T0 Model: Critical Correction to the Wavelength-Dependent Redshift Formula

Mathematical Analysis and Physical Implications

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May 25, 2025

Abstract

This document addresses a critical sign error discovered in the wavelength-dependent redshift formula of the T0 model. Through systematic mathematical analysis and numerical verification, we demonstrate that the correct formula is $z(\lambda) = z_0(1 - \ln(\lambda/\lambda_0))$ rather than the previously published $z(\lambda) = z_0(1 + \ln(\lambda/\lambda_0))$. This correction has profound implications for experimental predictions, with the corrected formula showing only 2% deviation from the exact solution compared to 40% error in the incorrect version. The correction enhances the physical consistency of the T0 model and provides clearer experimental signatures for distinguishing it from standard cosmological models.

Contents

1 Introduction and Problem Statement

A systematic review of the T0 model literature has revealed an inconsistency in the wavelength-dependent redshift formula across multiple documents. This correction note provides:

1. Rigorous mathematical derivation of the correct formula
2. Numerical verification of accuracy
3. Physical interpretation and consistency analysis
4. Impact assessment on experimental predictions
5. Required corrections for existing documents

Critical Issue Identified

Incorrect Formula Found in Multiple Documents:

$$z(\lambda) = z_0 \left(1 + \beta_{\mathrm{T}} \ln \frac{\lambda}{\lambda_0}\right) \tag{1}$$

Correct Formula (This Work):

$$z(\lambda) = z_0 \left(1 - \beta_{\mathrm{T}} \ln \frac{\lambda}{\lambda_0}\right) \tag{2}$$

With $\beta_{\mathrm{T}} = 1$ in natural units: $z(\lambda) = z_0 \left(1 - \ln \frac{\lambda}{\lambda_0}\right)$

2 Mathematical Derivation of the Correct Formula

2.1 Starting from First Principles

The T0 model predicts photon energy loss through interaction with time field gradients:

$$\frac{dE}{dr} = -g_T \omega^2 \frac{2G}{r^2} \tag{3}$$

where $g_T = \xi = \alpha_{\mathrm{EM}} = 1$ in natural units.
Dimensional verification:

- $[dE/dr] = [E]/[E^{-1}] = [E^2]$
- $[g_T \omega^2 2G/r^2] = [1][E^2][E^{-2}]/[E^{-2}] = [E^2]$ ✓

2.2 Integration Over Cosmic Distances

For propagation from source at r_1 to observer at r_2 with $r_2 \gg r_1$:

$$\begin{aligned}
\Delta E &= -\int_{r_1}^{r_2} g_T \omega^2 \frac{2G}{r^2} dr && (4)\\
&= -g_T \omega^2 2G \int_{r_1}^{r_2} \frac{1}{r^2} dr && (5)\\
&= -g_T \omega^2 2G \left[-\frac{1}{r}\right]_{r_1}^{r_2} && (6)\\
&= g_T \omega^2 2G \left(\frac{1}{r_1} - \frac{1}{r_2}\right) && (7)\\
&\approx g_T \omega^2 \frac{2G}{r_1} \quad (\text{for } r_2 \gg r_1) && (8)
\end{aligned}$$

2.3 Redshift Definition and Wavelength Dependence

The redshift is defined as:

$$z = \frac{\Delta E}{E} = \frac{\Delta E}{\omega} \tag{9}$$

Substituting our result:

$$z = \frac{g_T \omega^2 (2G/r)}{\omega} = g_T \omega \frac{2G}{r} \tag{10}$$

In natural units where $\omega = 1/\lambda$:

$$z(\lambda) = g_T \frac{1}{\lambda} \frac{2G}{r} = \frac{g_T 2G/r}{\lambda} \tag{11}$$

Defining the reference redshift $z_0 = (g_T 2G/r)/\lambda_0$:

$$\boxed{z(\lambda) = z_0 \frac{\lambda_0}{\lambda}} \tag{12}$$

This is the **exact** wavelength-dependent redshift formula.

2.4 Logarithmic Approximation

For small wavelength variations around λ_0, we can approximate:

Let $\lambda = \lambda_0(1 + \varepsilon)$ where ε is small. Then:

$$\begin{aligned}
z(\lambda) &= z_0 \frac{\lambda_0}{\lambda_0(1+\varepsilon)} = \frac{z_0}{1+\varepsilon} && (13)\\
&\approx z_0(1-\varepsilon) \quad (\text{Taylor expansion for small } \varepsilon) && (14)\\
&= z_0 \left(1 - \frac{\lambda - \lambda_0}{\lambda_0}\right) && (15)
\end{aligned}$$

For the logarithmic form, we use the approximation $\ln(1+\varepsilon) \approx \varepsilon$ for small ε:

$$\ln \frac{\lambda}{\lambda_0} = \ln(1+\varepsilon) \approx \varepsilon = \frac{\lambda - \lambda_0}{\lambda_0} \tag{16}$$

Therefore:

$$\boxed{z(\lambda) \approx z_0 \left(1 - \ln \frac{\lambda}{\lambda_0}\right)} \tag{17}$$

3 Numerical Verification

3.1 Test Parameters

We verify the approximation accuracy using realistic parameters:

- $\xi = 1.32 \times 10^{-4}$ (universal T0 parameter)
- $\lambda_0 = 500$ nm (reference wavelength)
- $\lambda_1 = 400$ nm (blue light, higher energy)
- $\lambda_2 = 600$ nm (red light, lower energy)

3.2 Comparison of Formulas

Formula	Blue (400nm)	Red (600nm)	Error vs Exact
Exact: $z_0(\lambda_0/\lambda)$	$1.250z_0$	$0.833z_0$	0%
Correct: $z_0(1 - \ln(\lambda/\lambda_0))$	$1.223z_0$	$0.818z_0$	$\sim 2\%$
Incorrect: $z_0(1 + \ln(\lambda/\lambda_0))$	$0.777z_0$	$1.182z_0$	$\sim 40\%$

Table 1: Numerical comparison of redshift formulas

Key findings:

- The correct formula has only $\sim 2\%$ error compared to the exact solution
- The incorrect formula has $\sim 40\%$ error - completely unacceptable
- The numerical verification conclusively demonstrates the correct sign

3.3 Detailed Numerical Analysis

For blue light ($\lambda_1 = 400$ nm):

$$\text{Exact:} \quad z_1 = z_0 \frac{500}{400} = 1.250z_0 \tag{18}$$

$$\text{Correct:} \quad z_1 = z_0(1 - \ln(0.8)) = z_0(1 + 0.223) = 1.223z_0 \tag{19}$$

$$\text{Incorrect:} \quad z_1 = z_0(1 + \ln(0.8)) = z_0(1 - 0.223) = 0.777z_0 \tag{20}$$

Relative errors:

$$\text{Correct:} \quad \frac{1.223 - 1.250}{1.250} = -2.1\% \tag{21}$$

$$\text{Incorrect:} \quad \frac{0.777 - 1.250}{1.250} = -37.9\% \tag{22}$$

4 Physical Interpretation and Consistency

4.1 Physical Logic Test

The correct formula $z(\lambda) = z_0(1 - \ln(\lambda/\lambda_0))$ predicts:

For blue light ($\lambda < \lambda_0$):

- $\ln(\lambda/\lambda_0) < 0$
- $z > z_0$ (enhanced redshift)
- Higher energy photons lose more energy ✓

For red light ($\lambda > \lambda_0$):

- $\ln(\lambda/\lambda_0) > 0$
- $z < z_0$ (reduced redshift)
- Lower energy photons lose less energy ✓

This behavior is **physically consistent**: more energetic photons interact more strongly with time field gradients and experience greater energy loss.

4.2 Consistency with Energy Loss Mechanism

The T0 energy loss rate $dE/dr \propto -\omega^2$ implies:

- Higher frequency $\omega \to$ greater energy loss rate
- Greater energy loss $\to$ larger redshift
- Therefore: higher frequency $\to$ larger redshift

Since $\omega = 1/\lambda$, this means shorter wavelengths should show enhanced redshift, exactly as predicted by the correct formula.

5 Experimental Implications

5.1 Observable Predictions

The corrected redshift formula makes specific testable predictions:

Theorem 5.1 (Wavelength-Dependent Redshift Signature)**.** *For any astrophysical source observed in the T0 model:*

$$\frac{\partial z}{\partial \ln \lambda} = -z_0 \tag{23}$$

This provides a direct experimental signature distinguishing T0 from standard cosmology.

5.2 Multi-Wavelength Spectroscopy

Experimental protocol:

1. Observe multiple emission lines from distant sources
2. Measure redshift $z(\lambda)$ for each wavelength
3. Plot z vs $\ln \lambda$ - should show linear relationship with slope $-z_0$
4. Compare with standard cosmology prediction (no wavelength dependence)

Required precision: $\Delta z/z \sim 10^{-3}$ to detect the logarithmic wavelength dependence.

5.3 Quantitative Example

For a source at redshift $z_0 = 1$:

$$z(400\text{ nm}) = 1 \times (1 - \ln(0.8)) = 1.223 \tag{24}$$
$$z(600\text{ nm}) = 1 \times (1 - \ln(1.2)) = 0.818 \tag{25}$$
$$\Delta z = 1.223 - 0.818 = 0.405 \tag{26}$$

This 40% variation across the visible spectrum should be easily detectable with modern instruments.

6 Impact on T0 Model Documentation

6.1 Documents Requiring Correction

The following T0 model documents contain the incorrect redshift formula and require updates:

1. `QMRelTimeMassPart1ZEn.tex` - Main theoretical framework
2. `MathZeitMasseLagrangeEn.tex` - Mathematical formulations
3. `DerivationVonBetaEn.tex` - Beta parameter derivation
4. `TempEinheitenCMBEn.tex` - CMB temperature analysis
5. `DynMassePhotonenNichtlokalEn.tex` - Photon dynamics

6.2 Specific Corrections Required

Correction 6.1 (Global Formula Replacement)**.** *Replace all instances of:*

$$z(\lambda) = z_0 \left(1 + \beta_T \ln \frac{\lambda}{\lambda_0}\right) \tag{27}$$

With:

$$z(\lambda) = z_0 \left(1 - \beta_T \ln \frac{\lambda}{\lambda_0}\right) \tag{28}$$

In natural units where $\beta_T = 1$:

$$z(\lambda) = z_0 \left(1 - \ln \frac{\lambda}{\lambda_0}\right) \tag{29}$$

6.3 Verification of Other Calculations

All numerical results and experimental predictions based on the wavelength-dependent redshift formula must be recalculated using the correct expression. This includes:

- CMB temperature evolution predictions
- Multi-wavelength astrophysical observations
- Precision cosmology parameter estimates
- Dark energy alternative explanations

7 Theoretical Significance

7.1 Enhanced Model Consistency

The correction strengthens the T0 model by:

- **Mathematical rigor**: Exact derivation from first principles
- **Physical consistency**: Logical energy loss mechanism
- **Numerical accuracy**: 2% vs 40% approximation error
- **Experimental testability**: Clear observable predictions

7.2 Distinguishing Features from Standard Cosmology

The corrected T0 redshift formula provides several advantages over standard cosmological models:

Feature	Standard Cosmology	T0 Model (Corrected)
Wavelength dependence	None	$z \propto -\ln\lambda$
Physical mechanism	Spatial expansion	Energy loss to time field
High-energy behavior	Same redshift	Enhanced redshift
Experimental signature	Flat spectrum	Logarithmic variation
Free parameters	Multiple	None (fixed by $\beta_{\mathrm{T}} = 1$)

Table 2: Comparison with standard cosmology

8 Mathematical Proof of Correctness

8.1 Rigorous Derivation

Proof of Correct Sign. Starting from the exact formula eq. (12):

$$z(\lambda) = z_0 \frac{\lambda_0}{\lambda} \tag{30}$$

For $\lambda = \lambda_0 e^x$ where x is small:

$$z(\lambda) = z_0 \frac{\lambda_0}{\lambda_0 e^x} = z_0 e^{-x} \tag{31}$$

$$\approx z_0(1 - x) \quad \text{(Taylor expansion)} \tag{32}$$

$$= z_0 \left(1 - \ln \frac{\lambda}{\lambda_0}\right) \tag{33}$$

This rigorously establishes the negative sign in the logarithmic term. □

8.2 Alternative Verification

We can also verify using the derivative:

$$\frac{d}{d\lambda}\left[z_0\frac{\lambda_0}{\lambda}\right] = -z_0\frac{\lambda_0}{\lambda^2} \tag{34}$$

$$\frac{d}{d\lambda}\left[z_0\left(1-\ln\frac{\lambda}{\lambda_0}\right)\right] = -z_0\frac{1}{\lambda} \tag{35}$$

For $\lambda \approx \lambda_0$: $-z_0/\lambda_0^2 \approx -z_0/\lambda_0$, confirming consistency.

9 Conclusions

9.1 Summary of Findings

This analysis has conclusively established:

1. **Correct Formula**: $z(\lambda) = z_0(1 - \ln(\lambda/\lambda_0))$
2. **Mathematical Accuracy**: 2% error vs 40% for incorrect formula
3. **Physical Consistency**: Higher energy photons show enhanced redshift
4. **Experimental Testability**: Clear observable signatures in multi-wavelength spectroscopy
5. **Documentation Impact**: Multiple T0 documents require correction

9.2 Implications for T0 Model Development

The correction:

- **Strengthens** the theoretical foundation through mathematical rigor
- **Enhances** experimental testability with clearer predictions
- **Improves** physical consistency with energy loss mechanisms
- **Provides** distinctive signatures distinguishing T0 from standard cosmology

9.3 Recommendations

Immediate actions:

1. Update all T0 documentation with the correct redshift formula
2. Recalculate numerical predictions using the corrected expression
3. Verify consistency across all derived results
4. Prepare experimental proposals for multi-wavelength tests

Long-term implications:

1. Design precision spectroscopic experiments
2. Develop statistical analysis methods for wavelength-dependent redshift

3. Investigate connections to other wavelength-dependent phenomena
4. Explore implications for fundamental physics beyond cosmology

Final Statement

The correction from $z(\lambda) = z_0(1 + \ln(\lambda/\lambda_0))$ to $z(\lambda) = z_0(1 - \ln(\lambda/\lambda_0))$ represents more than a sign change—it establishes the T0 model as a mathematically rigorous, physically consistent, and experimentally testable alternative to standard cosmological models. This correction enhances rather than weakens the T0 framework, providing clearer predictions and stronger theoretical foundations for future development.

Acknowledgments

The author acknowledges the importance of rigorous mathematical verification in theoretical physics development. This correction demonstrates the value of systematic analysis and cross-verification in establishing robust theoretical frameworks.

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