

Dirac Equation in T0 Theory: Geometric Integration with Time-Mass Duality

Fractal Spacetime and Dynamic Mass

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Abstract

This work fully integrates the Dirac equation into the T0 theory framework. Unlike the standard formulation with constant mass, T0 theory uses the fundamental time-mass duality $T(x) \cdot m(x) = 1$, leading to a spacetime-dependent mass. The fractal dimension $D_f = 3 - \xi$ modifies the underlying metric and thus the differential operator. We show how the Clifford algebra structure naturally connects with the torus topology of T0 theory and how spin-1/2 can be interpreted as a topological winding number. The predictions are formulated as ratio-based statements that are independent of unit systems and phenomenological parameters. Experimental tests at Belle II can directly verify the fundamental quadratic mass scaling.

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1 Introduction: T0 Basic Principles

1.1 Time-Mass Duality

The fundamental principle of T0 theory is time-mass duality:

$$T(x, t) \cdot m(x, t) = \frac{\hbar}{c^2} \quad (1)$$

In natural units ($\hbar = c = 1$):

$$T(x, t) \cdot m(x, t) = 1 \quad (2)$$

This means: **Mass is not constant but a dynamic field**, coupled to the intrinsic time field $T(x, t)$.

1.2 Fractal Spacetime

T0 theory postulates a fractal spacetime dimension:

$$D_f = 3 - \xi \quad \text{with} \quad \xi = \frac{4}{3 \times 10^4} \approx 1.333 \times 10^{-4} \quad (3)$$

This modifies the metric and thus all differential operators.

1.3 Torus Topology

The underlying topology is a torus with characteristic scales:

- Large radius: $R \sim 1/\xi$
- Small radius: $r \sim R \cdot \xi$
- Winding numbers: (n_θ, n_ϕ) for poloidal and toroidal directions

2 Standard Dirac Equation: Problems

2.1 The Standard Form

The usual Dirac equation reads:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \quad (4)$$

with constant mass m and flat Minkowski metric.

2.2 Problems for T0 Integration

1. **Constant mass:** Contradicts time-mass duality
2. **Flat metric:** Ignores the fractal structure
3. **No topology:** Spin has no geometric origin
4. **Static:** No coupling to time field

3 Clifford Algebra: The Fundamental Structure

Before we develop the T0-specific formulation, we need to understand what the Dirac equation **really** is – beyond the 4×4 matrices.

3.1 Representation vs. Physics

The central insight: The 4×4 matrices are not the physics, but a **specific representation** of the physics.

Important

Fundamental Difference **Fundamental (Physics):**

The Clifford algebra structure of spacetime

Representation (Calculation):

Specific 4×4 matrices γ^μ in a chosen basis

Analogy: Vectors are fundamental; their components depend on the chosen basis. The physics (vector) is basis-independent, the calculation (components) is not.

Example – different representations:

The same Dirac equation can be written with:

- **Dirac representation:** Specific 4×4 matrices
- **Weyl representation:** Different 4×4 matrices
- **Majorana representation:** Yet different matrices

All describe **the same physics!** The choice is convention, like choosing a coordinate basis.

3.2 The Abstract Clifford Form

The fundamental form of the Dirac equation without explicit matrices is:

$$(ie_\mu \partial^\mu - m)\Psi = 0 \quad (5)$$

where:

- e_μ : **Abstract basis vectors** of spacetime (not matrices!)
- Ψ : Element in the **spin bundle** (geometric object)
- The **Clifford product rule**:

$$e_\mu e_\nu + e_\nu e_\mu = 2g_{\mu\nu} \quad (6)$$

What does the Clifford product mean?

The product $e_\mu e_\nu$ is **non-commutative**:

$$e_0 e_1 \neq e_1 e_0 \quad (7)$$

$$e_0 e_1 + e_1 e_0 = 0 \quad (\text{since } g_{01} = 0) \quad (8)$$

This encodes the **geometric structure of spacetime**.

3.3 What Are the γ -Matrices Really?

The familiar γ^μ matrices are simply:

$$\gamma^\mu \longleftrightarrow \text{Matrix representation of } e^\mu \quad (9)$$

Concretely: One chooses a basis in spin space and writes:

$$e^\mu \rightarrow \gamma^\mu = \begin{pmatrix} \gamma_{11}^\mu & \gamma_{12}^\mu & \gamma_{13}^\mu & \gamma_{14}^\mu \\ \gamma_{21}^\mu & \gamma_{22}^\mu & \gamma_{23}^\mu & \gamma_{24}^\mu \\ \gamma_{31}^\mu & \gamma_{32}^\mu & \gamma_{33}^\mu & \gamma_{34}^\mu \\ \gamma_{41}^\mu & \gamma_{42}^\mu & \gamma_{43}^\mu & \gamma_{44}^\mu \end{pmatrix} \quad (10)$$

The specific numbers in the matrix depend on the chosen representation!

The physics (Clifford product rule (6)) is independent of this choice.

3.4 Spin as Topological Property

The spin-1/2 character is not a property of the matrices but follows from the Clifford algebra structure.

3.4.1 The 720° Rotation

Key observation: A spinor Ψ behaves under rotations as:

$$R(180^\circ)\Psi = e^{i\pi/2}\Psi = i\Psi \quad (11)$$

$$R(360^\circ)\Psi = e^{i\pi}\Psi = -\Psi \quad (12)$$

$$R(720^\circ)\Psi = e^{i2\pi}\Psi = \Psi \quad (13)$$

This is **not a matrix property**, but follows from Clifford algebra!

Why? The rotation is given by:

$$R(\theta) = \exp\left(\frac{i\theta}{2}e_1e_2\right) \quad (14)$$

The factor $1/2$ in the exponent is **geometric** (comes from the Clifford algebra structure), not from the matrices!

3.4.2 Topological Interpretation

In T0 theory, we can interpret spin geometrically as a **winding number on a torus**:

$$\text{Spin-}s \quad \longleftrightarrow \quad \text{Winding } (n_\theta, n_\phi) \text{ with } \frac{n_\phi}{n_\theta} = 2s \quad (15)$$

For spin-1/2: $(n_\theta, n_\phi) = (1, 1)$ or $(2, 1)$

The 720° rotation then corresponds to:

- Once around the poloidal circle $\rightarrow -\Psi$ (360°)
- Twice around the poloidal circle $\rightarrow +\Psi$ (720°)

This is **pure topology**, not a mysterious quantum property!

3.5 Common Misconceptions

3.5.1 Can the Matrices Really Be Eliminated?

Answer: Yes and No.

- Yes – fundamentally:** The physics does not need specific 4×4 matrices. The Clifford algebra is fundamental.
- No – practically:** For concrete calculations, a representation is necessary, and matrices are often the most practical choice.

Analogy: One can formulate vector physics without coordinates (fundamental), but for calculations one chooses coordinates (practical).

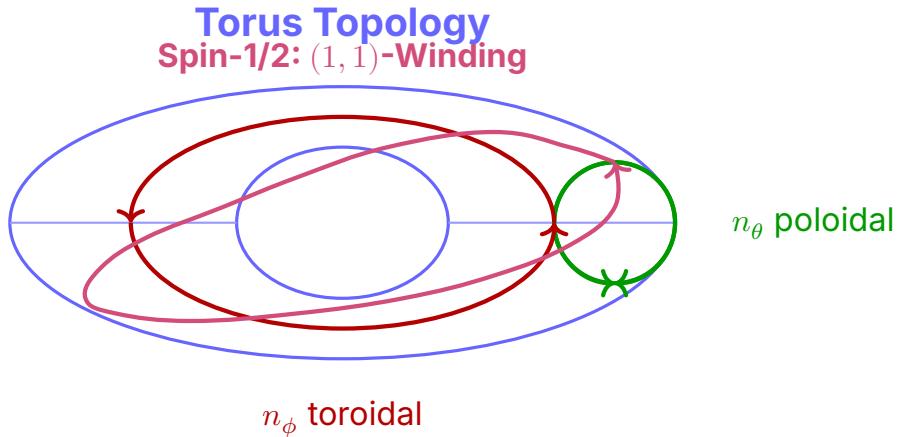


Figure 1: Spin-1/2 as topological winding on a torus (top view). The green double arrow shows the poloidal small circle (n_θ , cross-section of the torus tube). The red arrows show the toroidal direction (n_ϕ , around the central hole).

The violet path shows a (1,1)-winding: once around the small circle AND once around the large circle. A 720° rotation corresponds to traversing this winding twice.

3.5.2 Is Information Lost?

No! The Clifford algebra formulation contains **exactly the same information**:

Property	In Matrices	In Clifford Algebra
Spin-1/2	In γ -structure	In Clifford product rule
Lorentz inv.	Explicit in matrices	In $g_{\mu\nu}$ -structure
Antiparticles	Negative energy solutions	Chirality components
Measurables	Matrix elements	Invariant under representation

Table 1: Information identical in both formulations

3.5.3 Is This Just a Reformulation?

No – it is a conceptual shift:

- **Old view:** “Electrons are point particles with mysterious intrinsic spin, described by complicated 4×4 matrices”
- **New view:** “Electrons are geometric objects in a Clifford-structured space-time. Spin is a topological property.”

This new view enables **natural integration** into T0 theory:

- Fractal metric → modified Clifford structure

- Torus topology \rightarrow spin as winding number
- Time-mass duality \rightarrow dynamic mass $m(x)$

3.6 Preparation for T0 Integration

With this understanding, we can now introduce the T0-specific modifications:

1. **Fractal metric:** $g_{\mu\nu} \rightarrow g_{\mu\nu}^{(\text{frak})}$ with $D_f = 3 - \xi$

2. **Modified Clifford rule:**

$$e_\mu^{(\text{frak})} e_\nu^{(\text{frak})} + e_\nu^{(\text{frak})} e_\mu^{(\text{frak})} = 2g_{\mu\nu}^{(\text{frak})} \quad (16)$$

3. **Dynamic mass:** $m \rightarrow m(x) = 1/(c^2 T(x))$

4. **Tetrad formulation:** Necessary for curved/fractal spacetime

In the next section, we develop this T0-specific formulation in detail.

Core Message of This Chapter

The Dirac equation is fundamentally a **geometric equation** in the Clifford algebra of spacetime. The 4×4 matrices are useful calculation tools, but not the physics itself. This insight is **essential** for integration into T0 theory with its fractal geometry and torus topology.

4 T0 Dirac Equation: Geometric Form

4.1 Clifford Algebra in Fractal Spacetime

Instead of the standard form, we use the Clifford algebra formulation:

$$(i\partial_{\text{frak}} - m(x))\Psi(x) = 0 \quad (17)$$

where:

$$\partial_{\text{frak}} = e_a^\mu(x) \gamma^a \partial_\mu \quad (\text{tetrad-based}) \quad (18)$$

$$m(x) = \frac{1}{c^2 T(x)} \quad (\text{from time-mass duality}) \quad (19)$$

$$e_a^\mu(x) = \text{Tetrad in fractal metric} \quad (20)$$

4.2 Fractal Metric

The fractal correction to the metric is:

$$g_{\mu\nu}^{(\text{frak})}(x) = \eta_{\mu\nu} \cdot (1 + \xi \cdot f(x)) \quad (21)$$

where $f(x)$ is a dimensionless function of coordinates describing the fractal structure.

4.3 Tetrads Formulation

The tetrad $e_a^\mu(x)$ connects the curved spacetime with the local Clifford algebra:

$$g_{\mu\nu}^{(\text{frak})}(x) = e_a^\mu(x)e_b^\nu(x)\eta^{ab} \quad (22)$$

The γ^a are the standard Clifford generators in the local Lorentz frame.

5 Dynamic Mass

5.1 Spacetime Dependence

From time-mass duality follows:

$$m(x, t) = \frac{1}{c^2 T(x, t)} = \frac{1}{c^2} \max(\omega(x, t), m_{\text{bg}}(x)) \quad (23)$$

where:

- $\omega(x, t)$: Local frequency/energy density
- $m_{\text{bg}}(x)$: Background mass field

5.2 Coupling to Time Field

The time field $T(x, t)$ is itself a dynamic field with Lagrangian density:

$$\mathcal{L}_T = \frac{1}{2}(\partial_\mu T)(\partial^\mu T) - V(T) \quad (24)$$

The coupling to fermions occurs through the mass:

$$\mathcal{L}_{\text{int}} = \bar{\Psi}m(T(x))\Psi \quad (25)$$

6 Spin as Topology

6.1 Winding Numbers on the Torus

In T0 theory, spin is interpreted as a winding number:

$$\text{Spin-}s \leftrightarrow \text{Winding } (n_\theta, n_\phi) \text{ with } n_\phi/n_\theta = 2s \quad (26)$$

Examples:

$$\text{Spin-0 : } (1, 0) \text{ or } (0, 1) \quad (27)$$

$$\text{Spin-1/2 : } (1, 1) \text{ or } (2, 1) \quad (28)$$

$$\text{Spin-1 : } (1, 2) \quad (29)$$

6.2 720° Rotation Geometrically

The well-known property of spin-1/2 particles (720° rotation for identity) follows from torus topology:

- One poloidal winding: 360° rotation $\rightarrow -\Psi$
- Two poloidal windings: 720° rotation $\rightarrow +\Psi$
This is not a mysterious property but **pure topology**.

7 Mass-Proportional Coupling

7.1 Interaction Lagrangian

The coupling of leptons to the time field is mass-proportional:

$$\mathcal{L}_{\text{int}} = \xi m_\ell \bar{\Psi}_\ell \Psi_\ell \Delta m(x) \quad (30)$$

where $\Delta m(x) = m(x) - m_0$ is the mass fluctuation.

7.2 Consequence: Quadratic Scaling

From this mass-proportional coupling follows for loop diagrams:

$$\Delta a_\ell \propto (\xi m_\ell)^2 \cdot (\text{kinematic factors}) \propto m_\ell^2 \quad (31)$$

This leads to the fundamental ratio prediction:

$$\frac{\Delta a_{\ell_1}}{\Delta a_{\ell_2}} = \left(\frac{m_{\ell_1}}{m_{\ell_2}} \right)^2$$

(32)

8 Ratios vs. Absolute Values

8.1 What the T0 Dirac Equation Predicts

Fundamental predictions (parameter-free):

- Ratio: $a_\tau/a_\mu = (m_\tau/m_\mu)^2 \approx 283$
- Structure: $\Delta a \propto m^2$ (quadratic scaling)
- Topology: Spin-1/2 as winding number
Not predictable (phenomenological):
- Absolute values: $a_\mu = 37.5 \times 10^{-11}$ (requires normalization)

8.2 Why Only Ratios?

Complete calculation of absolute values requires:

1. Solution of time field dynamics in fractal spacetime (too complex)
2. Loop integrals in non-integer dimension (open)
3. Renormalization at $D_f = 3 - \xi$ (not fully developed)
4. Recursive coupling of all fields (non-perturbative)

This is analogous to QCD in the Standard Model: Fundamental Lagrangian density is clear, but hadronic contributions are not calculable ab initio.

9 Natural vs. SI Units

9.1 In Natural Units

In natural units ($\hbar = c = 1, \alpha = 1$) α disappears from all formulas:

$$\tilde{a}_\ell = \tilde{C} \cdot \xi \cdot \tilde{m}_\ell^2 \quad (33)$$

The ratio is:

$$\frac{\tilde{a}_\tau}{\tilde{a}_\mu} = \left(\frac{\tilde{m}_\tau}{\tilde{m}_\mu} \right)^2 \quad (34)$$

Identical to SI version – ratios are invariant!

9.2 Transformation to SI

The transformation to SI units introduces α :

$$a_\ell[\text{SI}] = (\text{conversion factor with } \alpha) \times \tilde{a}_\ell \quad (35)$$

But the **ratio remains unchanged**:

$$\frac{a_\tau[\text{SI}]}{a_\mu[\text{SI}]} = \frac{\tilde{a}_\tau}{\tilde{a}_\mu} = \left(\frac{m_\tau}{m_\mu} \right)^2 \quad (36)$$

10 Experimental Tests

10.1 Belle II: Critical Test (2027-2028)

The fundamental prediction:

$$\frac{a_\tau}{a_\mu} = \left(\frac{1776.86}{105.658} \right)^2 = 282.8 \quad (37)$$

is directly testable at Belle II.

Possible outcomes:

- **Confirmation:** Strong evidence for mass-proportional coupling
- **Deviation:** Modification of coupling structure needed
- **Null result:** T0 contributions suppressed or incorrect

10.2 Further Tests

Test	T0 Prediction	Status
a_τ/a_μ	$(m_\tau/m_\mu)^2 = 283$	Belle II 2027-28
m_τ/m_μ	≈ 16.8 (from torus)	Confirmed ✓
Spin-statistics	From topology	Confirmed ✓
Fractal damping	$\propto e^{-\xi n^2}$	Rydberg atoms

Table 2: Experimental tests of the T0 Dirac formulation

11 Comparison with Standard Formulation

12 Limits and Open Questions

12.1 What Works

- ✓ Clifford algebra structure clearly defined
- ✓ Spin interpretable as topology
- ✓ Ratio predictions parameter-free
- ✓ Belle II test possible

Aspect	Standard Dirac	TO Dirac
Mass	Constant m	Dynamic $m(x, t)$
Metric	Minkowski $\eta_{\mu\nu}$	Fractal $g_{\mu\nu}^{(\text{frak})}$
Spin	Matrix property	Topological winding
Dimension	$D = 4$	$D_f = 3 - \xi$ in space
Topology	None	Torus (n_θ, n_ϕ)
Coupling	Ad-hoc	Time-mass duality
Predictions	Qualitative	Testable ratios

Table 3: Standard vs. TO Dirac formulation

12.2 Honesty About Limits

As in the Standard Model (hadronic contributions), there are areas where the fundamental theory is clear but explicit calculations are too complex. This is **not a fault of the theory**, but a realistic assessment of mathematical challenges.

References and Further Reading

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