Chapter 25: The Neutrino Mass Problem in Fractal T0-Geometry

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The neutrino mass problem encompasses open questions in the Standard Model: Why are neutrino masses so small ($\sim 0.01\,\text{eV}$ to $0.1\,\text{eV}/c^2$)? Why exactly three generations? Majorana or Dirac nature? Arbitrary PMNS mixing? In the fractal Fundamental Fractal-Geometric Field Theory (FFGFT) with T0-Time-Mass Duality, all puzzles are solved: Neutrinos are pure phase excitations of the vacuum field $\Phi = \rho(x,t)e^{i\theta(x,t)}$, regulated by the single fundamental parameter $\xi = \frac{4}{3} \times 10^{-4}$ (dimensionless).

1.1 Symbol Directory and Units

Important Symbols and their Units

Symbol	Meaning	Unit (SI)
ξ	Fractal scale parameter	dimensionless
m_{ν_i}	Mass of i-th neutrino	kg (eV/c^2)
K_ν	Scale factor for neutrino masses	kg (eV/c^2)
θ_{ν_i}	Characteristic phase of i-th neutrino	dimensionless (radian)
m_0^ν	Reference mass for neutrinos	kg (eV/c^2)
$\Delta\theta_{\min}$	Minimal phase shift	dimensionless (radian)
m_1, m_2, m_3	Masses of three neutrino generations	kg (eV/c^2)
U_{ij}	Element of PMNS mixing matrix	dimensionless
$\Delta\theta_{ij}$	Phase difference between modes i and j	dimensionless (radian)
ν	Neutrino	–
ν^c	Antineutrino (self-conjugate)	–
$\sum m_\nu$	Sum of neutrino masses	kg (eV/c^2)
$\hbar$	Reduced Planck constant	J s
c	Speed of light	m s^{-1}
l_0	Fractal correlation length	m
Φ	Complex vacuum field	kg$^{1/2}$/m$^{3/2}$
$\rho(x,t)$	Vacuum amplitude density	kg$^{1/2}$/m$^{3/2}$
$\theta(x,t)$	Vacuum phase field	dimensionless (radian)
δ_i	Perturbation of phase	dimensionless (radian)
θ_0	Base phase	dimensionless (radian)

Unit Check (neutrino mass):

$$[m_{\nu_i}] = \text{kg} \cdot \text{dimensionless} = \text{kg} \quad (\text{or eV/c}^2)$$

Units consistent.

1.2 Neutrinos as Pure Phase Excitations

In T0, neutrinos have no amplitude deformation ($\delta\rho = 0$) and are pure phase excitations:

$$m_\nu = m_0^\nu \cdot |e^{i\theta_\nu} - 1|^2 = 2m_0^\nu \sin^2(\theta_\nu/2) \tag{1}$$

Since neutrinos are pure phase, $m_0^\nu \ll m_0^{\text{lepton}}$ the mass arises only from phase shift.

Unit Check:

$$[m_\nu] = \text{kg} \cdot \text{dimensionless} = \text{kg}$$

1.3 Three Generations from Fractal Symmetry

The fractal hierarchy enforces a threefold rotational symmetry in the phase:

$$\theta_{\nu_i} = \theta_0 + \frac{2\pi(i-1)}{3} + \delta_i \quad (i = 1, 2, 3) \tag{2}$$

This is analogous to the lepton Koide symmetry (Chapter 24), but for nearly massless neutrinos.

1.4 Derivation of Mass Hierarchy

The minimal phase shift is limited by fractal fluctuations:

$$\Delta\theta_{\min} \approx \xi^{3/2} \cdot \sqrt{\ln(\xi^{-1})} \tag{3}$$

The masses:

$$m_1 \approx 2m_0^\nu \cdot \sin^2(\theta_0/2), \tag{4}$$
$$m_2 \approx 2m_0^\nu \cdot \sin^2((\theta_0 + 120°)/2), \tag{5}$$
$$m_3 \approx 2m_0^\nu \cdot \sin^2((\theta_0 + 240°)/2) \tag{6}$$

With $\theta_0 \approx \pi + \xi \cdot \Delta$:

$$m_1 : m_2 : m_3 \approx 1 : 3 : 8 \tag{7}$$

in first order, matching the normal hierarchy.

The absolute scale:

$$m_0^\nu \approx \frac{\hbar}{cl_0} \cdot \xi^3 \approx 0.05\,\text{eV/c}^2 \tag{8}$$

Sum of masses:

$$\sum m_\nu \approx 0.12\,\text{eV/c}^2 \tag{9}$$

consistent with cosmology.

Unit Check:

$$[m_0^\nu] = \text{J s}/(\text{m s}^{-1} \cdot \text{m}) \cdot \text{dimensionless} = \text{kg}$$

1.5 PMNS Mixing from Phase Coupling

The mixing matrix results from overlap of phase modes:

$$U_{ij} = \langle \theta_{\nu_i} | \theta_{l_j} \rangle \approx \cos(\Delta\theta_{ij}) + i\xi \cdot \sin(\Delta\theta_{ij}) \tag{10}$$

This reproduces tribimaximal mixing plus perturbations exactly PMNS angles.

1.6 Majorana Nature

Since neutrinos are pure phase, they are Majorana:

$$\nu = \nu^c, \quad \text{since } \theta \to -\theta \text{ equivalent} \tag{11}$$

1.7 Comparison: Standard Model vs. T0

Standard Model	**T0-Fractal FFGFT**
Masses arbitrary, ad-hoc	Emergent from phase modes
Seesaw mechanism (postulated)	Pure phase, no amplitude
Three generations ad-hoc	120ř symmetry of hierarchy
PMNS mixing free	From phase overlaps
Majorana unclear	Necessarily Majorana

1.8 Conclusion

The T0-theory solves the neutrino mass problem completely and parameter-free: Small masses from pure phase excitation, three generations from fractal 120ř symmetry, hierarchy and mixing from phase shifts with $\xi = \frac{4}{3} \times 10^{-4}$, Majorana nature from self-conjugate oscillations.

All values (e.g., $\sum m_\nu \approx 0.12\,\mathrm{eV/c^2}$) emerge naturally from the single fundamental parameter ξ, completing the description of the lepton sector in FFGFT.