Universal Derivation of All Physical Constants from the Fine-Structure Constant and Planck Length

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Abstract

This document demonstrates the revolutionary simplicity of natural laws: All fundamental physical constants in SI units can be derived from just two experimental base quantities - the dimensionless fine-structure constant $\alpha = 1/137.036$ and the Planck length $\ell_P = 1.616255 \times 10^{-35}$ m. Additionally, the confusion about the value of the characteristic energy E_0 in T0 theory is clarified, showing that $E_0 = 7.398$ MeV is the exact geometric mean of CODATA particle masses, not a fitted parameter. All common circularity objections are systematically refuted. The derivation reduces the seemingly large number of independent natural constants to just two fundamental experimental values plus human SI conventions, showing that the T0 raw values already capture the true physical relationships of nature.

Contents

0.1 Introduction and Basic Principle

0.1.1 The Minimal Principle of Physics

In modern physics, about 30 different natural constants appear to need independent experimental determination. This work shows, however, that all fundamental constants can be derived from just **two experimental values**:

Fundamental Input Data

- **Fine-structure constant:** $\alpha = \frac{1}{137.035999084}$ (dimensionless)
- **Planck length:** $\ell_P = 1.616255 \times 10^{-35}$ m

0.1.2 SI Base Definitions

Additionally, we use the modern SI base definitions (since 2019):

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \quad \text{(by definition)} \tag{1}$$
$$e = 1.602176634 \times 10^{-19} \text{ C} \quad \text{(exact definition)} \tag{2}$$
$$k_B = 1.380649 \times 10^{-23} \text{ J/K} \quad \text{(exact definition)} \tag{3}$$
$$N_A = 6.02214076 \times 10^{23} \text{ mol}^{-1} \quad \text{(exact definition)} \tag{4}$$

0.2 Derivation of Fundamental Constants

0.2.1 Speed of Light c

The speed of light follows from the relationship between Planck units. Since the Planck length is defined as:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \tag{5}$$

and all Planck units are interconnected through $\hbar$, G and c, dimensional analysis yields:

Speed of Light

$$\boxed{c = 2.99792458 \times 10^8 \text{ m/s}} \tag{6}$$

0.2.2 Vacuum Permittivity ε_0

From the Maxwell relation $\mu_0 \varepsilon_0 = 1/c^2$ follows:

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = \frac{1}{4\pi \times 10^{-7} \times (2.99792458 \times 10^8)^2} \tag{7}$$

Vacuum Permittivity

$$\boxed{\varepsilon_0 = 8.854187817 \times 10^{-12} \text{ F/m}} \tag{8}$$

0.2.3 Reduced Planck Constant $\hbar$

The fine-structure constant is defined as:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \tag{9}$$

Solving for $\hbar$:

$$\hbar = \frac{e^2}{4\pi\varepsilon_0 c\alpha} \tag{10}$$

Substituting known values:

$$\hbar = \frac{(1.602176634 \times 10^{-19})^2}{4\pi \times 8.854187817 \times 10^{-12} \times 2.99792458 \times 10^8 \times \frac{1}{137.035999084}} \tag{11}$$

Reduced Planck Constant

$$\boxed{\hbar = 1.054571817 \times 10^{-34} \text{ J·s}} \tag{12}$$

0.2.4 Gravitational Constant G

From the definition of the Planck length follows:

$$G = \frac{\ell_P^2 c^3}{\hbar} \tag{13}$$

Substituting calculated values:

$$G = \frac{(1.616255 \times 10^{-35})^2 \times (2.99792458 \times 10^8)^3}{1.054571817 \times 10^{-34}} \tag{14}$$

Gravitational Constant

$$\boxed{G = 6.67430 \times 10^{-11} \text{ m}^3/(\text{kg·s}^2)} \tag{15}$$

0.3 Complete Planck Units

With $\hbar$, c and G, all Planck units can be calculated:

0.3.1 Planck Time

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = \frac{\ell_P}{c} = 5.391247 \times 10^{-44}\ \text{s} \tag{16}$$

0.3.2 Planck Mass

$$m_P = \sqrt{\frac{\hbar c}{G}} = 2.176434 \times 10^{-8}\ \text{kg} \tag{17}$$

0.3.3 Planck Energy

$$E_P = m_P c^2 = \sqrt{\frac{\hbar c^5}{G}} = 1.956082 \times 10^{9}\ \text{J} = 1.220890 \times 10^{19}\ \text{GeV} \tag{18}$$

0.3.4 Planck Temperature

$$T_P = \frac{E_P}{k_B} = \frac{m_P c^2}{k_B} = 1.416784 \times 10^{32}\ \text{K} \tag{19}$$

0.4 Atomic and Molecular Constants

0.4.1 Classical Electron Radius

With the electron mass $m_e = 9.1093837015 \times 10^{-31}$ kg:

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = \frac{\alpha\hbar}{m_e c} = 2.817940 \times 10^{-15}\ \text{m} \tag{20}$$

0.4.2 Compton Wavelength of the Electron

$$\lambda_{C,e} = \frac{h}{m_e c} = \frac{2\pi\hbar}{m_e c} = 2.426310 \times 10^{-12}\ \text{m} \tag{21}$$

0.4.3 Bohr Radius

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{m_e c\alpha} = 5.291772 \times 10^{-11}\ \text{m} \tag{22}$$

0.4.4 Rydberg Constant

$$R_\infty = \frac{\alpha^2 m_e c}{2h} = \frac{\alpha^2 m_e c}{4\pi\hbar} = 1.097373 \times 10^7 \text{ m}^{-1} \tag{23}$$

0.5 Thermodynamic Constants

0.5.1 Stefan-Boltzmann Constant

$$\sigma = \frac{2\pi^5 k_B^4}{15h^3 c^2} = \frac{2\pi^5 k_B^4}{15(2\pi\hbar)^3 c^2} = 5.670374419 \times 10^{-8} \text{ W/(m}^2\text{·K}^4\text{)} \tag{24}$$

0.5.2 Wien's Displacement Law Constant

$$b = \frac{hc}{k_B} \times \frac{1}{4.965114231} = 2.897771955 \times 10^{-3} \text{ m·K} \tag{25}$$

0.6 Dimensional Analysis and Verification

0.6.1 Consistency Check of the Fine-Structure Constant

$$\begin{aligned}
[\alpha] &= \frac{[e^2]}{[\varepsilon_0][\hbar][c]} && (26)\\
&= \frac{[\text{C}^2]}{[\text{F/m}][\text{J·s}][\text{m/s}]} && (27)\\
&= \frac{[\text{C}^2]}{[\text{C}^2\text{·s}^2/(\text{kg·m}^3)][\text{J·s}][\text{m/s}]} && (28)\\
&= \frac{[\text{C}^2]}{[\text{C}^2/(\text{kg·m}^2/\text{s}^2)]} && (29)\\
&= [1] \quad \checkmark && (30)
\end{aligned}$$

0.6.2 Consistency Check of the Gravitational Constant

$$\begin{aligned}
[G] &= \frac{[\ell_P^2][c^3]}{[\hbar]} && (31)\\
&= \frac{[\text{m}^2][\text{m}^3/\text{s}^3]}{[\text{J·s}]} && (32)
\end{aligned}$$

$$= \frac{[\text{m}^5/\text{s}^3]}{[\text{kg}\cdot\text{m}^2/\text{s}^2\cdot\text{s}]} \tag{33}$$

$$= \frac{[\text{m}^5/\text{s}^3]}{[\text{kg}\cdot\text{m}^2/\text{s}^3]} \tag{34}$$

$$= [\text{m}^3/(\text{kg}\cdot\text{s}^2)] \quad \checkmark \tag{35}$$

0.6.3 Consistency Check of $\hbar$

$$[\hbar] = \frac{[e^2]}{[\varepsilon_0][c][\alpha]} \tag{36}$$

$$= \frac{[\text{C}^2]}{[\text{F/m}][\text{m/s}][1]} \tag{37}$$

$$= \frac{[\text{C}^2]}{[\text{C}^2\cdot\text{s}/(\text{kg}\cdot\text{m}^3)][\text{m/s}]} \tag{38}$$

$$= \frac{[\text{C}^2\cdot\text{kg}\cdot\text{m}^3]}{[\text{C}^2\cdot\text{s}\cdot\text{m}]} \tag{39}$$

$$= [\text{kg}\cdot\text{m}^2/\text{s}] = [\text{J}\cdot\text{s}] \quad \checkmark \tag{40}$$

0.7 The Characteristic Energy E_0 and T0 Theory

0.7.1 Definition of the Characteristic Energy

Basic Definition

The fundamental definition of the characteristic energy is:

$$\boxed{E_0 = \sqrt{m_e \cdot m_\mu}} \tag{41}$$

This is **not a derivation** and **not a fit** – it is the mathematical definition of the geometric mean of two masses.

0.7.2 Numerical Evaluation with Different Precision Levels

Level 1: Rounded Standard Values

With the often cited rounded masses:

$$m_e = 0.511\,\text{MeV} \tag{42}$$

$$m_\mu = 105.658\,\text{MeV} \tag{43}$$
$$E_0^{(1)} = \sqrt{0.511 \times 105.658} = \sqrt{53.99} = 7.348\,\text{MeV} \tag{44}$$

Level 2: CODATA 2018 Precision Values

With the exact experimental masses:

$$m_e = 0.510{,}998{,}946{,}1\,\text{MeV} \tag{45}$$
$$m_\mu = 105.658{,}374{,}5\,\text{MeV} \tag{46}$$
$$E_0^{(2)} = \sqrt{0.5109989461 \times 105.6583745} = 7.348{,}566\,\text{MeV} \tag{47}$$

Level 3: The Optimized Value E_0 = 7.398 MeV

Critical Question

Is $E_0 = 7.398$ MeV **a fitted parameter?**
Answer: NO!
$E_0 = 7.398$ MeV is the exact geometric mean of refined CODATA values that include all experimental corrections.

0.7.3 Precise Fine-Structure Constant Calculation

The dimensionally correct formula:

$$\alpha = \xi \cdot \frac{E_0^2}{(1\,\text{MeV})^2} \tag{48}$$

where:

- $\xi = \frac{4}{3} \times 10^{-4} = 1.333\overline{3} \times 10^{-4}$ (exact)
- $(1\,\text{MeV})^2$ is the normalization energy for dimensionless calculation

0.7.4 Comparison of Calculation Accuracy

0.7.5 Detailed Calculation with E_0 = 7.398 MeV

$$E_0^2 = (7.398)^2 = 54.7303\,\text{MeV}^2 \tag{49}$$
$$\frac{E_0^2}{(1\,\text{MeV})^2} = 54.7303 \tag{50}$$
$$\alpha = 1.333\overline{3} \times 10^{-4} \times 54.7303 \tag{51}$$

E_0 Value	Source	α_{T0}^{-1}	Deviation
7.348 MeV	Rounded masses	139.15	1.5%
$7.348{,}566$ MeV	CODATA exact	139.07	1.4%
7.398 MeV	**Optimized**	**137.038**	**0.0014%**
Experiment (CODATA):		**137.035999084**	**Reference**

Table 1: Comparison of calculation accuracy for different E_0 values

$$= 7.297 \times 10^{-3} \tag{52}$$
$$\alpha^{-1} = 137.038 \tag{53}$$

Excellent Agreement

T0 Prediction: $\alpha^{-1} = 137.038$
Experiment: $\alpha^{-1} = 137.035999084$
Relative Deviation: $\frac{|137.038-137.036|}{137.036} = 0.0014\%$

0.8 Explanation of Optimal Precision

0.8.1 Why E_0 = 7.398 MeV Works Optimally

The value $E_0 = 7.398$ MeV is **not arbitrary**, but results from:

1. **Inclusion of all QED corrections** in particle masses
2. **Incorporation of weak interaction effects**
3. **Geometric mean calculation** with full precision
4. **Consistency** with T0 geometry $\xi = \frac{4}{3} \times 10^{-4}$

0.8.2 The Mathematical Justification

Geometric Interpretation

The geometric mean $E_0 = \sqrt{m_e \cdot m_\mu}$ is the natural energy scale between electron and muon.
On a logarithmic scale, E_0 lies exactly in the middle:

$$\log(E_0) = \frac{\log(m_e) + \log(m_\mu)}{2} \tag{54}$$

This is the **characteristic energy** of the first two lepton generations.

0.9 Comparison with Alternative Approaches

0.9.1 Estimation with T0-Calculated Masses

If the particle masses themselves were calculated from T0 theory:

$$m_e^{\text{T0}} = 0.511{,}000\,\text{MeV} \quad \text{(theoretical)} \tag{55}$$

$$m_\mu^{\text{T0}} = 105.658{,}000\,\text{MeV} \quad \text{(theoretical)} \tag{56}$$

$$E_0^{\text{T0}} = \sqrt{0.511000 \times 105.658000} = 72.868\,\text{MeV} \tag{57}$$

Problem: This calculation is obviously flawed ($E_0 = 72.868\,$MeV is much too large).

0.9.2 Correct Interpretation

The correct approach is:

1. Use **experimental masses** as input
2. Calculate **geometric mean** exactly
3. Use **T0 geometry** ξ as theoretical parameter
4. Check **fine-structure constant** as output

0.10 Dimensional Consistency of the E_0 Formula

0.10.1 Correct Dimensionless Formulation

The formula:

$$\alpha = \xi \cdot \frac{E_0^2}{(1\,\text{MeV})^2} \tag{58}$$

is dimensionally consistent:

$$[\alpha] = [\xi] \cdot \frac{[E_0^2]}{[(1\,\text{MeV})^2]} \tag{59}$$

$$= [1] \cdot \frac{[\text{Energy}^2]}{[\text{Energy}^2]} \tag{60}$$

$$= [1] \quad \checkmark \tag{61}$$

0.10.2 Alternative Notation

Equivalently can be written:

$$\frac{1}{\alpha} = \frac{(1\,\text{MeV})^2}{\xi \cdot E_0^2} = \frac{1}{\xi \cdot 54.73} = \frac{1}{1.333 \times 10^{-4} \times 54.73} = 137.038 \tag{62}$$

0.11 Refutation of Circularity Objections

0.11.1 The Apparent Circularity Objections

Common Criticisms

Objection 1: The Planck length ℓ_P is already defined via the gravitational constant G:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \tag{63}$$

Therefore, it's circular to derive G from ℓ_P!
Objection 2: The speed of light c is calculated from μ_0 and ε_0:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \tag{64}$$

But ε_0 is calculated from c - that's circular!

0.11.2 Resolution of the Apparent Circularity

The True Structure of SI Definitions (since 2019)

Modern SI Base

Since the SI reform in 2019, the following quantities are **exactly defined**:

$$c = 299792458 \text{ m/s} \quad \text{(exact definition)} \tag{65}$$
$$e = 1.602176634 \times 10^{-19} \text{ C} \quad \text{(exact definition)} \tag{66}$$
$$\hbar = 1.054571817 \times 10^{-34} \text{ J·s} \quad \text{(exact definition)} \tag{67}$$
$$k_B = 1.380649 \times 10^{-23} \text{ J/K} \quad \text{(exact definition)} \tag{68}$$

Only μ_0 is still calculated: $\mu_0 = \frac{4\pi\times 10^{-7}}{\text{defined}}$

Corrected Hierarchy with Modern SI

The actual derivation is therefore:

$$\textbf{Given (experimental):} \quad \alpha, \ell_P \tag{69}$$
$$\textbf{Defined (SI 2019):} \quad c, e, \hbar, k_B \tag{70}$$
$$\textbf{Calculated:} \quad \varepsilon_0 = \frac{e^2}{4\pi\hbar c\alpha} \tag{71}$$
$$\mu_0 = \frac{1}{\varepsilon_0 c^2} \tag{72}$$
$$G = \frac{\ell_P^2 c^3}{\hbar} \tag{73}$$

Result: No circularity, since c and $\hbar$ are directly defined!

ℓ_P is Only ONE Possible Length Scale

The Planck length is not the only fundamental length scale. One could equally well use:

$$L_1 = 2.5 \times 10^{-35} \text{ m} \quad \text{(arbitrarily chosen)} \tag{74}$$
$$L_2 = 1.0 \times 10^{-35} \text{ m} \quad \text{(round number)} \tag{75}$$
$$L_3 = \pi \times 10^{-35} \text{ m} \quad \text{(with } \pi\text{)} \tag{76}$$
$$L_4 = e \times 10^{-35} \text{ m} \quad \text{(with } e\text{)} \tag{77}$$

The Mathematics Works with ANY Length Scale

The general formula is:

$$G = \frac{L^2 \times c^3}{\hbar} \tag{78}$$

Crucial: Only with the specific length $\ell_P = 1.616255 \times 10^{-35}$ m does one obtain the correct experimental value of G.

The SI Reference is What Matters

Length Scale L	Calculated G	Status
2.5×10^{-35} m	1.04×10^{-10} m^3/(kg·s^2)	Wrong
1.0×10^{-35} m	1.67×10^{-11} m^3/(kg·s^2)	Wrong
$\pi \times 10^{-35}$ m	1.64×10^{-10} m^3/(kg·s^2)	Wrong
$\ell_P = 1.616 \times 10^{-35}$ m	**6.674×10^{-11} m^3/(kg·s^2)**	**Correct**

Table 2: G-values for different length scales

0.11.3 The True Hierarchy

Correct Interpretation

ℓ_P is not defined via G - rather both are manifestations of the same fundamental geometry!
The true order:

1. Fundamental 3D space geometry $\rightarrow \xi = \frac{4}{3} \times 10^{-4}$
2. From this follows ℓ_P as natural scale
3. From this follows G as emergent property
4. SI units provide the reference to human measures

0.11.4 Experimental Confirmation of Non-Circularity

Independent Measurement of ℓ_P

The Planck length can in principle be measured independently of G through:

1. **Quantum gravity experiments:** Direct measurement of the minimal length scale
2. **Black hole Hawking radiation:** ℓ_P determines the evaporation rate

3. **Cosmological observations:** ℓ_P influences quantum fluctuations of inflation
4. **High-energy scattering experiments:** At Planck energies, ℓ_P becomes directly accessible

Independent Measurement of α

The fine-structure constant is measured through:

1. **Quantum Hall effect:** $\alpha = \frac{e^2}{h} \times \frac{R_K}{Z_0}$
2. **Anomalous magnetic moment:** α from QED corrections
3. **Atom interferometry:** α from recoil measurements
4. **Spectroscopy:** α from hydrogen spectrum
   None of these methods uses G or ℓ_P!

0.11.5 Mathematical Proof of Non-Circularity

Definition Hierarchy

$$\textbf{Given:} \quad \alpha \text{ (experimental)}, \quad \ell_P \text{ (experimental)} \tag{79}$$
$$\textbf{Defined:} \quad \mu_0 \text{ (SI convention)}, \quad e \text{ (SI convention)} \tag{80}$$
$$\textbf{Calculated:} \quad c = f_1(\mu_0), \quad \varepsilon_0 = f_2(\mu_0, c) \tag{81}$$
$$\hbar = f_3(e, \varepsilon_0, c, \alpha) \tag{82}$$
$$G = f_4(\ell_P, c, \hbar) \tag{83}$$

Each quantity depends only on previously defined quantities!

Circularity Test

A circular argument exists if:

$$A \xrightarrow{\text{defined}} B \xrightarrow{\text{defined}} C \xrightarrow{\text{defined}} A \tag{84}$$

In our case:

$$\alpha, \ell_P \xrightarrow{\text{calculated}} \hbar \xrightarrow{\text{calculated}} G \nrightarrow \alpha, \ell_P \tag{85}$$

Result: No circularity present!

0.11.6 The Philosophical Argument

Reference Scales are Necessary

Fundamental Insight

All physics needs reference scales!
Nature is dimensionally structured. To get from dimensionless relationships to measurable quantities, we need:

- An **energy scale** (from α)
- A **length scale** (from ℓ_P)
- **SI conventions** (human measures)

This is not a weakness of the theory, but a necessity of any dimensional physics!

0.12 Further Considerations

0.12.1 Connection to the T0 Model

Within the T0 model, even α and ℓ_P can be derived from more fundamental geometric principles:

$$\xi = \frac{4}{3} \times 10^{-4} \quad \text{(3D space geometry)} \tag{86}$$

$$\alpha = \xi \times E_0^2 \quad \text{with } E_0 = \sqrt{m_e \times m_\mu} \tag{87}$$

$$\ell_P = \xi \times \ell_{fundamental} \tag{88}$$

This would reduce the number of fundamental parameters to just **one**: the geometric parameter ξ.