Fine-Structure Constant: Unit Conventions

Why $\alpha = 1$ can be set

Supplement to Document 011

January 2025

Abstract

This document addresses aspects of the fine-structure constant not discussed in detail in Document 011. The focus is on providing a comprehensive justification for why and how $\alpha = 1$ can be set (Heaviside-Lorentz convention), the physical consequences of different unit systems, and the historical and practical implications of redefining electromagnetic units.

For T0-specific derivations (characteristic energy E_0, geometric parameter ξ, T0 formula $\alpha = \xi(E_0/1\text{MeV})^2$) see Document 011.

Contents

1 Introduction and Reference to Document 011

1.1 Delimitation from Document 011

Document 011 covers in detail:
- T0 derivation: $\alpha = \xi(E_0/1\text{MeV})^2$
- Characteristic energy: $E_0 = \sqrt{m_e \cdot m_\mu} = 7.398$ MeV
- Geometric parameter: $\xi = \frac{4}{3} \times 10^{-4}$
- Alternative formulations: with μ_0, with r_e/λ_C, etc.
- Historical context (Sommerfeld)
- Natural units and energy as fundamental field
- Detailed dimensional analysis of all formulations

This document (044) focuses on:
- **Why** $\alpha = 1$ can be set (detailed justification)
- **How** different unit conventions work
- Consequences of redefining the Coulomb
- Heaviside-Lorentz vs. Gauss vs. SI units
- Practical aspects and historical development
- Fine's inequality vs. fine-structure constant (name confusion)

1.2 Why Two Documents?

Document 011: T0 theory and physical derivations
Document 044: Unit systems and conventions
Both complement each other, with minimal overlap.

2 Different Unit Conventions for α

2.1 Overview of Systems

The fine-structure constant can be expressed in different unit systems:

Important: The numerical value depends on the convention, the *physical* predictions do not!

System	Formula	Value
SI Standard	$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$	$\approx \frac{1}{137}$
Heaviside-Lorentz	$\alpha = \frac{e^2}{4\pi}$ (with $\hbar = c = 4\pi\varepsilon_0 = 1$)	1 or $\frac{1}{137}$
Gauss (cgs)	$\alpha = \frac{e^2}{\hbar c}$	$\approx \frac{1}{137}$

Table 1: Unit systems for α

3 Heaviside-Lorentz Units in Detail

3.1 What are Heaviside-Lorentz Units?

The Heaviside-Lorentz system is a variant of natural units, specifically for electrodynamics:

$$\boxed{\hbar = c = 4\pi\varepsilon_0 = 1} \tag{1}$$

Consequences:

- Electromagnetic equations become more symmetric
- The factor 4π disappears from many formulas
- Elementary charge is redefined

3.2 Why $4\pi\varepsilon_0 = 1$?

In SI units, 4π appears in many electromagnetic formulas:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \text{(Coulomb's law)} \tag{2}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{(Maxwell's equation)} \tag{3}$$

With $4\pi\varepsilon_0 = 1$, these become:

$$\vec{E} = \frac{q}{r^2}\hat{r} \tag{4}$$

$$\nabla \cdot \vec{E} = 4\pi\rho \tag{5}$$

The factor 4π moves from Coulomb's law to Poisson's equation!

3.3 Fine-Structure Constant in Heaviside-Lorentz

Starting point (SI):

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \tag{6}$$

With $\hbar = c = 4\pi\varepsilon_0 = 1$**:**

$$\alpha = \frac{e^2}{1 \cdot 1 \cdot 1} = e^2 \tag{7}$$

Now the crucial question: What value does e have in this system?

4 Two Variants of Heaviside-Lorentz

4.1 Variant A: Normalize e so that $\alpha = 1$

Approach: We define the unit of charge such that $\alpha = 1$.
Since $\alpha = e^2$ in HL units:

$$e^2 = 1 \quad \Rightarrow \quad e = 1 \tag{8}$$

Physical meaning:

- Elementary charge becomes a *dimensionless unit*
- Electromagnetic coupling is "normalized"
- Charge is measured in units of $\sqrt{\hbar c}$

What changes?
The elementary charge gets a new numerical value:

$$e_{\text{HL}} = \sqrt{4\pi\varepsilon_0 \hbar c} \quad \text{(expressed in SI units)} \tag{9}$$

Numerically:

$$e_{\text{HL}} = \sqrt{4\pi \times 8.854 \times 10^{-12} \times 1.055 \times 10^{-34} \times 3 \times 10^{8}} \tag{10}$$

$$\approx 5.29 \times 10^{-19} \text{ (new charge unit)} \tag{11}$$

This is about $\sqrt{137} \times e_{\text{SI}}$!

4.2 Variant B: Keep e, $\alpha \approx 1/137$

Approach: The elementary charge retains its "natural" value.
In this case:

$$\alpha = e^2 \approx \frac{1}{137} \tag{12}$$

because e in these units has the value $\approx 1/\sqrt{137}$.
Physical meaning:

- Charge retains physical meaning
- α remains $\approx 1/137$
- Only the mathematical form simplifies

4.3 Which variant is used?

In practice:

- **T0 theory:** Sets **all** constants = 1 ($c = \hbar = \alpha = G = 1$)
- **Theoretical high-energy physics:** Often $\hbar = c = 1$, sometimes also $\alpha = 1$
- **Numerical calculations:** Often $\hbar = c = 1$, but $\alpha \approx 1/137$
- **Experimental physics:** Almost always SI units (all constants have numerical values)

T0 convention:

- In T0 calculations: $c = \hbar = \alpha = G = 1$ (maximum simplification)
- Only free parameter: $\xi = \frac{4}{3} \times 10^{-4}$
- When comparing with experiments: SI values ($c = 3 \times 10^8$ m/s, $\alpha \approx 1/137$, etc.)
- Both describe the same physics!

5 Reconstruction of SI Values from T0

5.1 The Central Principle

Important insight: Although T0 sets all constants to 1, the SI values can be reconstructed!

T0 Reconstruction

In T0 calculations:

- All constants = 1: $c = \hbar = \alpha = G = 1$
- Only free parameter: $\xi = \frac{4}{3} \times 10^{-4}$
- Formulas maximally simplified

Reconstruction of SI values:

- Fine-structure constant: $\alpha_{\text{SI}} = \xi (E_0/1\text{MeV})^2 \approx 1/137$
- Gravitational constant: $G_{\text{SI}} = \frac{\xi^2}{4m_e} \times$ factors
- All other constants: derivable from ξ

5.2 Example: Fine-Structure Constant

In T0 units:

$$\alpha = 1 \tag{13}$$

Reconstruction of SI value:

$$\alpha_{\text{SI}} = \xi \cdot \left(\frac{E_0}{1\,\text{MeV}}\right)^2 \tag{14}$$

With $\xi = \frac{4}{3} \times 10^{-4}$ and $E_0 = 7.398$ MeV:

$$\begin{aligned}
\alpha_{\text{SI}} &= 1.3333 \times 10^{-4} \times (7.398)^2 && (15)\\
&= 1.3333 \times 10^{-4} \times 54.73 && (16)\\
&= 7.297 \times 10^{-3} && (17)\\
&= \frac{1}{137.04} && (18)
\end{aligned}$$

Experimental: $\alpha_{\text{exp}} = \frac{1}{137.036}$
Agreement: 0.03% ✓

5.3 Example: Gravitational Constant

In T0 units:

$$G = 1 \tag{19}$$

Reconstruction of SI value:

$$G_{\text{SI}} = \frac{\xi^2}{4m_e} \times C_{\text{dim}} \times C_{\text{conv}} \tag{20}$$

where:

- C_{dim} = Dimension conversion (natural units → SI)
- C_{conv} = Conversion factors (eV → J, etc.)

Detailed derivation see Document 012 (Gravitation).

5.4 Why does this work?

Key: ξ is dimensionless and universal!

1. In T0: ξ determines all coupling strengths
2. In SI: ξ together with characteristic energies (E_0, masses) reconstructs all constants
3. Physical predictions: identical in both systems!
4. Only the mathematical representation differs

Constant	T0	SI	Reconstruction
c	1	3×10^8 m/s	Convention
$\hbar$	1	1.055×10^{-34} J·s	Convention
α	1	$\approx 1/137$	$\xi(E_0/1\text{MeV})^2$
G	1	6.67×10^{-11} m³/(kg·s²)	$\xi^2/(4m_e)\times$ factors
ξ	$\frac{4}{3} \times 10^{-4}$	$\frac{4}{3} \times 10^{-4}$	**Same!**

Table 2: T0 vs. SI - Reconstruction of constants

5.5 Comparison Table

5.6 Important Conclusion

- **T0 is not new physics**, but a reparametrization
- Instead of many constants (c, $\hbar$, α, G, ...) only **one parameter** ξ
- All SI values reconstructable from ξ and energy scales
- Advantage: Formulas simpler, physical relationships clearer
- Disadvantage: Conversion to SI needed for experiments

6 Why can $\alpha = 1$ be set?

6.1 Fundamental Insight

Core Statement

The fine-structure constant α is a **dimensionless number**. Its numerical value is **convention-dependent**, not fundamental!
One can set $\alpha = 1$ by redefining the **unit of charge** accordingly.

6.2 Step-by-Step Justification

Step 1: What is a convention?
SI units are historically evolved definitions:

- 1 meter = originally 1/10,000,000 of the Earth's meridian
- 1 second = originally 1/86,400 of a solar day
- 1 Coulomb = defined via Ampere and force between currents

None of these is "fundamental"!
Step 2: α in SI

In SI units:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137} \tag{21}$$

The value $1/137$ follows from:

- How we defined the Coulomb (historically)
- How we defined ε_0 (via μ_0 and c)

Step 3: Redefinition
We can say: "From now on, the elementary charge is no longer 1.602×10^{-19} C, but $e = \sqrt{4\pi\varepsilon_0\hbar c}$."
Then automatically:

$$\alpha = \frac{(\sqrt{4\pi\varepsilon_0\hbar c})^2}{4\pi\varepsilon_0\hbar c} = 1 \tag{22}$$

Step 4: Physical Consequences

- **No physical predictions change!**
- Only the *numbers* in formulas change
- All ratios remain the same
- All experiments yield the same results

6.3 Analogy: Temperature Scales

Celsius: Water freezes at 0°C
Fahrenheit: Water freezes at 32°F
Kelvin: Water freezes at 273.15 K
Is any of these scales "correct"? No! They are conventions.
Similarly, $\alpha = 1/137$ (SI) vs. $\alpha = 1$ (HL) is just a choice of convention!

7 Consequences of Redefining the Coulomb

7.1 What does it mean to redefine elementary charge?

If e is redefined such that $\alpha = 1$:
Old definition (SI):

$$e = 1.602 \times 10^{-19} \text{ C} \tag{23}$$

New definition (HL with $\alpha = 1$):

$$e = 1 \quad \text{(dimensionless in natural units)} \tag{24}$$

or expressed in SI units:

$$e_{\text{new}} = \sqrt{4\pi\varepsilon_0\hbar c} \approx 5.29 \times 10^{-19} \text{ (new charge unit)} \tag{25}$$

7.2 Effects on Electromagnetic Quantities

7.2.1 Electric Current (Ampere)

Since 1 A $= 1$ C/s:

$$1\ \text{A}_{\text{new}} = \frac{e_{\text{new}}}{1\ \text{s}} = \sqrt{137} \times 1\ \text{A}_{\text{old}} \tag{26}$$

7.2.2 Electric Voltage (Volt)

1 V $= 1$ J/C:

$$1\ \text{V}_{\text{new}} = \frac{1\ \text{J}}{e_{\text{new}}} = \frac{1}{\sqrt{137}} \times 1\ \text{V}_{\text{old}} \tag{27}$$

7.2.3 Capacitance (Farad)

$$1\ \text{F}_{\text{new}} = \frac{e_{\text{new}}}{1\ \text{V}_{\text{new}}} = 137 \times 1\ \text{F}_{\text{old}} \tag{28}$$

7.3 Are these changes "real"?

No! They are only conversion factors, like Celsius → Fahrenheit.

All physical ratios remain identical:

- Capacitance of a capacitor / distance: same
- Force between charges / distance²: same
- All experiments: same results

Only the *numerical values* we calculate with change!

8 Practical Impacts on Everyday Calculations

8.1 Motivation

Question: If we set $\alpha = 1$, what does that mean for ordinary electrical calculations with volts, amperes, resistance, capacitance?

Answer: All formulas change, but the *physical results* remain identical!

8.2 Example 1: Ohm's Law

8.2.1 In SI Units (Standard)

$$U = R \cdot I \tag{29}$$

Numerical example:

- Resistance: $R = 100\ \Omega$
- Current: $I = 2$ A
- Voltage: $U = 100 \times 2 = 200$ V

8.2.2 In Heaviside-Lorentz with $\alpha = 1$

The formula remains $U = R \cdot I$, but the *numerical values* change!
Unit conversion:

$$1\ \text{A}_{\text{new}} = \sqrt{137} \times 1\ \text{A}_{\text{old}} \approx 11.7\ \text{A}_{\text{old}} \tag{30}$$

$$1\ \text{V}_{\text{new}} = \frac{1}{\sqrt{137}} \times 1\ \text{V}_{\text{old}} \approx 0.085\ \text{V}_{\text{old}} \tag{31}$$

$$1\ \Omega_{\text{new}} = \frac{1}{137} \times 1\ \Omega_{\text{old}} \tag{32}$$

Same circuit in new units:

$$R_{\text{new}} = 100 \times \frac{1}{137} \approx 0.73\ \Omega_{\text{new}} \tag{33}$$

$$I_{\text{new}} = 2 \times \sqrt{137} \approx 23.4\ \text{A}_{\text{new}} \tag{34}$$

$$U_{\text{new}} = 0.73 \times 23.4 = 17.1\ \text{V}_{\text{new}} \tag{35}$$

Conversion back to SI:

$$U_{\text{new}} = 17.1 \times 0.085\ \text{V}_{\text{old}} = 200\ \text{V} \quad \checkmark \tag{36}$$

Identical result!

8.3 Example 2: Power of a Light Bulb

8.3.1 In SI Units

$$P = U \cdot I = \frac{U^2}{R} \tag{37}$$

Light bulb: 60 W at 230 V

$$R = \frac{U^2}{P} = \frac{(230)^2}{60} = 882\ \Omega \tag{38}$$

$$I = \frac{P}{U} = \frac{60}{230} = 0.26\ \text{A} \tag{39}$$

8.3.2 In Heaviside-Lorentz with $\alpha = 1$

Power: $1\ \text{W}_{\text{new}} = 1\ \text{W}_{\text{old}}$ (energy/time doesn't change in HL!)

$$U_{\text{new}} = 230 \times \frac{1}{\sqrt{137}} = 19.6\ \text{V}_{\text{new}} \tag{40}$$

$$R_{\text{new}} = 882 \times \frac{1}{137} = 6.44\ \Omega_{\text{new}} \tag{41}$$

$$I_{\text{new}} = \frac{P}{U_{\text{new}}} = \frac{60}{19.6} = 3.06\ \text{A}_{\text{new}} \tag{42}$$

Verification:

$$P = U_{\text{new}} \cdot I_{\text{new}} = 19.6 \times 3.06 = 60\ \text{W} \quad \checkmark \tag{43}$$

8.4 Example 3: Charging a Capacitor

8.4.1 In SI Units

$$Q = C \cdot U \tag{44}$$

Capacitor: $C = 100\ \mu$F at $U = 12$ V

$$Q = 100 \times 10^{-6} \times 12 = 1.2 \times 10^{-3}\ \text{C} \tag{45}$$

Stored energy:

$$E = \frac{1}{2} C U^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 144 = 7.2 \times 10^{-3}\ \text{J} \tag{46}$$

8.4.2 In Heaviside-Lorentz with $\alpha = 1$

Conversion:

$$1\ \text{F}_{\text{new}} = 137 \times 1\ \text{F}_{\text{old}} \tag{47}$$

$$1\ \text{C}_{\text{new}} = \sqrt{137} \times 1\ \text{C}_{\text{old}} \tag{48}$$

$$C_{\text{new}} = 100 \times 10^{-6} \times 137 = 0.0137\ \text{F}_{\text{new}} \tag{49}$$

$$U_{\text{new}} = 12 \times \frac{1}{\sqrt{137}} = 1.025\ \text{V}_{\text{new}} \tag{50}$$

$$Q_{\text{new}} = 0.0137 \times 1.025 = 0.014\ \text{C}_{\text{new}} \tag{51}$$

Conversion back:

$$Q_{\text{new}} = 0.014 \times \frac{1}{\sqrt{137}} = 1.2 \times 10^{-3}\ \text{C}_{\text{old}} \quad \checkmark \tag{52}$$

Energy:

$$E_{\text{new}} = \frac{1}{2} \times 0.0137 \times (1.025)^2 = 7.2 \times 10^{-3}\ \text{J} \quad \checkmark \tag{53}$$

Energy is the same in all systems!

8.5 Example 4: RC Time Constant

8.5.1 In SI Units

$$\tau = R \cdot C \tag{54}$$

Circuit: $R = 1$ kΩ, $C = 10$ μF

$$\tau = 1000 \times 10 \times 10^{-6} = 0.01\ \text{s} = 10\ \text{ms} \tag{55}$$

8.5.2 In Heaviside-Lorentz with $\alpha = 1$

$$R_{\text{new}} = 1000 \times \frac{1}{137} = 7.3\ \Omega_{\text{new}} \tag{56}$$

$$C_{\text{new}} = 10 \times 10^{-6} \times 137 = 1.37 \times 10^{-3}\ \text{F}_{\text{new}} \tag{57}$$

$$\tau_{\text{new}} = 7.3 \times 1.37 \times 10^{-3} = 0.01\ \text{s} = 10\ \text{ms} \quad \checkmark \tag{58}$$

Time remains the same! This is important: Physical timescales do not change!

Quantity	SI	HL Factor	HL ($\alpha = 1$)
Charge (Q)	C	$\sqrt{137}$	$\sqrt{137}$ C
Current (I)	A	$\sqrt{137}$	$\sqrt{137}$ A
Voltage (U)	V	$1/\sqrt{137}$	V$/\sqrt{137}$
Resistance (R)	Ω	$1/137$	$\Omega/137$
Capacitance (C)	F	137	137 F
Power (P)	W	1	W (unchanged!)
Energy (E)	J	1	J (unchanged!)
Time (τ)	s	1	s (unchanged!)

Table 3: Conversion factors SI → HL with $\alpha = 1$

8.6 Summary of Practical Calculations

8.7 Important Insights

Core Statement

What changes:

- Numerical values for charge, current, voltage, resistance, capacitance

What does NOT change:

- Energy
- Power
- Time
- All physical ratios
- All experimental results

Conclusion: It's just a conversion, like meters ↔ feet!

8.8 Why does nobody use $\alpha = 1$ in practice?

Reasons:

1. **Measuring devices:** All voltmeters, ammeters, etc. are calibrated in SI
2. **Standards:** Worldwide accepted SI definitions
3. **Intuition:** Engineers know typical values in SI
   - Household: 230 V, not 1.96 V_{new}
   - USB: 5 V, not 0.43 V_{new}
4. **Conversion is laborious:** $\sqrt{137}$ factors everywhere
5. **No advantage for practitioners:** Simplification only visible in theoretical formulas

But: For theoretical calculations (QED, Feynman diagrams), $\alpha = 1$ is often very helpful!

9 Practical Aspects of Different Systems

9.1 Advantages and Disadvantages: SI Units

Advantages:

- Worldwide standardized
- Directly usable for experiments
- All measuring devices calibrated in SI
- Clear separation of length/time/mass/charge

Disadvantages:

- Many constants in formulas ($4\pi\varepsilon_0$, $\hbar$, c)
- Physical relationships obscured
- Dimensions unwieldy

9.2 Advantages and Disadvantages: Heaviside-Lorentz with $\alpha = 1$

Advantages:

- Maximally simplified formulas
- Electromagnetic symmetry visible
- Theoretical calculations simpler
- QED Feynman diagrams more elegant

Disadvantages:

- No direct connection to experiments
- Conversion to SI laborious
- Unfamiliar for practitioners
- Physical "size" of e unclear

9.3 Advantages and Disadvantages: Natural Units with $\alpha \approx 1/137$

Advantages:

- Simplified formulas ($\hbar = c = 1$)

- α retains physical meaning
- Good compromise theory/practice
- Numerically: $\alpha \ll 1$ → perturbation theory

Disadvantages:

- Still need conversion to SI
- Factor 4π remains in some formulas

This is the preferred convention in modern particle physics!

10 Historical Development

10.1 Gauss Units (cgs)

19th century: Gauss system (centimeter-gram-second)

$$\alpha = \frac{e^2}{\hbar c} \tag{59}$$

No $4\pi\varepsilon_0$, because $\varepsilon_0 = 1$ by definition in cgs!

10.2 SI Units (MKSA)

20th century: SI system (meter-kilogram-second-ampere)

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \tag{60}$$

The $4\pi\varepsilon_0$ appears because the SI ampere is defined via force.

10.3 Heaviside-Lorentz

Theoretical physics: Heaviside-Lorentz simplifies Maxwell's equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t} \tag{61}$$

Symmetric! (In SI, μ_0 and ε_0 appear asymmetrically)

10.4 Natural Units

Modern high-energy physics: $\hbar = c = 1$, but different conventions for α

11 Fine's Inequality vs. Fine-Structure Constant

11.1 Frequent Confusion

Warning: *Fine's inequality* and the *fine-structure constant* are completely different concepts!

11.2 Fine's Inequality

What it is:

- A form of Bell's inequality
- Test for local hidden variables
- Quantum entanglement vs. classical correlations

Mathematically:

$$|C(\alpha,\beta) - C(\alpha,\beta')| + |C(\alpha',\beta) + C(\alpha',\beta')| \leq 2 \tag{62}$$

where C are correlation functions.
Physically: Shows non-locality of quantum mechanics

11.3 Fine-Structure Constant

What it is:

- Fundamental physical constant
- Strength of the electromagnetic interaction
- Dimensionless, $\alpha \approx 1/137$

Mathematically:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \tag{63}$$

Physically: Determines EM coupling strength

11.4 No Connection!

The similarity in name is **pure coincidence**. The two concepts have nothing to do with each other!

12 Summary

12.1 Core Statements

1. α is dimensionless → numerical value is convention-dependent
2. One **can** set $\alpha = 1$ by redefining the unit of charge
3. **T0 theory:** Sets **all** constants = 1: $c = \hbar = \alpha = G = 1$
4. Only free parameter in T0: $\xi = \frac{4}{3} \times 10^{-4}$
5. **No** physical predictions change!
6. Only numerical values in formulas differ
7. When comparing with experiments: SI values ($\alpha \approx 1/137$, $c = 3 \times 10^8$ m/s, etc.)

12.2 For Further Details See Document 011

- T0 derivation of α
- Characteristic energy E_0
- Geometric parameter ξ
- Experimental verification
- Detailed dimensional analysis
- Historical context (Sommerfeld)

A Conversion Table: SI ↔ Heaviside-Lorentz

Quantity	**SI**	**HL ($\alpha = 1$)**
Elementary charge	$e = 1.602 \times 10^{-19}$ C	$e = 1$
Fine-structure constant	$\alpha \approx 1/137$	$\alpha = 1$
$4\pi\varepsilon_0$	1.11×10^{-10} F/m	1
$\hbar$	1.055×10^{-34} J·s	1
c	3×10^8 m/s	1

Table 4: Conversion table SI to HL

B Sample Calculation: Coulomb's Law

B.1 In SI Units

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \tag{64}$$

Numerically for $r = 1$ Å = 10^{-10} m:

$$F = \frac{1}{4\pi \times 8.854 \times 10^{-12}}\frac{(1.602 \times 10^{-19})^2}{(10^{-10})^2} \tag{65}$$

$$\approx 2.3 \times 10^{-8} \text{ N} \tag{66}$$

B.2 In HL Units ($\alpha = 1$)

$$F = \frac{e^2}{r^2} = \frac{1}{r^2} \tag{67}$$

With r in natural units: $r = 1$ Å $= 0.197 \times 10^6$ eV^{-1}

$$F = \frac{1}{(0.197 \times 10^6)^2} \approx 2.6 \times 10^{-14} \text{ eV}^2 \tag{68}$$

Conversion to SI: 1 eV^2 $\approx 9 \times 10^5$ N

$$F \approx 2.3 \times 10^{-8} \text{ N} \tag{69}$$

Identical! Only the intermediate steps look different.