

$x$	term variable	
$\alpha$	type variable	
$t$	$::=$	term
	$x$	variable
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : \kappa\} \Rightarrow t$	type abstraction
	$t_1 t_2$	application
	$t [\tau]$	type application
$\kappa$	$::=$	kind
	$*$	star
	$\kappa_1 \rightarrow \kappa_2$	kind arrow
$\tau$	$::=$	type
	$\alpha$	type variable
	$\tau_1 \rightarrow \tau_2$	$\equiv (\rightarrow) \tau_1 \tau_2$
	$\forall(\alpha : \kappa), \tau$	universal quantification
	$\lambda(\alpha : \kappa), \tau$	operator abstraction
	$\tau_1 \tau_2$	operator application
$\Gamma$	$::=$	typing environment
	$\emptyset$	empty
	$\Gamma, x : \tau$	variable
	$\Gamma, \alpha : \kappa$	type variable
$v$	$::=$	value
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : \kappa\} \Rightarrow v$	type abstraction

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Initial environment:  $\Gamma = \emptyset,$   
 $(\rightarrow) : * \rightarrow * \rightarrow *$

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$\boxed{\Gamma \vdash t : \tau}$  Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T\_VAR} \\
\\
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \rightarrow \tau_2} \quad \text{T\_ABS} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : \kappa \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha : \kappa\} \Rightarrow t) : \forall(\alpha : \kappa), \tau} \quad \text{T\_TYABS} \\
\\
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \tau_2 \equiv \tau'_2 \quad \Gamma \vdash t_2 : \tau'_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T\_APP} \\
\\
\frac{\Gamma \vdash t : \forall(\alpha : \kappa), \tau_2 \quad \Gamma \vdash \tau_1 : \kappa \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T\_TYAPP}
\end{array}$$

$\boxed{\Gamma \vdash \tau : \kappa}$  Kinding rules

$$\begin{array}{c}
\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \quad \text{K\_VAR} \\
\\
\frac{\Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash (\lambda(\alpha : \kappa_1), \tau) : \kappa_1 \rightarrow \kappa_2} \quad \text{K\_ABS} \\
\\
\frac{\Gamma \vdash \tau_1 : \kappa_2 \rightarrow \kappa_1 \quad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \tau_2 : \kappa_1} \quad \text{K\_APP} \\
\\
\frac{\Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash (\forall(\alpha : \kappa), \tau) : *} \quad \text{K\_FORALL}
\end{array}$$

$\tau_1 \equiv \tau_2$     Type equivalence

$$\begin{array}{c}
\frac{}{\tau \equiv \tau} \quad \text{EQ\_REFL} \\
\\
\frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \quad \text{EQ\_SYM} \\
\\
\frac{\tau_1 \equiv \tau_2 \quad \tau_2 \equiv \tau_3}{\tau_1 \equiv \tau_3} \quad \text{EQ\_TRANS} \\
\\
\frac{}{\alpha \equiv \alpha} \quad \text{EQ\_VAR} \\
\\
\frac{\tau_1 \equiv \tau_2}{(\forall(\alpha : \kappa), \tau_1) \equiv (\forall(\alpha : \kappa), \tau_2)} \quad \text{EQ\_FORALL} \\
\\
\frac{\tau_1 \equiv \tau_2}{(\lambda(\alpha : \kappa), \tau_1) \equiv (\lambda(\alpha : \kappa), \tau_2)} \quad \text{EQ\_ABS} \\
\\
\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2}{\tau_1 \tau_2 \equiv \tau'_1 \tau'_2} \quad \text{EQ\_APP} \\
\\
\frac{[\alpha \mapsto \tau_2] \tau_1 \triangleright \tau'_1}{(\lambda(\alpha : \kappa), \tau_1) \tau_2 \equiv \tau'_1} \quad \text{EQ\_APPABS}
\end{array}$$

$t \longrightarrow t'$     Operational semantics

$$\begin{array}{c}
\frac{t_2 \longrightarrow t'_2}{t_1 t_2 \longrightarrow t_1 t'_2} \quad \text{E\_APP1} \\
\\
\frac{t \longrightarrow t'}{t v \longrightarrow t' v} \quad \text{E\_APP2} \\
\\
\frac{[x \mapsto v] t \triangleright t'}{(\lambda(x : \tau) \Rightarrow t) v \longrightarrow t'} \quad \text{E\_APPABS} \\
\\
\frac{t \longrightarrow t'}{(\lambda\{\alpha : \kappa\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : \kappa\} \Rightarrow t')} \quad \text{E\_TABS} \\
\\
\frac{t \longrightarrow t'}{t [\tau] \longrightarrow t' [\tau]} \quad \text{E\_TAPP} \\
\\
\frac{[\alpha \mapsto \tau] v \triangleright v'}{(\lambda\{\alpha : \kappa\} \Rightarrow v) [\tau] \longrightarrow v'} \quad \text{E\_TAPPABS}
\end{array}$$

$[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau_3$     Type substitution

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau] \alpha \triangleright \tau} \text{SUBST\_VAR1} \\
\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau] \alpha_2 \triangleright \alpha_2} \text{SUBST\_VAR2} \\
\frac{}{[\alpha \mapsto \tau_1](\lambda(\alpha : \kappa), \tau_2) \triangleright (\lambda(\alpha : \kappa), \tau_2)} \text{SUBST\_ABS1} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\lambda(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda(\alpha_2 : \kappa), \tau'_2)} \text{SUBST\_ABS2} \\
\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1](\tau_2 \tau_3) \triangleright \tau'_2 \tau'_3} \text{SUBST\_APP} \\
\frac{}{[\alpha \mapsto \tau_1](\forall(\alpha : \kappa), \tau_2) \triangleright (\forall(\alpha : \kappa), \tau_2)} \text{SUBST\_FORALL1} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\forall(\alpha_2 : \kappa), \tau_2) \triangleright (\forall(\alpha_2 : \kappa), \tau'_2)} \text{SUBST\_FORALL2} \\
\boxed{[x \mapsto t] t_1 \triangleright t_2} \quad \text{substitution} \\
\frac{}{[x \mapsto t] x \triangleright t} \text{SUBST\_VAR1} \\
\frac{x_1 \neq x_2}{[x_1 \mapsto t] x_2 \triangleright x_2} \text{SUBST\_VAR2} \\
\frac{}{[x \mapsto t_1](\lambda(x : \tau) \Rightarrow t_2) \triangleright (\lambda(x : \tau) \Rightarrow t_2)} \text{SUBST\_ABS1} \\
\frac{x_1 \neq x_2 \quad x_2 \notin fv(t_1) \quad [x_1 \mapsto t_1] t_2 \triangleright t'_2}{[x_1 \mapsto t_1](\lambda(x_2 : \tau) \Rightarrow t_2) \triangleright (\lambda(x_2 : \tau) \Rightarrow t'_2)} \text{SUBST\_ABS2} \\
\frac{x_1 \neq x_2 \quad x_3 \notin fv(t_1, t_2) \quad [x_2 \mapsto x_3] t_2 \triangleright t'_2 \quad [x_1 \mapsto t_1] t'_2 \triangleright t''_2}{[x_1 \mapsto t_1](\lambda(x_2 : \tau) \Rightarrow t_2) \triangleright (\lambda(x_3 : \tau) \Rightarrow t''_2)} \text{SUBST\_ABS3} \\
\frac{[x \mapsto t_1] t_2 \triangleright t'_2}{[x \mapsto t_1](\lambda\{\alpha : \kappa\} \Rightarrow t_2) \triangleright (\lambda\{\alpha : \kappa\} \Rightarrow t'_2)} \text{SUBST\_TABS} \\
\frac{[x \mapsto t_1] t_2 \triangleright t'_2 \quad [x \mapsto t_1] t_3 \triangleright t'_3}{[x \mapsto t_1](t_2 t_3) \triangleright t'_2 t'_3} \text{SUBST\_APP} \\
\frac{[x \mapsto t_1] t_2 \triangleright t'_2}{[x \mapsto t_1](t_2 [\tau]) \triangleright (t'_2 [\tau])} \text{SUBST\_TAPP} \\
\boxed{[\alpha \mapsto \tau] t_1 \triangleright t_2} \quad \text{substitution of type variable in term} \\
\frac{}{[\alpha \mapsto \tau] x \triangleright x} \text{TTSUBST\_VAR} \\
\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \triangleright (\lambda(x : \tau'_2) \Rightarrow t_2)} \text{TTSUBST\_ABS} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau] t_1 \triangleright t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : \kappa\} \Rightarrow t_1) \triangleright (\lambda\{\alpha_2 : \kappa\} \Rightarrow t_2)} \text{TTSUBST\_TABS} \\
\frac{[\alpha \mapsto \tau] t_1 \triangleright t'_1 \quad [\alpha \mapsto \tau] t_2 \triangleright t'_2}{[\alpha \mapsto \tau](t_1 t_2) \triangleright t'_1 t'_2} \text{TTSUBST\_APP} \\
\frac{[\alpha \mapsto \tau_1] t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 [\tau_2]) \triangleright (t_2 [\tau'_2])} \text{TTSUBST\_TAPP}
\end{array}$$

Definition rules: 43 good 0 bad  
Definition rule clauses: 78 good 0 bad