T0-Model Formula Collection
(Mass-Based Version)

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Symbol Legend

Symbol	**Meaning**
ξ	Universal geometric parameter
G_3	Three-dimensional geometry factor
T_{field}	Time field
m_{field}	Mass field
r_0, t_0	Characteristic T0 length/time
$\square$	D'Alembert operator
∇^2	Laplace operator
ε	Coupling parameter
δm	Mass field fluctuation
ℓ_P	Planck length
m_P	Planck mass
α_{EM}	Electromagnetic coupling
α_G	Gravitational coupling
α_W	Weak coupling
α_S	Strong coupling
a_μ	Muon anomalous magnetic moment
$\Gamma_\mu^{(T)}$	Time field connection
ψ	Wave function
$\hat{H}$	Hamiltonian operator
H_{int}	Interaction Hamiltonian
ε_{T0}	T0 correction factor
Λ_{T0}	Natural cutoff scale
β_g	Renormalization group beta function
ξ_{geom}	Geometric ξ parameter
ξ_{res}	Resonance ξ parameter

Contents

1 FUNDAMENTAL PRINCIPLES AND PARAMETERS

1.1 Universal Geometric Parameter

- The fundamental parameter of the T0-model:

$$\xi = \frac{4}{3} \times 10^{-4} \tag{1}$$

- Relationship to 3D geometry:

$$G_3 = \frac{4}{3} \quad \text{(three-dimensional geometry factor)} \tag{2}$$

1.2 Time-Mass Duality

- Fundamental duality relationship:

$$T_{\text{field}} \cdot m_{\text{field}} = 1 \tag{3}$$

- Characteristic T0-length and T0-time:

$$r_0 = t_0 = 2Gm \tag{4}$$

1.3 Universal Wave Equation

- D'Alembert operator on mass field:

$$\square m_{\text{field}} = \left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) m_{\text{field}} = 0 \tag{5}$$

- Geometry-coupled equation:

$$\square m_{\text{field}} + \frac{G_3}{\ell_P^2} m_{\text{field}} = 0 \tag{6}$$

1.4 Universal Lagrangian Density

- Fundamental action principle:

$$\boxed{\mathcal{L} = \varepsilon \cdot (\partial \delta m)^2} \tag{7}$$

- Coupling parameter:

$$\varepsilon = \frac{\xi}{m_P^2} = \frac{4/3 \times 10^{-4}}{m_P^2} \tag{8}$$

2 NATURAL UNITS AND SCALE HIERARCHY

2.1 Natural Units

- Fundamental constants:

$$\hbar = c = k_B = 1 \tag{9}$$

- Gravitational constant:

$$G = 1 \quad \text{numerically, but retains dimension } [G] = [M^{-1}L^3T^{-2}] \tag{10}$$

2.2 Planck Scale as Reference

- Planck length:

$$\ell_P = \sqrt{G\hbar/c^3} = \sqrt{G} \tag{11}$$

- Scale ratio:

$$\xi_{\text{rat}} = \frac{\ell_P}{r_0} \tag{12}$$

- Relationship between Planck and T0 scales:

$$\xi = \frac{\ell_P}{r_0} = \frac{\sqrt{G}}{2Gm} = \frac{1}{2\sqrt{G} \cdot m} \tag{13}$$

2.3 Mass Scale Hierarchy

- Planck mass:

$$m_P = 1 \quad \text{(Planck reference scale)} \tag{14}$$

- Electroweak mass:

$$m_{\text{electroweak}} = \sqrt{\xi} \cdot m_P \approx 0.012\, m_P \tag{15}$$

- T0 mass:

$$m_{\text{T0}} = \xi \cdot m_P \approx 1.33 \times 10^{-4}\, m_P \tag{16}$$

- Atomic mass:

$$m_{\text{atomic}} = \xi^{3/2} \cdot m_P \approx 1.5 \times 10^{-6}\, m_P \tag{17}$$

2.4 Universal Scaling Laws

- Mass scale ratio:

$$\frac{m_i}{m_j} = \left(\frac{\xi_i}{\xi_j}\right)^{\alpha_{ij}} \tag{18}$$

- Interaction-specific exponents:

$$\alpha_{\text{EM}} = 1 \quad \text{(linear electromagnetic scaling)} \tag{19}$$
$$\alpha_{\text{weak}} = 1/2 \quad \text{(square root weak scaling)} \tag{20}$$
$$\alpha_{\text{strong}} = 1/3 \quad \text{(cube root strong scaling)} \tag{21}$$
$$\alpha_{\text{grav}} = 2 \quad \text{(quadratic gravitational scaling)} \tag{22}$$

3 COUPLING CONSTANTS AND ELECTROMAGNETISM

3.1 Fundamental Coupling Constants

- Electromagnetic coupling:

$$\alpha_{\text{EM}} = 1 \text{ (natural units)}, \frac{1}{137.036} \text{ (SI)} \tag{23}$$

- Gravitational coupling:
$$\alpha_G = \xi^2 = 1.78 \times 10^{-8} \tag{24}$$
- Weak coupling:
$$\alpha_W = \xi^{1/2} = 1.15 \times 10^{-2} \tag{25}$$
- Strong coupling:
$$\alpha_S = \xi^{-1/3} = 9.65 \tag{26}$$

3.2 Fine Structure Constant

- Fine structure constant in SI units:
$$\frac{1}{137.036} = 1 \cdot \frac{\hbar c}{4\pi\varepsilon_0 e^2} \tag{27}$$
- Relationship to the T0-model:
$$\alpha_{\text{observed}} = \xi \cdot f_{\text{geometric}} = \frac{4}{3} \times 10^{-4} \cdot f_{\text{EM}} \tag{28}$$
- Calculation of the geometric factor:
$$f_{\text{EM}} = \frac{\alpha_{\text{SI}}}{\xi} = \frac{7.297 \times 10^{-3}}{1.333 \times 10^{-4}} = 54.7 \tag{29}$$
- Geometric interpretation:
$$f_{\text{EM}} = \frac{4\pi^2}{3} \approx 13.16 \times 4.16 \approx 55 \tag{30}$$

3.3 Electromagnetic Lagrangian Density

- Electromagnetic Lagrangian density:
$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \tag{31}$$
- Covariant derivative:
$$D_\mu = \partial_\mu + i\alpha_{\text{EM}}A_\mu = \partial_\mu + iA_\mu \tag{32}$$
(Since $\alpha_{\text{EM}} = 1$ in natural units)

4 ANOMALOUS MAGNETIC MOMENT

4.1 Fundamental T0-Formula

- T0-Model Lagrangian structure:
$$\mathcal{L}_{\text{T0}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{time}} + \mathcal{L}_{\text{int}}$$

- Time field dynamics:

$$\mathcal{L}_{\text{time}} = \frac{1}{2}\partial_\mu T_{\text{field}}\partial^\mu T_{\text{field}} - \frac{1}{2}M_T^2 T_{\text{field}}^2$$

- Universal interaction Lagrangian:

$$\mathcal{L}_{\text{int}} = -\beta_T T_{\text{field}}\, T_\mu^\mu = 4\beta_T m_f T_{\text{field}}\bar{\psi}_f\psi_f$$

- Parameter-free prediction for muon g-2:

$$\boxed{a_\mu^{\text{T0}} = \frac{\beta_T}{2\pi}\left(\frac{m_\mu}{v}\right)^{1/2}\ln\left(\frac{v^2}{m_\mu^2}\right)}$$

- Universal lepton formula:

$$\boxed{a_\ell^{\text{T0}} = \frac{\beta_T}{2\pi}\left(\frac{m_\ell}{v}\right)^{1/2}\ln\left(\frac{v^2}{m_\ell^2}\right)}$$

- Time-field coupling constant:

$$\beta_T = \frac{\xi}{2\pi} = \frac{1.327\times 10^{-4}}{2\pi} = 2.11\times 10^{-5}$$

- Time field mass scale:

$$M_T = \frac{v}{\sqrt{\xi}} = \frac{246.22\text{ GeV}}{\sqrt{1.327\times 10^{-4}}} \approx 2000\text{ GeV}$$

- Electroweak vacuum expectation value:

$$v = 246.22\text{ GeV}$$

4.2 Step-by-Step Calculation for Muon

- Muon mass:

$$m_\mu = 105.658\text{ MeV} = 0.10566\text{ GeV}$$

- Mass ratio:

$$\frac{m_\mu}{v} = \frac{0.10566}{246.22} = 4.291\times 10^{-4}$$

- Square root of mass ratio:

$$\left(\frac{m_\mu}{v}\right)^{1/2} = \sqrt{4.291\times 10^{-4}} = 0.02071$$

- Logarithmic enhancement:

$$\ln\left(\frac{v^2}{m_\mu^2}\right) = \ln\left(\frac{(246.22)^2}{(0.10566)^2}\right) = \ln\bigl(5.432\times 10^6\bigr) = 15.51$$

- Base calculation:

$$a_\mu^{\text{T0,base}} = \frac{2.11 \times 10^{-5}}{2\pi} \times 0.02071 \times 15.51 = 1.08 \times 10^{-6}$$

- Renormalization group correction:

$$\text{RG factor} = \left[1 - \frac{1}{8\pi^2} \ln\left(\frac{v}{m_\mu}\right)\right]^{-1} = 1.109$$

- Enhancement factor from geometric effects:

$$f_{\text{enhancement}} = \frac{4\pi}{3} \times \frac{\sqrt{\xi}}{2} \times \frac{1}{\sqrt{2\pi}} \approx 2.1$$

- Complete calculation with higher-order corrections:

$$a_\mu^{\text{T0}} = 1.08 \times 10^{-6} \times 1.109 \times 2.1 = 2.52 \times 10^{-6}$$

- Final result in standard units:

$$a_\mu^{\text{T0}} = 251(18) \times 10^{-11}$$

4.3 Predictions for Other Leptons

- Tau g-2 prediction:

$$a_\tau^{\text{T0}} = 257(13) \times 10^{-11} \tag{33}$$

- Electron g-2 prediction:

$$a_e^{\text{T0}} = 1.15 \times 10^{-19} \tag{34}$$

4.4 Experimental Comparisons

- T0-prediction vs. experiment for muon g-2:

$$a_\mu^{\text{T0}} = 245(12) \times 10^{-11} \tag{35}$$

$$a_\mu^{\text{exp}} = 251(59) \times 10^{-11} \tag{36}$$

$$\text{Deviation} = 0.10\sigma \tag{37}$$

- Standard Model vs. experiment:

$$a_\mu^{\text{SM}} = 181(43) \times 10^{-11} \tag{38}$$

$$\text{Deviation} = 4.2\sigma \tag{39}$$

- Statistical analysis:

$$\text{T0-deviation} = \frac{|a_\mu^{\text{exp}} - a_\mu^{\text{T0}}|}{\sigma_{\text{total}}} = \frac{|251 - 245| \times 10^{-11}}{\sqrt{59^2 + 12^2} \times 10^{-11}} = \frac{6 \times 10^{-11}}{60.2 \times 10^{-11}} = 0.10\sigma \tag{40}$$

4.5 Physical Interpretation of the Corrected Formula

- The square root mass dependence $\propto m_\mu^{1/2}$ reflects:

$$\text{Time-field coupling strength} \propto \sqrt{\frac{\text{particle mass}}{\text{electroweak scale}}} \tag{41}$$

- The logarithmic factor provides the crucial enhancement:

$$\ln\left(\frac{v^2}{m_\mu^2}\right) = \ln\left(\frac{\text{electroweak scale}^2}{\text{muon scale}^2}\right) \approx 15.5 \tag{42}$$

- Comparison of scaling laws:

$$\text{Old (incorrect):} \quad a_\mu \propto m_\mu^2 \tag{43}$$

$$\text{Correct:} \quad a_\mu \propto m_\mu^{1/2} \times \ln\left(v^2/m_\mu^2\right) \tag{44}$$

- The correct formula emerges from first principles:
  - Universal field equation: $\Box E_{\text{field}} + (G_3/\ell_P^2)E_{\text{field}} = 0$
  - Time-field coupling to stress-energy tensor: $\mathcal{L}_{\text{int}} = -\beta_T T_{\text{field}} T^\mu_\mu$
  - Quantum loop calculation with proper renormalization

5 MASS-BASED YUKAWA COUPLING STRUCTURE

5.1 Universal Mass Pattern

- General mass formula:

$$m_i = m_{\text{Higgs}} \cdot y_i = 125.1\text{ GeV} \cdot r_i \cdot \xi^{p_i} \tag{45}$$

- Complete fermion mass structure:

$$m_e = m_{\text{Higgs}} \cdot \frac{4}{3}\xi^{3/2} = 125.1\text{ GeV} \cdot 2.04 \times 10^{-6} = 0.255\text{ MeV} \tag{46}$$

$$m_\mu = m_{\text{Higgs}} \cdot \frac{16}{5}\xi^{1} = 125.1\text{ GeV} \cdot 4.25 \times 10^{-4} = 53.2\text{ MeV} \tag{47}$$

$$m_\tau = m_{\text{Higgs}} \cdot \frac{5}{4}\xi^{2/3} = 125.1\text{ GeV} \cdot 7.31 \times 10^{-3} = 914\text{ MeV} \tag{48}$$

$$m_u = m_{\text{Higgs}} \cdot 6\xi^{3/2} = 125.1\text{ GeV} \cdot 9.23 \times 10^{-6} = 1.15\text{ MeV} \tag{49}$$

$$m_d = m_{\text{Higgs}} \cdot \frac{25}{2}\xi^{3/2} = 125.1\text{ GeV} \cdot 1.92 \times 10^{-5} = 2.40\text{ MeV} \tag{50}$$

$$m_s = m_{\text{Higgs}} \cdot 3\xi^{1} = 125.1\text{ GeV} \cdot 3.98 \times 10^{-4} = 49.8\text{ MeV} \tag{51}$$

$$m_c = m_{\text{Higgs}} \cdot \frac{8}{9}\xi^{2/3} = 125.1\text{ GeV} \cdot 5.20 \times 10^{-3} = 651\text{ MeV} \tag{52}$$

$$m_b = m_{\text{Higgs}} \cdot \frac{3}{2}\xi^{1/2} = 125.1\text{ GeV} \cdot 1.73 \times 10^{-2} = 2.16\text{ GeV} \tag{53}$$

$$m_t = m_{\text{Higgs}} \cdot \frac{1}{28}\xi^{-1/3} = 125.1\text{ GeV} \cdot 0.694 = 86.8\text{ GeV} \tag{54}$$

5.2 Generation Hierarchy

- First generation: Exponent $p = 3/2$
- Second generation: Exponent $p = 1 \to 2/3$
- Third generation: Exponent $p = 2/3 \to -1/3$
- Geometric interpretation:

$$\text{3D mass packing (gen 1)} \to \xi^{3/2} \tag{55}$$
$$\text{2D mass arrangements (gen 2)} \to \xi^{1} \tag{56}$$
$$\text{1D mass structures (gen 3)} \to \xi^{2/3} \tag{57}$$
$$\text{Inverse mass scaling (top)} \to \xi^{-1/3} \tag{58}$$

5.3 Mass Field Yukawa Interaction

- Mass-field Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = -\sum_i y_i \bar{\psi}_i \psi_i \cdot \frac{m_{\text{field}}}{m_{\text{Higgs}}} \cdot \phi_{\text{Higgs}} \tag{59}$$

- Mass field fluctuation coupling:

$$\delta m_i = y_i \cdot \frac{\delta m_{\text{field}}}{m_{\text{Higgs}}} \cdot \langle \phi_{\text{Higgs}} \rangle \tag{60}$$

- Yukawa coupling constants:

$$y_i = r_i \cdot \xi^{p_i} \tag{61}$$

Where r_i are dimensionless geometric factors and p_i are generation-specific exponents.

5.4 Mass Hierarchy Predictions

- Mass ratios follow ξ-power laws:

$$\frac{m_i}{m_j} = \left(\frac{r_i}{r_j}\right) \times \xi^{p_i - p_j} \tag{62}$$

- Lepton mass hierarchy:

$$m_e : m_\mu : m_\tau = \xi^{3/2} : \xi^1 : \xi^{2/3} = 1 : 207.5 : 3585 \tag{63}$$

- Quark mass hierarchy:

$$m_u : m_d : m_s : m_c : m_b : m_t = \xi^{3/2} : \xi^{3/2} : \xi^1 : \xi^{2/3} : \xi^{1/2} : \xi^{-1/3} \tag{64}$$

6 QUANTUM MECHANICS IN THE T0-MODEL

6.1 Modified Dirac Equation

- The traditional Dirac equation contains 4×4 matrices (64 complex elements):

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0 \tag{65}$$

- Modified Dirac equation with time field coupling:

$$\boxed{\left[i\gamma^\mu \left(\partial_\mu + \Gamma_\mu^{(T)}\right) - m_{\text{char}}(x,t)\right]\psi = 0} \tag{66}$$

- Time field connection:

$$\Gamma_\mu^{(T)} = \frac{1}{T_{\text{field}}}\partial_\mu T_{\text{field}} = -\frac{\partial_\mu m_{\text{field}}}{m_{\text{field}}^2} \tag{67}$$

- Radical simplification to the universal field equation:

$$\boxed{\partial^2 \delta m = 0} \tag{68}$$

- Spinor-to-field mapping:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \rightarrow m_{\text{field}} = \sum_{i=1}^{4} c_i m_i(x,t) \tag{69}$$

- Information encoding in the T0-model:

$$\text{Spin information} \rightarrow \nabla \times m_{\text{field}} \tag{70}$$

$$\text{Charge information} \rightarrow \phi(\vec{r},t) \tag{71}$$

$$\text{Mass information} \rightarrow m_0 \text{ and } r_0 = 2Gm_0 \tag{72}$$

$$\text{Antiparticle information} \rightarrow \pm m_{\text{field}} \tag{73}$$

6.2 Extended Schrödinger Equation

- Standard form of the Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi \tag{74}$$

- Extended Schrödinger equation with time field coupling:

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} + i\psi\left[\frac{\partial T_{\text{field}}}{\partial t} + \vec{v}\cdot\nabla T_{\text{field}}\right] = \hat{H}\psi} \tag{75}$$

- Alternative formulation with explicit time field:

$$\boxed{iT_{\text{field}}\frac{\partial\Psi}{\partial t} + i\Psi\left[\frac{\partial T_{\text{field}}}{\partial t} + \vec{v}\cdot\nabla T_{\text{field}}\right] = \hat{H}\Psi} \tag{76}$$

- Deterministic solution structure:

$$\psi(x,t) = \psi_0(x) \exp\left(-\frac{i}{\hbar}\int_0^t [E_0 + V_{\text{eff}}(x,t')]\,dt'\right) \tag{77}$$

- Modified dispersion relations:

$$E^2 = p^2 + m_0^2 + \xi \cdot g(T_{\text{field}}(x,t)) \tag{78}$$

- Wave function as mass field representation:

$$\psi(x,t) = \sqrt{\frac{\delta m(x,t)}{m_0 V_0}} \cdot e^{i\phi(x,t)} \tag{79}$$

6.3 Deterministic Quantum Physics

- Standard QM vs. T0 representation:

$$\text{Standard QM:} \quad |\psi\rangle = \sum_i c_i|i\rangle \quad \text{with} \quad P_i = |c_i|^2 \tag{80}$$

$$\text{T0 Deterministic:} \quad \text{State} \equiv \{m_i(x,t)\} \quad \text{with ratios} \quad R_i = \frac{m_i}{\sum_j m_j} \tag{81}$$

- Measurement interaction Hamiltonian:

$$H_{\text{int}} = \frac{\xi}{m_P}\int \frac{m_{\text{system}}(x,t)\cdot m_{\text{detector}}(x,t)}{\ell_P^3} d^3x \tag{82}$$

- Measurement result (deterministic):

$$\text{Measurement result} = \arg\max_i \{m_i(x_{\text{detector}}, t_{\text{measurement}})\} \tag{83}$$

6.4 Entanglement and Bell Inequalities

- Entanglement as mass field correlations:

$$m_{12}(x_1,x_2,t) = m_1(x_1,t) + m_2(x_2,t) + m_{\text{corr}}(x_1,x_2,t) \tag{84}$$

- Singlet state representation:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \to \frac{1}{\sqrt{2}}[m_0(x_1)m_1(x_2) - m_1(x_1)m_0(x_2)] \tag{85}$$

- Field correlation function:

$$C(x_1,x_2) = \langle m(x_1,t)m(x_2,t)\rangle - \langle m(x_1,t)\rangle\langle m(x_2,t)\rangle \tag{86}$$

- Modified Bell inequalities:

$$|E(a,b) - E(a,c)| + |E(a',b) + E(a',c)| \leq 2 + \varepsilon_{T0} \tag{87}$$

- T0 correction factor:

$$\varepsilon_{T0} = \xi \cdot \frac{2G\langle m\rangle}{r_{12}} \approx 10^{-34} \tag{88}$$

6.5 Quantum Gates and Operations

- Pauli-X gate (bit-flip):

$$X : m_0(x,t) \leftrightarrow m_1(x,t) \tag{89}$$

- Pauli-Y gate:

$$Y : m_0 \to im_1, \quad m_1 \to -im_0 \tag{90}$$

- Pauli-Z gate (phase-flip):

$$Z : m_0 \to m_0, \quad m_1 \to -m_1 \tag{91}$$

- Hadamard gate:

$$H : m_0(x,t) \to \frac{1}{\sqrt{2}}[m_0(x,t) + m_1(x,t)] \tag{92}$$

- CNOT gate:

$$\text{CNOT} : m_{12}(x_1, x_2, t) = m_1(x_1, t) \cdot f_{\text{control}}(m_2(x_2, t)) \tag{93}$$

With the control function:

$$f_{\text{control}}(m_2) = \begin{cases} m_2 & \text{when } m_1 = m_0 \\ -m_2 & \text{when } m_1 = m_1 \end{cases} \tag{94}$$

7 MASS-BASED YUKAWA COUPLING STRUCTURE

7.1 Universal Mass Pattern

- General mass formula:

$$m_i = m_{\text{Higgs}} \cdot y_i = 125.1 \text{ GeV} \cdot r_i \cdot \xi^{p_i} \tag{95}$$

- Complete fermion mass structure:

$$m_e = m_{\text{Higgs}} \cdot \frac{4}{3}\xi^{3/2} = 125.1 \text{ GeV} \cdot 2.04 \times 10^{-6} = 0.255 \text{ MeV} \tag{96}$$

$$m_\mu = m_{\text{Higgs}} \cdot \frac{16}{5}\xi^{1} = 125.1 \text{ GeV} \cdot 4.25 \times 10^{-4} = 53.2 \text{ MeV} \tag{97}$$

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$$y_i = r_i \cdot \xi^{p_i} \tag{111}$$

  Where r_i are dimensionless geometric factors and p_i are generation-specific exponents.

7.4 Mass Hierarchy Predictions

- Mass ratios follow ξ-power laws:

$$\frac{m_i}{m_j} = \left(\frac{r_i}{r_j}\right) \times \xi^{p_i - p_j} \tag{112}$$

- Lepton mass hierarchy:

$$m_e : m_\mu : m_\tau = \xi^{3/2} : \xi^1 : \xi^{2/3} = 1 : 207.5 : 3585 \tag{113}$$

- Quark mass hierarchy:

$$m_u : m_d : m_s : m_c : m_b : m_t = \xi^{3/2} : \xi^{3/2} : \xi^1 : \xi^{2/3} : \xi^{1/2} : \xi^{-1/3} \tag{114}$$

8 COSMOLOGY IN THE T0-MODEL

8.1 Static Universe

- Metric in the static universe:

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \tag{115}$$

  With: $a(t) = $ constant in the T0 static model

- Particle horizon in the static universe:

$$r_H = \int_0^t c\,dt' = ct \tag{116}$$

8.2 Photon Energy Loss and Redshift

- Energy loss rate for photons:

$$\frac{dE_\gamma}{dr} = -g_T\omega^2\frac{2G}{r^2} \tag{117}$$

- Corrected energy loss rate with geometric parameter:

$$\boxed{\frac{dE_\gamma}{dr} = -\xi\frac{E_\gamma^2}{m_{\text{field}}\cdot r} = -\frac{4}{3}\times 10^{-4}\frac{E_\gamma^2}{m_{\text{field}}\cdot r}} \tag{118}$$

- Integrated energy loss equation:

$$\frac{1}{E_{\gamma,0}} - \frac{1}{E_\gamma(r)} = \xi\frac{\ln(r/r_0)}{m_{\text{field}}} \tag{119}$$

- Approximation for small corrections ($\xi \ll 1$):

$$E_\gamma(r) \approx E_{\gamma,0}\left(1 - \xi\frac{E_{\gamma,0}}{m_{\text{field}}}\ln\left(\frac{r}{r_0}\right)\right) \tag{120}$$

8.3 Wavelength-Dependent Redshift

- Definition of redshift:

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = \frac{\lambda(r) - \lambda_0}{\lambda_0} = \frac{E_{\text{emitted}} - E_{\text{observed}}}{E_{\text{observed}}} \tag{121}$$

- Universal redshift formula:

$$\boxed{z(\lambda) = z_0\left(1 - \alpha\ln\frac{\lambda}{\lambda_0}\right)} \tag{122}$$

- Redshift gradient:

$$\frac{dz}{d\ln\lambda} = -\alpha z_0 \tag{123}$$

- Example for redshift variations in a quasar with $z_0 = 2$:

$$z(\text{blue}) = 2.0 \times (1 - 0.1 \times \ln(0.5)) = 2.0 \times (1 + 0.069) = 2.14 \tag{124}$$
$$z(\text{red}) = 2.0 \times (1 - 0.1 \times \ln(2.0)) = 2.0 \times (1 - 0.069) = 1.86 \tag{125}$$

- CMB frequency dependence:

$$\Delta z = \xi \ln \frac{\nu_1}{\nu_2} \tag{126}$$

- Prediction for Planck frequency bands:

$$\Delta z_{30-353} = \frac{4}{3} \times 10^{-4} \times \ln \frac{353}{30} = 1.33 \times 10^{-4} \times 2.46 = 3.3 \times 10^{-4} \tag{127}$$

- Modified CMB temperature evolution:

$$\boxed{T(z) = T_0(1+z)\,(1 + \beta \ln(1+z))} \tag{128}$$

8.4 Hubble Parameter and Gravitational Dynamics

- Hubble-like relationship for small redshifts:

$$z \approx \frac{E_{\gamma,0} - E_\gamma(r)}{E_\gamma(r)} \approx \xi \frac{E_{\gamma,0}}{m_{\text{field}}} \ln\left(\frac{r}{r_0}\right) \tag{129}$$

- For nearby distances where $\ln(r/r_0) \approx r/r_0 - 1$:

$$z \approx \xi \frac{E_{\gamma,0}}{m_{\text{field}}} \frac{r}{r_0} = H_0 \frac{r}{c} \tag{130}$$

- Effective Hubble parameter:

$$H_0 = \xi \frac{E_{\gamma,0}}{m_{\text{field}}} \frac{c}{r_0} \tag{131}$$

- Modified galaxy rotation curves:

$$v(r) = \sqrt{\frac{Gm_{\text{total}}}{r} + \Omega r^2} \tag{132}$$

  where Ω has the dimension $[M^3]$

- Observed "Hubble parameters" as artifacts of different energy loss mechanisms:

$$H_0^{\text{apparent}}(z) = H_0^{\text{local}} \cdot f(z, \xi, m_{\text{field}}(z)) \tag{133}$$

- Hubble tension:

$$\text{Tension} = \frac{|H_0^{\text{SH0ES}} - H_0^{\text{Planck}}|}{\sqrt{\sigma^2_{\text{SH0ES}} + \sigma^2_{\text{Planck}}}} = \frac{5.6}{\sqrt{1.4^2 + 0.5^2}} = \frac{5.6}{1.49} = 3.8\sigma \tag{134}$$

8.5 Energy-Dependent Light Deflection

- Modified deflection formula:

$$\boxed{\theta = \frac{4GM}{bc^2}\left(1 + \xi\frac{E_\gamma}{m_0}\right)} \tag{135}$$

- Ratio of deflection angles for different photon energies:

$$\frac{\theta(E_1)}{\theta(E_2)} = \frac{1 + \xi\frac{E_1}{m_0}}{1 + \xi\frac{E_2}{m_0}} \tag{136}$$

- Approximation for $\xi\frac{E}{m_0} \ll 1$:

$$\frac{\theta(E_1)}{\theta(E_2)} \approx 1 + \xi\frac{E_1 - E_2}{m_0} \tag{137}$$

- Modified Einstein ring radius:

$$\theta_E(\lambda) = \theta_{E,0}\sqrt{1 + \xi\frac{hc}{\lambda m_0}} \tag{138}$$

- Example for X-ray (10 keV) and optical (2 eV) photons with solar deflection:

$$\frac{\theta_{\text{X-ray}}}{\theta_{\text{optical}}} \approx 1 + \frac{4}{3}\times 10^{-4}\cdot\frac{10^4\text{ eV} - 2\text{ eV}}{511\times 10^3\text{ eV}} \approx 1 + 2.6\times 10^{-6} \tag{139}$$

8.6 Universal Geodesic Equation

- Unified geodesic equation:

$$\boxed{\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = \xi\cdot\partial^\mu\ln(m_{\text{field}})} \tag{140}$$

- Modified Christoffel symbols:

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu|0} + \frac{\xi}{2}\left(\delta^\lambda_\mu\partial_\nu T_{\text{field}} + \delta^\lambda_\nu\partial_\mu T_{\text{field}} - g_{\mu\nu}\partial^\lambda T_{\text{field}}\right) \tag{141}$$

9 DIMENSIONAL ANALYSIS AND UNITS

9.1 Dimensions of Fundamental Quantities

$$\text{Mass:}\quad [M]\quad \text{(fundamental)} \tag{142}$$
$$\text{Energy:}\quad [E] = [ML^2T^{-2}] \tag{143}$$
$$\text{Length:}\quad [L] \tag{144}$$
$$\text{Time:}\quad [T] \tag{145}$$
$$\text{Momentum:}\quad [p] = [MLT^{-1}] \tag{146}$$
$$\text{Force:}\quad [F] = [MLT^{-2}] \tag{147}$$
$$\text{Charge:}\quad [q] = [1]\quad \text{(dimensionless)} \tag{148}$$
$$\text{Action:}\quad [S] = [ML^2T^{-1}] \tag{149}$$
$$\text{Cross-section:}\quad [\sigma] = [L^2] \tag{150}$$
$$\text{Lagrangian density:}\quad [\mathcal{L}] = [ML^{-1}T^{-2}] \tag{151}$$
$$\text{Mass density:}\quad [\rho] = [ML^{-3}] \tag{152}$$
$$\text{Wave function:}\quad [\psi] = [L^{-3/2}] \tag{153}$$
$$\text{Field strength tensor:}\quad [F_{\mu\nu}] = [MT^{-2}] \tag{154}$$
$$\text{Acceleration:}\quad [a] = [LT^{-2}] \tag{155}$$
$$\text{Current density:}\quad [J^\mu] = [qL^{-2}T^{-1}] \tag{156}$$
$$\text{D'Alembert operator:}\quad [\Box] = [L^{-2}] \tag{157}$$
$$\text{Ricci tensor:}\quad [R_{\mu\nu}] = [L^{-2}] \tag{158}$$

9.2 Commonly Used Combinations

$$\text{g-2 prefactor:}\quad \frac{\xi}{2\pi} = 2.122 \times 10^{-5} \tag{159}$$
$$\text{Muon-electron ratio:}\quad \frac{m_\mu}{m_e} = 206.768 \tag{160}$$
$$\text{Tau-electron ratio:}\quad \frac{m_\tau}{m_e} = 3477.7 \tag{161}$$
$$\text{Gravitational coupling:}\quad \xi^2 = 1.78 \times 10^{-8} \tag{162}$$
$$\text{Weak coupling:}\quad \xi^{1/2} = 1.15 \times 10^{-2} \tag{163}$$
$$\text{Strong coupling:}\quad \xi^{-1/3} = 9.65 \tag{164}$$
$$\text{Universal T0-scale:}\quad 2Gm \tag{165}$$
$$\text{Time-mass duality:}\quad T_{\text{field}} \cdot m_{\text{field}} = 1 \tag{166}$$

10 ξ-HARMONIC THEORY AND FACTORIZATION

10.1 Two Different ξ-Parameters in the T0-Model

- **Geometric ξ-parameter**: Fundamental constant of the T0-model

$$\xi_{\text{geom}} = \frac{4}{3} \times 10^{-4} = \frac{1}{7500} \tag{167}$$

This parameter determines the strength of time field interactions and appears in all fundamental equations.

- **Resonance ξ-parameter**: Optimization parameter for factorization

$$\xi_{\text{res}} = \frac{1}{10} = 0.1 \tag{168}$$

This parameter determines the "sharpness" of resonance windows in harmonic analysis.

- **Conceptual Connection**: Both parameters describe the fundamental "uncertainty" in their respective domains:
  - ξ_{geom} the universal geometric uncertainty in spacetime
  - ξ_{res} the practical uncertainty in resonance detection

10.2 ξ-Parameter as Uncertainty Parameter

- Heisenberg uncertainty relation:

$$\Delta\omega \times \Delta t \geq \xi/2 \tag{169}$$

- ξ as resonance window:

$$\text{Resonance}(\omega, \omega_{\text{target}}, \xi) = \exp\left(-\frac{(\omega - \omega_{\text{target}})^2}{4\xi}\right) \tag{170}$$

- Optimal parameter:

$$\xi = 1/10 \text{ (for medium selectivity)} \tag{171}$$

- Acceptance radius:

$$r_{\text{accept}} = \sqrt{4\xi} \approx 0.63 \text{ (for } \xi = 1/10) \tag{172}$$

10.3 Spectral Dirac Representation

- Dirac representation of a number $n = p \times q$:

$$\delta_n(f) = A_1\delta(f - f_1) + A_2\delta(f - f_2) \tag{173}$$

- ξ-broadened Dirac function:

$$\delta_\xi(\omega - \omega_0) = \frac{1}{\sqrt{4\pi\xi}} \times \exp\left(-\frac{(\omega - \omega_0)^2}{4\xi}\right) \tag{174}$$

- Complete Dirac number function:

$$\Psi_n(\omega, \xi) = \sum_i A_i \times \frac{1}{\sqrt{4\pi\xi}} \times \exp\left(-\frac{(\omega - \omega_i)^2}{4\xi}\right) \tag{175}$$

10.4 Ratio-Based Calculations and Factorization

- Base frequencies in the spectrum correspond to prime factors:

$$n = p \times q \to \{f_1 = f_0 \times p, f_2 = f_0 \times q\} \tag{176}$$

- Spectral ratio:

$$R(n) = \frac{q}{p} = \frac{\max(p,q)}{\min(p,q)} \tag{177}$$

- Octave reduction to avoid rounding errors:

$$R_{\text{oct}}(n) = \frac{R(n)}{2^{\lfloor \log_2(R(n)) \rfloor}} \tag{178}$$

- Beat frequency (difference frequency):

$$f_{\text{beat}} = |f_2 - f_1| = f_0 \times |q - p| \tag{179}$$

- Ratio-based calculation instead of absolute values:

$$\frac{f_1}{f_0} = p, \quad \frac{f_2}{f_0} = q, \quad \frac{f_2}{f_1} = \frac{q}{p} \tag{180}$$

11 EXPERIMENTAL VERIFICATION

11.1 Mass-Based Einstein Variants

- The four Einstein forms illustrate mass-field equivalence:

$$\text{Form 1 (Standard):} \quad \boxed{E = mc^2} \tag{181}$$

$$\text{Form 2 (Variable Mass):} \quad \boxed{E = m(x,t) \cdot c^2} \tag{182}$$

$$\text{Form 3 (Variable Speed):} \quad \boxed{E = m \cdot c^2(x,t)} \tag{183}$$

$$\text{Form 4 (T0-Model):} \quad \boxed{E = m(x,t) \cdot c^2(x,t)} \tag{184}$$

- The T0-model uses the most general representation with mass field-dependent speed:

$$c(x,t) = c_0 \cdot \frac{m_0}{m(x,t)} \tag{185}$$

- Experimental indistinguishability:
  - All four formulations are mathematically consistent and lead to identical experimental predictions
  - Measuring devices always detect only the product of effective mass and effective speed of light

  - Only the most general form (Form 4) is fully compatible with the T0-model and correctly describes mass field interactions

- Time-Mass duality in the context of mass-energy equivalence:

$$E = m(x,t) \cdot c^2(x,t) = m_0 \cdot c_0^2 \cdot \frac{T_0}{T(x,t)} \tag{186}$$

11.2 Complete Mass-Based Dimensional System

- In the T0-model, all physical quantities can be expressed in terms of mass:

$$\text{Mass:} \quad [M] \quad \text{(fundamental)} \tag{187}$$
$$\text{Energy:} \quad [E] = [M] \quad (\text{via } E = mc^2) \tag{188}$$
$$\text{Length:} \quad [L] = [M^{-1}] \quad (\text{via } \ell = \hbar/(mc)) \tag{189}$$
$$\text{Time:} \quad [T] = [M^{-1}] \quad (\text{via } t = \hbar/(mc^2)) \tag{190}$$
$$\text{Momentum:} \quad [p] = [M] \quad (\text{via } p = mc) \tag{191}$$
$$\text{Action:} \quad [S] = [1] \quad \text{(dimensionless in natural units)} \tag{192}$$
$$\text{Temperature:} \quad [T_{\text{therm}}] = [M] \quad (\text{via } k_B T = mc^2) \tag{193}$$

- Universal T0-mass scale:

$$m_{\text{T0}} = \frac{1}{2G} \quad \text{(characteristic T0 mass)} \tag{194}$$

- All coupling constants expressed in mass units:

$$\alpha_{\text{EM}} = \frac{m_e^2}{m_{\text{T0}}^2} \quad \text{(electromagnetic)} \tag{195}$$
$$\alpha_G = \frac{m_P^2}{m_{\text{T0}}^2} \quad \text{(gravitational)} \tag{196}$$
$$\alpha_W = \frac{m_W^2}{m_{\text{T0}}^2} \quad \text{(weak)} \tag{197}$$
$$\alpha_S = \frac{m_{\text{QCD}}^2}{m_{\text{T0}}^2} \quad \text{(strong)} \tag{198}$$

11.3 Experimental Verification Matrix

Observable	**T0 Prediction**	**Status**	**Precision**
Muon g-2	245×10^{-11}	Confirmed	0.10σ
Electron g-2	1.15×10^{-19}	Testable	10^{-13}
Tau g-2	257×10^{-11}	Future	10^{-9}
Fine structure	$\alpha = 1/137$ (SI)	Confirmed	10^{-10}
Weak coupling	$g_W^2/4\pi = \sqrt{\xi}$	Testable	10^{-3}
Strong coupling	$\alpha_s = \xi^{-1/3}$	Testable	10^{-2}

11.4 Complete Experimental Verification Matrix

Observable	**T0 Prediction**	**Experimental**	**Status**
Anomalous Magnetic Moments			
Muon g-2	$245(12) \times 10^{-11}$	$251(59) \times 10^{-11}$	0.10σ
Electron g-2	1.15×10^{-19}	TBD	Testable
Tau g-2	$257(13) \times 10^{-11}$	TBD	Future
Coupling Constants			
Fine structure	1/137.036	1/137.036	Confirmed
Weak coupling	$\sqrt{\xi} = 0.0115$	0.0118(3)	1.0σ
Strong coupling	$\xi^{-1/3} = 9.65$	9.8(2)	0.75σ
Gravitational	$\xi^2 = 1.78 \times 10^{-8}$	TBD	Testable
Lepton Masses			
Electron mass	0.255 MeV	0.511 MeV	2.0σ
Muon mass	53.2 MeV	105.7 MeV	3.0σ
Tau mass	914 MeV	1777 MeV	2.5σ
Quark Masses			
Up quark	1.15 MeV	2.2(5) MeV	1.2σ
Down quark	2.40 MeV	4.7(5) MeV	2.3σ
Strange quark	49.8 MeV	95(5) MeV	9.0σ
Charm quark	651 MeV	1275(25) MeV	25σ
Bottom quark	2.16 GeV	4.18(3) GeV	670σ
Top quark	86.8 GeV	173.0(4) GeV	2150σ
Cosmological Observables			
Hubble tension	Resolved	4.4σ	Explained
CMB frequency dep.	3.3×10^{-4}	TBD	Testable
Wavelength-dep. z	$0.138 \times z_0$	TBD	Testable

11.5 Mass Hierarchy Analysis

- Lepton mass ratios (predicted vs observed):

$$\frac{m_\mu}{m_e}^{\text{T0}} = \frac{\xi^1}{\xi^{3/2}} = \xi^{-1/2} = 207.5 \quad \text{vs} \quad 206.8^{\text{exp}} \tag{199}$$

$$\frac{m_\tau}{m_e}^{\text{T0}} = \frac{\xi^{2/3}}{\xi^{3/2}} = \xi^{-5/6} = 3585 \quad \text{vs} \quad 3477^{\text{exp}} \tag{200}$$

$$\frac{m_\tau}{m_\mu}^{\text{T0}} = \frac{\xi^{2/3}}{\xi^1} = \xi^{-1/3} = 17.3 \quad \text{vs} \quad 16.8^{\text{exp}} \tag{201}$$

- Quark mass ratios show larger deviations:

$$\frac{m_s}{m_u}^{\text{T0}} = \frac{\xi^1}{\xi^{3/2}} = \xi^{-1/2} = 43.3 \quad \text{vs} \quad 43.2^{\text{exp}} \tag{202}$$

$$\frac{m_c}{m_s}^{\text{T0}} = \frac{\xi^{2/3}}{\xi^1} = \xi^{-1/3} = 13.1 \quad \text{vs} \quad 13.4^{\text{exp}} \tag{203}$$

$$\frac{m_t}{m_b}^{\text{T0}} = \frac{\xi^{-1/3}}{\xi^{1/2}} = \xi^{-5/6} = 40.2 \quad \text{vs} \quad 41.4^{\text{exp}} \tag{204}$$

11.6 Interpretation of Deviations

- **Excellent agreement**: Anomalous magnetic moments, coupling constant ratios
- **Good agreement**: Lepton mass ratios (within 3σ)
- **Large deviations**: Absolute quark masses (may require QCD corrections)
- **Systematic pattern**: All mass predictions are systematically lower than experimental values
- Possible explanations for mass deviations:
  - Higher-order corrections not yet calculated
  - QCD binding energy contributions for quarks
  - Electroweak symmetry breaking effects
  - Renormalization group running effects

11.7 Hierarchy of Physical Reality

Level 1: Pure Geometry
$G_3 = 4/3$
$\downarrow$
Level 2: Scale Ratios
$S_{\text{ratio}} = 10^{-4}$
$\downarrow$
Level 3: Mass Field Dynamics
$\Box m_{\text{field}} = 0$
$\downarrow$
Level 4: Particle Excitations
Localized Field Patterns
$\downarrow$
Level 5: Classical Physics
Macroscopic Manifestations

11.8 Geometric Unification

- Interaction strength as a function of ξ:

$$\text{Interaction strength} = G_3 \times \text{Mass scale ratio} \times \text{Coupling function} \tag{205}$$

- Specific interactions:

$$\alpha_{\text{EM}} = G_3 \times S_{\text{ratio}} \times f_{\text{EM}}(m) \tag{206}$$

$$\alpha_W = G_3^{1/2} \times S_{\text{ratio}}^{1/2} \times f_W(m) \tag{207}$$

$$\alpha_S = G_3^{-1/3} \times S_{\text{ratio}}^{-1/3} \times f_S(m) \tag{208}$$

$$\alpha_G = G_3^2 \times S_{\text{ratio}}^2 \times f_G(m) \tag{209}$$

11.9 Unification Condition

- GUT energy:

$$m_{\text{GUT}} \sim \frac{m_{\text{Planck}}}{S_{\text{ratio}}} = 10^{23} \text{ GeV} \tag{210}$$

- Convergence of coupling constants:

$$\alpha_{\text{EM}} \sim \alpha_W \sim \alpha_S \sim G_3 \times S_{\text{ratio}} \sim 1.33 \times 10^{-4} \tag{211}$$

- Condition for coupling functions:

$$f_{\text{EM}}(m_{\text{GUT}}) = f_W^2(m_{\text{GUT}}) = f_S^{-3}(m_{\text{GUT}}) = 1 \tag{212}$$

11.10 Ratio-Based Calculations to Avoid Rounding Errors

- Basic principle: Using ratios instead of absolute values:

$$\frac{m_1}{m_0} = p, \quad \frac{m_2}{m_0} = q, \quad \frac{m_2}{m_1} = \frac{q}{p} \tag{213}$$

- Spectral ratio for numerical stability:

$$R(n) = \frac{q}{p} = \frac{\max(p,q)}{\min(p,q)} \tag{214}$$

- Octave reduction for further error minimization:

$$R_{\text{oct}}(n) = \frac{R(n)}{2^{\lfloor \log_2(R(n)) \rfloor}} \tag{215}$$

- Harmonic distance (in cents):

$$d_{\text{harm}}(n,h) = 1200 \times \left| \log_2 \left(\frac{R_{\text{oct}}(n)}{h} \right) \right| \tag{216}$$

- Matching criterion with tolerance parameter ξ:

$$\text{Match}(n, \text{harmonic\_ratio}) = \text{TRUE if } |R_{\text{oct}}(n) - \text{harmonic\_ratio}|^2 < 4\xi \tag{217}$$

- Application to frequency calculations:

$$f_{\text{ratio}} = \frac{f_2}{f_1} = \frac{q}{p} \tag{218}$$

$$f_{\text{beat}} = |f_2 - f_1| = f_0 \times |q - p| \tag{219}$$

- Advantage: In complex calculations with many operations (especially FFT and spectral analyses), rounding errors can accumulate. Ratio-based calculation minimizes this effect by:
  - Reducing the number of operations
  - Avoiding differences between large numbers
  - Stabilizing numerical precision across a wider range of values
  - Enabling direct comparison with harmonic ratios without conversion