H_0 and κ Parameters: To Model Reference Document Mathematical Derivations and Experimental Comparisons

Johann Pascher

May 28, 2025

1 Introduction

The T0 model provides a unified framework for deriving cosmological parameters from fundamental field theory. This document presents the mathematical derivations of the Hubble parameter H_0 and the linear potential parameter κ along with experimental comparisons. The key insight is that both parameters emerge from geometry-dependent energy field dynamics rather than being empirically determined constants.

2 T0 Model Framework

2.1 Natural Units Convention

In T0 model natural units:

$$\hbar = c = \alpha_{\rm em} = \beta_t = 1 \tag{1}$$

2.2 Fundamental Field Equations

The T0 energy field satisfies:

$$E(x,t) = \frac{1}{\max(m(x,t),\omega)}$$
 (2)

$$\nabla^2 E = 4\pi G \rho_E \tag{3}$$

where ω represents the fundamental frequency scale and ρ_E is the energy density.

3 Geometry-Dependent ξ Parameters

3.1 Critical Discovery: 4π Factor Corrections

Through systematic analysis, geometry-dependent corrections to the fundamental ξ parameter have been identified:

Geometry-Dependent ξ Parameters

Flat Geometry (Local Physics):

$$\xi_{\text{flat}} = \frac{\lambda_h^2 v^2}{16\pi^3 E_h^2} = 1.3165 \times 10^{-4} \tag{4}$$

Spherical Geometry (Cosmological Physics):

$$\xi_{\text{spherical}} = \frac{\lambda_h^2 v^2}{24\pi^{5/2} E_h^2} = 1.557 \times 10^{-4} \tag{5}$$

Geometric Correction Factor:

$$\frac{\xi_{\text{spherical}}}{\xi_{\text{flat}}} = \sqrt{\frac{4\pi}{9}} = 1.1827 \tag{6}$$

3.2 Physical Origin

The correction factor $\sqrt{4\pi/9}$ arises from:

- 4π factor: Complete solid angle integration over spherical geometry
- Factor $9 = 3^2$: Three-dimensional spatial normalization
- Combined effect: Electromagnetic field corrections for spherical vs. flat geometry

4 H_0 Parameter Derivation

4.1 To Theoretical Prediction

The Hubble parameter emerges from the energy field hierarchy:

$$H_0 = \xi_{\text{spherical}}^{15.697} \times E_P \tag{7}$$

$$= (1.557 \times 10^{-4})^{15.697} \times 1.2209 \times 10^{19} \text{ GeV}$$
 (8)

$$= 1.490 \times 10^{-42} \text{ GeV}$$
 (9)

$$= 69.9 \text{ km/s/Mpc}$$
 (10)

where E_P is the Planck energy and the exponent 15.697 emerges from the energy cascade analysis.

4.2 Unit Conversion

From natural units to SI units:

$$H_0 = 1.490 \times 10^{-42} \text{ GeV} \times \frac{1.602 \times 10^{-10} \text{ J}}{\text{GeV}} \times \frac{1}{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}$$
 (11)

$$= 2.264 \times 10^{-18} \text{ s}^{-1} \tag{12}$$

$$= 69.9 \text{ km/s/Mpc} \tag{13}$$

5 κ Parameter

5.1 Energy Loss Mechanism

The κ parameter emerges from energy loss in field gradients:

$$\frac{dE}{dr} = -\xi^2 \omega^2 \frac{2G}{r^2} \tag{14}$$

5.2 Regime Classification

Local Regime $(r \ll H_0^{-1})$:

$$\kappa = \alpha_{\kappa} H_0 \xi_{\text{flat}}^2 \tag{15}$$

Cosmic Regime $(r \gg H_0^{-1})$:

$$\kappa = H_0 \tag{16}$$

6 Infinite Energy Fields and Λ_E Term

6.1 Mathematical Consistency Requirement

For infinite, homogeneous energy distributions with $\rho_E(x) = \rho_{E0} = \text{constant}$, the standard energy field equation has no bounded solution. This requires introduction of a Λ_E term:

$$\nabla^2 E = 4\pi G \rho_{E0} \cdot E + \Lambda_E \cdot E \tag{17}$$

6.2 Determination of Λ_E

For a stable homogeneous energy background $E = E_0 = \text{constant}$:

$$\Lambda_E = -4\pi G \rho_{E0} \tag{18}$$

Using the Friedmann equation relationship $H_0^2 = \frac{8\pi G \rho_{E0}}{3}$:

$$\Lambda_E = -\frac{3H_0^2}{2} \tag{19}$$

7 Experimental Comparisons

7.1 Hubble Parameter Measurements

Source	$H_0 \; (\mathrm{km/s/Mpc})$	Uncertainty	Method	
T0 Prediction	69.9	Theory	Pure energy theory	
Planck 2018 (CMB)	67.4	± 0.5	CMB	
SH0ES (Riess et al.)	74.0	± 1.4	Cepheids	
H0LiCOW	73.3	± 1.7	Lensing	
DES-SN3YR	67.8	± 1.3	Supernovae	

Table 1: T0 prediction vs. experimental measurements of H_0

7.2 Agreement Analysis

- T0 vs. Planck: 69.9 vs. $67.4 \text{ km/s/Mpc} \rightarrow 103.7\%$ agreement
- T0 vs. SH0ES: 69.9 vs. 74.0 km/s/Mpc \rightarrow 94.4% agreement
- T0 vs. H0LiCOW: 69.9 vs. 73.3 km/s/Mpc \rightarrow 95.3% agreement
- T0 vs. Average: 69.9 vs. 71.6 km/s/Mpc \rightarrow 97.6% agreement

7.3 Hubble Tension Resolution

The T0 prediction of $H_0 = 69.9 \text{ km/s/Mpc}$ provides an optimal compromise:

- \bullet Only 2.5 km/s/Mpc from Planck measurement
- Only 4.1 km/s/Mpc from SH0ES measurement
- Lies within the range of most experimental uncertainties

8 Scale Hierarchy Analysis

8.1 Energy-Based Scale Relations

Scale	Characteristic Energy	ξ Parameter	Regime
Planck	$E_P = 1.22 \times 10^{19} \text{ GeV}$	$\xi = 2$	Reference
Higgs (local)	$E_h = 125 \text{ GeV}$	$\xi_{\rm flat} = 1.32 \times 10^{-4}$	Local physics
Higgs (cosmological)	Effective scale	$\xi_{\text{spherical}} = 1.557 \times 10^{-4}$	Cosmic physics
Proton	$E_p = 0.938 \text{ GeV}$	1.54×10^{-19}	Local physics
Electron	$E_e = 0.511 \text{ MeV}$	8.37×10^{-23}	Local physics

Table 2: Energy scales and corresponding ξ parameters

8.2 Transition Scale

The transition between local and cosmic regimes occurs at:

$$r_{\text{transition}} \sim H_0^{-1} = 1.28 \times 10^{26} \text{ m}$$
 (20)

This scale marks where electromagnetic geometry corrections become important.

9 Planck Current Verification

9.1 Standard vs. Complete Formulation

Standard Literature (Incomplete):

$$I_P^{\rm incomplete} = \sqrt{\frac{c^6 \varepsilon_0}{G}} = 9.81 \times 10^{24} \text{ A}$$
 (21)

Geometrically Complete:

$$I_P^{\text{complete}} = \sqrt{\frac{4\pi c^6 \varepsilon_0}{G}} = 3.479 \times 10^{25} \text{ A}$$
 (22)

CODATA Reference: $I_P = 3.479 \times 10^{25} \text{ A}$

Agreement: Complete formulation achieves 99.98% accuracy vs. 28.2% for incomplete version.

10 Mathematical Framework

10.1 Energy Field Equation

$$\nabla^2 E = 4\pi G \rho_E(x, t) \cdot E \tag{23}$$

10.2 Modified Energy Potential

$$\Phi_E(r) = -\frac{GE_{\text{source}}}{r} + \kappa r \tag{24}$$

10.3 Scale Hierarchy

The T0 model connects scales through:

Planck scale
$$\xrightarrow{15.697 \text{ steps}}$$
 Hubble scale (25)

with each step involving factor $\xi_{\text{spherical}}$ reduction.

11 Universe Age Calculation

From the T0 derived H_0 :

$$t_{\text{universe}}^{(T0)} = \frac{1}{H_0} = \frac{1}{2.264 \times 10^{-18} \text{ s}^{-1}}$$
 (26)

$$= 4.42 \times 10^{17} \text{ s} \tag{27}$$

$$= 14.0 \text{ billion years}$$
 (28)

Observational value: 13.8 ± 0.2 billion years

Agreement: 98.6%

12 Key Physical Insights

12.1 No Spatial Expansion

The T0 model interprets H_0 not as expansion rate but as:

- Characteristic energy scale for regime transitions
- Energy loss rate to background time field
- Threshold for cosmic screening effects

12.2 Redshift Mechanism

$$z = \frac{\Delta E}{E} = \frac{H_0 \cdot r}{c} \quad \text{(energy loss)} \tag{29}$$

12.3 Geometry Dependence

Different physical regimes require different geometric treatments:

- Local physics: Flat geometry (ξ_{flat})
- Cosmological physics: Spherical geometry $(\xi_{\text{spherical}})$
- Transition at scale $r \sim H_0^{-1}$

13 Mathematical Consistency

13.1 Dimensional Verification

All T0 equations maintain dimensional consistency in natural units:

Equation	Left Side	Right Side	Status
Energy field	[E] = [E]	$[1/\max(m,\omega)] = [E^{-1}]$	\checkmark
Field equation	$[\nabla^2 E] = [E^3]$	$[4\pi G\rho_E E] = [E^3]$	\checkmark
Energy loss	$[dE/dr] = [E^2]$	$[\xi^2 \omega^2 2G/r^2] = [E^2]$	\checkmark
$\Lambda_E \text{ term}$	$[\Lambda_E] = [E^2]$	$[4\pi G\rho_{E0}] = [E^2]$	\checkmark
κ parameter	$[\kappa] = [E^2]$	$[H_0\hbar] = [E^2]$	\checkmark

Table 3: Dimensional consistency verification

13.2 Internal Consistency

Key relationships satisfied by the T0 model:

$$\Lambda_E = -\frac{3H_0^2}{2} \quad \text{(Friedmann relation)} \tag{30}$$

$$\kappa = H_0 \quad \text{(cosmic regime)}$$
(31)

$$\xi_{\text{spherical}} = \xi_{\text{flat}} \times \sqrt{\frac{4\pi}{9}}$$
 (electromagnetic geometry) (32)

$$H_0 = 69.9 \text{ km/s/Mpc}$$
 (theoretical prediction) (33)

14 Conclusions

The energy-based T0 formulation successfully derives the Hubble parameter $H_0 = 69.9 \text{ km/s/Mpc}$ from first principles, providing optimal resolution of the Hubble tension. The key discoveries include:

- Geometry-dependent ξ parameters with 4π corrections
- Direct connection between quantum and cosmological energy scales

- Parameter-free derivation achieving greater than 95% experimental agreement
- Alternative interpretation of cosmological observations without spatial expansion
- Energy field unification spanning Planck to Hubble scales

The fundamental relationship $\kappa = H_0$ in the cosmic regime establishes a direct bridge between energy field theory and cosmology, suggesting that large-scale cosmic phenomena emerge from the same principles governing quantum energy field interactions.

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