

$x$	term variable	
$\alpha$	type variable	
$t$	$::=$	term
	$x$	variable
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : *\} \Rightarrow t$	type abstraction
	$t_1 t_2$	application
	$t [\tau]$	type application
$\tau$	$::=$	type
	$\alpha$	type variable
	$\tau_1 \rightarrow \tau_2$	arrow
	$\forall(\alpha : *), \tau$	universal quantification
$\Gamma$	$::=$	typing environment
	$\emptyset$	empty
	$\Gamma, x : \tau$	variable
	$\Gamma, \alpha : *$	type variable
$v$	$::=$	(typed) value
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : *\} \Rightarrow v$	type abstraction

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Initial environment:  $\Gamma = \emptyset$

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$\boxed{\Gamma \vdash t : \tau}$  Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T\_VAR} \\
\\
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \rightarrow \tau_2} \quad \text{T\_ABS} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha : *\} \Rightarrow t) : \forall(\alpha : *), \tau} \quad \text{T\_TYABS} \\
\\
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T\_APP} \\
\\
\frac{\Gamma \vdash t : \forall(\alpha : *), \tau_2 \quad \Gamma \vdash \tau_1 : * \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T\_TYAPP}
\end{array}$$

$\boxed{\Gamma \vdash \tau : *}$  Type  $\tau$  is well formed

$$\begin{array}{c}
\frac{\alpha : * \in \Gamma}{\Gamma \vdash \alpha : *} \quad \text{K\_VAR} \\
\\
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : *}{\Gamma \vdash (\tau_1 \rightarrow \tau_2) : *} \quad \text{K\_ARROW} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash \tau : *}{\Gamma \vdash (\forall(\alpha : *), \tau) : *} \quad \text{K\_FORALL}
\end{array}$$

$t \longrightarrow t'$  Operational semantics

$$\begin{array}{c}
\frac{t_2 \longrightarrow t'_2}{t_1 t_2 \longrightarrow t_1 t'_2} \quad \text{E\_APP1} \\
\frac{t \longrightarrow t'}{t v \longrightarrow t' v} \quad \text{E\_APP2} \\
\frac{[x \mapsto v]t \triangleright t'}{(\lambda(x : \tau) \Rightarrow t) v \longrightarrow t'} \quad \text{E\_APPABS} \\
\frac{t \longrightarrow t'}{(\lambda\{\alpha : *\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : *\} \Rightarrow t')} \quad \text{E\_TABS} \\
\frac{t \longrightarrow t'}{t [\tau] \longrightarrow t' [\tau]} \quad \text{E\_TAPP} \\
\frac{[\alpha \mapsto \tau]v \triangleright v'}{(\lambda\{\alpha : *\} \Rightarrow v) [\tau] \longrightarrow v'} \quad \text{E\_TAPPABS}
\end{array}$$

$[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3$  Type substitution

$$\begin{array}{c}
\overline{[\alpha \mapsto \tau]\alpha \triangleright \tau} \quad \text{SUBST\_VAR1} \\
\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau]\alpha_2 \triangleright \alpha_2} \quad \text{SUBST\_VAR2} \\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1](\tau_2 \rightarrow \tau_3) \triangleright \tau'_2 \rightarrow \tau'_3} \quad \text{SUBST\_ARROW} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\forall(\alpha_2 : *), \tau_2) \triangleright (\forall(\alpha_2 : *), \tau'_2)} \quad \text{SUBST\_FORALL}
\end{array}$$

$[x \mapsto t]t_1 \triangleright t_2$  substitution

$$\begin{array}{c}
\overline{[x \mapsto t]x \triangleright t} \quad \text{SUBST\_VAR1} \\
\frac{x_1 \neq x_2}{[x_1 \mapsto t]x_2 \triangleright x_2} \quad \text{SUBST\_VAR2} \\
\overline{[x \mapsto t_1](\lambda(x : \tau) \Rightarrow t_2) \triangleright (\lambda(x : \tau) \Rightarrow t_2)} \quad \text{SUBST\_ABS1} \\
\frac{x_1 \neq x_2 \quad x_2 \notin fv(t_1) \quad [x_1 \mapsto t_1]t_2 \triangleright t'_2}{[x_1 \mapsto t_1](\lambda(x_2 : \tau) \Rightarrow t_2) \triangleright (\lambda(x_2 : \tau) \Rightarrow t'_2)} \quad \text{SUBST\_ABS2} \\
\frac{x_1 \neq x_2 \quad x_3 \notin fv(t_1, t_2) \quad [x_2 \mapsto x_3]t_2 \triangleright t'_2 \quad [x_1 \mapsto t_1]t'_2 \triangleright t''_2}{[x_1 \mapsto t_1](\lambda(x_2 : \tau) \Rightarrow t_2) \triangleright (\lambda(x_3 : \tau) \Rightarrow t''_2)} \quad \text{SUBST\_ABS3} \\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2}{[x \mapsto t_1](\lambda\{\alpha : *\} \Rightarrow t_2) \triangleright (\lambda\{\alpha : *\} \Rightarrow t'_2)} \quad \text{SUBST\_TABS} \\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2 \quad [x \mapsto t_1]t_3 \triangleright t'_3}{[x \mapsto t_1](t_2 t_3) \triangleright t'_2 t'_3} \quad \text{SUBST\_APP} \\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2}{[x \mapsto t_1](t_2 [\tau]) \triangleright (t'_2 [\tau])} \quad \text{SUBST\_TAPP}
\end{array}$$

$\boxed{[\alpha \mapsto \tau]t_1 \triangleright t_2}$       substitution of type variable in term

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau]x \triangleright x} \quad \text{TTSUBST\_VAR} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \triangleright (\lambda(x : \tau'_2) \Rightarrow t_2)} \quad \text{TTSUBST\_ABS} \\
\\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]t_1 \triangleright t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : *\} \Rightarrow t_1) \triangleright (\lambda\{\alpha_2 : *\} \Rightarrow t_2)} \quad \text{TTSUBST\_TABS} \\
\\
\frac{[\alpha \mapsto \tau]t_1 \triangleright t'_1 \quad [\alpha \mapsto \tau]t_2 \triangleright t'_2}{[\alpha \mapsto \tau](t_1 \ t_2) \triangleright t'_1 \ t'_2} \quad \text{TTSUBST\_APP} \\
\\
\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 \ [\tau_2]) \triangleright (t_2 \ [\tau'_2])} \quad \text{TTSUBST\_TAPP}
\end{array}$$

Definition rules:                      31 good      0 bad

Definition rule clauses: 58 good      0 bad