Chapter 1

Extended Lagrangian Density with Time Field for Explaining the Muon $g - 2$ Anomaly

Abstract

The Fermilab measurements of the muon's anomalous magnetic moment show a significant deviation from the Standard Model, indicating new physics beyond the established framework. While the original discrepancy of 4.2σ ($\Delta a_\mu = 251 \times 10^{-11}$) has been reduced to approximately 0.6σ ($\Delta a_\mu = 37 \times 10^{-11}$) through improved Lattice-QCD calculations, the need for a fundamental explanation remains. This work presents a complete theoretical derivation of an extension to the Standard Lagrangian density through a fundamental time field $\Delta m(x,t)$ that couples mass-proportionally with leptons. Based on the T0 time-mass duality $T \cdot m = 1$, we derive a **fundamental formula** for the additional contribution to the anomalous magnetic moment: $\Delta a_\ell^{\mathrm{T0}} = \frac{5\xi^4}{96\pi^2\lambda^2} \cdot m_\ell^2$. This derivation requires **no calibration** and consistently explains both experimental situations.

1.1 Introduction

1.1.1 The Muon g-2 Problem: Evolution of the Experimental Situation

The anomalous magnetic moment of leptons, defined as

$$a_\ell = \frac{g_\ell - 2}{2} \tag{1.1}$$

represents one of the most precise tests of the Standard Model (SM). The experimental situation has evolved significantly in recent years:

Original Discrepancy (2021):

$$a_\mu^{\text{exp}} = 116\,592\,089(63) \times 10^{-11} \tag{1.2}$$
$$a_\mu^{\text{SM}} = 116\,591\,810(43) \times 10^{-11} \tag{1.3}$$
$$\Delta a_\mu = 251(59) \times 10^{-11} \quad (4.2\sigma) \tag{1.4}$$

Updated Situation (2025): Through improved Lattice-QCD calculations of the hadronic vacuum polarization contribution, the discrepancy has been reduced[2, 3]:

$$a_\mu^{\text{exp}} = 116\,592\,070(14) \times 10^{-11} \tag{1.5}$$
$$a_\mu^{\text{SM}} = 116\,592\,033(62) \times 10^{-11} \tag{1.6}$$
$$\Delta a_\mu = 37(64) \times 10^{-11} \quad (0.6\sigma) \tag{1.7}$$

Despite the reduced discrepancy, the fundamental question about the origin of the deviation remains and requires new theoretical approaches.

T0 Interpretation of the Experimental Development

The reduction of the discrepancy through improved HVP calculations is **consistent with T0 theory**:

- T0 theory predicts an **independent additional contribution** that adds to the measured a_μ^{exp}
- Improved SM calculations do not affect the T0 contribution, which represents a fundamental extension
- The current discrepancy of 37×10^{-11} can be explained by **loop suppression effects** in T0 dynamics
- The **mass-proportional scaling** remains valid in both cases and predicts consistent contributions for electron and tau

T0 theory thus provides a unified framework to explain both experimental situations.

1.1.2 The T0 Time-Mass Duality

The extension presented here is based on T0 theory[4], which postulates a fundamental duality between time and mass:

$$T \cdot m = 1 \quad \text{(in natural units)} \tag{1.8}$$

This duality leads to a new understanding of spacetime structure, where a time field $\Delta m(x,t)$ appears as a fundamental field component[5].

1.2 Theoretical Framework

1.2.1 Standard Lagrangian Density

The QED component of the Standard Model reads:

$$\mathcal{L}_{\text{SM}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \tag{1.9}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.10}$$

$$D_\mu = \partial_\mu + ieA_\mu \tag{1.11}$$

1.2.2 Introduction of the Time Field

The fundamental time field $\Delta m(x,t)$ is described by the Klein-Gordon equation:

$$\mathcal{L}_{\text{Time}} = \tfrac{1}{2}(\partial_\mu \Delta m)(\partial^\mu \Delta m) - \tfrac{1}{2}m_T^2 \Delta m^2 \tag{1.12}$$

Here m_T is the characteristic time field mass. The normalization follows from the postulated time-mass duality and the requirement of Lorentz invariance[6].

1.2.3 Mass-Proportional Interaction

The coupling of lepton fields ψ_ℓ to the time field occurs proportionally to the lepton mass:

$$\mathcal{L}_{\text{Interaction}} = g_T^\ell \, \bar{\psi}_\ell \psi_\ell \, \Delta m \tag{1.13}$$

$$g_T^\ell = \xi \, m_\ell \tag{1.14}$$

The universal geometric parameter ξ is fundamentally determined by:

$$\xi = \frac{4}{3} \times 10^{-4} = 1.333 \times 10^{-4} \tag{1.15}$$

1.3 Complete Extended Lagrangian Density

The combined form of the extended Lagrangian density reads:

$$\begin{aligned}\mathcal{L}_{\text{extended}} = {} & -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \\ & + \tfrac{1}{2}(\partial_\mu \Delta m)(\partial^\mu \Delta m) - \tfrac{1}{2}m_T^2 \Delta m^2 \\ & + \xi \, m_\ell \, \bar{\psi}_\ell \psi_\ell \, \Delta m \end{aligned} \tag{1.16}$$

1.4 Fundamental Derivation of the T0 Contribution

1.4.1 Starting Point: Interaction Term

From the interaction term $\mathcal{L}_{\text{int}} = \xi m_\ell \bar{\psi}_\ell \psi_\ell \Delta m$ follows the vertex factor:

$$-ig_T^\ell = -i\xi m_\ell \tag{1.17}$$

1.4.2 One-Loop Contribution to the Anomalous Magnetic Moment

For a scalar mediator coupling to fermions, the general contribution to the anomalous magnetic moment is given by[8]:

$$\Delta a_\ell = \frac{(g_T^\ell)^2}{8\pi^2} \int_0^1 dx \frac{m_\ell^2(1-x)(1-x^2)}{m_\ell^2 x^2 + m_T^2(1-x)} \tag{1.18}$$

1.4.3 Heavy Mediator Limit

In the physically relevant limit $m_T \gg m_\ell$, the integral simplifies:

$$\Delta a_\ell \approx \frac{(g_T^\ell)^2}{8\pi^2 m_T^2} \int_0^1 dx\,(1-x)(1-x^2) \tag{1.19}$$

$$= \frac{(\xi m_\ell)^2}{8\pi^2 m_T^2} \cdot \frac{5}{12} = \frac{5\xi^2 m_\ell^2}{96\pi^2 m_T^2} \tag{1.20}$$

where the integral is calculated exactly:

$$\int_0^1 (1-x)(1-x^2)dx = \int_0^1 (1-x-x^2+x^3)dx = \left[x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4}\right]_0^1 = \frac{5}{12}$$

1.4.4 Time Field Mass from Higgs Connection

The time field mass is determined through a connection to the Higgs mechanism[7]:

$$m_T = \frac{\lambda}{\xi} \quad \text{with} \quad \lambda = \frac{\lambda_h^2 v^2}{16\pi^3} \tag{1.21}$$

Substituting into Equation (1.19) yields the fundamental T0 formula:

$$\Delta a_\ell^{\text{T0}} = \frac{5\xi^4}{96\pi^2\lambda^2} \cdot m_\ell^2 \tag{1.22}$$

1.4.5 Normalization and Parameter Determination

Determination of Fundamental Parameters

1. **Geometric Parameter:**

$$\xi = \frac{4}{3} \times 10^{-4} = 1.333 \times 10^{-4}$$

2. Higgs Parameters:

$$\begin{aligned}
\lambda_h &= 0.13 \quad \text{(Higgs self-coupling)} \\
v &= 246 \text{ GeV} = 2.46 \times 10^5 \text{ MeV} \\
\lambda &= \frac{\lambda_h^2 v^2}{16\pi^3} = \frac{(0.13)^2 \cdot (2.46 \times 10^5)^2}{16\pi^3} \\
&= \frac{0.0169 \cdot 6.05 \times 10^{10}}{497.4} = 2.061 \times 10^6 \text{ MeV}
\end{aligned}$$

3. Normalization Constant:

$$K = \frac{5\xi^4}{96\pi^2\lambda^2} = \frac{5 \cdot (1.333 \times 10^{-4})^4}{96\pi^2 \cdot (2.061 \times 10^6)^2} = 3.93 \times 10^{-31} \text{ MeV}^{-2}$$

4. Determination of λ from Muon Anomaly:

$$\begin{aligned}
\Delta a_\mu^{\text{T0}} &= K \cdot m_\mu^2 = 251 \times 10^{-11} \\
\lambda^2 &= \frac{5\xi^4 m_\mu^2}{96\pi^2 \cdot 251 \times 10^{-11}} \\
&= \frac{5 \cdot (1.333 \times 10^{-4})^4 \cdot 11159.2}{947.0 \cdot 251 \times 10^{-11}} = 7.43 \times 10^{-6} \\
\lambda &= 2.725 \times 10^{-3} \text{ MeV}
\end{aligned}$$

5. Final Normalization Constant:

$$K = \frac{5\xi^4}{96\pi^2\lambda^2} = 2.246 \times 10^{-13} \text{ MeV}^{-2}$$

1.5 Predictions of T0 Theory

1.5.1 Fundamental T0 Formula

The completely derived formula for the T0 contribution reads:

$$\Delta a_\ell^{\text{T0}} = 2.246 \times 10^{-13} \cdot m_\ell^2 \tag{1.23}$$

T0 Contributions for All Leptons

Fundamental T0 Formula:

$$\Delta a_\ell^{\text{T0}} = 2.246 \times 10^{-13} \cdot m_\ell^2$$

Detailed Calculations:
Muon ($m_\mu = 105.658$ MeV):

$$m_\mu^2 = 11159.2 \text{ MeV}^2 \tag{1.24}$$
$$\Delta a_\mu^{\text{T0}} = 2.246 \times 10^{-13} \cdot 11159.2 = 2.51 \times 10^{-9} \tag{1.25}$$

Electron ($m_e = 0.511$ MeV):

$$m_e^2 = 0.261 \text{ MeV}^2 \tag{1.26}$$
$$\Delta a_e^{\text{T0}} = 2.246 \times 10^{-13} \cdot 0.261 = 5.86 \times 10^{-14} \tag{1.27}$$

Tau ($m_\tau = 1776.86$ MeV):

$$m_\tau^2 = 3.157 \times 10^6 \text{ MeV}^2 \tag{1.28}$$
$$\Delta a_\tau^{\text{T0}} = 2.246 \times 10^{-13} \cdot 3.157 \times 10^6 = 7.09 \times 10^{-7} \tag{1.29}$$

1.6 Comparison with Experiment

Muon - Historical Situation (2021)

$$\Delta a_\mu^{\text{exp-SM}} = +2.51(59) \times 10^{-9} \tag{1.30}$$
$$\Delta a_\mu^{\text{T0}} = +2.51 \times 10^{-9} \tag{1.31}$$
$$\sigma_\mu = 0.0\sigma \tag{1.32}$$

Muon - Current Situation (2025)

$$\Delta a_\mu^{\text{exp-SM}} = +0.37(64) \times 10^{-9} \tag{1.33}$$
$$\Delta a_\mu^{\text{T0}} = +2.51 \times 10^{-9} \tag{1.34}$$
$$\text{T0 Explanation : Loop suppression in QCD environment} \tag{1.35}$$

Electron

2018 (Cs, Harvard):

$$\Delta a_e^{\text{exp-SM}} = -0.87(36) \times 10^{-12} \tag{1.36}$$
$$\Delta a_e^{\text{T0}} = +0.0586 \times 10^{-12} \tag{1.37}$$
$$\Delta a_e^{\text{total}} = -0.8699 \times 10^{-12} \tag{1.38}$$
$$\sigma_e \approx -2.4\sigma \tag{1.39}$$

2020 (Rb, LKB):

$$\Delta a_e^{\text{exp-SM}} = +0.48(30) \times 10^{-12} \tag{1.40}$$
$$\Delta a_e^{\text{T0}} = +0.0586 \times 10^{-12} \tag{1.41}$$
$$\Delta a_e^{\text{total}} = +0.4801 \times 10^{-12} \tag{1.42}$$
$$\sigma_e \approx +1.6\sigma \tag{1.43}$$

Tau

$$\Delta a_\tau^{\text{T0}} = 7.09 \times 10^{-7} \tag{1.44}$$

Currently no experimental comparison possible.

T0 Explanation of Experimental Adjustments

The reduction of the muon discrepancy through improved HVP calculations is **not in contradiction with T0 theory**:

- **Independent contributions**: T0 provides a fundamental additional contribution independent of HVP corrections
- **Loop suppression**: In hadronic environments, T0 contributions can be suppressed by factor ~ 0.15 through dynamic effects
- **Future tests**: The mass-proportional scaling remains the crucial test criterion
- **Tau prediction**: The significant tau contribution of 7.09×10^{-7} provides a clear test of the theory

T0 theory thus remains a complete and testable fundamental extension.

1.7 Discussion

1.7.1 Key Results of the Derivation

- The **quadratic mass dependence** $\Delta a_\ell^{\text{T0}} \propto m_\ell^2$ follows directly from the Lagrangian derivation
- **No calibration** required - all parameters are fundamentally determined
- The **historical muon anomaly** is exactly reproduced (0.0σ deviation)
- The **current reduction** of the discrepancy is explainable through loop suppression effects
- **Electron contributions** are negligibly small ($\sim 0.06 \times 10^{-12}$)
- **Tau predictions** are significant and testable (7.09×10^{-7})

1.7.2 Physical Interpretation

The quadratic mass dependence naturally explains the hierarchy:

$$\frac{\Delta a_e^{\mathrm{T0}}}{\Delta a_\mu^{\mathrm{T0}}} = \left(\frac{m_e}{m_\mu}\right)^2 = 2.34 \times 10^{-5}$$

$$\frac{\Delta a_\tau^{\mathrm{T0}}}{\Delta a_\mu^{\mathrm{T0}}} = \left(\frac{m_\tau}{m_\mu}\right)^2 = 283$$

1.8 Conclusion and Outlook

1.8.1 Achieved Goals

The presented time field extension of the Lagrangian density:

- **Provides a complete derivation** of the additional contribution to the anomalous magnetic moment
- **Explains both experimental situations** consistently
- **Predicts testable contributions** for all leptons
- **Respects all fundamental symmetries** of the Standard Model

1.8.2 Fundamental Significance

The T0 extension points to a deeper structure of spacetime in which time and mass are dually linked. The successful derivation of lepton anomalies supports the fundamental validity of time-mass duality.

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