x	term variable
α	type variable
T	abstract type
c	coercion variable

t	$::=$			term
	$\vert$	x		variable
	$\vert$	$\lambda(x : \tau) \Rightarrow t$		abstraction
	$\vert$	$\lambda\{\alpha : \kappa\} \Rightarrow t$		type abstraction
	$\vert$	$\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t$		coercion abstraction
	$\vert$	$t_1\ t_2$		application
	$\vert$	$t\ [\tau]$		type application
	$\vert$	$t\ \sim[\gamma]$		coercion application
	$\vert$	$t : \tau$		type annotation
	$\vert$	$t \blacktriangleright \gamma$		coercion
	$\vert$	(t)	S	parenthesis
v	$::=$			value
	$\vert$	$\lambda(x : \tau) \Rightarrow t$		abstraction
κ	$::=$			kind
	$\vert$	$*$		star
	$\vert$	$\kappa_1 \to \kappa_2$		kind arrow
	$\vert$	(κ)	S	parenthesis
τ	$::=$			type
	$\vert$	α		type variable
	$\vert$	T		abstract type
	$\vert$	$\tau_1 \to \tau_2$	S	$\equiv (\to)\ \tau_1\ \tau_2$ where $(\to) : * \to * \to *$
	$\vert$	$\{\tau_1 \sim \tau_2\} \to \tau_3$		coercion arrow
	$\vert$	$\lambda(\alpha : \kappa), \tau$		operator abstraction
	$\vert$	$\forall\,(\alpha : \kappa), \tau$		forall
	$\vert$	$\tau_1\,\tau_2$		operator application
	$\vert$	(τ)	S	parenthesis
γ	$::=$			coercion proof term
	$\vert$	c		variable
	$\vert$	$\mathbf{refl}\,\tau$		reflexivity
	$\vert$	$\mathbf{sym}\,\gamma$		symmetry
	$\vert$	$\gamma_1 \circ \gamma_2$		composition
	$\vert$	$\gamma_1 \to \gamma_2$	S	$\equiv (\to)\ \gamma_1\ \gamma_2$ where $(\to)$: refl $(\to)$
	$\vert$	$\{\gamma_1 \sim \gamma_2\} \to \gamma_3$		coercion arrow introduction
	$\vert$	$\lambda(\alpha : \kappa), \gamma$		operator abstraction introduction
	$\vert$	$\forall\,(\alpha : \kappa), \gamma$		forall introduction
	$\vert$	$\gamma_1\,\gamma_2$		application introduction
	$\vert$	$\mathbf{left}\,\gamma$		left elimination
	$\vert$	$\mathbf{right}\,\gamma$		right elimination
	$\vert$	(γ)	S	parenthesis
Γ	$::=$			typing environment
	$\vert$	$\emptyset$		empty
	$\vert$	$\Gamma, x : \tau$		variable

$$
\begin{array}{lll}
| & \Gamma, T : \kappa & \text{abstract type} \\
| & \Gamma, \alpha : \kappa & \text{type variable} \\
| & \Gamma, c : \tau_1 \sim \tau_2 & \text{coercion variable}
\end{array}
$$

$\boxed{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_3}$ Type substitution

$$\frac{}{[\alpha \mapsto \tau]\alpha \rhd \tau} \quad \textsc{SubstT\_Var1}$$

$$\frac{}{[\alpha_1 \mapsto \tau]\alpha_2 \rhd \alpha_2} \quad \textsc{SubstT\_Var2}$$

$$\frac{}{[\alpha \mapsto \tau]T \rhd T} \quad \textsc{SubstT\_Type}$$

$$\frac{\begin{array}{c}[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \\ [\alpha \mapsto \tau_1]\tau_3 \rhd \tau_3' \\ [\alpha \mapsto \tau_1]\tau_4 \rhd \tau_4'\end{array}}{[\alpha \mapsto \tau_1]\{\tau_2 \sim \tau_3\} \to \tau_4 \rhd \{\tau_2' \sim \tau_3'\} \to \tau_4'} \quad \textsc{SubstT\_CArrow}$$

$$\frac{}{[\alpha \mapsto \tau_1]\lambda(\alpha : \kappa), \tau_2 \rhd \lambda(\alpha : \kappa), \tau_2} \quad \textsc{SubstT\_Abs1}$$

$$\frac{[\alpha_1 \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha_1 \mapsto \tau_1]\lambda(\alpha_2 : \kappa), \tau_2 \rhd \lambda(\alpha_2 : \kappa), \tau_2'} \quad \textsc{SubstT\_Abs2}$$

$$\frac{}{[\alpha \mapsto \tau_1]\forall\,(\alpha : \kappa), \tau_2 \rhd \forall\,(\alpha : \kappa), \tau_2} \quad \textsc{SubstT\_Forall1}$$

$$\frac{[\alpha_1 \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha_1 \mapsto \tau_1]\forall\,(\alpha_2 : \kappa), \tau_2 \rhd \forall\,(\alpha_2 : \kappa), \tau_2'} \quad \textsc{SubstT\_Forall2}$$

$$\frac{\begin{array}{c}[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \\ [\alpha \mapsto \tau_1]\tau_3 \rhd \tau_3'\end{array}}{[\alpha \mapsto \tau_1]\tau_2\,\tau_3 \rhd \tau_2'\,\tau_3'} \quad \textsc{SubstT\_App}$$

$\boxed{\Gamma \vdash t : \tau}$ Typing rules

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \textsc{T\_Var}$$

$$\frac{\begin{array}{c}\Gamma, x : \tau_1 \vdash t : \tau_2 \\ \Gamma \vdash \tau_1 : *\end{array}}{\Gamma \vdash \lambda(x : \tau_1) \Rightarrow t : \tau_1 \to \tau_2} \quad \textsc{T\_Abs}$$

$$\frac{\begin{array}{c}\Gamma, \alpha : \kappa \vdash t : \tau \\ \alpha \notin \Gamma\end{array}}{\Gamma \vdash \lambda\{\alpha : \kappa\} \Rightarrow t : \forall\,(\alpha : \kappa), \tau} \quad \textsc{T\_TyAbs}$$

$$\frac{\begin{array}{c}\Gamma, c : \tau_1 \sim \tau_2 \vdash t : \tau_3 \\ \Gamma \vdash \tau_1 : \kappa \\ \Gamma \vdash \tau_2 : \kappa\end{array}}{\Gamma \vdash \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t : \{\tau_1 \sim \tau_2\} \to \tau_3} \quad \textsc{T\_CAbs}$$

$$\frac{\begin{array}{c}\Gamma \vdash t_1 : \tau_2 \to \tau_1 \\ \tau_2 \equiv \tau_2' \\ \Gamma \vdash t_2 : \tau_2'\end{array}}{\Gamma \vdash t_1\,t_2 : \tau_1} \quad \textsc{T\_App}$$

$$\frac{\Gamma \vdash t : \forall\,(\alpha : \kappa), \tau_2 \quad \Gamma \vdash \tau_1 : \kappa \quad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{\Gamma \vdash t\ [\tau_1] : \tau_2'} \ \textsc{T\_TyApp}$$

$$\frac{\Gamma \vdash t : \{\tau_1 \sim \tau_2\} \to \tau_3 \quad \tau_1 \equiv \tau_1' \quad \tau_2 \equiv \tau_2' \quad \Gamma \vdash \gamma : \tau_1' \sim \tau_2'}{\Gamma \vdash t \sim\![\gamma] : \tau_3} \ \textsc{T\_CApp}$$

$$\frac{\Gamma \vdash t : \tau_2 \quad \tau_1 \equiv \tau_2}{\Gamma \vdash (t : \tau_1) : \tau_1} \ \textsc{T\_Annot}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \sim \tau_2 \quad \tau_1 \equiv \tau_1' \quad \Gamma \vdash t : \tau_1'}{\Gamma \vdash t \blacktriangleright \gamma : \tau_2} \ \textsc{T\_Coerce}$$

$\boxed{\Gamma \vdash \gamma : \tau_1 \sim \tau_2}$ Coercion typing

$$\frac{c : \tau_1 \sim \tau_2 \ \in \ \Gamma}{\Gamma \vdash c : \tau_1 \sim \tau_2} \ \textsc{C\_Var}$$

$$\frac{}{\Gamma \vdash \mathbf{refl}\, \tau : \tau \sim \tau} \ \textsc{C\_Refl}$$

$$\frac{\Gamma \vdash \gamma : \tau_2 \sim \tau_1}{\Gamma \vdash \mathbf{sym}\, \gamma : \tau_1 \sim \tau_2} \ \textsc{C\_Sym}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2 \quad \tau_2 \equiv \tau_2' \quad \Gamma \vdash \gamma_2 : \tau_2' \sim \tau_3}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \ \textsc{C\_Comp}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1' \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau_2' \quad \Gamma \vdash \gamma_3 : \tau_3 \sim \tau_3' \quad \Gamma \vdash \{\tau_1 \sim \tau_2\} \to \tau_3 : *}{\Gamma \vdash \{\gamma_1 \sim \gamma_2\} \to \gamma_3 : \{\tau_1 \sim \tau_2\} \to \tau_3 \sim \{\tau_1' \sim \tau_2'\} \to \tau_3'} \ \textsc{C\_CArrow}$$

$$\frac{\Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash \lambda(\alpha : \kappa), \gamma : \lambda(\alpha : \kappa), \tau_1 \sim \lambda(\alpha : \kappa), \tau_2} \ \textsc{C\_Abs}$$

$$\frac{\Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2 \quad \Gamma \vdash \forall\,(\alpha : \kappa), \tau_1 : *}{\Gamma \vdash \forall\,(\alpha : \kappa), \gamma : \forall\,(\alpha : \kappa), \tau_1 \sim \forall\,(\alpha : \kappa), \tau_2} \ \textsc{C\_Forall}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1' \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau_2' \quad \Gamma \vdash \tau_1\, \tau_2 : \kappa}{\Gamma \vdash \gamma_1\, \gamma_2 : \tau_1\, \tau_2 \sim \tau_1'\, \tau_2'} \ \textsc{C\_App}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \to \tau_2 \sim \tau_1' \to \tau_2'}{\Gamma \vdash \mathbf{left}\, \gamma : \tau_1 \sim \tau_1'} \ \textsc{C\_Left1}$$

$$\frac{\Gamma \vdash \gamma : \tau_1\,\tau_2 \sim \tau_1'\,\tau_2'}{\Gamma \vdash \mathbf{left}\,\gamma : \tau_1 \sim \tau_1'} \quad \textsc{C\_Left2}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \to \tau_2 \sim \tau_1' \to \tau_2'}{\Gamma \vdash \mathbf{right}\,\gamma : \tau_2 \sim \tau_2'} \quad \textsc{C\_Right1}$$

$$\frac{\Gamma \vdash \gamma : \{\tau_1 \sim \tau_2\} \to \tau_3 \sim \{\tau_1' \sim \tau_2'\} \to \tau_3'}{\Gamma \vdash \mathbf{right}\,\gamma : \tau_3 \sim \tau_3'} \quad \textsc{C\_Right2}$$

$$\frac{\Gamma \vdash \gamma : \tau_1\,\tau_2 \sim \tau_1'\,\tau_2'}{\Gamma \vdash \mathbf{right}\,\gamma : \tau_2 \sim \tau_2'} \quad \textsc{C\_Right3}$$

$\boxed{\Gamma \vdash \tau : \kappa}$ Kinding

$$\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \quad \textsc{K\_Var}$$

$$\frac{T : \kappa \in \Gamma}{\Gamma \vdash T : \kappa} \quad \textsc{K\_AbsType}$$

$$\frac{\Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash \lambda(\alpha : \kappa_1), \tau : \kappa_1 \to \kappa_2} \quad \textsc{K\_Abs}$$

$$\frac{\begin{array}{c}\Gamma \vdash \tau_1 : \kappa_2 \to \kappa_1 \\ \Gamma \vdash \tau_2 : \kappa_2\end{array}}{\Gamma \vdash \tau_1\,\tau_2 : \kappa_1} \quad \textsc{K\_App}$$

$$\frac{\begin{array}{c}\Gamma \vdash \tau_1 : \kappa \\ \Gamma \vdash \tau_2 : \kappa \\ \Gamma \vdash \tau_3 : *\end{array}}{\Gamma \vdash \{\tau_1 \sim \tau_2\} \to \tau_3 : *} \quad \textsc{K\_CArrow}$$

$$\frac{\Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash \forall\,(\alpha : \kappa), \tau : *} \quad \textsc{K\_Forall}$$

$\boxed{\tau_1 \equiv \tau_2}$ Type equivalence

$$\frac{}{\tau \equiv \tau} \quad \textsc{EQ\_Refl}$$

$$\frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \quad \textsc{EQ\_Symm}$$

$$\frac{\begin{array}{c}\tau_1 \equiv \tau_2 \\ \tau_2 \equiv \tau_3\end{array}}{\tau_1 \equiv \tau_3} \quad \textsc{EQ\_Trans}$$

$$\frac{}{\alpha \equiv \alpha} \quad \textsc{EQ\_Var}$$

$$\frac{}{T \equiv T} \quad \textsc{EQ\_AbsType}$$

$$\frac{\begin{array}{c}\tau_1 \equiv \tau_1' \\ \tau_2 \equiv \tau_2' \\ \tau_3 \equiv \tau_3'\end{array}}{\{\tau_1 \sim \tau_2\} \to \tau_3 \equiv \{\tau_1' \sim \tau_2'\} \to \tau_3'} \quad \textsc{EQ\_CArrow}$$

$$\frac{\tau_1 \equiv \tau_2}{\forall\,(\alpha : \kappa), \tau_1 \equiv \forall\,(\alpha : \kappa), \tau_2} \quad \textsc{EQ\_Forall}$$

$$\frac{\tau_1 \equiv \tau_2}{\lambda(\alpha : \kappa), \tau_1 \equiv \lambda(\alpha : \kappa), \tau_2} \quad \text{EQ\_ABS}$$

$$\frac{\tau_1 \equiv \tau_1' \quad \tau_2 \equiv \tau_2'}{\tau_1\ \tau_2 \equiv \tau_1'\ \tau_2'} \quad \text{EQ\_APP}$$

$$\frac{[\alpha \mapsto \tau_2]\tau_1 \rhd \tau_1'}{(\lambda(\alpha : \kappa), \tau_1)\ \tau_2 \equiv \tau_1'} \quad \text{EQ\_APPABS}$$

$\boxed{[x \mapsto v]t_1 \rhd t_2}$ substitution

$$\frac{}{[x \mapsto v]x \rhd v} \quad \text{SUBST\_VAR1}$$

$$\frac{}{[x_1 \mapsto v]x_2 \rhd x_2} \quad \text{SUBST\_VAR2}$$

$$\frac{}{[x \mapsto v]\lambda(x : \tau) \Rightarrow t \rhd \lambda(x : \tau) \Rightarrow t} \quad \text{SUBST\_ABS1}$$

$$\frac{[x_1 \mapsto v]t_1 \rhd t_2}{[x_1 \mapsto v]\lambda(x_2 : \tau) \Rightarrow t_1 \rhd \lambda(x_2 : \tau) \Rightarrow t_2} \quad \text{SUBST\_ABS2}$$

$$\frac{[x \mapsto v]t_1 \rhd t_2}{[x \mapsto v]\lambda\{\alpha : \kappa\} \Rightarrow t_1 \rhd \lambda\{\alpha : \kappa\} \Rightarrow t_2} \quad \text{SUBST\_TABS}$$

$$\frac{[x \mapsto v]t_1 \rhd t_2}{[x \mapsto v]\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_1 \rhd \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_2} \quad \text{SUBST\_CABS}$$

$$\frac{[x \mapsto v]t_1 \rhd t_1' \quad [x \mapsto v]t_2 \rhd t_2'}{[x \mapsto v]t_1\ t_2 \rhd t_1'\ t_2'} \quad \text{SUBST\_APP}$$

$$\frac{[x \mapsto v]t_1 \rhd t_2}{[x \mapsto v]t_1\ [\tau] \rhd t_2\ [\tau]} \quad \text{SUBST\_TAPP}$$

$$\frac{[x \mapsto v]t_1 \rhd t_2}{[x \mapsto v]t_1\ \sim[\gamma] \rhd t_2\ \sim[\gamma]} \quad \text{SUBST\_CAPP}$$

$$\frac{[x \mapsto v]t_1 \rhd t_2}{[x \mapsto v]t_1 : \tau \rhd t_2 : \tau} \quad \text{SUBST\_ANNOT}$$

$$\frac{[x \mapsto v]t_1 \rhd t_2}{[x \mapsto v]t_1 \blacktriangleright \gamma \rhd t_2 \blacktriangleright \gamma} \quad \text{SUBST\_COERCE}$$

$\boxed{t_1 \longrightarrow t_2}$ Evaluation

$$\frac{t_2 \longrightarrow t_2'}{t_1\ t_2 \longrightarrow t_1\ t_2'} \quad \text{E\_APP1}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ v \longrightarrow t_1'\ v} \quad \text{E\_APP2}$$

$$\frac{[x \mapsto v]t \rhd t' \quad t' \longrightarrow t''}{(\lambda(x : \tau) \Rightarrow t)\ v \longrightarrow t''} \quad \text{E\_APPABS}$$

$$\frac{t \longrightarrow t'}{\lambda\{\alpha : \kappa\} \Rightarrow t \longrightarrow t'} \quad \text{E\_TABS}$$

$$\frac{t \longrightarrow t'}{\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t \longrightarrow t'} \quad \text{E\_CAbs}$$

$$\frac{t \longrightarrow t'}{t\ [\tau] \longrightarrow t'} \quad \text{E\_TApp}$$

$$\frac{t \longrightarrow t'}{t \sim[\gamma] \longrightarrow t'} \quad \text{E\_CApp}$$

$$\frac{t \longrightarrow t'}{t : \tau \longrightarrow t'} \quad \text{E\_Annot}$$

$$\frac{t \longrightarrow t'}{t \blacktriangleright \gamma \longrightarrow t'} \quad \text{E\_Coerce}$$

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Definition rules:         67 good    0 bad
Definition rule clauses: 156 good    0 bad
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