

x	term variable	
α	type variable	
T	abstract type	
c	coercion variable	
t	$::=$	term
	x	variable
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : \kappa\} \Rightarrow t$	type abstraction
	$\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t$	coercion abstraction
	$t_1 t_2$	application
	$t [\tau]$	type application
	$t \sim[\gamma]$	coercion application
	$t : \tau$	type annotation
	$t \blacktriangleright \gamma$	coercion
	(t)	S parenthesis
v	$::=$	value
	$\lambda(x : \tau) \Rightarrow t$	abstraction
κ	$::=$	kind
	$*$	star
	$\kappa_1 \rightarrow \kappa_2$	kind arrow
	(κ)	S parenthesis
τ	$::=$	type
	α	type variable
	T	abstract type
	$\tau_1 \rightarrow \tau_2$	arrow
	$\{\tau_1 \sim \tau_2\} \rightarrow \tau_3$	coercion arrow
	$\lambda(\alpha : \kappa), \tau$	operator abstraction
	$\forall(\alpha : \kappa), \tau$	forall
	$\tau_1 \tau_2$	operator application
	(τ)	S parenthesis
γ	$::=$	coercion proof term
	c	variable
	refl τ	reflexivity
	sym γ	symmetry
	$\gamma_1 \circ \gamma_2$	composition
	$\gamma_1 \rightarrow \gamma_2$	arrow introduction
	$\{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3$	coercion arrow introduction
	$\lambda(\alpha : \kappa), \gamma$	operator abstraction introduction
	$\forall(\alpha : \kappa), \gamma$	forall introduction
	$\gamma_1 \gamma_2$	application introduction
	left γ	left elimination
	right γ	right elimination
	(γ)	S parenthesis
Γ	$::=$	typing environment
	\emptyset	empty
	$\Gamma, x : \tau$	variable

	$\Gamma, T : \kappa$	abstract type
	$\Gamma, \alpha : \kappa$	type variable
	$\Gamma, c : \tau_1 \sim \tau_2$	coercion variable

$\boxed{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3}$ Type substitution

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau]\alpha \triangleright \tau} \text{ SUBST_VAR1} \\
\\
\frac{}{[\alpha_1 \mapsto \tau]\alpha_2 \triangleright \alpha_2} \text{ SUBST_VAR2} \\
\\
\frac{}{[\alpha \mapsto \tau]T \triangleright T} \text{ SUBST_TYPE} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1]\tau_2 \rightarrow \tau_3 \triangleright \tau'_2 \rightarrow \tau'_3} \text{ SUBST_ARROW} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3 \quad [\alpha \mapsto \tau_1]\tau_4 \triangleright \tau'_4}{[\alpha \mapsto \tau_1]\{\tau_2 \sim \tau_3\} \rightarrow \tau_4 \triangleright \{\tau'_2 \sim \tau'_3\} \rightarrow \tau'_4} \text{ SUBST_CARROW} \\
\\
\frac{}{[\alpha \mapsto \tau_1]\lambda(\alpha : \kappa), \tau_2 \triangleright \lambda(\alpha : \kappa), \tau_2} \text{ SUBST_ABS1} \\
\\
\frac{[\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1]\lambda(\alpha_2 : \kappa), \tau_2 \triangleright \lambda(\alpha_2 : \kappa), \tau'_2} \text{ SUBST_ABS2} \\
\\
\frac{}{[\alpha \mapsto \tau_1]\forall(\alpha : \kappa), \tau_2 \triangleright \forall(\alpha : \kappa), \tau_2} \text{ SUBST_FORALL1} \\
\\
\frac{[\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1]\forall(\alpha_2 : \kappa), \tau_2 \triangleright \forall(\alpha_2 : \kappa), \tau'_2} \text{ SUBST_FORALL2} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1]\tau_2 \tau_3 \triangleright \tau'_2 \tau'_3} \text{ SUBST_APP}
\end{array}$$

$\boxed{\Gamma \vdash t : \tau}$ Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \text{ T_VAR} \\
\\
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash \lambda(x : \tau_1) \Rightarrow t : \tau_1 \rightarrow \tau_2} \text{ T_ABS} \\
\\
\frac{\Gamma, \alpha : \kappa \vdash t : \tau \quad \alpha \notin \Gamma}{\Gamma \vdash \lambda\{\alpha : \kappa\} \Rightarrow t : \forall(\alpha : \kappa), \tau} \text{ T_TYABS} \\
\\
\frac{\Gamma, c : \tau_1 \sim \tau_2 \vdash t : \tau_3 \quad \Gamma \vdash \tau_1 : \kappa \quad \Gamma \vdash \tau_2 : \kappa}{\Gamma \vdash \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3} \text{ T_CABS}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \tau_2 \equiv \tau'_2 \quad \Gamma \vdash t_2 : \tau'_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T_APP} \\
\\
\frac{\Gamma \vdash t : \forall(\alpha : \kappa), \tau_2 \quad \Gamma \vdash \tau_1 : \kappa \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T_TYAPP} \\
\\
\frac{\Gamma \vdash t : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \quad \tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2 \quad \Gamma \vdash \gamma : \tau'_1 \sim \tau'_2}{\Gamma \vdash t \sim [\gamma] : \tau_3} \quad \text{T_CAPP} \\
\\
\frac{\Gamma \vdash t : \tau_2 \quad \tau_1 \equiv \tau_2}{\Gamma \vdash (t : \tau_1) : \tau_1} \quad \text{T_ANNOT} \\
\\
\frac{\Gamma \vdash \gamma : \tau_1 \sim \tau_2 \quad \tau_1 \equiv \tau'_1 \quad \Gamma \vdash t : \tau'_1}{\Gamma \vdash t \blacktriangleright \gamma : \tau_2} \quad \text{T_COERCE}
\end{array}$$

$$\boxed{\Gamma \vdash \gamma : \tau_1 \sim \tau_2}$$

Coercion typing

$$\begin{array}{c}
\frac{c : \tau_1 \sim \tau_2 \in \Gamma}{\Gamma \vdash c : \tau_1 \sim \tau_2} \quad \text{C_VAR} \\
\\
\frac{}{\Gamma \vdash \mathbf{refl} \tau : \tau \sim \tau} \quad \text{C_REFL} \\
\\
\frac{\Gamma \vdash \gamma : \tau_2 \sim \tau_1}{\Gamma \vdash \mathbf{sym} \gamma : \tau_1 \sim \tau_2} \quad \text{C_SYM} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2 \quad \tau_2 \equiv \tau'_2 \quad \Gamma \vdash \gamma_2 : \tau'_2 \sim \tau_3}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \quad \text{C_COMP} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \tau_1 \rightarrow \tau_2 : *}{\Gamma \vdash \gamma_1 \rightarrow \gamma_2 : \tau_1 \rightarrow \tau_2 \sim \tau'_1 \rightarrow \tau'_2} \quad \text{C_ARROW} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \gamma_3 : \tau_3 \sim \tau'_3 \quad \Gamma \vdash \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 : *}{\Gamma \vdash \{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3 : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \sim \{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3} \quad \text{C_CARROW} \\
\\
\frac{\Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash \lambda(\alpha : \kappa), \gamma : \lambda(\alpha : \kappa), \tau_1 \sim \lambda(\alpha : \kappa), \tau_2} \quad \text{C_ABS} \\
\\
\frac{\Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2 \quad \Gamma \vdash \forall(\alpha : \kappa), \tau_1 : *}{\Gamma \vdash \forall(\alpha : \kappa), \gamma : \forall(\alpha : \kappa), \tau_1 \sim \forall(\alpha : \kappa), \tau_2} \quad \text{C_FORALL}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \tau_1 \tau_2 : \kappa}{\Gamma \vdash \gamma_1 \gamma_2 : \tau_1 \tau_2 \sim \tau'_1 \tau'_2} \text{C_APP} \\
\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau'_1 \rightarrow \tau'_2}{\Gamma \vdash \mathbf{left} \gamma : \tau_1 \sim \tau'_1} \text{C_LEFT1} \\
\frac{\Gamma \vdash \gamma : \tau_1 \tau_2 \sim \tau'_1 \tau'_2}{\Gamma \vdash \mathbf{left} \gamma : \tau_1 \sim \tau'_1} \text{C_LEFT2} \\
\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau'_1 \rightarrow \tau'_2}{\Gamma \vdash \mathbf{right} \gamma : \tau_2 \sim \tau'_2} \text{C_RIGHT1} \\
\frac{\Gamma \vdash \gamma : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \sim \{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3}{\Gamma \vdash \mathbf{right} \gamma : \tau_3 \sim \tau'_3} \text{C_RIGHT2} \\
\frac{\Gamma \vdash \gamma : \tau_1 \tau_2 \sim \tau'_1 \tau'_2}{\Gamma \vdash \mathbf{right} \gamma : \tau_2 \sim \tau'_2} \text{C_RIGHT3}
\end{array}$$

$\boxed{\Gamma \vdash \tau : \kappa}$ Kinding

$$\begin{array}{c}
\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \text{K_VAR} \\
\frac{T : \kappa \in \Gamma}{\Gamma \vdash T : \kappa} \text{K_ABSTYPE} \\
\frac{\Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash \lambda(\alpha : \kappa_1), \tau : \kappa_1 \rightarrow \kappa_2} \text{K_ABS} \\
\frac{\Gamma \vdash \tau_1 : \kappa_2 \rightarrow \kappa_1 \quad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \tau_2 : \kappa_1} \text{K_APP} \\
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : *}{\Gamma \vdash \tau_1 \rightarrow \tau_2 : *} \text{K_ARROW} \\
\frac{\Gamma \vdash \tau_1 : \kappa \quad \Gamma \vdash \tau_2 : \kappa \quad \Gamma \vdash \tau_3 : *}{\Gamma \vdash \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 : *} \text{K_CARROW} \\
\frac{\Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash \forall(\alpha : \kappa), \tau : *} \text{K_FORALL}
\end{array}$$

$\boxed{\tau_1 \equiv \tau_2}$ Type equivalence

$$\begin{array}{c}
\frac{}{\tau \equiv \tau} \text{EQ_REFL} \\
\frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \text{EQ_SYMM} \\
\frac{\tau_1 \equiv \tau_2 \quad \tau_2 \equiv \tau_3}{\tau_1 \equiv \tau_3} \text{EQ_TRANS} \\
\frac{}{\alpha \equiv \alpha} \text{EQ_VAR}
\end{array}$$

$$\begin{array}{c}
\frac{}{T \equiv T} \quad \text{EQ_ABSTYPE} \\
\\
\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2}{\tau_1 \rightarrow \tau_2 \equiv \tau'_1 \rightarrow \tau'_2} \quad \text{EQ_ARROW} \\
\\
\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2 \quad \tau_3 \equiv \tau'_3}{\{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \equiv \{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3} \quad \text{EQ_CARROW} \\
\\
\frac{\tau_1 \equiv \tau_2}{\forall (\alpha : \kappa), \tau_1 \equiv \forall (\alpha : \kappa), \tau_2} \quad \text{EQ_FORALL} \\
\\
\frac{\tau_1 \equiv \tau_2}{\lambda(\alpha : \kappa), \tau_1 \equiv \lambda(\alpha : \kappa), \tau_2} \quad \text{EQ_ABS} \\
\\
\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2}{\tau_1 \tau_2 \equiv \tau'_1 \tau'_2} \quad \text{EQ_APP} \\
\\
\frac{[\alpha \mapsto \tau_2] \tau_1 \triangleright \tau'_1}{(\lambda(\alpha : \kappa), \tau_1) \tau_2 \equiv \tau'_1} \quad \text{EQ_APPAbs} \\
\\
\boxed{[x \mapsto v] t_1 \triangleright t_2} \quad \text{substitution} \\
\\
\frac{}{[x \mapsto v] x \triangleright v} \quad \text{SUBST_VAR1} \\
\\
\frac{}{[x_1 \mapsto v] x_2 \triangleright x_2} \quad \text{SUBST_VAR2} \\
\\
\frac{}{[x \mapsto v] \lambda(x : \tau) \Rightarrow t \triangleright \lambda(x : \tau) \Rightarrow t} \quad \text{SUBST_ABS1} \\
\\
\frac{[x_1 \mapsto v] t_1 \triangleright t_2}{[x_1 \mapsto v] \lambda(x_2 : \tau) \Rightarrow t_1 \triangleright \lambda(x_2 : \tau) \Rightarrow t_2} \quad \text{SUBST_ABS2} \\
\\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] \lambda\{\alpha : \kappa\} \Rightarrow t_1 \triangleright \lambda\{\alpha : \kappa\} \Rightarrow t_2} \quad \text{SUBST_TABS} \\
\\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_1 \triangleright \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_2} \quad \text{SUBST_CABS} \\
\\
\frac{[x \mapsto v] t_1 \triangleright t'_1 \quad [x \mapsto v] t_2 \triangleright t'_2}{[x \mapsto v] t_1 t_2 \triangleright t'_1 t'_2} \quad \text{SUBST_APP} \\
\\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 [\tau] \triangleright t_2 [\tau]} \quad \text{SUBST_TAPP} \\
\\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 \sim [\gamma] \triangleright t_2 \sim [\gamma]} \quad \text{SUBST_CAPP} \\
\\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 : \tau \triangleright t_2 : \tau} \quad \text{SUBST_ANNOT} \\
\\
\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 \blacktriangleright \gamma \triangleright t_2 \blacktriangleright \gamma} \quad \text{SUBST_COERCE}
\end{array}$$

$t_1 \longrightarrow t_2$ Evaluation

$$\begin{array}{c}
\frac{t_2 \longrightarrow t'_2}{t_1 t_2 \longrightarrow t_1 t'_2} \quad \text{E_APP1} \\
\\
\frac{t_1 \longrightarrow t'_1}{t_1 v \longrightarrow t'_1 v} \quad \text{E_APP2} \\
\\
\frac{[x \mapsto v]t \triangleright t' \quad t' \longrightarrow t''}{(\lambda(x : \tau) \Rightarrow t) v \longrightarrow t''} \quad \text{E_APPABS} \\
\\
\frac{t \longrightarrow t'}{\lambda\{\alpha : \kappa\} \Rightarrow t \longrightarrow t'} \quad \text{E_TABS} \\
\\
\frac{t \longrightarrow t'}{\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t \longrightarrow t'} \quad \text{E_CABS} \\
\\
\frac{t \longrightarrow t'}{t [\tau] \longrightarrow t'} \quad \text{E_TAPP} \\
\\
\frac{t \longrightarrow t'}{t \sim[\gamma] \longrightarrow t'} \quad \text{E_CAPP} \\
\\
\frac{t \longrightarrow t'}{t : \tau \longrightarrow t'} \quad \text{E_ANNOT} \\
\\
\frac{t \longrightarrow t'}{t \blacktriangleright \gamma \longrightarrow t'} \quad \text{E_COERCE}
\end{array}$$

Definition rules: 71 good 0 bad
Definition rule clauses: 169 good 0 bad