

x term variable
 α type variable
 c coercion variable
 i index metavariable

$t ::=$
 $| x$ variable
 $| \lambda(x : \tau) \Rightarrow t$ abstraction
 $| \lambda\{\alpha : *\} \Rightarrow t$ type abstraction
 $| \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t$ coercion abstraction
 $| t_1 t_2$ application
 $| t [\tau]$ type application
 $| t \sim[\gamma]$ coercion application
 $| t \blacktriangleright \gamma$ coercion

$\tau ::=$
 $| \alpha$ type variable
 $| \tau_1 \rightarrow \tau_2$ arrow
 $| \{\tau_1 \sim \tau_2\} \rightarrow \tau_3$ coercion arrow
 $| \forall(\alpha : *), \tau$ universal quantification

$\gamma ::=$
 $| c$ variable
 $| \mathbf{refl} \tau$ reflexivity
 $| \mathbf{sym} \gamma$ symmetry
 $| \gamma_1 \circ \gamma_2$ composition
 $| \gamma_1 \rightarrow \gamma_2$ arrow introduction
 $| \{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3$ coercion arrow introduction
 $| \forall(\alpha : *), \gamma$ universal quantification introduction
 $| \gamma @ \tau$ instantiation (quantification elimination)
 $| \mathbf{elim}_i \gamma$ generalized elimination

$\Gamma ::=$
 $| \emptyset$ empty
 $| \Gamma, x : \tau$ variable
 $| \Gamma, \alpha : *$ type variable
 $| \Gamma, c : \tau_1 \sim \tau_2$ coercion variable

$v ::=$
 $| \lambda(x : \tau) \Rightarrow t$ abstraction

Initial environment: $\Gamma = \emptyset$

$\boxed{\Gamma \vdash t : \tau}$ Typing rules

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T_VAR}$$

$$\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \rightarrow \tau_2} \quad \text{T_ABS}$$

$$\begin{array}{c}
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha : *\} \Rightarrow t) : \forall(\alpha : *), \tau} \quad \text{T_TYABS} \\
\\
\frac{\Gamma, c : \tau_1 \sim \tau_2 \vdash t : \tau_3 \quad \Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : *}{\Gamma \vdash (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3} \quad \text{T_CABS} \\
\\
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T_APP} \\
\\
\frac{\Gamma \vdash t : \forall(\alpha : *), \tau_2 \quad \Gamma \vdash \tau_1 : * \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T_TYAPP} \\
\\
\frac{\Gamma \vdash t : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \quad \Gamma \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash t \sim[\gamma] : \tau_3} \quad \text{T_CAPP} \\
\\
\frac{\Gamma \vdash \gamma : \tau_1 \sim \tau_2 \quad \Gamma \vdash t : \tau_1}{\Gamma \vdash (t \blacktriangleright \gamma) : \tau_2} \quad \text{T_COERCE}
\end{array}$$

$$\boxed{\Gamma \vdash \gamma : \tau_1 \sim \tau_2}$$

Coercion typing

$$\begin{array}{c}
\frac{c : \tau_1 \sim \tau_2 \in \Gamma}{\Gamma \vdash c : \tau_1 \sim \tau_2} \quad \text{C_VAR} \\
\\
\frac{\Gamma \vdash \tau : *}{\Gamma \vdash \mathbf{refl} \tau : \tau \sim \tau} \quad \text{C_REFL} \\
\\
\frac{\Gamma \vdash \gamma : \tau_2 \sim \tau_1}{\Gamma \vdash \mathbf{sym} \gamma : \tau_1 \sim \tau_2} \quad \text{C_SYM} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau_3}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \quad \text{C_COMP} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \tau_1 \rightarrow \tau_2 : *}{\Gamma \vdash (\gamma_1 \rightarrow \gamma_2) : (\tau_1 \rightarrow \tau_2) \sim (\tau'_1 \rightarrow \tau'_2)} \quad \text{C_ARROW} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \gamma_3 : \tau_3 \sim \tau'_3 \quad \Gamma \vdash \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 : *}{\Gamma \vdash (\{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3) : (\{\tau_1 \sim \tau_2\} \rightarrow \tau_3) \sim (\{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3)} \quad \text{C_CARROW} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash \gamma : \tau_1 \sim \tau_2 \quad \Gamma \vdash \forall(\alpha : *), \tau_1 : *}{\Gamma \vdash (\forall(\alpha : *), \gamma) : (\forall(\alpha : *), \tau_1) \sim (\forall(\alpha : *), \tau_2)} \quad \text{C_FORALL} \\
\\
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \gamma : (\forall(\alpha_1 : *), \tau_2) \sim (\forall(\alpha_2 : *), \tau_3) \quad [\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha_2 \mapsto \tau_1] \tau_3 \triangleright \tau'_3}{\Gamma \vdash \gamma @ \tau_1 : \tau'_2 \sim \tau'_3} \quad \text{C_INST} \\
\\
\frac{i \in \{1, 2\} \quad \Gamma \vdash \gamma : (\tau_1 \rightarrow \tau_2) \sim (\tau'_1 \rightarrow \tau'_2)}{\Gamma \vdash \mathbf{elim}_i \gamma : \tau_i \sim \tau'_i} \quad \text{C_ELIMARROW} \\
\\
\frac{i \in \{1, 2, 3\} \quad \Gamma \vdash \gamma : (\{\tau_1 \sim \tau_2\} \rightarrow \tau_3) \sim (\{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3)}{\Gamma \vdash \mathbf{elim}_i \gamma : \tau_i \sim \tau'_i} \quad \text{C_ELIMCARROW}
\end{array}$$

$$\boxed{\Gamma \vdash \tau : *}$$

Type τ is well formed

$$\begin{array}{c}
\frac{\alpha : * \in \Gamma}{\Gamma \vdash \alpha : *} \quad \text{K_VAR} \\
\\
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : *}{\Gamma \vdash (\tau_1 \rightarrow \tau_2) : *} \quad \text{K_ARROW}
\end{array}$$

$$\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : * \quad \Gamma \vdash \tau_3 : *}{\Gamma \vdash (\{\tau_1 \sim \tau_2\} \rightarrow \tau_3) : *} \quad \text{K_CARROW}$$

$$\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash \tau : *}{\Gamma \vdash (\forall (\alpha : *), \tau) : *} \quad \text{K_FORALL}$$

$t \longrightarrow t'$ Operational semantics

$$\frac{t_2 \longrightarrow t'_2}{t_1 t_2 \longrightarrow t_1 t'_2} \quad \text{E_APP1}$$

$$\frac{t \longrightarrow t'}{t v \longrightarrow t' v} \quad \text{E_APP2}$$

$$\frac{[x \mapsto v] t \triangleright t'}{(\lambda(x : \tau) \Rightarrow t) v \longrightarrow t'} \quad \text{E_APPAbs}$$

$$\frac{t \longrightarrow t'}{(\lambda\{\alpha : *\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : *\} \Rightarrow t')} \quad \text{E_TABS}$$

$$\frac{t \longrightarrow t'}{t [\tau] \longrightarrow t' [\tau]} \quad \text{E_TAPP}$$

$$\frac{[\alpha \mapsto \tau] v \triangleright v'}{(\lambda\{\alpha : *\} \Rightarrow v) [\tau] \longrightarrow v'} \quad \text{E_TAPPAbs}$$

$$\frac{t \longrightarrow t'}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \longrightarrow (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t')} \quad \text{E_CABS}$$

$$\frac{t \longrightarrow t'}{t \sim [\gamma] \longrightarrow t' \sim [\gamma]} \quad \text{E_CAPP}$$

$$\frac{[c \mapsto \gamma] t \triangleright t'}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \sim [\gamma] \longrightarrow t'} \quad \text{E_CAPPAbs}$$

$$\frac{t \longrightarrow t'}{(t \blacktriangleright \gamma) \longrightarrow t'} \quad \text{E_COERCE}$$

$[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau_3$ Type substitution

$$\frac{}{[\alpha \mapsto \tau] \alpha \triangleright \tau} \quad \text{SUBST_VAR1}$$

$$\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau] \alpha_2 \triangleright \alpha_2} \quad \text{SUBST_VAR2}$$

$$\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1] (\tau_2 \rightarrow \tau_3) \triangleright \tau'_2 \rightarrow \tau'_3} \quad \text{SUBST_ARROW}$$

$$\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3 \quad [\alpha \mapsto \tau_1] \tau_4 \triangleright \tau'_4}{[\alpha \mapsto \tau_1] (\{\tau_2 \sim \tau_3\} \rightarrow \tau_4) \triangleright (\{\tau'_2 \sim \tau'_3\} \rightarrow \tau'_4)} \quad \text{SUBST_CARROW}$$

$$\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1] (\forall (\alpha_2 : *), \tau_2) \triangleright (\forall (\alpha_2 : *), \tau'_2)} \quad \text{SUBST_FORALL}$$

$[x \mapsto v] t_1 \triangleright t_2$ substitution

$$\frac{}{[x \mapsto v] x \triangleright v} \quad \text{SUBST_VAR1}$$

$$\begin{array}{c}
\frac{x_1 \neq x_2}{[x_1 \mapsto v]x_2 \triangleright x_2} \quad \text{SUBST_VAR2} \\
\\
\frac{}{[x \mapsto v](\lambda(x : \tau) \Rightarrow t) \triangleright (\lambda(x : \tau) \Rightarrow t)} \quad \text{SUBST_ABS1} \\
\\
\frac{x_1 \neq x_2 \quad [x_1 \mapsto v]t_1 \triangleright t_2}{[x_1 \mapsto v](\lambda(x_2 : \tau) \Rightarrow t_1) \triangleright (\lambda(x_2 : \tau) \Rightarrow t_2)} \quad \text{SUBST_ABS2} \\
\\
\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](\lambda\{\alpha : *\} \Rightarrow t_1) \triangleright (\lambda\{\alpha : *\} \Rightarrow t_2)} \quad \text{SUBST_TABS} \\
\\
\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_1) \triangleright (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_2)} \quad \text{SUBST_CABS} \\
\\
\frac{[x \mapsto v]t_1 \triangleright t'_1 \quad [x \mapsto v]t_2 \triangleright t'_2}{[x \mapsto v](t_1 \ t_2) \triangleright t'_1 \ t'_2} \quad \text{SUBST_APP} \\
\\
\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](t_1 \ [\tau]) \triangleright (t_2 \ [\tau])} \quad \text{SUBST_TAPP} \\
\\
\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](t_1 \ \sim[\gamma]) \triangleright (t_2 \ \sim[\gamma])} \quad \text{SUBST_CAPP} \\
\\
\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](t_1 \ \blacktriangleright \gamma) \triangleright (t_2 \ \blacktriangleright \gamma)} \quad \text{SUBST_COERCE} \\
\\
\boxed{[\alpha \mapsto \tau]t_1 \triangleright t_2} \quad \text{substitution of type variable in term} \\
\\
\frac{}{[\alpha \mapsto \tau]x \triangleright x} \quad \text{TTSUBST_VAR} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \triangleright (\lambda(x : \tau'_2) \Rightarrow t_2)} \quad \text{TTSUBST_ABS} \\
\\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]t_1 \triangleright t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : *\} \Rightarrow t_1) \triangleright (\lambda\{\alpha_2 : *\} \Rightarrow t_2)} \quad \text{TTSUBST_TABS} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda\{c : \tau_2 \sim \tau_3\} \Rightarrow t_1) \triangleright (\lambda\{c : \tau'_2 \sim \tau'_3\} \Rightarrow t_2)} \quad \text{TTSUBST_CABS} \\
\\
\frac{[\alpha \mapsto \tau]t_1 \triangleright t'_1 \quad [\alpha \mapsto \tau]t_2 \triangleright t'_2}{[\alpha \mapsto \tau](t_1 \ t_2) \triangleright t'_1 \ t'_2} \quad \text{TTSUBST_APP} \\
\\
\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 \ [\tau_2]) \triangleright (t_2 \ [\tau'_2])} \quad \text{TTSUBST_TAPP} \\
\\
\frac{[\alpha \mapsto \tau]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](t_1 \ \sim[\gamma_1]) \triangleright (t_2 \ \sim[\gamma_2])} \quad \text{TTSUBST_CAPP} \\
\\
\frac{[\alpha \mapsto \tau]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](t_1 \ \blacktriangleright \gamma_1) \triangleright (t_2 \ \blacktriangleright \gamma_2)} \quad \text{TTSUBST_COERCE} \\
\\
\boxed{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2} \quad \text{substitution of type variable in coercion term}
\end{array}$$

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau]c \triangleright c} \quad \text{ACSUBST_VAR} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](\mathbf{refl} \ \tau_2) \triangleright \mathbf{refl} \ \tau'_2} \quad \text{ACSUBST_REFL}
\end{array}$$

$$\begin{array}{c}
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](\mathbf{sym} \gamma_1) \triangleright \mathbf{sym} \gamma_2} \quad \text{ACSUBST_SYM} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2}{[\alpha \mapsto \tau](\gamma_1 \circ \gamma_2) \triangleright \gamma'_1 \circ \gamma'_2} \quad \text{ACSUBST_COMP} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2}{[\alpha \mapsto \tau](\gamma_1 \rightarrow \gamma_2) \triangleright \gamma'_1 \rightarrow \gamma'_2} \quad \text{ACSUBST_ARROW} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2 \quad [\alpha \mapsto \tau]\gamma_3 \triangleright \gamma'_3}{[\alpha \mapsto \tau](\{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3) \triangleright (\{\gamma'_1 \sim \gamma'_2\} \rightarrow \gamma'_3)} \quad \text{ACSUBST_CARROW} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha_1 \mapsto \tau](\forall(\alpha_2 : *), \gamma_1) \triangleright (\forall(\alpha_2 : *), \gamma_2)} \quad \text{ACSUBST_FORALL} \\
\frac{[\alpha \mapsto \tau_1]\gamma_1 \triangleright \gamma_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3}{[\alpha \mapsto \tau_1](\gamma_1 @ \tau_2) \triangleright \gamma_2 @ \tau_3} \quad \text{ACSUBST_INST} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](\mathbf{elim}_i \gamma_1) \triangleright \mathbf{elim}_i \gamma_2} \quad \text{ACSUBST_ELIM}
\end{array}$$

$\boxed{[c \mapsto \gamma]t_1 \triangleright t_2}$ substitution of coercion variable in term

$$\begin{array}{c}
\frac{}{[c \mapsto \gamma]x \triangleright x} \quad \text{CTSUBST_VAR} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](\lambda(x : \tau) \Rightarrow t_1) \triangleright (\lambda(x : \tau) \Rightarrow t_2)} \quad \text{CTSUBST_ABS} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](\lambda\{\alpha : *\} \Rightarrow t_1) \triangleright (\lambda\{\alpha : *\} \Rightarrow t_2)} \quad \text{CTSUBST_TABS} \\
\frac{}{[c \mapsto \gamma](\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \triangleright (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t)} \quad \text{CTSUBST_CABS1} \\
\frac{c_1 \neq c_2 \quad [c_1 \mapsto \gamma]t_1 \triangleright t_2}{[c_1 \mapsto \gamma](\lambda\{c_2 : \tau_1 \sim \tau_2\} \Rightarrow t_1) \triangleright (\lambda\{c_2 : \tau_1 \sim \tau_2\} \Rightarrow t_2)} \quad \text{CTSUBST_CABS2} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t'_1 \quad [c \mapsto \gamma]t_2 \triangleright t'_2}{[c \mapsto \gamma](t_1 t_2) \triangleright t'_1 t'_2} \quad \text{CTSUBST_APP} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](t_1 [\tau]) \triangleright (t_2 [\tau])} \quad \text{CTSUBST_TAPP} \\
\frac{[c \mapsto \gamma_1]t_1 \triangleright t_2 \quad [c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2}{[c \mapsto \gamma_1](t_1 \sim [\gamma_2]) \triangleright (t_2 \sim [\gamma'_2])} \quad \text{CTSUBST_CAPP} \\
\frac{[c \mapsto \gamma_1]t_1 \triangleright t_2 \quad [c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2}{[c \mapsto \gamma_1](t_1 \blacktriangleright \gamma_2) \triangleright (t_2 \blacktriangleright \gamma'_2)} \quad \text{CTSUBST_COERCE}
\end{array}$$

$\boxed{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}$ substitution of coercion variable in coercion term

$$\begin{array}{c}
\frac{}{[c \mapsto \gamma]c \triangleright \gamma} \quad \text{CCSUBST_VAR1} \\
\frac{c_1 \neq c_2}{[c_1 \mapsto \gamma]c_2 \triangleright c_2} \quad \text{CCSUBST_VAR2} \\
\frac{}{[c \mapsto \gamma](\mathbf{refl} \tau) \triangleright \mathbf{refl} \tau} \quad \text{CCSUBST_REFL}
\end{array}$$

$$\begin{array}{c}
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\mathbf{sym} \gamma_2) \triangleright \mathbf{sym} \gamma_3} \quad \text{CCSUBST_SYM} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3}{[c \mapsto \gamma_1](\gamma_2 \circ \gamma_3) \triangleright \gamma'_2 \circ \gamma'_3} \quad \text{CCSUBST_COMP} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3}{[c \mapsto \gamma_1](\gamma_2 \rightarrow \gamma_3) \triangleright \gamma'_2 \rightarrow \gamma'_3} \quad \text{CCSUBST_ARROW} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3 \quad [c \mapsto \gamma_1]\gamma_4 \triangleright \gamma'_4}{[c \mapsto \gamma_1](\{\gamma_2 \sim \gamma_3\} \rightarrow \gamma_4) \triangleright (\{\gamma'_2 \sim \gamma'_3\} \rightarrow \gamma'_4)} \quad \text{CCSUBST_CARROW} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\forall (\alpha : *), \gamma_2) \triangleright (\forall (\alpha : *), \gamma_3)} \quad \text{CCSUBST_FORALL} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\gamma_2 @ \tau) \triangleright \gamma_3 @ \tau} \quad \text{CCSUBST_INST} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\mathbf{elim}_i \gamma_2) \triangleright \mathbf{elim}_i \gamma_3} \quad \text{CCSUBST_ELIM}
\end{array}$$

Definition rules: 83 good 0 bad
 Definition rule clauses: 160 good 0 bad