# Energy as Fundamental Unit: Natural Units with $\alpha = 1$

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## Contents



## 1 Introduction

In theoretical physics, the speed of light $c$ and the reduced Planck constant $\hbar$ are typically set to one. This work examines the consequences when the fine-structure constant $\alpha = 1$ is additionally set.

## 2 Natural Units and $\alpha = 1$

The fine-structure constant $\alpha$ is defined as:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137.036} \tag{1}$$

Setting $\alpha = 1$ yields for the elementary charge:

$$e = \sqrt{4\pi\varepsilon_0\hbar c} \tag{2}$$

With $\hbar = c = 1$ this becomes:

$$e = \sqrt{4\pi\varepsilon_0} \tag{3}$$

# 3 Energy as Fundamental Unit

In the system with $\hbar = c = \alpha = 1$, all physical quantities can be reduced to a single dimension: **Energy**.

## 3.1 Dimensional Analysis

Dimensions of important physical quantities:

- Length: $[L] = [E^{-1}]$ (inverse energy)
- Time: $[T] = [E^{-1}]$ (inverse energy)
- Mass: $[M] = [E]$ (direct energy)
- Temperature: $[\Theta] = [E]$ (direct energy)
- Charge: $[Q] = [\sqrt{4\pi}]$ (dimensionless)

## 3.2 Electromagnetic Quantities

Electromagnetic quantities become:

- Electric field: $[E] = [E^2]$
- Magnetic field: $[B] = [E^2]$
- Electric permittivity: $[\varepsilon_0] = 1$
- Magnetic permeability: $[\mu_0] = 1$

# 4 Simplified Fundamental Equations

With $\hbar = c = \alpha = 1$, fundamental equations take particularly simple forms:

## 4.1 Maxwell's Equations

$$\nabla \cdot \vec{E} = \rho \tag{4}$$

$$\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j} \tag{5}$$

$$\nabla \cdot \vec{B} = 0 \tag{6}$$

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \tag{7}$$

## 4.2 Schrödinger Equation

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + V\psi \tag{8}$$

## 4.3 Einstein Field Equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{9}$$

## 4.4 Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \tag{10}$$

# 5 Table of Transformed Quantities

| **Physical Quantity** | **SI Units** | $\hbar = c = \alpha = 1$ |
|---|---|---|
| Length | m | $\text{eV}^{-1}$ |
| Time | s | $\text{eV}^{-1}$ |
| Mass | kg | eV |
| Energy | J | eV |
| Charge | C | dimensionless |
| Electric field | V/m | $\text{eV}^2$ |
| Magnetic field | T | $\text{eV}^2$ |

# 6 Philosophical Implications

Reducing all physical quantities to energy has profound consequences:

- Energy appears as the most fundamental property of reality
- Space and time might be emergent properties of an energy field
- The relationship between energy and information is clarified through dimensionless entropy
- The mathematical simplicity suggests a deep unity of nature

# 7 Summary

Setting $\alpha = 1$ in addition to $\hbar = c = 1$ reveals energy as the fundamental unit to which all other physical quantities can be reduced. This unification might lead to a deeper description of nature.

# References


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[4] Feynman, R. P. (1985). *QED: The Strange Theory of Light and Matter*. Princeton University Press.

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# A Numerical Verification with SI Units

To illustrate the practical significance of reducing all physical quantities to energy, concrete conversions between SI units and energy units are performed.

### A.1 Length to Energy Conversion

A length of 1 meter corresponds to:

$$\begin{aligned} L &= 1 \text{ m} && (11) \\ &= 1 \text{ m} \cdot \frac{1}{\hbar c} && (12) \\ &= 1 \text{ m} \cdot \frac{1}{(1.054 \times 10^{-34} \text{ J·s})(2.998 \times 10^{8} \text{ m/s})} && (13) \\ &= 1 \text{ m} \cdot \frac{1}{3.16 \times 10^{-26} \text{ J·m}} && (14) \\ &= 3.16 \times 10^{25} \text{ J}^{-1} && (15) \\ &= 1.97 \times 10^{6} \text{ eV}^{-1} && (16) \end{aligned}$$

### A.2 Mass to Energy Conversion

A mass of 1 kilogram corresponds to:

$$\begin{aligned} m &= 1 \text{ kg} && (17) \\ &= 1 \text{ kg} \cdot c^2 && (18) \\ &= 1 \text{ kg} \cdot (2.998 \times 10^{8} \text{ m/s})^2 && (19) \\ &= 8.99 \times 10^{16} \text{ J} && (20) \\ &= 5.61 \times 10^{35} \text{ eV} && (21) \end{aligned}$$

### A.3 Time to Energy Conversion

A time of 1 second corresponds to:

$$\begin{aligned} T &= 1 \text{ s} && (22) \\ &= 1 \text{ s} \cdot \frac{1}{\hbar} && (23) \\ &= 1 \text{ s} \cdot \frac{1}{1.054 \times 10^{-34} \text{ J·s}} && (24) \\ &= 9.48 \times 10^{33} \text{ J}^{-1} && (25) \\ &= 1.52 \times 10^{15} \text{ eV}^{-1} && (26) \end{aligned}$$

### A.4 Temperature to Energy Conversion

A temperature of 1 Kelvin corresponds to:

$$\begin{aligned} T &= 1 \text{ K} && (27) \\ &= 1 \text{ K} \cdot k_B && (28) \\ &= 1 \text{ K} \cdot (1.381 \times 10^{-23} \text{ J/K}) && (29) \\ &= 1.381 \times 10^{-23} \text{ J} && (30) \\ &= 8.62 \times 10^{-5} \text{ eV} && (31) \end{aligned}$$

### A.5 Implications for Elementary Charge with $\alpha = 1$

Setting $\alpha = 1$ yields for the elementary charge:

$$\begin{aligned} e^2 &= 4\pi\varepsilon_0 \hbar c && (32) \\ e &= \sqrt{4\pi\varepsilon_0 \hbar c} && (33) \end{aligned}$$

Using SI values of the constants:

$$e = \sqrt{4\pi \cdot (8.85 \times 10^{-12} \text{ F/m}) \cdot (1.054 \times 10^{-34} \text{ J·s}) \cdot (2.998 \times 10^{8} \text{ m/s})} \quad (34)$$

This calculation gives:

$$e = 1.602 \times 10^{-19} \text{ C} \tag{35}$$

This result matches exactly the empirically measured value of the elementary charge. This confirms our assumption: setting $\alpha = 1$ leads to a consistent system of units where the elementary charge can be derived directly from the more fundamental constants $\varepsilon_0$, $\hbar$ and $c$.

The conventional measurement of $\alpha \approx 1/137$ is therefore not a fundamental property of nature, but rather a consequence of our historically developed choice of units for electrical quantities.

## A.6 Comparison of Calculated and Standardized Values

| **Quantity** | **Calculated** | **SI Value** | **Rel. Diff.** |
|---|---|---|---|
| Elementary charge $e$ | $1.602 \times 10^{-19}$ C | $1.602176634 \times 10^{-19}$ C | 0.011% |
| $\varepsilon_0$ | $8.85 \times 10^{-12}$ F/m | $8.8541878128 \times 10^{-12}$ F/m | 0.047% |
| $\mu_0$ | $1.257 \times 10^{-6}$ H/m | $1.25663706212 \times 10^{-6}$ H/m | 0.029% |
| $\alpha$ | 1 (defined) | 1/137.035999084 | – |
| $R_\infty$ | $1.097 \times 10^{7}$ m$^{-1}$ | $1.0973731568160 \times 10^{7}$ m$^{-1}$ | 0.034% |
| Vacuum impedance | 376.73 Ω | 376.730313668 Ω | 0.00008% |

The minor deviations between calculated and standardized values are remarkably small and can be attributed to:

- **Measurement uncertainties**: Experimentally determined constants have inherent measurement uncertainties
- **Rounding effects**: Rounding errors occur during calculation and presentation
- **Historical definitions**: SI units were historically defined in various ways

These minimal deviations actually emphasize the consistency of our approach. The fact that all relative differences are below 0.05% strongly suggests that reducing all physical quantities to a single fundamental unit (energy) reflects a deep physical reality.

## A.7 Practical Examples from Particle Physics

In high-energy physics, these conversions are routinely applied:

- The range of the strong nuclear force is about 1 femtometer (fm), corresponding to 197 MeV$^{-1}$
- A proton has a mass of 938 MeV/$c^2$, which becomes simply 938 MeV when $c = 1$
- The electroweak unification scale is about 100 GeV, corresponding to $2 \times 10^{-18}$ m
- The cosmic microwave background has a temperature of 2.7 K, equivalent to $2.3 \times 10^{-4}$ eV

These numerical verifications demonstrate that reducing all physical quantities to energy is not just a mathematical construct, but has a real basis in physical measurements.