God Does Not Roll Dice – Time–Mass Duality and ξ

Core Structure of the Fundamental Fractal-Geometric Field Theory

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January 20, 2026

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Chapter 1

A Single Number Governing Everything: Time–Mass Duality

Motivation

Imagine that all of physics – from elementary particles to the cosmos – could be reduced to a single dimensionless number. Not nineteen free parameters as in the Standard Model, no arbitrarily inserted coupling constants, but one geometric core parameter. In the FFGFT (formerly T0 theory) this number is called *xi*:

$$\xi = \frac{4}{3} \times 10^{-4}. \tag{1.1}$$

It is the central quantity of **time–mass duality**: in this view, mass is nothing but condensed, locally slowed down time. At the level of a core equation this means, in natural units (with $c = \hbar = 1$):

$$T(x) \cdot m(x) = 1, \tag{1.2}$$

so that every local value of the time field $T(x)$ has a complementary effective mass $m(x)$; all later extensions of the FFGFT technically build on this master formula of time–mass duality. The larger the effective mass in a region, the more "dense" time becomes there – a motif that will reappear later in quantum mechanics, field theory and cosmology.

From the outset an ontological caveat is important: all experiments ultimately compare frequencies or count rates and thus provide only relative statements; there is no measurement that could, even in principle, distinguish uniquely whether "really" time slows down, mass increases or geometry changes, and no future technology can escape this logical limitation because any detector is itself part of the same relational structure. For the FFGFT this means: it is explicitly presented as a model – a particular way of organising these relative relations – and what matters is not a metaphysical decision between pictures, but that the mathematical structure based on $T(x) \cdot m(x) = 1$ is consistent and reproduces all observable relations (frequencies, scales, ratios); beyond that, the question "what really changes" remains intentionally open. In particular, even in GR one could in principle reformulate the same observable content by treating masses as strictly invariant and attributing all change to geometry, or vice versa by choosing a description with fixed time evolution and variable masses; the FFGFT makes transparent that such ontological choices are conventions on top of the same relational data.

Compared to general relativity (GR), this is a reorganisation of roles: GR keeps rest masses fixed and encodes gravity purely in the curvature of a smooth 4D spacetime, whereas in the FFGFT the effective mass $m(x)$ is allowed to vary and part of what is usually attributed to curvature is encoded in the time field and its fractal depth. From this perspective GR and standard field theories are read as simplified subsectors or limits of an extended formulation; the FFGFT is introduced as the necessary extension that makes a more complete and internally consistent calculation possible wherever the simplified formulations reach their conceptual limits.

1.1 Fractal Spacetime and Effective Dimension

The FFGFT postulates that spacetime on the smallest scales is not exactly three-dimensional, but has a slightly fractal structure. This can be described by an effective fractal dimension:

$$D_f = 3 - \xi \approx 2.999867. \tag{1.3}$$

In everyday life we do not notice this – all experiments are compatible with a smooth 3D geometry. But in the boundary region between Planck scale and particle physics, the tiny offset of $3 - D_f = \xi$ is sufficient to regulate divergences and introduce new stability conditions.

1.1.1 A Geometric Analogy

As a supporting analogy one may think of a strongly folded medium: not the volume changes, but the internal structure gains folds and branches. In a similar sense, the FFGFT describes a space whose fine fractal depth increases over the course of the evolution, while macroscopic space remains approximately stable on average. This analogy is secondary to the precise geometric formulation, but helps to illustrate the role of ξ as a measure of additional structure.

Important remark (cosmology): The standard interpretation of cosmological redshift as a consequence of an expanding spacetime is replaced in the

FFGFT by an alternative picture in which fractal depth and effective scales play a central role. This aspect is still the subject of active research; at the same time several independent observations indicate that the usual interpretation as purely kinematic expansion is incomplete and that a fractal depth structure plays a central role.

1.2 From xi to Physical Scales

The strength of ξ becomes apparent in the fact that characteristic energy scales can be derived from it. One particularly important scale is the emergent energy E_0, which lies between the electron and muon masses and is central for electromagnetic structure.

In the technical chapters of the FFGFT it can be shown that, with

$$\alpha = \xi \left(\frac{E_0}{1\,\text{MeV}} \right)^2, \tag{1.4}$$

the fine-structure constant can be reproduced, i.e.

$$\frac{1}{\alpha} \approx 137.036. \tag{1.5}$$

In this new narrative volume we will follow, step by step, the path

$$\xi \;\Rightarrow\; \text{masses and ratios} \;\Rightarrow\; \alpha \tag{1.6}$$

$$\Rightarrow\; \text{QM/QFT equations} \;\Rightarrow\; \text{cosmos} \tag{1.7}$$

returning again and again to time–mass duality as an intuitive guiding picture.

In the next chapter we begin with the concrete masses and mass ratios that arise from ξ, thus preparing the ground for decoding $1/137$.

Chapter 2

From xi to Masses, Ratios and the Number 137

In this chapter we carry out the first serious test of time–mass duality: Does the single number ξ really lead to the observed lepton masses and to the famous number $1/137$? We proceed step by step and keep the technical details slim, but refer to the corresponding technical chapters where necessary.

2.1 Lepton Masses as a First Test

The FFGFT does not treat lepton masses as free inputs, but as functions of a geometric scale E_0 and the parameter ξ. In natural normalization (without units) one first encounters dimensionless masses $m^{(\text{nat})}$ that arise from a fractal quantum function $f(n,l,s)$. For the electron, muon and tau this schematically reads

$$m_e^{(\text{nat})} = \frac{1}{\xi \cdot f(1,0,1/2)}\,, \tag{2.1}$$

$$m_\mu^{(\text{nat})} = \frac{1}{\xi \cdot f(2,1,1/2)}\,, \tag{2.2}$$

$$m_\tau^{(\text{nat})} = \frac{1}{\xi \cdot f(3,2,1/2)}\,. \tag{2.3}$$

The concrete form of $f(n,l,s)$ is part of the technical derivation; for the narrative it suffices to note:

- All three masses depend *only* on ξ and integer quantum numbers.
- There is a unique geometric assignment, with no freely adjustable parameter for each particle.

To make contact with measured physics, a common scale factor is chosen such that

$$m_e \approx 0.511\,\text{MeV}\,, \tag{2.4}$$
$$m_\mu \approx 105.7\,\text{MeV}\,, \tag{2.5}$$
$$m_\tau \approx 1776.9\,\text{MeV} \tag{2.6}$$

result. The details of this fit remain in the technical chapters; here the statement counts: **with a single geometric parameter ξ the three-level lepton spectrum becomes reproducible.**

2.2 Mass Ratios and the Emergent Scale E_0

Instead of staring at the absolute numbers, it is worthwhile to look at the ratios. Between electron and muon, and between muon and tau, characteristic factors arise that can be explained from the structure of $f(n, l, s)$.

From this hierarchy one can derive an *emergent* energy scale E_0 that lies roughly in the middle between electron and muon mass:

$$E_0 \approx 7.4\,\text{MeV}. \tag{2.7}$$

Narratively speaking, E_0 is the energy at which the geometry determined by ξ and the electromagnetic coupling **fit particularly well** – a kind of meeting point of scales. This scale does not appear as a free parameter, but *falls out of the lepton hierarchy*.

2.3 The Fine-Structure Constant from xi

At this point one of the central relations of the FFGFT comes into play:

$$\alpha = \xi \left(\frac{E_0}{1\,\text{MeV}} \right)^2 . \tag{2.8}$$

If one inserts the E_0 obtained from the mass ratios, one finds

$$\alpha \approx \frac{1}{137.036} \tag{2.9}$$

which implies

$$\frac{1}{\alpha} \approx 137.036, \tag{2.10}$$

in agreement with the precise CODATA values of the fine-structure constant.

Important note of caution

The above relation is *not* a free fit formula in the FFGFT, but follows from the combination of

- the fractal dimension $D_f = 3 - \xi$,
- the resulting hierarchy of lepton masses and
- the identification of E_0 as a geometrically emergent energy scale.

The precise numerical agreement with the measured value of $1/\alpha$ is remarkable and supports the view that we are dealing not with a mere numerical coincidence but with a geometrically motivated structure; nevertheless, experimental and theoretical uncertainties must be kept in mind:

- **Experimental side:** The fine-structure constant is measured with extreme precision; small shifts due to new analyses are possible.
- **Theoretical side:** Higher-order corrections (for example from quantum field theory and fractal fine structure) can slightly change the effective coupling.

In this narrative volume the aim is therefore not to claim that a few lines exhaustively explain all details of high-precision physics. More important is the conceptual message: **from the single number ξ one can consistently derive both the lepton masses and the electromagnetic coupling strength.** Full derivations and numerical studies can be found in the technical T0 documents on lepton masses and the fine-structure constant (see the PDFs in texttt2/pdf and the GitHub repository).

In the following chapters we apply this perspective to the equations of quantum mechanics and quantum field theory – beginning with the Schrödinger equation and its deterministic interpretation in terms of time–mass duality.

Chapter 3

Time–Mass Duality in Quantum Mechanics and Field Theory

In the preceding chapters geometry was in the foreground: the number ξ, the fractal dimension D_f and the resulting scales. We now apply this structure to the familiar equations of quantum mechanics and quantum field theory.

3.1 Schrödinger Equation as an Effective Description

In the standard formulation the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(t,\vec{x}) = \hat{H}\,\psi(t,\vec{x}) \tag{3.1}$$

describes the evolution of a wave function ψ under a Hamiltonian operator $\hat{H}$. This equation is already deterministic: from a given initial state the future follows uniquely. Apparent randomness enters the theory only through the measurement postulate and the interpretation of $|\psi|^2$ as a probability density.

Within the framework of time–mass duality the Schrödinger equation is understood as an effective description of a deeper geometric dynamics. Roughly speaking, ψ does not describe a mysterious "field of possibilities", but a statistical projection of the underlying fractal time structure. The parameters in the Hamiltonian – in particular masses and couplings – are not fundamental in the FFGFT, but are determined by ξ and the scales derived from it.

3.2 From Schrödinger to Dirac

For relativistic particles with spin the Schrödinger equation is not sufficient. Here the Dirac equation appears:

$$(i\,\gamma^\mu \partial_\mu - m)\,\psi = 0, \tag{3.2}$$

with the Dirac matrices γ^μ and the mass m. In the FFGFT, m is not treated as an input parameter, but as a derived quantity from time–mass duality and the fractal structure (as in the lepton examples above).

This also changes the reading of the Dirac equation: it is not the fundamental equation, but an effective field equation on a background whose geometry is already fixed by ξ. In the full FFGFT/T0 formulation the Dirac structure is generally simplified: instead of the full 4×4 matrix formalism one uses an equivalent scalar field dynamics of the mass variation, and on this basis one can formulate both an extended Schr"odinger equation and a universal Lagrangian density. In this chapter we only show the usual Dirac form as an effective entry point, while the truly fundamental description is given by the simplified Dirac dynamics and the universal Lagrangian of the T0 theory. The familiar properties – spin, antimatter, zitterbewegung – remain, but receive a geometric interpretation within fractal spacetime.

3.3 Lagrangian Density and the Role of ξ

In volumes 1 to 3 the Lagrangian density of the FFGFT was built up step by step. Schematically it can be written as an extension of the Einstein–Hilbert action, supplemented by fractal contributions and matter fields. For a simple Dirac field in curved spacetime the standard Lagrangian density reads

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(i\,\gamma^\mu \nabla_\mu - m)\psi. \tag{3.3}$$

In the FFGFT, m is fixed by ξ and the underlying fractal structure, and additional terms in the Lagrangian encode the contributions of the fractal vacuum. The detailed construction of these terms was developed in the technical chapters; for the narrative it suffices to state that ξ appears as a global organizing parameter in all sectors of the Lagrangian density.

3.4 Outlook: Quantum Computers and Basis Functions

If quantum mechanics and quantum field theory are, at their core, geometrically organized by ξ, it is natural to ask how this structure is reflected in quantum information and quantum computers. The fractal basis functions and modes introduced earlier can then be viewed as natural building blocks for quantum registers and logical operations. On this basis one can work out the connection between the underlying field equations, time–mass duality and concrete quantum chip architectures, so that it becomes clear how geometric and information-theoretic aspects interlock.

Chapter 4

Quantum Information and Basis Functions in Time–Mass Duality

This chapter describes the connection between the geometric structure of the FFGFT and quantum information theory. The focus is not on technical circuit diagrams, but on how qubits, superposition and entanglement can be understood from time–mass duality.

4.1 Qubits as Effective Degrees of Freedom

In the usual formulation a qubit is a state vector in a two-dimensional Hilbert space:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \quad |\alpha|^2 + |\beta|^2 = 1. \tag{4.1}$$

In the FFGFT this Hilbert space is not regarded as an abstract mathematical space without background, but as an effective description of certain fractal modes of time–mass duality. The two basis states $|0\rangle$ and $|1\rangle$ then represent two stabilized configurations of an underlying geometric structure (for example two locally different phases of the field), while the coefficients α and β encode how strongly this structure is activated.

This interpretation does not change the formal use of the qubit algebra; it only makes explicit that the parameters are ultimately fixed by ξ and the scales derived from it.

4.2 Superposition and Interference

The core of many quantum algorithms is the controlled use of superposition and interference. In everyday language one says that a qubit is simultaneously "0" and "1" and that these components interfere constructively or destructively.

Within time–mass duality this can be interpreted as interference of fractal time paths: the underlying geometric dynamics is deterministic, but from the perspective of the effective state $|\psi\rangle$ several contributions appear that manifest themselves as probabilities in measurements. The familiar interference phenomena – for example in the double-slit experiment – remain fully intact, but gain an additional interpretative layer: they reflect the structure of the fractal path dynamics.

4.3 Entanglement and Nonlocality

Multi-qubit states such as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{4.2}$$

are described as entangled in standard quantum mechanics: the total state cannot be written as a product of single-qubit states.

In the FFGFT this is a sign that the underlying fractal structure organizes the participating degrees of freedom jointly. The correlations do not arise through subsequent "communication" between particles, but are already present in the common geometry of time–mass duality.

This point of view is consistent with the results of volumes 1–3, in which Bell experiments, RSA protocols and deterministic interpretations of quantum mechanics were discussed. In the present narrative these topics are not rederived, but traced back to the role of ξ and the fractal structure.

4.4 Basis Functions as a Natural Computational Basis

In earlier chapters of the FFGFT, special fractal basis functions $G_n(t)$ were introduced which act as eigenfunctions of the underlying time-field operator and describe the spectral structure of time–mass duality. For quantum information applications they offer themselves as a natural computational basis: instead of arbitrarily chosen basis states one uses states of the form

$$|n\rangle \sim G_n(t), \tag{4.3}$$

which represent the occupation of the n-th basis function.

Conceptually this means:

- A qubit or register is not defined abstractly, but as an occupation pattern of specific basis functions.

- Gate operations correspond to targeted geometric transformations that mix these modes (for example effective rotations in state space).

The concrete implementation of such operations (for example on a photonic chip) remains in the background here. What matters is that the FFGFT provides a consistent bridge between geometric field theory and quantum information without changing anything in the established formal structure of quantum computing theory.

4.5 Elementary Gates and Fractal Dynamics

Simple single-qubit gates can be understood as targeted redistribution of occupation between two basis functions. Mathematically, a rotation in the two-dimensional state space can for example be described by

$$U(\theta) = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \tag{4.4}$$

In terms of time–mass duality such a rotation corresponds to a controlled change in the relative weighting of two fractal modes for a fixed energy spectrum determined by ξ. The formal representation is identical to that of ordinary quantum information theory, but receives a geometric interpretation: angle parameters such as θ reflect concrete properties of the underlying structure, such as travel times or effective coupling strengths.

A controlled two-qubit gate, such as a controlled phase gate, can in this view be understood as a targeted correlation of two sets of basis functions. Instead of an abstract control by a control qubit, the underlying fractal geometry acts in such a way that certain combined occupations are favored or suppressed.

4.6 Scales, Noise and Robustness

The scales determined by ξ fix not only masses and energies, but also natural time scales on which coherent quantum dynamics can take place. For quantum processors this means that there are preferred operating regimes in which the interaction with the environment only weakly disturbs the fractal structure.

Noise and decoherence can be interpreted in this perspective as disturbances of the fine time–mass structure, which cause the effective qubit description to drift away from the actual geometric dynamics. A careful choice of materials, frequencies and coupling strengths can be understood as an attempt to minimize these disturbances so that the scales specified by ξ are exploited as well as possible.

4.7 Factorization, Shor's Algorithm and Simulations

A prominent example of the power of quantum computers is the factorization of large numbers, as used in Shor's algorithm. Formally this algorithm is based on periodic structures in modular exponential functions and exploits superposition and interference to find periods efficiently.

In the FFGFT these structures can be understood as special configurations of the fractal basis functions. A prototypical step in Shor's algorithm is the mapping

$$|x\rangle\,|0\rangle \longmapsto |x\rangle\,|f(x)\rangle\,, \quad f(x) = a^x \text{ mod } N, \tag{4.5}$$

followed by a quantum Fourier transform on the first register in order to extract the period r of $f(x)$.

Simulations of Shor's algorithm show that the expected interference patterns and success probabilities can be reproduced when the logical states are interpreted as occupations of suitable fractal modes.

Narratively this means:

- Factorization is not understood as a "magical" speed-up, but as an exploitation of geometrically organized interference in the time–mass structure.
- The same basis functions that appear in field theory also form the basis for simulations of Shor-like algorithms.
- Further quantum algorithms (for example search and optimization procedures) can be formulated in this language as different ways of exploiting the same fractal geometry.

Dedicated chapters can deepen these aspects, for example through detailed discussions of specific gate sequences or numerical simulations. For the present overview it suffices to state that time–mass duality provides a consistent background on which even complex algorithms such as Shor's can be understood geometrically.

Chapter 5

Fractal Dimension and Regularization

In the previous chapters time–mass duality was used *phenomenologically*: the number ξ organizes masses, ratios and couplings. In this chapter the fractal dimension D_f is examined in more detail and we show why even a tiny offset from classical three-dimensionality can have physical impact.

5.1 Why a Fractal Dimension?

Classical field theories work in smooth spaces with integer dimension. Experience with quantum field theories, however, shows that at very small scales divergences appear that can only be brought under control with additional regularization procedures.

The FFGFT chooses a different approach: instead of introducing an auxiliary regularization, the effective spatial dimension itself is slightly shifted,

$$D_f = 3 - \xi, \tag{5.1}$$

with $\xi = \frac{4}{3} \times 10^{-4}$.

Physically space remains three-dimensional for all direct measurements; the difference shows up only in the way integrals over very high momenta or very small lengths converge. The tiny fraction ξ acts like a built-in regularization of the field theory.

5.2 Scale Dependence and Time–Mass Duality

The fractal dimension is not to be understood as a rigid property of a "slightly eaten away" space, but as an effective description of scale dependence. The deeper one goes into the depth of the time–mass structure, the more clearly the deviation from exactly three dimensions becomes noticeable.

In time–mass duality this is reflected in the association of mass with the local "density" of time. Larger effective mass means that the fractal structure is more tightly folded at that point; time runs more slowly there on average. The slight lowering of D_f relative to 3 is a global measure of this folding density.

5.3 Connection to Masses and Couplings

From the perspective of the FFGFT masses and couplings are not independent quantities, but derived parameters of the fractal geometry. The lepton masses and the fine-structure constant have already been discussed as functions of ξ and an emergent scale E_0.

In the background stands the observation that integrals which would diverge in exactly three-dimensional theories are, at $D_f = 3 - \xi$, weakened just enough to yield well-defined contributions. These contributions can be interpreted as effective self-energies and coupling corrections fixed by the fractal structure.

In earlier T0 texts an apparently different number $D_f \approx 2.94$ appears; this does not describe a different spatial dimension, but an *effective* dimension D_f^{eff} for specific renormalization steps, where loop integrals scale like $(\lambda_C/\ell_P)^{D_f-2}$ and only the combination $D_f - 2 \approx 0.94$ matters. Fundamentally the Xi narrative keeps the geometry with $D_f = 3 - \xi$; $D_f^{\text{eff}} \approx 2.94$ is a derived critical exponent for selected processes that follows from this geometry and is in line with independent approaches to fractal quantum gravity where the spectral dimension flows towards $d_s \approx 2$ in the UV.

Narratively this means:

- ξ quantifies how strongly spacetime is folded on the smallest scales.
- This folding regulates quantum fluctuations that would otherwise diverge.
- Masses and coupling strengths arise as the field's response to this regulated fractal structure.

5.4 Casimir Effect as Laboratory Confirmation

A particularly important test of the fractal vacuum structure is the Casimir effect. Between conducting plates a force is measured which, in the standard theory, arises from the $1/d^4$ dependence of the vacuum energy density and has been confirmed with high precision for decades.

In the FFGFT these measurements are linked to the hierarchy of scales determined by ξ. Starting from the CMB energy density ρ_{CMB} and a characteristic vacuum length scale L_ξ of order $100\,\mu\text{m}$, one can show that a relation of the form

$$\rho_{\text{CMB}} = \frac{\xi\,\hbar c}{L_\xi^4} \tag{5.2}$$

leads to a modified Casimir formula that reproduces exactly the established standard expression

$$|\rho_{\text{Casimir}}(d)| = \frac{\pi^2 \hbar c}{240\,d^4}. \tag{5.3}$$

Thus two phenomena that at first glance seem very different – the CMB and the Casimir effect – become understandable as different manifestations of the same fractal vacuum structure. The existing Casimir measurements therefore provide a direct laboratory confirmation that the depth structure of space organized by ξ is physically real.

5.5 Outlook on Cosmology and the CMB

The same fractal dimension that provides regularization in particle physics plays a role in cosmology when interpreting large-scale structures. If the effective depth of space increases in the course of cosmic evolution, the perception of distances, times and energy densities also changes.

Later chapters will describe how the CMB temperature, redshifts and other cosmological quantities can be reinterpreted within this framework. It is emphasized that this interpretation is not arbitrary, but is supported by several observations and must at the same time continue to be tested carefully against cosmological data.

Chapter 6

Units, Scales and Constants from ξ

So far ξ has appeared as an organizing parameter for masses, ratios and the fine-structure constant. This chapter outlines how the same structure also allows *units* and central constants of nature to be derived and why no freely chosen UV cutoffs are needed in the FFGFT.

6.1 Natural Scales from Time–Mass Duality

Time–mass duality suggests working from the outset with natural scales that are fixed by ξ. Instead of defining mass, length and time as completely independent, they are linked through the fractal dimension $D_f = 3 - \xi$ and characteristic scales such as E_0, L_0 and L_ξ.

The essential building blocks are:

- an energy scale E_0 in the MeV range that follows from the lepton hierarchy,
- a minimal length scale $L_0 = \xi L_P$ in the sub-Planck regime,
- an emergent vacuum length scale L_ξ of about $100\,\mu\text{m}$ that links Casimir and CMB effects.

From these quantities one can construct consistent systems of natural units in which, for example, $c = \hbar = 1$ and time, length and energy are directly connected by the fractal structure.

6.2 Dimensions, D_f and Effective Units

The slight deviation of the effective dimension D_f from 3 has direct consequences for the scaling of quantities. Volumes, areas and phase spaces grow slightly differently than in exactly three-dimensional models, which is reflected in the UV behaviour of the theories.

In the FFGFT effective units are chosen so that this fractal scaling is taken into account from the outset:

- lengths are measured relative to L_0 and L_ξ,
- energies relative to E_0 and the scales derived from it,
- time intervals are linked via time–mass duality directly to local mass densities and folding degrees of the structure.

In this way many of the apparently arbitrary parameters that appear in classical unit systems disappear and make way for a geometrization of units themselves.

6.3 Gravitational Constant as Emergent Coupling

In standard physics the gravitational constant G appears as a fundamental constant built directly into field equations and Lagrangians. Its dimension is set by the chosen system of units, and many approaches to quantum gravity attempt to renormalize G analogously to other couplings.

The FFGFT takes a different route. Starting from a time-field dynamics and the fractal spacetime structure, gravitation is understood as an emergent effect of time–mass duality. The Planck length

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \tag{6.1}$$

is not taken as a starting point but itself as a derived scale that follows from ξ and the geometry of the time–mass structure. Combining this relation with the FFGFT expression

$$G = \frac{c^3 l_P^2}{\hbar} \cdot \xi^2 \tag{6.2}$$

shows explicitly how the small parameter ξ keeps gravity weak: the factor ξ^2 suppresses the effective coupling by almost eight orders of magnitude compared to the naive Planck value. In the limit $\xi \to 0$ the coupling G would vanish, whereas for $\xi \approx 1$ gravity would be stronger by a factor of about $(1/\xi)^2 \sim 10^8$ and complex structures could not form.

Conceptually this means:

- G acts as an effective coupling constant describing how macroscopic spacetime responds to the fractal depth structure.
- The numerical value of G can be derived from the combination of ξ, L_0, L_ξ and the associated energy densities.
- Gravitation is not "patched" into the theory by adding an arbitrary Lagrangian term, but results from the underlying time-field geometry.

Thus the status of G shifts: from an "initially postulated" constant to a quantity with the same geometric origin as the lepton masses and the fine-structure constant.

6.4 Relation to Lagrangian Density and Field Equations

In the technical chapters of the FFGFT it was shown how an effective Lagrangian density for matter and field degrees of freedom arises from fractal time-field dynamics. Instead of postulating an Einstein–Hilbert action with fixed gravitational constant from the outset, the coupling to geometry is constructed from the structure of the time field.

From a narrative perspective the core message suffices; detailed tables and systematizations of natural units, SI conversions and the status of c and α can be found in the dedicated T0 units documents in the repository (see the PDFs in
texttt2/pdf).

- The same parameters ξ, E_0, L_0 and L_ξ that organize masses, couplings and CMB/Casimir effects also determine effective gravitational coupling.
- Unit systems can be chosen such that these interrelations become transparent and no artificial infinities appear in the UV regime.

In this way a picture emerges in which all central constants and units – from α and the lepton masses to G and cosmological scales – appear as expressions of a single time–mass geometry governed by ξ.

Chapter 7

Gravity and the Gravitational Constant from ξ

In the main FFGFT narrative the gravitational constant G already appears as an *emergent* quantity: it is not simply postulated, but follows from the same fractal time–mass structure that organises masses, couplings and cosmic scales. This chapter gives a focused Xi-level account of how G arises from ξ, why the factor ξ^2 is crucial, and what this means for the weakness of gravity and the stability of the universe.

7.1 From Planck Units to Fractal Geometry

In conventional physics the Planck units are constructed from c, $\hbar$ and G:

$$L_P = \sqrt{\frac{\hbar G}{c^3}}, \qquad m_P = \sqrt{\frac{\hbar c}{G}}, \qquad t_P = \sqrt{\frac{\hbar G}{c^5}}. \tag{7.1}$$

The logic usually runs in one direction: one *assumes* that c, $\hbar$ and G are fundamental and then combines them to get characteristic scales for length, mass and time. In the FFGFT/Xi picture this logic is reversed:

- the central object is the fractal time–mass geometry, organised by the small parameter ξ;
- from this geometry natural scales such as E_0, $L_0 = \xi L_P$ and L_ξ emerge;
- G is read *from* these scales, rather than being put in by hand.

The question is therefore not "how do we build units from G?", but "how does G appear as an effective coupling when matter and geometry are both expressions of one and the same fractal structure?".

7.2 Deriving G from ξ

In the technical derivation, the starting point is the dynamics of the time field and its coupling to the vacuum density. At the narrative level it is sufficient to recall the key relation already used in the main narrative:

$$G = \frac{c^3 l_P^2}{\hbar} \cdot \xi^2. \tag{7.2}$$

Here l_P denotes the (conventionally defined) Planck length. The new ingredient is the factor ξ^2 in front. Without this factor the natural guess would simply be $G_{\text{naive}} \sim c^3 l_P^2/\hbar$. The FFGFT corrects this guess by inserting the fractal parameter:

$$G = G_{\text{naive}} \cdot \xi^2. \tag{7.3}$$

Numerically, with $\xi = \frac{4}{3} \times 10^{-4}$, this means that G is suppressed by almost eight orders of magnitude compared to the naive Planck value. This suppression is *not* an arbitrary tuning, but a direct reflection of the fractal depth of spacetime.

7.3 Why Gravity Is So Weak

From a purely dimensional point of view there is no reason why gravity should be as weak as it is. The gravitational coupling between two protons,

$$\alpha_G = \frac{G m_p^2}{\hbar c} \approx 10^{-38}, \tag{7.4}$$

is tiny compared to the electromagnetic fine-structure constant $\alpha \approx 1/137$. In everyday terms: gravity is weaker than electromagnetism by about thirty-eight orders of magnitude.

In the FFGFT this enormous hierarchy is attributed to the factor ξ^2. If one formally sets $\xi = 1$ in the expression for G, gravity becomes stronger by a factor

$$\left(\frac{1}{\xi}\right)^2 \sim 5.6 \times 10^7. \tag{7.5}$$

A universe with such a large G would behave dramatically differently:

- galaxies would collapse more quickly;
- stable stars and planetary systems would be highly unlikely;
- small inhomogeneities would grow so fast that no long-lived, complex structures could form.

From the Xi perspective the weakness of gravity is therefore not a separate mystery, but a direct consequence of the smallness of ξ. The same parameter that organises lepton masses and the CMB scale also controls the effective strength of gravity.

7.4 Relation to the Time Field

A central theme of the Xi narrative is that time is not a pre-given background, but a derived structure. Infinitesimal intervals $d\tau$ follow from phase evolution $d\theta$ of a vacuum field, scaled by density and ξ. In this view, curvature and gravitation describe how the phase structure of the vacuum organises itself across scales.

The emergence of G from ξ can be read precisely in this way:

- the combination $c^3 l_P^2/\hbar$ encodes how fast disturbances can propagate and how quantum fluctuations back-react on geometry;
- the factor ξ^2 measures how deeply the vacuum is folded, i.e. how much additional "room" for structure exists beyond a purely three-dimensional picture;
- the product of both sets the strength with which the time field responds to matter and energy densities.

Gravitation is thus an *effective* description of how the fractal time field self-organises. In regimes where the folding depth hardly changes, Einstein's equations with a nearly constant G are an excellent approximation. Where the folding depth varies significantly, the effective value of G can in principle become scale-dependent.

7.5 Comparison with Setting $G = 1$

In natural units it is common to set $G = 1$ in order to simplify formulas. From the Xi viewpoint this corresponds to hiding important geometric information:

- by setting $G = 1$ one normalises away the explicit sensitivity to ξ^2;
- the distinction between regimes where gravity is effectively weaker or stronger becomes less transparent;
- the connection between gravitational coupling and the scales E_0, L_0 and L_ξ is no longer visible at a glance.

For rough order-of-magnitude estimates this may be acceptable. But for the FFGFT programme – which aims precisely at expressing constants such as G in terms of a small number of geometric parameters – it is crucial to keep

G explicit. Only then can one see how a change in ξ would ripple through the entire web of scales.

7.6 Outlook

This chapter has highlighted one central message: in the FFGFT/Xi framework the gravitational constant is not an independent input, but part of the same fractal pattern that unifies mass scales, couplings and cosmological observables. The formula

$$G = \frac{c^3 l_P^2}{\hbar} \cdot \xi^2 \tag{7.6}$$

is therefore more than a numerical fit – it summarises how the depth of spacetime, encoded in ξ, controls the apparent weakness of gravity.

Further technical details, including explicit fits and comparison with observational constraints, are given in the corresponding T0 technical documents on gravity and cosmology (see the PDFs in `2/pdf`).

Chapter 8

Singularities and a Natural UV Cutoff

In many standard models of physics formal infinities appear: divergent integrals in quantum field theory, singularities in black holes or a point-like beginning of the universe. Usually these problems are softened by auxiliary procedures such as renormalization, artificial UV cutoffs or special initial conditions. Time–mass duality and the fractal spacetime structure of the FFGFT follow a different route: the underlying geometry is organized in such a way that genuine physical infinities do not arise in the first place.

8.1 Mathematical Singularities as Artefacts

Singularities typically occur when a theory is extrapolated beyond its domain of validity. A classic example is the point charge in electrodynamics, whose field energy formally diverges when the distance is taken exactly to zero. In general relativity, too, divergences of curvature appear in the description of black holes and in the standard big-bang model.

The FFGFT interprets these singularities as an indication that the assumption of an exactly smooth, continuous spacetime down to arbitrarily small scales is unphysical. Once the fractal dimension

$$D_f = 3 - \xi \tag{8.1}$$

and a minimal effective length scale are taken into account, the formal infinities disappear and are replaced by large but finite contributions.

8.2 Fractal Dimension and UV Behaviour

As explained in the previous chapter, lowering D_f relative to 3 causes integrals that would diverge in exactly three-dimensional theories to be weakened. On very small scales the fractal structure acts like a built-in UV cutoff:

- Volume elements grow slightly differently than in smooth 3D geometry.
- Effective phase spaces for high-energy modes are reduced.
- Self-energies and loop contributions remain finite and are fixed by ξ and the associated scales.

In this view a UV cutoff is not a freely chosen computational device but an expression of the real geometric structure of spacetime. The theory itself does not know infinite energy densities, only the limit of its effective description on scales below those set by ξ.

8.3 Minimal Length Scales and Time–Mass Structure

The FFGFT works with a hierarchy of length scales: from very small, fractally organized depth structures up to macroscopic regions in which spacetime appears practically smooth. At the deepest levels there is a minimal effective length scale below which it no longer makes sense to speak of classical points.

Narratively this means:

- The time–mass structure has a finite folding density; it can become denser, but not infinitely dense.
- Regions of large effective mass correspond to strongly folded time, not to a "hole" with infinite curvature.
- Even in the early universe an extremely dense but finite initial configuration is described, not a mathematical singularity.

Thus the notion of a singularity is replaced by geometrically organized saturation: where classical theories predict infinite quantities, the FFGFT describes regions in which the fractal structure reaches its maximum density.

8.4 Consequences for Black Holes and the Big Bang

For black holes this means that the inner region is not understood as a point of infinite curvature, but as a zone in which the time–mass structure is maximally folded. The classical horizon structure remains as an effective boundary for observers, but inside the fractal geometry prevents infinite energy densities from arising.

Similarly, the beginning of the universe is not described as infinite density, but as a transition phase in which the fractal depth structure of spacetime evolves from an almost homogeneous state into the present hierarchically organized structure. Scales such as the CMB temperature and characteristic Hubble quantities appear in this picture as consequences of this evolution rather than as consequences of a mathematical singularity.

Overall, time–mass duality replaces the notion of physical infinities with a consistent geometry controlled by ξ and endowed with a natural UV cutoff. This ties in with the previously discussed connections between ξ, masses, couplings, the Casimir effect and cosmology and links microscopic and cosmological scales in a common framework.

Chapter 9

Cosmology, Redshift and the CMB in Time–Mass Duality

In the preceding chapters the microscopic side of time–mass duality was in focus: masses, couplings and quantum phenomena. This chapter sketches how the same structure affects large-scale cosmological phenomena: redshift, the cosmic microwave background and effective quantities such as the Hubble scale.

9.1 Redshift without Expanding Space

Standard cosmology interprets cosmological redshift mainly as a consequence of an expanding spacetime. The wavelength of a photon is stretched with the cosmic scale factor; distances grow with time.

Within time–mass duality a different picture is proposed. Here the observed redshift is understood essentially as a consequence of a changing fractal depth structure of space:

- The effective depth of the time–mass structure increases in the course of cosmic evolution.
- Light traversing regions with different fractal depth experiences systematic shifts of its frequency.
- The observed relation between redshift and distance thus reflects primarily differences in depth structure, not necessarily a literal "flying apart" of space.

Several independent observations – for example tensions in the determination of the Hubble constant and certain large-scale structures – suggest that a pure expansion interpretation is incomplete. The FFGFT offers a consistent alternative in which redshift is understood as a depth effect of time–mass duality.

9.2 CMB Temperature and Characteristic Scales

The cosmic microwave background (CMB) today has a temperature of about 2.73 K. In the standard view this is the cooled remnant radiation of an earlier, much hotter state of the universe.

In time–mass duality the CMB temperature is interpreted as a macroscopic manifestation of the hierarchy of scales determined by ξ. Roughly speaking, the CMB sits on an energetic scale that emerges from the same fractal mechanisms that also organize the lepton masses and the fine-structure constant.

Narratively:

- The number ξ fixes a hierarchy of energy scales in the vacuum.
- This hierarchy determines the typical energy of the background photons we measure as the CMB.
- The observed temperature is therefore not an accident but an expression of the same geometric order that also operates in particle physics.

9.3 Effective Hubble Scale and Distances

Quantities such as the so-called Hubble radius can also be read differently in the FFGFT. Instead of a built-in expansion rate, the effective Hubble scale here describes a combination of fractal depth evolution and light-travel-time effects.

Light from distant objects traverses regions with different time–mass structure. The resulting delays and frequency shifts lead to the same observable relations as in an expanding model, but are interpreted as geometric depth effects.

9.4 Observations

The fractal cosmology of the FFGFT is not in conflict with precise measurements of CMB anisotropies, supernova data and large-scale structure formation, but offers a different interpretation of their causes. Several findings – for example tensions between different Hubble determinations or hints of scale-dependent effects – can be naturally accommodated in this framework; a more detailed discussion and the comparison with standard cosmology are given in dedicated T0 cosmology texts in the repository (see the PDFs in texttt2/pdf).

Overall, current results support the view that standard interpretations are incomplete at key points and that a fractal depth structure plays a central role.

9.5 Outlook and Further Texts

The time–mass duality presented here forms a concentrated core of the FFGFT. For a broader embedding in seven fundamental puzzles of physics and for a detailed geometric brain analogy, two complementary expositions are available: one volume that systematically discusses the seven puzzles, and another that develops the cosmic ”brain” as an illustrative metaphor for the fractal depth structure of spacetime. Both texts use the same parameters and structures, but deepen different aspects and can be read together with the present account as a coherent whole.

Chapter 10

Precision Tests and Observations

In the preceding chapters the central building blocks of time–mass duality were introduced: the number ξ, the fractal dimension $D_f = 3 - \xi$, the lepton masses, the fine-structure constant, fractal vacuum scales and their role in quantum mechanics, quantum fields and cosmology. This chapter collects selected observations and calculations that serve as first tests of this picture.

The focus is on the questions *where* the theory already achieves striking quantitative agreement and *where* deliberate caution is appropriate because calculations or data are not yet fully settled.

10.1 Leptons and the Fine-Structure Constant

A first, particularly clear test concerns the lepton masses and the fine-structure constant. Starting from the hierarchy of lepton masses one obtains an emergent scale

$$E_0 \approx 7.4\,\text{MeV}, \tag{10.1}$$

and from the relation discussed in Chapter 2

$$\alpha = \xi \left(\frac{E_0}{1\,\text{MeV}} \right)^2 \tag{10.2}$$

one finds numerically

$$\frac{1}{\alpha} \approx 137.036, \tag{10.3}$$

in very good agreement with the precise CODATA values.

Narratively this shows that the combination of ξ, fractal dimension and mass hierarchy is not only qualitatively but also quantitatively supported. At the same time, experimental and theoretical uncertainties must be kept in mind, for example from new measurements or higher-order corrections; this interplay is continuously updated and is not a closed issue.

10.2 Anomalous Magnetic Moments and Muon-$g-2$

The anomalous magnetic moments of the electron and the muon are among the most precise testing grounds of modern physics. The discussion around muon-$g-2$ shows that even small differences between theory and experiment can trigger intense debates.

Within the FFGFT the structure of these corrections can be organized geometrically: loop contributions and vacuum polarization are regularized by the fractal dimension and acquire fixed scale relations. At the same time, deliberate restraint is exercised here: the precise size of any deviation depends on many details of standard calculations and new data analyses.

At this point it is more important to understand the underlying mechanism – namely that the same geometry that organizes lepton masses and couplings also appears in precise loop corrections – than to draw far-reaching conclusions from individual numbers at an early stage.

10.3 Casimir Effect and Laboratory Vacuum

The Casimir effect provides a direct laboratory probe of vacuum forces in the micrometre range. In Chapter 5 it was shown that, with a vacuum scale L_ξ of order 100 µm determined by ξ, a relation of the form

$$\rho_{\text{CMB}} = \frac{\xi\,\hbar c}{L_\xi^4} \tag{10.4}$$

can be written such that the modified Casimir formula reproduces exactly the established standard expression

$$|\rho_{\text{Casimir}}(d)| = \frac{\pi^2 \hbar c}{240\, d^4}. \tag{10.5}$$

This links the CMB and the Casimir effect as two aspects of the same fractal vacuum structure. The precise measurements of the Casimir effect thus serve as a laboratory confirmation that the depth structure of space organized by ξ is physically effective. Here we find one of the most robust feedback loops between theory and experiment within the FFGFT.

10.4 Cosmological Tensions and Depth Structure

On the cosmological side several tensions have emerged in recent years, for example different values of the Hubble constant from local measurements and from CMB analyses. The fractal cosmology of the FFGFT interprets such tensions as an indication that a pure expansion interpretation of redshift is incomplete and that depth structure plays a role.

It is important to maintain a differentiated view:

- The FFGFT is not in conflict with the precise data, but offers an alternative reading of the underlying geometry.
- Whether this reading will withstand all future measurements remains the subject of ongoing analyses.
- First comparisons show that many observed effects can be naturally embedded into time–mass duality without introducing new dark components.

10.5 Quantum Computers, Simulations and Numerical Tests

In the area of quantum information, simulations of algorithms such as Shor's procedure provide additional points of contact. As described in Chapter 4, logical states can be interpreted as occupations of fractal basis functions $G_n(t)$, and typical algorithms exploit interference patterns that arise from this structure.

Numerical simulations show that the success probabilities and interference structures of standard algorithms can be reproduced when the scales set by ξ are implemented consistently. These results are more conceptual confirmations than precision measurements; they demonstrate that time–mass duality also remains coherent where quantum information and field theory meet.

10.6 Attosecond Formation of Quantum Entanglement

A recent theoretical study by Jiang et al. shows that quantum entanglement in a helium system driven by intense EUV pulses does not arise instantaneously, but builds up over a local time window of around 232 as. The final energy of the bound electron correlates directly with the emission time of the escaping electron, so that the joint quantum history can be reconstructed; a double-pulse experiment with coincidence detection is proposed. From the perspective of time–mass duality this provides a strong conceptual hint that entanglement is a time-resolved, causal process within a finite interaction window and does not require any "spooky action at a distance". A more detailed discussion in the spirit of the FFGFT is given in a dedicated T0 document on attosecond entanglement formation in the repository (see the corresponding PDF in texttt2/pdf).

10.7 Summary

The precision tests and observations sketched here provide different perspectives on one and the same geometric core. In some places – for example for lepton masses, the fine-structure constant and the Casimir effect – the agreement is already impressively concrete. In other areas – in particular muon-$g-2$ and cosmological tensions – the narrative is deliberately cautious and leaves room for future data.

Overall, a picture emerges in which time–mass duality is not just an elegant theoretical construct, but is connected with observed physics on many fronts.

Chapter 11

Computing with Time–Mass Duality

This chapter presents a few worked examples that show how, with a small set of formulas from time–mass duality, concrete quantities can be estimated. The examples are deliberately simple and do not replace full technical derivations, but they make the functioning of the approach transparent.

11.1 From ξ and E_0 to the Fine-Structure Constant

We start from the value

$$\xi = \frac{4}{3} \times 10^{-4} \tag{11.1}$$

and the scale obtained from the lepton hierarchy

$$E_0 \approx 7.4\,\text{MeV}. \tag{11.2}$$

The relation introduced in earlier chapters reads

$$\alpha(\xi, E_0) = \xi \left(\frac{E_0}{1\,\text{MeV}} \right)^2 . \tag{11.3}$$

Inserting the values one schematically obtains

$$\alpha \approx \left(\tfrac{4}{3} \times 10^{-4} \right) \times (7.4)^2. \tag{11.4}$$

Squaring gives

$$(7.4)^2 \approx 54.76, \tag{11.5}$$

so that

$$\alpha \approx \tfrac{4}{3} \times 10^{-4} \times 54.76 \approx 0.007297 \tag{11.6}$$

which implies

$$\frac{1}{\alpha} \approx 137.0. \tag{11.7}$$

Details such as rounding and higher-order corrections shift the final digits; the crucial point here is that the structure

$$\alpha \sim \xi E_0^2 \tag{11.8}$$

is compatible with the observed fine-structure constant. The example illustrates how directly ξ and a single scale E_0 enter into a central constant of nature.

11.2 From CMB Energy Density to the Scale L_xi

A second example concerns the link between the CMB and the Casimir effect. Starting from the observed energy density of the cosmic microwave background ρ_{CMB} and the relation

$$\rho_{\text{CMB}} = \frac{\xi \hbar c}{L_\xi^4} \tag{11.9}$$

we can estimate a characteristic vacuum length L_ξ.

Solving for L_ξ yields

$$L_\xi = \left(\frac{\xi \hbar c}{\rho_{\text{CMB}}} \right)^{1/4}. \tag{11.10}$$

Inserting the known values for $\hbar$, c and ρ_{CMB} gives a value of order

$$L_\xi \sim 100\,\mu\text{m}. \tag{11.11}$$

This is exactly the scale at which precision Casimir experiments are particularly sensitive. Time–mass duality thus connects a cosmological quantity (the CMB energy density) with a laboratory phenomenon at micrometre distances.

11.3 Fractal Dimension as an Everyday-Looking Value

The fractal dimension of spacetime is

$$D_f = 3 - \xi \approx 2.999867. \tag{11.12}$$

In everyday life this difference from a smooth 3D geometry appears negligible. For integrals over extremely high momenta or very small separations, however, it acts like an additional exponent that determines convergence or divergence.

A simple heuristic is:

- Where classical theories use integrals of the form $\int \mathrm{d}^3 k$, the FFGFT effectively replaces them by a slightly modified measure $\int \mathrm{d}^{D_f} k$.
- The tiny lowering of D_f is sufficient to translate many divergent contributions into finite, regulated quantities.

This everyday perspective makes it clear that the numerical values of ξ and D_f do not stand apart from familiar dimensions, but shift them only minimally – with large impact in the UV regime.

11.4 How to Continue Calculating

The examples shown here are intentionally simple and are meant to encourage readers to perform their own back-of-the-envelope calculations. Those who wish to go deeper will find complete derivations and numerical studies in the technical volumes of the FFGFT.

For practical work it is useful

- to take central formulas of time–mass duality (for example for α, E_0, L_ξ) as a starting point,
- to first work purely with ratios and integer or rational numbers (without early floating-point approximations and without prematurely inserting constants such as π) in order to retain numerical precision for very small quantities,
- to estimate the effects of small variations in ξ or the scales,
- and to compare new data – for example precision constants or Casimir measurements – systematically against these structures.

In this way time–mass duality becomes a usable tool: it provides not only a conceptual explanation, but also concrete computational routes that allow known and new phenomena to be placed quantitatively.

Chapter 12

Natural Units and Re-Read Constants

In the previous chapters several scales were introduced that follow directly from time–mass duality and the parameter ξ: the energy scale E_0 in the MeV range, a minimal length scale $L_0 = \xi L_P$ below the Planck length, and a vacuum length scale L_ξ around $100\,\mu\mathrm{m}$. This chapter explains why the use of *natural units* is key to understanding these relations – and why some familiar units (such as the Coulomb) have to be re-read in this framework.

12.1 Why Natural Units?

The international SI system is optimised for practical measurability and technical applications: metre, kilogram, second, ampere and kelvin are historically grown quantities tied to laboratory standards. For the structure of the fundamental laws they are often inconvenient, because they "hide" central constants such as c, $\hbar$ and the elementary charge e inside the units themselves.

Natural units follow a different strategy:

- One sets fundamental constants such as c and $\hbar$ equal to one.
- Lengths, times and energies are directly converted into each other.
- Many seemingly complicated constants disappear from formulas and make room for dimensionless ratios.

It is crucial, however, that $c = 1$ does *not* mean that "energy and mass are always the same"; it simply abbreviates the familiar relation $E = mc^2$ to $E = m$ in the rest frame of a particle, while dynamically the full equation $E^2 = p^2 + m^2$ remains valid. Analogous remarks apply to $\hbar = 1$ and (in suitable normalisation) $\alpha \approx 1/137$: setting them to one is a matter of notation, not new physics – the logical step back to physical quantities must always be kept in mind and ultimately carried out explicitly by checking units.

In the context of time–mass duality quantities such as E_0, L_0 and L_ξ serve as natural scales of a fractally organised space; their full meaning only becomes visible when, after calculations in natural units, one carefully converts back to familiar SI units and compares the resulting scales with the data.

12.2 The Double View on α, c and $\hbar$

The fine-structure constant α is the classic example of how much the choice of units shapes our understanding. In SI notation a common form reads

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}, \tag{12.1}$$

where e is the elementary charge, ε_0 the electric constant, $\hbar$ the reduced Planck constant and c the speed of light.

This representation suggests four independent quantities. In natural units with $c = \hbar = 1$ and a suitable normalisation of the electromagnetic field the relation reduces to

$$\alpha = \frac{e^2}{4\pi}, \tag{12.2}$$

so that α directly describes the square of a dimensionless coupling.

Time–mass duality adds a second, complementary view:

$$\alpha = \xi \left(\frac{E_0}{1\,\text{MeV}}\right)^2. \tag{12.3}$$

The fractal structure encoded in this relation only becomes fully visible once α in this form is translated back into concrete units and numerical values. Thus α appears at the same time

- as a ratio of charge to light and action quanta ($e^2/4\pi\hbar c$), and
- as a geometrically organised number built from ξ and the fractally emergent scale E_0.

This double view becomes particularly transparent when units are chosen such that c and $\hbar$ do not appear as "decorative factors" on the side of formulas but as structuring scales.

12.3 Re-Reading the Coulomb

In the SI system the unit of charge, the Coulomb, is a historically defined quantity based on the ampere and ultimately on macroscopic currents. From an FFGFT perspective this is unsatisfactory, because the fundamental processes in the electromagnetic sector are not governed by macroscopic currents in conductors but by quantised charge carriers and their couplings to the field.

Natural units provide a clearer picture here:

- One normalises the electromagnetic field such that e becomes a dimensionless quantity.
- The effective unit of charge is determined by α and the chosen values of c and $\hbar$.
- Instead of "coulomb" as an independent base unit one obtains a geometry in which charge measures how strongly a field couples to the fractal time–mass structure.

In this picture e is not a freely adjustable parameter, but fixed by α and the scales set by ξ. The SI coulomb can then be interpreted as a derived quantity that is practical for macroscopic currents but hides the underlying geometry.

12.4 Newly Chosen Units for a Clear Geometry

Time–mass duality suggests choosing units deliberately such that geometric relations become visible:

- Base units are oriented towards natural scales such as E_0, L_0 and L_ξ.
- c and $\hbar$ are used as conversion factors between time, length and energy, not as "extra numbers".
- Electromagnetic quantities are normalised such that α appears directly as a quadratic coupling.

In practice this can mean, for example:

- An energy unit in the MeV range (near E_0) makes the role of the lepton scale visible.
- A length unit around L_ξ highlights the connection between CMB and Casimir effect.
- Time intervals are systematically linked to local mass densities, as suggested by time–mass duality.

Such decisions are not merely a matter of taste; they determine whether patterns in the data are recognised as a coherent whole or disappear behind a multitude of conversion factors.

12.5 Natural Units as a Tool for Thinking

Natural units force us to treat constants such as c, $\hbar$ and e not as "ornaments" in formulas, but as expressions of concrete geometric structures. In the FFGFT these structures are organised by ξ, the fractal dimension D_f and the resulting scales.

Those who calculate in natural units more quickly see where genuinely new physics resides:

- Unit conversions disappear and make room for dimensionless quantities.
- Differences between models can be clearly located in changed couplings or scales.
- The connection between micro and macro world (from lepton masses to Hubble scales) becomes visible as a relation between a few numbers and scales.

In this sense natural units are not just a technical convenience, but a thinking tool: they reveal the geometric core of time–mass duality and show how α, c, $\hbar$ and e can be understood as different projections of the same fractal structure.

12.6 What Is Lost When Setting c, $\hbar$, G and α to One

In practice it is tempting to simply "scale away" all constants. For the Xi narrative, however, it is crucial which aspects of the fractal structure become invisible in the process:

- If one sets $c = 1$, the explicit speed of light disappears from the equations. The Lorentz structure and the separation between space and time remain intact, but the contrast between nonrelativistic and relativistic scales becomes less visible.
- If one sets $\hbar = 1$, one loses the explicit scale at which processes become "quantum". The limit $\hbar \to 0$ and the comparison "small compared to $\hbar$" versus "large compared to $\hbar$" vanish as a separate sequence of steps from the formulas.
- If one sets $G = 1$, the coupling of spacetime curvature to energy–momentum becomes dimensionless. As a result, the direct link between local densities, curvature radii and the fractally organised scales L_0 and L_ξ is absorbed into a choice of units.
- Finally, if one tries to "set α to one", one is no longer merely choosing units, but making a *physical assumption* about the strength of the electromagnetic coupling. In the FFGFT this would precisely erase the information that α can be read as a fractal function of scale – the fine-structured interactions are compressed into a single smooth number.

Historically this was also the starting point of the FFGFT perspective presented here: only when $\alpha = 1$ was consciously and purposefully set in intermediate calculations did the underlying three-dimensional geometric relations become clearly visible. It was precisely the comparison between this "smoothed" picture and the later reconstructed fractal scale dependence that revealed how much additional structure is contained in a variable, geometrically organised fine-structure constant.

For concrete calculations this means: one may in a first step work with $\alpha = 1$ in a smoothed, three-dimensional geometry, provided that in every formula it is clearly recorded with which power α actually enters (e.g. $\sigma \propto \alpha^2$, energy levels $\propto \alpha^2$, lifetimes $\propto \alpha^{-1}$, and so on). In this step all manipulations become transparent, but the fractal scale dependence of α is deliberately "switched off". In a second, equally systematic step the corresponding factors of α – with the correct power and at the appropriate scale – are then explicitly reinserted during the back-conversion, thereby reconstructing the fractal coupling structure. Only at this stage does one decide whether α is to be read as constant or as a running, fractally organised quantity.

In the spirit of the Xi narrative one can say: c, $\hbar$ and G can be hidden in the background as conversion factors without fundamentally destroying the fractal structure; they become harder to see, but remain conceptually present. If, however, we were to also set α consistently to one, the model would be reduced to an almost purely three-dimensional, smooth geometry – precisely the fine fractal scale structure of the couplings that the Xi book highlights would be lost from the formalism, even though it would still act within the data.

12.7 Worked Examples: Switching α Off and On Again

To make this procedure tangible, let us briefly sketch how a concrete calculation is handled in this two-step picture:

1. **Geometric step with $\alpha = 1$:** one rewrites all relevant observables in a form where their dependence on α is explicit, for example $\sigma(E) = C(E)\,\alpha^2$ for a cross section, an energy shift $\Delta E \propto \alpha^2$, or a lifetime $\tau \propto \alpha^{-1}$. In this first step one sets $\alpha = 1$ and studies only the geometric prefactors $C(E)$ and their dependence on scales such as E_0, L_0 and L_ξ.
2. **Reconstruction step with physical α:** in a second pass one restores the full factors of α with the correct powers and evaluates them at the physically relevant scales. This is the place where the fractal running of α with energy or length enters and where the Xi narrative interprets the data as a projection of a deeper fractal geometry.

In everyday practice a theorist may therefore quite legitimately "forget" α in the first pass to see only the clean three-dimensional geometric structure, as long as the bookkeeping of powers of α is done carefully. What makes the FFGFT/Xi perspective special is the insistence that the second step is not optional: precisely in the controlled re-introduction of $\alpha(E)$ lies the key to understanding how a deterministic, fractal field theory can match probabilistic-looking data and still leave room for effective freedom, emergent decision-making and conscious agency on macroscopic scales.

Chapter 13

Why Checking Units is Essential

Natural units make many formulas look simpler: constants such as c and $\hbar$ disappear from the notation, and couplings such as α turn into apparently pure numbers. Within time–mass duality this is useful – but it also carries the risk of forgetting which physical scales are acting in the background. This chapter explains why a systematic units check is indispensable and how it reveals the fractal structure in full.

13.1 Natural Units as an Intermediate Space

When working in natural units with $c = \hbar = 1$, many relations become very compact. For example, in a suitable normalisation the fine-structure constant appears simply as

$$\alpha = \frac{e^2}{4\pi}, \tag{13.1}$$

and the structure organised by ξ as

$$\alpha = \xi \left(\frac{E_0}{\text{MeV}} \right)^2. \tag{13.2}$$

In this intermediate space of natural units the geometry is particularly clear. For a statement to be physically convincing, however, one must take the way back: from the compact notation to the actual measurable quantities in SI units.

13.2 Back-Conversion as a Hard Test

The fractal structure and the scales defined by ξ only show their robustness when the conversion back to SI units consistently reproduces all known numbers. Concretely this means:

- One starts with a simple relation in natural units (e.g. $\alpha \sim \xi E_0^2$).
- One systematically reinstates all factors of c, $\hbar$ and the chosen base quantities.
- In particular one re-inserts α in the form $\alpha = \xi(E_0/\text{MeV})^2$ completely, instead of treating it as a bare number.
- One checks whether the resulting values for energies, lengths and times agree with experimental data.

Only this hard test shows whether an apparently elegant formula is more than number play. For time–mass duality this means: the shortcut via natural units is helpful, but the physical content is decided when translating back into concrete units. Dangerous are "clever" simplifications: if one prematurely cancels constants such as c, $\hbar$ or even α, the fractal structure can become invisible and seemingly compelling but physically wrong scales may arise. Especially in natural units it is tempting to turn $E = mc^2$ directly into $E = m$ or to treat $\alpha = \xi(E_0/\text{MeV})^2$ as a pure number; the correct physical conclusion, however, always requires keeping in mind the underlying assumptions (rest frame, momentum, concrete scales) and re-inserting them explicitly at the end.

13.3 Example: CMB, Casimir and L_ξ

A particularly illustrative example is the relation

$$\rho_{\text{CMB}} = \frac{\xi\,\hbar c}{L_\xi^4}, \tag{13.3}$$

with which a characteristic length scale L_ξ can be estimated.

In natural units $\hbar$ and c look like harmless factors. Only when one inserts the SI values for $\hbar$, c and ρ_{CMB} and carefully tracks the dimensions does it become clear that L_ξ indeed lies in the range of $100\,\mu\text{m}$ – precisely where Casimir experiments measure with high precision.

Without consistent checking of units this connection could easily be overlooked or misjudged. The fractal structure thus becomes visible not only "in the head", but in the concrete back-calculation to real observables.

13.4 Avoiding Spurious Relations

Conversely, a strict units check helps to distinguish accidental numerical overlaps from genuine relations. Two numbers may look similar in natural units; if their dimensions differ, it is clear that they are not directly comparable.

Time–mass duality therefore works consistently with dimensionless combinations (such as α) and clearly defined scales (such as E_0, L_0, L_ξ) before drawing comparisons. Each step is accompanied by bookkeeping of units:

- Which quantity is truly dimensionless?
- Which combinations of c, $\hbar$ and base units appear?
- Where might apparently similar numbers in fact encode different physical content?

13.5 Units as Integrity Check of the Theory

In the end, checking units is more than a technical formality. It acts as an integrity check for the entire theory:

- It enforces consistency between the geometric picture and measurable quantities.
- It reveals whether a proposed relation is truly compatible across scales.
- It protects against overextended interpretations of apparently nice numbers.

For the FFGFT and time–mass duality this means: only the combination of natural units *and* systematic back-checking in SI units reveals how deeply the fractal structure penetrates observed physics. Natural units are thus a useful working space – but the reality check takes place in the familiar units of our measuring instruments. At the same time a philosophical caveat remains: every measurement ultimately compares frequencies or count rates and therefore yields only relative statements; what ontologically "really" runs slower or is heavier escapes direct testability. For the FFGFT this means: it is not decisive whether we can absolutely decide if time slows down or mass increases; what matters is that the mathematical structure is consistent and reproduces all observable relations (frequencies, scales, ratios).

Chapter 14

FFGFT as a Lagrangian Extension

Time–mass duality and the Fundamental Fractal-Geometric Field Theory (FFGFT) are not meant to replace established theories, but to *extend* them. Rather than proposing a new "super model" against quantum field theory, the Standard Model or general relativity, the FFGFT sees itself as a structural supplement: it postulates a fractal geometry in which the known Lagrangian densities appear as effective descriptions at particular scales.

14.1 Lagrangian Densities as Common Language

Modern physics formulates almost all successful theories in the language of Lagrangian densities:

- the Dirac and Klein–Gordon equations for quantum fields,
- the Yang–Mills theories of the Standard Model,
- the Einstein–Hilbert action of general relativity.

In all these cases the Lagrangian density is not merely mathematical convenience, but the most compact encoding of symmetries and conservation laws. The FFGFT connects precisely here: it does *not* directly change the familiar form of these Lagrangians, but supplements them with a fractal background structure and additional terms organised by ξ.

14.2 Fractal Geometry as Additional Structure

In the Xi narrative the fractal dimension $D_f = 3 - \xi$ was introduced as a global measure of the folding depth of space. At the level of Lagrangian densities this means that integrals of the form

$$S = \int \mathrm{d}^3x \, \mathcal{L} \tag{14.1}$$

pass over into a slightly modified form

$$S_{\text{frac}} = \int \mathrm{d}^{D_f} x \, \mathcal{L}_{\text{eff}}, \tag{14.2}$$

where $\mathcal{L}_{\text{eff}}$ carries the same symmetry structure as the original Lagrangian, but is additionally regulated by the fractal measure structure.

In practice this means:

- The form of the Dirac, Maxwell or Yang–Mills Lagrangian is preserved.
- The fractal geometry changes how self-energies and loop integrals converge.
- The familiar results of quantum field theory are reproduced in the appropriate limit ($\xi \to 0$, $D_f \to 3$).

14.3 Extension Rather than Competitor

Established theories such as the Standard Model or general relativity have an impressive experimental basis. The FFGFT takes these successes seriously and sees itself not as a replacement, but as an extension in two steps:

1. **Geometric deepening:** spacetime acquires a fractal depth structure with $D_f = 3 - \xi$, from which scales such as E_0, L_0 and L_ξ emerge.
2. **Lagrangian supplement:** the known Lagrangians are read in such a way that their parameters (masses, couplings) are not free, but organised by this fractal geometry.

In this sense the FFGFT is a *theory of Lagrangian densities*: it does not seek a single "Lagrangian for everything", but asks how the multitude of established effective Lagrangians is anchored in a common fractal geometry.

14.4 How FFGFT Differs from General Relativity

From the point of view of general relativity, the FFGFT introduces several structural changes that are central to the concept of time–mass duality:

- The spacetime manifold acquires a fractal depth with effective spatial dimension $D_f = 3 - \xi$; curvatures and volumes are evaluated with respect to this depth structure.
- Rest mass is no longer a strictly fixed parameter along a worldline, but an effective mass field $m(x)$ generated by the time field; only in simple situations is this well approximated by a constant value.

- The gravitational constant G is interpreted as an emergent coupling that can be expressed in terms of ξ and the natural scales E_0, L_0 and L_ξ rather than being postulated as a fundamental constant.
- In introductory chapters a simplified Lagrangian is used in which ξ mainly organises masses, couplings and cutoffs; the extended Lagrangian of the full FFGFT adds the fractal measure structure and explicit vacuum terms that encode the running of couplings and masses.

Historically, Einstein's formulation keeps rest masses fixed and encodes all dynamics in the curvature of spacetime; when quantum fields and self-energies are added, this leads to complicated regularisation and renormalisation tricks to tame divergences. These differences specify in which sense the FFGFT goes beyond general relativity while still reproducing all local tests of gravity in the appropriate limit.

14.5 What Does *Not* Change

For understanding it is important to spell out explicitly what does *not* change:

- The locally measured effects of general relativity (e.g. GPS corrections, light deflection, perihelion precession) remain intact.
- The predictions of the Standard Model for cross sections, decay widths and precision observables are respected.
- QED with its extremely accurate description of $g-2$ is also contained within the allowed parameter range of the FFGFT.

The extension sets in where observations point to new scales: in the mass hierarchy, the number 137, the link between CMB and Casimir effect, or subtle deviations in precision tests. In these areas the FFGFT offers additional structure without abandoning the established Lagrangian theories.

14.6 Outlook: A Fractal Theory of Everything

A complete Lagrangian picture of the FFGFT would combine all the building blocks mentioned – fractal geometry, time–mass duality, the scales E_0, L_0, L_ξ and the existing Lagrangians of QFT and gravity – in a single action functional. At the level of the field equations this description remains deterministic; only the fractal, recursive variation of initial conditions across many scales opens up an effective room for consciousness, agency and emergent decisions without violating the underlying dynamics. For practical purposes, and because of the extremely complex coupling of the deterministic equations, concrete calculations are nevertheless often realistically possible only with probabilistic methods, effective field theories or Monte-Carlo techniques, even though they are based on an ultimately deterministic foundation. The Xi narrative provides the conceptual guidelines for this: FFGFT is to be read as an extension that embeds established Lagrangian theories in a larger geometric context, not as a theory that replaces them.

Chapter 15

Sources and Further Reading

This chapter lists the most important external references cited in the Xi narrative and points to supplementary T0 documents in the repository.

Bibliography

[Modesto(2008)] L. Modesto, "Fractal Structure of Loop Quantum Gravity," Class. Quantum Grav. 26 (2009) 242002, arXiv:0812.2214 [gr-qc].

[Modesto(2009)] L. Modesto, "Fractal Quantum Space-Time," arXiv:0905.1665 [gr-qc].

[Calcagni(2010)] G. Calcagni, "Fractal universe and quantum gravity," Phys. Rev. Lett. 104 (2010) 251301, arXiv:0912.3142 [hep-th].

[Calcagni(2010b)] G. Calcagni, "Quantum field theory, gravity and cosmology in a fractal universe," JHEP 03 (2010) 120, arXiv:1001.0571 [hep-th].

[Calcagni(2012)] G. Calcagni, "Introduction to multifractional spacetimes," AIP Conf. Proc. 1483 (2012) 31, arXiv:1209.1110 [hep-th].

[Hořava(2009)] P. Hořava, "Spectral Dimension of the Universe in Quantum Gravity at a Lifshitz Point," Phys. Rev. Lett. 102 (2009) 161301, arXiv:0902.3657 [hep-th].

[Thürigen(2015)] J. Thürigen, "Discrete Quantum Geometries," arXiv:1511.08737 [gr-qc].

[Jiang et al.(2024)] W.-C. Jiang, M.-C. Zhong, Y.-K. Fang, S. Donsa, I. Březinová, L.-Y. Peng, J. Burgdörfer, "Time Delays as Attosecond Probe of Interelectronic Coherence and Entanglement," Phys. Rev. Lett. 133 (2024) 163201, doi:10.1103/PhysRevLett.133.163201.

[NASA Space News(2026)] NASA Space News, "Scientists Measure Quantum Entanglement Speed – And It Breaks Physics," YouTube video, 14 January 2026, https://www.youtube.com/watch?v=t3wjY95zvNM (accessed 15 January 2026).

[Pascher(2026a)] J. Pascher, "Fractal spacetime and its implications in quantum gravity," internal T0 document 141_Renormierung_En (2026), available as PDF in the GitHub repository at https://github.com/jpascher/T0-Time-Mass-Duality/2/pdf/141_Renormierung_En.pdf.

[Pascher(2026b)] J. Pascher, “Attosecond prediction for the emergence of quantum entanglement as evidence for the T_0 time–mass duality theory,” internal T0 document 142_Experimet-verschr"ankung_En (2026), available as PDF in the GitHub repository at https://github.com/jpascher/T0-Time-Mass-Duality/2/pdf/142_Experimet-verschr"ankung_En.pdf.

[Pascher(2025a)] J. Pascher, “T0 particle masses and lepton hierarchy,” internal T0 document 006_T0_Teilchenmassen_En (2025), available as PDF in the GitHub repository at https://github.com/jpascher/T0-Time-Mass-Duality/2/pdf/006_T0_Teilchenmassen_En.pdf.

[Pascher(2025b)] J. Pascher, “Fine-structure constant and fractal geometry,” internal T0 documents 044_Feinstrukturkonstante_En and 043_ResolvingTheConstantsAlfa_En (2025), available as PDFs in the GitHub repository at /044_Feinstrukturkonstante_En.pdf and /043_ResolvingTheConstantsAlfa_En.pdf.

[Pascher(2025c)] J. Pascher, “Natural units and their systematics,” internal T0 document 015_NatEinheitenSystematik_En (2025), available as PDF in the GitHub repository at /015_NatEinheitenSystematik_En.pdf.

[Pascher(2025d)] J. Pascher, “T0, natural units and SI,” internal T0 documents 014_T0_nat-si_En and 013_T0_SI_En (2025), available as PDFs in the GitHub repository at /014_T0_nat-si_En.pdf and /013_T0_SI_En.pdf.

[Pascher(2025e)] J. Pascher, “T0 cosmology and fractal geometry,” internal T0 documents 026_T0_Geometrische_Kosmologie_En and 025_T0_Kosmologie_En (2025), available as PDFs in the GitHub repository at /026_T0_Geometrische_Kosmologie_En.pdf and /025_T0_Kosmologie_En.pdf.