Reformulation of Lagrangian Densities in Time-Mass Duality

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Introduction

I will attempt to develop a consistent reformulation of the fundamental Lagrangian densities based on the time-mass duality theory. The goal is to create a mathematically coherent and physically meaningful formulation that captures all essential aspects of the theory.

1 Fundamental Principles

Let us begin with the fundamental principles of time-mass duality:

- Intrinsic Time: $T = \frac{\hbar}{mc^2}$
- Modified Time Derivative: $\partial_{t/T} = \frac{\partial}{\partial(t/T)} = T\frac{\partial}{\partial t}$
- Duality between: Standard Picture (time dilation, constant mass) and Alternative Picture (absolute time, variable mass)

2 Modified Lagrangian Density for Scalar Fields

The standard Lagrangian density for a scalar field (e.g., the Higgs field) is:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - V(\phi) \tag{1}$$

In time-mass duality, this becomes:

$$\mathcal{L}_{\text{scalar-T}} = \frac{1}{2}(D_{T\mu}\phi)(D_T^\mu \phi) - \frac{1}{2}m^2\phi^2 - V(\phi) \tag{2}$$

where the modified covariant derivative is defined as:

$$D_{T\mu}\phi = T(x)\partial_\mu \phi + \phi\partial_\mu T(x) \tag{3}$$

Explicitly written:

$$\mathcal{L}_{\text{scalar-T}} = \frac{1}{2}T(x)^2\left(\frac{\partial\phi}{\partial t}\right)^2 + T(x)\phi\frac{\partial\phi}{\partial t}\frac{\partial T(x)}{\partial t} - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m^2\phi^2 - V(\phi) \quad (4)$$

3 Complete Higgs Lagrangian Density

For the Higgs field as a complex doublet, we obtain:

$$\mathcal{L}_{\text{Higgs-T}} = (D_{T\mu}\Phi_T)^\dagger(D_T^\mu\Phi_T) - V_T(\Phi_T) \quad (5)$$

with the covariant derivative:

$$D_{T\mu}\Phi_T = T(x)(\partial_\mu + ig\tau^a W_\mu^a + ig'\frac{Y}{2}B_\mu)\Phi_T + \Phi_T\partial_\mu T(x) \quad (6)$$

The Higgs potential retains its form:

$$V_T(\Phi_T) = -\mu^2\Phi_T^\dagger\Phi_T + \lambda(\Phi_T^\dagger\Phi_T)^2 \quad (7)$$

4 Reformulated Yukawa Coupling

The Yukawa coupling is modified to:

$$\mathcal{L}_{\text{Yukawa-T}} = -y_f\bar{\psi}_L\Phi_T\psi_R + \text{h.c.} \quad (8)$$

The transformation function $\mathcal{T}(\gamma)$ is not explicitly needed here, as mass variation is implicitly accounted for by $T(x)$.

5 Lagrangian Density for Fermions

The Dirac Lagrangian density for fermions becomes:

$$\mathcal{L}_{\text{Dirac-T}} = \bar{\psi}(i\gamma^\mu D_{T\mu} - m)\psi \quad (9)$$

with:

$$D_{T\mu}\psi = T(x)D_\mu\psi + \psi\partial_\mu T(x) \quad (10)$$

where D_μ is the usual covariant derivative with gauge fields.

6 Gauge Boson Lagrangian Density

For gauge bosons, the Lagrangian density is modified to:

$$\mathcal{L}_{\text{Gauge-T}} = -\frac{1}{4}T(x)^2 F_{\mu\nu}F^{\mu\nu} \tag{11}$$

with the unchanged field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \tag{12}$$

7 Unified Formulation of the Complete Lagrangian Density

The total Lagrangian density is now:

$$\mathcal{L}_{\text{Total-T}} = \mathcal{L}_{\text{Higgs-T}} + \mathcal{L}_{\text{Dirac-T}} + \mathcal{L}_{\text{Yukawa-T}} + \mathcal{L}_{\text{Gauge-T}} \tag{13}$$

8 Field Equations from the Modified Lagrangian Density

The field equations are derived by applying the Euler-Lagrange equations:

For the Higgs field:

$$D_{T\mu}D_T^\mu \Phi_T + \frac{\partial V_T}{\partial \Phi_T^\dagger} = 0 \tag{14}$$

For fermions:

$$(i\gamma^\mu D_{T\mu} - m)\psi = 0 \tag{15}$$

For gauge bosons:

$$\partial_\mu(T(x)^2 F^{\mu\nu}) + ig[A_\mu, T(x)^2 F^{\mu\nu}] = j^\nu \tag{16}$$

9 Incorporation of Gravity via Modified Einstein-Hilbert Action

The Einstein-Hilbert action is modified to:

$$S_{\text{Grav-T}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,T(x)R \tag{17}$$

where R is the Ricci scalar, adjusted by $T(x)$.

10 Summary and Consistency Check

The reformulation is based on the consistent introduction of $T(x)$ into all derivatives and field terms. The theory should:

- Remain Lorentz-invariant under consideration of the duality
- Correctly describe phenomena such as time dilation and mass variation
- Predict testable deviations from the Standard Model

11 Comprehensive Lagrangian Density of the Time-Mass Duality Theory

The complete Lagrangian density is:

$$\mathcal{L}_{\text{Total-T}} = \mathcal{L}_{\text{Higgs-T}} + \mathcal{L}_{\text{Fermion-T}} + \mathcal{L}_{\text{Gauge-T}} + \mathcal{L}_{\text{Yukawa-T}} \tag{18}$$

11.1 Higgs Sector

$$\mathcal{L}_{\text{Higgs-T}} = (D_{T\mu}\Phi_T)^\dagger (D_T^\mu \Phi_T) - V_T(\Phi_T) \tag{19}$$

with:

- $D_{T\mu}\Phi_T = T(x)(\partial_\mu + ig\tau^a W_\mu^a + ig'\frac{Y}{2}B_\mu)\Phi_T + \Phi_T \partial_\mu T(x)$
- $V_T(\Phi_T) = -\mu^2 \Phi_T^\dagger \Phi_T + \lambda(\Phi_T^\dagger \Phi_T)^2$
- $T(x) = \frac{\hbar}{mc^2}$

11.2 Fermion Sector

$$\mathcal{L}_{\text{Fermion-T}} = \sum_f \bar{\psi}_f (i\gamma^\mu D_{T\mu} - m_f)\psi_f \tag{20}$$

11.3 Gauge Boson Sector

$$\mathcal{L}_{\text{Gauge-T}} = -\frac{1}{4}T(x)^2 (G_{\mu\nu}^a G^{a\mu\nu} + W_{\mu\nu}^a W^{a\mu\nu} + B_{\mu\nu}B^{\mu\nu}) \tag{21}$$

11.4 Yukawa Sector

$$\mathcal{L}_{\text{Yukawa-T}} = -\sum_f y_f \bar{\psi}_{fL} \Phi_T \psi_{fR} + \text{h.c.} \tag{22}$$

11.5 Energy-Momentum Relation

The modified energy-momentum relation is:

$$E^2 = (pc)^2 + (mc^2)^2 + \alpha \frac{\hbar c}{T} \tag{23}$$