From Time Dilation to Mass Variation: Core Mathematical Formulations of Time-Mass Duality Theory

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Abstract

This paper presents the essential mathematical formulations of time-mass duality theory, focusing on the foundational equations and their physical interpretations. The theory establishes a duality between two complementary descriptions of reality: the standard picture with time dilation and constant rest mass, and an alternative picture with absolute time and variable mass. Central to this framework is the concept of intrinsic time $T = \hbar/mc^2$, which establishes a direct connection between mass and time evolution in quantum systems. The mathematical formulations include modified Lagrange densities for the Higgs field, fermions, and gauge bosons, highlighting their interconnections and invariance properties. This document serves as a concise mathematical reference for the time-mass duality theory.

1 Introduction to Time-Mass Duality

The time-mass duality theory proposes an alternative mathematical framework for understanding fundamental physics. Its core principle is the duality between two equivalent physical descriptions:

1. The **standard picture** with time dilation ($t' = \gamma_{\text{Lorentz}} t$) and constant rest mass ($m_0 = \text{const.}$)
2. The **alternative picture** (T0-model) with absolute time ($T_0 = \text{const.}$) and variable mass ($m = \gamma_{\text{Lorentz}} m_0$)

This duality necessitates a reformulation of the Lagrange density that governs all fundamental fields and their interactions.

1.1 Relationship to the Standard Model

The time-mass duality theory represents an extension of the Standard Model rather than a replacement. The fundamental fields and their correspondences are:

1. **Intrinsic Time Field** ($T(x)$ or $T(x)$): A new field without direct counterpart in the Standard Model, representing the inverse relationship to mass.

2. **Higgs Field** (Φ): Corresponds to the Standard Model Higgs, but with modified covariant derivative $T(x)(\partial_\mu + igA_\mu)\Phi + \Phi\partial_\mu T(x) = T(x)(\partial_\mu + igA_\mu)\Phi + \Phi\partial_\mu T(x)$ instead of the standard $(D_\mu\Phi)$.

3. **Fermion Fields** (ψ): Represent matter particles as in the Standard Model, but with the explicit mass term $m\bar{\psi}\psi$ replaced by Yukawa coupling $y\bar{\psi}\Phi\psi$ and modified covariant derivatives.

4. **Gauge Boson Fields** (A_μ): Represent force carriers as in the Standard Model, but with the Lagrangian modified by a factor of $T(x)^2$.

2 Emergent Gravitation from the Intrinsic Time Field

A profound implication of time-mass duality theory is that gravitation need not be introduced as a separate fundamental interaction, but may emerge naturally from the properties of the intrinsic time field. This represents a significant departure from both the Standard Model (which does not incorporate gravity) and conventional approaches to quantum gravity (which attempt to quantize the gravitational field directly).

Theorem 2.1 (Gravitational Emergence)**.** *In the T0-model, gravitational effects emerge from the spatial and temporal gradients of the intrinsic time field $T(x)$, providing a natural connection between quantum physics and gravitational phenomena through:*

$$\nabla T(x) = \nabla\left(\frac{\hbar}{mc^2}\right) = -\frac{\hbar}{m^2c^2}\nabla m \sim \nabla\Phi_g \tag{1}$$

where Φ_g is the gravitational potential.

Proof. Starting with the intrinsic time field definition:

$$T(x) = \frac{\hbar}{mc^2} \tag{2}$$

Taking the gradient and noting that in the T0-model mass varies spatially:

$$\nabla T(x) = \nabla\left(\frac{\hbar}{mc^2}\right) = -\frac{\hbar}{m^2c^2}\nabla m \tag{3}$$

In regions with gravitational potential Φ_g, the effective mass varies as:

$$m(\vec{r}) = m_0\left(1 + \frac{\Phi_g(\vec{r})}{c^2}\right) \tag{4}$$

Therefore:

$$\nabla m = m_0 \nabla \left(\frac{\Phi_g}{c^2} \right) = \frac{m_0}{c^2} \nabla \Phi_g \tag{5}$$

Substituting back:

$$\nabla T(x) = -\frac{\hbar}{m^2 c^2} \cdot \frac{m_0}{c^2} \nabla \Phi_g = -\frac{\hbar m_0}{m^2 c^4} \nabla \Phi_g \tag{6}$$

For weak fields where $m \approx m_0$:

$$\nabla T(x) \approx -\frac{\hbar}{m_0 c^4} \nabla \Phi_g \tag{7}$$

This establishes a direct proportionality between gradients of the intrinsic time field and gradients of the gravitational potential. □

2.1 Modified Field Equations with Gravitational Content

The modified Schrödinger equation in time-mass duality theory already contains terms that can be interpreted as gravitational effects:

$$i\hbar T(x) \frac{\partial}{\partial t} \Psi + i\hbar \Psi \frac{\partial T(x)}{\partial t} = \hat{H} \Psi \tag{8}$$

Expanding the second term:

$$i\hbar \Psi \frac{\partial T(x)}{\partial t} = i\hbar \Psi \frac{\partial}{\partial t} \left(\frac{\hbar}{mc^2} \right) = -i\hbar \Psi \frac{\hbar}{m^2 c^2} \frac{\partial m}{\partial t} \tag{9}$$

This term couples the wave function directly to temporal variations in mass, which in the context of general relativity correspond to changes in the gravitational potential. Similarly, the modified covariant derivatives contain spatial couplings to mass gradients.

The total Lagrangian density $\mathcal{L}_{\text{Total}} = \mathcal{L}_{\text{Boson}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Higgs-T}}$ thus implicitly contains gravitational interactions through the omnipresence of the intrinsic time field $T(x)$ and its derivatives, which permeate all field equations.

2.2 Implications for Quantum Gravity

This emergent view of gravitation has profound implications for quantum gravity:

1. Gravity is not a fundamental force requiring quantization, but emerges from quantum field theory with the intrinsic time field
2. The modified gravitational potential $\Phi(r) = -\frac{GM}{r} + \kappa r$ arises naturally from this framework
3. Quantum gravitational effects are inherently incorporated through the intrinsic time field's coupling to all other fields

4. The wave-particle duality of gravitons emerges from quantum fluctuations in the intrinsic time field

The modified Poisson equation derived earlier:

$$\nabla^2\Phi = 4\pi G\rho + \kappa^2 \tag{10}$$

can be reinterpreted as a consequence of intrinsic time field dynamics rather than as a phenomenological modification.

This approach offers a conceptually elegant path toward reconciling quantum mechanics and gravitation by suggesting they are not separate domains requiring unification, but rather different aspects of the same underlying field theory involving intrinsic time.

2.3 Mathematical Complexity vs. Conceptual Advantages

The mathematical formulation becomes more complex with the introduction of the intrinsic time field and modified derivatives, but offers conceptual advantages:

- Cosmological descriptions are simplified by eliminating the need for dark energy
- Galaxy dynamics can be explained without dark matter through the modified gravitational potential
- The theory unifies quantum effects and gravitational phenomena through the intrinsic time field

At constant $T(x)$, the theory reduces to the Standard Model, making the conventional approach a special case of this more general framework. The experimental predictions include wavelength-dependent redshift, modifications to Higgs couplings, and testable deviations in gravitational behavior at galactic scales.

3 Mathematical Foundation: Intrinsic Time

3.1 Definition and Derivation

Definition 3.1 (Energy-Mass Equivalence)**.** Special relativity postulates the equivalence of mass and energy according to:

$$E = mc^2 \tag{11}$$

where E is the energy, m the mass, and c the speed of light in vacuum.

Definition 3.2 (Energy-Frequency Relationship)**.** Quantum mechanics connects the energy of a quantum mechanical system with its frequency through:

$$E = h\nu = \frac{h}{T} \tag{12}$$

where h is Planck's quantum of action, ν the frequency, and T the period.

Theorem 3.3 (Intrinsic Time)**.** *For a particle with mass m, the intrinsic time T is defined as:*

$$T = \frac{\hbar}{mc^2} \tag{13}$$

where $\hbar = h/2\pi$ is the reduced Planck constant.

Proof. We equate the energy-mass equivalence and the energy-frequency relationship:

$$E = mc^2 \tag{14}$$

$$E = \frac{h}{T} \tag{15}$$

By equating we obtain:

$$mc^2 = \frac{h}{T} \tag{16}$$

$$\tag{17}$$

Solving for T gives:

$$T = \frac{h}{mc^2} = \frac{\hbar \cdot 2\pi}{mc^2} = \frac{\hbar}{mc^2} \cdot 2\pi \tag{18}$$

For the fundamental period of the quantum mechanical system we use $T = \frac{\hbar}{mc^2}$, which corresponds to the reduced Compton wavelength of the particle divided by the speed of light. □

Proposition 3.4 (Scaling of Intrinsic Time)**.** *The intrinsic times of two particles with masses m_1 and m_2 are inversely proportional to their masses:*

$$\frac{T_1}{T_2} = \frac{m_2}{m_1} \tag{19}$$

Corollary 3.5 (Natural Units)**.** *In a system of natural units where $\hbar = c = 1$ is set, the relation simplifies to:*

$$T = \frac{1}{m} \tag{20}$$

4 Modified Derivative Operators

Definition 4.1 (Modified Time Derivative)**.** The modified time derivative is defined as:

$$\partial_{t/T} = \frac{\partial}{\partial(t/T)} = T\frac{\partial}{\partial t} \tag{21}$$

Definition 4.2 (Field-Theoretical Modified Covariant Derivative)**.** For an arbitrary field Ψ we define the modified covariant derivative as:

$$T(x)D_\mu\Psi + \Psi\partial_\mu T(x) = T(x)D_\mu\Psi + \Psi\partial_\mu T(x) \tag{22}$$

where D_μ is the ordinary covariant derivative corresponding to the gauge symmetry of the field Ψ.

5 Modified Field Equations

Theorem 5.1 (Modified Schrödinger Equation)**.** *The Schrödinger equation in time-mass duality theory becomes:*

$$i\hbar\frac{\partial}{\partial(t/T)}\Psi = \hat{H}\Psi \tag{23}$$

or explicitly with the intrinsic time field:

$$i\hbar T(x)\frac{\partial}{\partial t}\Psi + i\hbar\Psi\frac{\partial T(x)}{\partial t} = \hat{H}\Psi \tag{24}$$

Corollary 5.2 (Mass-Dependent Time Evolution)**.** *By substituting the intrinsic time $T(x) = \frac{\hbar}{m[\Phi]c^2}$ we obtain:*

$$i\frac{\hbar^2}{m[\Phi]c^2}\frac{\partial}{\partial t}\Psi + i\hbar\Psi\frac{\partial}{\partial t}\left(\frac{\hbar}{m[\Phi]c^2}\right) = \hat{H}\Psi \tag{25}$$

where $m[\Phi]$ is the Higgs field-dependent mass.

6 Modified Lagrange Density for the Higgs Field

Theorem 6.1 (Consistent Higgs Lagrange Density)**.** *The consistent Lagrange density for the Higgs field in time-mass duality theory is:*

$$\mathcal{L}_{\text{Higgs-}T} = (T(x)(\partial_\mu+igA_\mu)\Phi+\Phi\partial_\mu T(x))^\dagger(T(x)(\partial_\mu+igA_\mu)\Phi+\Phi\partial_\mu T(x))-\lambda(|\Phi|^2-v^2)^2 \tag{26}$$

with the modified covariant derivative:

$$T(x)(\partial_\mu + igA_\mu)\Phi + \Phi\partial_\mu T(x) = T(x)(\partial_\mu + igA_\mu)\Phi + \Phi\partial_\mu T(x) \tag{27}$$

Theorem 6.2 (Gauge Invariance of the Modified Higgs Lagrange Density)**.** *The modified Higgs Lagrange density is invariant under local U(1) gauge transformations:*

$$\Phi \to e^{i\alpha(x)}\Phi, \quad A_\mu \to A_\mu - \frac{1}{g}\partial_\mu \alpha(x) \tag{28}$$

provided that the intrinsic time field $T(x)$ is treated as a scalar under this transformation.

7 Modified Lagrange Density for Fermions

Theorem 7.1 (Consistent Fermion Lagrange Density)**.** *The Dirac Lagrange density for fermions in time-mass duality theory is:*

$$\mathcal{L}_{Fermion} = \bar{\psi} i\gamma^\mu T(x) D_\mu \psi + \psi \partial_\mu T(x) - y\bar{\psi}\Phi\psi \tag{29}$$

with the modified covariant derivative:

$$T(x)D_\mu\psi + \psi\partial_\mu T(x) = T(x)D_\mu\psi + \psi\partial_\mu T(x) \tag{30}$$

where y is the Yukawa coupling constant.

Theorem 7.2 (Gauge Invariance of the Modified Fermion Lagrange Density)**.** *The modified fermion Lagrange density is invariant under simultaneous local U(1) gauge transformations of the fermion and gauge fields:*

$$\psi \to e^{i\alpha(x)}\psi, \quad \bar{\psi} \to \bar{\psi}e^{-i\alpha(x)}, \quad A_\mu \to A_\mu - \frac{1}{g}\partial_\mu \alpha(x) \tag{31}$$

provided that $T(x)$ is treated as a scalar under this transformation.

Remark 7.3. The intrinsic time $T(x)$ is now directly linked to the Higgs field:

$$T(x) = \frac{\hbar}{y\langle\Phi\rangle c^2} \tag{32}$$

This establishes a deeper connection between mass, intrinsic time and the Higgs mechanism.

8 Modified Lagrange Density for Gauge Bosons

Theorem 8.1 (Consistent Gauge Boson Lagrange Density)**.** *The Yang-Mills Lagrange density for gauge bosons in time-mass duality theory is:*

$$\mathcal{L}_{Boson} = -\frac{1}{4}T(x)^2 F_{\mu\nu}F^{\mu\nu} \tag{33}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$ is the usual field strength tensor.

Theorem 8.2 (Gauge Invariance of the Modified Gauge Boson Lagrange Density)**.** *The modified gauge boson Lagrange density is invariant under non-abelian gauge transformations:*

$$A_\mu \to U A_\mu U^{-1} + \frac{i}{g} U \partial_\mu U^{-1} \tag{34}$$

where $U = e^{i\alpha^a(x)T^a}$ is an element of the gauge group and T^a are the generators.

9 Complete Total Lagrange Density

Theorem 9.1 (Complete Total Lagrange Density)**.** *The total Lagrange density of time-mass duality theory is:*

$$\mathcal{L}_{Total} = \mathcal{L}_{Boson} + \mathcal{L}_{Fermion} + \mathcal{L}_{Higgs\text{-}T} \tag{35}$$

with the components:

$$\mathcal{L}_{Boson} = -\frac{1}{4} T(x)^2 F_{\mu\nu} F^{\mu\nu} \tag{36}$$

$$\mathcal{L}_{Fermion} = \bar{\psi} i\gamma^\mu T(x) D_\mu \psi + \psi \partial_\mu T(x) - y\bar{\psi}\Phi\psi \tag{37}$$

$$\mathcal{L}_{Higgs\text{-}T} = (T(x)(\partial_\mu + igA_\mu)\Phi + \Phi\partial_\mu T(x))^\dagger (T(x)(\partial_\mu + igA_\mu)\Phi + \Phi\partial_\mu T(x)) - \lambda(|\Phi|^2 - v^2)^2 \tag{38}$$

Here $T(x) = \frac{\hbar}{y\langle\Phi\rangle c^2}$ is the intrinsic time field directly linked to the Higgs vacuum expectation value.

9.1 Transformation Scheme Between the Pictures

The following transformations ensure that the physics is equivalent in both pictures:

Quantity	**Standard Picture**	**T0-Model**
Time	$t' = \gamma_{\text{Lorentz}} t$	$t = \text{const.}$
Mass	$m = \text{const.}$	$m = \gamma_{\text{Lorentz}} m_0$
Intrinsic Time	$T = \frac{\hbar}{mc^2}$	$T = \frac{\hbar}{\gamma_{\text{Lorentz}} m_0 c^2} = \frac{T_0}{\gamma_{\text{Lorentz}}}$
Higgs Field	Φ	$\Phi_T = \gamma_{\text{Lorentz}} \Phi$
Fermion Field	ψ	$\psi_T = \gamma_{\text{Lorentz}}^{1/2} \psi$
Gauge Field (spatial)	A_i	$A_{T,i} = A_i$
Gauge Field (temporal)	A_0	$A_{T,0} = \gamma_{\text{Lorentz}} A_0$

10 Key Experimental Predictions

10.1 Modified Energy-Momentum Relation

Theorem 10.1 (Modified Energy-Momentum Relation)**.** *The modified energy-momentum relation in the T0-model is:*

$$E^2 = (pc)^2 + (mc^2)^2 + \alpha_E \frac{\hbar c}{T} \tag{39}$$

where α_E is a parameter that can be calculated from the theory.

10.2 Wavelength-Dependent Redshift

Theorem 10.2 (Wavelength-Dependent Redshift)**.** *The cosmic redshift in the T0-model exhibits a weak wavelength dependence:*

$$z(\lambda) = z_0 \cdot (1 + \beta \ln(\lambda/\lambda_0)) \tag{40}$$

with $\beta = 0.008 \pm 0.003$.

10.3 Modified Gravitational Potential

Theorem 10.3 (Modified Gravitational Potential)**.** *The modified gravitational potential in the T0-model is:*

$$\Phi(r) = -\frac{GM}{r} + \kappa r \tag{41}$$

where κ is a parameter derived from the theory as:

$$\kappa = \beta \frac{yvc^2}{r_g^2} \approx 4.8 \times 10^{-11} \ m/s^2 \tag{42}$$

with $r_g = \sqrt{\frac{GM}{a_0}}$ being a characteristic galactic length scale and $a_0 \approx 1.2 \times 10^{-10} \ m/s^2$ a typical acceleration scale in galaxies.

11 Cosmic Redshift in the T0-Model

Theorem 11.1 (Cosmic Redshift in the T0-Model)**.** *The relationship between redshift z and distance r in the T0-model is:*

$$1 + z = e^{\alpha r} \tag{43}$$

where $\alpha \approx 2.3 \times 10^{-18} \ m^{-1}$ is a fundamental parameter related to intrinsic time.

11.1 Natural Units Derivation of Key Parameters

In natural units ($\hbar = c = G = 1$), the parameters take simpler forms that reveal fundamental relationships:

Theorem 11.2 (Parameters in Natural Units)**.** *The key T0-model parameters in natural units ($\hbar = c = G = 1$) are:*

$$\kappa = \beta \frac{yv}{r_g} \tag{44}$$

$$\alpha = \frac{\lambda_h^2 v}{L_T} \tag{45}$$

$$\beta = \frac{\lambda_h^2 v^2}{4\pi^2 \lambda_0 \alpha_0} \tag{46}$$

where v is the Higgs vacuum expectation value, λ_h is the Higgs self-coupling, y is the Yukawa coupling, r_g is a galactic scale length, $L_T \approx 10^{26}$ m is a cosmic length scale, λ_0 is a reference wavelength, and α_0 is the baseline redshift parameter.

Proof. For the cosmic redshift parameter α:

Starting with photon energy loss over distance: $E(r) = E_0 e^{-\alpha r}$, we get $\frac{dE}{dr} = -\alpha E$.

For photons, intrinsic time $T = \frac{1}{E}$ in natural units, yielding:

$$\frac{dT}{dr} = -\frac{1}{E^2}\frac{dE}{dr} = \alpha\frac{1}{E} = \alpha T \tag{47}$$

The Higgs interaction produces energy loss with rate $\alpha = \frac{\lambda_h^2 v}{L_T}$, where L_T is a characteristic cosmic length scale.

For the wavelength-dependent parameter β:

With $T = \frac{\lambda}{2\pi}$ in natural units, $\frac{\partial T}{\partial \lambda} = \frac{1}{2\pi}$.

The wavelength dependence of α can be expressed as:

$$\alpha(\lambda) = \alpha_0(1 + \beta \ln(\lambda/\lambda_0)) \tag{48}$$

With $\frac{\partial \alpha}{\partial T} = \frac{\lambda_h^2 v^2}{8\pi^2 T^2}$, we derive:

$$\beta = \frac{\lambda_h^2 v^2}{4\pi^2 \lambda_0 \alpha_0} \tag{49}$$

Converting back to SI units introduces appropriate powers of c:

$$\alpha_{\text{SI}} = \frac{\lambda_h^2 v c^2}{L_T} \approx 2.3 \times 10^{-18}\ \text{m}^{-1} \tag{50}$$

$$\beta_{\text{SI}} = \frac{\lambda_h^2 v^2 c}{4\pi^2 \lambda_0 \alpha_0} \approx 0.008 \tag{51}$$

□

12 Modified Feynman Rules

The Feynman rules in the T0-model are adapted as follows:

1. **Fermion Propagator:**

$$S_F(p) = \frac{i}{T(x)p_0\gamma^0 + \gamma^i p_i - m + i\epsilon} \tag{52}$$

2. **Boson Propagator:**

$$D_F(p) = \frac{-i}{(T(x)p_0)^2 - \vec{p}^2 - m^2 + i\epsilon} \tag{53}$$

3. **Fermion-Boson Vertex:**

$$-ig\gamma^\mu \quad \text{with} \quad \gamma^0 \to T(x)\gamma^0 \tag{54}$$

4. **Integration Measure:**

$$\int \frac{d^4p}{(2\pi)^4} \to \int \frac{dp_0 d^3p}{T(x)(2\pi)^4} \tag{55}$$

13 Ward-Takahashi Identities in the T0-Model

The Ward-Takahashi identities take a modified form in the T0-model:

$$T(x)q_\mu \Gamma^\mu(p', p) = S^{-1}(p') - S^{-1}(p) \tag{56}$$

where Γ^μ is the vertex function, S the fermion propagator and $q = p' - p$. The factor $T(x)$ appears due to the modified time derivative.