$\begin{array}{ll} termvar, \ x & \text{term variable} \\ variant, \ V & \text{variant} \\ typvar, \ X & \text{type variable} \\ exc, \ Exc & \text{exception} \\ effect, \ E & \text{effect} \\ n & \\ m & \end{array}$

```
t
               ::=
                                                                                _{\rm term}
                                                                                    variable
                      \boldsymbol{x}
                      V
                                                                                    type constructors
                      \lambda(x:T) \to t
                                                                                    abstraction
                      \lambda(X:K) \to t
                                                                                    type abstraction
                      \lambda(E:\varphi) \to t
                                                                                    effect abstraction
                      t t'
                                                                                    application
                      t[T]
                                                                                    type application
                                                                                    effect application
                      t[[eff]]
                      \mathbf{let} \ x = t_1 \mathbf{in} \ t_2
                                                                                    let binding
                      \mathbf{let}\,\mathbf{rec}\,x:\,T=abs\,\mathbf{in}\;t
                                                                                    recursive let binding
                      \mathbf{match}\ t\ \mathbf{with}\ p_1 \to t_1|\,..\,|p_n \to t_n\,\mathbf{end}
                                                                                    pattern matching
                      (t: [eff]T)
                                                                                    annotation
                      fail [T]Exc t_1 ... t_n
                                                                                    fail
                      try t with pe_1 \rightarrow t_1 | ... | pe_n \rightarrow t_n end
                                                                                    try
                                                                          S
                      t;t'
                                                                                    sequence == let (_{-}: Unit) = t in t'
                                                                          S
                      (t)
                      [x_1 \mapsto t_1, \dots, x_n \mapsto t_n]t
                                                                          Μ
                      [X \mapsto T]t
                                                                          Μ
                      [E \mapsto eff]t
                                                                          Μ
                      {\bf failure}\ exnval
                                                                          Μ
                      TConstr V v_1 \dots v_n
                                                                          Μ
               ::=
                                                                                value
                      TConstr V v_1 \dots v_n
                                                                                    type constructors
                      \lambda(x:T) \to t
                                                                                    abstraction
abs
                                                                                lambda abstractions
               ::=
                      \lambda(x:T) \to t
                                                                                    abstraction
                      \lambda(X:K) \to abs
                                                                                    type abstraction
                      \lambda(E:\varphi) \to abs
                                                                                    effect abstraction
                      (abs: [eff]T)
                                                                                    type annotation
               ::=
                                                                                pattern
p
                      V p_1 \dots p_n
                                                                                   variant
                                                                                    wildcard variable
               ::=
                                                                                try pattern
pe
                                                                                   Exception pattern
                      Exc x_1 \dots x_n
                                                                                runtime value of exceptions
exnval
               ::=
                      Exc v_1 \dots v_n
                                                                                effects elements
effelm
               ::=
                      E
                                                                                    effect
                      IO
                                                                                   IO effect
                      \mathbf{Exn}[exn]
                                                                                    exception
```

```
::=
                                                                exceptions
exn
                  Exc_1 | ... | Exc_n
                                                                effect
eff
                 ::=
                        effelm_1, ..., effelm_n
                                                           Μ
                        eff_1 \cup eff_2 \cup ... \cup eff_n
                        eff_1 \setminus [exn]
                                                           Μ
                        (eff)
                                                           S
K
                                                                kinds
                 ::=
                                                                   star
                        K \to K'
                                                                   kind arrow
T
                                                                type
                        X
                                                                   variable
                        Unit
                                                                   Unit type (contained in the module opened by default)
                         T 
ightarrow T'
                                                           S
                                                                   pure function == T - [] - i T'
                        T - [eff] - > T'
                                                                   function
                        \lambda(X:K), T
                                                                   operator abstraction
                        \forall (X:K), T
                                                                   for all
                        \forall (E:\varphi), T
                                                                   effect forall
                         T T'
                                                                   operator application
                                                           S
                        (T)
                        [X \mapsto T]T'
                                                           Μ
                        [E \mapsto \mathit{eff}] T
                                                           Μ
Γ
                                                                 type environment
                        empty
                                                                   empty
                        \Gamma, x_1: T_1, \ldots, x_n: T_n
                                                                    vars
                                                           S
                        \Gamma, V: T
                                                                    type constructors (contained in the above values enviro
                        \Gamma, X : K
                                                                    tvars
                        \Gamma, T : \{ V_1 : T_1 ... V_n : T_n \}
                                                                    variants
                        \Gamma, Exc exc Ty
                                                                    exceptions
                        \Gamma, E
                                                                   effects
                        \Gamma_1 \cup ... \cup \Gamma_n
                                                           Μ
excTy
                        T_1 \dots T_n
terminals
```

```
\equiv
                            &
formula
                           judgement
                           formula_1 .. formula_n
                           \mathbf{not} (formula)
                           T \equiv T'
                           x:\,T\,\in\,\Gamma
                           X:K\,\in\,\Gamma
                           V:\,T\,\in\,\Gamma
                           T: \{Variant\} \in \Gamma
                           E \in \Gamma
                           Exc\ exc\ Ty\ \in\ \Gamma
                           \mathbf{set}(eff) = \mathbf{set}(eff')
                           \mathbf{set}(exn) = \mathbf{set}(exn')
                           V \in Variant \triangleright T_1 ... T_n
                            Variant is empty
substs
                   ::=
                           x_1 \leftarrow v_1, \dots, x_n \leftarrow v_n
                           substs_1 \cup ... \cup substs_n
                                                                  Μ
patterns
                   ::=
                    p_1 \dots p_n
VArgs
                           T_1 \dots T_n
Variant
                           V_1 VArgs_1 \dots V_n VArgs_n
                           empty
                            Variant \backslash \, V
                                                                  Μ
Jtype
                           \Gamma \vdash t : [\mathit{eff}] T
                                                                            Typing
Jkind
                   ::=
                           \Gamma \vdash \, T : K
                                                                            Kinding
JEff
                   ::=
                    | \Gamma \vdash \mathit{eff}
                                                                            Effects typing
```

```
JEffElm
                             ::=
                                   \Gamma \vdash \mathit{effelm}
                             Effects elements typing
JP atterns \, Typing
                            ::=
                                    Variant & \Gamma \vdash patterns : T \rhd \Gamma_1 ... \Gamma_n
                                                                                                Patterns matching typing
JPattern\,Typing
                             ::=
                                    Variant \& \Gamma \vdash p : T \rhd \Gamma'
                                                                                                Pattern matching typing
                              JExnPattern\,Typing
                             ::=
                                   \Gamma \vdash pe \rhd Exc \& \Gamma'
                                                                                                Exception pattern matching
Jequiv
                             ::=
                                   T \equiv T'
                                                                                                Type equivalence
J\!E\!f\!f\!Equiv
                                   eff \equiv eff'
                                                                                                Effects equivalence
JEffElmEquiv
                             ::=
                                   effelm \equiv effelm'
                                                                                                Effect element equivalence
Jop
                                   t \longrightarrow t'
                                                                                                Evaluation
JExnMatches
                             ::=
                                   exnval matches pe \rhd \{x_1 \leftarrow v_1, ..., x_n \leftarrow v_n\}
                                                                                                Exception pattern matching
JMatches
                             ::=
                                   v  matches p \rhd \{substs\}
                                                                                                Pattern matching with subst
judgement
                             ::=
                                   Jtype
                                   Jkind
                                   JEff
                                   JEffElm
                                   JPatterns Typing
                                   JPattern\,Typing
                                   JExnPattern\,Typing
                                   Jequiv
                                   JEffEquiv
                                   JEffElmEquiv
                                   Jop
                                   JExnMatches
                                   JMatches
user\_syntax
                             ::=
                                   termvar
```

varianttypvarexceffectnmtvabsppeexnvaleffelmexneffKT Γ excTyterminalsformulasubstspatternsVArgsVariant

$\Gamma \vdash t : [eff] T$ Typing

$$\frac{x:T\in\Gamma}{\Gamma\vdash x:[]T}\quad \text{T-VAR}$$

$$\frac{V:T\in\Gamma}{\Gamma\vdash V:[]T}\quad \text{T-VARIANT}$$

$$\frac{\Gamma,x_1:T_1\vdash t:[eff]T}{\Gamma\vdash T_1:*}\quad \text{T-ABS}$$

$$\frac{\Gamma\vdash \lambda(x_1:T_1)\to t:[]T_1-[eff]->T}{\Gamma\vdash t:[eff_1]T_1-[eff_2]->T_2}\quad \text{T-APP}$$

$$\frac{\Gamma\vdash t:[eff_1]T_1-[eff_2]->T_2}{\Gamma\vdash t':[eff_1]\cup eff_2\cup eff_3]T_2}\quad \text{T-APP}$$

$$\frac{\Gamma,X:K\vdash t:[eff_1]T}{\Gamma\vdash \lambda(X:K)\to t:[eff]\forall\,(X:K),T}\quad \text{T-TABS}$$

$$\frac{\Gamma,E\vdash t:[eff]T}{\Gamma\vdash \lambda(E:\varphi)\to t:[eff]\forall\,(E:\varphi),T}\quad \text{T-EABS}$$

$$\frac{\Gamma\vdash t:[eff]\forall\,(X:K),T_2}{\Gamma\vdash T_1:K}\quad \text{T-TAPP}$$

$$\frac{\Gamma\vdash t:[eff][X\mapsto T_1]T_2}{\Gamma\vdash t:[eff][X\mapsto T_1]T_2}\quad \text{T-TAPP}$$

$$\frac{\Gamma\vdash t:[eff][X\mapsto T_1]T_2}{\Gamma\vdash eff']:[eff][E\mapsto eff']T}\quad \text{T-EAPP}$$

 $\Gamma \vdash eff$ Effects typing

$$\frac{\Gamma \vdash effelm_1 \quad .. \quad \Gamma \vdash effelm_n}{\Gamma \vdash effelm_1, \dots, effelm_n} \quad \text{Eff_Eff}$$

 $\Gamma \vdash effelm$ Effects elements typing

$$\frac{E \in \Gamma}{\Gamma \vdash E} \quad \text{EffElm_Eff}$$

$$\frac{\Gamma \vdash \mathbf{IO}}{\Gamma \vdash \mathbf{IO}} \quad \text{EffElm_IO}$$

$$\frac{Exc_1 \ exc Ty_1 \in \Gamma \quad .. \quad Exc_n \ exc Ty_n \in \Gamma}{\Gamma \vdash \mathbf{Exn} \left[Exc_1 | .. | Exc_n \right]} \quad \text{EffElm_Exn}$$

Variant & $\Gamma \vdash patterns : T \rhd \Gamma_1 ... \Gamma_n$ Patterns matching typing

$$\frac{Variant \& \Gamma \vdash p_1 : T \rhd \Gamma_1 \quad .. \quad Variant \& \Gamma \vdash p_n : T \rhd \Gamma_n}{Variant \& \Gamma \vdash p_1 ... p_n : T \rhd \Gamma_1 ... \Gamma_n} \quad PsTy_Patterns$$

Variant & $\Gamma \vdash p : T \rhd \Gamma'$ Pattern matching typing

 $V \in Variant \triangleright T_1 ... T_n$

 $T_1: \{Variant_1\} \in \Gamma \quad \dots \quad T_n: \{Variant_n\} \in \Gamma$

 $\frac{Variant_1 \& \Gamma \vdash p_1 : T_1 \rhd \Gamma_1 \quad .. \quad Variant_n \& \Gamma \vdash p_n : T_n \rhd \Gamma_n}{Variant \& \Gamma \vdash V p_1 ... p_n : T \rhd \Gamma_1 \cup ... \cup \Gamma_n} \quad \text{PTY_VARIANT}$

$$\overline{\textit{Variant \& } \Gamma \vdash x : T \rhd \mathbf{empty}, x : T} \quad \mathsf{PTY_WILDCARD}$$

 $\Gamma \vdash pe \rhd Exc \& \Gamma'$ Exception pattern matching typing

$$\frac{Exc\ T_1 ... T_n \in \Gamma}{\Gamma \vdash Exc\ x_1 ... x_n \rhd Exc\ \&\ \Gamma, x_1 : T_1, ..., x_n : T_n} \quad \text{PETY_EXC}$$

 $T \equiv T'$ Type equivalence

$$\overline{T \equiv T} \qquad \text{Q-Refl}$$

$$\overline{T \equiv T'} \qquad \text{Q-Symm}$$

$$T_1 \equiv T_2$$

$$T_2 \equiv T_3$$

$$T_1 \equiv T_3 \qquad \text{Q-Trans}$$

$$T_{11} \equiv T_{21}$$

$$eff_1 \equiv eff_2$$

$$T_{12} \equiv T_{22}$$

$$T_{11} - [eff_1] - > T_{12} \equiv T_{21} - [eff_2] - > T_{22}$$

$$Q_{-ARROW}$$

$$T_1 \equiv T_2$$

$$\forall (X:K), T_1 \equiv \forall (X:K), T_2$$

$$Q_{-ALL}$$

$$T_1 \equiv T_2$$

$$\forall (E:\varphi), T_1 \equiv \forall (E:\varphi), T_2$$

$$Q_{-EALL}$$

$$T_1 \equiv T_2$$

$$\forall (E:\varphi), T_1 \equiv \forall (E:\varphi), T_2$$

$$Q_{-ALL}$$

$$T_1 \equiv T_2$$

$$\forall (E:\varphi), T_1 \equiv \forall (E:\varphi), T_2$$

$$Q_{-ALL}$$

$$\begin{split} T_{11} &\equiv T_{21} \\ T_{12} &\equiv T_{22} \\ \hline T_{11} \ T_{12} &\equiv T_{21} \ T_{22} \end{split} \quad \mathbf{Q}_\mathbf{APP} \\ \hline (\lambda(X:K), T_{11}) \ T_{12} &\equiv [X \mapsto T_{12}] \ T_{11} \end{split} \quad \mathbf{Q}_\mathbf{APPABS} \end{split}$$

 $eff \equiv eff'$ Effects equivalence

$$\frac{eff \equiv eff}{eff} \quad \text{EffEQ_RefL}$$

$$\frac{\text{set } (effelm_1, ..., effelm_n) = \text{set } (effelm'_1, ..., effelm'_n)}{effelm_1, ..., effelm_n \equiv effelm'_1, ..., effelm'_n} \quad \text{EffEQ_EQ}$$

 $effelm \equiv effelm'$ Effect element equivalence

 $|t \longrightarrow t'|$ Evaluation

 $\overline{(\mathbf{failure}\ exnval)[[eff]] \longrightarrow \mathbf{failure}\ exnval} \quad \text{E_EAPPFAILURE}$

```
\frac{t \longrightarrow t'}{t\lceil [eff] \rceil \longrightarrow t'\lceil [eff] \rceil} \quad \text{E\_EAPP}
                                                                                           \frac{}{(\lambda(E:\varphi)\to t)[[\mathit{eff}]]\longrightarrow [E\mapsto \mathit{eff}]t}\quad \text{E\_EAppEAbs}
                                                                                                                                                                                                                                                                    E_LetFailure
                                                                      \overline{\mathbf{let} \ x = \mathbf{failure} \ exnval \ \mathbf{in} \ t_2 \longrightarrow \mathbf{failure} \ exnval}
                                                                                                             \frac{t_1 \longrightarrow t'_1}{\mathbf{let} \ x = t_1 \ \mathbf{in} \ t_2 \longrightarrow \mathbf{let} \ x = t'_1 \ \mathbf{in} \ t_2} \quad \mathbf{E\_LET1}
                                                                                                                           \overline{\mathbf{let}\,x = v\,\mathbf{in}\,t \longrightarrow [x \mapsto v]t} \quad \text{E\_Let2}
                                                                                                                                                                                                                                                                                                                   E_LETREC
                                             \overline{\mathbf{let}\,\mathbf{rec}\,x:T=abs\,\mathbf{in}\,t\longrightarrow[x\mapsto(\mathbf{let}\,\mathbf{rec}\,x:T=abs\,\mathbf{in}\,abs)]t}
                                                                                                                                                                                                                                                                                                                    E_MatchFailure
              	extbf{match failure} \ exnval \ 	extbf{with} \ p_1 	o t_1 | ... | p_n 	o t_n \ 	extbf{end} \longrightarrow 	extbf{failure} \ exnval \ 	extbf{example}
         \frac{t \longrightarrow t'}{\mathbf{match} \ t \ \mathbf{with} \ p_1 \to t_1 | ... | p_n \to t_n \, \mathbf{end} \longrightarrow \mathbf{match} \ t' \, \mathbf{with} \ p_1 \to t_1 | ... | p_n \to t_n \, \mathbf{end}}
                        \frac{v \text{ matches } p_1 \rhd \{x_1 \leftarrow v_1, \dots, x_n \leftarrow v_n\}}{\text{match } v \text{ with } p_1 \rightarrow t_1 | \dots | p_n \rightarrow t_n \text{ end } \longrightarrow [x_1 \mapsto v_1, \dots, x_n \mapsto v_n] t_1}
                                                                                                                                                                                                                                                                                                               E_MatchFound
                                                                                           \mathbf{not} (v \mathbf{ matches } p_1 \rhd \{x_1 \leftarrow v_1, ..., x_n \leftarrow v_n\})
\frac{1}{\operatorname{match} v \operatorname{with} p_1 \to t_1 | p_2 \to t_2 | ... | p_n \to t_n \operatorname{end} \longrightarrow \operatorname{match} v \operatorname{with} p_2 \to t_2 | ... | p_n \to t_n \operatorname{end}}{\operatorname{match} v \operatorname{with} p_2 \to t_2 | ... | p_n \to t_n \operatorname{end}}
                                                                                                                                                                                                                                                                                                                                                                                    E_MATCHSTEP
                                                                                                                                          \frac{t \longrightarrow t'}{(t: [\mathit{eff}]T) \longrightarrow t'} \quad \text{E-Annot}
                                             \frac{t \longrightarrow t'}{\mathbf{fail} [T] \mathit{Exc} \ v_1 \ldots v_n \ t \ t_1 \ldots t_n \longrightarrow \mathbf{fail} [T] \mathit{Exc} \ v_1 \ldots v_n \ t' \ t_1 \ldots t_n} \quad \text{E\_FAILUREARGS}

\frac{}{\mathbf{fail} [T] Exc \ v_1 \dots v_n \longrightarrow \mathbf{failure} \ Exc \ v_1 \dots v_n} \qquad \text{E_FAILURE}

                      \frac{t \longrightarrow t'}{\textbf{try } t \, \textbf{with } pe_1 \rightarrow t_1 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try } t' \, \textbf{with } pe_1 \rightarrow t_1 | ... | pe_n \rightarrow t_n \, \textbf{end}}
                                                                                                                                                                                                                                                                                                                                                           E_TRY
                                                                     \overline{\mathbf{try}\ v\ \mathbf{with}\ pe_1 \to t_1|..|pe_n \to t_n\ \mathbf{end} \longrightarrow v}
                                                                                                                                                                                                                                                            E_TryNoFailure
                                                     \mathbf{not} \ (\mathit{exnval} \ \mathbf{matches} \ \mathit{pe}_1 \rhd \{\mathit{x}_1 \leftarrow \mathit{v}_1, \, \ldots, \mathit{x}_n \leftarrow \mathit{v}_n\})
                                                                                                                                                                                                                                                                                  E_TryNotFound
                                          \frac{1}{\text{try failure } exnval \text{ with } pe_1 \rightarrow t_1 \text{ end } \longrightarrow \text{failure } exnval}
        \frac{exnval\ \mathbf{matches}\ pe_1\rhd\{x_1\leftarrow v_1,\,..\,,x_n\leftarrow v_n\}}{\mathbf{try}\ \mathbf{failure}\ exnval\ \mathbf{with}\ pe_1\rightarrow t_1|\,..\,|pe_n\rightarrow t_n\ \mathbf{end}\longrightarrow[x_1\mapsto v_1,\,..\,,x_n\mapsto v_n]t_1}
                                                                                                                             not (exnval matches pe_1 \rhd \{x_1 \leftarrow v_1, ..., x_n \leftarrow v_n\})
 \overline{\text{try failure } \textit{exnval } \textbf{with } pe_1 \rightarrow t_1 | pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textit{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textit{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textit{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textit{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textit{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textbf{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textbf{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textbf{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textbf{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textbf{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textbf{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textbf{exnval } \textbf{with } pe_2 \rightarrow t_2 | ... | pe_n \rightarrow t_n \, \textbf{end} \longrightarrow \textbf{try failure } \textbf{exnval } 
        exnval matches pe \rhd \{x_1 \leftarrow v_1, ..., x_n \leftarrow v_n\} Exception pattern matching with substitution creation
                                                                                                                                                                                                                                                                          EXNMATCHES_MATCHES
                            \overline{Exc\ v_1 \dots v_n\ \mathbf{matches}\ Exc\ x_1 \dots x_n \rhd \{x_1 \leftarrow v_1, \dots, x_n \leftarrow v_n\}}
          v  matches p \rhd \{substs\}
                                                                                                                        Pattern matching with substitution creation
                                                                                                                 \frac{}{v \text{ matches } x \rhd \{x \leftarrow v\}} \quad \text{Matches\_Any}
                   \frac{v_1 \ \mathbf{matches} \ p_1 \rhd \{substs_1\} \quad .. \quad v_n \ \mathbf{matches} \ p_n \rhd \{substs_n\}}{\mathbf{TConstr} \ V \ v_1 \ .. \ v_n \ \mathbf{matches} \ V \ p_1 \ .. \ p_n \rhd \{substs_1 \ \cup \ .. \ \cup \ substs_n\}}
                                                                                                                                                                                                                                                                                               MATCHES_MATCHES
                                                                                                                       76 good
                                                                                                                                                                             0 bad
Definition rules:
```

Definition rule clauses: 155 good