Natural Unit Systems: Universal Energy Conversion and Fundamental Length Scale Hierarchy

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Abstract

This foundational document establishes the natural unit system used throughout the T0 model framework. By setting fundamental constants to unity and adopting energy as the base dimension, all physical quantities can be expressed as powers of energy. This document serves as the reference for unit conversions and dimensional analysis across all T0 model applications.

Contents

1 List of Symbols and Notation

Symbol	Meaning	Units/Notes
	Fundamental Constants	
$\hbar$	Reduced Planck constant	Set to 1
c	Speed of light	Set to 1
G	Gravitational constant	Set to 1
k_B	Boltzmann constant	Set to 1
e	Elementary charge	$[E^0]$ (dimensionless)
ε_0, μ_0	Vacuum permittivity, permeability	Set to 1 in QED units
	Units	
l_P, t_P, m_P, E_P, T_P	Planck length, time, mass, energy, temp.	Natural base units
m_e, a_0, E_h	Electron mass, Bohr radius, Hartree energy	Atomic units
	Coupling Constants	
α_{EM}	Fine-structure constant	$e^2/(4\pi) = 1$ (nat.), $\approx 1/137$ (SI)
$\alpha_s, \alpha_W, \alpha_G$	Strong, weak, gravitational coupling	Dimensionless
	Physical Quantities	
E, m, Θ	Energy, mass, temperature	$[E]$
L, r, λ, t	Length, radius, wavelength, time	$[E^{-1}]$
p, ω, ν	Momentum, angular freq., frequency	$[E]$
F	Force	$[E^2]$
v	Velocity	Dimensionless
q	Electric charge	$[E^0]$ (dimensionless)
	Special Scales & Notation	
r_0, ξ	T0 length, scaling parameter	$\xi l_P, \xi \approx 1.33 \times 10^{-4}$
$\lambda_{C,e}, r_e$	Compton wavelength, classical e radius	$\hbar/(m_e c), e^2/(4\pi\varepsilon_0 m_e c^2)$
$[X], [E^n]$	Dimension of X, energy dimension	Dimensional analysis
$\sim, \leftrightarrow$	Approximately, conversion	Order of magnitude, units

Table 1: Symbols and notation

2 Introduction

Natural units are unit systems where fundamental physical constants are set to unity to simplify calculations and reveal the underlying mathematical structure of physical laws. The most well-known systems are **Planck units** (for gravitation and quantum physics) and **atomic units** (for quantum chemistry).

This document establishes the complete framework for the natural unit system used in the T0 model, which is based on Planck units with energy as the fundamental dimension. The key insight is that energy $[E]$ serves as the universal dimension from which all other physical quantities derive.

2.1 Comparison with Other Natural Unit Systems

System	Constants Set to 1	Base Units	Applications	Notes
Planck Units	$\hbar, c, G, k_B = 1$	l_P, t_P, m_P, E_P	Quantum gravity, cosmology	Universal significance
Atomic Units	$m_e, e, \hbar, \frac{1}{4\pi\varepsilon_0} = 1$	a_0, E_h	Quantum chemistry, atoms	Chemistry applications
Particle Physics	$\hbar, c = 1$	GeV	High energy physics	Practical for colliders
T0 Model	$\hbar, c, G, k_B = 1$	Energy $[E]$	Unified physics	Energy as base dimension

Table 2: Comparison of natural unit systems

3 Fundamentals of Natural Unit Systems

3.1 Planck Units

The Planck units were proposed by Max Planck in 1899 [?, ?] and are based on the fundamental natural constants:

$$G = 1 \quad \text{(gravitational constant)} \tag{1}$$
$$c = 1 \quad \text{(speed of light)} \tag{2}$$
$$\hbar = 1 \quad \text{(reduced Planck constant)} \tag{3}$$

Planck recognized that these units *"retain their meaning for all times and for all, including extraterrestrial and non-human cultures necessarily"* [?].

3.2 Atomic Units

The atomic units, introduced by Hartree in 1927 [?], set:

$$m_e = 1 \quad \text{(electron mass)} \tag{4}$$
$$e = 1 \quad \text{(elementary charge)} \tag{5}$$
$$\hbar = 1 \tag{6}$$
$$\frac{1}{4\pi\varepsilon_0} = 1 \quad \text{(Coulomb constant)} \tag{7}$$

3.3 Quantum Optical Units

For quantum field theory applications, quantum optical units are commonly used:

$$c = 1 \quad \text{(speed of light)} \tag{8}$$
$$\hbar = 1 \quad \text{(reduced Planck constant)} \tag{9}$$
$$\varepsilon_0 = 1 \quad \text{(permittivity)} \tag{10}$$
$$\mu_0 = 1 \quad \text{(permeability, because } c = 1/\sqrt{\varepsilon_0\mu_0}\text{)} \tag{11}$$

3.4 Advantages of Natural Units

Natural units offer several key advantages:

- **Simplified equations** (e.g., $E = m$ instead of $E = mc^2$)
- **No superfluous constants** in calculations
- **Universal scaling** for fundamental physics
- **Reveals fundamental relationships** between physical quantities
- **Provides dimensional consistency** checks
- **Eliminates arbitrary conversion factors**
- **Highlights the universal role** of energy

4 Mathematical Proof of Energy Equivalence

4.1 Fundamental Dimensional Relations

In natural units, all physical quantities have dimensions that can be expressed as powers of energy $[E]$ [?, ?]:

$$[L] = [E]^{-1} \quad \text{(from } \hbar c = 1\text{)} \tag{12}$$
$$[T] = [E]^{-1} \quad \text{(from } \hbar = 1\text{)} \tag{13}$$
$$[M] = [E] \quad \text{(from } c = 1\text{)} \tag{14}$$

4.2 Conversion of Fundamental Quantities

Length: From the relation $\hbar c = 1$ it follows:

$$[L] = \frac{[\hbar][c]}{[E]} = [E]^{-1} \tag{15}$$

Time: From $\hbar = 1$ and $E = \hbar\omega$ it follows:

$$[T] = \frac{[\hbar]}{[E]} = [E]^{-1} \tag{16}$$

Mass: From $E = mc^2$ and $c = 1$ it follows:

$$[M] = [E] \tag{17}$$

Velocity:

$$[v] = \frac{[L]}{[T]} = \frac{[E]^{-1}}{[E]^{-1}} = [E]^0 = \text{dimensionless} \tag{18}$$

Momentum:

$$[p] = [M][v] = [E] \cdot [E]^0 = [E] \tag{19}$$

Force:

$$[F] = [M][a] = [E] \cdot [E]^{-1} = [E]^2 \tag{20}$$

Charge: In Planck units from $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$:

$$[q] = [E]^{1/2} \tag{21}$$

4.3 Generalization

Any physical quantity G can be represented as a product of powers of the fundamental constants:

$$G = c^a \cdot \hbar^b \cdot G^c \cdot k_B^d \cdot \ldots \tag{22}$$

In natural units this becomes:

$$[G] = [E]^n \quad \text{for a specific } n \in \mathbb{Q} \tag{23}$$

Physical Quantity	SI Dimension	Natural Dimension	Derivation
Energy	$[ML^2T^{-2}]$	$[E]$	Base dimension
Mass	$[M]$	$[E]$	$E = mc^2, c = 1$
Temperature	$[\Theta]$	$[E]$	$E = k_BT, k_B = 1$
Length	$[L]$	$[E^{-1}]$	$l_P = \sqrt{\hbar G/c^3} = 1$
Time	$[T]$	$[E^{-1}]$	$t_P = \sqrt{\hbar G/c^5} = 1$
Momentum	$[MLT^{-1}]$	$[E]$	$p = mv, v = [E^0]$
Force	$[MLT^{-2}]$	$[E^2]$	$F = ma = [E][E] = [E^2]$
Power	$[ML^2T^{-3}]$	$[E^2]$	$P = E/t = [E]/[E^{-1}] = [E^2]$
Charge	$[AT]$	$[E^0]$	Dimensionless in Planck units
Electric Field	$[MLT^{-3}A^{-1}]$	$[E^2]$	$\vec{E} = \vec{F}/q$
Magnetic Field	$[MT^{-2}A^{-1}]$	$[E^2]$	$\vec{B} = \vec{F}/(qv)$

Table 3: Universal energy dimensions of physical quantities

4.4 Fundamental Relationships

The key relationships in natural units become:

$$E = m \quad \text{(mass-energy equivalence)} \tag{24}$$
$$E = T \quad \text{(temperature-energy equivalence)} \tag{25}$$
$$[L] = [T] = [E^{-1}] \quad \text{(space-time unity)} \tag{26}$$
$$\omega = E \quad \text{(frequency-energy equivalence)} \tag{27}$$
$$p = E \quad \text{(momentum-energy equivalence for massless particles)} \tag{28}$$

5 Length Scale Hierarchy

5.1 Standard Length Scales

Physical systems organize themselves around characteristic length scales:

5.2 The T0 Length Scale

The T0 model introduces a sub-Planckian length scale:

Definition 5.1 (T0 Length)**.**

$$r_0 = \xi \cdot l_P \tag{29}$$

where $\xi \approx 1.33 \times 10^{-4}$ *is a dimensionless parameter.*

This gives:

$$r_0 = \xi \cdot l_P = 1.33 \times 10^{-4} \times 1.616 \times 10^{-35}\,\text{m} \tag{30}$$
$$= 2.15 \times 10^{-39}\,\text{m} \tag{31}$$

In natural units with $l_P = 1$:

$$r_0 = \xi \approx 1.33 \times 10^{-4} \tag{32}$$

Scale	Symbol	SI Value (m)	Natural Units ($l_P = 1$)
Planck Length	l_P	1.616×10^{-35}	1
Compton (electron)	$\lambda_{C,e}$	2.426×10^{-12}	1.5×10^{23}
Classical electron radius	r_e	2.818×10^{-15}	1.7×10^{20}
Bohr radius	a_0	5.292×10^{-11}	3.3×10^{24}
Nuclear scale	$\sim 10^{-15}$	10^{-15}	6.2×10^{19}
Atomic scale	$\sim 10^{-10}$	10^{-10}	6.2×10^{24}
Human scale	~ 1	1	6.2×10^{34}
Earth radius	$R_\oplus$	6.371×10^{6}	3.9×10^{41}
Solar System	$\sim 10^{12}$	10^{12}	6.2×10^{46}
Galactic scale	$\sim 10^{21}$	10^{21}	6.2×10^{55}

Table 4: Standard length scales in natural units

6 Unit Conversions

6.1 Energy as Reference

Using the electronvolt (eV) as the practical energy unit:

Physical Quantity	Conversion to SI	Example (1 GeV)
Energy	$1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$	$1.602 \times 10^{-10}\,\mathrm{J}$
Mass	$E(\mathrm{eV}) \times 1.783 \times 10^{-36}\,\mathrm{kg\,eV^{-1}}$	$1.783 \times 10^{-27}\,\mathrm{kg}$
Length	$E(\mathrm{eV})^{-1} \times 1.973 \times 10^{-7}\,\mathrm{m\,eV}$	$1.973 \times 10^{-16}\,\mathrm{m}$
Time	$E(\mathrm{eV})^{-1} \times 6.582 \times 10^{-16}\,\mathrm{s\,eV}$	$6.582 \times 10^{-25}\,\mathrm{s}$
Temperature	$E(\mathrm{eV}) \times 1.161 \times 10^{4}\,\mathrm{K\,eV^{-1}}$	$1.161 \times 10^{13}\,\mathrm{K}$

Table 5: Conversion factors from natural to SI units

6.2 Planck Scale Conversions

Converting between Planck units and SI:

Planck Unit	Natural Value	SI Value
Length (l_P)	1	$1.616 \times 10^{-35}\,\mathrm{m}$
Time (t_P)	1	$5.391 \times 10^{-44}\,\mathrm{s}$
Mass (m_P)	1	$2.176 \times 10^{-8}\,\mathrm{kg}$
Energy (E_P)	1	$1.220 \times 10^{19}\,\mathrm{GeV}$
Temperature (T_P)	1	$1.417 \times 10^{32}\,\mathrm{K}$

Table 6: Planck unit conversions

7 Mathematical Framework

7.1 Simplified Equations

In natural units, fundamental equations become elegantly simple:

7.1.1 Quantum Mechanics

$$\text{Schrödinger equation:} \quad i\frac{\partial \psi}{\partial t} = H\psi \tag{33}$$

$$\text{Uncertainty principle:} \quad \Delta E \Delta t \geq \frac{1}{2} \tag{34}$$

$$\text{de Broglie relation:} \quad \lambda = \frac{1}{p} \tag{35}$$

7.1.2 Special Relativity

$$\text{Mass-energy:} \quad E = m \tag{36}$$

$$\text{Energy-momentum:} \quad E^2 = p^2 + m^2 \tag{37}$$

$$\text{Lorentz factor:} \quad \gamma = \frac{1}{\sqrt{1 - v^2}} \tag{38}$$

7.1.3 General Relativity

$$\text{Einstein equations:} \quad G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{39}$$

$$\text{Schwarzschild radius:} \quad r_s = 2M \tag{40}$$

7.1.4 Electromagnetism

$$\text{Coulomb's law:} \quad F = \frac{q_1 q_2}{4\pi r^2} \tag{41}$$

$$\text{Fine structure constant:} \quad \alpha = \frac{e^2}{4\pi}(\text{with } 4\pi\varepsilon_0 = 1) \tag{42}$$

7.1.5 Thermodynamics

$$\text{Stefan-Boltzmann:} \quad j = \sigma T^4 \tag{43}$$

$$\text{Wien's law:} \quad \lambda_{max} T = b \tag{44}$$

$$\text{Boltzmann distribution:} \quad P \propto e^{-E/T} \tag{45}$$

8 Advantages and Applications

8.1 Advantages of Natural Units

- **Simplified equations** (e.g., $E = m$ instead of $E = mc^2$)
- **No superfluous constants** in calculations
- **Universal scaling** for fundamental physics
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8.2 Disadvantages

- **Unintuitive for macroscopic applications**
- **Conversion to SI requires knowledge** of fundamental constants
- **Initial unfamiliarity** for those used to SI units
- **Engineering preference** for practical SI units

8.3 Practical Applications

- Particle physics calculations
- Quantum field theory
- General relativity and cosmology
- High-energy astrophysics
- String theory and quantum gravity
- Fundamental constant relationships

9 Working with Natural Units

9.1 Working with Natural Units

To convert a calculation from SI to natural units:

1. Express all quantities in terms of energy (eV or GeV)
2. Set $\hbar = c = G = k_B = 1$
3. Perform the calculation
4. Convert results back to SI if needed

9.2 Dimensional Check

Always verify dimensional consistency:

- All terms in an equation must have the same energy dimension
- Check that exponents are consistent
- Use dimensional analysis to verify results

9.3 Fundamental Forces in Natural Units

The four fundamental forces can be characterized by their dimensionless coupling constants:

Force	**Dimensionless Coupling**	**Typical Value**	**Range**
Electromagnetic	α_{EM}	$\sim 1/137$	∞
Strong	α_s	~ 0.118 at $Q^2 = M_Z^2$	$\sim 1 \times 10^{-15}$ m
Weak	$\alpha_W = g^2/(4\pi)$	$\sim 1/30$	$\sim 1 \times 10^{-18}$ m
Gravitation	$\alpha_G = Gm^2/(\hbar c)$	m^2/m_P^2	∞

Table 7: Fundamental forces characterized by coupling constants

SI Unit	SI Dimension	Natural Dimension	Conversion	Accuracy
Meter	$[L]$	$[E^{-1}]$	$1\,\mathrm{m} \leftrightarrow (197\,\mathrm{MeV})^{-1}$	$< 0.001\%$
Second	$[T]$	$[E^{-1}]$	$1\,\mathrm{s} \leftrightarrow (6.58 \times 10^{-22}\,\mathrm{MeV})^{-1}$	$< 0.00001\%$
Kilogram	$[M]$	$[E]$	$1\,\mathrm{kg} \leftrightarrow 5.61 \times 10^{26}\,\mathrm{MeV}$	$< 0.001\%$
Ampere	$[I]$	$[E]^{1/2}$	$1\,\mathrm{A} \leftrightarrow (6.24 \times 10^{18}\,\mathrm{eV})^{1/2}/\mathrm{s}$	$< 0.005\%$
Kelvin	$[\Theta]$	$[E]$	$1\,\mathrm{K} \leftrightarrow 8.62 \times 10^{-5}\,\mathrm{eV}$	$< 0.01\%$
Volt	$[ML^2T^{-3}I^{-1}]$	$[E]$	$1\,\mathrm{V} \leftrightarrow 1\,\mathrm{eV}/e$	$< 0.0001\%$
Coulomb	$[TI]$	$[E^0]$	$1\,\mathrm{C} \leftrightarrow 6.24 \times 10^{18}\,e$	$< 0.0001\%$

Table 8: Comprehensive unit conversions from SI to natural units

9.4 Comprehensive Unit Conversions

10 Conclusion

This natural unit system provides the foundation for all T0 model calculations. By establishing energy as the universal dimension and setting fundamental constants to unity, we reveal the underlying unity of physical laws across all scales from the sub-Planckian T0 length to cosmological distances.

Key principles:

1. Energy is the fundamental dimension
2. All physical quantities are powers of energy
3. The T0 length extends physics below the Planck scale
4. Natural units simplify fundamental equations
5. Dimensional consistency is paramount

This framework serves as the basis for all further developments in the T0 model, providing both computational tools and conceptual insights into the nature of physical reality.

References

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