```
term variable
\boldsymbol{x}
      type variable
\alpha
T
      type constructor
      coercion variable
c
t
           ::=
                                                        _{\text{term}}
                                                            variable
                   \lambda(x:\tau) \Rightarrow t
                                                            abstraction
                   \lambda\{\alpha:\kappa\}\Rightarrow t
                                                            type abstraction
                   \lambda\{c: \tau_1 \sim \tau_2\} \Rightarrow t
                                                            coercion abstraction
                                                            application
                    t [\tau]
                                                            type application
                    t \sim [\gamma]
                                                            coercion application
                                                            type annotation
                                                            coercion
                                                        kind
\kappa
                                                            star
                                                            kind arrow
                   \kappa_1 \to \kappa_2
                                                        type
                                                            type variable
                                                            type constructor
                                                            \equiv (\rightarrow) \ \tau_1 \ \tau_2
                    \{\tau_1 \sim \tau_2\} \to \tau_3  \lambda(\alpha:\kappa), \tau 
                                                            coercion arrow
                                                            operator abstraction
                   \forall (\alpha : \kappa), \tau
                                                            universal quantification
                                                            operator application
                                                        coercion proof term
           ::=
                                                            variable
                   \operatorname{\mathbf{refl}} \tau
                                                            reflexivity
                                                            symmetry
                   \operatorname{sym} \gamma
                                                            composition
                   \gamma_1 \circ \gamma_2
                                                             \equiv (\rightarrow) \gamma_1 \gamma_2 \text{ where } (\rightarrow) : \text{refl } (\rightarrow)
                    \gamma_1 \rightarrow \gamma_2
                   \{\gamma_1 \sim \gamma_2\} \to \gamma_3
                                                            coercion arrow introduction
                   \lambda(\alpha:\kappa),\gamma
                                                            operator abstraction introduction
                   \forall (\alpha : \kappa), \gamma
                                                            universal quantification introduction
                                                            application introduction
                   \gamma_1 \gamma_2
                   left \gamma
                                                            left elimination
                   \mathbf{right} \, \gamma
                                                            right elimination
Γ
                                                         typing environment
                   Ø
                                                            empty
                   \Gamma, x : \tau
                                                            variable
                   \Gamma, T : \kappa
                                                            type constructor
                                                            type variable
                                                            coercion variable
                                                        typed value
tv
                   \lambda(x:\tau) \Rightarrow t
\lambda\{\alpha:\kappa\} \Rightarrow tv
                                                            abstraction
```

type abstraction

Initial environment:
$$\Gamma = \emptyset$$
, $(\rightarrow): * \rightarrow * \rightarrow *$

$\Gamma \vdash t : \tau$ Typing rules

$$\frac{x:\tau\in\Gamma}{\Gamma\vdash x:\tau} \quad \text{T-VAR}$$

$$\frac{\Gamma,x:\tau_1\vdash t:\tau_2}{\Gamma\vdash\tau_1:*}$$

$$\frac{\Gamma\vdash \tau_1:*}{\Gamma\vdash (\lambda(x:\tau_1)\Rightarrow t):\tau_1\to\tau_2} \quad \text{T-Abs}$$

$$\alpha\notin\Gamma$$

$$\frac{\alpha\notin\Gamma}{\Gamma,\alpha:\kappa\vdash t:\tau}$$

$$\frac{\Gamma\vdash (\lambda\{\alpha:\kappa\}\Rightarrow t):\forall(\alpha:\kappa),\tau}{\Gamma\vdash (\lambda\{\alpha:\kappa\}\Rightarrow t):\forall(\alpha:\kappa),\tau} \quad \text{T-TYAbs}$$

$$\frac{\Gamma,c:\tau_1\sim\tau_2\vdash t:\tau_3}{\Gamma\vdash\tau_1:\kappa}$$

$$\frac{\Gamma\vdash\tau_1:\kappa}{\Gamma\vdash\tau_2:\kappa}$$

$$\frac{\Gamma\vdash t_1:\tau_2\to\tau_1}{\Gamma\vdash t_1:t_2:\tau_1} \quad \text{T-App}$$

$$\frac{\Gamma\vdash t_1:\tau_2\to\tau_1}{\Gamma\vdash t:\tau_1:\kappa}$$

$$\frac{\Gamma\vdash t_1:\tau_2\to\tau_1}{\Gamma\vdash t:\tau_1:\kappa} \quad \text{T-App}$$

$$\frac{\Gamma\vdash t:\forall(\alpha:\kappa),\tau_2}{\Gamma\vdash t:\tau_1:\tau_2} \quad \text{T-TYApp}$$

$$\frac{\Gamma\vdash t:\{\tau_1\sim\tau_2\}\to\tau_3}{\Gamma\vdash \tau:\tau_1\sim\tau_2} \quad \text{T-TYApp}$$

$$\frac{\Gamma\vdash t:\{\tau_1\sim\tau_2\}\to\tau_3}{\Gamma\vdash \tau:\tau_1\sim\tau_2} \quad \text{T-TYApp}$$

$$\frac{\Gamma\vdash t:\tau_1\simeq\tau_1'}{\tau_1\equiv\tau_1'}$$

$$\frac{\tau_1\equiv\tau_1'}{\Gamma\vdash (t:\tau_1):\tau_1} \quad \text{T-Annot}$$

$$\frac{\Gamma\vdash \tau:\tau_1\sim\tau_2}{\Gamma\vdash t:\tau_1'}$$

$$\frac{\tau_1\equiv\tau_1'}{\Gamma\vdash (t\vdash\tau_1):\tau_2} \quad \text{T-Coerce}$$

 $\Gamma \vdash \gamma : \tau_1 \sim \tau_2$ Coercion typing

$$\frac{c:\tau_1 \sim \tau_2 \in \Gamma}{\Gamma \vdash c:\tau_1 \sim \tau_2} \quad \text{C-Var}$$

$$\frac{\Gamma \vdash refl \tau : \tau \sim \tau}{\Gamma \vdash refl \tau : \tau \sim \tau} \quad \text{C_AREFL}$$

$$\frac{\Gamma \vdash refl \tau : \tau \sim \tau}{\Gamma \vdash sym\gamma : \tau_1 \sim \tau_2} \quad \text{C_SYM}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \quad \text{C_COMP}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \quad \text{C_COMP}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \gamma_2 : \tau_2 \sim \tau_2'} \quad \Gamma \vdash \gamma_3 : \tau_3 \sim \tau_3'$$

$$\Gamma \vdash \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 : *$$

$$\frac{\alpha \notin \Gamma}{\Gamma \vdash \lambda(\alpha : \kappa), \gamma : \lambda(\alpha : \kappa), \tau_1 \sim \lambda(\alpha : \kappa), \tau_2} \quad \text{C_ABS}$$

$$\frac{\alpha \notin \Gamma}{\Gamma \vdash \lambda(\alpha : \kappa), \gamma : \lambda(\alpha : \kappa), \tau_1 \sim \lambda(\alpha : \kappa), \tau_2} \quad \text{C_ABS}$$

$$\frac{\alpha \notin \Gamma}{\Gamma \vdash \gamma : \tau_1 \sim \tau_2} \quad \Gamma \vdash \forall (\alpha : \kappa), \tau_1 : *$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \gamma_1 : \tau_2 : \kappa} \quad \Gamma \vdash \gamma_1 : \tau_2 \sim \tau_1' \tau_2' \quad \text{C_APP}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau_1' \rightarrow \tau_2'}{\Gamma \vdash \text{left } \gamma : \tau_1 \sim \tau_1'} \quad \text{C_APP}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau_1' \rightarrow \tau_2'}{\Gamma \vdash \text{left } \gamma : \tau_1 \sim \tau_1'} \quad \text{C_AEFT1}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau_1' \rightarrow \tau_2'}{\Gamma \vdash \text{left } \gamma : \tau_1 \sim \tau_1'} \quad \text{C_AEFT2}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau_1' \rightarrow \tau_2'}{\Gamma \vdash \text{left } \gamma : \tau_1 \sim \tau_1'} \quad \text{C_AEFT2}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau_1' \rightarrow \tau_2'}{\Gamma \vdash \text{left } \gamma : \tau_2 \sim \tau_1' \rightarrow \tau_2'} \quad \text{C_AIGHT1}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau_1' \rightarrow \tau_2'}{\Gamma \vdash \text{left } \gamma : \tau_2 \sim \tau_1'} \quad \text{C_AIGHT2}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau_1' \tau_2'}{\Gamma \vdash \text{left } \gamma : \tau_2 \sim \tau_1' \rightarrow \tau_2'} \quad \text{C_AIGHT2}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau_1' \tau_2'}{\Gamma \vdash \text{left } \gamma : \tau_2 \sim \tau_1' \rightarrow \tau_2'} \quad \text{C_AIGHT2}$$

 $\Gamma \vdash \tau : \kappa$ Kinding rules

$$\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \quad \text{K_-VAR}$$

$$\frac{T : \kappa \in \Gamma}{\Gamma \vdash T : \kappa} \quad \text{K_-TYPECONSTR}$$

$$\alpha \notin \Gamma$$

$$\Gamma, \alpha : \kappa_{1} \vdash \tau : \kappa_{2}$$

$$\Gamma \vdash (\lambda(\alpha : \kappa_{1}), \tau) : \kappa_{1} \to \kappa_{2}$$

$$\Gamma \vdash \tau_{1} : \kappa_{2} \to \kappa_{1}$$

$$\Gamma \vdash \tau_{2} : \kappa_{2}$$

$$\Gamma \vdash \tau_{1} : \kappa_{2} \to \kappa_{1}$$

$$\Gamma \vdash \tau_{1} : \kappa$$

$$\Gamma \vdash \tau_{1} : \kappa$$

$$\Gamma \vdash \tau_{2} : \kappa$$

$$\Gamma \vdash \tau_{3} : *$$

$$\Gamma \vdash (\{\tau_{1} \sim \tau_{2}\} \to \tau_{3}) : *$$

$$\alpha \notin \Gamma$$

$$\Gamma, \alpha : \kappa \vdash \tau : *$$

$$\Gamma \vdash (\forall (\alpha : \kappa), \tau) : *$$

$$K_{-}FORALL$$

$\tau_1 \equiv \tau_2$ Type equivalence

$t \longrightarrow tv$ Operational semantics

$$\frac{t_2 \longrightarrow tv_2}{t_1 tv_2 \longrightarrow tv_3} \\
\frac{t_1 t_2 \longrightarrow tv_3}{t_1 t_2 \longrightarrow tv_3}$$
 E_APP1

$$t \mapsto tv_1 \\ tv_1 tv_2 \to tv_3 \\ tv tv_2 \to tv_3 \\ \hline t tv tv_2 \to tv_3 \\ \hline t tv tv_1 \mid t \rhd t' \\ \hline (\lambda(x:\tau) \Rightarrow t) tv_1 \to tv_2 \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to t) tv_1 \to tv_2 \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to (\lambda(\alpha:\kappa) \Rightarrow tv) \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to (\lambda(\alpha:\kappa) \Rightarrow tv) \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to (\lambda(\alpha:\kappa) \Rightarrow tv) \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to (\lambda(\alpha:\kappa) \Rightarrow tv) \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to (\lambda(\alpha:\kappa) \Rightarrow tv) \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to (\lambda(\alpha:\kappa) \Rightarrow tv) \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to tv \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to tv \\ \hline (t \to tv) \\ \hline (t \to tv) \\ \hline (t \to tv) \to tv \\ \hline (\lambda(\alpha:\kappa) \Rightarrow t) \to tv \\ \hline (t \to tv) \to tv$$

$$\begin{array}{c} [\alpha \mapsto \tau] \; \alpha \triangleright \tau \\ \\ \alpha_1 \neq \alpha_2 \\ [\alpha_1 \mapsto \tau] \; T_2 \triangleright \alpha_2 \\ \\ [\alpha \mapsto \tau] \; T \triangleright \tau \\ \\ [\alpha \mapsto \tau] \; T \triangleright \tau \\ \\ [\alpha \mapsto \tau] \; T_2 \triangleright \tau_2' \\ [\alpha \mapsto \tau] \; \tau_2 \triangleright \tau_2' \\ [\alpha \mapsto \tau] \; \tau_3 \triangleright \tau_3' \\ [\alpha \mapsto \tau] \; \tau_4 \triangleright \tau_4' \\ \\ [\alpha \mapsto \tau_1] \; \tau_2 \triangleright \tau_2' \\ [\alpha \mapsto \tau_1] \; \tau_2 \triangleright \tau_2' \\ [\alpha_1 \mapsto \tau_1] \; (\lambda(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda'(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda'(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda'(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda'(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda'(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda'(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda'(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda'(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda'(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda'(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda'(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda'(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda'(\alpha_2 : \kappa), \tau_2') \\ \\ [\alpha_1 \mapsto \tau_1] \; (\lambda_2 \mapsto \tau_2) \\ \\ [\alpha_1 \mapsto \tau_1] \; \tau_2 \triangleright \tau_2' \\ \\ [\alpha_1 \mapsto \tau_1] \; \tau_2 \triangleright \tau_2' \\ \\ [\alpha_1 \mapsto \tau_1] \; \tau_2 \triangleright \tau_2' \\ \\ [\alpha_1 \mapsto \tau_1] \; \tau_2 \triangleright \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_1 \mapsto \tau_2 \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_1 \mapsto \tau_2 \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_1 \mapsto \tau_2 \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_1 \mapsto \tau_2 \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_1 \mapsto \tau_2 \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_1 \mapsto \tau_2 \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_1 \mapsto \tau_2 \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_1 \mapsto \tau_2 \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2' \\ \\ [\alpha_1 \mapsto \tau_2] \; \tau_2 \mapsto \tau_2'$$

$$\frac{[x \mapsto tv] \ t_1 \rhd t_2}{[x \mapsto tv] \ (t_1 \blacktriangleright \gamma) \rhd (t_2 \blacktriangleright \gamma)} \quad \text{Subst_Coerce}$$

 $\left| \left[\alpha \mapsto \tau \right] \right| t_1 \rhd t_2 \left| \right|$

substitution of type variable in term

Definition rules: 80 good 3 bad Definition rule clauses: 207 good 3 bad