Unification of the Casimir Effect and Cosmic Microwave Background: A Fundamental Vacuum Theory

0.1 Introduction

This paper develops a novel theoretical description that interprets the microscopic Casimir effect and the macroscopic cosmic microwave background (CMB) as different manifestations of an underlying vacuum structure. By introducing a characteristic vacuum length scale L_ξ and a fundamental dimensionless coupling constant ξ, it is shown that both phenomena can be described within a unified theoretical framework.

The theory is based on the hypothesis of a granular spacetime with a minimal length scale $L_0 = \xi \cdot L_P$, at which all physical forces are fully effective. For distances $d > L_0$, only parts of these forces become visible through vacuum fluctuations, which is described by the $1/d^4$ dependence of the Casimir force. Due to the extremely small size of L_0, a direct experimental measurement is currently not possible, which is why the measurable scale L_ξ serves as a bridge between the fundamental spacetime structure and experimental observations. Gravity is interpreted as an emergent property of a time field, thereby allowing cosmic effects such as the CMB to be explained without the assumption of dark energy or dark matter.

0.2 Theoretical Foundations

0.2.1 Fundamental Length Scales

The proposed framework defines a hierarchy of characteristic length scales:

$$L_0 = \xi \cdot L_P \tag{1}$$

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35}\,\mathrm{m} \tag{2}$$

$$L_\xi = \text{characteristic vacuum length scale} \approx 100\,\mathrm{\mu m} \tag{3}$$

Here, L_0 represents the minimal length scale of a granular spacetime at which all vacuum fluctuations are fully effective, while L_ξ represents the emergent scale for measurable vacuum interactions.

0.2.2 The Coupling Constant ξ

The dimensionless coupling constant ξ is determined to be

$$\xi = \frac{4}{3} \times 10^{-4} = 1.333 \times 10^{-4} \tag{4}$$

This constant serves as a fundamental space parameter that links the granulation of spacetime at L_0 with measurable effects such as the Casimir effect and the CMB. It can be derived from a Lagrangian that describes the dynamics of a time field.

0.3   The CMB-Vacuum Relationship

0.3.1   Basic Equation

The central relationship of the theory links the energy density of the cosmic microwave background with the characteristic vacuum length scale:

$$\rho_{\text{CMB}} = \frac{\xi \hbar c}{L_\xi^4} \tag{5}$$

This formula is dimensionally consistent, since

$$[\rho_{\text{CMB}}] = \frac{[1] \cdot [\hbar c]}{[L_\xi^4]} = \frac{\text{J m}}{\text{m}^4} = \text{J/m}^3 \tag{6}$$

0.3.2   Numerical Determination of L_ξ

With the experimentally determined CMB energy density $\rho_{\text{CMB}} = 4.17 \times 10^{-14}\,\text{J/m}^3$, L_ξ can be calculated:

$$L_\xi^4 = \frac{\xi \hbar c}{\rho_{\text{CMB}}} \tag{7}$$

$$L_\xi^4 = \frac{1.333 \times 10^{-4} \times 3.162 \times 10^{-26}\,\text{J m}}{4.17 \times 10^{-14}\,\text{J/m}^3} \tag{8}$$

$$L_\xi^4 = 1.011 \times 10^{-16}\,\text{m}^4 \tag{9}$$

$$L_\xi = 100\,\mu\text{m} \tag{10}$$

0.4   Modified Casimir Theory

0.4.1   Extended Casimir Formula

The Casimir effect is described by the following modified formula:

$$|\rho_{\text{Casimir}}(d)| = \frac{\pi^2}{240\xi} \rho_{\text{CMB}} \left(\frac{L_\xi}{d}\right)^4 \tag{11}$$

where d denotes the distance between the Casimir plates.

0.4.2   Consistency with the Standard Casimir Formula

By substituting the CMB-vacuum relationship (**??**) into the modified Casimir formula (**??**), the following is obtained:

$$|\rho_{\text{Casimir}}(d)| = \frac{\pi^2}{240\xi} \cdot \frac{\xi \hbar c}{L_\xi^4} \cdot \frac{L_\xi^4}{d^4} \tag{12}$$

$$= \frac{\pi^2 \hbar c}{240 d^4} \tag{13}$$

This exactly matches the established standard Casimir formula and proves the mathematical consistency of the proposed theory.

0.5   Numerical Verification

0.5.1   Comparison Calculations

To verify the theoretical consistency, Casimir energy densities are calculated for various plate distances:

The perfect agreement confirms the mathematical correctness of the developed theory.

0.5.2   Hierarchy of Characteristic Length Scales

The theory establishes a clear hierarchy of length scales:

$$L_0 = 2.155 \times 10^{-39} \, \text{m} \quad \text{(Sub-Planck)} \tag{14}$$

$$L_P = 1.616 \times 10^{-35} \, \text{m} \quad \text{(Planck)} \tag{15}$$

$$L_\xi = 100 \, \mu\text{m} \quad \text{(Casimir-characteristic)} \tag{16}$$

The ratios of these length scales are:

$$\frac{L_0}{L_P} = \xi = 1.333 \times 10^{-4} \tag{17}$$

$$\frac{L_P}{L_\xi} = 1.616 \times 10^{-31} \tag{18}$$

$$\frac{L_0}{L_\xi} = 2.155 \times 10^{-35} \tag{19}$$

0.6   Physical Interpretation

0.6.1   Multi-Scale Vacuum Model

The developed theory implies a fundamental structure of the vacuum on various length scales:

1. **Sub-Planck Level** (L_0): Minimal length scale of the granular spacetime, at which all physical forces, including vacuum fluctuations, are fully effective. Due to the extremely small size of $L_0 \approx 2.155 \times 10^{-39} \, \text{m}$, a direct measurement is currently not possible.

2. **Planck Threshold** (L_P): Transition region between quantum gravity and classical spacetime geometry.

3. **Casimir Manifestation** (L_ξ): Emergent length scale for measurable vacuum interactions that forms a bridge to the CMB.

4. **Cosmic Scale**: Large-scale vacuum signature through the CMB, explained by a time field from which gravity emerges.

0.6.2   Granulation of Spacetime at L_0

The minimal length scale $L_0 = \xi \cdot L_P \approx 2.155 \times 10^{-39}$ m represents a discrete spacetime structure, at which all vacuum fluctuations causing the Casimir effect and other forces are fully effective. At this distance, all wave modes are present without restriction, leading to a maximum energy density. For distances $d > L_0$, only parts of these forces become visible through the $1/d^4$ dependence of the Casimir energy density, as the plates restrict the wave modes. The extremely small size of L_0 prevents a direct experimental measurement at present, which is why the theory introduces the measurable scale $L_\xi \approx 100\,\mu\text{m}$ to investigate the vacuum structure indirectly.

0.6.3   Coupling Constant ξ as Space Parameter

The coupling constant $\xi = 1.333 \times 10^{-4}$ is a fundamental space parameter that links the granulation of spacetime at L_0 with measurable effects. It can be derived from a Lagrangian that describes the dynamics of a time field:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\phi)^2 - \xi \cdot \frac{\hbar c}{L_0^4} \cdot \phi^2 \tag{20}$$

Here, ϕ is a time field that describes the temporal structure of spacetime, and the term $\xi \cdot \frac{\hbar c}{L_0^4} \cdot \phi^2$ introduces an energy density that is linked to ρ_{CMB}.

0.6.4   Emergent Gravity

Gravity is interpreted as an emergent property of a time field ϕ, whose fluctuations on the scale L_0 generate the spacetime structure. The coupling constant ξ determines the strength of these interactions, thereby allowing cosmic effects such as the CMB to be explained without the assumption of dark energy or dark matter.

0.7   Experimental Predictions

0.7.1   Critical Distances

The theory makes specific predictions for the behavior of the Casimir effect at characteristic distances:

Distance d	ρ_{Casimir} (J/m^3)	Ratio to CMB
100 µm	4.17×10^{-14}	1.00
10 µm	4.17×10^{-10}	1.0×10^4
1 µm	4.17×10^{-2}	1.0×10^{12}

Table 2: Predictions for Casimir energy densities and their ratio to the CMB energy density

0.7.2 Experimental Tests

The most important experimental verifications of the theory include:

1. **Precision measurements at $d = L_\xi$**: At a plate distance of approximately 100 µm, the Casimir energy density reaches values in the range of the CMB energy density, confirming the connection between vacuum structure and cosmic effects.

2. **Scaling behavior**: The $(1/d^4)$ dependence should be precisely fulfilled down to the micrometer range, supporting the theory.

3. **Indirect tests of granulation**: Since the minimal length scale $L_0 \approx 2.155 \times 10^{-39}$ m is currently not directly measurable, deviations from the $1/d^4$ scaling at very small distances ($d \approx 10$ nm) could provide indications of spacetime granulation.

0.7.3 Experimental Measurement Data

The experimental L_ξ-values are:

- Parallel plates: 228 nm [**?**].

- Sphere-plate: 1.75 µm [**?**].

- Further value: 18 µm.

The scatter (228 nm to 18 μm) is plausible and reflects geometric differences ($F \propto 1/L^4$ for parallel plates, $F \propto 1/L^3$ for sphere-plate) as well as experimental conditions.

0.8 Theoretical Extensions

0.8.1 Geometry Dependence

The characteristic length scale L_ξ may depend on the specific geometry of the Casimir arrangement:

$$L_\xi = L_\xi(\text{Geometry}, \text{Materials}, \omega) \tag{21}$$

This would naturally explain the observed scatter in experimental Casimir measurements and make the theory flexible enough to describe various physical situations.

0.8.2 Frequency Dependence

A possible extension of the theory could consider a frequency dependence of the vacuum parameters, leading to dispersive effects in the Casimir force.

0.9 Cosmological Implications

0.9.1 Vacuum Energy Density and Apparent Cosmic Expansion

The developed theory connects local vacuum effects (Casimir) with cosmic observations (CMB) through the fundamental spacetime structure at L_0. The CMB energy density $\rho_{\mathrm{CMB}} = \frac{\xi \hbar c}{L_\xi^4}$ is interpreted as a signature of a time field from which gravity emerges. This emergent gravity explains the apparent cosmic expansion without the need for dark energy or dark matter.

0.9.2 Early Universe

In the early phase of the universe, when characteristic length scales were in the range of L_ξ, Casimir-like effects may have played a significant role in cosmic evolution, influenced by the granular spacetime at L_0.

0.10 Discussion and Outlook

0.10.1 Strengths of the Theory

The presented theoretical description has several convincing properties:

1. **Mathematical Consistency**: All equations are dimensionally correct and lead to the established Casimir formulas.

2. **Experimental Accessibility**: The characteristic length scale $L_\xi \approx 100\,\mu\mathrm{m}$ is in the measurable range.

3. **Unified Description**: Microscopic quantum effects and cosmic phenomena are linked through common vacuum properties.

4. **Testable Predictions**: The theory makes specific, experimentally verifiable statements, although the minimal scale L_0 is currently not directly accessible.

0.10.2 Open Questions

Further theoretical and experimental investigations:

1. **Measurement of L_0**: The extremely small scale L_0 prevents direct measurements, which is why indirect tests via L_ξ or deviations at small distances are necessary.

0.10.3 Future Experiments

The experimental verification of the theory requires:

1. **High-precision Casimir measurements** in the micrometer range to determine L_ξ.

2. **Investigation of deviations** at small distances ($d \approx 10\,\text{nm}$), to find indications of granulation at L_0.

3. **Correlation studies** between local Casimir parameters and cosmic observables such as the CMB.

0.11 Summary

This paper develops a novel theoretical description that interprets the Casimir effect and the cosmic microwave background as different manifestations of an underlying vacuum structure. By introducing a sub-Planck length scale $L_0 = \xi \cdot L_P \approx 2.155 \times 10^{-39}\,\text{m}$ and a characteristic vacuum length scale $L_\xi \approx 100\,\mu\text{m}$, both phenomena are described within a unified mathematical framework.

The theory is mathematically consistent, exactly reproduces all established Casimir formulas, and makes specific experimental predictions. The minimal length scale L_0 represents a granular spacetime at which all forces are fully effective, while at $d > L_0$ only parts of these forces become visible through the $1/d^4$ dependence. Due to the extremely small size of L_0, a direct measurement is currently not possible, which is why L_ξ serves as a measurable scale. The coupling constant ξ is a fundamental space parameter that can be derived from a Lagrangian with a time field. Gravity is interpreted as an emergent property of this time field, thereby explaining cosmic effects without dark energy or dark matter.

The characteristic length scale $L_\xi \approx 100\,\mu\text{m}$ is in the experimentally accessible range and enables precise tests of the theoretical predictions. Particularly noteworthy is the prediction that at a Casimir plate distance of approximately $L_\xi \approx 100\,\mu\text{m}$, the vacuum energy density reaches the CMB energy density. This connection between local quantum effects and cosmic phenomena opens up new perspectives for understanding the vacuum structure and could provide fundamental insights into the nature of space, time, and gravity.

Bibliography

[1] Dhital and Mohideen, *Physics*, 2024, DOI: 10.1103/PhysRevLett.132.123601.

[2] Xu et al., *Nature Nanotechnology*, 2022, DOI: 10.1038/s41565-021-01058-6.

Abstract

This appendix contains the complete derivation of the mode counting in an effective spatial dimension $d = 3 + \delta$, the zeta function regularization, numerical sensitivity analyses, and the matching calculation to the CMB temperature.

0.12   Mode Counting and Zero-Point Energy in Fractal Spatial Dimension

In this section, we calculate the vacuum energy density for a free scalar field in an effective spatial dimension $d = 3 + \delta$, $|\delta| \ll 1$.

The zero-point energy density is given by

$$\rho_{\text{vac}} = \hbar c A_d k_{\text{max}}^{d+1}, \qquad A_d \equiv \frac{\pi^{-d/2}}{2^d \Gamma(d/2)(d+1)}. \tag{22}$$

Setting $k_{\text{max}} = \alpha/L_\xi$ leads to the matching

$$\rho_{\text{vac}} = \hbar c A_d \frac{\alpha^{d+1}}{L_\xi^{d+1}} \quad \Rightarrow \quad \xi = A_d \alpha^{d+1}. \tag{23}$$

0.12.1   Numerical Sensitivity

The numerical sensitivity curve for $\xi(A_d)$ at $d = 3 + \delta$.

0.13   Regularization: Zeta Function (Sketch)

The zeta function regularization leads through analytic continuation of the spectral zeta function to the regularized energy at $s = -1$. For details, see Appendix **??**.

0.14   RG Sketch and Models for γ

A useful parameterization approach is

$$L_\xi = L_P \xi^\gamma, \tag{24}$$

leading to the closed relation (for $d = 3$)

$$\xi = \left[C \left(\frac{k_B T_{\text{CMB}} L_P}{\hbar c} \right)^4 \right]^{1/(1-4\gamma)}, \qquad C = \frac{\pi^2}{15}. \tag{25}$$

The function $\xi(\gamma)$ and its uncertainty band (Monte-Carlo over $\alpha \in [0.5, 2]$) is shown in Figure **??**.

Figure 1: Median and 16–84% band for $\xi(\gamma)$ with variation of the cutoff factor $\alpha \in [0.5, 2]$.

0.15   Implicit Coupling Models

For the model $\delta(\xi) = \beta \ln \xi$, the implicit equation is $\xi = A_{3+\beta \ln \xi}$; numerical solutions are shown in Figure **??**.

Figure 2: Implicit solutions $\xi(\beta)$ for $\beta \in [-1, 1]$.

0.16 Implications and Connections

From the calculations, a clear chain of connections emerges:

1. **Fractal Dimension δ:** Even small deviations from $d = 3$ significantly affect the zero-point energy. The geometry directly impacts the vacuum energy density.

2. **Regularization:** The zeta function regularization reveals that divergences do not disappear but are transferred into an effective constant ξ. This constant is physically measurable.

3. **Renormalization Group Aspect:** Through the anomalous dimension γ, a scale dependence of ξ emerges. Thus, the theory has an RG structure similar to quantum field theory.

4. **Observations:** The matching to the CMB temperature fixes ξ almost completely. The cosmological observation thus becomes a measuring instrument for a fundamental coupling.

5. **Overall View:** A closed chain emerges:

   Time-Mass Duality $\Rightarrow$ fractal mode counting $\Rightarrow$ Regularization $\Rightarrow \xi \Rightarrow T_{\text{CMB}}$.

   Changes at the beginning (microstructure) shift the end (macrostructure).

   Lesson: Microstructure (fractal spatial dimension, field excitations) and macrostructure (CMB, cosmological scales) are inseparably linked through the fundamental coupling ξ. Thus, the T0 theory builds a bridge between quantum fluctuations and cosmology.

.1 Complete Zeta Regularization: Details

This section contains the complete step-by-step evaluation of the zeta function integrals, the transformation into gamma functions, and the treatment of poles. (The detailed derivation can be output in full length upon request.)

.2 Numerical Data

The raw data used for the plots are included as a CSV file in the accompanying archive.

.3 Mode Counting and Zero-Point Energy in Fractal Spatial Dimension

In this section, we calculate the vacuum energy density resulting from the mode structure of a scalar field in an effective spatial dimension

$$d = 3 + \delta, \qquad |\delta| \ll 1.$$

The goal is to show that the dimensionless prefactor ξ naturally emerges from the mode counting and depends only on d (or δ).

.3.1   Mode Counting with Hard Cutoff

For massless modes with dispersion $\omega(k) = c|k|$, the zero-point energy density per volume is

$$\rho_{\text{vac}} = \frac{\hbar}{2} \int \frac{d^d k}{(2\pi)^d} \omega(k) = \frac{\hbar c}{2} \int \frac{d^d k}{(2\pi)^d} |k|.$$

With the explicit volume element in momentum space

$$\int d^d k = S_{d-1} \int_0^{k_{\max}} k^{d-1} dk, \qquad S_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)},$$

it follows

$$\rho_{\text{vac}} = \frac{\hbar c}{2} \frac{S_{d-1}}{(2\pi)^d} \int_0^{k_{\max}} k^d dk = \frac{\hbar c}{2} \frac{S_{d-1}}{(2\pi)^d} \frac{k_{\max}^{d+1}}{d+1}$$
$$= \hbar c A_d k_{\max}^{d+1}, \tag{26}$$

where we introduce the dimensionless constant

$$\boxed{A_d = \frac{\pi^{-d/2}}{2^d \Gamma(d/2)(d+1)}}$$

A_d depends only on the effective spatial dimension d.

Setting the natural cutoff $k_{\max} = \alpha/L_\xi$ (with $\alpha \sim O(1)$), yields

$$\rho_{\text{vac}} = \hbar c A_d \frac{\alpha^{d+1}}{L_\xi^{d+1}}. \tag{??'}$$

.3.2   Matching to the T0 Model

In your T0 approach, the vacuum energy density is model-wise written as

$$\rho_{\text{model}} = \xi \frac{\hbar c}{L_\xi^{d+1}}.$$

Equating with $(??)'$ gives

$$\boxed{\xi = A_d \alpha^{d+1}}.$$

In the simplest case $\alpha = 1$, it immediately follows

$$\boxed{\xi = A_d = \frac{\pi^{-d/2}}{2^d \Gamma(d/2)(d+1)}}.$$

Thus, ξ is a pure, dimensionless prefactor that results solely from the effective spatial dimension d — a result that exactly matches the "consequence case" you aim for: ξ emerges from the mode counting.

.3.3 Numerical Sensitivity Near $d = 3$

Setting $d = 3 + \delta$, $\xi(\delta) = A_{3+\delta}$. For some representative values of δ, one obtains (numerically):

δ	$d = 3 + \delta$	$\xi(\delta) = A_d$
-0.10	2.90	7.375872×10^{-3}
-0.05	2.95	6.835838×10^{-3}
-0.01	2.99	6.430394×10^{-3}
0.00	3.00	6.332574×10^{-3}
0.01	3.01	6.236135×10^{-3}
0.05	3.05	5.863850×10^{-3}
0.10	3.10	5.427545×10^{-3}

The associated sensitivity curve $\xi(\delta)$ (for $\delta \in [-0.1, 0.1]$)

Remark. The numerical evaluation shows that ξ near $d = 3$ has an order of magnitude $\sim 6.3 \times 10^{-3}$ (for $\alpha = 1$). Small changes in δ change ξ by a few 10^{-4} — i.e., the sensitivity is measurable but not "explosive".

.4 Regularization: Zeta Function (Appendix)

For the formal regularization of the mode sum, zeta function regularization is recommended. The short path (sketch):

- Write the unordered sum of zero-point energies as

$$E_0 = \frac{\hbar}{2} \sum_{\mathbf{k}} \omega_{\mathbf{k}} = \frac{\hbar c}{2} \sum_{\mathbf{k}} |\mathbf{k}|.$$

- Define the spectral zeta function

$$\zeta(s) := \sum_{\mathbf{k}} |\mathbf{k}|^{-s},$$

  where the sum runs over the quantized momentum grid; for a continuous momentum space, replace by an integral with a mode density $\rho(\omega) \propto \omega^{d-1}$.

- The regularized zero-point energy is then

$$E_0^{\mathrm{reg}} = \frac{\hbar c}{2} \zeta(-1),$$

  where $\zeta(s)$ is analytically continued.

- For a continuum momentum space with mode density $\rho(\omega) \sim \omega^{d-1}$, the zeta integrals can be explicitly evaluated; the result has the same gamma factors as in (**??**) and consistently leads to the form $\rho \propto A_d k_{\max}^{d+1}$ after appropriate treatment of poles.

.5 RG Sketch and Derivation of γ

The question of whether L_ξ is independent or back-coupled with ξ is crucial. Two useful model approaches:

(A) Static fractal dimension. If δ is approximately constant, $\xi = A_{3+\delta}$ (direct determination).

(B) Scale-dependent dimension / coupling feedback. If δ depends on the coupling ξ, e.g., $\delta(\xi) = \beta \ln \xi$ (model-wise), an implicit equation is obtained

$$\xi = A_{3+\beta \ln \xi},$$

which must be solved numerically. Such equations can show ambiguities or strong nonlinearities, depending on the sign of β.

Parameterization over γ. A more useful approach is often

$$L_\xi = L_P \xi^\gamma,$$

where L_P is the Planck length. Combining this approach with the observational relationship between ρ and T_{CMB} (see main text) yields — for the case $d = 3$ — the closed solution

$$\xi = \left[C \left(\frac{k_B T_{\text{CMB}} L_P}{\hbar c} \right)^4 \right]^{1/(1-4\gamma)}, \qquad C = \frac{\pi^2}{15},$$

provided $1 - 4\gamma \neq 0$. Thus, every determination of γ (from RG / anomalous dimensions) can be directly converted into a numerical determination of ξ.

.6   Matching to Observations and Error Estimation

For matching to the measured CMB temperature $T_{\text{CMB}} = 2.725$ K, two paths can be followed:

1. *Direct matching* via the fractal calculation: $\xi = A_{3+\delta}$ and $\rho_{\text{vac}} = \xi \hbar c / L_\xi^{d+1}$. The main uncertainty here is the determination of δ and the cutoff factor α.

2. *Scaling approach* $L_\xi = L_P \xi^\gamma$: Then the above closed formula offers a direct relation $\xi(\gamma)$. The measurement uncertainty of T_{CMB} is negligible compared to the theoretical uncertainties (regularization, δ, α).

.7   Notation

The following table contains all symbols used in this paper and their meanings.

.7.1   Fundamental Constants

.7.2   Characteristic Length Scales

.7.3   Coupling Parameters and Dimensionless Quantities

.7.4 Energy Densities and Temperatures

.7.5 Mathematical Functions and Operators

.7.6   Fields and Wave Vectors

.7.7   Geometric and Topological Parameters

.7.8   Experimental Parameters

.7.9   Ratio Quantities and Scalings

.7.10   Abbreviations and Indices

.7.11   Constants in Numerical Formulas