IMSE 780: Methods of Operations Research
Midterm Exam
Fall 2023

- Test Period: 48 hours.
- Format: Take home, open book, open note. Use any resources you want, so long as you DO NOT DISCUSS THE EXAM MATERIAL WITH ANYONE else except the professor.
- Python portions should be turned in as a Colab or Jupyter notebook, preferably only one notebook per submission (make clear which cells correspond to which problem). You may either upload the .ipynb file or give editor permissions to the professor and include the link in your submission.
- Submit your work via Canvas in any combination of documents (typed, scanned, or photographed), notebook files, and links. Using multiple formats may require multiple submissions on Canvas.
- You MUST sign this sheet and submit a scanned/photographed copy to receive any credit.
- Good luck!

I _____ affirm that I have not given or received any unauthorized aid on this exam, and acknowledge that sharing/discussing exam material with anyone besides the professor during the exam window constitutes a violation of the university honor policy.

Signature: _____

Date: _____

1. (6pts) The following matrix systems each display a system of equations ready for an iteration of the simplex method. For each system, tell me:

   - What variables make up the current basis?
   - What is the objective value at the current basic solution?
   - Is the current basis optimal?
   - If the current basis is not optimal, further provide the following:
     - What variable would you next bring into the basis?
     - Given your selection of incoming variable, what variable should exit the basis?
     - Given your selection of incoming and outgoing variables, what are the values for all the variables at the next basic solution?

   (a)

$$\begin{bmatrix} 1 & 3 & 0 & -3 & 0 & 1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 1 & 1 \\ 0 & -4 & 1 & 5 & 0 & 3 & 0 \\ 0 & 6 & 0 & 1 & 1 & 8 & 0 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 23 \\ 15 \\ 40 \\ 26 \end{bmatrix}$$

   (b)

$$\begin{bmatrix} 1 & 0 & 5 & 6 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \\ 4 \\ 4 \\ 3 \end{bmatrix}$$

2. (5pts) For each below description of a pair of primal and dual LPs, please determine if such a relation is possible and, if possible, construct a pair of LPs that match the description. Your sample LPs should have no more than two variables and two constraints each.

   - Both the primal and dual have an optimal objective value of 0.
   - Both the primal and dual have no feasible solutions.
   - Both the primal and dual have feasible solutions and an unbounded objective.
   - The primal has feasible solutions and an unbounded objective, while the dual has no feasible solutions.
   - The primal has feasible solutions and an unbounded objective, while the dual has an optimal objective value of 0.

3. (4pts) Consider the Wyndor LP model presented in section 4.1 of the class notes. Suppose the processing time for Product 2 in Plant 3 is doubled, so that each batch of Product 2 requires 4 hours of production time in Plant 3. Is the optimal basis from the original version of the problem still optimal with this change? Justify your answer.

4. (6pts) The coach of an age group swim team needs to assign swimmers to a 200-yard medley relay team to send to the Junior Olympics. Since most of his best swimmers are very fast in more than one stroke, it is not clear which swimmer should be assigned to each of the four strokes. The five fastest swimmers and the best times (in seconds) they have achieved in each of the strokes (for 50 yards) are:

Stroke	Carl	Chris	David	Tony	Ken
Backstroke	37.7	32.9	33.8	37.0	35.4
Breaststroke	43.4	33.1	42.2	34.7	41.8
Butterfly	33.3	28.5	38.9	30.4	33.6
Freestyle	29.2	26.4	29.6	28.5	31.1

The coach wishes to determine how to assign four swimmers to the four different strokes to minimize the sum of the corresponding best times.

Using Python and your modeling library of choice, model the above as an integer program and present the optimal assignment of swimmers to strokes.

5. (3pts) Recall our homework 6 question 1, where you were to build a model that helps Wyndor decide whether or not to build new facilities. There is something peculiar about the model most of you formulated, as well as the model I provided in the "HW6 Grading Notes" notebook shared in class:

   https://colab.research.google.com/drive/19fV6KEsDaX656jcHkBX7ZI0YeSFJKmVI?usp=sharing

   The way it is set up, it is possible that the optimal solution has you building more new facilities than necessary. In this case, the optimal objective value of \$39M/week is attainable with $y_1 = 0, y_2 = 0, y_3 = 1$, but it is also attainable with $y_1 = 1, y_2 = 1, y_3 = 1$. Thus our model might suggest the company go through the effort to build these new facilities for no marginal gain!

   This was fine for the assumptions of the homework problem, but suppose we want to do better. Using the model in the above notebook as a base, how could you alter the objective function so that the y_i variables only equal 1 if building the corresponding facility strictly improves weekly profit? Please give the new objective and briefly explain how using it achieves our goal. (Note: this can be done without adding any more variables or constraints to the model.)

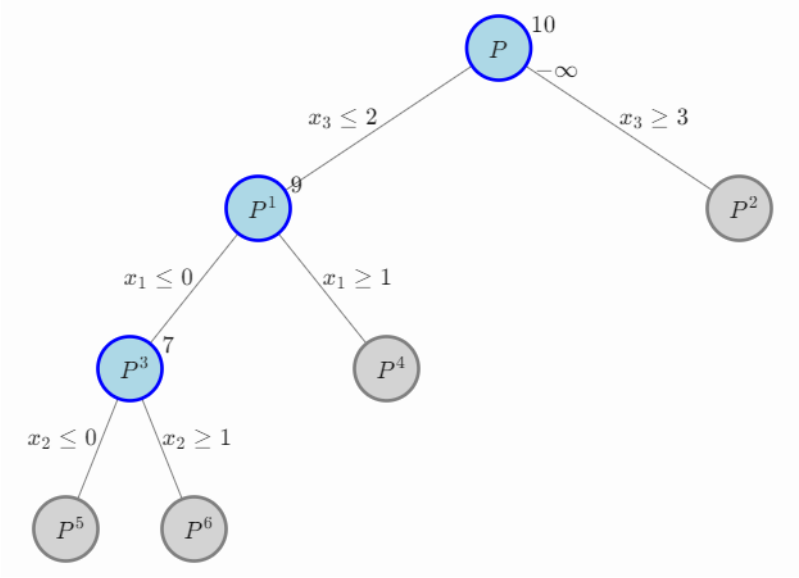
6. (6pts) The athletic teams at Kansas State University compete in a conference called the Big 12. Starting next year, the Big 12 conference will (confusingly) consist of 16 schools from across the country. Many of these schools are geographically distant from one another. Suppose that, in an effort to reduce travel distances and aid in scheduling, the conference wishes to divide its teams into two eight-team divisions. They would like to do so while minimizing the average distance between teams in the same division.

   Your company has been tasked with determining the makeup of these divisions. One of your coworkers has been working on the problem, which he decided to formulate as an integer program. Unfortunately, he was unable to finish before going on an extended vacation, so you will need to complete the model for him. To get you started, he has compiled his work in the following notebook:

   https://colab.research.google.com/gist/jpavelka/fa73fc8d863991c0a2ed9448d34f95f2

   So far, your coworker has identified the libraries required (the first cell in the notebook) and done the work to determine the distances between schools and plot them out on a map (second cell). He needs your help completing the IP formulation in the third cell. So far he has only set up the decision variables, so you'll need to define the constraints and objective function. Please complete the formulation to determine the best divisions.

7. (2pts) Suppose you are solving an integer program (maximization problem) using branch and bound, and your current tree looks like this:



Under the following node selection strategies, which node would you explore next?

(a) Depth-first selection.

(b) Best node selection.

8. (3pts) Consider the following integer program:

$$\begin{aligned} \max \quad & 3x_1 + 2x_2 \\ \text{s.t.} \quad & 6x_1 + 3x_2 \leq 12 \\ & 2x_1 + 4x_2 \leq 6 \\ & x_1, x_2 \in \mathbb{I}_+ \end{aligned}$$

Create a valid inequality for this IP that is *not* satisfied by the LP relaxation optimal solution. (Hint: You may wish to plot this and/or enumerate all the feasible integer points.)