

$x$	term variable	
$\alpha$	type variable	
$T$	type	
$t$	::=	term
	$x$	variable
	$\lambda(x : \tau) \rightarrow t$	abstraction
	$\lambda\{\alpha : \kappa\} \rightarrow t$	type abstraction
	$t_1 t_2$	application
	$t[\tau]$	type application
	$t : \tau$	type annotation
	$(t)$	S parenthesis
$v$	::=	value
	$\lambda(x : \tau) \rightarrow t$	abstraction
$\kappa$	::=	kind
	$*$	star
	$\kappa_1 \rightarrow \kappa_2$	kind arrow
	$(\kappa)$	S parenthesis
$\tau$	::=	type
	$\alpha$	type variable
	$T$	type
	$\tau_1 \rightarrow \tau_2$	arrow
	$\lambda(\alpha : \kappa), \tau$	operator abstraction
	$\forall(\alpha : \kappa), \tau$	forall
	$\tau_1 \tau_2$	operator application
	$(\tau)$	S parenthesis
$\Gamma$	::=	typing environment
	$\emptyset$	empty
	$\Gamma, x : \tau$	variable
	$\Gamma, T : \kappa$	abstract type
	$\Gamma, \alpha : \kappa$	type variable

$\boxed{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau_3}$     Type substitution

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau] \alpha \triangleright \tau} \text{ SUBSTT\_VAR1} \\
\\
\frac{}{[\alpha_1 \mapsto \tau] \alpha_2 \triangleright \alpha_2} \text{ SUBSTT\_VAR2} \\
\\
\frac{}{[\alpha \mapsto \tau] T \triangleright T} \text{ SUBSTT\_TYPE} \\
\\
\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1] \tau_2 \rightarrow \tau_3 \triangleright \tau'_2 \rightarrow \tau'_3} \text{ SUBSTT\_ARROW} \\
\\
\frac{}{[\alpha \mapsto \tau_1] \lambda(\alpha : \kappa), \tau_2 \triangleright \lambda(\alpha : \kappa), \tau_2} \text{ SUBSTT\_ABS1} \\
\\
\frac{[\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1] \lambda(\alpha_2 : \kappa), \tau_2 \triangleright \lambda(\alpha_2 : \kappa), \tau'_2} \text{ SUBSTT\_ABS2}
\end{array}$$

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau_1] \forall (\alpha : \kappa), \tau_2 \triangleright \forall (\alpha : \kappa), \tau_2} \text{SUBSTIT\_FORALL1} \\
\frac{[\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1] \forall (\alpha_2 : \kappa), \tau_2 \triangleright \forall (\alpha_2 : \kappa), \tau'_2} \text{SUBSTIT\_FORALL2} \\
\frac{\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1] \tau_2 \tau_3 \triangleright \tau'_2 \tau'_3}}{\text{SUBSTIT\_APP}}
\end{array}$$

$\boxed{\Gamma \vdash t : \tau}$  Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \text{T\_VAR} \\
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash \lambda(x : \tau_1) \rightarrow t : \tau_1 \rightarrow \tau_2} \text{T\_ABS} \\
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1 t_2 : \tau_1} \text{T\_APP} \\
\frac{\Gamma, \alpha : \kappa \vdash t : \tau \quad \alpha \notin \Gamma}{\Gamma \vdash \lambda\{\alpha : \kappa\} \rightarrow t : \forall (\alpha : \kappa), \tau} \text{T\_TYABS} \\
\frac{\Gamma \vdash t : \forall (\alpha : \kappa), \tau_2 \quad \Gamma \vdash \tau_1 : \kappa \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t[\tau_1] : \tau'_2} \text{T\_TYAPP} \\
\frac{\Gamma \vdash t : \tau_2 \quad \tau_1 \equiv \tau_2}{\Gamma \vdash (t : \tau_1) : \tau_1} \text{T\_ANNOT}
\end{array}$$

$\boxed{\Gamma \vdash \tau : \kappa}$  Kinding

$$\begin{array}{c}
\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \text{K\_VAR} \\
\frac{T : \kappa \in \Gamma}{\Gamma \vdash T : \kappa} \text{K\_ABSTYPE} \\
\frac{\Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash \lambda(\alpha : \kappa_1), \tau : \kappa_1 \rightarrow \kappa_2} \text{K\_ABS} \\
\frac{\Gamma \vdash \tau_1 : \kappa_2 \rightarrow \kappa_1 \quad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \tau_2 : \kappa_1} \text{K\_APP} \\
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : *}{\Gamma \vdash \tau_1 \rightarrow \tau_2 : *} \text{K\_ARROW} \\
\frac{\Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash \forall (\alpha : \kappa), \tau : *} \text{K\_FORALL}
\end{array}$$

$\boxed{\tau_1 \equiv \tau_2}$  Type equivalence

$$\begin{array}{c}
\frac{}{\tau \equiv \tau} \text{EQ\_REFL} \\
\frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \text{EQ\_SYMM} \\
\frac{\tau_1 \equiv \tau_2 \quad \tau_2 \equiv \tau_3}{\tau_1 \equiv \tau_3} \text{EQ\_TRANS} \\
\frac{}{\alpha \equiv \alpha} \text{EQ\_VAR} \\
\frac{}{T \equiv T} \text{EQ\_ABSTYPE} \\
\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2}{\tau_1 \rightarrow \tau_2 \equiv \tau'_1 \rightarrow \tau'_2} \text{EQ\_ARROW} \\
\frac{\tau_1 \equiv \tau_2}{\forall (\alpha : \kappa), \tau_1 \equiv \forall (\alpha : \kappa), \tau_2} \text{EQ\_FORALL} \\
\frac{\tau_1 \equiv \tau_2}{\lambda(\alpha : \kappa), \tau_1 \equiv \lambda(\alpha : \kappa), \tau_2} \text{EQ\_ABS} \\
\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2}{\tau_1 \tau_2 \equiv \tau'_1 \tau'_2} \text{EQ\_APP} \\
\frac{[\alpha \mapsto \tau_2] \tau_1 \triangleright \tau'_1}{(\lambda(\alpha : \kappa), \tau_1) \tau_2 \equiv \tau'_1} \text{EQ\_APPAbs}
\end{array}$$

$$\boxed{[x \mapsto v] t_1 \triangleright t_2} \quad \text{substitution}$$

$$\begin{array}{c}
\frac{}{[x \mapsto v] x \triangleright v} \text{SUBST\_VAR1} \\
\frac{}{[x_1 \mapsto v] x_2 \triangleright x_2} \text{SUBST\_VAR2} \\
\frac{}{[x \mapsto v] \lambda(x : \tau) \rightarrow t \triangleright \lambda(x : \tau) \rightarrow t} \text{SUBST\_ABS1} \\
\frac{[x_1 \mapsto v] t_1 \triangleright t_2}{[x_1 \mapsto v] \lambda(x_2 : \tau) \rightarrow t_1 \triangleright \lambda(x_2 : \tau) \rightarrow t_2} \text{SUBST\_ABS2} \\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] \lambda\{\alpha : \kappa\} \rightarrow t_1 \triangleright \lambda\{\alpha : \kappa\} \rightarrow t_2} \text{SUBST\_TABS} \\
\frac{[x \mapsto v] t_1 \triangleright t'_1 \quad [x \mapsto v] t_2 \triangleright t'_2}{[x \mapsto v] t_1 t_2 \triangleright t'_1 t'_2} \text{SUBST\_APP} \\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1[\tau] \triangleright t_2[\tau]} \text{SUBST\_TAPP} \\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 : \tau \triangleright t_2 : \tau} \text{SUBST\_ANNOT}
\end{array}$$

$$\boxed{t_1 \longrightarrow t_2} \quad \text{Evaluation}$$

$$\begin{array}{c}
\frac{t_2 \longrightarrow t'_2}{t_1 \ t_2 \longrightarrow t_1 \ t'_2} \quad \text{E\_APP1} \\
\\
\frac{t_1 \longrightarrow t'_1}{t_1 \ v \longrightarrow t'_1 \ v} \quad \text{E\_APP2} \\
\\
\frac{\begin{array}{c} [x \mapsto v] t \triangleright t' \\ t' \longrightarrow t'' \end{array}}{(\lambda(x : \tau) \rightarrow t) v \longrightarrow t''} \quad \text{E\_APPABS} \\
\\
\frac{t \longrightarrow t'}{\lambda\{\alpha : \kappa\} \rightarrow t \longrightarrow t'} \quad \text{E\_TABS} \\
\\
\frac{t \longrightarrow t'}{t[\tau] \longrightarrow t'} \quad \text{E\_TAPP} \\
\\
\frac{t \longrightarrow t'}{(t : \tau) \longrightarrow t'} \quad \text{E\_ANNOT}
\end{array}$$

Definition rules: 45 good 0 bad  
Definition rule clauses: 94 good 0 bad