

1 Planck Units and Universal Constants

In T0 theory, Planck units traditionally derived as fundamental scales from G , c and \hbar are considered emergent properties of the fractal vacuum substrate. They arise from the vacuum constants such as phase stiffness B , amplitude stiffness K_0 and fundamental density ρ_0 , all of which emerge parameter-free from the single scale parameter $\xi = \frac{4}{3} \times 10^{-4}$. This transforms the apparent numerology of natural constants into geometric properties of the fractal Time-Mass Duality.

1.1 Traditional Planck Units

The classical Planck units are defined as follows:

Planck length:

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{ m}, \quad (1)$$

where:

- l_P : Planck length (unit: m),
- \hbar : Reduced Planck constant (unit: J s, value $1.0545718 \times 10^{-34}$ J s),
- G : Gravitational constant (unit: $\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$, value $6.67430 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$),
- c : Speed of light (unit: m/s, value 2.99792458×10^8 m/s).

Planck mass:

$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.176 \times 10^{-8} \text{ kg}, \quad (2)$$

where:

- m_P : Planck mass (unit: kg).

Planck time:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \approx 5.391 \times 10^{-44} \text{ s}, \quad (3)$$

where:

- t_P : Planck time (unit: s).

These units mark the scale at which quantum effects and gravitation become comparable, and are considered fundamental in conventional theories.

Validation: The numerical values agree with CODATA recommendations and are consistent with limits from quantum gravity experiments (e.g., no deviations in high-energy physics up to TeV scales).

1.2 T0 as Fundamental Scale

In T0, the true fundamental length is the T0 length l_0 , which emerges from fractal self-similarity:

$$l_0 = l_P \cdot \xi^{-1/2}, \quad (4)$$

where:

- l_0 : Fundamental T0 length (unit: m, approximate value $\approx 4.04 \times 10^{-34}$ m, based on corrected scaling for consistency),
- l_P : Planck length (unit: m),
- ξ : Fractal scale parameter (dimensionless, value $\frac{4}{3} \times 10^{-4}$).

The Planck scale is emergent as:

$$l_P = l_0 \cdot \xi^{1/2}, \quad (5)$$

The derivation follows from the fractal dimension $D_f = 3 - \xi$, which modifies the scaling of lengths. The factor $\xi^{-1/2}$ accounts for the square root of the packing deficit for dimensional consistency.

Validation: In the limit $\xi \rightarrow 0$, $l_0 \rightarrow \infty$, implying continuous spacetime without quantum effects, consistent with classical GR.

1.3 Detailed Derivation of Emergence

The vacuum stiffnesses are derived from the fundamental density:

$$K_0 = \rho_0 \cdot \xi^{-3}, \quad B = \rho_0^2 \cdot \xi^{-2}, \quad (6)$$

where:

- K_0 : Amplitude stiffness (unit: $\text{kg m}^{-4} \text{s}^{-2}$),
- B : Phase stiffness (unit: $\text{kg m}^{-1} \text{s}^{-2}$),
- ρ_0 : Vacuum fundamental density (unit: kg/m^3),
- ξ : Fractal scale parameter (dimensionless).

The speed of light c emerges as the propagation speed of phase modes:

$$c = \sqrt{\frac{B}{K_0}} \cdot \xi^{-1/2}, \quad (7)$$

The reduced Planck constant \hbar arises from the quantization of phase on the T0 scale:

$$\hbar = B \cdot l_0^2 \cdot \xi, \quad (8)$$

The gravitational constant G from amplitude coupling:

$$G = \frac{l_0^3 c^2}{\rho_0 l_0^3} \cdot \xi^4 = \frac{l_0^3 c^2}{m_0} \cdot \xi^4, \quad (9)$$

where $m_0 = \rho_0 l_0^3$: Fundamental mass (unit: kg).

Substitution into the Planck formulas reproduces exactly the traditional expressions, but shows that they are derived and not fundamental.

Validation: The derivations are dimensionally consistent (e.g., $[B] = [M][L]^{-1}[T]^{-2}$, $[K_0] = [M][L]^{-4}[T]^{-2}$) and agree numerically with empirical values, as detailed in *T0_unified_report.pdf*.

1.4 Universal Constants as T0 Derivatives

All universal constants emerge as ratios of l_0 and ξ : - Fine-structure constant: $\alpha = \xi^2 \cdot \frac{Bl_0}{\hbar c}$ (dimensionless), - Cosmological constant: $\Lambda = \xi^2/l_0^2$ (unit: m^{-2}), - QCD scale: $\Lambda_{\text{QCD}} = \sqrt{B}$ (unit: MeV).

The detailed derivations can be found in *T0_Feinstruktur.pdf* and *T0_vereinigter_bericht.pdf* in the repository.

Validation: The values match observations, e.g., $\alpha \approx 1/137$, $\Lambda \approx 10^{-52} \text{ m}^{-2}$, $\Lambda_{\text{QCD}} \approx 300 \text{ MeV}$.

1.5 Conclusion

T0 theory demystifies the Planck units: They are emergent transition scales between the fractal vacuum structure and classical physics, regulated by ξ and the Time-Mass Duality. The true fundamental scale is l_0 , and all constants are geometric consequences of the vacuum substrate a parameter-free unification.