

$x$	term variable	
$\alpha$	type variable	
$T$	type constructor	
$c$	coercion variable	
$i$	index metavariable	
$t$	$::=$	term
	$x$	variable
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha\} \Rightarrow t$	type abstraction
	$\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t$	coercion abstraction
	$t_1 t_2$	application
	$t [\tau]$	type application
	$t \sim[\gamma]$	coercion application
	$t : \tau$	type annotation
	$t \blacktriangleright \gamma$	coercion
$\tau$	$::=$	type
	$\alpha$	type variable
	$T$	type constructor
	$\tau_1 \rightarrow \tau_2$	arrow
	$\{\tau_1 \sim \tau_2\} \rightarrow \tau_3$	coercion arrow
	$\forall \alpha, \tau$	universal quantification
$\gamma$	$::=$	coercion proof term
	$c$	variable
	<b>refl</b> $\tau$	reflexivity
	<b>sym</b> $\gamma$	symmetry
	$\gamma_1 \circ \gamma_2$	composition
	$\gamma_1 \rightarrow \gamma_2$	arrow introduction
	$\{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3$	coercion arrow introduction
	$\forall \alpha, \gamma$	universal quantification introduction
	$\gamma @ \tau$	instantiation (quantification elimination)
	<b>elim</b> <sub>i</sub> $\gamma$	generalized elimination
$\Gamma$	$::=$	typing environment
	$\emptyset$	empty
	$\Gamma, x : \tau$	variable
	$\Gamma, T$	type constructor
	$\Gamma, \alpha$	type variable
	$\Gamma, c : \tau_1 \sim \tau_2$	coercion variable
$tv$	$::=$	typed value
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha\} \Rightarrow tv$	type abstraction
	$\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow tv$	coercion abstraction
	$tv [\tau]$	type application
	$tv \sim[\gamma]$	coercion application

	$tv : \tau$	type annotation
	$tv \blacktriangleright \gamma$	coercion

$v$	$::=$	value
	$\lambda x \Rightarrow t$	abstraction

---

Initial environment:  $\Gamma = \emptyset$

---

$\boxed{\Gamma \vdash t : \tau}$     Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T\_VAR} \\
\\
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \rightarrow \tau_2} \quad \text{T\_ABS} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha\} \Rightarrow t) : \forall \alpha, \tau} \quad \text{T\_TYABS} \\
\\
\frac{\Gamma, c : \tau_1 \sim \tau_2 \vdash t : \tau_3 \quad \Gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2}{\Gamma \vdash (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3} \quad \text{T\_CABS} \\
\\
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T\_APP} \\
\\
\frac{\Gamma \vdash t : \forall \alpha, \tau_2 \quad \Gamma \vdash \tau_1 \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T\_TYAPP} \\
\\
\frac{\Gamma \vdash t : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \quad \Gamma \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash t \sim[\gamma] : \tau_3} \quad \text{T\_CAPP} \\
\\
\frac{\Gamma \vdash t : \tau_1}{\Gamma \vdash (t : \tau_1) : \tau_1} \quad \text{T\_ANNOT} \\
\\
\frac{\Gamma \vdash \gamma : \tau_1 \sim \tau_2 \quad \Gamma \vdash t : \tau_1}{\Gamma \vdash (t \blacktriangleright \gamma) : \tau_2} \quad \text{T\_COERCE}
\end{array}$$

$\boxed{\Gamma \vdash \gamma : \tau_1 \sim \tau_2}$     Coercion typing

$$\begin{array}{c}
\frac{c : \tau_1 \sim \tau_2 \in \Gamma}{\Gamma \vdash c : \tau_1 \sim \tau_2} \quad \text{C\_VAR} \\
\\
\frac{\Gamma \vdash \tau}{\Gamma \vdash \mathbf{refl} \tau : \tau \sim \tau} \quad \text{C\_REFL} \\
\\
\frac{\Gamma \vdash \gamma : \tau_2 \sim \tau_1}{\Gamma \vdash \mathbf{sym} \gamma : \tau_1 \sim \tau_2} \quad \text{C\_SYM} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau_3}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \quad \text{C\_COMP} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \tau_1 \rightarrow \tau_2}{\Gamma \vdash (\gamma_1 \rightarrow \gamma_2) : (\tau_1 \rightarrow \tau_2) \sim (\tau'_1 \rightarrow \tau'_2)} \quad \text{C\_ARROW}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \gamma_3 : \tau_3 \sim \tau'_3}{\Gamma \vdash \{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3 : (\{\tau_1 \sim \tau_2\} \rightarrow \tau_3) \sim (\{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3)} \quad \text{C\_CARROW} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha \vdash \gamma : \tau_1 \sim \tau_2 \quad \Gamma \vdash \forall \alpha, \tau_1}{\Gamma \vdash (\forall \alpha, \gamma) : (\forall \alpha, \tau_1) \sim (\forall \alpha, \tau_2)} \quad \text{C\_FORALL} \\
\\
\frac{\Gamma \vdash \tau_1 \quad \Gamma \vdash \gamma : (\forall \alpha_1, \tau_2) \sim (\forall \alpha_2, \tau_3)}{[\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha_2 \mapsto \tau_1] \tau_3 \triangleright \tau'_3} \quad \text{C\_INST} \\
\Gamma \vdash \gamma @ \tau_1 : \tau'_2 \sim \tau'_3 \\
\\
\frac{i \in \{1, 2\} \quad \Gamma \vdash \gamma : (\tau_1 \rightarrow \tau_2) \sim (\tau'_1 \rightarrow \tau'_2)}{\Gamma \vdash \mathbf{elim}_i \gamma : \tau_i \sim \tau'_i} \quad \text{C\_ELIMARROW} \\
\\
\frac{i \in \{1, 2, 3\} \quad \Gamma \vdash \gamma : (\{\tau_1 \sim \tau_2\} \rightarrow \tau_3) \sim (\{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3)}{\Gamma \vdash \mathbf{elim}_i \gamma : \tau_i \sim \tau'_i} \quad \text{C\_ELIMCARROW}
\end{array}$$

$\boxed{\Gamma \vdash \tau}$  Type  $\tau$  is well formed

$$\begin{array}{c}
\frac{\alpha \in \Gamma}{\Gamma \vdash \alpha} \quad \text{K\_VAR} \\
\\
\frac{T \in \Gamma}{\Gamma \vdash T} \quad \text{K\_TYPECONSTR} \\
\\
\frac{\Gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2}{\Gamma \vdash \tau_1 \rightarrow \tau_2} \quad \text{K\_ARROW} \\
\\
\frac{\Gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2 \quad \Gamma \vdash \tau_3}{\Gamma \vdash \{\tau_1 \sim \tau_2\} \rightarrow \tau_3} \quad \text{K\_CARROW} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha \vdash \tau}{\Gamma \vdash \forall \alpha, \tau} \quad \text{K\_FORALL}
\end{array}$$

$\boxed{t \longrightarrow tv}$  Operational semantics

$$\begin{array}{c}
\frac{t_2 \longrightarrow tv_2 \quad t_1 \, tv_2 \longrightarrow tv_3}{t_1 \, t_2 \longrightarrow tv_3} \quad \text{E\_APP1} \\
\\
\frac{t \longrightarrow tv_1 \quad tv_1 \, tv_2 \longrightarrow tv_3}{t \, tv_2 \longrightarrow tv_3} \quad \text{E\_APP2} \\
\\
\frac{[x \mapsto tv_1] t \triangleright t' \quad t' \longrightarrow tv_2}{(\lambda(x : \tau) \Rightarrow t) \, tv_1 \longrightarrow tv_2} \quad \text{E\_APPABS} \\
\\
\frac{t \longrightarrow tv}{(\lambda\{\alpha\} \Rightarrow t) \longrightarrow (\lambda\{\alpha\} \Rightarrow tv)} \quad \text{E\_TABS} \\
\\
\frac{t \longrightarrow tv}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \longrightarrow (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow tv)} \quad \text{E\_CABS} \\
\\
\frac{t \longrightarrow tv}{t \, [\tau] \longrightarrow tv \, [\tau]} \quad \text{E\_TAPP} \\
\\
\frac{[\alpha \mapsto \tau] t \triangleright t' \quad t' \longrightarrow tv}{(\lambda\{\alpha\} \Rightarrow t) \, [\tau] \longrightarrow tv} \quad \text{E\_TAPPABS} \\
\\
\frac{t \longrightarrow tv}{t \sim [\gamma] \longrightarrow tv \sim [\gamma]} \quad \text{E\_CAPP}
\end{array}$$

$$\frac{[c \mapsto \gamma]t \triangleright t' \quad t' \longrightarrow tv}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \sim[\gamma] \longrightarrow tv} \quad \text{E\_CAPPABS}$$

$$\frac{t \longrightarrow tv}{(t : \tau) \longrightarrow (tv : \tau)} \quad \text{E\_ANNOT}$$

$$\frac{t \longrightarrow tv}{(t \blacktriangleright \gamma) \longrightarrow (tv \blacktriangleright \gamma)} \quad \text{E\_COERCE}$$

$\boxed{tv \longrightarrow v}$     type erasure

$$\overline{(\lambda(x : \tau) \Rightarrow t) \longrightarrow (\lambda x \Rightarrow t)} \quad \text{ERASE\_ABS}$$

$$\frac{tv \longrightarrow v}{(\lambda\{\alpha\} \Rightarrow tv) \longrightarrow v} \quad \text{ERASE\_TABS}$$

$$\frac{tv \longrightarrow v}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow tv) \longrightarrow v} \quad \text{ERASE\_CABS}$$

$$\frac{tv \longrightarrow v}{(tv [\tau]) \longrightarrow v} \quad \text{ERASE\_TAPP}$$

$$\frac{tv \longrightarrow v}{(tv \sim[\gamma]) \longrightarrow v} \quad \text{ERASE\_CAPP}$$

$$\frac{tv \longrightarrow v}{(tv : \tau) \longrightarrow v} \quad \text{ERASE\_ANNOT}$$

$$\frac{tv \longrightarrow v}{(tv \blacktriangleright \gamma) \longrightarrow v} \quad \text{ERASE\_COERCE}$$

$\boxed{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3}$     Type substitution

$$\overline{[\alpha \mapsto \tau]\alpha \triangleright \tau} \quad \text{SUBST\_VAR1}$$

$$\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau]\alpha_2 \triangleright \alpha_2} \quad \text{SUBST\_VAR2}$$

$$\overline{[\alpha \mapsto \tau]T \triangleright T} \quad \text{SUBST\_TYPE}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1](\tau_2 \rightarrow \tau_3) \triangleright \tau'_2 \rightarrow \tau'_3} \quad \text{SUBST\_ARROW}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3 \quad [\alpha \mapsto \tau_1]\tau_4 \triangleright \tau'_4}{[\alpha \mapsto \tau_1](\{\tau_2 \sim \tau_3\} \rightarrow \tau_4) \triangleright (\{\tau'_2 \sim \tau'_3\} \rightarrow \tau'_4)} \quad \text{SUBST\_CARROW}$$

$$\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\forall \alpha_2, \tau_2) \triangleright (\forall \alpha_2, \tau'_2)} \quad \text{SUBST\_FORALL}$$

$\boxed{[x \mapsto tv]t_1 \triangleright t_2}$     substitution

$$\overline{[x \mapsto tv]x \triangleright tv} \quad \text{SUBST\_VAR1}$$

$$\frac{x_1 \neq x_2}{[x_1 \mapsto tv]x_2 \triangleright x_2} \quad \text{SUBST\_VAR2}$$

$$\overline{[x \mapsto tv](\lambda(x : \tau) \Rightarrow t) \triangleright (\lambda(x : \tau) \Rightarrow t)} \quad \text{SUBST\_ABS1}$$

$$\frac{x_1 \neq x_2 \quad [x_1 \mapsto tv]t_1 \triangleright t_2}{[x_1 \mapsto tv](\lambda(x_2 : \tau) \Rightarrow t_1) \triangleright (\lambda(x_2 : \tau) \Rightarrow t_2)} \text{ SUBST\_ABS2}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](\lambda\{\alpha\} \Rightarrow t_1) \triangleright (\lambda\{\alpha\} \Rightarrow t_2)} \text{ SUBST\_TABS}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_1) \triangleright (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_2)} \text{ SUBST\_CABS}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t'_1 \quad [x \mapsto tv]t_2 \triangleright t'_2}{[x \mapsto tv](t_1 \ t_2) \triangleright t'_1 \ t'_2} \text{ SUBST\_APP}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](t_1 \ [\tau]) \triangleright (t_2 \ [\tau])} \text{ SUBST\_TAPP}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](t_1 \sim [\gamma]) \triangleright (t_2 \sim [\gamma])} \text{ SUBST\_CAPP}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](t_1 : \tau) \triangleright (t_2 : \tau)} \text{ SUBST\_ANNOT}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](t_1 \blacktriangleright \gamma) \triangleright (t_2 \blacktriangleright \gamma)} \text{ SUBST\_COERCE}$$

$$\boxed{[\alpha \mapsto \tau]t_1 \triangleright t_2} \quad \text{substitution of type variable in term}$$

$$\frac{}{[\alpha \mapsto \tau]x \triangleright x} \text{ TT\_SUBST\_VAR}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \triangleright (\lambda(x : \tau'_2) \Rightarrow t_2)} \text{ TT\_SUBST\_ABS}$$

$$\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]t_1 \triangleright t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2\} \Rightarrow t_1) \triangleright (\lambda\{\alpha_2\} \Rightarrow t_2)} \text{ TT\_SUBST\_TABS}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda\{c : \tau_2 \sim \tau_3\} \Rightarrow t_1) \triangleright (\lambda\{c : \tau'_2 \sim \tau'_3\} \Rightarrow t_2)} \text{ TT\_SUBST\_CABS}$$

$$\frac{[\alpha \mapsto \tau]t_1 \triangleright t'_1 \quad [\alpha \mapsto \tau]t_2 \triangleright t'_2}{[\alpha \mapsto \tau](t_1 \ t_2) \triangleright t'_1 \ t'_2} \text{ TT\_SUBST\_APP}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 \ [\tau_2]) \triangleright (t_2 \ [\tau'_2])} \text{ TT\_SUBST\_TAPP}$$

$$\frac{[\alpha \mapsto \tau]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](t_1 \sim [\gamma_1]) \triangleright (t_2 \sim [\gamma_2])} \text{ TT\_SUBST\_CAPP}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 : \tau_2) \triangleright (t_2 : \tau'_2)} \text{ TT\_SUBST\_ANNOT}$$

$$\frac{[\alpha \mapsto \tau]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](t_1 \blacktriangleright \gamma_1) \triangleright (t_2 \blacktriangleright \gamma_2)} \text{ TT\_SUBST\_COERCE}$$

$$\boxed{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2} \quad \text{substitution of type variable in coercion term}$$

$$\frac{}{[\alpha \mapsto \tau]c \triangleright c} \text{ ACSUBST\_VAR}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](\mathbf{refl} \ \tau_2) \triangleright \mathbf{refl} \ \tau'_2} \text{ ACSUBST\_REFL}$$

$$\begin{array}{c}
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](\mathbf{sym} \gamma_1) \triangleright \mathbf{sym} \gamma_2} \quad \text{ACSUBST\_SYM} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2}{[\alpha \mapsto \tau](\gamma_1 \circ \gamma_2) \triangleright \gamma'_1 \circ \gamma'_2} \quad \text{ACSUBST\_COMP} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2}{[\alpha \mapsto \tau](\gamma_1 \rightarrow \gamma_2) \triangleright \gamma'_1 \rightarrow \gamma'_2} \quad \text{ACSUBST\_ARROW} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2 \quad [\alpha \mapsto \tau]\gamma_3 \triangleright \gamma'_3}{[\alpha \mapsto \tau](\{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3) \triangleright (\{\gamma'_1 \sim \gamma'_2\} \rightarrow \gamma'_3)} \quad \text{ACSUBST\_CARROW} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha_1 \mapsto \tau](\forall \alpha_2, \gamma_1) \triangleright (\forall \alpha_2, \gamma_2)} \quad \text{ACSUBST\_FORALL} \\
\frac{[\alpha \mapsto \tau_1]\gamma_1 \triangleright \gamma_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3}{[\alpha \mapsto \tau_1](\gamma_1 @ \tau_2) \triangleright \gamma_2 @ \tau_3} \quad \text{ACSUBST\_INST} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](\mathbf{elim}_i \gamma_1) \triangleright \mathbf{elim}_i \gamma_2} \quad \text{ACSUBST\_ELIM}
\end{array}$$

$\boxed{[c \mapsto \gamma]t_1 \triangleright t_2}$  substitution of coercion variable in term

$$\begin{array}{c}
\frac{}{[c \mapsto \gamma]x \triangleright x} \quad \text{CTSUBST\_VAR} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](\lambda(x : \tau) \Rightarrow t_1) \triangleright (\lambda(x : \tau) \Rightarrow t_2)} \quad \text{CTSUBST\_ABS} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](\lambda\{\alpha\} \Rightarrow t_1) \triangleright (\lambda\{\alpha\} \Rightarrow t_2)} \quad \text{CTSUBST\_TABS} \\
\frac{}{[c \mapsto \gamma](\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \triangleright (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t)} \quad \text{CTSUBST\_CABS1} \\
\frac{c_1 \neq c_2 \quad [c_1 \mapsto \gamma]t_1 \triangleright t_2}{[c_1 \mapsto \gamma](\lambda\{c_2 : \tau_1 \sim \tau_2\} \Rightarrow t_1) \triangleright (\lambda\{c_2 : \tau_1 \sim \tau_2\} \Rightarrow t_2)} \quad \text{CTSUBST\_CABS2} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t'_1 \quad [c \mapsto \gamma]t_2 \triangleright t'_2}{[c \mapsto \gamma](t_1 t_2) \triangleright t'_1 t'_2} \quad \text{CTSUBST\_APP} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](t_1 [\tau]) \triangleright (t_2 [\tau])} \quad \text{CTSUBST\_TAPP} \\
\frac{[c \mapsto \gamma_1]t_1 \triangleright t_2 \quad [c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2}{[c \mapsto \gamma_1](t_1 \sim [\gamma_2]) \triangleright (t_2 \sim [\gamma'_2])} \quad \text{CTSUBST\_CAPP} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](t_1 : \tau) \triangleright (t_2 : \tau)} \quad \text{CTSUBST\_ANNOT} \\
\frac{[c \mapsto \gamma_1]t_1 \triangleright t_2 \quad [c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2}{[c \mapsto \gamma_1](t_1 \blacktriangleright \gamma_2) \triangleright (t_2 \blacktriangleright \gamma'_2)} \quad \text{CTSUBST\_COERCE}
\end{array}$$

$\boxed{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}$  substitution of coercion variable in coercion term

$$\begin{array}{c}
\frac{}{[c \mapsto \gamma]c \triangleright \gamma} \quad \text{CCSUBST\_VAR1} \\
\frac{c_1 \neq c_2}{[c_1 \mapsto \gamma]c_2 \triangleright c_2} \quad \text{CCSUBST\_VAR2}
\end{array}$$

$$\begin{array}{c}
\frac{}{[c \mapsto \gamma](\mathbf{refl} \tau) \triangleright \mathbf{refl} \tau} \text{CCSUBST\_REFL} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\mathbf{sym} \gamma_2) \triangleright \mathbf{sym} \gamma_3} \text{CCSUBST\_SYM} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3}{[c \mapsto \gamma_1](\gamma_2 \circ \gamma_3) \triangleright \gamma'_2 \circ \gamma'_3} \text{CCSUBST\_COMP} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3}{[c \mapsto \gamma_1](\gamma_2 \rightarrow \gamma_3) \triangleright \gamma'_2 \rightarrow \gamma'_3} \text{CCSUBST\_ARROW} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3 \quad [c \mapsto \gamma_1]\gamma_4 \triangleright \gamma'_4}{[c \mapsto \gamma_1](\{\gamma_2 \sim \gamma_3\} \rightarrow \gamma_4) \triangleright (\{\gamma'_2 \sim \gamma'_3\} \rightarrow \gamma'_4)} \text{CCSUBST\_CARROW} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\forall \alpha, \gamma_2) \triangleright (\forall \alpha, \gamma_3)} \text{CCSUBST\_FORALL} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\gamma_2 @ \tau) \triangleright \gamma_3 @ \tau} \text{CCSUBST\_INST} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\mathbf{elim}_i \gamma_2) \triangleright \mathbf{elim}_i \gamma_3} \text{CCSUBST\_ELIM}
\end{array}$$

Definition rules: 97 good 0 bad

Definition rule clauses: 186 good 0 bad