

Simpson's Paradox

Jérôme Pasquier

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 <https://github.com/jpasquier/pres-simpson>

Professional Experience and Education

Current Position:

- **Biostatistician, CSL Behring (2023–Present)**

Expertise in pre-clinical research applications, including study design, drug candidate selection, and assay development.

Previous Positions:

- **Biostatistician, Unisanté, University of Lausanne (2015–2023)**

Supported statistical aspects of research, including writing protocols, performing analyses, and publishing articles.

- **Scientific Collaborator, Federal Statistical Office (2013–2015, 2009–2012)**

Focused on survey design, data analysis, and quality control for national projects.

- **Teaching Assistant, University of Fribourg (2003–2008)**

Assisted in mathematics instruction and interdisciplinary research.

Education:

- **PhD in Mathematics, University of Fribourg (2003–2008)**

Thesis: Stochastic models for gene regulation.

Nearly 60% of hospitalized COVID-19 patients in Israel fully vaccinated, data shows

Erica Carbajal - Thursday, August 19th, 2021



Of 514 patients in Israel hospitalized with COVID-19 as of Aug. 15, 59 percent were fully vaccinated, according to an Aug. 16 article from [Science](#) that cited national data tracked by Israel's largest health management organization. The figures suggest breakthrough infections may be more common than the term implies, the report suggests.

The country has one of the world's highest COVID-19 vaccination levels, with about 78 percent of those ages 12 and older fully vaccinated, mostly with the Pfizer vaccine. At the same time, Israel now has one of the highest infection rates in the world, potentially a sign of waning vaccine immunity as the highly contagious delta variant spreads, [Science](#) reports.

What Do the Data Show?

Indeed, data show that approximately 60% of patients with severe COVID-19 infection were fully vaccinated.

severe cases	severe unvax	severe vax
515	214 (41.6%)	301 (58.4%)

Data were downloaded from the [Israeli Ministry of Health Dashboard](#) on August 15, 2021, by Jeffrey Morris and can be found in a [blog post](#) by him.

Age Group	Population (%)		Severe Cases (per 100k ¹)		Effectiveness
	Unvax	Vax	Unvax	Vax	
All (≥ 12)	1302912 (18.8%)	5634634 (81.2%)	214 (16.4)	301 (5.3)	67.5%

The effectiveness of the vaccine is defined as the reduction in infection rate².

If a vaccine has an effectiveness of 80 percent:

- It does not mean that the vaccine will only work 80% of the time.
- It means that in a vaccinated population, there will be 80% fewer people infected.

$$\text{Prob}(\text{Severe} \mid \text{Vax}) = (1 - \text{Effectiveness}) \times \text{Prob}(\text{Severe} \mid \text{Unvax})$$

$$\text{Effectiveness} = 1 - \frac{\text{Prob}(\text{Severe} \mid \text{Vax})}{\text{Prob}(\text{Severe} \mid \text{Unvax})} \quad \text{Here: } \text{Effectiveness} = 1 - \frac{5.3}{16.4} = 67.5\%$$

¹ $\text{severe_}(\text{un})\text{vax_per_100k} = \frac{\text{severe_}(\text{un})\text{vax}}{\text{population_}(\text{un})\text{vax}} \times 10^5.$

² See [World Health Organization website](#).

Let's Break Down the Data by Age Group

Age Group	Population (%)		Severe Cases (per 100k)		Effectiveness
	Unvax	Vax	Unvax	Vax	
All (≥ 12)	1302912 (18.8%)	5634634 (81.2%)	214 (16.4)	301 (5.3)	67.5%
<50	1116834 (24.2%)	3501118 (75.8%)	43 (3.9)	11 (0.3)	91.8%
≥ 50					

We have good effectiveness in the <50 age group!

So, I guess that the effectiveness is very low in the ≥ 50 age group.

Let's Break Down the Data by Age Group

Age Group	Population (%)		Severe Cases (per 100k)		Effectiveness
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All (≥ 12)	1302912 (18.8%)	5634634 (81.2%)	214 (16.4)	301 (5.3)	67.5%
<50	1116834 (24.2%)	3501118 (75.8%)	43 (3.9)	11 (0.3)	91.8%
≥ 50	186078 (8%)	2133516 (92%)	171 (91.9)	290 (13.6)	85.2%

WTF?

Let's Break Down the Data by Age Group

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The effectiveness measured in the whole population is lower than the effectiveness measured in each age group.

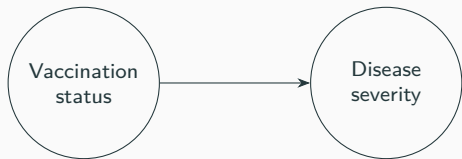
It seems paradoxical. Why do we have an effectiveness lower than 70% in the whole population while we have an effectiveness higher than 85% in each age group?

This paradoxical effect is what we call **Simpson's Paradox**.

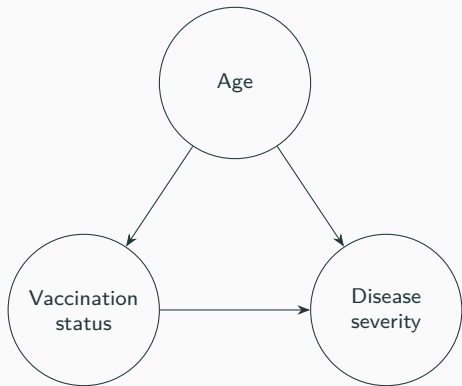
Let's Break Down the Data by Age Group

Age Group	Population (%)		Severe Cases (per 100k)		Effectiveness
	Unvax	Vax	Unvax	Vax	
12-15	383649 (67.5%)	184549 (32.5%)	1 (0.3)	0 (0)	100%
16-19	127745 (22.9%)	429109 (77.1%)	2 (1.6)	0 (0)	100%
20-29	265871 (21.1%)	991408 (78.9%)	4 (1.5)	0 (0)	100%
30-39	194213 (16.7%)	968837 (83.3%)	12 (6.2)	2 (0.2)	96.7%
40-49	145355 (13.6%)	927214 (86.4%)	24 (16.5)	9 (1)	94.1%
50-59	84545 (10.2%)	747949 (89.8%)	34 (40.2)	22 (2.9)	92.7%
60-69	65205 (8.9%)	665717 (91.1%)	50 (76.7)	58 (8.7)	88.6%
70-79	20512 (4.2%)	464336 (95.8%)	39 (190.1)	92 (19.8)	89.6%
80-89	12683 (5.7%)	208911 (94.3%)	32 (252.3)	100 (47.9)	81%
90+	3132 (6.3%)	46602 (93.7%)	16 (510.8)	18 (38.6)	92.4%

So What's the Problem?



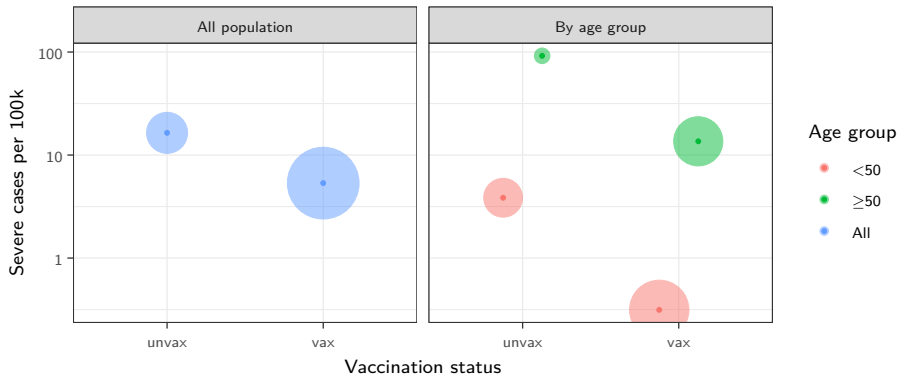
Confounding



Unbalanced Data

The proportion of vaccinated people in each age group is different.

Age Group	Vaccinated (%)
<50	75.8%
≥50	92.0%



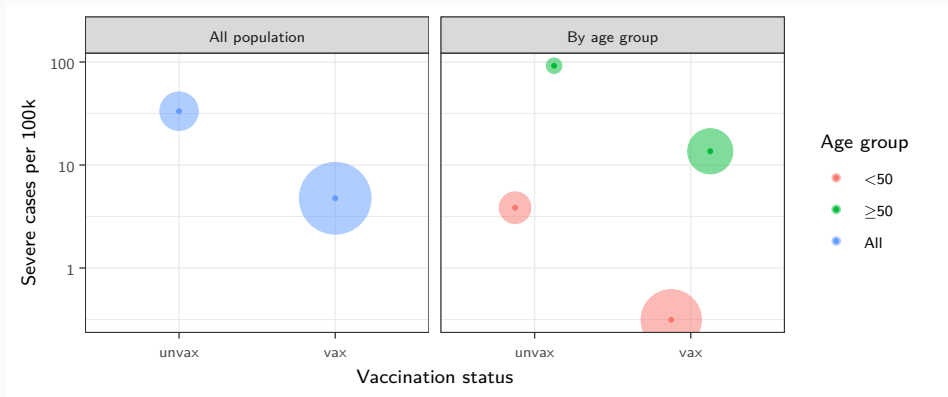
Balanced Vaccination

Age Group	Population (%)		Severe Cases (per 100k)		Effectiveness
	Unvax	Vax	Unvax	Vax	
All (≥ 12)	1302912 (18.8%)	5634634 (81.2%)	214 (16.4)	301 (5.3)	67.5%
<50	1116834 (24.2%)	3501118 (75.8%)	43 (3.9)	11 (0.3)	91.8%
≥ 50	186078 (8%)	2133516 (92%)	171 (91.9)	290 (13.6)	85.2%

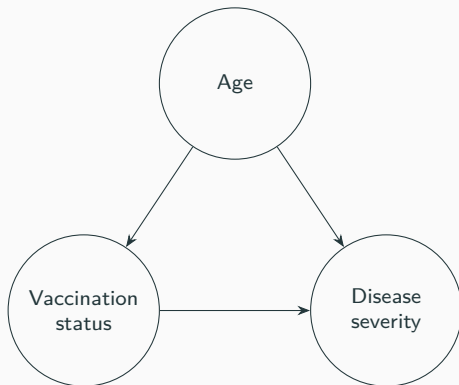
Let's maintain the original effectiveness in each age group while harmonizing the vaccination rate. The number of serious cases is recalculated accordingly. Finally, the effectiveness is recalculated for the population as a whole.

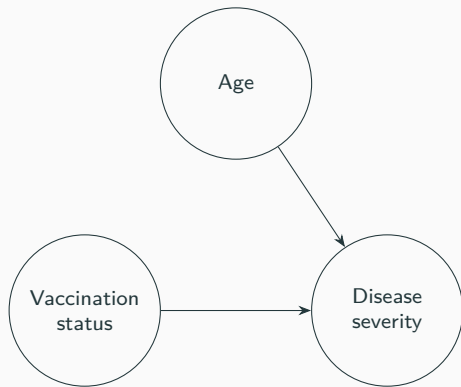
Age Group	Population (%)		Severe Cases (per 100k)		Effectiveness
	Unvax	Vax	Unvax	Vax	
All	1302912 (18.8%)	5634634 (81.2%)	434 (33.3)	268 (4.8)	85.7%
<50	867278 (18.8%)	3750673 (81.2%)	33 (3.9)	12 (0.3)	91.8%
≥ 50	435633 (18.8%)	1883960 (81.2%)	400 (91.9)	256 (13.6)	85.2%

Balanced Vaccination



Confounding





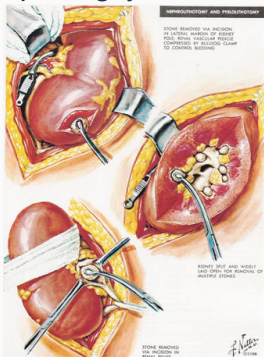
Balancing the data removed the confounding.

Comparison of treatment of renal calculi by open surgery, percutaneous nephrolithotomy, and extracorporeal shockwave lithotripsy

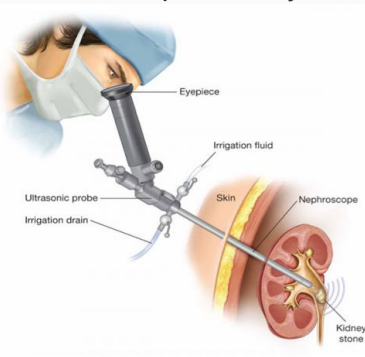
Charig et al.
Br Med J (Clin Res Ed)
1986

C R CHARIG, D R WEBB, S R PAYNE, J E A WICKHAM

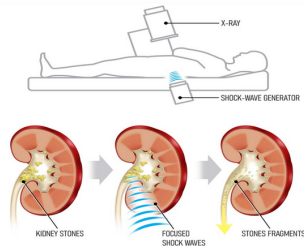
Open Surgery



Percutaneous Nephrolithotomy



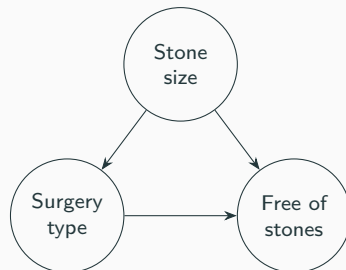
Extracorporeal Shock Wave Lithotripsy



Kidney Stones Data

Data from the Article:

Surgery	Stone Size	N	Success	Failure	Rate
open	small	87	81	6	0.93
pcnl	small	270	234	36	0.87
open	large	263	192	71	0.73
pcnl	large	80	55	25	0.69
open	overall	350	273	77	0.78
pcnl	overall	350	289	61	0.83



Statistics (Risk):

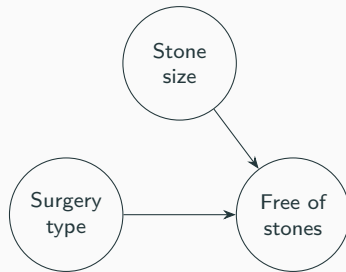
Stone Size	Success Open	Success PCNL	Risk Difference	Risk Ratio	Odds Ratio
small	81/87	234/270	-0.06	0.93	0.48
large	192/263	55/80	-0.04	0.94	0.81
overall	273/350	289/350	0.05	1.06	1.34

Kidney Stones Data - Balanced

Balanced Data:

(Keep the rate in groups, balance the data, and recalculate the number of successes and failures.)

Surgery	Stone Size	N	Success	Failure	Rate
open	small	175.00	163.00	12.00	0.93
pcnl	small	175.00	152.00	23.00	0.87
open	large	175.00	128.00	47.00	0.73
pcnl	large	175.00	120.00	55.00	0.69
open	overall	350.00	291.00	59.00	0.83
pcnl	overall	350.00	272.00	78.00	0.78



Statistics (Risk):

Stone Size	Success Open	Success PCNL	Risk Difference	Risk Ratio	Odds Ratio
small	163/175	152/175	-0.06	0.93	0.49
large	128/175	120/175	-0.05	0.94	0.80
overall	291/350	272/350	-0.05	0.93	0.71

Kidney Stones Data - Logistic Regression

Model 1: $\text{logit}(\text{free of stones}) = \beta_0 + \beta_1 \times \text{surgery type}$

Model 2: $\text{logit}(\text{free of stones}) = \beta_0 + \beta_1 \times \text{surgery type} + \beta_2 \times \text{stone size}$

Original Data, Model 1

Term	OR	2.5%	97.5%
(Intercept)	3.55	2.77	4.59
PCNL Surgery	1.34	0.92	1.95

Balanced Data, Model 1

Term	OR	2.5%	97.5%
(Intercept)	4.93	3.76	6.59
PCNL Surgery	0.71	0.48	1.03

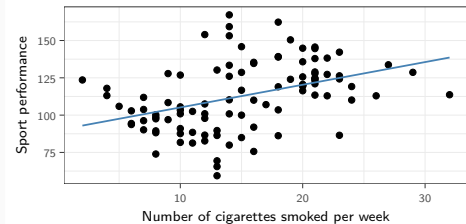
Original Data, Model 2

Term	OR	2.5%	97.5%
(Intercept)	2.81	2.17	3.68
PCNL Surgery	0.70	0.45	1.09
Small Stone	3.53	2.22	5.68

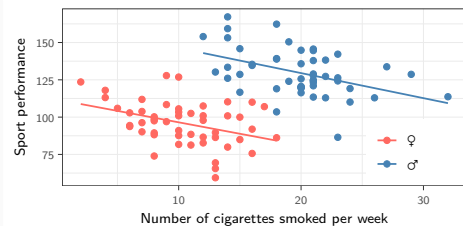
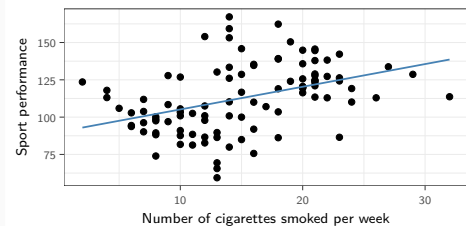
Balanced Data, Model 2

Term	OR	2.5%	97.5%
(Intercept)	2.94	2.17	4.05
PCNL Surgery	0.69	0.47	1.02
Small Stone	3.73	2.47	5.73

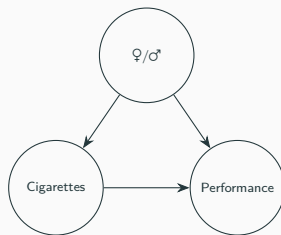
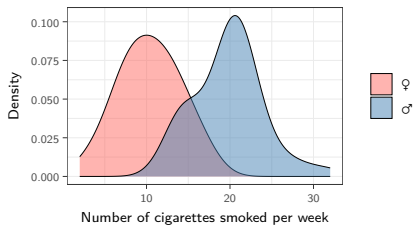
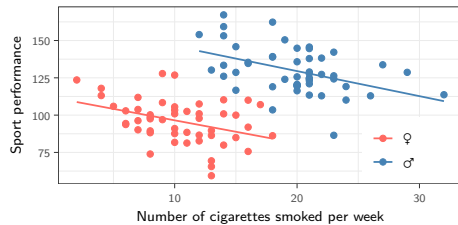
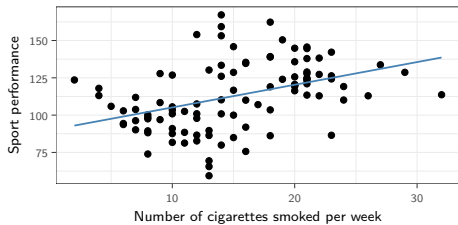
Latest Example (Seen on an ARTE Program)



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- Simpson's paradox is (normally) avoided in randomized trials.
- However, it can be a serious problem in observational studies.
- Good statistics require more than just mathematics. Statisticians need to work closely with specialists in the field from which the data comes.
- Mathematics allows us to understand the world as long as we remember that it exists.

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Thank you for your attention!

References

- [1] Stefanos Bonovas and Daniele Piovani. "Simpson's Paradox in Clinical Research: A Cautionary Tale". In: *Journal of Clinical Medicine* 12.4 (Feb. 2023), p. 1633.
- [2] Erica Carbajal. *Nearly 60% of Hospitalized COVID-19 Patients in Israel Fully Vaccinated, Data Shows*. <https://www.beckershospitalreview.com/public-health/nearly-60-of-hospitalized-covid-19-patients-in-israel-fully-vaccinated-study-finds.html>. Aug. 2021.
- [3] C. R. Charig et al. "Comparison of Treatment of Renal Calculi by Open Surgery, Percutaneous Nephrolithotomy, and Extracorporeal Shockwave Lithotripsy.". In: *Br Med J (Clin Res Ed)* 292.6524 (Mar. 1986), pp. 879–882.
- [4] Jeffrey Morris. *Israeli Data: How Can Efficacy vs. Severe Disease Be Strong When 60% of Hospitalized Are Vaccinated?* <https://www.covid-datascience.com/post/israeli-data-how-can-efficacy-vs-severe-disease-be-strong-when-60-of-hospitalized-are-vaccinated>. Aug. 2021.
- [5] Denis van Waerebeke and Cédric Piktoroff. *Voyages au pays des maths - Le paradoxe de Simpson*. ARTE, 2022.
- [6] WHO. *Vaccine Efficacy, Effectiveness and Protection*. <https://www.who.int/news-room/feature-stories/detail/vaccine-efficacy-effectiveness-and-protection>. July 2021.