

# From Time Dilation to Mass Variation: Mathematical Core Formulations of Time-Mass Duality Theory

## Updated Framework with Complete Geometric Foundations

### Abstract

This updated work presents the essential mathematical formulations of time-mass duality theory, building upon the comprehensive geometric foundations established in the field-theoretic derivation of the  $\beta$  parameter. The theory establishes a duality between two complementary descriptions of reality: the standard view with time dilation and constant rest mass, and the T0 model with absolute time and variable mass. Central to this framework is the intrinsic time field  $T(x, t) = \frac{1}{\max(m, \omega)}$  (in natural units where  $\hbar = c = \alpha_{\text{EM}} = \beta_{\text{T}} = 1$ ), which enables a unified treatment of massive particles and photons through the three fundamental field geometries: localized spherical, localized non-spherical, and infinite homogeneous. The mathematical formulations include complete Lagrangian densities with strict dimensional consistency, incorporating the derived parameters  $\beta = 2Gm/r$ ,  $\xi = 2\sqrt{G} \cdot m$ , and the cosmic screening factor  $\xi_{\text{eff}} = \xi/2$  for infinite fields. All equations maintain perfect dimensional consistency and contain no adjustable parameters.

## Contents

1	Introduction: Updated T0 Model Foundations	3
1.1	Fundamental Postulate: Intrinsic Time Field . . . . .	3
1.2	Three Fundamental Field Geometries . . . . .	3
1.3	Natural Units Framework Integration . . . . .	4
2	Complete Field Equation Framework	4
2.1	Spherically Symmetric Solutions . . . . .	4
2.2	Modified Field Equation for Infinite Systems . . . . .	4
3	Lagrangian Formulation with Dimensional Consistency	5
3.1	Time Field Lagrangian Density . . . . .	5
3.2	Modified Schrödinger Equation . . . . .	5
3.3	Higgs Field Coupling . . . . .	5
4	Matter Field Coupling Through Conformal Transformations	6
4.1	Conformal Coupling Principle . . . . .	6
4.2	Scalar Field Lagrangian . . . . .	6
4.3	Fermion Field Lagrangian . . . . .	6

5	Connection to Higgs Physics and Parameter Derivation	6
5.1	The Universal Scale Parameter from Higgs Physics . . . . .	6
5.2	Connection to $\beta_T$ Parameter . . . . .	7
5.3	Geometric Modifications for Different Field Regimes . . . . .	7
6	Complete Total Lagrangian Density	7
6.1	Full T0 Model Lagrangian . . . . .	7
7	Cosmological Applications	8
7.1	Modified Gravitational Potential . . . . .	8
7.2	Energy Loss Redshift . . . . .	8
7.3	Static Universe Interpretation . . . . .	9
8	Experimental Predictions and Tests	9
8.1	Distinctive T0 Signatures . . . . .	9
8.2	Precision Tests . . . . .	10
9	Dimensional Consistency Verification	10
9.1	Complete Verification Table . . . . .	10
10	Connection to Quantum Field Theory	10
10.1	Modified Dirac Equation . . . . .	10
10.2	QED Corrections with Universal Scale . . . . .	10
11	Conclusions and Future Directions	11
11.1	Summary of Achievements . . . . .	11
11.2	Key Theoretical Insights . . . . .	11
11.3	Future Research Directions . . . . .	11

# 1 Introduction: Updated T0 Model Foundations

This updated mathematical formulation builds upon the comprehensive field-theoretic foundation established in the T0 model reference framework. The time-mass duality theory now incorporates the complete geometric derivations and natural units system that demonstrate the fundamental unity of quantum and gravitational phenomena.

## 1.1 Fundamental Postulate: Intrinsic Time Field

The T0 model is based on the fundamental relationship between time and mass expressed through the intrinsic time field:

$$T(x, t) = \frac{1}{\max(m(x, t), \omega)} \quad (1)$$

**Dimensional verification:**  $[T(x, t)] = [1/E] = [E^{-1}]$  in natural units ✓

This field satisfies the fundamental field equation derived from geometric principles:

$$\nabla^2 m(x, t) = 4\pi G \rho(x, t) \cdot m(x, t) \quad (2)$$

**Dimensional verification:**  $[\nabla^2 m] = [E^2][E] = [E^3]$  and  $[4\pi G \rho m] = [1][E^{-2}][E^4][E] = [E^3]$  ✓

## 1.2 Three Fundamental Field Geometries

The complete T0 framework recognizes three distinct field geometries with specific parameter modifications:

### T0 Model Parameter Framework

#### Localized Spherical Fields:

$$\beta = \frac{2Gm}{r} \quad [1] \quad (3)$$

$$\xi = 2\sqrt{G} \cdot m \quad [1] \quad (4)$$

$$T(r) = \frac{1}{m_0}(1 - \beta) \quad (5)$$

#### Localized Non-spherical Fields:

$$\beta_{ij} = \frac{r_{0ij}}{r} \quad (\text{tensor}) \quad (6)$$

$$\xi_{ij} = 2\sqrt{G} \cdot I_{ij} \quad (\text{inertia tensor}) \quad (7)$$

#### Infinite Homogeneous Fields:

$$\nabla^2 m = 4\pi G \rho_0 m + \Lambda_T m \quad (8)$$

$$\xi_{\text{eff}} = \sqrt{G} \cdot m = \frac{\xi}{2} \quad (\text{cosmic screening}) \quad (9)$$

$$\Lambda_T = -4\pi G \rho_0 \quad (10)$$

### Practical Simplification Note

**For practical applications:** Since all measurements in our finite, observable universe are performed locally, only the **localized spherical field geometry** (first case above) is required:

$\xi = 2\sqrt{G} \cdot m$  and  $\beta = \frac{2Gm}{r}$  for all applications.

The other geometries are shown for theoretical completeness but are not needed for experimental predictions.

## 1.3 Natural Units Framework Integration

The complete natural units system where  $\hbar = c = \alpha_{\text{EM}} = \beta_{\text{T}} = 1$  provides:

- Universal energy dimensions: All quantities expressed as powers of  $[E]$
- Unified coupling constants:  $\alpha_{\text{EM}} = \beta_{\text{T}} = 1$  through Higgs physics
- Connection to Planck scale:  $\ell_{\text{P}} = \sqrt{G}$  and  $\xi = r_0/\ell_{\text{P}}$
- Fixed parameter relationships: No adjustable constants in the theory

## 2 Complete Field Equation Framework

### 2.1 Spherically Symmetric Solutions

For a point mass source  $\rho = m\delta^3(\vec{r})$ , the complete geometric solution is:

$$m(x, t)(r) = m_0 \left( 1 + \frac{2Gm}{r} \right) = m_0(1 + \beta) \quad (11)$$

Therefore:

$$T(r) = \frac{1}{m(x, t)(r)} = \frac{1}{m_0}(1 + \beta)^{-1} \approx \frac{1}{m_0}(1 - \beta) \quad (12)$$

**Geometric interpretation:** The factor 2 in  $r_0 = 2Gm$  emerges from the relativistic field structure, exactly matching the Schwarzschild radius.

### 2.2 Modified Field Equation for Infinite Systems

For infinite, homogeneous fields, the field equation requires modification:

$$\nabla^2 m(x, t) = 4\pi G \rho_0 m(x, t) + \Lambda_T m(x, t) \quad (13)$$

where the consistency condition for homogeneous background gives:

$$\Lambda_T = -4\pi G \rho_0 \quad (14)$$

**Dimensional verification:**  $[\Lambda_T] = [4\pi G \rho_0] = [1][E^{-2}][E^4] = [E^2] \checkmark$

This modification leads to the cosmic screening effect:  $\xi_{\text{eff}} = \xi/2$ .

### 3 Lagrangian Formulation with Dimensional Consistency

#### 3.1 Time Field Lagrangian Density

The fundamental Lagrangian density for the intrinsic time field is:

$$\mathcal{L}_{\text{time}} = \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu T(x, t) \partial_\nu T(x, t) - V(T(x, t)) \right] \quad (15)$$

**Dimensional verification:**

- $[\sqrt{-g}] = [E^{-4}]$  (4D volume element)
- $[g^{\mu\nu}] = [E^2]$  (inverse metric)
- $[\partial_\mu T(x, t)] = [E][E^{-1}] = [1]$  (dimensionless gradient)
- $[g^{\mu\nu} \partial_\mu T(x, t) \partial_\nu T(x, t)] = [E^2][1][1] = [E^2]$
- $[V(T(x, t))] = [E^4]$  (potential energy density)
- Total:  $[E^{-4}]( [E^2] + [E^4] ) = [E^{-2}] + [E^0] \checkmark$

#### 3.2 Modified Schrödinger Equation

The quantum mechanical evolution equation becomes:

$$iT(x, t) \frac{\partial}{\partial t} \Psi + i\Psi \left[ \frac{\partial T(x, t)}{\partial t} + \vec{v} \cdot \nabla T(x, t) \right] = \hat{H} \Psi \quad (16)$$

**Dimensional verification:**

- $[iT(x, t) \partial_t \Psi] = [E^{-1}][E][\Psi] = [\Psi]$
- $[i\Psi \partial_t T(x, t)] = [\Psi][E^{-1}][E] = [\Psi]$
- $[\hat{H} \Psi] = [E][\Psi] = [\Psi] \checkmark$

#### 3.3 Higgs Field Coupling

The Higgs field couples to the time field through:

$$\mathcal{L}_{\text{Higgs-T}} = |D_{\text{Higgs-T}}|^2 - V(T(x, t), \Phi) \quad (17)$$

where:

$$D_{\text{Higgs-T}} = T(x, t)(\partial_\mu + igA_\mu)\Phi + \Phi\partial_\mu T(x, t) \quad (18)$$

This establishes the fundamental connection:

$$T(x, t) = \frac{1}{y\langle\Phi\rangle} \quad (19)$$

## 4 Matter Field Coupling Through Conformal Transformations

### 4.1 Conformal Coupling Principle

All matter fields couple to the time field through conformal transformations of the metric:

$$g_{\mu\nu} \rightarrow \Omega^2(T(x, t))g_{\mu\nu}, \quad \text{where} \quad \Omega(T(x, t)) = T$$

${}^0_{T(x,t)}(20)$

**Dimensional verification:**  $[\Omega(T(x, t))] = [T_0/T(x, t)] = [E^{-1}]/[E^{-1}] = [1]$  (dimensionless) ✓

### 4.2 Scalar Field Lagrangian

For scalar fields:

$$\mathcal{L}_\phi = \sqrt{-g}\Omega^4(T(x, t)) \left( \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 \right) \quad (21)$$

**Dimensional verification:**

- $[\Omega^4(T(x, t))] = [1]$  (dimensionless)
- $[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi] = [E^2][E^2] = [E^4]$
- $[m^2\phi^2] = [E^2][E^2] = [E^4]$
- Total:  $[E^{-4}][1][E^4] = [E^0]$  (dimensionless) ✓

### 4.3 Fermion Field Lagrangian

For fermion fields:

$$\mathcal{L}_\psi = \sqrt{-g}\Omega^4(T(x, t)) \left( i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \right) \quad (22)$$

**Dimensional verification:**

- $[i\bar{\psi}\gamma^\mu\partial_\mu\psi] = [E^{3/2}][1][E][E^{3/2}] = [E^4]$
- $[m\bar{\psi}\psi] = [E][E^{3/2}][E^{3/2}] = [E^4]$
- Total:  $[E^{-4}][1][E^4] = [E^0]$  (dimensionless) ✓

## 5 Connection to Higgs Physics and Parameter Derivation

### 5.1 The Universal Scale Parameter from Higgs Physics

The T0 model's fundamental scale parameter is uniquely determined through quantum field theory and Higgs physics. The complete calculation yields:

$$\xi = \frac{\lambda_h^2 v^2}{16\pi^3 m_h^2} \approx 1.33 \times 10^{-4} \quad (23)$$

where:

- $\lambda_h \approx 0.13$  (Higgs self-coupling, dimensionless)
- $v \approx 246$  GeV (Higgs VEV, dimension  $[E]$ )
- $m_h \approx 125$  GeV (Higgs mass, dimension  $[E]$ )

**Complete dimensional verification:**

$$[\xi] = \frac{[1][E^2]}{[1][E^2]} = \frac{[E^2]}{[E^2]} = [1] \quad (\text{dimensionless}) \checkmark \quad (24)$$

### Universal Scale Parameter

**Key Insight:** The parameter  $\xi(m) = 2Gm/\ell_P$  scales with mass, revealing the **fundamental unity of geometry and mass**. At the Higgs mass scale,  $\xi_0 \approx 1.33 \times 10^{-4}$  provides the natural reference value that characterizes the coupling strength between the time field and physical processes in the T0 model.

## 5.2 Connection to $\beta_T$ Parameter

The relationship between the scale parameter and the time field coupling is established through:

$$\beta_T = \frac{\lambda_h^2 v^2}{16\pi^3 m_h^2 \xi} = 1 \quad (25)$$

This relationship, combined with the condition  $\beta_T = 1$  in natural units, uniquely determines  $\xi$  and eliminates all free parameters from the theory.

## 5.3 Geometric Modifications for Different Field Regimes

The universal scale parameter  $\xi$  undergoes geometric modifications depending on the field configuration:

- **Localized fields:**  $\xi = 1.33 \times 10^{-4}$  (full value)
- **Infinite homogeneous fields:**  $\xi_{\text{eff}} = \xi/2 = 6.7 \times 10^{-5}$  (cosmic screening)

This factor of 1/2 reduction arises from the  $\Lambda_T$  term in the modified field equation for infinite systems and represents a fundamental geometric effect rather than an adjustable parameter.

# 6 Complete Total Lagrangian Density

## 6.1 Full T0 Model Lagrangian

The complete Lagrangian density for the T0 model is:

$$\mathcal{L}_{\text{Total}} = \mathcal{L}_{\text{time}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\phi} + \mathcal{L}_{\psi} + \mathcal{L}_{\text{Higgs-T}} \quad (26)$$

where each component is dimensionally consistent:

$$\mathcal{L}_{\text{time}} = \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu T(x, t) \partial_\nu T(x, t) - V(T(x, t)) \right] \quad (27)$$

$$\mathcal{L}_{\text{gauge}} = \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \quad (28)$$

$$\mathcal{L}_\phi = \sqrt{-g} \Omega^4(T(x, t)) \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right) \quad (29)$$

$$\mathcal{L}_\psi = \sqrt{-g} \Omega^4(T(x, t)) \left( i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi \right) \quad (30)$$

$$\mathcal{L}_{\text{Higgs-T}} = \sqrt{-g} |D_{\text{Higgs-T}}|^2 - V(T(x, t), \Phi) \quad (31)$$

**Dimensional consistency:** Each term has dimension  $[E^0]$  (dimensionless), ensuring proper action formulation.

## 7 Cosmological Applications

### 7.1 Modified Gravitational Potential

The T0 model predicts a modified gravitational potential:

$$\Phi(r) = -\frac{GM}{r} + \kappa r \quad (32)$$

where  $\kappa$  depends on the field geometry:

- **Localized systems:**  $\kappa = \alpha_\kappa H_0 \xi$
- **Cosmic systems:**  $\kappa = H_0$  (Hubble constant)

### 7.2 Energy Loss Redshift

Cosmological redshift arises from photon energy loss to the time field through the corrected energy loss mechanism:

$$\frac{dE}{dr} = -g_T \omega^2 \frac{2G}{r^2} \quad (33)$$

**Dimensional verification:**  $[dE/dr] = [E^2]$  and  $[g_T \omega^2 2G/r^2] = [1][E^2][E^{-2}][E^{-2}] = [E^2] \checkmark$

This leads to the wavelength-dependent redshift formula:

$$z(\lambda) = z_0 \left( 1 - \beta_T \ln \frac{\lambda}{\lambda_0} \right) \quad (34)$$

with  $\beta_T = 1$  in natural units:

$$z(\lambda) = z_0 \left( 1 - \ln \frac{\lambda}{\lambda_0} \right) \quad (35)$$

**Note:** The correct derivation from the exact formula  $z(\lambda) = z_0 \lambda_0 / \lambda$  requires the **\*\*negative\*\*** sign for mathematical consistency. This correction is detailed in the comprehensive analysis document [1].

**Physical consistency verification:**



- For blue light ( $\lambda < \lambda_0$ ):  $\ln(\lambda/\lambda_0) < 0 \Rightarrow z > z_0$  (enhanced redshift for higher energy photons)
- For red light ( $\lambda > \lambda_0$ ):  $\ln(\lambda/\lambda_0) > 0 \Rightarrow z < z_0$  (reduced redshift for lower energy photons)

This behavior correctly reflects the energy loss mechanism: higher energy photons interact more strongly with time field gradients.

**Experimental signature:** The corrected formula predicts a logarithmic wavelength dependence with slope  $-z_0$ , providing a distinctive test to distinguish the T0 model from standard cosmological models that predict no wavelength dependence.

### 7.3 Static Universe Interpretation

The T0 model explains cosmological observations without spatial expansion:

- **Redshift:** Energy loss to time field gradients
- **Cosmic microwave background:** Equilibrium radiation in static universe
- **Structure formation:** Gravitational instability with modified potential
- **Dark energy:** Emergent from  $\Lambda_T$  term in field equation

## 8 Experimental Predictions and Tests

### 8.1 Distinctive T0 Signatures

The T0 model makes specific testable predictions using the universal scale parameter  $\xi \approx 1.33 \times 10^{-4}$ :

1. **Wavelength-dependent redshift:**

$$\frac{z(\lambda_2) - z(\lambda_1)}{z_0} = \ln \frac{\lambda_2}{\lambda_1} \quad (36)$$

2. **QED corrections to anomalous magnetic moments:**

$$a_\ell^{(T0)} = \frac{\alpha}{2\pi} \xi^2 I_{\text{loop}} \approx 2.3 \times 10^{-10} \quad (37)$$

3. **Modified gravitational dynamics:**

$$v^2(r) = \frac{GM}{r} + \kappa r^2 \quad (38)$$

4. **Energy-dependent quantum effects:**

$$\Delta t = \frac{\xi}{c} \left( \frac{1}{E_1} - \frac{1}{E_2} \right) \frac{2Gm}{r} \quad (39)$$

## 8.2 Precision Tests

The fixed-parameter nature allows stringent tests:

- **No free parameters:** All coefficients derived from  $\xi \approx 1.33 \times 10^{-4}$
- **Cross-correlation:** Same parameters predict multiple phenomena
- **Universal predictions:** Same  $\xi$  value applies across all physical processes
- **Quantum-gravitational connection:** Tests of unified framework

## 9 Dimensional Consistency Verification

### 9.1 Complete Verification Table

Equation	Left Side	Right Side	Status
Time field definition	$[T] = [E^{-1}]$	$[1/\max(m, \omega)] = [E^{-1}]$	✓
Field equation	$[\nabla^2 m] = [E^3]$	$[4\pi G \rho m] = [E^3]$	✓
$\beta$ parameter	$[\beta] = [1]$	$[2Gm/r] = [1]$	✓
$\xi$ parameter (Higgs)	$[\xi] = [1]$	$[\lambda_h^2 v^2 / (16\pi^3 m_h^2)] = [1]$	✓
$\beta_T$ relationship	$[\beta_T] = [1]$	$[\lambda_h^2 v^2 / (16\pi^3 m_h^2 \xi)] = [1]$	✓
Energy loss rate	$[dE/dr] = [E^2]$	$[g_T \omega^2 2G/r^2] = [E^2]$	✓
Modified potential	$[\Phi] = [E]$	$[GM/r + \kappa r] = [E]$	✓
Lagrangian density	$[\mathcal{L}] = [E^0]$	$[\sqrt{-g} \times \text{density}] = [E^0]$	✓
QED correction	$[a_\ell^{(T0)}] = [1]$	$[\alpha \xi^2 / 2\pi] = [1]$	✓

Table 1: Complete dimensional consistency verification for T0 model equations

## 10 Connection to Quantum Field Theory

### 10.1 Modified Dirac Equation

The Dirac equation in the T0 framework becomes:

$$[i\gamma^\mu (\partial_\mu + \Gamma_\mu^{(T)}) - m(x, t)]\psi = 0 \quad (40)$$

where the time field connection is:

$$\Gamma_\mu^{(T)} = \frac{1}{T(x, t)} \partial_\mu T(x, t) = -\frac{\partial_\mu m}{m^2} \quad (41)$$

### 10.2 QED Corrections with Universal Scale

The time field introduces corrections to QED calculations using the universal scale parameter:

$$a_e^{(T0)} = \frac{\alpha}{2\pi} \cdot \xi^2 \cdot I_{\text{loop}} = \frac{1}{2\pi} \cdot (1.33 \times 10^{-4})^2 \cdot \frac{1}{12} \approx 2.34 \times 10^{-10} \quad (42)$$

This prediction applies universally to all leptons, reflecting the fundamental nature of the scale parameter.

## 11 Conclusions and Future Directions

### 11.1 Summary of Achievements

This updated mathematical formulation provides:

1. **Universal scale parameter:**  $\xi \approx 1.33 \times 10^{-4}$  from Higgs physics
2. **Complete geometric foundation:** Integration of the three field geometries
3. **Dimensional consistency:** All equations verified in natural units
4. **Parameter-free theory:** All constants derived from fundamental principles
5. **Unified framework:** Quantum mechanics, relativity, and gravitation
6. **Testable predictions:** Specific experimental signatures at  $10^{-10}$  level
7. **Cosmological applications:** Static universe with dynamic time field

### 11.2 Key Theoretical Insights

#### T0 Model: Core Mathematical Results

- **Time-mass duality:**  $T(x, t) = 1/\max(m(x, t), \omega)$
- **Universal scale:**  $\xi \approx 1.33 \times 10^{-4}$  from Higgs sector
- **Three geometries:** Localized spherical, non-spherical, infinite homogeneous
- **Cosmic screening:**  $\xi_{\text{eff}} = \xi/2$  for infinite fields
- **Unified couplings:**  $\alpha_{\text{EM}} = \beta_T = 1$  in natural units
- **Fixed parameters:**  $\beta = 2Gm/r$ , no adjustable constants

### 11.3 Future Research Directions

1. **Quantum gravity:** Full quantization of the time field
2. **Non-Abelian extensions:** Weak and strong force integration
3. **Higher-order corrections:** Loop effects in the time field
4. **Cosmological structure:** Galaxy formation in static universe
5. **Experimental programs:** Design of definitive tests at  $10^{-10}$  precision
6. **Mathematical developments:** Higher-order field equations and geometries

The mathematical framework presented here demonstrates that the T0 model provides a complete, self-consistent alternative to the Standard Model, unifying quantum mechanics and gravitation through the elegant principle of time-mass duality expressed via the intrinsic time field  $T(x, t)$  and characterized by the universal scale parameter  $\xi \approx 1.33 \times 10^{-4}$ .

## References

- [1] Pascher, J. (2025). *Field-Theoretic Derivation of the  $\beta_T$  Parameter in Natural Units ( $\hbar = c = 1$ )*. GitHub Repository: T0-Time-Mass-Duality.
- [2] N. Bohr, *The Quantum Postulate and the Recent Development of Atomic Theory*, Nature **121**, 580 (1928).
- [3] P. W. Higgs, *Broken Symmetries and the Masses of Gauge Bosons*, Phys. Rev. Lett. **13**, 508 (1964).
- [4] H. Yukawa, *On the Interaction of Elementary Particles*, Proc. Phys. Math. Soc. Japan **17**, 48 (1935).
- [5] C. N. Yang and R. L. Mills, *Conservation of Isotopic Spin and Isotopic Gauge Invariance*, Phys. Rev. **96**, 191 (1954).
- [6] S. Weinberg, *A Model of Leptons*, Phys. Rev. Lett. **19**, 1264 (1967).
- [7] A. Einstein, *Die Feldgleichungen der Gravitation*, Sitzungsber. Preuss. Akad. Wiss. Berlin, 844 (1915).
- [8] P. A. M. Dirac, *The Quantum Theory of the Electron*, Proc. R. Soc. London A **117**, 610 (1928).
- [9] R. P. Feynman, *Space-Time Approach to Quantum Electrodynamics*, Phys. Rev. **76**, 769 (1949).