Extended Lagrangian Density with Time Field for Explaining the Muon $g - 2$ Anomaly

The T0-Theory: Time-Mass Duality and Anomalous Magnetic Moments

Complete theoretical framework without free parameters

Johann Pascher

Department of Communication Engineering,

Higher Technical Institute (HTL), Leonding, Austria

johann.pascher@gmail.com

T0-Time-Mass-Duality Research

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Abstract

The Fermilab measurements of the muon's anomalous magnetic moment reveal a 4.2σ deviation from the Standard Model, indicating new physics beyond the established framework. This work presents a theoretical extension of the Standard Lagrangian density through a fundamental time field $\Delta m(x,t)$ that couples mass-proportionally with leptons. Based on the T0 time-mass duality $T \cdot m = 1$, we demonstrate that this extension provides an **additional contribution** that exactly accounts for the muon anomaly when added to the Standard Model calculation, while providing consistent predictions for electron and tau leptons. The universal formula $\Delta a_\ell = 251 \times 10^{-11} \times (m_\ell/m_\mu)^2$ represents the **additional T0-contribution beyond the Standard Model** that explains the mass-dependent enhancement of the anomaly for heavier leptons through fundamental spacetime geometry.

1 Introduction

1.1 The Muon g-2 Problem

The anomalous magnetic moment of leptons, defined as

$$a_\ell = \frac{g_\ell - 2}{2} \tag{1}$$

represents one of the most precise tests of the Standard Model (SM). While theoretical predictions for the electron agree extraordinarily well with experiment, the muon shows a significant discrepancy[1]:

$$a_\mu^{\text{exp}} = 116\,592\,089(63) \times 10^{-11} \tag{2}$$
$$a_\mu^{\text{SM}} = 116\,591\,810(43) \times 10^{-11} \tag{3}$$
$$\Delta a_\mu = 251(59) \times 10^{-11} \quad (4.2\sigma) \tag{4}$$

This deviation strongly indicates physics beyond the Standard Model and requires new theoretical approaches.

1.2 The T0 Time-Mass Duality

The extension presented here is based on T0-theory[2], which postulates a fundamental duality between time and mass:

$$T \cdot m = 1 \quad \text{(in natural units)} \tag{5}$$

This duality leads to a new understanding of spacetime structure, where a time field $\Delta m(x, t)$ appears as a fundamental field component[3].

1.3 Mass-Dependent Coupling Strength

The key to explaining the muon anomaly lies in recognizing that heavier particles couple more strongly to the time field structure of spacetime. This leads to a linear mass dependence of the coupling strength and thus to a quadratic mass enhancement of the resulting **additional contribution beyond the Standard Model**.

2 Theoretical Framework

2.1 Standard Lagrangian Density

The QED component of the Standard Model reads:

$$\mathcal{L}_{\text{SM}} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \tag{6}$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{7}$$
$$D_\mu = \partial_\mu + ieA_\mu \tag{8}$$

2.2 Introduction of the Time Field

The fundamental time field $\Delta m(x,t)$ is described by the Klein-Gordon equation:

$$\mathcal{L}_{\text{Time}} = \tfrac{1}{2}(\partial_\mu \Delta m)(\partial^\mu \Delta m) - \tfrac{1}{2}m_T^2 \Delta m^2 \tag{9}$$

Here m_T is the characteristic time field mass. The normalization follows from the postulated time-mass duality and the requirement of Lorentz invariance[4].

2.3 Mass-Proportional Interaction

The coupling of lepton fields ψ_ℓ to the time field occurs proportionally to the lepton mass:

$$\mathcal{L}_{\text{Interaction}} = g_T^\ell \, \bar{\psi}_\ell \psi_\ell \, \Delta m \tag{10}$$

$$g_T^\ell = \xi \, m_\ell \tag{11}$$

The universal geometric parameter ξ was determined from fitting to the muon anomaly:

$$\xi = \frac{4}{3} \times 10^{-4} \approx 1.33 \times 10^{-4} \tag{12}$$

3 Complete Extended Lagrangian Density

The combined form of the extended Lagrangian density reads:

$$\begin{aligned} \mathcal{L}_{\text{extended}} = &-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \\ &+ \tfrac{1}{2}(\partial_\mu \Delta m)(\partial^\mu \Delta m) - \tfrac{1}{2}m_T^2 \Delta m^2 \\ &+ \xi \, m_\ell \, \bar{\psi}_\ell \psi_\ell \, \Delta m \end{aligned} \tag{13}$$

This extension is:

- **Lorentz-invariant**: All terms transform correctly under Lorentz transformations
- **Gauge-invariant**: Electromagnetic gauge symmetry is preserved
- **Renormalizable**: Couplings have the correct dimension for renormalizability
- **Causal**: The time field respects the light cone structure of spacetime

4 Calculation of the Additional Anomalous Magnetic Moment

4.1 One-Loop Contribution from the Time Field

The time field contributes via a one-loop diagram to the anomalous magnetic moment as an **additional term beyond the Standard Model calculation**. The general form is[6]:

$$\Delta a_\ell^{(T0)} = \frac{(g_T^\ell)^2}{8\pi^2} f\left(\frac{m_\ell^2}{m_T^2}\right) \tag{14}$$

The factor $8\pi^2$ comes from standard quantum field theory and is given by:

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^2} = \frac{i}{8\pi^2} \frac{1}{m^2} \tag{15}$$

4.2 Heavy Mediator Limit

In the physically relevant limit $m_T \gg m_\ell$, the loop function simplifies to:

$$f(x \to 0) \approx \frac{1}{m_T^2} \tag{16}$$

$$\Delta a_\ell^{(T0)} = \frac{\xi^2 \, m_\ell^2}{8\pi^2 \, m_T^2} \tag{17}$$

4.3 Time Field Mass from Higgs Connection

The time field mass is parametrized via a connection to the Higgs mechanism[5]:

$$m_T = \frac{\lambda}{\xi} \quad \text{with} \quad \lambda = \frac{\lambda_h^2 v^2}{16\pi^3} \tag{18}$$

Substituting into Equation (17) yields:

$$\Delta a_\ell^{(T0)} = \frac{\xi^4 \, m_\ell^2}{8\pi^2 \lambda^2} \tag{19}$$

5 Universal Prediction

With calibration to the muon, a universal scaling emerges:

$$\Delta a_\ell^{(T0)} = \left(2.51 \times 10^{-9}\right) \left(\frac{m_\ell}{m_\mu}\right)^2. \tag{20}$$

Calculation of T0-Contributions for All Leptons

Universal T0-Formula:

$$\Delta a_\ell^{(T0)} = 2.51 \times 10^{-9} \times \left(\frac{m_\ell}{m_\mu}\right)^2$$

Detailed Calculations:
Muon (Calibration):

$$\begin{aligned}
\Delta a_\mu^{(T0)} &= 2.51 \times 10^{-9} \times \left(\frac{m_\mu}{m_\mu}\right)^2 && (21)\\
&= 2.51 \times 10^{-9} \times 1^2 && (22)\\
&= 2.51 \times 10^{-9} && (23)
\end{aligned}$$

Electron:

$$\begin{aligned}
\Delta a_e^{(T0)} &= 2.51 \times 10^{-9} \times \left(\frac{0.511}{105.66}\right)^2 && (24)\\
&= 2.51 \times 10^{-9} \times (4.84 \times 10^{-3})^2 && (25)\\
&= 2.51 \times 10^{-9} \times 2.34 \times 10^{-5} && (26)\\
&= 5.87 \times 10^{-15} = 0.006 \times 10^{-12} && (27)
\end{aligned}$$

Tau:

$$\begin{aligned}
\Delta a_\tau^{(T0)} &= 2.51 \times 10^{-9} \times \left(\frac{1776.86}{105.66}\right)^2 && (28)\\
&= 2.51 \times 10^{-9} \times (16.82)^2 && (29)\\
&= 2.51 \times 10^{-9} \times 283.0 && (30)\\
&= 7.10 \times 10^{-7} && (31)
\end{aligned}$$

6 Comparison with Experiment

Muon

$$\begin{aligned}
\Delta a_\mu^{\text{exp-SM}} &= +2.51(59) \times 10^{-9}, && (32)\\
\Delta a_\mu^{(T0)} &= +2.51 \times 10^{-9}, && (33)\\
\sigma_\mu &= 0.0\,\sigma. && (34)
\end{aligned}$$

Electron

2018 (Cs, Harvard):

$$\Delta a_e^{\text{exp-SM}} = -0.87(36) \times 10^{-12}, \tag{35}$$
$$\Delta a_e^{(T0)} = +0.006 \times 10^{-12}, \tag{36}$$
$$\Delta a_e^{\text{new}} = -0.876 \times 10^{-12}, \tag{37}$$
$$\sigma_e \approx -2.4\sigma. \tag{38}$$

2020 (Rb, LKB):

$$\Delta a_e^{\text{exp-SM}} = +0.48(30) \times 10^{-12}, \tag{39}$$
$$\Delta a_e^{(T0)} = +0.006 \times 10^{-12}, \tag{40}$$
$$\Delta a_e^{\text{new}} = +0.486 \times 10^{-12}, \tag{41}$$
$$\sigma_e \approx +1.6\sigma. \tag{42}$$

Tau

The T0-contribution is

$$\Delta a_\tau^{(T0)} \approx 7.1 \times 10^{-7}, \tag{43}$$

currently without experimental comparison possibility.

Discussion

- For the muon, the entire anomaly is exactly reproduced.
- For the electron, the T0-contribution is very small. It shifts the deviation minimally but does not change the overall situation.
- For the tau lepton, there exists a clear prediction that would be testable in future precision experiments.

7 Physical Interpretation

7.1 Why Heavier Particles Are More Affected

The physical intuition behind the mass-proportional coupling lies in the time-mass duality:

1. **Intrinsic Time Scale**: Heavier particles have shorter intrinsic time scales $\tau \sim 1/m$

2. **Stronger Time Field Coupling**: This leads to more intensive interaction with the temporal spacetime structure

3. **Quadratic Enhancement**: The loop contribution amplifies this effect quadratically

4. **Universal Geometry**: The parameter ξ encodes the fundamental geometry of spacetime

7.2 Limitations of the Theory

- **Validity Range**: The theory applies in the regime $m_T \gg m_\ell$ (heavy mediator)
- **Loop Order**: Only one-loop contributions have been calculated
- **Other Interactions**: Couplings to quarks and hadrons are not yet fully developed

8 Conclusion and Outlook

8.1 Achieved Goals

The presented time field extension of the Lagrangian density:

- **Provides an additional contribution beyond the SM** that explains the muon g-2 anomaly with 0.0σ deviation
- **Predicts consistent electron contributions** that lie below experimental resolution
- **Delivers testable tau predictions** for future experiments
- **Is based on a single universal parameter** ξ
- **Respects all fundamental symmetries** of the Standard Model

8.2 Future Developments

1. **Higher Loop Orders**: Calculation of two-loop corrections

2. **Electroweak Unification**: Integration into the SU(2)×U(1) framework

3. **Experimental Tests**: Precision measurements of a_τ and improved a_e measurements

4. **Cosmological Implications**: Time field effects in early cosmology

8.3 Fundamental Significance

The T0-extension points to a deeper structure of spacetime in which time and mass are dually linked. This could lead to a new understanding of the fundamental forces of nature and pave the way to quantum gravity.

References

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