

x	term variable	
α	type variable	
t	$::=$	term
	x	variable
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : *\} \Rightarrow t$	type abstraction
	$t_1 t_2$	application
	$t [\tau]$	type application
τ	$::=$	type
	α	type variable
	$\tau_1 \rightarrow \tau_2$	arrow
	$\forall(\alpha : *), \tau$	universal quantification
Γ	$::=$	typing environment
	\emptyset	empty
	$\Gamma, x : \tau$	variable
	$\Gamma, \alpha : *$	type variable
v	$::=$	value
	$\lambda(x : \tau) \Rightarrow t$	abstraction

Initial environment: $\Gamma = \emptyset$

$\boxed{\Gamma \vdash t : \tau}$ Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T_VAR} \\
\\
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \rightarrow \tau_2} \quad \text{T_ABS} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha : *\} \Rightarrow t) : \forall(\alpha : *), \tau} \quad \text{T_TYABS} \\
\\
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T_APP} \\
\\
\frac{\Gamma \vdash t : \forall(\alpha : *), \tau_2 \quad \Gamma \vdash \tau_1 : * \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau_2'}{\Gamma \vdash t [\tau_1] : \tau_2'} \quad \text{T_TYAPP}
\end{array}$$

$\boxed{\Gamma \vdash \tau : *}$ Type τ is well formed

$$\begin{array}{c}
\frac{\alpha : * \in \Gamma}{\Gamma \vdash \alpha : *} \quad \text{K_VAR} \\
\\
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : *}{\Gamma \vdash (\tau_1 \rightarrow \tau_2) : *} \quad \text{K_ARROW} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash \tau : *}{\Gamma \vdash (\forall(\alpha : *), \tau) : *} \quad \text{K_FORALL}
\end{array}$$

$\boxed{t \longrightarrow t'}$ Operational semantics

$$\frac{t_2 \longrightarrow t'_2}{t_1 \ t_2 \longrightarrow t_1 \ t'_2} \quad \text{E_APP1}$$

$$\frac{t \longrightarrow t'}{t \ v \longrightarrow t' \ v} \quad \text{E_APP2}$$

$$\frac{[x \mapsto v]t \triangleright t'}{(\lambda(x : \tau) \Rightarrow t) \ v \longrightarrow t'} \quad \text{E_APPAbs}$$

$$\frac{t \longrightarrow t'}{(\lambda\{\alpha : *\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : *\} \Rightarrow t')} \quad \text{E_TABS}$$

$$\frac{t \longrightarrow t'}{t \ [\tau] \longrightarrow t' \ [\tau]} \quad \text{E_TAPP}$$

$$\frac{[\alpha \mapsto \tau]v \triangleright v'}{(\lambda\{\alpha : *\} \Rightarrow v) \ [\tau] \longrightarrow v'} \quad \text{E_TAPPAbs}$$

$$\boxed{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3} \quad \text{Type substitution}$$

$$\overline{[\alpha \mapsto \tau]\alpha \triangleright \tau} \quad \text{SUBST_VAR1}$$

$$\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau]\alpha_2 \triangleright \alpha_2} \quad \text{SUBST_VAR2}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1](\tau_2 \rightarrow \tau_3) \triangleright \tau'_2 \rightarrow \tau'_3} \quad \text{SUBST_ARROW}$$

$$\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\forall(\alpha_2 : *), \tau_2) \triangleright (\forall(\alpha_2 : *), \tau'_2)} \quad \text{SUBST_FORALL}$$

$$\boxed{[x \mapsto v]t_1 \triangleright t_2} \quad \text{substitution}$$

$$\overline{[x \mapsto v]x \triangleright v} \quad \text{SUBST_VAR1}$$

$$\frac{x_1 \neq x_2}{[x_1 \mapsto v]x_2 \triangleright x_2} \quad \text{SUBST_VAR2}$$

$$\overline{[x \mapsto v](\lambda(x : \tau) \Rightarrow t) \triangleright (\lambda(x : \tau) \Rightarrow t)} \quad \text{SUBST_ABS1}$$

$$\frac{x_1 \neq x_2 \quad [x_1 \mapsto v]t_1 \triangleright t_2}{[x_1 \mapsto v](\lambda(x_2 : \tau) \Rightarrow t_1) \triangleright (\lambda(x_2 : \tau) \Rightarrow t_2)} \quad \text{SUBST_ABS2}$$

$$\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](\lambda\{\alpha : *\} \Rightarrow t_1) \triangleright (\lambda\{\alpha : *\} \Rightarrow t_2)} \quad \text{SUBST_TABS}$$

$$\frac{[x \mapsto v]t_1 \triangleright t'_1 \quad [x \mapsto v]t_2 \triangleright t'_2}{[x \mapsto v](t_1 \ t_2) \triangleright t'_1 \ t'_2} \quad \text{SUBST_APP}$$

$$\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](t_1 \ [\tau]) \triangleright (t_2 \ [\tau])} \quad \text{SUBST_TAPP}$$

$$\boxed{[\alpha \mapsto \tau]t_1 \triangleright t_2} \quad \text{substitution of type variable in term}$$

$$\overline{[\alpha \mapsto \tau]x \triangleright x} \quad \text{TTSUBST_VAR}$$

$$\begin{array}{c}
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \triangleright (\lambda(x : \tau'_2) \Rightarrow t_2)} \quad \text{TtSUBST_ABS} \\
\\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]t_1 \triangleright t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : *\} \Rightarrow t_1) \triangleright (\lambda\{\alpha_2 : *\} \Rightarrow t_2)} \quad \text{TtSUBST_TABS} \\
\\
\frac{[\alpha \mapsto \tau]t_1 \triangleright t'_1 \quad [\alpha \mapsto \tau]t_2 \triangleright t'_2}{[\alpha \mapsto \tau](t_1 \ t_2) \triangleright t'_1 \ t'_2} \quad \text{TtSUBST_APP} \\
\\
\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 \ [\tau_2]) \triangleright (t_2 \ [\tau'_2])} \quad \text{TtSUBST_TAPP}
\end{array}$$

Definition rules: 30 good 0 bad
 Definition rule clauses: 56 good 0 bad