Chapter 17: Alternative to GR + ΛCDM in Fractal T0-Geometry

1 Chapter 17: Alternative to GR + ΛCDM in Fractal T0-Geometry

The fractal Fundamental Fractal-Geometric Field Theory (FFGFT) with T0-Time-Mass Duality represents a fundamental, parameter-free alternative to General Relativity (GR) combined with the ΛCDM model. All observed cosmological and gravitational phenomena are explained by the single fundamental scale parameter $\xi = \frac{4}{3} \times 10^{-4}$ (dimensionless) without separate dark components, inflation, or singularities.

1.1 Symbol Directory and Units

Important Symbols and their Units

Symbol	Meaning	Unit (SI)
ξ	Fractal scale parameter	dimensionless
$a(t)$	Scale factor	dimensionless
$\dot{a}$	Time derivative of scale factor	s^{-1}
G	Gravitational constant	$\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
$\rho_m, \rho_r, \rho_\Lambda$	Densities (matter, radiation, vacuum)	$\mathrm{kg}\,\mathrm{m}^{-3}$
k	Curvature parameter	dimensionless
p_m, p_r	Pressures (matter, radiation)	Pa
Λ	Cosmological constant	m^{-2}
R	Ricci scalar	m^{-2}
g	Metric determinant	dimensionless
ρ_0	Vacuum equilibrium density	$\mathrm{kg}^{1/2}/\mathrm{m}^{3/2}$
$\mathcal{L}_m$	Matter Lagrangian density	$\mathrm{J}\,\mathrm{m}^{-3}$
l_0	Fractal correlation length	m
c	Speed of light	$\mathrm{m}\,\mathrm{s}^{-1}$
$\langle\delta^2\rangle$	Mean squared density fluctuation	dimensionless
H_0	Hubble constant	s^{-1}
Ω_b	Baryon density parameter	dimensionless

1.2 The ΛCDM Model and its Problems

The standard model is based on the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\Lambda) - \frac{k}{a^2}, \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + \rho_r + 3p_m + 3p_r) + \frac{\Lambda}{3}, \tag{2}$$

with typically six or more free parameters $(\Omega_m, \Omega_r, \Omega_\Lambda, \Omega_k, H_0, w)$ and additional assumptions such as an inflaton field and hypothetical dark matter particles.

Unit Check (first Friedmann equation):

$$\left[\left(\frac{\dot{a}}{a}\right)^2\right] = \mathrm{s}^{-2}$$

$$\left[\frac{8\pi G}{3}\rho_m\right] = \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2} \cdot \mathrm{kg}\,\mathrm{m}^{-3} = \mathrm{s}^{-2}$$

Units consistent.

Problems:

- Cosmological constant problem: $\rho_\Lambda^{\text{QFT}}/\rho_\Lambda^{\text{obs}} \approx 10^{120}$,
- Coincidence problem: Why $\Omega_\Lambda \approx \Omega_m$ exactly today? (fine-tuning),
- No natural explanation for flat galaxy rotation curves without postulated dark matter.

1.3 Fractal T0-Action Complete Derivation

The fundamental action in T0 is an extension of the Einstein-Hilbert action with fractal terms:

$$S = \int \sqrt{-g} \left[\frac{R}{16\pi G} + \xi \cdot \rho_0^2 \left((\partial_\mu \ln a)^2 + \sum_{k=1}^{\infty} \xi^k (\nabla^k \ln a)^2 \right) + \mathcal{L}_m \right] d^4x, \tag{3}$$

where the infinite sum term encodes self-similarity across fractal hierarchy levels k.

Unit Check:

$$\begin{aligned} [S] &= \mathrm{J\,s} \\ [\xi \rho_0^2 (\partial_\mu \ln a)^2] &= \text{dimensionless} \cdot \mathrm{kg\,m^{-3}} \cdot \mathrm{m^{-2}} = \mathrm{J\,m^{-3}} \end{aligned}$$

Units consistent for all terms.

By resummation of the fractal series (geometric series for small ξ):

$$\sum_{k=1}^{\infty} \xi^k (\nabla^k \ln a)^2 \approx \frac{\xi (\nabla \ln a)^2}{1 - \xi (\nabla l_0)^2}, \tag{4}$$

where $l_0 \approx 2.4 \times 10^{-32}\,\mathrm{m}$ is the fundamental correlation length derived from ξ.

1.4 Derivation of Modified Friedmann Equations

Assuming an FRW metric $ds^2 = -dt^2 + a^2(t) d\vec{x}^2$ and variation with respect to $a(t)$ yields the modified Friedmann equations:

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_m + \xi \cdot \frac{c^2}{l_0^2 a^4} \left(1 + \xi \ln a + \xi^{1/2} \langle \delta^2 \rangle \right), \tag{5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho_m + 3p_m) + \xi \cdot \frac{c^2}{l_0^2 a^4} \left(1 - 3\xi \ln a - 2\xi^{1/2} \langle \delta^2 \rangle \right). \tag{6}$$

The fractal term $\xi c^2/(l_0^2 a^4)$ dominates in the early universe and regulates the singularity, while $\langle \delta^2 \rangle$ accounts for the backreaction of structure formation.

Unit Check:

$$\left[\xi \frac{c^2}{l_0^2 a^4} \right] = \text{dimensionless} \cdot \mathrm{m^2\,s^{-2}/m^2} = \mathrm{s^{-2}}$$

1.5 Complete Solution for the Late Universe

For the late universe ($a \gg 1$):

$$H^2(a) \approx H_0^2 \left(\Omega_b a^{-3} + \xi^2 \left(1 + \xi^{1/2} \frac{\langle \delta^2 \rangle}{a^3} \right) \right), \tag{7}$$

where Ω_b is the baryonic density parameter (no dark matter needed).

The effective vacuum term $\Omega_\Lambda^{\text{eff}} \approx 0.7$ emerges naturally from fractal dynamics, matching observations, without fine-tuning.

Unit Check:

$$[H_0^2 \xi^2] = \text{s}^{-2} \cdot \text{dimensionless} = \text{s}^{-2}$$

1.6 Comparison with ΛCDM

ΛCDM	**Fractal T0-Geometry**
6+ free parameters	Only $\xi = \frac{4}{3} \times 10^{-4}$
Separate dark matter	Fractal modification of gravitation
Separate dark energy	Dynamic vacuum from Time-Mass Duality
Ad-hoc inflation	Natural phase transition
Initial singularity	Regulated pre-vacuum
Fine-tuning problems	Natural emergence from ξ

1.7 Conclusion

The T0-theory is not just an alternative, but a deeper, unified description: GR + ΛCDM emerge as effective limiting cases of fractal Time-Mass Duality for $\xi \to 0$. All cosmological observations from CMB anisotropies through supernovae to galaxy structures are reproduced parameter-free, while fundamental problems such as the cosmological constant problem and singularities are naturally solved.

Through the single parameter ξ, T0 reduces cosmology to an elegant geometric principle: the dynamic self-organization of a fractal vacuum.