State of the Art and Challenges in Operations of Rideshare Transportation Systems

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Outline

1. Introduction and Motivation
2. State-of-the-Art of Ridesharing Routing Systems
3. State-of-the-Art of Ridesharing Repositioning Systems
4. Research Gaps

1

Introduction & Motivation

Ridesharing Control Systems and Related Fields

Background of Ridesharing System Controls

- The efficiency and efficacy of ridesharing systems requires a fleet of vehicles to be controlled such that they can serve as many customers as possible (revenue/service level) with as few resources as possible (cost/efficiency)
  - Costs Primarily Include:
    - Number of Vehicles
    - Fuel Costs
- The success of ridesharing systems relies heavily on participation by a user base. [54]

Pickups and drop offs from a particular studied neighborhood in NYC [11].

Background of Ridesharing System Controls

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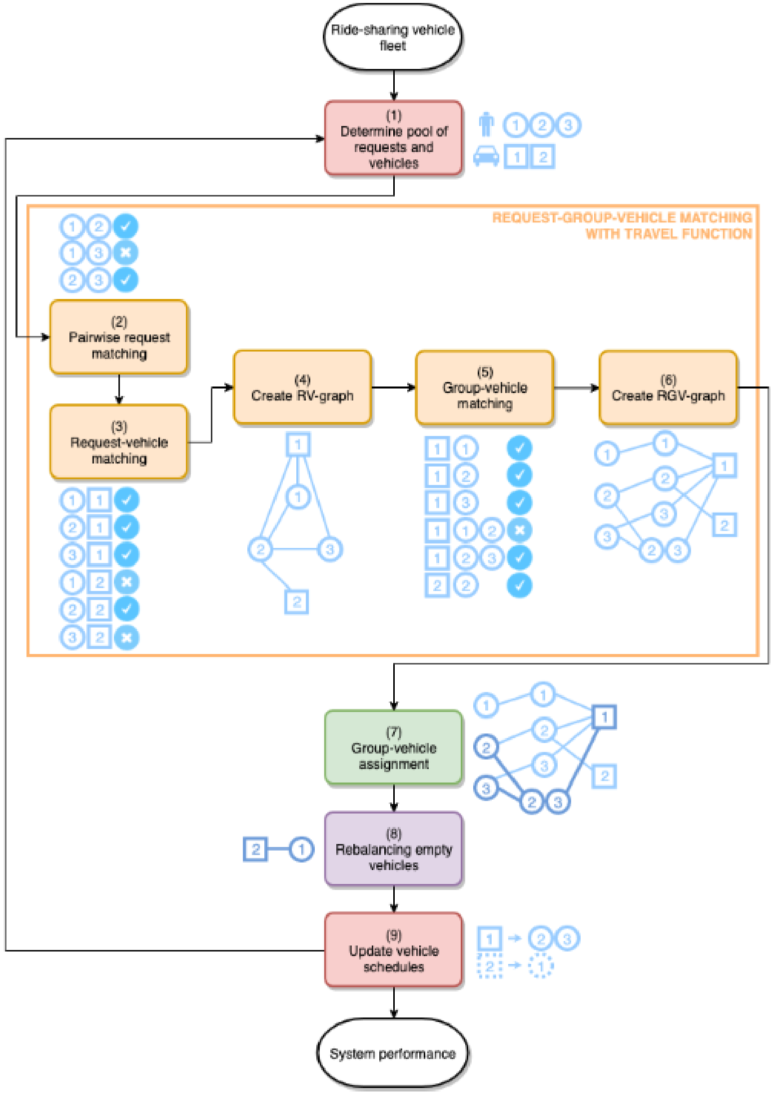
- Ridesharing control level systems have been commonly researched and designed beginning with dial a ride problems as early as the 1970s [17]
- Gains in computing power and optimization problem solving utilities/methodologies have allowed for significant growth in the complexity and accuracy of control schemes
- Large-scale commercial Transportation Network Companies (TNCs) have proven somewhat viable in the market: Uber, Lyft, Didi, etc. [55]

A typical rideshare GUI [55].

Motivation for Studying Routing and Repositioning

Topic	Content
Ridesharing Motivation	• Pooled ridesharing has been shown to decrease pollution, parking requirements, and traffic in simulations [56] • The efficiency of a ridesharing system, and its ability to meet demand is primarily dependent on fleet size and how vehicles are utilized within a network area [57]
Definitions	• Routing is ordering of pickup and drop-offs vehicles make and how the vehicle traverses a road network to reach them • Repositioning is moving empty vehicles to more optimal locations to account for future demand
Routing and Repositioning Motivation	• Routing and repositioning systems are important not only generally across ride share systems but are important to adjust according to the goals associated with each system [3] • Routing and repositioning directly impact the customer/rider experience through level-of-service (LoS) trip characteristics [58]



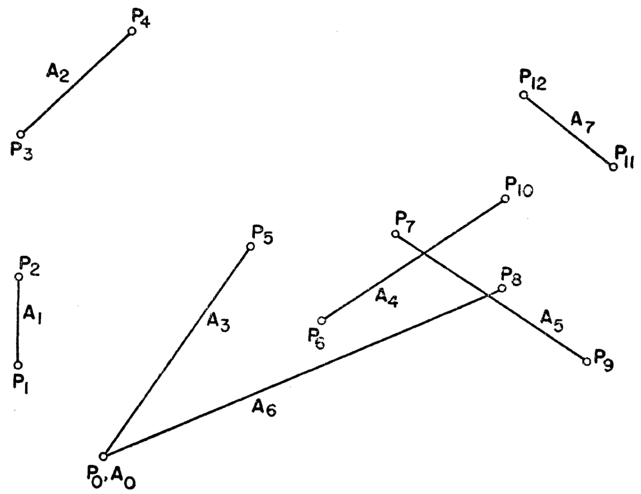
Outline of system architecture for an example rideshare simulation [58].

2

State-of-the-Art of Ridesharing Routing Systems

State-of-the-Art: Routing Systems – Problem Types

Problem Type	Characteristics
Vehicle Routing Problem (VRP)	• Originally formulated in 1959 for routing trucks and freight [12] • Most commonly used for depot/customer routing • NP-Hard problem formulation • Based on traveling salesman problem which has unclear origins
Traveling Salesman Problem (TSP) [32]	• Formulated to identify best set of trips to take within a tour • Not necessarily associated with actual turn-by-turn routing
Dial-a-ride problem (DARP)	• Formulated during the 1970's fuel crisis as means of reducing fuel usage [17] • Specific to joining riders in shared vehicles • Requires different formulation of VRP due to no centralized depot



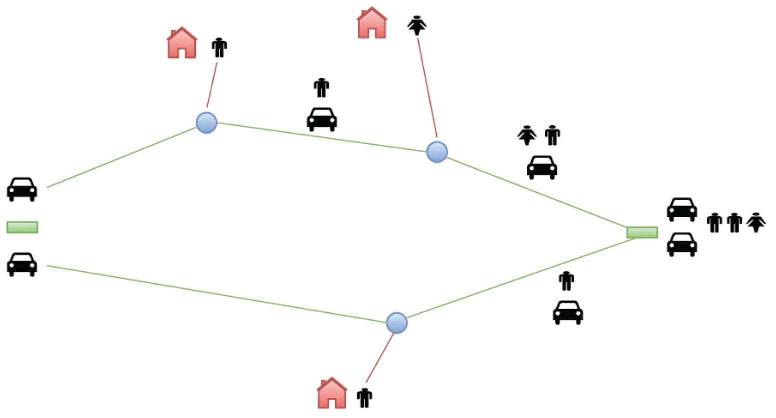
An example VRP solution [12] .

State-of-the-Art: Routing Systems – VRP vs DARP

Component	VRP	DARP

Objective Function	• Typically targeted towards freight and therefore cost is primary basis for optimization [12]	• Targeted towards environmental savings [14]
		  • Riders are introduced (vs material in freight)
		  • Rider satisfaction becomes highly important
		    • Can be objective or constraint

Constraints	• Usually requires satisfying delivery time windows [12] • Uses preplanning and often offline optimization for destination locations [12,18]	• Typically planned dynamically with little ability to preplan [13] • Can sometimes need to meet floating targets [13]
	• Refueling and charging operations can be conducted with freight on board but due to time constraints not typically with passengers	
Solution Methods	• Similar solution methods for each • Fundamentally same problem but with different time and event horizons	



A diagram displaying a floating target pickup procedure [13].

State-of-the-Art: Routing Systems-Importance and Problem Overview

- Vehicles in a ridesharing system must be told how to get from their current location to a pickup location, and then from the pickup location to the destination location [59]
  - Many different optimization targets possible [59]
  - Significant differences in constraints relative to problem basis type and powertrain
    - DARP vs Freight, Gas vs Electric vehicles [9]

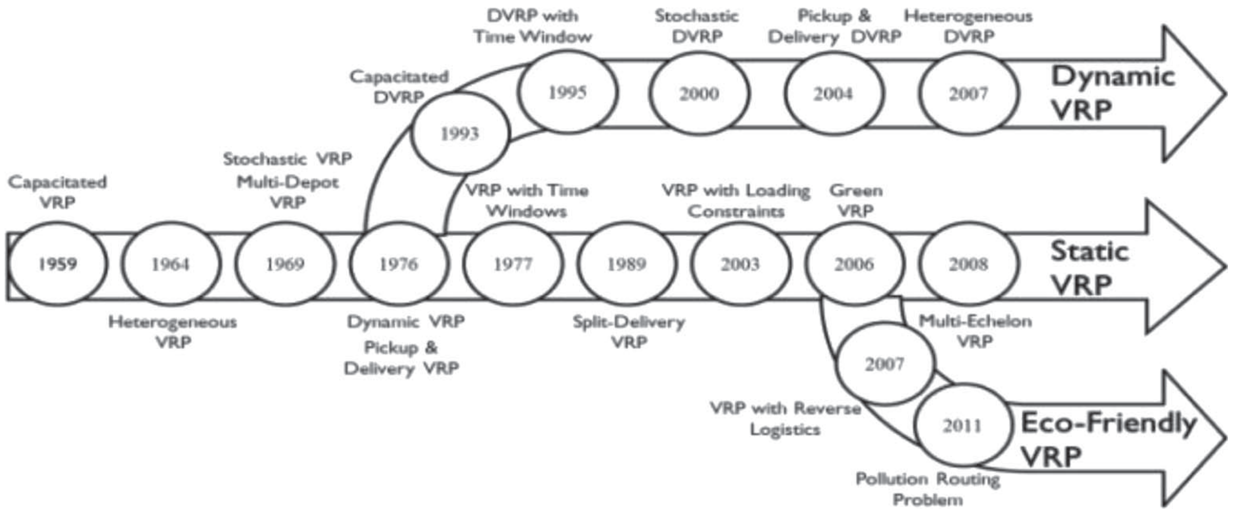
- Notes
  - Routing algorithms attempt to solve NP-Hard problems and must do so in real time to function in ridesharing systems [29]
  - Routing has massive effects on trip service level characteristics [26]

State-of-the-Art: Routing Systems – Optimization Formulation

Factor Type	Info

Cost	• Environmental [15]
	  • Distance
	  • Speed
	  • Route characteristics
	  • Traffic
	• Fuel [15]
	• Electricity [9,65]

Time	• Window [64] • Travel time
Fleet Related	• Fleet size • Fleet composition [19]
Driver Related	• Driver Cost [66]

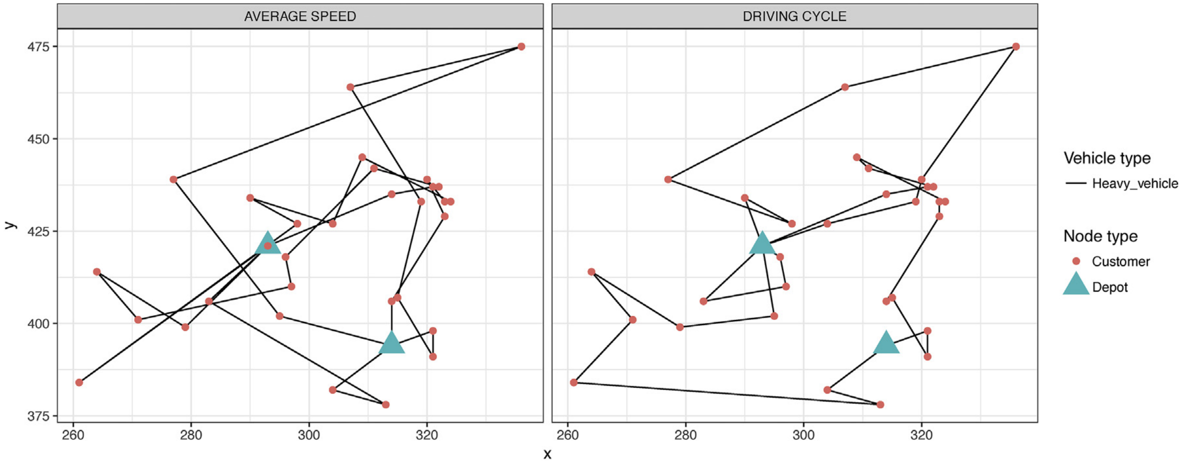


A timeline depicting the complexity and growth in routing objective functions over time [18].

State-of-the-Art: Routing Systems – Optimization Formulation Details

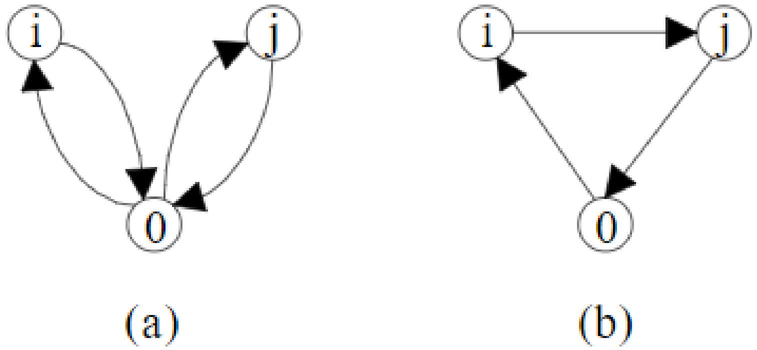
- Emissions estimation is critical for optimization, but its accuracy is correlated with level of detail [15]
  - Average speed estimation [61]
    - Use average speed in area as vehicle speed
    - Does not consider start-stop, acceleration, microscopic traffic
    - Time horizon
      - Smaller time periods yield more accurate results but require more frequent recalculation
  - GPS data
    - Use actual vehicle data to model likely emissions
    - [60]

- Heterogenous vs homogenous fleets
  - Some vehicles less efficient than others
  - Heterogenous fleets more accurate in case of existing TNC providers
- NOx vs CO2 [62]
  - CO2 emissions highly dependent on fuel consumption but NOx are not
- Driving cycles [63]
  - EPA estimates vs real world performance

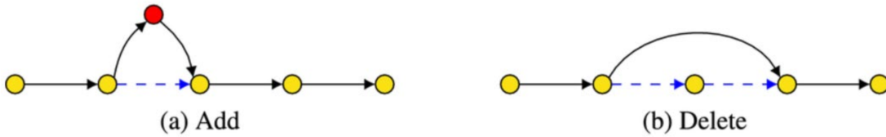


Comparison of optimal routes with emissions calculated first through average speed and then through driving cycle [15].

Method	• Details
Clarke Wright Savings Algorithm [22]	• $D_a = c_{0i} + c_{i0} + c_{0j} + c_{j0}$ • $D_b = c_{0i} + c_{ij} + c_{j0}$ • $S_{ij} = D_a - D_b = c_{i0} + c_{0j} + c_{ij}$ • Formulate Cost Table • Evaluate savings for each pair of destinations • Utilize highest saving pairs until capacity is exceeded
Insertion and deletion heuristics [23]	• Iterate through each tour • Add a constraint by which to check the addition or deletion of a stop to each potential tour $\sum_{i=0}^{n}\sum_{j=0}^{n} a_{ij}x_{ij} > b$ where b is a generalized additional constraint • Ability to “undo” previous decisions increases algorithm's ability to make near optimal decisions over that of a traditional insertion algorithm



Visualization of trip pair to be evaluated by Clarke Wright [22].



An illustration of an insertion and deletion step [70].

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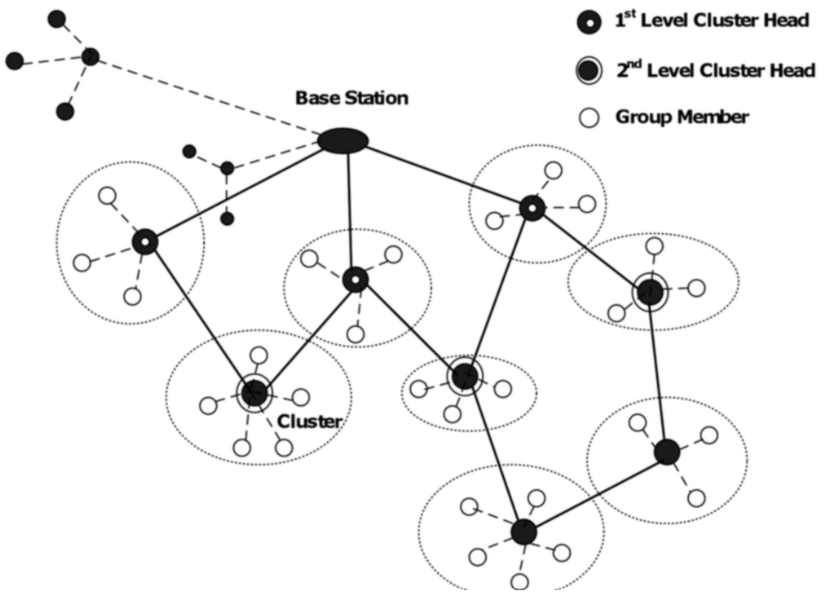
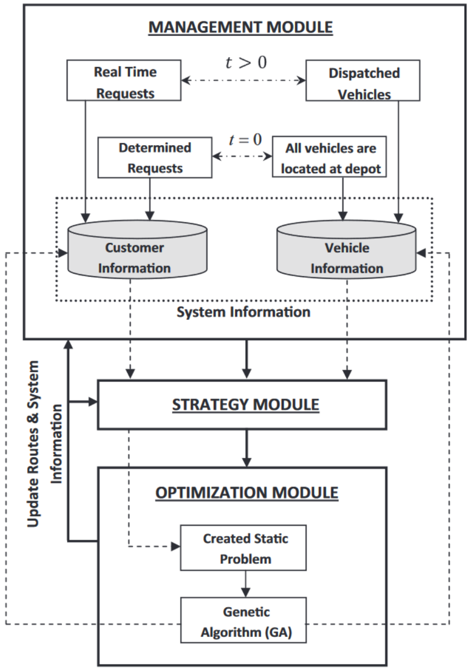
Method	• Details
Clustering [24]	• Built on Clarke-Wright savings algorithm • After checking the savings of all customers added to tour, consider each cluster as a new "customer" and include it in the next savings calculation iteration • The addition of clusters as new customers allows for customers in similar areas to be aggregated, reducing computational complexity
Purpose specific heuristics [65]	• Build on existing VRP problem with unlimited vehicle capacity • Define a vehicle capacity and partition clusters of stops • Calculate fuel consumption cap as $\left(1+\frac{2}{\beta}\right) * b * \left(\sum_{i=1}^{n} d_i\right) + \left(1+\frac{\beta}{2}\right) a\lvert C\rvert + \frac{2a\left(\sum_{i=1}^{n} d_i\right)}{Q}$ where Q is vehicle capacity, β is a positive, rational number, C is the length of an optimal tour, i is a cluster of stops, b and a are weighting coefficients • Calculate the minimum partition length and return to simulation



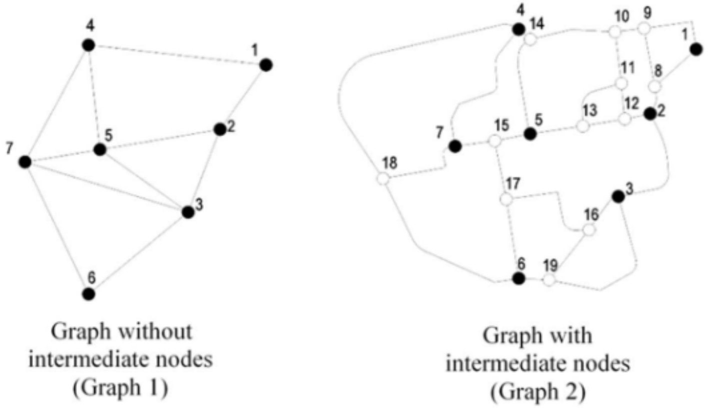
An example clustered routing destination set [71].

State-of-the-Art: Routing Systems – Solution Methods Details

Method	• Details
Metaheuristics [64]	• Modular separation of key areas of optimization process • Management module: contains statuses of vehicles and agents • Strategy module: divides information into time step chunks for optimization • Optimization module: executes genetic algorithm-based optimization procedure where phenotypes contain ordered pickup sets among possible options
Multiple Integer Linear Program (MILP) Solvers	• Formulate problem as MILP such that numerical exact solver can be used to obtain exact solution • $\min\{c_d * \sum_{i \in N} \sum_{j \in N} (d_{ij} * x_{ij}) + c_s * \sum_{i \in N_s} (y_i) + c_v * m$ where c_d is the travel distance cost, d_{ij} is the road length, x_{ij} is the decision variable as to whether to use an arc, c_s is the cost of purchasing and implementing a charging station, y_i is the decision variable relating to the location of a charging station on arc i, c_v is the vehicle purchase cost, and m is the number of assigned vehicles



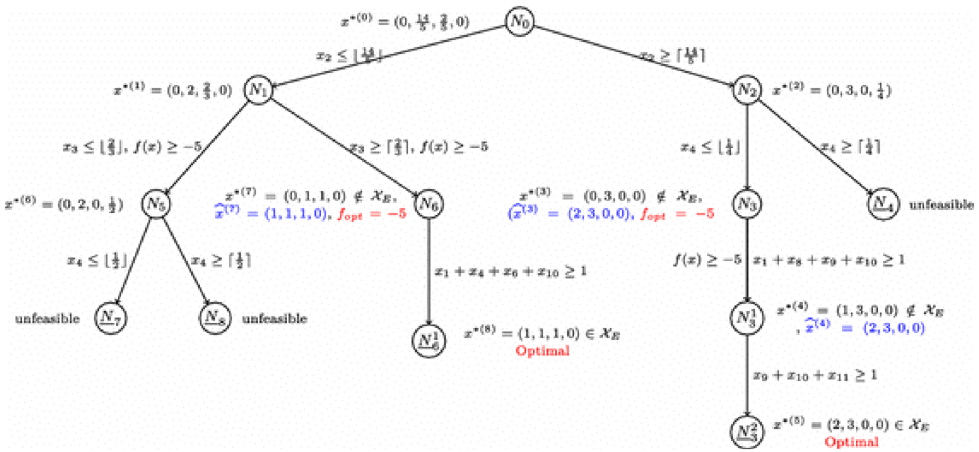
The system architecture of a metaheuristic-based system [64].



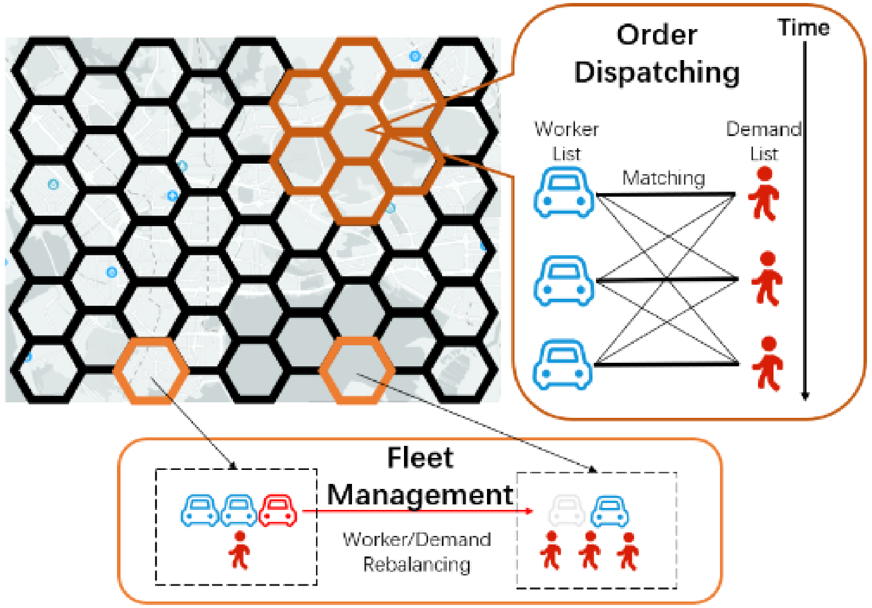
The intermediate node technique utilized to reduce complexity and allow CPLEX to more easily solve the MIP problem [68].

State-of-the-Art: Routing Systems – Solution Methods Details

Method	• Details
Branch cut algorithms [67,69]	• Problem is relaxed into linear program without integer constraints • Problem is solved with simple linear program solver • If solution violates integer constraints, heuristics are applied to remove subtours that violate these inequalities • Use solutions that do not yield integer values as upper bounds and solutions that do satisfy constraints as lower bounds • Add integer constraints back into problem until they are violated and cut the tree at this point
Reinforcement Learning Approaches [7]	• Service area divided into geometric grid with travel to adjacent cells representing actions • Travel time and cost utilized to represent cost of action • Reward function determined by high level criterion such as predicted demand in cell at timestamp $t+1$ • Cost and demand functions passed to meta-reinforcement learning algorithm • $\min \sum_{m=1}^{\lvert V \rvert} \sum_{i=0}^{\lvert C \rvert} \sum_{j=1}^{\lvert C \rvert+1} c_{ij} x_{ijm}$



A tree with cuts added, illustrating the addition of inequality constraints to evaluate solution branches [72].



Visualization of reinforcement learning grid applied to area [7].

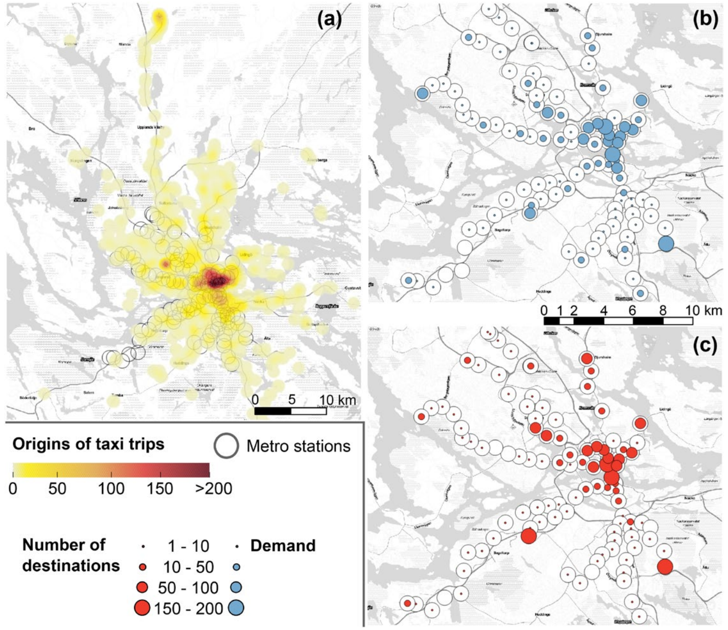
Research Gap Identification I: Routing Systems

Topic	Remarks	Gap Identification
Routing Systems	• Optimization most commonly targets cost reduction associated with vehicle travel time and distance • Targets based on technical or vehicle centric objectives • Approaches optimize technical objectives but do not consider passenger acceptance as variable	• **Optimization formulation is incomplete and does not consider other factors from human perspective that may effect passenger acceptance** • **Lack of a passenger acceptance model in rideshare to comprehensively understand the passengers' choices in rideshare** • **Lack of optimization approaches addressing the tradeoff between technical objectives and passenger acceptance**

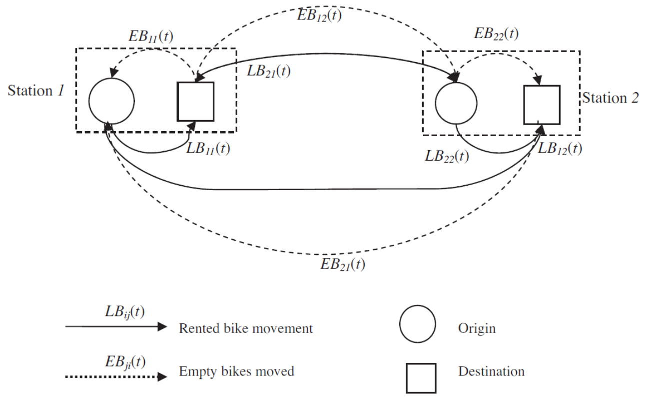
3

State-of-the-Art of Ridesharing Repositioning Systems

System Type	Application Details
SAV's [1, 3, 25, 27, 28, 30, 31, 33-39, 43, 45, 46]	• Typical target to maximize service level (minimize trip rejection rate) for occupants • Dynamic or static updates (DRS)
Bike sharing [40-44]	• Structured more like inventory management problem • Less dynamic, often utilize static demand forecasting, once a day repositioning • Utilize stations rather than individual pickup/dropoff locations



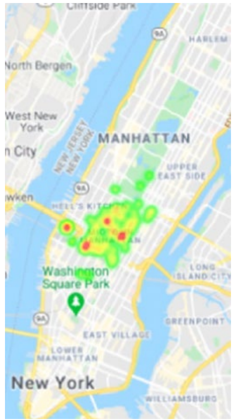
An example taxi redistribution network [28]



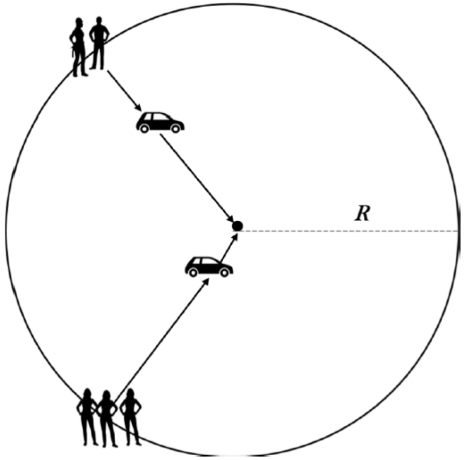
A subsection of a bike distribution system model [41].

- Dynamic ridesharing (DRS) operations typically have geographically unbalanced demand [3,29]
  - Trip destinations leave vehicles “stuck” in areas with low demand
  - Areas with high demand fall outside wait time constraint distances
  - Example: Rush hour

- Notes
  - Causes requests to be denied [3,26,30,35,39-42,44]
    - Wait time cap (LOS metric) is greater than distance from car to pickup locations
  - Lowers ability to share trips [36]
    - Lower trip density leads to less opportunities for matching trips



An example area with unbalanced demand [3].



An illustration of the unbalanced problem [29].

State-of-the-Art: Repositioning – Demand Forecasting

Demand Forecast Type	Details		
Demand known [25,40,31,42,45]	• Demand generated from publicly available datasets		
Demand predicted	Static	Zonal	Dynamic (Outline) [43]

	• Demand is estimated based on previous data and is not updated based on system state [25-28]	• Fixed vehicle requirements are applied to geographical zones within the area [45]	• Vehicle requirements are updated based on the current system state
			• Reactive
			  • Vehicle requirements are based on current system state
			• Without lookahead
			  • The state changes made by repositioning vehicles are not considered
			• With lookahead
			  • The effect of actions taken at current time are used to forecast future states

State-of-the-Art: Repositioning – Demand Forecasting

Demand Prediction Type	Features
Static	Min-cost-flow [45] Demand predicted zonally at beginning of day and evaluated with Hungarian bipartite graph minimization [47]
Dynamic Reactive	Travel to closest request [1] Aggregate trips to virtual stations and respond to demand imbalance [30] Vehicles sent towards nearest rejected request [31]
Dynamic w/o Lookahead	LSTM/Neural Network Prediction [27,34,38] Dynamic Transportation Problem Algorithm [33,37,39] Particle Filter Based Demand Estimation [35, 36,43]
Dynamic w/ Lookahead	Reward based on movement to areas with high future demand [3,30] Artificial requests added to network based on future demand prediction

Zonal demand aggregation [30].

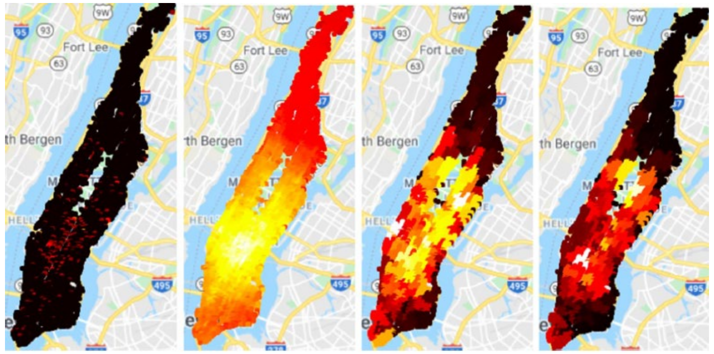
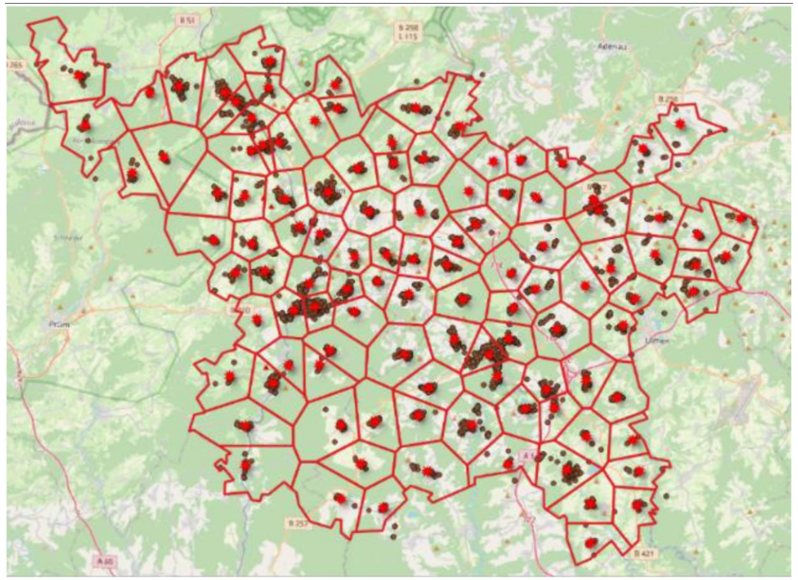


Illustration of changes in reward across NYC simulation area [3].

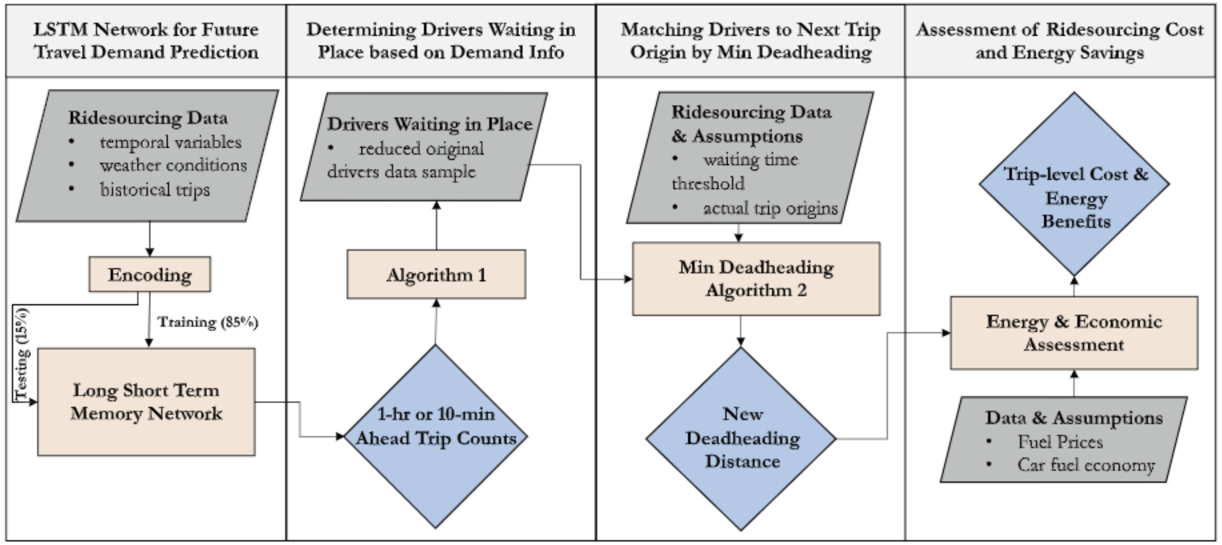
State-of-the-Art: Repositioning – Optimization Formulation

Method	Features
Heuristic	• Go to station with passenger with maximum waiting time [28] • Go to zones with rejected requests [3,31] • Evenly redistribute vehicles over area [47] • Distribute vehicles to locations in which recent requests occurred [47]

Cost Centric	• Minimize cost [3,25,26,40,43]
	    • Minimize repositioning cost [3,25, 31,43]
	        • Minimize fuel costs
	        • Minimize repositioning travel time [43]
	    • Minimize holding costs [41]
	    • Minimize assignment cost [1,3]
	    • Minimize deprecation cost [25]
	    • Maximize revenue [25,41]
	• Minimize deadheading [27]
	• Minimize travel time [31]



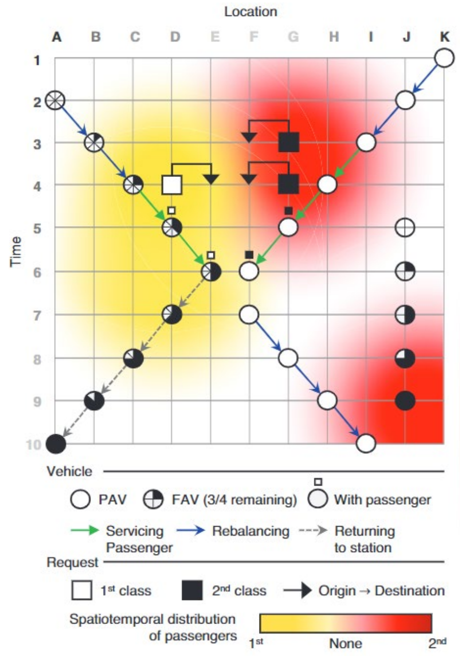
Zonal mapping of requests in network [47].



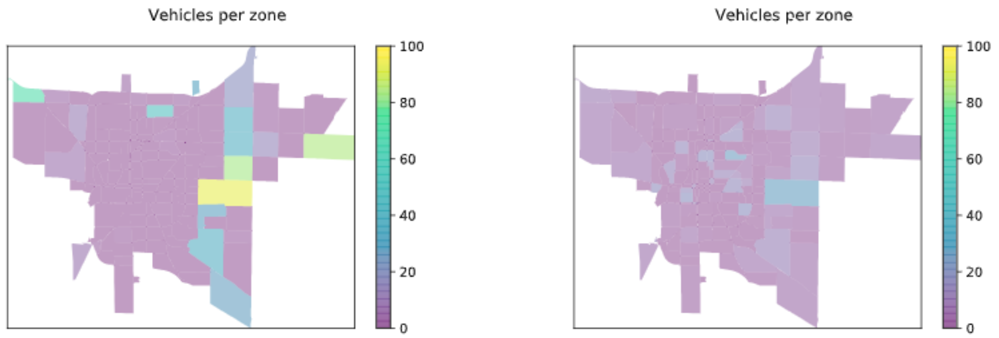
An illustration of the computational architecture proposed for a minimize dead-heading targeted system [27].

State-of-the-Art: Repositioning – Optimization Formulation

Method	**Features**
Service Centric	• Minimize passenger delay [26,43] • Minimize make span (time to service all requests) [42] • Minimize rejection rate [3,26,30,35,40-42,44] • Maximize number of expected requests that can be served by vehicles in the same zone at then end of a time horizon [36] • Minimize arrival time [38] • Minimize waiting passengers at node [43]
Geographic	• Balance trip flows and counter flows [33,34,37,39,45,47] • Reward vehicles for traveling to locations with expected future demand [38]



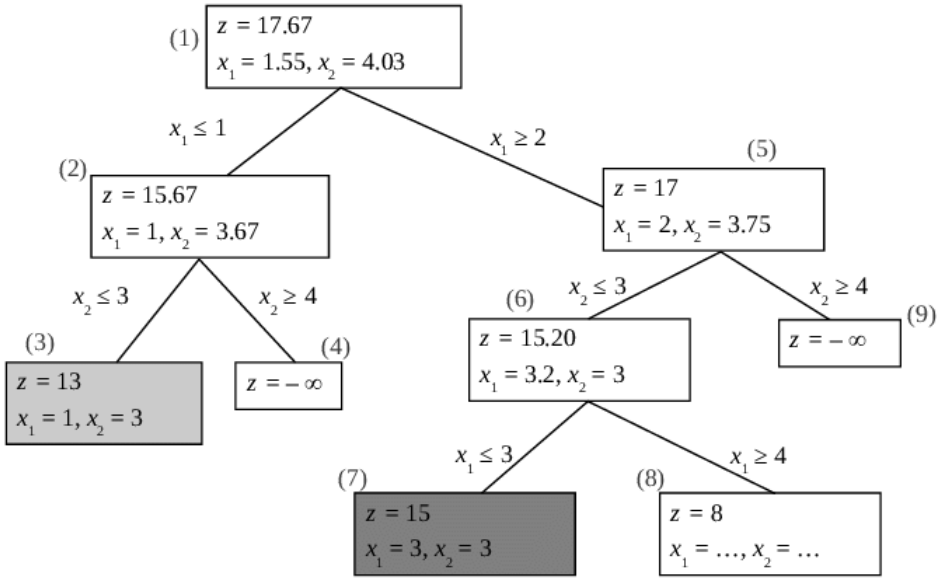
A spatiotemporal map illustrating the proposed use of a multi-class-based system formulated to address service level objectives [26].



Comparison of geographic vehicle distribution with and without repositioning applied [39].

State-of-the-Art: Repositioning – Solution Methods Details

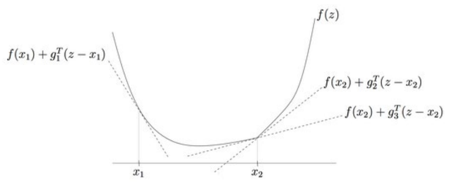
Method	• Details
Exhaustive calculation [27]	• Calculate all possible solutions and only utilize the best one
Heuristic	• Index based redistribution [28] • Indices assigned based on maximal passenger waiting time • Heuristic nearest neighbors [28] • Estimated demand flows computed and idle vehicles with minimum cost to satisfy flow requirements are moved[45] • Potential destination zones are ranked for each vehicle by demand supply ratio and vehicle is sent to highest rank zone
Integer linear programming [3,24,31,35,36,39]	• An exact solution is obtainable via branch and bound tree search [26] • An approximate optimum is calculated through branch-and-bound and incremental optimal method [36] • Complex integrality constraints removed, problem solved, and then optimal solution in first stage fixed and used to solve second stage (MILP) [40]



A graphical representation of a branch and bound ILP solution [74].

State-of-the-Art: Repositioning – Solution Methods Details

Method	• Details
Lagrangian Relaxation Algorithm [25,43]	• Relax constraints into objective function by turning them into penalty terms with Lagrangian coefficients • Iteratively calculate feasible solutions within an upper and lower bound • Update Lagrange multipliers • Use sub-gradient method [49]
Convex relaxation [50]	• Sample average approximation [51] formulated through Monte Carlo simulation used to relax constraint and transform problem into simple LP [34]
Model predictive control [34,38]	• Implement optimization over finite time horizon taking into account how actions effect that time horizon and iterate at each time step



An illustration of a sub gradient method, a process of evaluating a complex gradient with estimator lines [73].

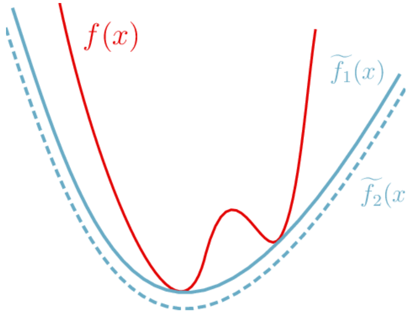
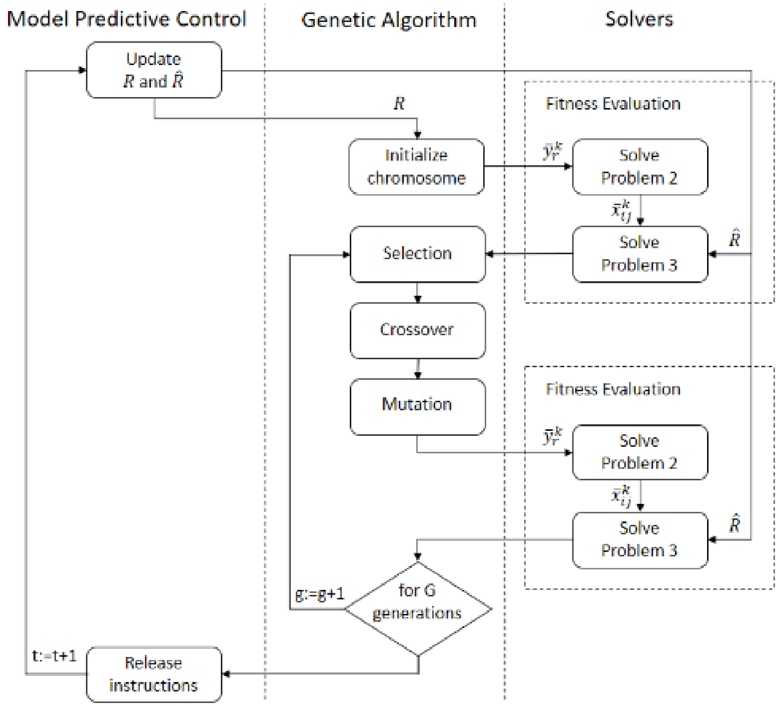


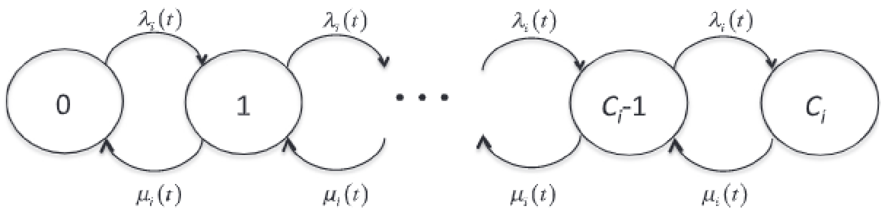
Illustration of a convex relaxation of f to $\tilde{f}$ [75].



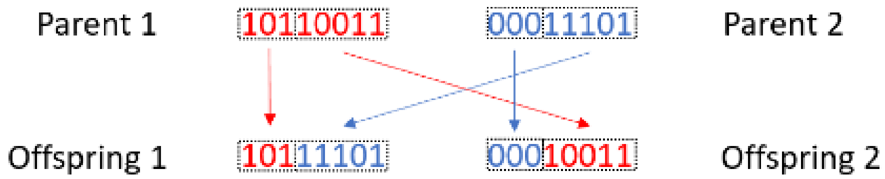
System architecture of combined MPC, GA solution structure for a rebalancing (repositioning) system [38].

State-of-the-Art: Repositioning – Solution Methods Details

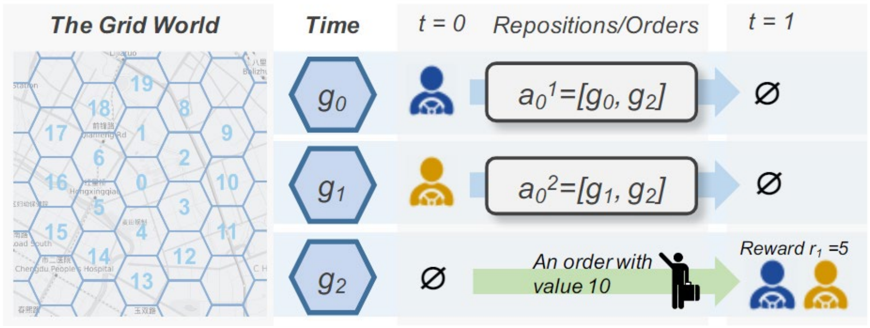
Method	• Details
Markov Chain Formulation [42]	• Decompose routing problem into clustering problem and routing problem for each individual vehicle • Assign clusters of stations $i \in S$ to each vehicle $v \in V$ such that rebalancing is completed as quickly as possible $\min H$ where $H = \max_{v \in V} h_v$ • For each v assigned set of stations, minimize H_v within its set of stations • Calculate total make span $\widetilde{H}$ as maximum of each individual vehicles make spans
Genetic Algorithm [38]	• Sample average approximation [51] formulated through Monte Carlo simulation used to relax constraint and transform problem into simple LP [34]
Reinforcement learning approaches (DQN) [30]	• Define reward function based on expected discounted returns calculated from average revenue of all vehicle agents arriving in each grid square $\mathbb{E}[\sum_{k=0}^{\infty} \lambda^k r_{t+k}^i]$ for vehicle i, time t, discount factor λ, action k • Contextual: Limit available actions by restricting movement and by prioritizing staying still over moving



Markov Chain Formulation structure [42].



An example mutation operation in a genetic algorithm solution method [38].



Grid world and action example [30].

Research Gap Identification II: Repositioning Systems

Topic	Remarks	Gap Identification
Repositioning Optimization	• Repositioning most often optimized to minimize the number of rejected requests within a network • Approaches minimize technical objectives such as cost and denied requests. • Repositioning is mostly considered as an independent optimization process at a sub-system level	• **No consideration of heterogenous tolerances among riders, varying by trip purpose, demographic characteristics etc.** • **Lack of tolerance model and optimization to trade off trip acceptance with technical objectives** • **Lack of consideration of repositioning as a sub-system subject to joint optimization with other sub-system components**

Research Gap Summary

Topic	Research Gaps
Routing	• Optimization formulation is incomplete and does not consider other factors “from human perspective” that effect people's choices • No tradeoff of passenger acceptance with technical objectives for complete picture of profitability • No passenger acceptance model used within optimization to understand tradeoffs between cost and ridership objectives
Repositioning	• No consideration of heterogenous wait time tolerances among riders, varying by trip purpose, demographic characteristics etc. • Lack of consideration of repositioning as subsystem subject to joint optimization with other component subsystems • Lack of tolerance model to trade off trip acceptance with technical objectives • No evaluation of PR system in the context of a complete transportation system including public transit, microtransit

Q & A

Thank you.

References

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