x term variable

α type variable

t	$::=$		term
	$\mid$	x	variable
	$\mid$	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\mid$	$\lambda\{\alpha : \kappa\} \Rightarrow t$	type abstraction
	$\mid$	$t_1\ t_2$	application
	$\mid$	$t\ [\tau]$	type application
κ	$::=$		kind
	$\mid$	$*$	star
	$\mid$	$\kappa_1 \to \kappa_2$	kind arrow
τ	$::=$		type
	$\mid$	α	type variable
	$\mid$	$\tau_1 \to \tau_2$	$\equiv (\to)\ \tau_1\ \tau_2$
	$\mid$	$\forall\,(\alpha : \kappa), \tau$	universal quantification
	$\mid$	$\lambda(\alpha : \kappa), \tau$	operator abstraction
	$\mid$	$\tau_1\, \tau_2$	operator application
Γ	$::=$		typing environment
	$\mid$	$\emptyset$	empty
	$\mid$	$\Gamma, x : \tau$	variable
	$\mid$	$\Gamma, \alpha : \kappa$	type variable
v	$::=$		value
	$\mid$	$\lambda(x : \tau) \Rightarrow t$	abstraction

Initial environment: $\Gamma = \emptyset, (\to) : * \to * \to *$

$\boxed{\Gamma \vdash t : \tau}$ Typing rules

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau}\ \textsc{T\_Var}$$

$$\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \qquad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \to \tau_2}\ \textsc{T\_Abs}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha : \kappa \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha : \kappa\} \Rightarrow t) : \forall\,(\alpha : \kappa), \tau}\ \textsc{T\_TyAbs}$$

$$\frac{\Gamma \vdash t_1 : \tau_2 \to \tau_1 \qquad \tau_2 \equiv \tau_2' \qquad \Gamma \vdash t_2 : \tau_2'}{\Gamma \vdash t_1\ t_2 : \tau_1}\ \textsc{T\_App}$$

$$\frac{\Gamma \vdash t : \forall\,(\alpha : \kappa), \tau_2 \qquad \Gamma \vdash \tau_1 : \kappa \qquad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{\Gamma \vdash t\ [\tau_1] : \tau_2'}\ \textsc{T\_TyApp}$$

$\boxed{\Gamma \vdash \tau : \kappa}$ Kinding rules

$$\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa}\ \textsc{K\_Var}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash (\lambda(\alpha : \kappa_1), \tau) : \kappa_1 \to \kappa_2} \quad \text{K\_ABS}$$

$$\frac{\Gamma \vdash \tau_1 : \kappa_2 \to \kappa_1 \qquad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1\, \tau_2 : \kappa_1} \quad \text{K\_APP}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash (\forall\, (\alpha : \kappa), \tau) : *} \quad \text{K\_FORALL}$$

$\boxed{\tau_1 \equiv \tau_2}$ Type equivalence

$$\frac{}{\tau \equiv \tau} \quad \text{EQ\_REFL}$$

$$\frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \quad \text{EQ\_SYM}$$

$$\frac{\tau_1 \equiv \tau_2 \qquad \tau_2 \equiv \tau_3}{\tau_1 \equiv \tau_3} \quad \text{EQ\_TRANS}$$

$$\frac{}{\alpha \equiv \alpha} \quad \text{EQ\_VAR}$$

$$\frac{\tau_1 \equiv \tau_2}{(\forall\, (\alpha : \kappa), \tau_1) \equiv (\forall\, (\alpha : \kappa), \tau_2)} \quad \text{EQ\_FORALL}$$

$$\frac{\tau_1 \equiv \tau_2}{(\lambda(\alpha : \kappa), \tau_1) \equiv (\lambda(\alpha : \kappa), \tau_2)} \quad \text{EQ\_ABS}$$

$$\frac{\tau_1 \equiv \tau_1' \qquad \tau_2 \equiv \tau_2'}{\tau_1\, \tau_2 \equiv \tau_1'\, \tau_2'} \quad \text{EQ\_APP}$$

$$\frac{[\alpha \mapsto \tau_2]\tau_1 \rhd \tau_1'}{(\lambda(\alpha : \kappa), \tau_1)\, \tau_2 \equiv \tau_1'} \quad \text{EQ\_APPABS}$$

$\boxed{t \longrightarrow t'}$ Operational semantics

$$\frac{t_2 \longrightarrow t_2'}{t_1\ t_2 \longrightarrow t_1\ t_2'} \quad \text{E\_APP1}$$

$$\frac{t \longrightarrow t'}{t\ v \longrightarrow t'\ v} \quad \text{E\_APP2}$$

$$\frac{[x \mapsto v]t \rhd t'}{(\lambda(x : \tau) \Rightarrow t)\ v \longrightarrow t'} \quad \text{E\_APPABS}$$

$$\frac{t \longrightarrow t'}{(\lambda\{\alpha : \kappa\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : \kappa\} \Rightarrow t')} \quad \text{E\_TABS}$$

$$\frac{t \longrightarrow t'}{t\ [\tau] \longrightarrow t'\ [\tau]} \quad \text{E\_TAPP}$$

$$\frac{[\alpha \mapsto \tau]v \rhd v'}{(\lambda\{\alpha : \kappa\} \Rightarrow v)\ [\tau] \longrightarrow v'} \quad \text{E\_TAPPABS}$$

$\boxed{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_3}$ Type substitution

$$\frac{}{[\alpha \mapsto \tau]\alpha \rhd \tau} \quad \text{SUBSTT\_VAR1}$$

$$\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau]\alpha_2 \rhd \alpha_2} \quad \text{SUBSTT\_VAR2}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha_1 \mapsto \tau_1](\lambda(\alpha_2 : \kappa), \tau_2) \rhd (\lambda(\alpha_2 : \kappa), \tau_2')} \quad \textsc{SubstT\_Abs}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha \mapsto \tau_1]\tau_3 \rhd \tau_3'}{[\alpha \mapsto \tau_1](\tau_2\,\tau_3) \rhd \tau_2'\,\tau_3'} \quad \textsc{SubstT\_App}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha_1 \mapsto \tau_1](\forall\,(\alpha_2 : \kappa), \tau_2) \rhd (\forall\,(\alpha_2 : \kappa), \tau_2')} \quad \textsc{SubstT\_Forall}$$

$\boxed{[x \mapsto v]t_1 \rhd t_2}$ substitution

$$\frac{}{[x \mapsto v]x \rhd v} \quad \textsc{Subst\_Var1}$$

$$\frac{x_1 \neq x_2}{[x_1 \mapsto v]x_2 \rhd x_2} \quad \textsc{Subst\_Var2}$$

$$\frac{}{[x \mapsto v](\lambda(x : \tau) \Rightarrow t) \rhd (\lambda(x : \tau) \Rightarrow t)} \quad \textsc{Subst\_Abs1}$$

$$\frac{x_1 \neq x_2 \qquad [x_1 \mapsto v]t_1 \rhd t_2}{[x_1 \mapsto v](\lambda(x_2 : \tau) \Rightarrow t_1) \rhd (\lambda(x_2 : \tau) \Rightarrow t_2)} \quad \textsc{Subst\_Abs2}$$

$$\frac{[x \mapsto v]t_1 \rhd t_2}{[x \mapsto v](\lambda\{\alpha : \kappa\} \Rightarrow t_1) \rhd (\lambda\{\alpha : \kappa\} \Rightarrow t_2)} \quad \textsc{Subst\_TAbs}$$

$$\frac{[x \mapsto v]t_1 \rhd t_1' \qquad [x \mapsto v]t_2 \rhd t_2'}{[x \mapsto v](t_1\,t_2) \rhd t_1'\,t_2'} \quad \textsc{Subst\_App}$$

$$\frac{[x \mapsto v]t_1 \rhd t_2}{[x \mapsto v](t_1\ [\tau]) \rhd (t_2\ [\tau])} \quad \textsc{Subst\_TApp}$$

$\boxed{[\alpha \mapsto \tau]t_1 \rhd t_2}$ substitution of type variable in term

$$\frac{}{[\alpha \mapsto \tau]x \rhd x} \quad \textsc{TtSubst\_Var}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha \mapsto \tau_1]t_1 \rhd t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \rhd (\lambda(x : \tau_2') \Rightarrow t_2)} \quad \textsc{TtSubst\_Abs}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau]t_1 \rhd t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : \kappa\} \Rightarrow t_1) \rhd (\lambda\{\alpha_2 : \kappa\} \Rightarrow t_2)} \quad \textsc{TtSubst\_TAbs}$$

$$\frac{[\alpha \mapsto \tau]t_1 \rhd t_1' \qquad [\alpha \mapsto \tau]t_2 \rhd t_2'}{[\alpha \mapsto \tau](t_1\,t_2) \rhd t_1'\,t_2'} \quad \textsc{TtSubst\_App}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \rhd t_2 \qquad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha \mapsto \tau_1](t_1\ [\tau_2]) \rhd (t_2\ [\tau_2'])} \quad \textsc{TtSubst\_TApp}$$

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Definition rules:        40 good    0 bad
Definition rule clauses: 74 good    0 bad
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