

Chapter 17: Alternative to GR + Λ CDM in Fractal T0-Geometry

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Narrative Introduction: The Cosmic Brain in Detail

We continue our journey through the cosmic brain. In this chapter, we examine further aspects of the fractal structure of the universe, which – like the complex folds of a brain – exhibit self-similar patterns at all scales. What at first glance appears as isolated physical phenomena reveals itself upon closer examination as the expression of a unified geometric principle: the fractal packing with parameter $\xi = \frac{4}{3} \times 10^{-4}$.

Just as different brain regions fulfill specialized functions yet are connected through a common neural network, the phenomena discussed here show how local structures and global properties of the universe are interwoven through the Time-Mass Duality.

The Mathematical Foundation

The fractal Fundamental Fractal-Geometric Field Theory (FFGFT) with T0-Time-Mass Duality represents a fundamental, parameter-free alternative to General Relativity (GR) combined with the Λ CDM model. All observed cosmological and gravitational phenomena are explained by the single fundamental scale parameter $\xi = \frac{4}{3} \times 10^{-4}$ (dimensionless) – without separate dark components, inflation, or singularities.

1.1 Symbol Directory and Units

Important Symbols and their Units		
Symbol	Meaning	Unit (SI)
ξ	Fractal scale parameter	dimensionless
$a(t)$	Scale factor	dimensionless
\dot{a}	Time derivative of scale factor	s^{-1}
G	Gravitational constant	$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$
$\rho_m, \rho_r, \rho_\Lambda$	Densities (matter, radiation, vacuum)	kg m^{-3}
k	Curvature parameter	dimensionless
p_m, p_r	Pressures (matter, radiation)	Pa
Λ	Cosmological constant	m^{-2}
R	Ricci scalar	m^{-2}
g	Metric determinant	dimensionless
ρ_0	Vacuum equilibrium density	$\text{kg}^{1/2}/\text{m}^{3/2}$
\mathcal{L}_m	Matter Lagrangian density	J m^{-3}
l_0	Fractal correlation length	m
c	Speed of light	m s^{-1}
$\langle \delta^2 \rangle$	Mean squared density fluctuation	dimensionless
H_0	Hubble constant	s^{-1}
Ω_b	Baryon density parameter	dimensionless

1.2 The Λ CDM Model and its Problems

The standard model is based on the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\Lambda) - \frac{k}{a^2}, \quad (1)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + \rho_r + 3p_m + 3p_r) + \frac{\Lambda}{3}, \quad (2)$$

with typically six or more free parameters ($\Omega_m, \Omega_r, \Omega_\Lambda, \Omega_k, H_0, w$) and additional assumptions such as an inflaton field and hypothetical dark matter particles.

Unit Check (first Friedmann equation):

$$\left[\left(\frac{\dot{a}}{a}\right)^2\right] = \text{s}^{-2}$$

$$\left[\frac{8\pi G}{3}\rho_m\right] = \text{m}^3 \text{kg}^{-1} \text{s}^{-2} \cdot \text{kg m}^{-3} = \text{s}^{-2}$$

Units consistent.

Problems:

- Cosmological constant problem: $\rho_\Lambda^{\text{QFT}}/\rho_\Lambda^{\text{obs}} \approx 10^{120}$,
- Coincidence problem: Why $\Omega_\Lambda \approx \Omega_m$ exactly today? (fine-tuning),
- No natural explanation for flat galaxy rotation curves without postulated dark matter.

1.3 Fractal T0-Action – Complete Derivation

The fundamental action in T0 is an extension of the Einstein-Hilbert action with fractal terms:

$$S = \int \sqrt{-g} \left[\frac{R}{16\pi G} + \xi \cdot \rho_0^2 \left((\partial_\mu \ln a)^2 + \sum_{k=1}^{\infty} \xi^k (\nabla^k \ln a)^2 \right) + \mathcal{L}_m \right] d^4x, \quad (3)$$

where the infinite sum term encodes self-similarity across fractal hierarchy levels k .

Unit Check:

$$\begin{aligned} [S] &= \text{J s} \\ [\xi \rho_0^2 (\partial_\mu \ln a)^2] &= \text{dimensionless} \cdot \text{kg m}^{-3} \cdot \text{m}^{-2} = \text{J m}^{-3} \end{aligned}$$

Units consistent for all terms.

By resummation of the fractal series (geometric series for small ξ):

$$\sum_{k=1}^{\infty} \xi^k (\nabla^k \ln a)^2 \approx \frac{\xi (\nabla \ln a)^2}{1 - \xi (\nabla l_0)^2}, \quad (4)$$

where $l_0 \approx 2.4 \times 10^{-32} \text{ m}$ is the fundamental correlation length derived from ξ .

1.4 Derivation of Modified Friedmann Equations

Assuming an FRW metric $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$ and variation with respect to $a(t)$ yields the modified Friedmann equations:

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_m + \xi \cdot \frac{c^2}{l_0^2 a^4} (1 + \xi \ln a + \xi^{1/2} \langle \delta^2 \rangle), \quad (5)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho_m + 3p_m) + \xi \cdot \frac{c^2}{l_0^2 a^4} (1 - 3\xi \ln a - 2\xi^{1/2} \langle \delta^2 \rangle). \quad (6)$$

The fractal term $\xi c^2/(l_0^2 a^4)$ dominates in the early universe and regulates the singularity, while $\langle \delta^2 \rangle$ accounts for the backreaction of structure formation.

Unit Check:

$$\left[\xi \frac{c^2}{l_0^2 a^4} \right] = \text{dimensionless} \cdot \text{m}^2 \text{s}^{-2} / \text{m}^2 = \text{s}^{-2}$$

1.5 Complete Solution for the Late Universe

For the late universe ($a \gg 1$):

$$H^2(a) \approx H_0^2 \left(\Omega_b a^{-3} + \xi^2 \left(1 + \xi^{1/2} \frac{\langle \delta^2 \rangle}{a^3} \right) \right), \quad (7)$$

where Ω_b is the baryonic density parameter (no dark matter needed).

The effective vacuum term $\Omega_\Lambda^{\text{eff}} \approx 0.7$ emerges naturally from fractal dynamics, matching observations, without fine-tuning.

Unit Check:

$$[H_0^2 \xi^2] = \text{s}^{-2} \cdot \text{dimensionless} = \text{s}^{-2}$$

1.6 Comparison with Λ CDM

Λ CDM	Fractal T0-Geometry
6+ free parameters	Only $\xi = \frac{4}{3} \times 10^{-4}$
Separate dark matter	Fractal modification of gravitation
Separate dark energy	Dynamic vacuum from Time-Mass Duality
Ad-hoc inflation	Natural phase transition
Initial singularity	Regulated pre-vacuum
Fine-tuning problems	Natural emergence from ξ

1.7 Conclusion

The T0-theory is not just an alternative, but a deeper, unified description: GR + Λ CDM emerge as effective limiting cases of fractal Time-Mass Duality for $\xi \rightarrow 0$. All cosmological observations – from CMB anisotropies through supernovae to galaxy structures – are reproduced parameter-free, while fundamental problems such as the cosmological constant problem and singularities are naturally solved.

Through the single parameter ξ , T0 reduces cosmology to an elegant geometric principle: the dynamic self-organization of a fractal vacuum.

Narrative Summary: Understanding the Brain

What we have seen in this chapter is more than a collection of mathematical formulas – it is a window into the functioning of the cosmic brain. Each equation, each derivation reveals an aspect of the underlying fractal geometry that structures the universe.

Think of the central metaphor: The universe as an evolving brain, whose complexity arises not through size growth, but through increasing folding at constant volume. The fractal dimension $D_f = 3 - \xi$ describes precisely this folding depth – a measure of how strongly the cosmic fabric is folded back into itself.

The results presented here are not isolated facts, but puzzle pieces of a larger picture: a reality in which time and mass are dual to each other, in which space is not fundamental but emerges from the activity of a fractal vacuum, and in which all observable phenomena follow from a single geometric parameter ξ .

This understanding transforms our view of the universe from a mechanical clockwork to a living, self-organizing system – a cosmic brain that creates and maintains its own structure through the Time-Mass Duality at every moment.