

Fractal Renormalization of the Fine-Structure Constant in the T0-Theory

Verification of Calculations with Error Analysis

Based on the Derivation by Johann Pascher

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Abstract

This document verifies the calculations of the fine-structure constant $\alpha \approx 1/137.036$ in the T0-Theory, based on the geometric constant $\xi = \frac{4}{3} \times 10^{-4}$, the characteristic energy $E_0 = 7.398 \text{ MeV}$, and the fractal dimension $D_f = 2.94$. Three methods are analyzed: the elementary derivation, the direct geometric calculation (Path 1), and the fractal renormalization (Path 2). Each calculation is accompanied by a note on whether it is correct or contains errors, with a detailed analysis of the issues.

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1 Introduction

The T0-Theory derives the fine-structure constant $\alpha \approx 1/137.036$ from geometric principles. This document verifies the calculations and highlights errors in the formulas for Path 1 and Path 2. The elementary derivation is identified as the most robust method.

2 Fundamental Constants of the T0-Theory

The fundamental parameters are:

$$\xi = \frac{4}{3} \times 10^{-4} \approx 1.333 \times 10^{-4}, \quad (1)$$

$$E_0 = 7.398 \text{ MeV}, \quad (2)$$

$$D_f = 2.94, \quad D_f^{-1} = \frac{1}{2.94} \approx 0.340136. \quad (3)$$

3 Elementary Derivation: $\alpha = \xi \cdot \frac{E_0^2}{(1 \text{ MeV})^2}$

3.1 Calculation

The simplest derivation is:

$$\alpha = \xi \cdot \frac{E_0^2}{(1 \text{ MeV})^2}. \quad (4)$$

With $\xi = 1.333 \times 10^{-4}$, $E_0 = 7.398 \text{ MeV}$:

$$E_0^2 = (7.398)^2 \approx 54.7296 \text{ MeV}^2, \quad (5)$$

$$\frac{E_0^2}{(1 \text{ MeV})^2} = 54.7296, \quad (6)$$

$$\alpha = 1.333 \times 10^{-4} \times 54.7296 \approx 0.007297, \quad (7)$$

$$\alpha^{-1} \approx \frac{1}{0.007297} \approx 137.0. \quad (8)$$

3.2 Error Analysis

Correctness

The calculation is **correct** and yields $\alpha^{-1} \approx 137.0$, which deviates by only 0.026% from the experimental value $\alpha^{-1} \approx 137.036$. The formula is dimensionally consistent and uses only two measurable parameters (ξ , E_0). The error of simplifying to $\alpha \propto \xi^{11/2}$ is avoided, as E_0 is an independent parameter.

4 Path 1: Direct Geometric Calculation

4.1 Calculation

The formula is:

$$\alpha^{-1} = 3\pi \times \frac{3}{4} \times 10^4 \times \ln(10^4) \times D_f^{-1} = 137.036, \quad (9)$$

with $\ln(10^4) \approx 9.210$, $D_f^{-1} \approx 0.340136$.

Step-by-step:

$$3\pi \approx 9.4248, \quad (10)$$

$$3\pi \times \frac{3}{4} = 9.4248 \times 0.75 \approx 7.0686, \quad (11)$$

$$7.0686 \times 10^4 = 70686, \quad (12)$$

$$70686 \times 9.2104 \approx 651019.3, \quad (13)$$

$$\alpha^{-1} \approx 651019.3 \times 0.340136 \approx 221291.7. \quad (14)$$

4.2 Error Analysis

Error

The calculation is **incorrect**. The computed value $\alpha^{-1} \approx 221291.7$ is far from 137.036. The factor 10^4 appears to be erroneous. Testing with 10^{-4} yields:

$$7.0686 \times 10^{-4} \times 9.2104 \times 0.340136 \approx 0.02214,$$

$$\alpha^{-1} \approx \frac{1}{0.02214} \approx 45.17,$$

which is also incorrect. The formula or coefficients (e.g., 10^4) are likely misdefined.

5 Path 2: Fractal Renormalization

5.1 Calculation

The formula is:

$$\alpha^{-1} = 1 + \Delta_{\text{frac}}, \tag{15}$$

$$\Delta_{\text{frac}} = \frac{3}{4\pi} \times \xi^{-2} \times D_{\text{frac}}^{-1}, \tag{16}$$

$$D_{\text{frac}} = \left(\frac{\lambda_C^{(\mu)}}{\ell_P} \right)^{D_f - 2}, \tag{17}$$

with $D_f = 2.94$, $\xi = \frac{4}{3} \times 10^{-4}$, and $\alpha^{-1} = 137.0$.

1. **Fractal Damping Factor**:

$$\lambda_C^{(\mu)} \approx \frac{1.973 \times 10^{-13}}{105.66} \approx 1.867 \times 10^{-15} \text{ m}, \quad (18)$$

$$\ell_P \approx 1.616 \times 10^{-35} \text{ m}, \quad (19)$$

$$\frac{\lambda_C^{(\mu)}}{\ell_P} \approx 1.155 \times 10^{20}, \quad (20)$$

$$D_{\text{frac}} = (1.155 \times 10^{20})^{0.94} \approx 6.93 \times 10^{18}, \quad (21)$$

$$D_{\text{frac}}^{-1} \approx \frac{1}{6.93 \times 10^{18}} \approx 1.443 \times 10^{-19}. \quad (22)$$

2. **Fractal Correction**:

$$\xi^{-2} = (7500)^2 = 5.625 \times 10^7, \quad (23)$$

$$\frac{3}{4\pi} \approx 0.23873, \quad (24)$$

$$\Delta_{\text{frac}} \approx 0.23873 \times 5.625 \times 10^7 \times 1.443 \times 10^{-19} \approx 1.938 \times 10^{-12}, \quad (25)$$

$$\alpha^{-1} \approx 1 + 1.938 \times 10^{-12} \approx 1. \quad (26)$$

5.2 Error Analysis

Error

The calculation is **incorrect**. The fractal correction yields $\Delta_{\text{frac}} \approx 1.938 \times 10^{-12}$, not 136 as stated in the original document. Thus, $\alpha^{-1} \approx 1$, far from 137.0. The error likely lies in the definition of Δ_{frac} or the values used for D_{frac} . Even using $D_{\text{frac}} = 6.7 \times 10^{18}$ (as in the original) does not yield the correct result.

6 Avoiding the Fallacy of $\alpha \propto \xi^{11/2}$

6.1 Calculation

An incorrect simplification would be:

$$\xi = 1.333 \times 10^{-4}, \quad (27)$$

$$\xi^{11/2} = (1.333 \times 10^{-4})^{5.5} \approx 2.34 \times 10^{-21}, \quad (28)$$

$$\alpha^{-1} \sim \frac{1}{2.34 \times 10^{-21}} \approx 10^{21}. \quad (29)$$

6.2 Error Analysis

Error

This simplification is **incorrect**. It ignores the physical significance of $E_0 = 7.398 \text{ MeV}$ as a measurable parameter (geometric mean of electron and muon masses). The correct formula $\alpha = \xi \cdot \frac{E_0^2}{(1 \text{ MeV})^2}$ respects dimensions and yields the correct result.

7 Summary

Summary

1. **Elementary Derivation:** $\alpha = \xi \cdot \frac{E_0^2}{(1 \text{ MeV})^2}$ is correct and yields $\alpha^{-1} \approx 137.0$, with only 0.026% deviation from the experimental value.
2. **Path 1:** The direct geometric calculation is incorrect, yielding $\alpha^{-1} \approx 221291.7$. The factor 10^4 is likely erroneous.
3. **Path 2:** The fractal renormalization is incorrect, as $\Delta_{\text{frac}} \approx 10^{-12}$ instead of 136, leading to $\alpha^{-1} \approx 1$.
4. **Fallacy of $\xi^{11/2}$:** This simplification is dimensionally incorrect and leads to absurd results ($\alpha^{-1} \sim 10^{21}$).
5. The elementary derivation is the most robust method, being transparent, dimensionally correct, and close to the experimental value.