

Linear Algebra

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Spectral Theorem

Singular Value Decomposition

Generalized Eigenvectors

Multilinear Maps

A multilinear map is a more general case of a linear map, which acts more than one vector in a way that is linear in each of its arguments.

$$f : V_1 \times V_2 \times \dots \times V_n \rightarrow W \quad (1)$$

Where V_1, V_2, \dots, V_n and W are vector spaces. An example would be the addition or subtraction of two or more vectors.

Multilinear Forms

Multilinear forms are multilinear maps that have a scalar output. An example is the dot product between two vectors, or summing over the elements of one or more vectors.

$$f : V_1 \times V_2 \times \dots \times V_n \rightarrow K \quad (2)$$

Where V_1, V_2, \dots, V_n and K is a scalar field.

Taking Derivatives

$$\frac{d}{d\mathbf{x}} (u^T x) = \left[\frac{d}{dx_1} (\sum_i u_i x_i), \dots, \frac{d}{dx_n} (\sum_i u_i x_i) \right] = u^T$$

$$\frac{d}{d\mathbf{x}} (x^T u) = \left[\frac{d}{dx_1} (\sum_i u_i x_i), \dots, \frac{d}{dx_n} (\sum_i u_i x_i) \right] = u^T$$

$$\frac{d}{d\mathbf{x}} (x^T x) = \left[\frac{d}{dx_1} (\sum_i x_i^2), \dots, \frac{d}{dx_n} (\sum_i x_i^2) \right] = 2x^T$$

$$\frac{d}{d\mathbf{x}} (\mathbf{A}x) = \begin{bmatrix} \underbrace{\frac{d}{dx_1} \left(\sum_i A_{1i} x_i \right)}_{A_{11}} & \dots & \underbrace{\frac{d}{dx_n} \left(\sum_i A_{1i} x_i \right)}_{A_{1n}} \\ \vdots & \ddots & \vdots \\ \underbrace{\frac{d}{dx_1} \left(\sum_i A_{ni} x_i \right)}_{A_{n1}} & \dots & \underbrace{\frac{d}{dx_n} \left(\sum_i A_{ni} x_i \right)}_{A_{nn}} \end{bmatrix} = \mathbf{A} \quad (3)$$

Combinatorics

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The Twentyfold Way

The twentyfold is a taxonomy of distribution problems developed by Kenneth Bogard in his book *Combinatorics through Guided Discovery*. It divides up the way in which k objects may be assigned to n individuals, subject to whether the objects are distinct or identical, and subject to conditions on how the objects are received.

The weakness (in my opinion) is that the language of "objects" and "recipients" is unclear because in practice it's not obvious which is which: if there are k students and n teachers, do the teachers receive students, or do the students receive a teacher?

The best way I can think of to resolve this is to say that an object can have only one recipient, but a recipient might receive more than one object. A more formal path is to think of the act of creating combinations in terms of functions. The elements of the domain are the objects. The elements of the range are the recipients. A function can be many-to-one, but it should not be one-to-many.

Favorite Teachers At a school with k students and n teachers, the students all have a favorite teacher. (They might all like the same one.). How many ways are there for the k students to pick a favorite?

Objects: k students. *Recipients:* n teachers. Many students might have one favorite teacher. There are n^k combinations.

Picking Team Mates Out of a choice of n athletes, a coach must assemble a team of k . How many ways are there to form a team?

Objects: n athletes. *Recipients:* team, not on the team. Many athletes can be assigned to one outcome of being on the team or not being on the team. There are $\binom{n}{k}$ combinations for the team, which is the same number as the $\binom{n}{n-k}$ selections for the bench.

Distinct Objects, Without Conditions

Distinct Recipients

The k objects are assigned to n recipients with no conditions as to the number of objects each recipient receives. This is the same as assigning the elements of a k -tuple from a selection of n with replacement.

$$S = \{(i_1, i_2, \dots, i_k) | i_j \in A, |A| = n\}$$

$$|S| = n^k \tag{4}$$

Binary Strings of Length k The k distinct positions of a binary string (i_1, i_2, \dots, i_k) of length k are assigned to an element of the set $A \in [0, 1]$. The number of possible binary strings of length k is 2^k .

4 COMBINATORICS

Subsets of a k -Element Set The subsets of a set of k distinct elements are formed by assigning each of its k distinguishable elements to one of the two labels $A \in [\text{included, excluded}]$. The number of possible subsets, including the empty subset and the full set, is 2^k .

Indistinct Recipients

Distinct Objects, At Most One is Assigned

Distinct Recipients

Exactly one of k distinct objects are assigned to a single one of n recipients.

Linear Regression

Ordinary Linear Regression

R^2 Value

Regression Diagnostics

t-Statistics

AIC and BIC

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