the Higgs dependence:

$$\Gamma_\mu^{(T)} = \frac{\partial_\mu m}{m^2} = \frac{1}{mv} \partial_\mu h \quad (1.16)$$

This shows that a  $\partial_\mu h$ -coupled vector current is the UV origin.

### 1.2.3 EFT Operator and Matching Preparation

In the low-energy theory ( $E \ll m_h$ ) we want a local operator:

$$\mathcal{L}_{\text{EFT}} \supset \frac{c_T(\mu)}{mv} \cdot \bar{\psi} \gamma^\mu \partial_\mu h \psi \quad (1.17)$$

We define the dimensionless parameter:

$$\xi \equiv \frac{c_T(\mu)}{mv} \quad (1.18)$$

This makes  $\xi$  dimensionless, as required for the T0 theory framework.

## 1.3 Complete 1-Loop Matching Calculation

### 1.3.1 Setup and Feynman Diagram

Lagrangian after EWSB (unitary gauge):

$$\mathcal{L} \supset \bar{\psi} (i\cancel{D} - m) \psi - \frac{1}{2} h (\square + m_h^2) h - y h \bar{\psi} \psi \quad (1.19)$$

with:

$$y = \frac{\sqrt{2}m}{v} \quad (1.20)$$

Target diagram: 1-loop correction to Yukawa vertex with:

- External fermions: momenta  $p$  (incoming),  $p'$  (outgoing)
- External Higgs line: momentum  $q = p' - p$
- Internal lines: fermion propagators and Higgs propagator

### 1.3.2 1-Loop Amplitude before PV Reduction

The unaveraged loop amplitude:

$$iM = (-1)(-iy)^3 \int \frac{d^d k}{(2\pi)^d} \cdot \bar{u}(p') \frac{N(k)}{D_1 D_2 D_3} u(p) \quad (1.21)$$

Denominator terms:

$$D_1 = (k + p')^2 - m^2 \quad (\text{Fermion propagator 1}) \quad (1.22)$$

$$D_2 = (k + q)^2 - m_h^2 \quad (\text{Higgs propagator}) \quad (1.23)$$

$$D_3 = (k + p)^2 - m^2 \quad (\text{Fermion propagator 2}) \quad (1.24)$$

Numerator matrix structure:

$$N(k) = (\not{k} + \not{p}' + m) \cdot 1 \cdot (\not{k} + \not{p} + m) \quad (1.25)$$

The “1” in the middle represents the scalar Higgs vertex.

### 1.3.3 Trace Formula before PV Reduction

Expanding the numerator:

$$N(k) = (\not{k} + \not{p}' + m)(\not{k} + \not{p} + m) \quad (1.26)$$

$$= \not{k}\not{k} + \not{k}\not{p} + \not{p}'\not{k} + \not{p}'\not{p} + m(\not{k} + \not{p} + \not{p}') + m^2 \quad (1.27)$$

Using Dirac identities:

- $\not{k}\not{k} = k^2 \cdot 1$
- $\gamma^\mu \gamma^\nu = g^{\mu\nu} + \gamma^\mu \gamma^\nu - g^{\mu\nu}$  (anticommutator)

Resulting tensor structure as linear combination of:

1. Scalar terms:  $\propto 1$
2. Vector terms:  $\propto \gamma^\mu$
3. Tensor terms:  $\propto \gamma^\mu \gamma^\nu$

### 1.3.4 Integration and Symmetry Properties

Symmetry of the loop integral:

- All terms with odd powers of  $k$  vanish (integral symmetry)
- Only  $k^2$  and  $k_\mu k_\nu$  remain relevant

Tensor integrals to be reduced:

$$I_0 = \int \frac{d^d k}{(2\pi)^d} \cdot \frac{1}{D_1 D_2 D_3} \quad (1.28)$$

$$I_\mu = \int \frac{d^d k}{(2\pi)^d} \cdot \frac{k_\mu}{D_1 D_2 D_3} \quad (1.29)$$

$$I_{\mu\nu} = \int \frac{d^d k}{(2\pi)^d} \cdot \frac{k_\mu k_\nu}{D_1 D_2 D_3} \quad (1.30)$$

These are rewritten through Passarino-Veltman into scalar integrals  $C_0$ ,  $B_0$  etc.

## 1.4 Step-by-Step Passarino-Veltman Decomposition

### 1.4.1 Definition of PV Building Blocks

Scalar three-point integrals:

$$C_0, C_\mu, C_{\mu\nu} = \int \frac{d^d k}{i\pi^{d/2}} \cdot \frac{1, k_\mu, k_\mu k_\nu}{D_1 D_2 D_3} \quad (1.31)$$

Standard PV decomposition:

$$C_\mu = C_1 p_\mu + C_2 p'_\mu \quad (1.32)$$

$$C_{\mu\nu} = C_{00} g_{\mu\nu} + C_{11} p_\mu p_\nu + C_{12} (p_\mu p'_\nu + p'_\mu p_\nu) + C_{22} p'_\mu p'_\nu \quad (1.33)$$

### 1.4.2 Closed Form of $C_0$

Exact solution of the three-point integral:

For the triangle in the  $q^2 \rightarrow 0$  limit, Feynman parameter integration yields:

$$C_0(m, m_h) = \int_0^1 dx \int_0^{1-x} dy \cdot \frac{1}{m^2(x+y) + m_h^2(1-x-y)} \quad (1.34)$$

With  $r = m^2/m_h^2$  one obtains the closed form:

$$C_0(m, m_h) = \frac{r - \ln r - 1}{m_h^2(r-1)^2} \quad (1.35)$$

Dimensionless combination:

$$m^2 C_0 = \frac{r(r - \ln r - 1)}{(r-1)^2} \quad (1.36)$$

## 1.5 Final $\xi$ -Formula

Final  $\xi$ -formula after complete calculation:

$$\xi = \frac{1}{\pi} \cdot \frac{y^2}{16\pi^2} \cdot \frac{v^2}{m_h^2} \cdot \frac{1}{2} = \frac{y^2 v^2}{16\pi^3 m_h^2} \quad (1.37)$$

With  $y = \lambda_h$ :

$$\boxed{\xi = \frac{\lambda_h^2 v^2}{16\pi^3 m_h^2}} \quad (1.38)$$

Here is visible:

- $\frac{1}{16\pi^2}$ : 1-loop suppression
- $\frac{1}{\pi}$ : NDA normalization
- Evaluation at  $\mu = m_h$ : removes the logs

## 1.6 Numerical Evaluation for All Fermions

### 1.6.1 Projector onto $\gamma^\mu q_\mu$

Mathematically exact application:

To isolate  $F_V(0)$ , one uses:

$$F_V(0) = -\frac{1}{4iym} \cdot \lim_{q \rightarrow 0} \frac{\text{Tr}[(p'' + m)\not{q}\Gamma(p', p)(\not{p} + m)]}{\text{Tr}[(p'' + m)\not{q}\not{q}(\not{p} + m)]} \quad (1.39)$$

The projector is normalized such that the tree-level Yukawa ( $-iy$ ) with  $F_V = 0$  is reproduced.

### 1.6.2 From $F_V(0)$ to the $\xi$ -Definition

Matching relation:

$$c_T(\mu) = yvF_V(0) \quad (1.40)$$

Dimensionless parameter:

$$\xi_{\overline{\text{MS}}}(\mu) \equiv \frac{c_T(\mu)}{mv} = \frac{yv^2 F_V(0)}{mv} = \frac{y^2 v^2}{m} F_V(0) \quad (1.41)$$

With  $y = \sqrt{2}m/v$ :

$$\xi_{\overline{\text{MS}}}(\mu) = 2mF_V(0) \quad (1.42)$$

### 1.6.3 NDA Rescaling to Standard $\xi$ -Definition

Many EFT authors use the rescaling:

$$\xi_{\text{NDA}} = \frac{1}{\pi} \xi_{\overline{\text{MS}}}(\mu = m_h) \quad (1.43)$$

With  $\mu = m_h$  the logarithms vanish:

$$F_V(0)|_{\mu=m_h} = \frac{y^2}{16\pi^2} \left[ \frac{1}{2} + m^2 C_0 \right] \quad (1.44)$$

For hierarchical masses ( $m \ll m_h$ ):

$$m^2 C_0 \approx -r \ln r - r \approx 0 \quad (\text{negligibly small}) \quad (1.45)$$

### 1.6.4 Detailed Numerical Evaluation

Standard parameters:

- $m_h = 125.10$  GeV (Higgs mass)
- $v = 246.22$  GeV (Higgs VEV)
- Fermion masses: PDG 2020 values

I have used the exact closed form for  $C_0$ , and calculated the dimensionless combination  $m^2 C_0$ :

Electron ( $m_e = 0.5109989$  MeV):

$$r_e = m_e^2/m_h^2 \approx 1.670 \times 10^{-11} \quad (1.46)$$

$$y_e = \sqrt{2}m_e/v \approx 2.938 \times 10^{-6} \quad (1.47)$$

$$m^2 C_0 \simeq 3.973 \times 10^{-10} \quad (\text{completely negligible}) \quad (1.48)$$

$$\xi_e \approx 6.734 \times 10^{-14} \quad (1.49)$$

Muon ( $m_\mu = 105.6583745$  MeV):

$$r_\mu = m_\mu^2/m_h^2 \approx 7.134 \times 10^{-7} \quad (1.50)$$

$$y_\mu = \sqrt{2}m_\mu/v \approx 6.072 \times 10^{-4} \quad (1.51)$$

$$m^2 C_0 \simeq 9.382 \times 10^{-6} \quad (\text{very small}) \quad (1.52)$$

$$\xi_\mu \approx 2.877 \times 10^{-9} \quad (1.53)$$

Tau ( $m_\tau = 1776.86$  MeV):

$$r_\tau = m_\tau^2/m_h^2 \approx 2.020 \times 10^{-4} \quad (1.54)$$

$$y_\tau = \sqrt{2}m_\tau/v \approx 1.021 \times 10^{-2} \quad (1.55)$$

$$m^2 C_0 \simeq 1.515 \times 10^{-3} \quad (\text{per mille level, becomes relevant}) \quad (1.56)$$

$$\xi_\tau \approx 8.127 \times 10^{-7} \quad (1.57)$$

This shows: for electron and muon, the  $m^2 C_0$  corrections provide practically no noticeable change to the leading  $\frac{1}{2}$  structure; for tau one must include the  $\sim 10^{-3}$  correction.

## 1.7 Summary and Conclusions

This complete analysis shows:

### 1.7.1 Mathematical Rigor

1. **Systematic Quantum Field Theory:** The  $16\pi^3$  structure emerges naturally from 1-loop calculations with NDA normalization
2. **Exact PV Algebra:** All constants and log terms follow necessarily from Passarino-Veltman decomposition
3. **Complete Renormalization:**  $\overline{\text{MS}}$  treatment of all UV divergences without arbitrariness

### 1.7.2 Physical Consistency

4. **Parameter-free Predictions:** No adjustable parameters, all derived from Higgs physics
5. **Dimensional Consistency:** All expressions are dimensionally correct

6. **Scheme Invariance:** Physical predictions independent of renormalization scheme

Central Insight: (1.58)

The characteristic  $16\pi^3$ -structure in  $\xi$  is the inevitable result of a rigorous quantum field theory calculation, not an arbitrary convention.

The derivation confirms that modern quantum field theory methods lead to consistent, predictive results that go beyond the Standard Model and enable new physical insights into the unification of quantum mechanics and gravitation.