galois

Formal Verification of Quantum Programs

Jennifer Paykin

Pacific Northwest National Labs

October 26, 2018

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- PhD from University of Pennsylvania 2018
- Now at Galois Inc in Portland OR
- Interested in programming language design, applications, & verified software
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With...

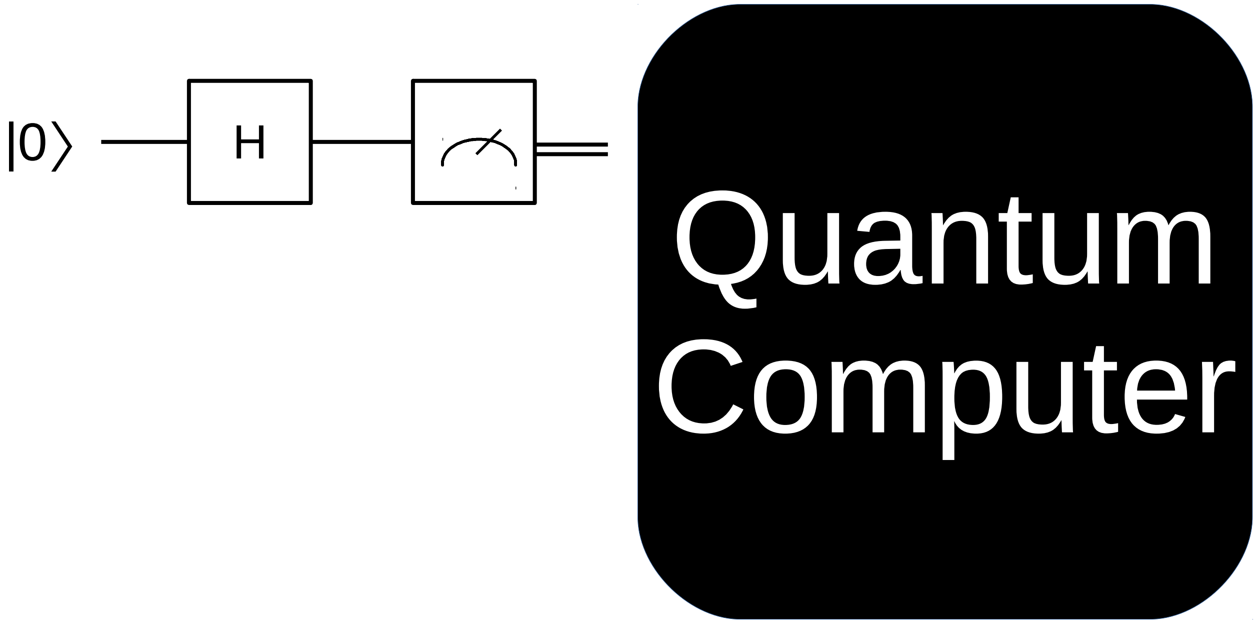
Robert Rand
University of Maryland

Steve Zdancewic

Dong-Ho Lee
University of Pennsylvania

Quantum Computer

Quantum
Computer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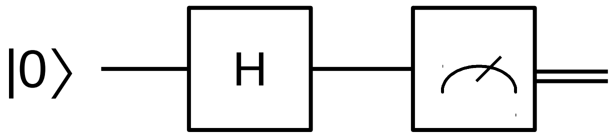
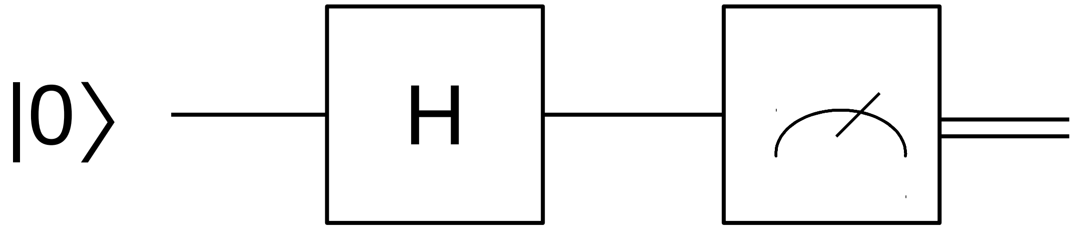


$$\begin{cases} 0 & \text{with probability 1/} \\ 1 & \text{with probability 1/} \end{cases}$$

Quantum Computer

Features of Circuits

- gates applied to wires



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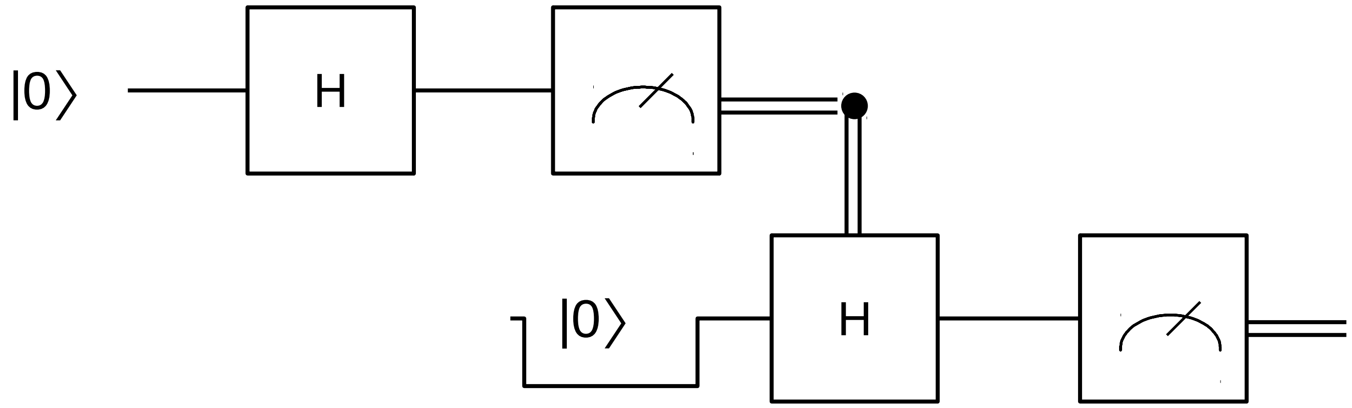

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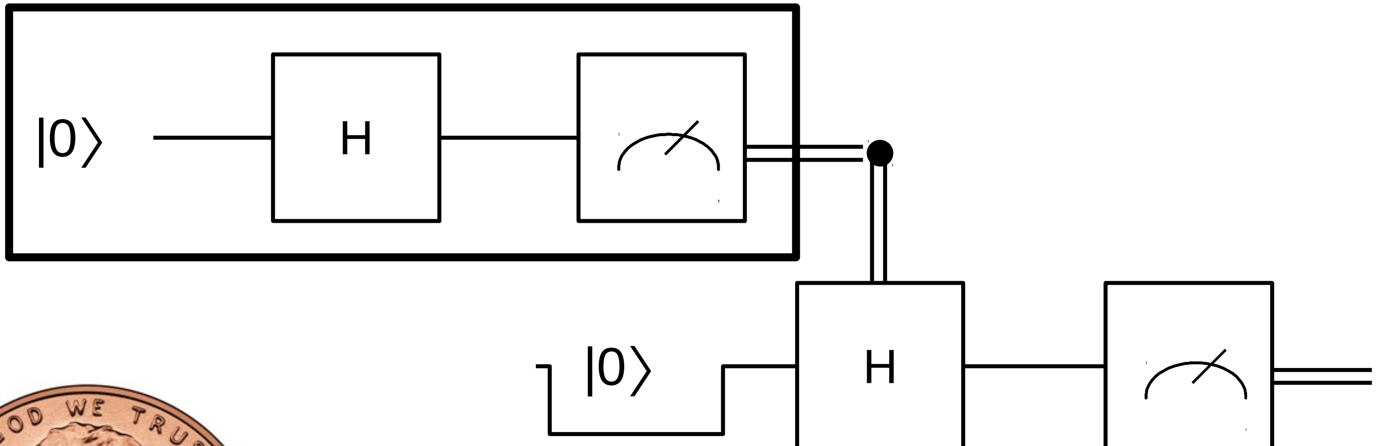

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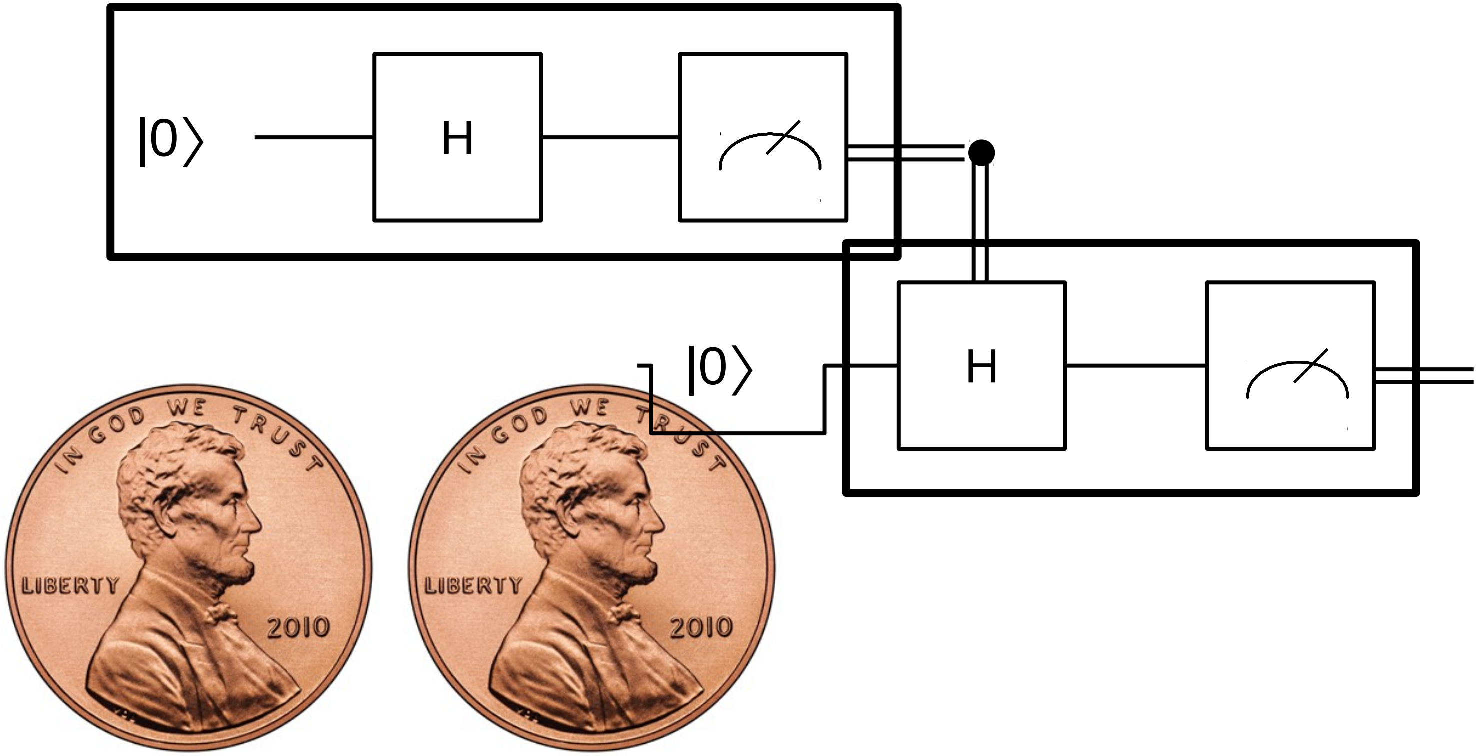

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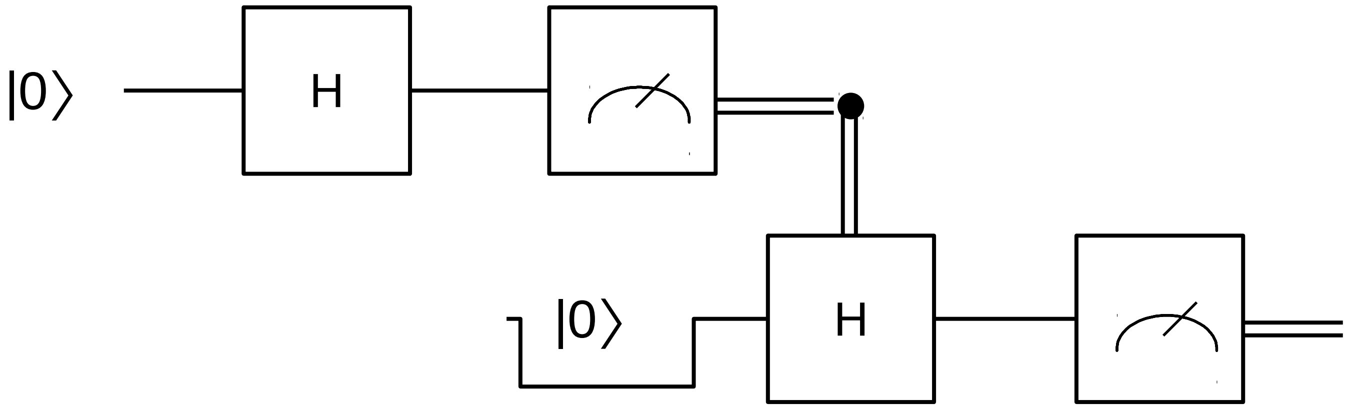

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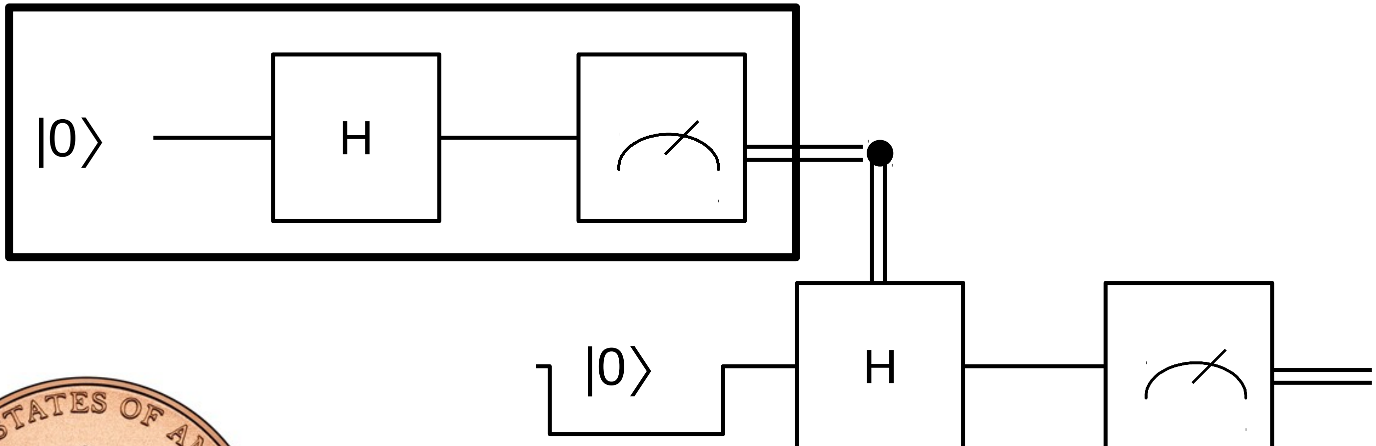

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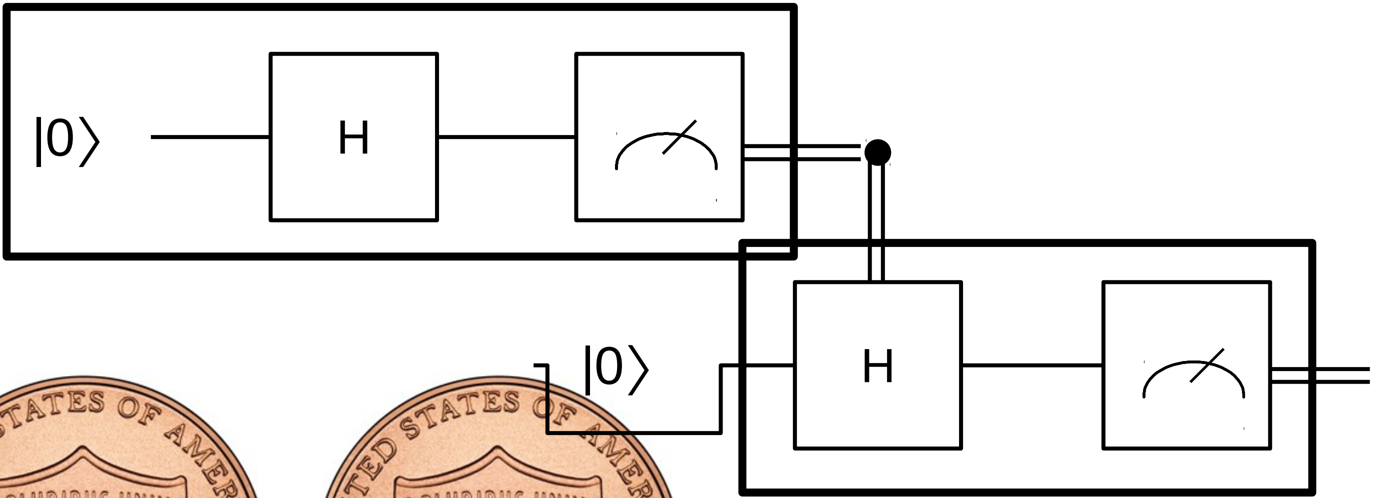

Features of Circuits

- gates applied to wires
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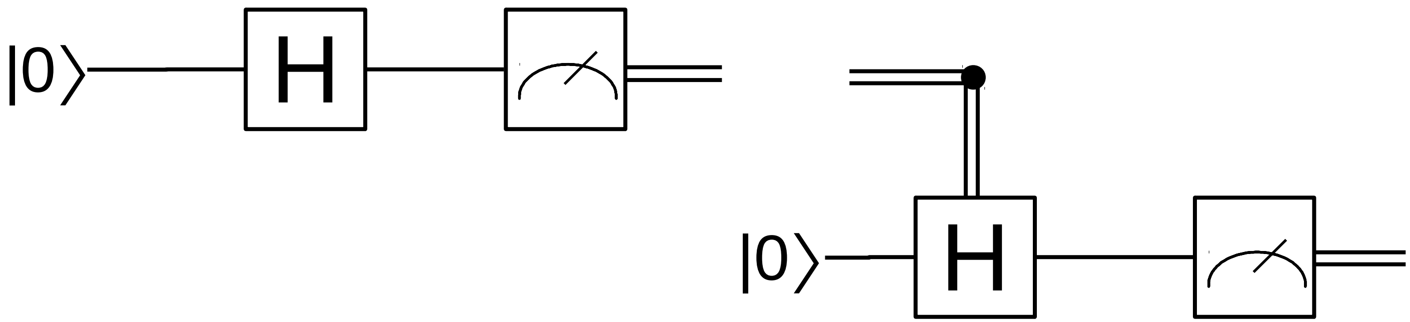

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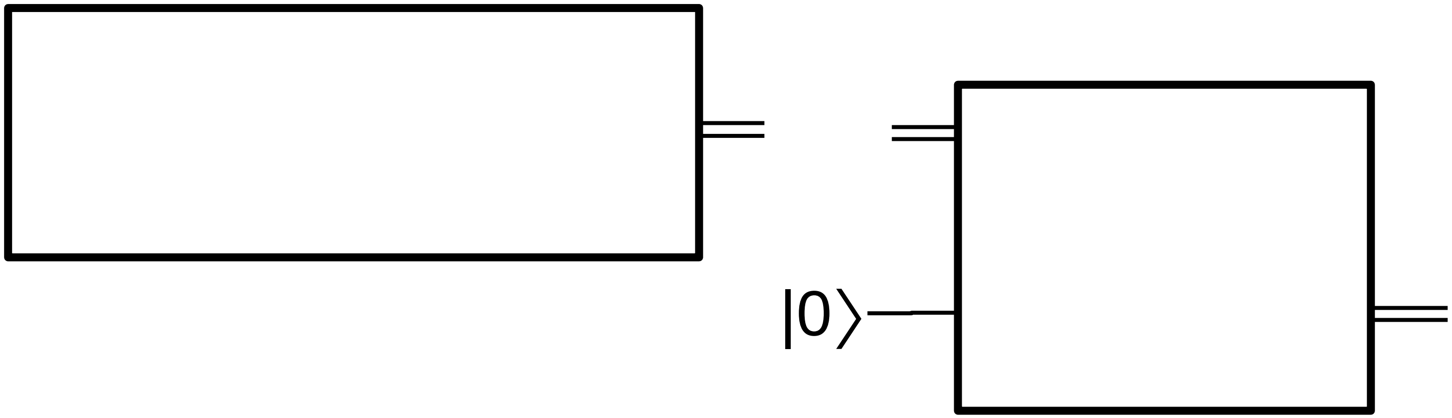

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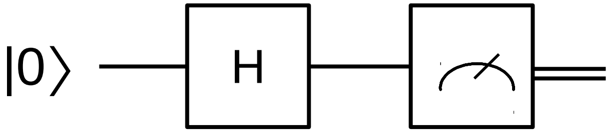

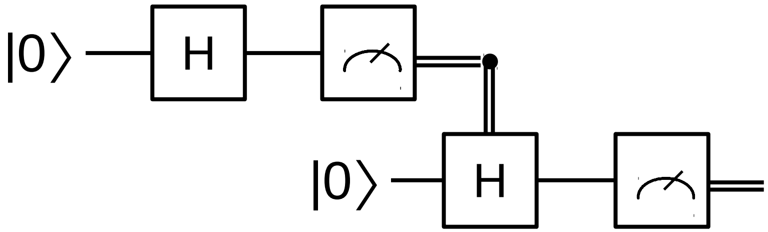

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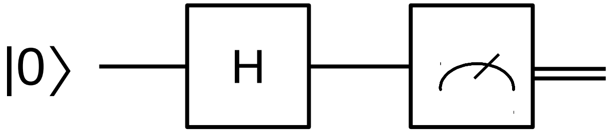

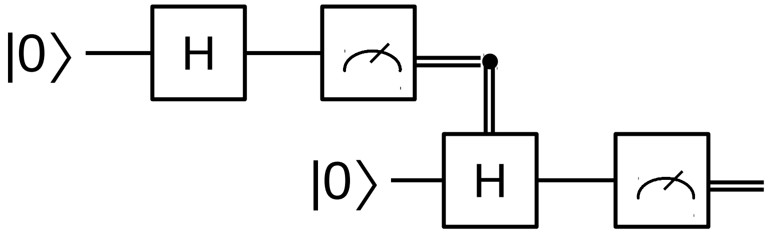

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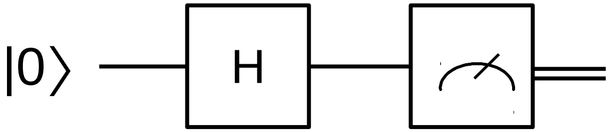

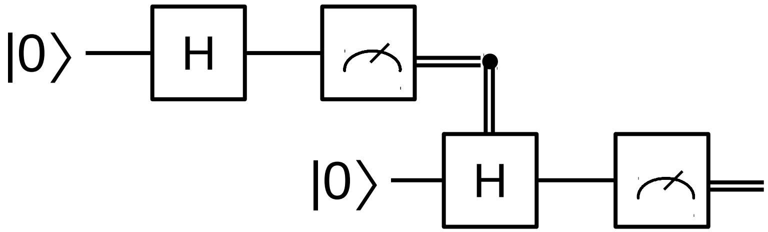

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- families of circuits
- semantics—mathematical meaning

$$\left[\!\left| \texttt{flip n} \right|\!\right]() = \begin{pmatrix} \frac{2^n - 1}{2^n} & 0 \\ 0 & \frac{1}{2^n} \end{pmatrix}$$

Quantum Circuit Languages

- Low-level – limited types, composition, and/or circuit families
  - PyQuil – Python framework for constructing Rigetti Quil circuits
  - QisKit – Python framework for constructing IBM QASM circuits
  - ProjectQ – open source framework for converting between low-level circuit languages, shared benchmarking, etc

Quantum Circuit Languages

- High level – types, composition, circuit families
  - Q# – standalone Microsoft language with simulation, optimization, debuggers
  - Quipper – functional language for circuit families embedded in Haskell
  - **QWIRE** – Strong type system with **semantics** embedded in Coq **proof assistant**

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- Algorithm is wrong → pen & paper proof

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- Implementation doesn’t mach specification → ??

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Formal Verification

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Formal Verification in Coq

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QWIRE in Coq

https://github.com/inQWIRE/QWIRE

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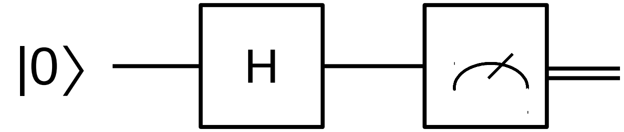
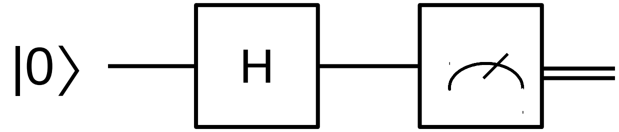
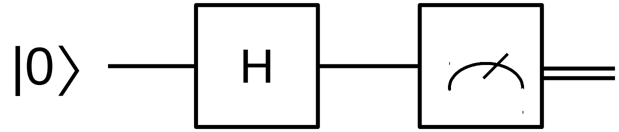


$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$



Formal Verification

$$\texttt{Theorem fair\_toss : } [\![\texttt{coin\_flip}]\!]\ \texttt{I} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$



Formal Verification

`Theorem fair_toss : ⟦coin_flip⟧ I =` $\begin{pmatrix}\frac{1}{2} & 0 \\ 0 & \frac{1}{2}\end{pmatrix}$

`Proof.`

tactics

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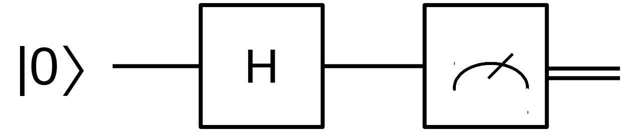

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=============================
⟦ coin_flip ⟧ I = fair_coin
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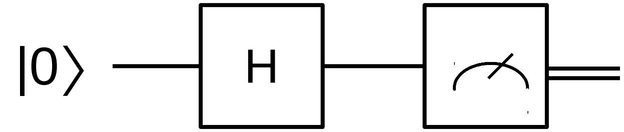
goal



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Proof.
  matrix_denote. (* unfold definitions *)


Qed.
```

* simplified for readability

```
  ============================
(I × I ⊗ I) × ((I ⊗ |0⟩⟨0| ⊗ I) × ((I ⊗ hadamard ⊗ I) ×
((I ⊗ |0⟩) × I × ((I ⊗ |0⟩)) †) × (I ⊗ hadamard ⊗ I) †)
× (I ⊗ |0⟩⟨0| ⊗ I) † .+ (I ⊗ |1⟩⟨1| ⊗ I) × ((I ⊗
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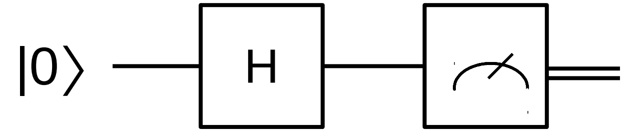
= fair_coin
```



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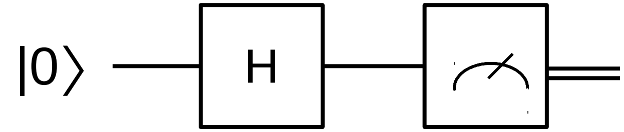

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Proof.
  matrix_denote. (* unfold definitions *)
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  solve_matrix. (* brute force *)
Qed.

============================
No more subgoals.
```

Rule #1: Only rely on brute force for small cases

```
Fixpoint flip (n : Nat) :=
  match n with

  | 0    ⇒

  | S n' ⇒

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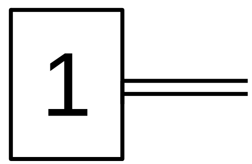

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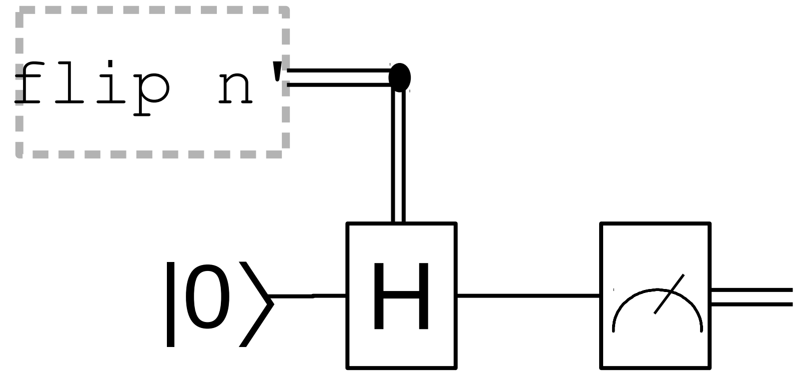

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Rule #2: Use induction to prove correctness of circuit families

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Lemma flip_corect : forall n,
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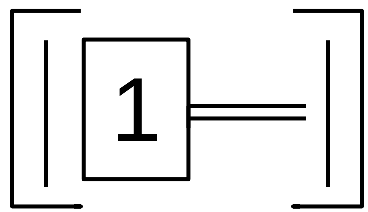

$$() = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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Proof.
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  - (* n = 0 *) (* easy brute force *)
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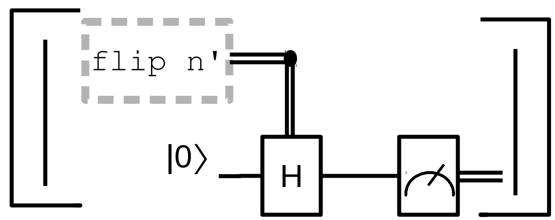

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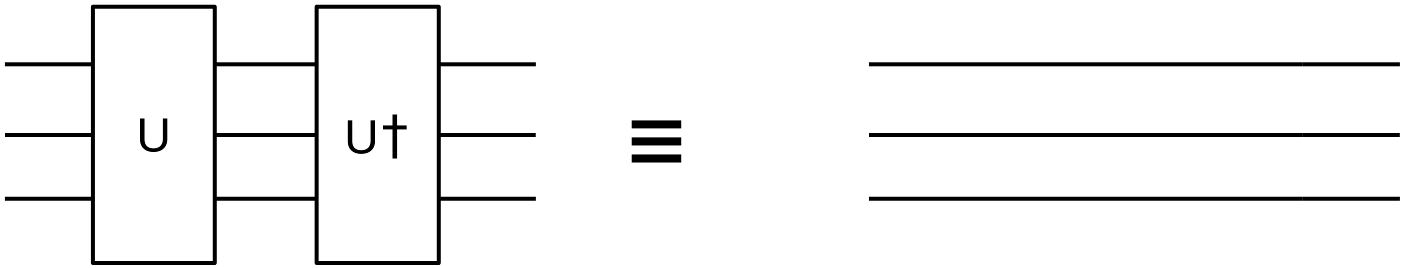
```
Proof.
  induction n.
  - (* n = 0 *)     (* easy brute force *)
  - (* n = n'+1 *) (* use IH to complete proof *)
Qed.
```

Rule #3: Use equational reasoning




```
Lemma unitary_adjoint_id : ∀ n (U : Unitary n),
  U† ;; U ≡ id n.

(* C1 ≡ C2 := ∀ ρ, ⟦C1⟧ρ = ⟦C2⟧ρ *)
```

QWIRE: circuit language in Coq

- https://github.com/inQWIRE/QWIRE
- Verify particular circuits & protocols
  - teleportation, Deutsch’s algorithm
- Verify entire families of circuits
  - Inductively-defined circuits
- Verified compiler & compiler optimizations
  - Boolean expressions to quantum oracles

QWIRE: circuit language in Coq

- Minimal circuit language that has desired features
  - Not meant for production systems
  - Implemented in Coq—interactive theorem prover
- Strong type system prevents ill-formed circuits
- Denotational semantics
  - Formalized in Coq

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  - Error rate analysis
  - Verified compiler passes

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- There are always trade-offs, but verification is often worthwhile!

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Thank you!

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https://github.com/inQWIRE/QWIRE

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