x	term variable
α	type variable
T	type constructor
c	coercion variable
i	index metavariable

t	::=		term
	$\vert$	x	variable
	$\vert$	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\vert$	$\lambda\{\alpha : \kappa\} \Rightarrow t$	type abstraction
	$\vert$	$\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t$	coercion abstraction
	$\vert$	$t_1\, t_2$	application
	$\vert$	$t\ [\tau]$	type application
	$\vert$	$t\ {\sim}[\gamma]$	coercion application
	$\vert$	$t : \tau$	type annotation
	$\vert$	$t \blacktriangleright \gamma$	coercion
κ	::=		kind
	$\vert$	$*$	star
	$\vert$	$\kappa_1 \to \kappa_2$	kind arrow
τ	::=		type
	$\vert$	α	type variable
	$\vert$	T	type constructor
	$\vert$	$\tau_1 \to \tau_2$	$\equiv (\to)\ \tau_1\ \tau_2$
	$\vert$	$\{\tau_1 \sim \tau_2\} \to \tau_3$	coercion arrow
	$\vert$	$\lambda(\alpha : \kappa), \tau$	operator abstraction
	$\vert$	$\forall\,(\alpha : \kappa), \tau$	universal quantification
	$\vert$	$\tau_1\, \tau_2$	operator application
γ	::=		coercion proof term
	$\vert$	c	variable
	$\vert$	$\mathbf{refl}\, \tau$	reflexivity
	$\vert$	$\mathbf{sym}\, \gamma$	symmetry
	$\vert$	$\gamma_1 \circ \gamma_2$	composition
	$\vert$	$\gamma_1 \to \gamma_2$	$\equiv (\to)\ \gamma_1\ \gamma_2$
	$\vert$	$\{\gamma_1 \sim \gamma_2\} \to \gamma_3$	coercion arrow introduction
	$\vert$	$\lambda(\alpha : \kappa), \gamma$	operator abstraction introduction
	$\vert$	$\forall\,(\alpha : \kappa), \gamma$	universal quantification introduction
	$\vert$	$\gamma_1\, \gamma_2$	application introduction
	$\vert$	$\gamma\ @\ \tau$	instantiation (quantification elimination)
	$\vert$	$\mathbf{elim}_{\mathrm{i}}\, \gamma$	generalized elimination
Γ	::=		typing environment
	$\vert$	$\emptyset$	empty
	$\vert$	$\Gamma, x : \tau$	variable
	$\vert$	$\Gamma, T : \kappa$	type constructor
	$\vert$	$\Gamma, \alpha : \kappa$	type variable
	$\vert$	$\Gamma, c : \tau_1 \sim \tau_2$	coercion variable

$$
\begin{array}{llll}
tv & ::= & & \text{typed value} \\
 & | & \lambda(x:\tau) \Rightarrow t & \quad\text{abstraction} \\
 & | & \lambda\{\alpha:\kappa\} \Rightarrow tv & \quad\text{type abstraction} \\
 & | & \lambda\{c:\tau_1 \sim \tau_2\} \Rightarrow tv & \quad\text{coercion abstraction} \\
 & | & tv\ [\tau] & \quad\text{type application} \\
 & | & tv\ {\sim}[\gamma] & \quad\text{coercion application} \\
 & | & tv : \tau & \quad\text{type annotation} \\
 & | & tv \blacktriangleright \gamma & \quad\text{coercion}
\end{array}
$$

$$
\begin{array}{llll}
v & ::= & & \text{value} \\
 & | & \lambda x \Rightarrow t & \quad\text{abstraction}
\end{array}
$$

$$
\text{Initial environment:} \quad \Gamma = \begin{array}{l} \emptyset, \\ (\to) : * \to * \to * \\ (\to) : (\to) \sim (\to) \end{array}
$$

$\boxed{\Gamma \vdash t : \tau}$ Typing rules

$$
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T\_Var}
$$

$$
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \qquad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \to \tau_2} \quad \text{T\_Abs}
$$

$$
\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha : \kappa \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha : \kappa\} \Rightarrow t) : \forall\,(\alpha : \kappa), \tau} \quad \text{T\_TyAbs}
$$

$$
\frac{\Gamma, c : \tau_1 \sim \tau_2 \vdash t : \tau_3 \qquad \Gamma \vdash \tau_1 : \kappa \qquad \Gamma \vdash \tau_2 : \kappa}{\Gamma \vdash (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) : \{\tau_1 \sim \tau_2\} \to \tau_3} \quad \text{T\_CAbs}
$$

$$
\frac{\Gamma \vdash t_1 : \tau_2 \to \tau_1 \qquad \tau_2 \equiv \tau_2' \qquad \Gamma \vdash t_2 : \tau_2'}{\Gamma \vdash t_1\ t_2 : \tau_1} \quad \text{T\_App}
$$

$$
\frac{\Gamma \vdash t : \forall\,(\alpha : \kappa), \tau_2 \qquad \Gamma \vdash \tau_1 : \kappa \qquad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{\Gamma \vdash t\ [\tau_1] : \tau_2'} \quad \text{T\_TyApp}
$$

$$
\frac{\Gamma \vdash t : \{\tau_1 \sim \tau_2\} \to \tau_3 \qquad \Gamma \vdash \gamma : \tau_1' \sim \tau_2' \qquad \tau_1 \equiv \tau_1' \qquad \tau_2 \equiv \tau_2'}{\Gamma \vdash t\ {\sim}[\gamma] : \tau_3} \quad \text{T\_CApp}
$$

$$
\frac{\Gamma \vdash t : \tau_2 \qquad \tau_1 \equiv \tau_2}{\Gamma \vdash (t : \tau_1) : \tau_1} \quad \text{T\_Annot}
$$

$$
\frac{\Gamma \vdash \gamma : \tau_1 \sim \tau_2 \qquad \Gamma \vdash t : \tau_1' \qquad \tau_1 \equiv \tau_1'}{\Gamma \vdash (t \blacktriangleright \gamma) : \tau_2} \quad \text{T\_Coerce}
$$

$\boxed{\Gamma \vdash \gamma : \tau_1 \sim \tau_2}$ Coercion typing

$$
\frac{c : \tau_1 \sim \tau_2 \in \Gamma}{\Gamma \vdash c : \tau_1 \sim \tau_2} \quad \text{C\_Var}
$$

$$
\frac{}{\Gamma \vdash \mathbf{refl}\,\tau : \tau \sim \tau} \quad \text{C\_Refl}
$$

$$
\frac{\Gamma \vdash \gamma : \tau_2 \sim \tau_1}{\Gamma \vdash \mathbf{sym}\,\gamma : \tau_1 \sim \tau_2} \quad \text{C\_Sym}
$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2 \qquad \tau_2 \equiv \tau_2' \qquad \Gamma \vdash \gamma_2 : \tau_2' \sim \tau_3}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \quad \text{C\_Comp}$$

$$\frac{\begin{array}{c}\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1' \qquad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau_2' \qquad \Gamma \vdash \gamma_3 : \tau_3 \sim \tau_3' \\ \Gamma \vdash \{\tau_1 \sim \tau_2\} \to \tau_3 : *\end{array}}{\Gamma \vdash (\{\gamma_1 \sim \gamma_2\} \to \gamma_3) : (\{\tau_1 \sim \tau_2\} \to \tau_3) \sim (\{\tau_1' \sim \tau_2'\} \to \tau_3')} \quad \text{C\_CArrow}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash (\lambda(\alpha : \kappa), \gamma) : (\lambda(\alpha : \kappa), \tau_1) \sim (\lambda(\alpha : \kappa), \tau_2)} \quad \text{C\_Abs}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2 \qquad \Gamma \vdash \forall\, (\alpha : \kappa), \tau_1 : *}{\Gamma \vdash (\forall\, (\alpha : \kappa), \gamma) : (\forall\, (\alpha : \kappa), \tau_1) \sim (\forall\, (\alpha : \kappa), \tau_2)} \quad \text{C\_Forall}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1' \qquad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau_2' \qquad \Gamma \vdash \tau_1\, \tau_2 : \kappa}{\Gamma \vdash \gamma_1\, \gamma_2 : \tau_1\, \tau_2 \sim \tau_1'\, \tau_2'} \quad \text{C\_App}$$

$$\frac{\begin{array}{c}\Gamma \vdash \tau_1 : \kappa \qquad \Gamma \vdash \gamma : (\forall\, (\alpha_1 : \kappa), \tau_2) \sim (\forall\, (\alpha_2 : \kappa), \tau_3) \\ [\alpha_1 \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha_2 \mapsto \tau_1]\tau_3 \rhd \tau_3'\end{array}}{\Gamma \vdash \gamma \mathbin{@} \tau_1 : \tau_2' \sim \tau_3'} \quad \text{C\_Inst}$$

$$\frac{i \in \{1, 2\} \qquad \Gamma \vdash \gamma : \tau_1\, \tau_2 \sim \tau_1'\, \tau_2'}{\Gamma \vdash \mathbf{elim}_{\mathrm{i}}\, \gamma : \tau_i \sim \tau_i'} \quad \text{C\_ElimApp}$$

$$\frac{i \in \{1, 2, 3\} \qquad \Gamma \vdash \gamma : (\{\tau_1 \sim \tau_2\} \to \tau_3) \sim (\{\tau_1' \sim \tau_2'\} \to \tau_3')}{\Gamma \vdash \mathbf{elim}_{\mathrm{i}}\, \gamma : \tau_i \sim \tau_i'} \quad \text{C\_ElimCArrow}$$

$\boxed{\Gamma \vdash \tau : \kappa}$ Kinding rules

$$\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \quad \text{K\_Var}$$

$$\frac{T : \kappa \in \Gamma}{\Gamma \vdash T : \kappa} \quad \text{K\_TypeConstr}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash (\lambda(\alpha : \kappa_1), \tau) : \kappa_1 \to \kappa_2} \quad \text{K\_Abs}$$

$$\frac{\Gamma \vdash \tau_1 : \kappa_2 \to \kappa_1 \qquad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1\, \tau_2 : \kappa_1} \quad \text{K\_App}$$

$$\frac{\Gamma \vdash \tau_1 : \kappa \qquad \Gamma \vdash \tau_2 : \kappa \qquad \Gamma \vdash \tau_3 : *}{\Gamma \vdash (\{\tau_1 \sim \tau_2\} \to \tau_3) : *} \quad \text{K\_CArrow}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash (\forall\, (\alpha : \kappa), \tau) : *} \quad \text{K\_Forall}$$

$\boxed{\tau_1 \equiv \tau_2}$ Type equivalence

$$\frac{}{\tau \equiv \tau} \quad \text{EQ\_Refl}$$

$$\frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \quad \text{EQ\_Sym}$$

$$\frac{\tau_1 \equiv \tau_2 \qquad \tau_2 \equiv \tau_3}{\tau_1 \equiv \tau_3} \quad \text{EQ\_Trans}$$

$$\frac{}{\alpha \equiv \alpha} \quad \text{EQ\_Var}$$

$$\frac{}{T \equiv T} \quad \text{EQ\_TypeConstr}$$

$$\frac{\tau_1 \equiv \tau_1' \qquad \tau_2 \equiv \tau_2' \qquad \tau_3 \equiv \tau_3'}{(\{\tau_1 \sim \tau_2\} \to \tau_3) \equiv (\{\tau_1' \sim \tau_2'\} \to \tau_3')} \quad \textsc{EQ\_CArrow}$$

$$\frac{\tau_1 \equiv \tau_2}{(\forall\,(\alpha : \kappa), \tau_1) \equiv (\forall\,(\alpha : \kappa), \tau_2)} \quad \textsc{EQ\_Forall}$$

$$\frac{\tau_1 \equiv \tau_2}{(\lambda(\alpha : \kappa), \tau_1) \equiv (\lambda(\alpha : \kappa), \tau_2)} \quad \textsc{EQ\_Abs}$$

$$\frac{\tau_1 \equiv \tau_1' \qquad \tau_2 \equiv \tau_2'}{\tau_1\ \tau_2 \equiv \tau_1'\ \tau_2'} \quad \textsc{EQ\_App}$$

$$\frac{[\alpha \mapsto \tau_2]\tau_1 \rhd \tau_1'}{(\lambda(\alpha : \kappa), \tau_1)\ \tau_2 \equiv \tau_1'} \quad \textsc{EQ\_AppAbs}$$

$\boxed{t \longrightarrow tv}$ Operational semantics

$$\frac{t_2 \longrightarrow tv_2 \qquad t_1\ tv_2 \longrightarrow tv_3}{t_1\ t_2 \longrightarrow tv_3} \quad \textsc{E\_App1}$$

$$\frac{t \longrightarrow tv_1 \qquad tv_1\ tv_2 \longrightarrow tv_3}{t\ tv_2 \longrightarrow tv_3} \quad \textsc{E\_App2}$$

$$\frac{[x \mapsto tv_1]t \rhd t' \qquad t' \longrightarrow tv_2}{(\lambda(x : \tau) \Rightarrow t)\ tv_1 \longrightarrow tv_2} \quad \textsc{E\_AppAbs}$$

$$\frac{t \longrightarrow tv}{(\lambda\{\alpha : \kappa\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : \kappa\} \Rightarrow tv)} \quad \textsc{E\_TAbs}$$

$$\frac{t \longrightarrow tv}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \longrightarrow (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow tv)} \quad \textsc{E\_CAbs}$$

$$\frac{t \longrightarrow tv}{t\ [\tau] \longrightarrow tv\ [\tau]} \quad \textsc{E\_TApp}$$

$$\frac{[\alpha \mapsto \tau]t \rhd t' \qquad t' \longrightarrow tv}{(\lambda\{\alpha : \kappa\} \Rightarrow t)\ [\tau] \longrightarrow tv} \quad \textsc{E\_TAppAbs}$$

$$\frac{t \longrightarrow tv}{t \sim\![\gamma] \longrightarrow tv \sim\![\gamma]} \quad \textsc{E\_CApp}$$

$$\frac{[c \mapsto \gamma]t \rhd t' \qquad t' \longrightarrow tv}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \sim\![\gamma] \longrightarrow tv} \quad \textsc{E\_CAppAbs}$$

$$\frac{t \longrightarrow tv}{(t : \tau) \longrightarrow (tv : \tau)} \quad \textsc{E\_Annot}$$

$$\frac{t \longrightarrow tv}{(t \blacktriangleright \gamma) \longrightarrow (tv \blacktriangleright \gamma)} \quad \textsc{E\_Coerce}$$

$\boxed{tv \longrightarrow v}$ type erasure

$$\frac{}{(\lambda(x : \tau) \Rightarrow t) \longrightarrow (\lambda x \Rightarrow t)} \quad \textsc{Erase\_Abs}$$

$$\frac{tv \longrightarrow v}{(\lambda\{\alpha : \kappa\} \Rightarrow tv) \longrightarrow v} \quad \textsc{Erase\_TAbs}$$

$$\frac{tv \longrightarrow v}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow tv) \longrightarrow v} \quad \textsc{Erase\_CAbs}$$

$$\frac{tv \longrightarrow v}{(tv\ [\tau]) \longrightarrow v} \quad \textsc{Erase\_TApp}$$

$$\frac{tv \longrightarrow v}{(tv\ \sim[\gamma]) \longrightarrow v} \quad \textsc{Erase\_CApp}$$

$$\frac{tv \longrightarrow v}{(tv : \tau) \longrightarrow v} \quad \textsc{Erase\_Annot}$$

$$\frac{tv \longrightarrow v}{(tv \blacktriangleright \gamma) \longrightarrow v} \quad \textsc{Erase\_Coerce}$$

$\boxed{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_3}$ Type substitution

$$\frac{}{[\alpha \mapsto \tau]\alpha \rhd \tau} \quad \textsc{SubstT\_Var1}$$

$$\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau]\alpha_2 \rhd \alpha_2} \quad \textsc{SubstT\_Var2}$$

$$\frac{}{[\alpha \mapsto \tau]T \rhd T} \quad \textsc{SubstT\_Type}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha \mapsto \tau_1]\tau_3 \rhd \tau_3' \qquad [\alpha \mapsto \tau_1]\tau_4 \rhd \tau_4'}{[\alpha \mapsto \tau_1](\{\tau_2 \sim \tau_3\} \to \tau_4) \rhd (\{\tau_2' \sim \tau_3'\} \to \tau_4')} \quad \textsc{SubstT\_CArrow}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha_1 \mapsto \tau_1](\lambda(\alpha_2 : \kappa), \tau_2) \rhd (\lambda(\alpha_2 : \kappa), \tau_2')} \quad \textsc{SubstT\_Abs}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha_1 \mapsto \tau_1](\forall\,(\alpha_2 : \kappa), \tau_2) \rhd (\forall\,(\alpha_2 : \kappa), \tau_2')} \quad \textsc{SubstT\_Forall}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha \mapsto \tau_1]\tau_3 \rhd \tau_3'}{[\alpha \mapsto \tau_1](\tau_2\,\tau_3) \rhd \tau_2'\,\tau_3'} \quad \textsc{SubstT\_App}$$

$\boxed{[x \mapsto tv]t_1 \rhd t_2}$ substitution

$$\frac{}{[x \mapsto tv]x \rhd tv} \quad \textsc{Subst\_Var1}$$

$$\frac{x_1 \neq x_2}{[x_1 \mapsto tv]x_2 \rhd x_2} \quad \textsc{Subst\_Var2}$$

$$\frac{}{[x \mapsto tv](\lambda(x : \tau) \Rightarrow t) \rhd (\lambda(x : \tau) \Rightarrow t)} \quad \textsc{Subst\_Abs1}$$

$$\frac{x_1 \neq x_2 \qquad [x_1 \mapsto tv]t_1 \rhd t_2}{[x_1 \mapsto tv](\lambda(x_2 : \tau) \Rightarrow t_1) \rhd (\lambda(x_2 : \tau) \Rightarrow t_2)} \quad \textsc{Subst\_Abs2}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](\lambda\{\alpha : \kappa\} \Rightarrow t_1) \rhd (\lambda\{\alpha : \kappa\} \Rightarrow t_2)} \quad \textsc{Subst\_TAbs}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_1) \rhd (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_2)} \quad \textsc{Subst\_CAbs}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_1' \qquad [x \mapsto tv]t_2 \rhd t_2'}{[x \mapsto tv](t_1\ t_2) \rhd t_1'\ t_2'} \quad \textsc{Subst\_App}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](t_1\ [\tau]) \rhd (t_2\ [\tau])} \quad \textsc{Subst\_TApp}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](t_1 \sim[\gamma]) \rhd (t_2 \sim[\gamma])} \quad \textsc{Subst\_CApp}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](t_1 : \tau) \rhd (t_2 : \tau)} \quad \textsc{Subst\_Annot}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](t_1 \blacktriangleright \gamma) \rhd (t_2 \blacktriangleright \gamma)} \quad \textsc{Subst\_Coerce}$$

$\boxed{[\alpha \mapsto \tau]t_1 \rhd t_2}$ substitution of type variable in term

$$\frac{}{[\alpha \mapsto \tau]x \rhd x} \quad \textsc{TtSubst\_Var}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha \mapsto \tau_1]t_1 \rhd t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \rhd (\lambda(x : \tau_2') \Rightarrow t_2)} \quad \textsc{TtSubst\_Abs}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau]t_1 \rhd t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : \kappa\} \Rightarrow t_1) \rhd (\lambda\{\alpha_2 : \kappa\} \Rightarrow t_2)} \quad \textsc{TtSubst\_TAbs}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha \mapsto \tau_1]\tau_3 \rhd \tau_3' \qquad [\alpha \mapsto \tau_1]t_1 \rhd t_2}{[\alpha \mapsto \tau_1](\lambda\{c : \tau_2 \sim \tau_3\} \Rightarrow t_1) \rhd (\lambda\{c : \tau_2' \sim \tau_3'\} \Rightarrow t_2)} \quad \textsc{TtSubst\_CAbs}$$

$$\frac{[\alpha \mapsto \tau]t_1 \rhd t_1' \qquad [\alpha \mapsto \tau]t_2 \rhd t_2'}{[\alpha \mapsto \tau](t_1\ t_2) \rhd t_1'\ t_2'} \quad \textsc{TtSubst\_App}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \rhd t_2 \qquad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha \mapsto \tau_1](t_1\ [\tau_2]) \rhd (t_2\ [\tau_2'])} \quad \textsc{TtSubst\_TApp}$$

$$\frac{[\alpha \mapsto \tau]t_1 \rhd t_2 \qquad [\alpha \mapsto \tau]\gamma_1 \rhd \gamma_2}{[\alpha \mapsto \tau](t_1 \sim[\gamma_1]) \rhd (t_2 \sim[\gamma_2])} \quad \textsc{TtSubst\_CApp}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \rhd t_2 \qquad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha \mapsto \tau_1](t_1 : \tau_2) \rhd (t_2 : \tau_2')} \quad \textsc{TtSubst\_Annot}$$

$$\frac{[\alpha \mapsto \tau]t_1 \rhd t_2 \qquad [\alpha \mapsto \tau]\gamma_1 \rhd \gamma_2}{[\alpha \mapsto \tau](t_1 \blacktriangleright \gamma_1) \rhd (t_2 \blacktriangleright \gamma_2)} \quad \textsc{TtSubst\_Coerce}$$

$\boxed{[\alpha \mapsto \tau]\gamma_1 \rhd \gamma_2}$ substitution of type variable in coercion term

$$\frac{}{[\alpha \mapsto \tau]c \rhd c} \quad \textsc{aCSubst\_Var}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha \mapsto \tau_1](\mathbf{refl}\,\tau_2) \rhd \mathbf{refl}\,\tau_2'} \quad \textsc{aCSubst\_Refl}$$

$$\frac{[\alpha \mapsto \tau]\gamma_1 \rhd \gamma_2}{[\alpha \mapsto \tau](\mathbf{sym}\,\gamma_1) \rhd \mathbf{sym}\,\gamma_2} \quad \textsc{aCSubst\_Sym}$$

$$\frac{[\alpha \mapsto \tau]\gamma_1 \rhd \gamma_1' \qquad [\alpha \mapsto \tau]\gamma_2 \rhd \gamma_2'}{[\alpha \mapsto \tau](\gamma_1 \circ \gamma_2) \rhd \gamma_1' \circ \gamma_2'} \quad \textsc{aCSubst\_Comp}$$

$$\frac{[\alpha \mapsto \tau]\gamma_1 \rhd \gamma_1' \qquad [\alpha \mapsto \tau]\gamma_2 \rhd \gamma_2' \qquad [\alpha \mapsto \tau]\gamma_3 \rhd \gamma_3'}{[\alpha \mapsto \tau](\{\gamma_1 \sim \gamma_2\} \to \gamma_3) \rhd (\{\gamma_1' \sim \gamma_2'\} \to \gamma_3')} \quad \textsc{aCSubst\_CArrow}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau]\gamma_1 \rhd \gamma_2}{[\alpha_1 \mapsto \tau](\lambda(\alpha_2 : \kappa), \gamma_1) \rhd (\lambda(\alpha_2 : \kappa), \gamma_2)} \quad \textsc{aCSubst\_Abs}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau]\gamma_1 \rhd \gamma_2}{[\alpha_1 \mapsto \tau](\forall\,(\alpha_2 : \kappa), \gamma_1) \rhd (\forall\,(\alpha_2 : \kappa), \gamma_2)} \quad \textsc{aCSubst\_Forall}$$

$$\frac{[\alpha \mapsto \tau]\gamma_1 \rhd \gamma_1' \qquad [\alpha \mapsto \tau]\gamma_2 \rhd \gamma_2'}{[\alpha \mapsto \tau](\gamma_1\ \gamma_2) \rhd \gamma_1'\ \gamma_2'} \quad \textsc{aCSubst\_App}$$

$$\frac{\begin{array}{c}[\alpha \mapsto \tau_1]\gamma_1 \rhd \gamma_2 \\ [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_3\end{array}}{[\alpha \mapsto \tau_1](\gamma_1\ @\ \tau_2) \rhd \gamma_2\ @\ \tau_3} \quad \textsc{aCSubst\_Inst}$$

$$\frac{[\alpha \mapsto \tau]\gamma_1 \rhd \gamma_2}{[\alpha \mapsto \tau](\mathbf{elim}_{\mathrm{i}}\,\gamma_1) \rhd \mathbf{elim}_{\mathrm{i}}\,\gamma_2} \quad \textsc{aCSubst\_Elim}$$

$\boxed{[c \mapsto \gamma]t_1 \rhd t_2}$ substitution of coercion variable in term

$$\frac{}{[c \mapsto \gamma]x \rhd x} \quad \textsc{CtSubst\_Var}$$

$$\frac{[c \mapsto \gamma]t_1 \rhd t_2}{[c \mapsto \gamma](\lambda(x : \tau) \Rightarrow t_1) \rhd (\lambda(x : \tau) \Rightarrow t_2)} \quad \textsc{CtSubst\_Abs}$$

$$\frac{[c \mapsto \gamma]t_1 \rhd t_2}{[c \mapsto \gamma](\lambda\{\alpha : \kappa\} \Rightarrow t_1) \rhd (\lambda\{\alpha : \kappa\} \Rightarrow t_2)} \quad \textsc{CtSubst\_TAbs}$$

$$\frac{}{[c \mapsto \gamma](\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \rhd (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t)} \quad \textsc{CtSubst\_CAbs1}$$

$$\frac{c_1 \neq c_2 \qquad [c_1 \mapsto \gamma]t_1 \rhd t_2}{[c_1 \mapsto \gamma](\lambda\{c_2 : \tau_1 \sim \tau_2\} \Rightarrow t_1) \rhd (\lambda\{c_2 : \tau_1 \sim \tau_2\} \Rightarrow t_2)} \quad \textsc{CtSubst\_CAbs2}$$

$$\frac{[c \mapsto \gamma]t_1 \rhd t_1' \qquad [c \mapsto \gamma]t_2 \rhd t_2'}{[c \mapsto \gamma](t_1\ t_2) \rhd t_1'\ t_2'} \quad \textsc{CtSubst\_App}$$

$$\frac{[c \mapsto \gamma]t_1 \rhd t_2}{[c \mapsto \gamma](t_1\ [\tau]) \rhd (t_2\ [\tau])} \quad \textsc{CtSubst\_TApp}$$

$$\frac{[c \mapsto \gamma_1]t_1 \rhd t_2 \qquad [c \mapsto \gamma_1]\gamma_2 \rhd \gamma_2'}{[c \mapsto \gamma_1](t_1\ {\sim}[\gamma_2]) \rhd (t_2\ {\sim}[\gamma_2'])} \quad \textsc{CtSubst\_CApp}$$

$$\frac{[c \mapsto \gamma]t_1 \rhd t_2}{[c \mapsto \gamma](t_1 : \tau) \rhd (t_2 : \tau)} \quad \textsc{CtSubst\_Annot}$$

$$\frac{[c \mapsto \gamma_1]t_1 \rhd t_2 \qquad [c \mapsto \gamma_1]\gamma_2 \rhd \gamma_2'}{[c \mapsto \gamma_1](t_1 \blacktriangleright \gamma_2) \rhd (t_2 \blacktriangleright \gamma_2')} \quad \textsc{CtSubst\_Coerce}$$

$\boxed{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_3}$ substitution of coercion variable in coercion term

$$\frac{}{[c \mapsto \gamma]c \rhd \gamma} \quad \textsc{CCSubst\_Var1}$$

$$\frac{c_1 \neq c_2}{[c_1 \mapsto \gamma]c_2 \rhd c_2} \quad \textsc{CCSubst\_Var2}$$

$$\frac{}{[c \mapsto \gamma](\mathbf{refl}\,\tau) \rhd \mathbf{refl}\,\tau} \quad \textsc{CCSubst\_Refl}$$

$$\frac{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_3}{[c \mapsto \gamma_1](\mathbf{sym}\,\gamma_2) \rhd \mathbf{sym}\,\gamma_3} \quad \textsc{CCSubst\_Sym}$$

$$\frac{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_2' \qquad [c \mapsto \gamma_1]\gamma_3 \rhd \gamma_3'}{[c \mapsto \gamma_1](\gamma_2 \circ \gamma_3) \rhd \gamma_2' \circ \gamma_3'} \quad \textsc{CCSubst\_Comp}$$

$$\frac{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_2' \qquad [c \mapsto \gamma_1]\gamma_3 \rhd \gamma_3' \qquad [c \mapsto \gamma_1]\gamma_4 \rhd \gamma_4'}{[c \mapsto \gamma_1](\{\gamma_2 \sim \gamma_3\} \to \gamma_4) \rhd (\{\gamma_2' \sim \gamma_3'\} \to \gamma_4')} \quad \textsc{CCSubst\_CArrow}$$

$$\frac{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_3}{[c \mapsto \gamma_1](\lambda(\alpha : \kappa), \gamma_2) \rhd (\lambda(\alpha : \kappa), \gamma_3)} \quad \textsc{CCSubst\_Abs}$$

$$\frac{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_3}{[c \mapsto \gamma_1](\forall\,(\alpha : \kappa), \gamma_2) \rhd (\forall\,(\alpha : \kappa), \gamma_3)} \quad \textsc{CCSubst\_Forall}$$

$$\frac{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_2' \qquad [c \mapsto \gamma_1]\gamma_3 \rhd \gamma_3'}{[c \mapsto \gamma_1](\gamma_2\,\gamma_3) \rhd \gamma_2'\,\gamma_3'} \quad \textsc{CCSubst\_App}$$

$$\frac{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_3}{[c \mapsto \gamma_1](\gamma_2 \mathbin{@} \tau) \rhd \gamma_3 \mathbin{@} \tau} \quad \textsc{CCSubst\_Inst}$$

$$\frac{[c \mapsto \gamma_1]\gamma_2 \rhd \gamma_3}{[c \mapsto \gamma_1](\mathbf{elim}_{\mathrm{i}}\,\gamma_2) \rhd \mathbf{elim}_{\mathrm{i}}\,\gamma_3} \quad \textsc{CCSubst\_Elim}$$

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Definition rules:        112 good    0 bad
Definition rule clauses: 212 good    0 bad
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