E1

E1

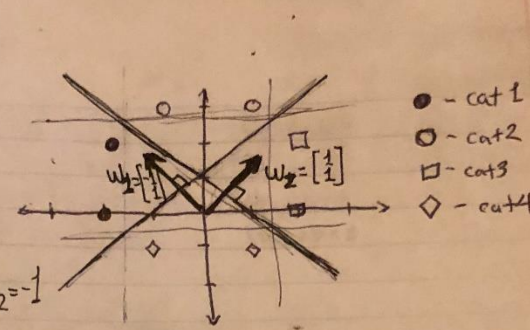
i

Category 1: $\left(\begin{bmatrix}-3\\0\end{bmatrix}, \begin{bmatrix}-2\\2\end{bmatrix}\right)$

Category 2: $\left(\begin{bmatrix}-1\\3\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}\right)$

Category 3: $\left(\begin{bmatrix}3\\0\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix}\right)$

Category 4: $\left(\begin{bmatrix}-1\\-1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right)$



$w_1=\begin{bmatrix}-1\\1\end{bmatrix}$ $\quad w_2=\begin{bmatrix}1\\1\end{bmatrix}$ $\quad b_1=-1$ $\quad b_2=-1$

$w_1^T p + b_1 = 0$ $\qquad w_2^T p + b_2 = 0$

$[-1\ 1]\begin{bmatrix}0\\1\end{bmatrix} + b_1 = 0$ $\qquad [1\ 1]\begin{bmatrix}0\\1\end{bmatrix} + b_2 = 0$

$1 + b_1 = 0$ $\qquad 1 + b_2 = 0$

$b_1 = -1$ $\qquad b_2 = -1$

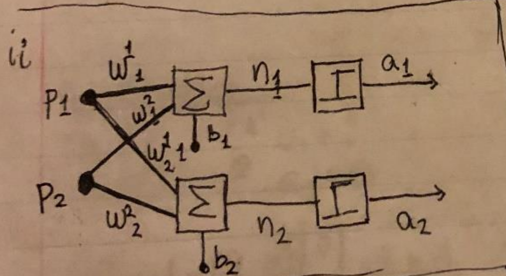
Best Decision Boundaries:

$n_1 = [-1\ 1]p - 1$

$n_2 = [1\ 1]p - 1$

The best decision boundary means it maximizes the margin between the boundary and the data (distance b/w data and boundary) and is orthogonal to the weight vector

ii



iii. $p=\begin{bmatrix}-3\\1\end{bmatrix}$ $\quad a_1 = \text{hardlim}(w_1^1 p_1 + w_1^2 p_2 + b_1)$

$= \text{hardlim}(3+1-1) = 1$

$a_2 = \text{hardlim}(w_2^1 p_1 + w_2^2 p_2 + b)$

$= \text{hardlim}(-3+1-1) = 0$

$t=\begin{bmatrix}0\\0\end{bmatrix}$ $\quad a=\begin{bmatrix}1\\0\end{bmatrix}$ $\quad e = t - a = \begin{bmatrix}-1\\0\end{bmatrix}$

${}_1w^{new} = {}_1w^{old} - p$

${}_1w^{new} = \begin{bmatrix}-1\\1\end{bmatrix} - \begin{bmatrix}-3\\1\end{bmatrix}$

$\boxed{{}_1w^{new} = \begin{bmatrix}2\\0\end{bmatrix}}$

$b_1^{new} = b_1^{old} + e$

$\boxed{b_1 = -1 - 1 = -2}$

${}_2w^{new} = {}_2w^{old}$

$\boxed{{}_2w^{new} = \begin{bmatrix}1\\1\end{bmatrix}}$

$b_2^{new} = b_2^{old} + e$

$\boxed{b_2^{new} = -1 + 0 =}$

E2

$Wp+b=0$

$\begin{bmatrix}1&1\\-1&1\end{bmatrix}\begin{bmatrix}p_1\\p_2\end{bmatrix}+\begin{bmatrix}-2\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$2p_1-2=0$

$p_1=1 \quad p_2=1$

$p_1+p_2-2=0$

$-p_1+p_2=0$

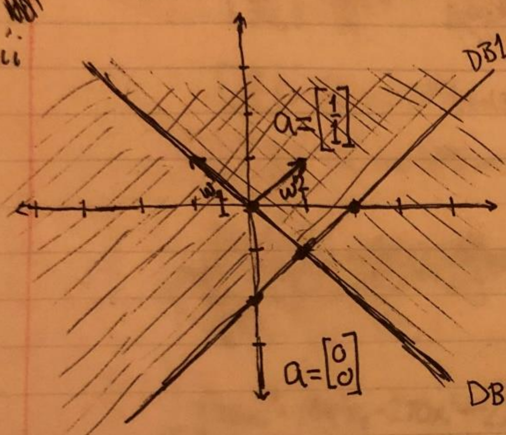
$p_2=p_1$

$\begin{bmatrix}1&1\\-1&1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}+\begin{bmatrix}-2\\0\end{bmatrix}=\begin{bmatrix}0\\-2\end{bmatrix}+\begin{bmatrix}-2\\0\end{bmatrix}=\begin{bmatrix}-2\\-2\end{bmatrix}$

E2

i. This network can classify 4 CLASSES as it is a layer with a hardlims function, which will produce either a 1 or -1 result for 2 inputs, so four possible outputs in total

ii



$w_1^T p+b_1=0$

$\begin{bmatrix}1\\-1\end{bmatrix}^T p+(-2)=0$

$\begin{bmatrix}1\\-1\end{bmatrix}^T p=2$

$p=\begin{bmatrix}2\\0\end{bmatrix},\begin{bmatrix}0\\-2\end{bmatrix}$

$w_2^T p+b_2=0$

$\begin{bmatrix}1&1\end{bmatrix}p=0$

$p=\begin{bmatrix}1\\-1\end{bmatrix},\begin{bmatrix}0\\0\end{bmatrix}$

$Wp+b=0$

$\begin{bmatrix}1&1\\-1&1\end{bmatrix}p+\begin{bmatrix}-2\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}p_1+p_2\\-p_1+p_2\end{bmatrix}+\begin{bmatrix}-2\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$-p_1=p_2$

$p_1+p_2=-2$

iii $a=\text{hardlims}(Wp+b)$

$=\text{hardlims}\left(\begin{bmatrix}1&1\\-1&1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}+\begin{bmatrix}-2\\0\end{bmatrix}\right)$

$=\text{hardlims}\left(\begin{bmatrix}0\\-2\end{bmatrix}+\begin{bmatrix}-2\\0\end{bmatrix}\right)$

$=\begin{bmatrix}0\\0\end{bmatrix}$

E3

$(-2x_2+3x_1)^2 = 4x_2^2 \dots$

$8x$

$(x_1+(x_2-5))^2 = x_1^2 + \cancel{2(x_2-5)+(x_2-5)^2}$ $\quad 2x_1$

$(3x_1-2x_2)^2 = 9x_1^2 - 12x_1x_2 + \dots$ $\quad 18x_1$

E3

$F(x) = [1+(x_1+x_2-5)^2][1+(3x_1-2x_2)^2]$

$(x_1+(x_2-5))^2$

$= x_1^2 + 2(x_2-5)x_1 + \dots$

i. $\nabla F(x) = \begin{bmatrix} \frac{\partial F}{\partial x_1} \\ \frac{\partial F}{\partial x_2} \end{bmatrix}$

$$= \begin{bmatrix} \frac{\partial F}{\partial x_1}(1+(x_1+x_2-5)^2)(1+(3x_1-2x_2)^2) + \frac{\partial F}{\partial x_1}(1+(3x_1-2x_2)^2)(1+(x_1+x_2-5)^2) \\ \frac{\partial F}{\partial x_2}(1+(x_1+x_2-5)^2)(1+(3x_1-2x_2)^2) + \frac{\partial F}{\partial x_2}(1+(3x_1-2x_2)^2)(1+(x_1+x_2-5)^2) \end{bmatrix}$$

$$= \begin{bmatrix} 36x_1^3 + 18x_1^2x_2 - 270x_1^2 - 22x_1x_2^2 + 470x_1 + 60x_1x_2 - 4x_2^3 + 80x_2^2 - 310x_2 - 10 \\ 6x_1^3 - 22x_1^2x_2 + 30x_1^2 - 310x_1 - 12x_1x_2^2 + 160x_1x_2 + 16x_2^3 - 120x_2^2 + 210x_2 - 10 \end{bmatrix}$$

$$\nabla^2 F(x) = \begin{bmatrix} 72x_1^2 + 36x_1x_2 - 540x_1 - 22x_2^2 + 470 + 60x_2 & 18x_1^2 - 44x_1x_2 + 60x_1 - 12x_2^2 + 160x_2 - 310 \\ 18x_1^2 - 44x_1x_2 + 60x_1 - 12x_2^2 + 160x_2 - 310 & -22x_1^2 - 24x_1x_2 + 160x_1 + 48x_2^2 - 240x_2 + 210 \end{bmatrix}$$

i. $x_0 = [10 \ \ 10]^T$ $\quad \nabla F(x_0) = \begin{bmatrix} 16590 \\ -6010 \end{bmatrix}$ $\quad \nabla^2 F(x_0) = \begin{bmatrix} 3730 & -1910 \\ -1910 & 1590 \end{bmatrix}$ $\quad \nabla^2 F(x) =$

$x_1 = x_0 - \nabla^2 F(x)^{-1} \nabla F(x)$

$= \begin{bmatrix} 10 \\ 10 \end{bmatrix} - \begin{bmatrix} 7.306 \\ 3.912 \end{bmatrix} = \begin{bmatrix} 2.694 \\ 6.088 \end{bmatrix}$

ii $x_0 = [2 \ 2]^T$ $\quad x_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -0.032 \\ -0.262 \end{bmatrix}$

$\nabla F(x_0) = \begin{bmatrix} 14 \\ -26 \end{bmatrix}$ $\quad x_1 = \begin{bmatrix} 1.968 \\ 1.738 \end{bmatrix}$

$\nabla^2 F(x) = \begin{bmatrix} -254 & -22 \\ -22 & 102 \end{bmatrix}$

$(x+y-5)(x+y-5)$

$= x(x+y-5) + y(x+y-5) - 5(x+y-5)$

$= \cancel{x^2} + \cancel{xy} - \cancel{5x} + \cancel{xy} + \cancel{y^2} - 5 - \cancel{5x} - 5y + 25 = x^2 + 2xy - 10x + y^2 - 5y + 2\cancel{4}$

iii. $\nabla F(x) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $F(x) = [(1 + (x_1 + x_2 - 5)^2][1 + (3x_1 - 2x_2)^2]$

$(3x-2y)(3x-2y)$

$= 9x_1^2(x_1^2 + 2x_1x_2 - 10x_1 + x_2^2 - 5x_2 + 21) - 12x_1x_2(x_1^2 + 2x_1x_2 - 10x_1 + x^2 - 5x_2 + 21)$ $9x^2 - 12xy + 4y + 1$

$+ 4x_2(x_1^2 + 2x_1x_2 - 10x_1 + x_2^2 - 5x_2 + 21) + (x_1^2 + 2x_1x_2 - 10x_1 + x_2^2 - 5x_2 + 21)$

$= 9x_1^4 + 18x_1^3x_2 - 90x_1^3 + 9x_1^2x_2^2 - 45x_1^2x_2 + 189 - 12x_1^3x_2 - 24x_1^2x_2^2 - 120x_1^2x_2$

A

Minimum $= (-12.4, 12.4)$

minimum $\Rightarrow \nabla F(x) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\nabla^2 F(x)$ is positive

E4

E4

x^* $[1\ 1]^T$ in direction $[-1\ 1]^T$ p

i. $F(x)=\frac{7}{2}x_1^2-6x_1x_2-x_2^2$ $\nabla F^2(x)=\begin{bmatrix}7 & -6\\ 6 & -2\end{bmatrix}$

$$\nabla F(x)\Big|_{x=x^*}=\begin{bmatrix}\frac{\partial F}{\partial x_1}\\ \frac{\partial F}{\partial x_2}\end{bmatrix}\Bigg|_{x=x^*}=\begin{bmatrix}7x_1-6x_2\\ 6x_1-2x_2\end{bmatrix}\Bigg|_{x=x^*}=\begin{bmatrix}1\\4\end{bmatrix}$$

$$\frac{p^T\nabla F(x)}{\|p\|}=\frac{[-1\ 1]\begin{bmatrix}1\\4\end{bmatrix}}{\sqrt{17}}=\frac{3}{\sqrt{17}}$$ First Directional Derivative

$$\frac{p^T\nabla^2F(x)p}{\|p\|^2}=\frac{[-1\ 1]\begin{bmatrix}7 & -6\\ 6 & -2\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}}{\sqrt{17}}=\frac{[-1\ 1]\begin{bmatrix}-13\\-8\end{bmatrix}}{\sqrt{17}}=\frac{5}{\sqrt{17}}$$ Second Directional Derivative

ii. $F(x)=5x_1^2-6x_1x_2+5x_2^2+4x_1+4x_2$

$$\nabla F(x)=\begin{bmatrix}10x_1-6x_2+4\\ -6x_1+10x_2+4\end{bmatrix}\Bigg|_{x^*=x^*}=\begin{bmatrix}8\\8\end{bmatrix}\quad \nabla^2F(x)=\begin{bmatrix}10 & -6\\ -6 & 10\end{bmatrix}$$

$$\frac{p^T\nabla F(x)}{\|p\|}=\frac{[-1\ 1]\begin{bmatrix}8\\8\end{bmatrix}}{\sqrt{128}}=\frac{0}{\sqrt{128}}=0$$ 1st DD

$$\frac{p^T\nabla^2F(x)p}{\|p\|}=\frac{[-1\ 1]\begin{bmatrix}10 & -6\\ -6 & 10\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}}{\sqrt{128}}=\frac{[-1\ 1]\begin{bmatrix}-16\\16\end{bmatrix}}{\sqrt{128}}=0$$ 2nd DD

iii. $F(x)=\frac{9}{2}x_1^2-2x_1x_2+3x_2^2+2x_1-x_2$

$$\nabla F(x)=\begin{bmatrix}9x_1-2x_2+2\\ -2x_1+6x_2-1\end{bmatrix}\Bigg|_{x=x^*}=\begin{bmatrix}9\\3\end{bmatrix}\quad \nabla^2F(x)=\begin{bmatrix}9 & -2\\ -2 & 6\end{bmatrix}$$

$$\frac{p^T\nabla F(x)}{\|p\|}=\frac{[-1\ 1]\begin{bmatrix}9\\3\end{bmatrix}}{\sqrt{90}}=\frac{-6}{3\sqrt{10}}=\frac{-2}{\sqrt{10}}$$ 1st DD

$$\frac{p^T\nabla^2F(x)p}{\|p\|}=\frac{[-1\ 1]\begin{bmatrix}9 & -2\\ -2 & 6\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}}{3\sqrt{10}}=\frac{[-1\ 1]\begin{bmatrix}-11\\8\end{bmatrix}}{3\sqrt{10}}=\frac{19}{3\sqrt{10}}$$ 2nd DD:

iv. $F(x) = -\frac{7}{2}x_1^2 - 6x_1x_2 + x_2^2$

$$\nabla F(x) = \begin{bmatrix} -7x_1 - 6x_2 \\ -6x_1 + 2x_2 \end{bmatrix}\Bigg|_{x=x^*} = \begin{bmatrix} -13 \\ -4 \end{bmatrix} \qquad \nabla^2 F(x) = \begin{bmatrix} -7 & -6 \\ -6 & 2 \end{bmatrix}$$

$$\frac{p^T \nabla F(x)}{\|p\|} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} -13 \\ -4 \end{bmatrix}}{\sqrt{185}} = \overset{\text{1st DD}}{\frac{9}{\sqrt{185}}} \qquad \frac{p^T \nabla^2 F(x) p}{\|p\|} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} -7 & -6 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}}{\sqrt{185}} = \frac{\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 8 \end{bmatrix}}{\sqrt{185}} = \overset{\text{2nd DD}}{\boxed{\frac{7}{\sqrt{185}}}}$$

E5

E5

Function 1

i. $\nabla F(x) = \begin{bmatrix} 7x_1 - 6x_2 \\ -6x_1 - 2x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$7x_1 - 6x_2 = 0$

$x_1 = \frac{6}{7}x_2$

$x_1 = 0$

$-6\left(\frac{6}{7}\right)x_2 - 2x_2 = 0$

$-\frac{22}{7}x_2 = 0$

$x_2 = 0$

$\boxed{X = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$

ii $\nabla^2 F([0\ 0]^T) = \begin{bmatrix} 7 & -6 \\ -6 & -2 \end{bmatrix}$

$A - I\lambda = \begin{bmatrix} 7-\lambda & -6 \\ -6 & -2-\lambda \end{bmatrix}$

$\Downarrow$

$\det = -14 + 5\lambda + \lambda^2 - 36 = 0$

$\lambda^2 - 5\lambda - 50 = 0$

$(\lambda - 10)(\lambda + 5) = 0$

$\lambda = 10, -5$

$\boxed{\text{Saddle Point}}$

iii. $\underline{\lambda = 10}$

$(A - 10I)\vec{v} = 0$

$\begin{bmatrix} -3 & -6 \\ -6 & -12 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-3v_1 - 6v_2 = 0 \quad v_1 = -2v_2$

$-6v_1 - 12v_2 = 0 \quad v_1 = -2v_2$

$v_2 = t \quad v_1 = -2t \quad v = \begin{bmatrix} -2 \\ 1 \end{bmatrix} t$

$\underline{\lambda = -5}$

$(A + 5I)\vec{v} = 0$

$\begin{bmatrix} 12 & -6 \\ -6 & 3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$

$12v_1 - 6v_2 = 0$

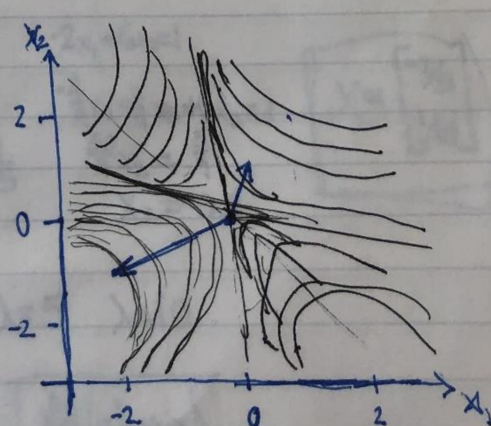
$-6v_1 + 3v_2 = 0$

$v_1 = \frac{1}{2}v_2$

$v_1 = \frac{1}{2}$

$v_2 = 1$

$V = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} t$



Function 2

i. $\nabla F(x) = \begin{bmatrix} 10x_1 - 6x_2 + 4 \\ -6x_1 + 10x_2 + 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$10x_1 - 6x_2 = -4$

$x_1 = 0.6x_2 - 0.4$

$x_1 = -1$

$-6x_1 + 10x_2 = -4$

$-3.6x_2 + 2.4 + 10x_2 = -4$

$6.4x_2 = -6.4$

$x_2 = -1$

$x = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

ii. $\nabla^2 F([-1, -1]) = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$\det\left(\begin{bmatrix} 10-\lambda & -6 \\ -6 & 10-\lambda \end{bmatrix}\right) = 0$

$100 - 20\lambda + \lambda^2 - 36 = 0$

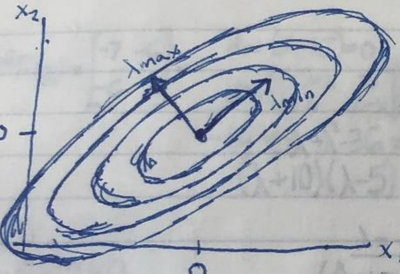
$\lambda^2 - 20\lambda + 64 = 0$

$(\lambda - 4)(\lambda - 16) = 0$

$\lambda_1 = 4 \quad \lambda_2 = 16$

$\boxed{\text{Minima}}$

iii.



$\lambda = 16$

$(A - 16I)\vec{v} = \vec{0}$

$\begin{bmatrix} 10-16 & -6 \\ -6 & 10-16 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-6v_1 - 6v_2 = 0$

$-6v_1 - 6v_2 = 0$

$v_2 = t \quad v_1 = -t$

$v = \begin{bmatrix} -1 \\ 1 \end{bmatrix} t$

$\lambda = 4$

$(A - 4I)\vec{v} = 0$

$\begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$

$6v_1 - 6v_2 = 0 \quad -6v_1 + 6v_2 = 0$

$v_2 = t \quad v_1 = t$

$v = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t$

Function 3

i. $\nabla F(x) = \begin{bmatrix} 9x_1 - 2x_2 + 2 \\ -2x_1 + 6x_2 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$9x_1 - 2x_2 + 2 = 0$

$x_1 = \frac{2}{9}x_2 - \frac{2}{9}$

$x_1 = -\frac{18}{90} = -\frac{1}{5}$

$-2x_1 + 6x_2 = 1$

$-\frac{4}{9}x_2 + \frac{4}{9} + 6x_2 = 1$

$\frac{50}{9}x_2 = \frac{5}{9}$

$x_2 = 0.1$

$\boxed{X = \begin{bmatrix} -1/5 \\ 1/10 \end{bmatrix}}$

ii. $\nabla^2 F(x) = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$\det\left(\begin{bmatrix} 9-\lambda & -2 \\ -2 & 6-\lambda \end{bmatrix}\right) = 0$

$54 - 15\lambda + \lambda^2 - 4 = 0$

$(\lambda - 10)(\lambda - 5) = 0$

$\lambda = 5 \quad \lambda = 10$

$\boxed{\text{Minima}}$

$\lambda=10$

iii. $(A-10\lambda)\vec{v}=0$

$\begin{bmatrix}9-10 & -2\\ -2 & 6-10\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$-v_1-2v_2=0 \quad v_1=2v_2$

$-2v_1-4v_2=0 \quad v_1=2v_2$

$v_2=t \quad v_1=2t \quad v=\begin{bmatrix}2\\ 1\end{bmatrix}t$

$\lambda=5$

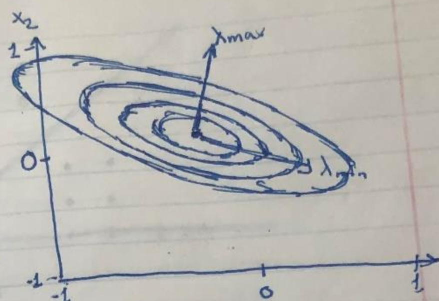
$(A-5\lambda)\vec{v}=0$

$\begin{bmatrix}9-5 & -2\\ -2 & 6-5\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$4v_1-2v_2=0 \quad v_1=\frac{1}{2}v_2$

$-2v_1-v_2=0 \quad v_1=\frac{1}{2}v_2$

$v_2=t \quad v_1=\frac{t}{2} \quad v=\begin{bmatrix}1\\ 2\end{bmatrix}t$



Equation 4

i $\nabla F(x)=\begin{bmatrix}-7x_1-6x_2\\ -6x_1+2x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$-7x_1-6x_2=0$

$x_1=-\frac{6}{7}x_2$

$x_1=0$

$-6x_1+2x_2=0$

$\frac{36}{7}x_2+\frac{14}{7}x_2=0$

$\frac{50}{7}x_2=0$

$\boxed{x=\begin{bmatrix}0\\ 0\end{bmatrix}}$

ii. $\nabla^2F(x)=\begin{bmatrix}-7 & -6\\ -6 & 2\end{bmatrix}$

$\det(A-\lambda I)=0$

$\det\left(\begin{bmatrix}-7-\lambda & -6\\ -6 & 2-\lambda\end{bmatrix}\right)=0$

$-14+5\lambda+\lambda^2-36=0$

$(\lambda+10)(\lambda-5)=0$

$\lambda_1=-10$

$\lambda_2=5$

$\boxed{\text{Saddle Point}}$

$\lambda=5$

iii. $(A-5\lambda)\vec{v}=0$

$\begin{bmatrix}-12 & -6\\ -6 & -3\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$-12v_1-6v_2=0 \quad v_1=-\frac{1}{2}v_2$

$-6v_1-3v_2=0 \quad v_1=-\frac{1}{2}v_2$

$v_2=t \quad v_1=-\frac{1}{2}t \quad v=\begin{bmatrix}-1\\ 2\end{bmatrix}t$

$\lambda=-10$

$(A+10\lambda)\vec{v}=0$

$\begin{bmatrix}3 & -6\\ -6 & 12\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$3v_1-6v_2=0 \quad v_1=2v_2$

$-6v_1+12v_2=0 \quad v_1=2v_2$

$v_2=t \quad v_1=2t \quad v=\begin{bmatrix}2\\ 1\end{bmatrix}t$