

x	term variable	
α	type variable	
T	type constructor	
t	$::=$	term
	x	variable
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : \kappa\} \Rightarrow t$	type abstraction
	$t_1 t_2$	application
	$t [\tau]$	type application
	$t : \tau$	type annotation
κ	$::=$	kind
	$*$	star
	$\kappa_1 \rightarrow \kappa_2$	kind arrow
τ	$::=$	type
	α	type variable
	T	type constructor
	$\tau_1 \rightarrow \tau_2$	$\equiv (\rightarrow) \tau_1 \tau_2$
	$\lambda(\alpha : \kappa), \tau$	operator abstraction
	$\forall(\alpha : \kappa), \tau$	universal quantification
	$\tau_1 \tau_2$	operator application
Γ	$::=$	typing environment
	\emptyset	empty
	$\Gamma, x : \tau$	variable
	$\Gamma, T : \kappa$	type constructor
	$\Gamma, \alpha : \kappa$	type variable
tv	$::=$	typed value
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : \kappa\} \Rightarrow tv$	type abstraction
	$tv [\tau]$	type application
	$tv : \tau$	type annotation
v	$::=$	value
	$\lambda x \Rightarrow t$	abstraction

Initial environment: $\Gamma = \emptyset,$
 $(\rightarrow) : * \rightarrow * \rightarrow *$

$\Gamma \vdash t : \tau$ Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T_VAR} \\
\\
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \rightarrow \tau_2} \quad \text{T_ABS}
\end{array}$$

$$\begin{array}{c}
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : \kappa \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha : \kappa\} \Rightarrow t) : \forall(\alpha : \kappa), \tau} \quad \text{T_TYABS} \\
\\
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \tau_2 \equiv \tau'_2 \quad \Gamma \vdash t_2 : \tau'_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T_APP} \\
\\
\frac{\Gamma \vdash t : \forall(\alpha : \kappa), \tau_2 \quad \Gamma \vdash \tau_1 : \kappa \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T_TYAPP} \\
\\
\frac{\Gamma \vdash t : \tau_2 \quad \tau_1 \equiv \tau_2}{\Gamma \vdash (t : \tau_1) : \tau_1} \quad \text{T_ANNOT}
\end{array}$$

$\boxed{\Gamma \vdash \tau : \kappa}$ Kinding rules

$$\begin{array}{c}
\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \quad \text{K_VAR} \\
\\
\frac{T : \kappa \in \Gamma}{\Gamma \vdash T : \kappa} \quad \text{K_TYPECONSTR} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash (\lambda(\alpha : \kappa_1), \tau) : \kappa_1 \rightarrow \kappa_2} \quad \text{K_ABS} \\
\\
\frac{\Gamma \vdash \tau_1 : \kappa_2 \rightarrow \kappa_1 \quad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \tau_2 : \kappa_1} \quad \text{K_APP} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash (\forall(\alpha : \kappa), \tau) : *} \quad \text{K_FORALL}
\end{array}$$

$\boxed{\tau_1 \equiv \tau_2}$ Type equivalence

$$\begin{array}{c}
\frac{}{\tau \equiv \tau} \quad \text{EQ_REFL} \\
\\
\frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \quad \text{EQ_SYM} \\
\\
\frac{\tau_1 \equiv \tau_2 \quad \tau_2 \equiv \tau_3}{\tau_1 \equiv \tau_3} \quad \text{EQ_TRANS} \\
\\
\frac{}{\alpha \equiv \alpha} \quad \text{EQ_VAR} \\
\\
\frac{}{T \equiv T} \quad \text{EQ_TYPECONSTR} \\
\\
\frac{\tau_1 \equiv \tau_2}{(\forall(\alpha : \kappa), \tau_1) \equiv (\forall(\alpha : \kappa), \tau_2)} \quad \text{EQ_FORALL} \\
\\
\frac{\tau_1 \equiv \tau_2}{(\lambda(\alpha : \kappa), \tau_1) \equiv (\lambda(\alpha : \kappa), \tau_2)} \quad \text{EQ_ABS} \\
\\
\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2}{\tau_1 \tau_2 \equiv \tau'_1 \tau'_2} \quad \text{EQ_APP} \\
\\
\frac{[\alpha \mapsto \tau_2] \tau_1 \triangleright \tau'_1}{(\lambda(\alpha : \kappa), \tau_1) \tau_2 \equiv \tau'_1} \quad \text{EQ_APPABS}
\end{array}$$

$\boxed{t \longrightarrow tv}$ Operational semantics

$$\frac{t_2 \longrightarrow tv_2 \quad t_1 tv_2 \longrightarrow tv_3}{t_1 t_2 \longrightarrow tv_3} \quad \text{E_APP1}$$

$$\begin{array}{c}
\frac{t \longrightarrow tv_1 \quad tv_1 tv_2 \longrightarrow tv_3}{t tv_2 \longrightarrow tv_3} \quad \text{E_APP2} \\
\frac{[x \mapsto tv_1]t \triangleright t' \quad t' \longrightarrow tv_2}{(\lambda(x : \tau) \Rightarrow t) tv_1 \longrightarrow tv_2} \quad \text{E_APPAbs} \\
\frac{t \longrightarrow tv}{(\lambda\{\alpha : \kappa\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : \kappa\} \Rightarrow tv)} \quad \text{E_TABS} \\
\frac{t \longrightarrow tv}{t [\tau] \longrightarrow tv [\tau]} \quad \text{E_TAPP} \\
\frac{[\alpha \mapsto \tau]t \triangleright t' \quad t' \longrightarrow tv}{(\lambda\{\alpha : \kappa\} \Rightarrow t) [\tau] \longrightarrow tv} \quad \text{E_TAPPAbs} \\
\frac{t \longrightarrow tv}{(t : \tau) \longrightarrow (tv : \tau)} \quad \text{E_ANNOT}
\end{array}$$

$\boxed{tv \longrightarrow v}$ type erasure

$$\begin{array}{c}
\overline{(\lambda(x : \tau) \Rightarrow t) \longrightarrow (\lambda x \Rightarrow t)} \quad \text{ERASE_ABS} \\
\frac{tv \longrightarrow v}{(\lambda\{\alpha : \kappa\} \Rightarrow tv) \longrightarrow v} \quad \text{ERASE_TABS} \\
\frac{tv \longrightarrow v}{(tv [\tau]) \longrightarrow v} \quad \text{ERASE_TAPP} \\
\frac{tv \longrightarrow v}{(tv : \tau) \longrightarrow v} \quad \text{ERASE_ANNOT}
\end{array}$$

$\boxed{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3}$ Type substitution

$$\begin{array}{c}
\overline{[\alpha \mapsto \tau]\alpha \triangleright \tau} \quad \text{SUBST_VAR1} \\
\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau]\alpha_2 \triangleright \alpha_2} \quad \text{SUBST_VAR2} \\
\overline{[\alpha \mapsto \tau]T \triangleright T} \quad \text{SUBST_TYPE} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\lambda(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda(\alpha_2 : \kappa), \tau'_2)} \quad \text{SUBST_ABS} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\forall(\alpha_2 : \kappa), \tau_2) \triangleright (\forall(\alpha_2 : \kappa), \tau'_2)} \quad \text{SUBST_FORALL} \\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1](\tau_2 \tau_3) \triangleright \tau'_2 \tau'_3} \quad \text{SUBST_APP}
\end{array}$$

$\boxed{[x \mapsto tv]t_1 \triangleright t_2}$ substitution

$$\begin{array}{c}
\overline{[x \mapsto tv]x \triangleright tv} \quad \text{SUBST_VAR1} \\
\frac{x_1 \neq x_2}{[x_1 \mapsto tv]x_2 \triangleright x_2} \quad \text{SUBST_VAR2} \\
\overline{[x \mapsto tv](\lambda(x : \tau) \Rightarrow t) \triangleright (\lambda(x : \tau) \Rightarrow t)} \quad \text{SUBST_ABS1}
\end{array}$$

$$\frac{x_1 \neq x_2 \quad [x_1 \mapsto tv]t_1 \triangleright t_2}{[x_1 \mapsto tv](\lambda(x_2 : \tau) \Rightarrow t_1) \triangleright (\lambda(x_2 : \tau) \Rightarrow t_2)} \text{ SUBST_ABS2}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](\lambda\{\alpha : \kappa\} \Rightarrow t_1) \triangleright (\lambda\{\alpha : \kappa\} \Rightarrow t_2)} \text{ SUBST_TABS}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t'_1 \quad [x \mapsto tv]t_2 \triangleright t'_2}{[x \mapsto tv](t_1 t_2) \triangleright t'_1 t'_2} \text{ SUBST_APP}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](t_1 [\tau]) \triangleright (t_2 [\tau])} \text{ SUBST_TAPP}$$

$$\frac{[x \mapsto tv]t_1 \triangleright t_2}{[x \mapsto tv](t_1 : \tau) \triangleright (t_2 : \tau)} \text{ SUBST_ANNOT}$$

$\boxed{[\alpha \mapsto \tau]t_1 \triangleright t_2}$ substitution of type variable in term

$$\frac{}{[\alpha \mapsto \tau]x \triangleright x} \text{ TT_SUBST_VAR}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \triangleright (\lambda(x : \tau'_2) \Rightarrow t_2)} \text{ TT_SUBST_ABS}$$

$$\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]t_1 \triangleright t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : \kappa\} \Rightarrow t_1) \triangleright (\lambda\{\alpha_2 : \kappa\} \Rightarrow t_2)} \text{ TT_SUBST_TABS}$$

$$\frac{[\alpha \mapsto \tau]t_1 \triangleright t'_1 \quad [\alpha \mapsto \tau]t_2 \triangleright t'_2}{[\alpha \mapsto \tau](t_1 t_2) \triangleright t'_1 t'_2} \text{ TT_SUBST_APP}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 [\tau_2]) \triangleright (t_2 [\tau'_2])} \text{ TT_SUBST_TAPP}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 : \tau_2) \triangleright (t_2 : \tau'_2)} \text{ TT_SUBST_ANNOT}$$

Definition rules: 51 good 0 bad
Definition rule clauses: 93 good 0 bad