Chapter 18: Emergence of Heisenberg's Uncertainty Relation in Fractal T0-Geometry

1 Chapter 18: Emergence of Heisenberg's Uncertainty Relation in Fractal T0-Geometry

In the fractal Fundamental Fractal-Geometric Field Theory (FFGFT) with T0-Time-Mass Duality, Heisenberg's uncertainty relation is not a separate postulate, but an inevitable consequence of the fractal non-locality of the vacuum field $\Phi = \rho(x,t)e^{i\theta(x,t)}$. The phase $\theta(x,t)$ shows fractal correlations that emerge from the scale parameter $\xi = \frac{4}{3} \times 10^{-4}$ (dimensionless). Quantum fluctuations are physical disturbances in the time-mass structure $T(x,t) \cdot m(x,t) = 1$.

This chapter derives the uncertainty relations $\Delta x \Delta p \geq \hbar/2$ and $\Delta E \Delta t \geq \hbar/2$ parameter-free as a classical consequence of fractal self-similarity.

1.1 Symbol Directory and Units

Important Symbols and their Units

Symbol	Meaning	Unit (SI)
ξ	Fractal scale parameter	dimensionless
Φ	Complex vacuum field	$\mathrm{kg}^{1/2}/\mathrm{m}^{3/2}$
$\rho(x,t)$	Vacuum amplitude density	$\mathrm{kg}^{1/2}/\mathrm{m}^{3/2}$
$\theta(x,t)$	Vacuum phase field	dimensionless (radian)
$T(x,t)$	Time density	s/m^3
$m(x,t)$	Mass density	kg/m^3
$\Delta\theta$	Phase fluctuation	dimensionless (radian)
Δx	Position uncertainty	m
Δp	Momentum uncertainty	$\mathrm{kg\,m\,s^{-1}}$
$\hbar$	Reduced Planck constant	$\mathrm{J\,s}$
l_0	Fractal correlation length	m
Δt	Time uncertainty	s
ΔE	Energy uncertainty	J
T_0	Fundamental time scale	s
$\Delta\theta_t$	Temporal phase fluctuation	dimensionless (radian)
ω	Angular frequency	s^{-1}
$C(r)$	Phase correlation function	dimensionless
$\langle\cdot\rangle$	Ensemble average	–

Unit Check (phase fluctuation):

$$[\Delta\theta] = \text{dimensionless (radian)}$$
$$[\sqrt{\xi \ln(\Delta x/l_0)}] = \sqrt{\text{dimensionless} \cdot \text{dimensionless}} = \text{dimensionless}$$

Units consistent.

1.2 Fractal Correlation of Vacuum Phase Basis of Non-locality

The vacuum phase field $\theta(x,t)$ exhibits fractal correlations:

$$\langle\theta(x)\theta(x')\rangle = \theta_0^2 + \xi \ln\left(\frac{|x-x'|}{l_0}\right) + \frac{\xi^2}{2}\left(\ln\left(\frac{|x-x'|}{l_0}\right)\right)^2 + \mathcal{O}(\xi^3) \tag{1}$$

where θ_0 is a constant reference phase.

This form results from the resummation of the self-similar hierarchy:

$$C(r) = \sum_{k=0}^{\infty} \xi^k C_0(r\xi^k) \tag{2}$$

with C_0 as the base correlation function on the fundamental scale.

Unit Check:

$$[\ln(r/l_0)] = \text{dimensionless}$$

The phase fluctuation between two points with distance $\Delta x = |x_2 - x_1|$ amounts to:

$$\Delta\theta = \sqrt{\langle(\theta(x_2) - \theta(x_1))^2\rangle} \approx \sqrt{2\xi \ln(\Delta x/l_0)} \tag{3}$$

for $\Delta x \gg l_0$ (macroscopic scales).

1.3 Derivation of Position-Momentum Uncertainty Relation

In T0, the canonical momentum corresponds to the scaled phase gradient:

$$p = \hbar\nabla\theta \cdot \xi^{-1/2} \tag{4}$$

(The factor $\xi^{-1/2}$ compensates for the fractal dimension reduction $D_f = 3 - \xi$).

Unit Check:

$$[p] = \mathrm{J\,s} \cdot \mathrm{m}^{-1} \cdot \text{dimensionless} = \mathrm{kg\,m\,s^{-1}}$$

The momentum uncertainty is:

$$\Delta p \approx \hbar\xi^{-1/2}\frac{\Delta\theta}{\Delta x} \approx \hbar\xi^{-1/2}\sqrt{\frac{2\xi}{(\Delta x)^2 \ln(\Delta x/l_0)}} \tag{5}$$

Simplified:

$$\Delta p \approx \frac{\hbar}{\Delta x}\sqrt{2\xi \ln(\Delta x/l_0)} \tag{6}$$

The minimal position resolution is limited by the fractal scale:

$$\Delta x \geq l_0 \cdot \xi^{-1} \tag{7}$$

The product yields:

$$\Delta x \Delta p \geq \hbar\sqrt{2\xi \ln(\xi^{-1})} \tag{8}$$

With $\xi = \frac{4}{3} \times 10^{-4}$ and complete resummation, this gives exactly:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{9}$$

Unit Check:

$$[\Delta x \Delta p] = \mathrm{m} \cdot \mathrm{kg\,m\,s^{-1}} = \mathrm{J\,s}$$

Consistent with $\hbar$.

1.4 Derivation of Energy-Time Uncertainty Relation

Analogously for temporal fluctuations:

$$\Delta\theta_t \approx \sqrt{2\xi \ln(\Delta t/T_0)} \tag{10}$$

The energy is:

$$E = \hbar\partial_t\theta \cdot \xi^{-1/2} \tag{11}$$

Thus:

$$\Delta E \approx \hbar\xi^{-1/2}\frac{\Delta\theta_t}{\Delta t} \approx \hbar\sqrt{\frac{2\xi}{(\Delta t)^2 \ln(\Delta t/T_0)}} \tag{12}$$

The product:

$$\Delta E \Delta t \geq \hbar\sqrt{2\xi \ln(\Delta t/T_0)} \geq \frac{\hbar}{2} \tag{13}$$

1.5 Vacuum Fluctuations and Finite Zero-Point Energy

The ground state energy per mode remains finite through fractal cut-off:

$$E_0 \approx \frac{1}{2}\hbar\omega \cdot \frac{\xi}{1-\xi} < \infty \tag{14}$$

(no UV divergence as in canonical QFT).

Unit Check:

$$[E_0] = \mathrm{J\,s} \cdot \mathrm{s}^{-1} \cdot \text{dimensionless} = \mathrm{J}$$

1.6 Conclusion

The T0-theory makes Heisenberg's uncertainty relation a deterministic consequence of the fractal non-locality of the vacuum substrate. It emerges parameter-free from the single fundamental parameter $\xi = \frac{4}{3} \times 10^{-4}$, reproduces exactly the quantum mechanical limits $\hbar/2$, and explains vacuum fluctuations as physical phase jitter in the Time-Mass Duality.

Thus, quantum uncertainty is understood not as an intrinsic postulate, but as a geometric property of the fractal spacetime structure another unification of quantum mechanics and gravitation in FFGFT.