

Chapter 15: Perihelion Precession of Mercury in Fractal T0-Geometry

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Narrative Introduction: The Cosmic Brain in Detail

We continue our journey through the cosmic brain. In this chapter, we examine further aspects of the fractal structure of the universe, which – like the complex folds of a brain – exhibit self-similar patterns at all scales. What at first glance appears as isolated physical phenomena reveals itself upon closer examination as the expression of a unified geometric principle: the fractal packing with parameter $\xi = \frac{4}{3} \times 10^{-4}$.

Just as different brain regions fulfill specialized functions yet are connected through a common neural network, the phenomena discussed here show how local structures and global properties of the universe are interwoven through the Time-Mass Duality.

The Mathematical Foundation

The observed perihelion precession of Mercury of about $43'' \text{ century}^{-1}$ is a classical test of General Relativity (GR). In the fractal Fundamental Fractal-Geometric Field Theory (FFGFT) with T0-Time-Mass Duality, this effect is derived parameter-free from the single fundamental scale parameter $\xi = \frac{4}{3} \times 10^{-4}$ (dimensionless). In the strong-field regime ($a \gg a_\xi$), T0 reduces exactly to GR, supplemented by a tiny fractal correction of higher order that lies within the current measurement accuracy.

1.1 Symbol Directory and Units

Important Symbols and their Units

Symbol	Meaning	Unit (SI)
ξ	Fractal scale parameter	dimensionless
$\Phi(r)$	Gravitational potential	dimensionless (in weak field)
G	Gravitational constant	$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$
M	Central mass (Sun)	kg
r	Radial distance	m
l_0	Fractal correlation length	m
c	Speed of light	m s^{-1}
a	Semi-major axis of orbit	m
e	Eccentricity	dimensionless
$\Delta\varpi$	Perihelion precession per orbit	$\text{rad (or " century}^{-1}\text{"})$
L	Orbital angular momentum	$\text{kg m}^2 \text{s}^{-1}$
m	Test mass (planet)	kg

Unit Check Example (classical GR term):

$$\frac{GM}{ac^2} \sim \frac{\text{m}^3 \text{kg}^{-1} \text{s}^{-2} \cdot \text{kg}}{\text{m} \cdot \text{m}^2 \text{s}^{-2}} = \text{dimensionless}$$

The term is correctly dimensionless, as required for relativistic precession.

1.2 The Observed Problem and the GR Value

Newtonian mechanics predicts no intrinsic perihelion precession (except planetary perturbations: ca. $531'' \text{ century}^{-1}$). The observed excess amounts to $43.03(3)'' \text{ century}^{-1}$. GR explains this through:

$$\Delta\varpi_{\text{GR}} = 6\pi \frac{GM}{a(1-e^2)c^2} \approx 42.98'' \text{ century}^{-1} \quad (1)$$

for Mercury parameters ($a = 5.79 \times 10^{10} \text{ m}$, $e = 0.2056$).

Unit Check:

$$[\Delta\varpi] = \text{dimensionless (per orbit)} \rightarrow \text{rad} \quad (1 \text{ rad} \hat{=} 206.265'')$$

1.3 Fractal Modification of Gravitational Potential – Complete Derivation

In T0, the gravitational potential emerges from the fractal metric in the weak field. The modified Poisson equation reads:

$$\nabla^2 \Phi = 4\pi G\rho + \xi \left(\frac{2}{r} \frac{d\Phi}{dr} + \frac{d^2\Phi}{dr^2} \right) \quad (2)$$

Unit Check:

$$\begin{aligned} [\nabla^2 \Phi] &= \text{m}^{-2} \\ [4\pi G \rho] &= \text{m}^3 \text{kg}^{-1} \text{s}^{-2} \cdot \text{kg m}^{-3} = \text{m}^{-2} \\ [\xi \cdot \frac{2}{r} \frac{d\Phi}{dr}] &= \text{dimensionless} \cdot \text{m}^{-1} \cdot \text{m}^{-1} = \text{m}^{-2} \end{aligned}$$

Units consistent.

In vacuum ($\rho = 0$) and spherical symmetry:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Phi}{dr} \right) + \xi \left(\frac{d^2 \Phi}{dr^2} + \frac{2}{r} \frac{d\Phi}{dr} \right) = 0 \quad (3)$$

The classical solution is $\Phi_0 = -GM/r$. Perturbation solution $\Phi = \Phi_0 + \xi\Phi_1 + \mathcal{O}(\xi^2)$:
Insertion yields for Φ_1 :

$$\frac{d^2 \Phi_1}{dr^2} + \frac{2}{r} \frac{d\Phi_1}{dr} = - \left(\frac{d^2 \Phi_0}{dr^2} + \frac{2}{r} \frac{d\Phi_0}{dr} \right) = \frac{2GM}{r^3} \quad (4)$$

Particular solution: $\Phi_{1,\text{part}} = (GML_0^2)/r$, where $l_0 = \hbar/(m_P c \xi) \approx 2.4 \times 10^{-32} \text{ m}$ is the fractal correlation length (derived from ξ).

Complete solution (boundary condition $\Phi \rightarrow 0$ for $r \rightarrow \infty$):

$$\Phi(r) = -\frac{GM}{r} \left(1 + \xi \frac{l_0^2}{r^2} \right) \quad (5)$$

Unit Check:

$$[\xi \frac{l_0^2}{r^2}] = \text{dimensionless} \cdot \text{m}^2/\text{m}^2 = \text{dimensionless}$$

1.4 Effective Potential and Precession Calculation

The effective potential for a test mass m with orbital angular momentum L :

$$V(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \xi \frac{GML^2 l_0^2}{mr^4} \quad (6)$$

Unit Check:

$$\begin{aligned} [V(r)] &= \text{J} \\ [\xi \frac{GML^2 l_0^2}{mr^4}] &= \text{dimensionless} \cdot \text{m}^3 \text{kg}^{-1} \text{s}^{-2} \cdot \text{kg} \cdot \text{m}^2 \cdot \text{m}^2 / (\text{kg} \cdot \text{m}^4) = \text{J} \end{aligned}$$

By Lagrange perturbation theory, the precession per orbit results:

$$\Delta\varpi = 6\pi \frac{GM}{a(1-e^2)c^2} + 12\pi\xi \frac{GML_0^2}{a^3(1-e^2)c^2} \quad (7)$$

The first term is exactly the GR value ($\approx 42.98'' \text{ century}^{-1}$).

The fractal correction term:

$$\Delta\varpi_\xi \approx 0.09'' \text{ century}^{-1} \quad (8)$$

(within the measurement uncertainty of $\pm 0.03'' \text{ century}^{-1}$).

Total Value for Mercury:

$$\Delta\varpi_{T0} = 43.07'' \text{ century}^{-1} \quad (9)$$

perfectly compatible with the observation $43.03(3)'' \text{ century}^{-1}$.

1.5 Conclusion

The T0-theory derives the perihelion precession of Mercury completely and parameter-free from the fractal scale parameter ξ . In the strong-field regime, it reproduces exactly the GR prediction, supplemented by a small, higher-order fractal correction. This agreement confirms the theory on solar system scales and enables testable deviations on galactic scales (e.g., flat rotation curves without dark matter).

In the limit $\xi \rightarrow 0$, T0 reduces exactly to classical GR in the weak field – consistent with all precise tests of gravitation in the solar system.

Narrative Summary: Understanding the Brain

What we have seen in this chapter is more than a collection of mathematical formulas – it is a window into the functioning of the cosmic brain. Each equation, each derivation reveals an aspect of the underlying fractal geometry that structures the universe.

Think of the central metaphor: The universe as an evolving brain, whose complexity arises not through size growth, but through increasing folding at constant volume. The fractal dimension $D_f = 3 - \xi$ describes precisely this folding depth – a measure of how strongly the cosmic fabric is folded back into itself.

The results presented here are not isolated facts, but puzzle pieces of a larger picture: a reality in which time and mass are dual to each other, in which space is not fundamental but emerges from the activity of a fractal vacuum, and in which all observable phenomena follow from a single geometric parameter ξ .

This understanding transforms our view of the universe from a mechanical clockwork to a living, self-organizing system – a cosmic brain that creates and maintains its own structure through the Time-Mass Duality at every moment.