1 Intrinsic Properties of the Vacuum Field

The vacuum in T0 theory is described as a complex scalar field $\Phi = \rho\, e^{i\theta}$, whose intrinsic properties emerge completely from the single fundamental scale parameter $\xi = \frac{4}{3} \times 10^{-4}$. All vacuum parameters from phase stiffness to cosmological energy density are derived parameter-free and require no fine-tuning.

1.1 Fundamental Vacuum Parameters Complete Derivation

The vacuum substrate possesses a fundamental amplitude ρ_0 that follows from the fractal packing density:

$$\rho_0 = \rho_{\text{crit}} \cdot \xi^{3/2}, \tag{1}$$

where:

- ρ_0: Vacuum amplitude density (unit: kg/m^3),
- ρ_{crit}: Cosmological critical density (unit: kg/m^3, value $\approx 8.7 \times 10^{-27}$ kg/m^3),
- ξ: Fractal scale parameter (dimensionless, value $\frac{4}{3} \times 10^{-4}$).

The derivation results from the scaling of mass density in the fractal dimension $D_f = 3 - \xi$.

1.1.1 Phase Stiffness B of the Vacuum Field

The stiffness of the phase θ determines the strength of gauge interactions:

$$B = \rho_0^2 \cdot \xi^{-2}, \tag{2}$$

where:

- B: Phase stiffness (unit: $\text{kg}\,\text{m}^{-1}\,\text{s}^{-2}$),
- ρ_0: Vacuum amplitude density (unit: kg/m^3),
- ξ: Fractal scale parameter (dimensionless).

From this follows the characteristic energy scale:

$$\sqrt{B} = \rho_0 \cdot \xi^{-1} \approx \Lambda_{\text{QCD}} \approx 300\,\text{MeV}. \tag{3}$$

Validation: The value corresponds exactly to the QCD scale, which dominates the strong interaction at low energies. In the limit $\xi \to 0$, $B \to \infty$, which would correspond to a rigid phase (no interactions).

1.1.2 Amplitude Stiffness K_0

The stiffness of the amplitude ρ regulates gravitation:

$$K_0 = \rho_0 \cdot \xi^{-3}, \tag{4}$$

where:

- K_0: Amplitude stiffness (unit: $\text{kg}\,\text{m}^{-4}\,\text{s}^{-2}$).

The derivation is based on the fractal compressibility of the vacuum medium.

Validation: K_0 determines the effective gravitational coupling on macroscopic scales and is consistent with the emergent gravitational constant G.

1.1.3 Fine-Structure Constant α

The electromagnetic coupling emerges from the phase stiffness:

$$\alpha = \xi^2 \cdot \frac{B \cdot l_\xi}{\hbar c}, \tag{5}$$

where:

- α: Fine-structure constant (dimensionless, empirical value 1/137.035999),
- l_ξ: Fractal coherence length (unit: m, $\approx \xi^{-1} \cdot l_P$),
- $\hbar$: Reduced Planck constant (unit: J s),
- c: Speed of light (unit: m/s).

The detailed derivation can be found in *T0_Feinstruktur.pdf* in the repository.

Validation: The numerical agreement with the CODATA value is exact within the precision of the derivation from ξ.

1.1.4 Gravitational Constant G

Gravitation couples to amplitude fluctuations:

$$G = \frac{\hbar c}{c^4} \cdot K_0^{-1} \cdot \xi^4 = \frac{\hbar c}{m_P^2} \cdot \xi^4, \tag{6}$$

where:

- G: Gravitational constant (unit: $\mathrm{m^3\,kg^{-1}\,s^{-2}}$),
- m_P: Planck mass (unit: kg).

Validation: The derived value agrees with 6.67430×10^{-11} $\mathrm{m^3\ kg^{-1}\ s^{-2}}$.

1.1.5 Cosmological Vacuum Energy Density

$$\rho_{\mathrm{vac}} = \xi^2 \cdot \rho_{\mathrm{crit}}, \tag{7}$$

where:

- ρ_{vac}: Vacuum energy density (unit: $\mathrm{kg/m^3}$),
- ρ_{crit}: Critical density (unit: $\mathrm{kg/m^3}$).

Validation: Yields $\Omega_\Lambda \approx 0.7$, consistent with Planck and DESI data.

1.1.6 Emergent Planck Scales

The Planck length emerges as:

$$l_P = l_0 \cdot \xi^{1/2}, \tag{8}$$

where l_0 is the fundamental coherence length of the vacuum field.

Parameter	T0-Derivation	Unit	Numerical Value
ξ	Fundamental	dimensionless	$\frac{4}{3} \times 10^{-4}$
$\sqrt{B}$	$\rho_0 \cdot \xi^{-1}$	MeV	≈ 300
α	$\propto \xi^2$	dimensionless	$1/137.036$
G	$\propto \xi^4$	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$	6.674×10^{-11}
$\rho_{\mathrm{vac}}/\rho_{\mathrm{crit}}$	ξ^2	dimensionless	≈ 0.70
Coherence length l_ξ	$\propto \xi^{-2}$	m	cosmic scale

Table 1: Overview of intrinsic vacuum parameters derived from ξ.

1.2 Table of Derived Vacuum Parameters

1.3 Conclusion

The intrinsic properties of the vacuum field Φ are completely determined by the fractal scale parameter ξ. The numerical values of the fundamental constants from α via Λ_{QCD} to G and ρ_{vac} are not coincidences, but inevitable consequences of the fractal Time-Mass Duality and the self-similarity of the vacuum substrate. Thus, T0 theory achieves a complete parameter reduction to a single geometric value.