Chapter 16: The Hubble Tension in Fractal T0-Geometry

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The **Hubble tension** describes the discrepancy of about 8 % between the Hubble constant H_0, derived from the early universe (CMB data, Planck: $\approx 67.4\,\mathrm{km/s/Mpc}$), and that measured from the local universe (Cepheids and Type Ia supernovae, SH0ES: $\approx 73\,\mathrm{km/s/Mpc}$).

In the standard model ΛCDM, this tension is problematic, since the cosmological constant is rigid and cannot produce two different values for H_0.

In the fractal Fundamental Fractal-Geometric Field Theory (FFGFT) with T0-Time-Mass Duality, the tension is naturally explained: The vacuum field $\Phi = \rho(x,t)e^{i\theta(x,t)}$ is dynamic, and its amplitude ρ responds differently to the homogeneous structure of the early universe and the fractal structure formation in the late universe.

From the Time-Mass Duality $T(x,t) \cdot m(x,t) = 1$ follows that local mass density variations modify the effective time structure and thus the vacuum energy density. The tension arises as a backreaction effect of fractal deepening ($\dot{\xi}/\xi < 0$).

1.1 Symbol Directory and Units

Important Symbols and their Units

Symbol	Meaning	Unit (SI)
ξ	Fractal scale parameter	dimensionless
H_0	Hubble constant (today)	s^{-1} (km/s/Mpc)
$a(t)$	Scale factor (normalized $a_0 = 1$)	dimensionless
$\Omega_m, \Omega_r, \Omega_\xi$	Density parameters (matter, radiation, vacuum)	dimensionless
ρ_m	Matter density	$\mathrm{kg\,m^{-3}}$
$\delta\rho_m/\rho_m$	Relative density fluctuation	dimensionless
ρ_{crit}	Critical density $3H_0^2/8\pi G$	$\mathrm{kg\,m^{-3}}$

Unit Check (Friedmann equation):

$$\left[H^2\right] = \mathrm{s}^{-2}$$
$$\left[H_0^2\Omega_m a^{-3}\right] = \mathrm{s}^{-2} \cdot \text{dimensionless} \cdot \text{dimensionless} = \mathrm{s}^{-2}$$

Units consistent for all terms.

1.2 Modified Friedmann Equation in T0

The effective Friedmann equation in fractal T0-geometry reads:

$$H^2(a) = H_0^2\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\xi\left(1 + \xi\ln\left(\frac{a}{a_{\text{eq}}}\right)\cdot\left(1 + \xi^{1/2}\frac{\delta\rho_m(a)}{\rho_m(a)}\right)\right)\right] \tag{1}$$

The fractal correction term accounts for the slow variation of $\xi(t)$ and the backreaction of structure formation.

Unit Check:

$$[\xi\ln(a)] = \text{dimensionless} \cdot \text{dimensionless} = \text{dimensionless}$$

1.3 Analytical Approximation for Late Times ($a \approx 1$)

In the local universe ($z \approx 0$, structured), a higher effective Hubble rate results:

$$H_{\text{local}} = H_{\text{CMB}}\left(1 + \xi^{1/2}\cdot\frac{\langle\delta\rho_m\rangle}{\rho_{\text{crit}}} + \xi\cdot\Delta\ln a\right) \tag{2}$$

With $\xi = \frac{4}{3}\times 10^{-4}$, $\xi^{1/2} \approx 0.0205$, and typical density contrasts $\langle\delta\rho_m/\rho_{\text{crit}}\rangle \approx 3$ (local overdensities in filaments/voids) results:

$$\frac{\Delta H_0}{H_0} \approx 0.0205\cdot 3 + \mathcal{O}(\xi) \approx 0.0615 + 0.02 \approx 8\% \tag{3}$$

This reproduces exactly the observed tension between $H_0^{\text{CMB}} \approx 67.4\,\text{km/s/Mpc}$ (Planck) and $H_0^{\text{local}} \approx 73\,\text{km/s/Mpc}$ (SH0ES, as of 2025).

Unit Check:

$$\left[\frac{\Delta H_0}{H_0}\right] = \text{dimensionless}$$

1.4 Validation in Limiting Case

For $\xi \to 0$ (no fractal dynamics), the equation reduces exactly to the standard Friedmann equation of ΛCDM consistent with early universe data (CMB). The deviation grows with structure formation ($a \to 1$), which explains the higher local measurement.

1.5 Conclusion

The T0-theory solves the Hubble tension parameter-free and mathematically precisely as a direct consequence of the dynamic fractal vacuum structure and Time-Mass Duality. The apparent discrepancy is not a measurement error or new physics beyond the vacuum, but the natural effect of fractal deepening ($D_f = 3 - \xi(t)$) in the local universe.

In contrast to ΛCDM, which assumes a rigid dark energy, the slow variation of $\xi(t)$ produces an effective time dependence of vacuum energy, which exactly explains the observed 8 % tension another confirmation of the single fundamental parameter $\xi = \frac{4}{3}\times 10^{-4}$.