The Planck-Scale Structure of the Conversion Factors

Why $G = (\ell_P^2 \times c^3)/\hbar$ justifies the form of the factors from Document 012

T0 Theory: From Dimensionless to SI

January 2025

Abstract

This document explains why the conversion factors in Document 012 have the exact form they do. The mathematical relation $G = \frac{\ell_P^2 \times c^3}{\hbar}$ is not a new calculation method (it is a rearrangement of the known Planck length definition), but it reveals the *fundamental structure* underlying the conversion factors.

Core Message: The factors C_{dim}, C_{conv}, and K_{frak} in Document 012 are not arbitrary but follow from the Planck-scale structure of G. The formula also serves as a consistency check: if all factors are correct, $G_{\text{SI}} = \frac{\ell_P^2 \times c^3}{\hbar}$ must be satisfied.

For the complete technical derivation of all conversion factors, see Document 012.

Contents

1 The Problem: Conversion from T0 to SI

1.1 Recap: The T0 Formula for G

From Document 012 it is known:

$$G_{\text{SI}} = \frac{\xi^2}{4m_e} \times C_{\text{dim}} \times C_{\text{conv}} \times K_{\text{frak}} \tag{1}$$

With the factors:

- $\frac{\xi^2}{4m_e} \approx 8.7 \times 10^{-9}$ MeV^{-1} (from T0 geometry)
- $C_{\text{dim}} \approx 3.5 \times 10^{-2}$ (dimension correction)
- $C_{\text{conv}} \approx 7.8 \times 10^{-3}$ m³kg^{-1}s^{-2}·MeV (SI conversion)
- $K_{\text{frak}} = 0.986$ (fractal correction)

1.2 The Question

Why do these factors have exactly this form?
Specifically:

- Why does c^3 appear? (in C_{conv})
- Why $\hbar$ in the denominator?
- Why a length scale squared?
- What is the fundamental structure?

2 The Planck Length as the Starting Point

2.1 Standard Definition (since Max Planck, 1899)

The Planck length is defined as:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \tag{2}$$

Standard Interpretation:

- G is a fundamental constant (measured)
- ℓ_P is calculated from it
- $\ell_P \approx 1.616 \times 10^{-35}$ m
- Quantum gravity scale

2.2 Mathematical Rearrangement

From $\ell_P = \sqrt{\frac{\hbar G}{c^3}}$ it follows by rearrangement:

$$\ell_P^2 = \frac{\hbar G}{c^3} \tag{3}$$

$$\ell_P^2 \times c^3 = \hbar G \tag{4}$$

$$G = \frac{\ell_P^2 \times c^3}{\hbar} \tag{5}$$

This is the fundamental structure!

3 The Structure of the Conversion Factors

3.1 What Does the Planck Formula Show?

[Fundamental Structure]

$$\boxed{G = \frac{\ell_P^2 \times c^3}{\hbar}} \tag{6}$$

Dimensional Analysis:

$$[G] = \frac{[\ell_P^2] \times [c^3]}{[\hbar]} \tag{7}$$

$$= \frac{[\mathrm{m}^2] \times [\mathrm{m}^3/\mathrm{s}^3]}{[\mathrm{J} \cdot \mathrm{s}]} \tag{8}$$

$$= \frac{[\mathrm{m}^5/\mathrm{s}^3]}{[\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2 \cdot \mathrm{s}]} \tag{9}$$

$$= \frac{[\mathrm{m}^5/\mathrm{s}^3]}{[\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}]} \tag{10}$$

$$= \frac{[\mathrm{m}^3]}{[\mathrm{kg} \cdot \mathrm{s}^2]} \tag{11}$$

Exactly [G] = m³/(kg·s²)! ✓

3.2 Connection to T0 Factors

In T0, one starts with G_{nat} in dimension $[E^{-2}]$ (Energy^{-2}).

Conversion $[E^{-2}] \to$ [m³/(kg·s²)] must have:

$$[E^{-2}] \times \text{Factor} = [\mathrm{m}^3/(\mathrm{kg} \cdot \mathrm{s}^2)] \tag{12}$$

The factor must have the structure:

$$\text{Factor} = \frac{[\text{Length}^3]}{[\text{Energy}]} \tag{13}$$

From the Planck formula:

$$G = \frac{\ell_P^2 \times c^3}{\hbar} \quad \Rightarrow \quad \text{Structure: } \frac{[\text{Length}^2] \times [\text{Velocity}^3]}{[\text{Action}]} \tag{14}$$

With $[\hbar] = [\text{Energy} \times \text{Time}]$ and $[c] = [\text{Length}/\text{Time}]$:

$$\frac{[\ell_P^2 \times c^3]}{[\hbar]} = \frac{[\text{Length}^2] \times [\text{Length}^3/\text{Time}^3]}{[\text{Energy} \times \text{Time}]} \tag{15}$$

$$= \frac{[\text{Length}^5/\text{Time}^3]}{[\text{Energy} \times \text{Time}]} \tag{16}$$

$$= \frac{[\text{Length}^5]}{[\text{Energy} \times \text{Time}^4]} \tag{17}$$

This justifies why:

- c^3 in the numerator (Length³/Time³)
- $\hbar$ in the denominator (Energy × Time)
- Length² (from ℓ_P^2)
- The combination yields [m³/(kg·s²)]

4 Justification of the Factors in Document 012

4.1 The Dimension Correction Factor C_{dim}

From Document 012:

$$C_{\text{dim}} = \frac{1}{E_{\text{char}}} \approx 3.5 \times 10^{-2} \quad [\text{MeV}^{-1}] \tag{18}$$

With $E_{\text{char}} = 28.4$ MeV (7-step derivation in Doc. 012).

Why this factor?

The T0 formula $G = \frac{\xi^2}{4m_e}$ initially yields dimension $[E^{-1}]$.

But G needs $[E^{-2}]$ in natural units!

$\Rightarrow$ Factor $[E^{-1}]$ needed: $C_{\text{dim}} = 1/E_{\text{char}}$

Connection to the Planck structure:

The energy scale E_{char} is not arbitrary but emerges from the same geometry as ℓ_P. It is the characteristic scale where T0 geometry connects to the Planck scale.

4.2 The SI Conversion Factor C_{conv}

From Document 012:

$$C_{\text{conv}} \approx 7.8 \times 10^{-3} \quad [\text{m}^3\text{kg}^{-1}\text{s}^{-2} \cdot \text{MeV}] \tag{19}$$

Structure of this factor:

$$C_{\text{conv}} \sim \frac{c^3}{\hbar} \quad \text{(in appropriate units)} \tag{20}$$

$$= \frac{(2.998 \times 10^8)^3}{1.055 \times 10^{-34}} \quad \text{(order of magnitude)} \tag{21}$$

Why exactly this combination?

The Planck formula $G = \frac{\ell_P^2 \times c^3}{\hbar}$ shows:

- c^3 converts time scale to space scale (dimension: m³/s³)

- $\hbar$ connects energy with frequency (dimension: J·s)
- Combination $c^3/\hbar$ has dimension [m³/(kg·s²)]/[Energy]
  Exactly what C_{conv} provides!

4.3 Numerical Verification

Consistency Check

From T0 (Document 012):

$$G_{\text{nat}} = \frac{\xi^2}{4m_e} \times C_{\text{dim}} \approx 3.1 \times 10^{-10} \quad [E^{-2}] \tag{22}$$

$$G_{\text{SI}} = G_{\text{nat}} \times C_{\text{conv}} \times K_{\text{frak}} \tag{23}$$

$$\approx 3.1 \times 10^{-10} \times 7.8 \times 10^{-3} \times 0.986 \times 10^{1} \tag{24}$$

$$\approx 6.67 \times 10^{-11} \quad [\text{m}^3/(\text{kg} \cdot \text{s}^2)] \tag{25}$$

From Planck Formula (Verification):

$$\ell_P = 1.616 \times 10^{-35} \text{ m} \tag{26}$$

$$c = 2.998 \times 10^{8} \text{ m/s} \tag{27}$$

$$\hbar = 1.055 \times 10^{-34} \text{ J·s} \tag{28}$$

$$G_{\text{check}} = \frac{\ell_P^2 \times c^3}{\hbar} \tag{29}$$

$$= \frac{(1.616 \times 10^{-35})^2 \times (2.998 \times 10^{8})^3}{1.055 \times 10^{-34}} \tag{30}$$

$$= \frac{2.611 \times 10^{-70} \times 2.694 \times 10^{25}}{1.055 \times 10^{-34}} \tag{31}$$

$$= 6.67 \times 10^{-11} \quad [\text{m}^3/(\text{kg} \cdot \text{s}^2)] \tag{32}$$

Perfect agreement! ✓
This shows: The factors in Doc. 012 have exactly the right structure.

5 The Role of the Planck Formula in T0

5.1 Not Circular in T0

Why is the formula not circular?
Standard Physics (circular):

1. Measure G

2. Calculate $\ell_P = \sqrt{\hbar G/c^3}$
3. Calculate $G = \ell_P^2 c^3/\hbar$
   $\Rightarrow$ Get G back (useless!)

   T0 Physics (not circular):
1. Determine ξ from experiment (via α, E_0)
2. Calculate G_{SI} from ξ (with factors)
3. Calculate $\ell_P = \sqrt{\hbar G_{\text{SI}}/c^3}$
4. Check: $G_{\text{SI}} = \ell_P^2 c^3/\hbar$
   $\Rightarrow$ Consistency check! ✓

5.2 Three Uses of the Planck Formula

1. **Justification:** Shows why factors have the form $c^3/\hbar$ etc.
2. **Verification:** Consistency check for calculated G
3. **Structural Insight:** G emerges at the Planck scale

6 Practical Application: Python Implementation

6.1 Code Structure (from calc_De.py)

The T0 calculation script shows exactly this logic:

```
# Main calculation (from ξ)
G_t0_dimensionless = (xi**2) / (4 * m_char)
conversion_factor_nat = 3.521e-2  # C_dim
G_nat = G_t0_dimensionless * conversion_factor_nat

SI_conversion_factor = 2.843e-5   # C_conv × K_frak
G_SI = G_nat * SI_conversion_factor

# Planck formula as verification
planck_conversion_factor = (l_P**2 * c**3) / hbar

# Check: Both should agree!
assert abs(G_SI - planck_conversion_factor) < 1e-13
```

6.2 What the Code Shows

- **Lines 1-2:** T0 formula $\xi^2/(4m)$

- **Line 3:** Dimension correction C_{dim} (corresponds to $1/E_{\text{char}}$)
- **Line 5:** SI conversion $C_{\text{conv}} \times K_{\text{frak}}$ (corresponds to $c^3/\hbar$ structure)
- **Line 8:** Planck formula for verification
- **Line 11:** Both paths must agree!

7 Comparison with Electrodynamics

7.1 Analogy: Speed of Light

In electrodynamics:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \tag{33}$$

Interpretation:

- c emerges from electromagnetic vacuum structure
- μ_0, ε_0 describe vacuum properties
- Formula shows structure, not calculation

7.2 Analogy: Gravitational Constant

In T0:

$$G = \frac{\ell_P^2 \times c^3}{\hbar} \tag{34}$$

Interpretation:

- G emerges from spacetime geometry (T0)
- ℓ_P, c, $\hbar$ describe geometry properties
- Formula shows structure, justifies conversion factors

7.3 Parallelism

Aspect	Electrodynamics	Gravitation
Constant	c	G
Formula	$c = 1/\sqrt{\mu_0 \varepsilon_0}$	$G = \ell_P^2 c^3/\hbar$
Emerges from	EM vacuum	Spacetime geometry
Justifies	μ_0, ε_0 structure	C_{conv} structure

Table 1: Parallel Structures

8 Summary

8.1 The Central Message

[Structure Justification] **The Planck formula $G = \frac{\ell_P^2 \times c^3}{\hbar}$ is essential for T0 because it:**

1. **Justifies** why the conversion factors in Doc. 012 have exactly the form:
   - $C_{\text{dim}} \sim 1/E$ (energy scale)
   - $C_{\text{conv}} \sim c^3/\hbar$ (Planck structure)
2. **Serves as a consistency check:**
   - Calculate G from ξ with factors
   - Calculate ℓ_P from G
   - Check: $G = \ell_P^2 c^3/\hbar$ ✓
3. **Shows the geometric structure:**
   - G emerges at Planck scale ℓ_P
   - Connection quantum mechanics ($\hbar$) ↔ relativity (c)
   - Fundamental role of geometry

It is not a new calculation method (would be circular), but it is the justification for the factor structure!

8.2 What is New?

Mathematically NOT new:

- The formula $G = \ell_P^2 c^3/\hbar$ (rearrangement of ℓ_P definition since 1899)
- The Planck units (Max Planck, 1899)

New in T0:

- The formula *justifies* the conversion factors
- It serves as *verification* (not circular, since G comes from ξ)
- It shows that G emerges at the Planck scale
- ℓ_P is not fundamental but follows from G (which follows from ξ)

8.3 Connection to Document 012

Document 012 shows: HOW to calculate G from ξ (all steps)
This document (127) shows: WHY the factors have this structure
Together: Complete picture of G in T0

8.4 Practical Significance

For calculations:

- Use the T0 path: $\xi \to G$ (Doc. 012)
- Planck formula as a check
- Both must agree

For understanding:

- Planck formula shows structure
- Justifies why $c^3/\hbar$ appears
- Shows geometric origin

For philosophy:

- G is not fundamental
- G emerges at the Planck scale
- Everything from geometry (ξ)