x	term variable
α	type variable
T	type constructor

$$
\begin{array}{llll}
t & ::= & & \text{term} \\
& | & x & \quad \text{variable} \\
& | & \lambda(x : \tau) \Rightarrow t & \quad \text{abstraction} \\
& | & \lambda\{\alpha\} \Rightarrow t & \quad \text{type abstraction} \\
& | & t_1\ t_2 & \quad \text{application} \\
& | & t\ [\tau] & \quad \text{type application} \\
& | & t : \tau & \quad \text{type annotation}
\end{array}
$$

$$
\begin{array}{llll}
\tau & ::= & & \text{type} \\
& | & \alpha & \quad \text{type variable} \\
& | & T & \quad \text{type constructor} \\
& | & \tau_1 \to \tau_2 & \quad \text{arrow} \\
& | & \forall\, \alpha, \tau & \quad \text{universal quantification}
\end{array}
$$

$$
\begin{array}{llll}
\Gamma & ::= & & \text{typing environment} \\
& | & \emptyset & \quad \text{empty} \\
& | & \Gamma, x : \tau & \quad \text{variable} \\
& | & \Gamma, T & \quad \text{type constructor} \\
& | & \Gamma, \alpha & \quad \text{type variable}
\end{array}
$$

$$
\begin{array}{llll}
tv & ::= & & \text{typed value} \\
& | & \lambda(x : \tau) \Rightarrow t & \quad \text{abstraction} \\
& | & \lambda\{\alpha\} \Rightarrow tv & \quad \text{type abstraction} \\
& | & tv\ [\tau] & \quad \text{type application} \\
& | & tv : \tau & \quad \text{type annotation}
\end{array}
$$

$$
\begin{array}{llll}
v & ::= & & \text{value} \\
& | & \lambda x \Rightarrow t & \quad \text{abstraction}
\end{array}
$$

Initial environment: $\Gamma = \emptyset$

$\boxed{\Gamma \vdash t : \tau}$ Typing rules

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau}\ \textsc{T\_Var}$$

$$\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \qquad \Gamma \vdash \tau_1}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \to \tau_2}\ \textsc{T\_Abs}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha\} \Rightarrow t) : \forall\, \alpha, \tau}\ \textsc{T\_TyAbs}$$

$$\frac{\Gamma \vdash t_1 : \tau_2 \to \tau_1 \qquad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1\ t_2 : \tau_1}\ \textsc{T\_App}$$

$$\frac{\Gamma \vdash t : \forall\, \alpha, \tau_2 \qquad \Gamma \vdash \tau_1 \qquad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{\Gamma \vdash t\ [\tau_1] : \tau_2'}\ \textsc{T\_TyApp}$$

$$\frac{\Gamma \vdash t : \tau_1}{\Gamma \vdash (t : \tau_1) : \tau_1} \quad \textsc{T\_Annot}$$

$\boxed{\Gamma \vdash \tau}$ Type τ is well formed

$$\frac{\alpha \in \Gamma}{\Gamma \vdash \alpha} \quad \textsc{K\_Var}$$

$$\frac{T \in \Gamma}{\Gamma \vdash T} \quad \textsc{K\_TypeConstr}$$

$$\frac{\Gamma \vdash \tau_1 \qquad \Gamma \vdash \tau_2}{\Gamma \vdash \tau_1 \to \tau_2} \quad \textsc{K\_Arrow}$$

$$\frac{\alpha \notin \Gamma \qquad \Gamma, \alpha \vdash \tau}{\Gamma \vdash \forall\, \alpha, \tau} \quad \textsc{K\_Forall}$$

$\boxed{t \longrightarrow tv}$ Operational semantics

$$\frac{t_2 \longrightarrow tv_2 \qquad t_1\ tv_2 \longrightarrow tv_3}{t_1\ t_2 \longrightarrow tv_3} \quad \textsc{E\_App1}$$

$$\frac{t \longrightarrow tv_1 \qquad tv_1\ tv_2 \longrightarrow tv_3}{t\ tv_2 \longrightarrow tv_3} \quad \textsc{E\_App2}$$

$$\frac{[x \mapsto tv_1]t \rhd t' \qquad t' \longrightarrow tv_2}{(\lambda(x : \tau) \Rightarrow t)\ tv_1 \longrightarrow tv_2} \quad \textsc{E\_AppAbs}$$

$$\frac{t \longrightarrow tv}{(\lambda\{\alpha\} \Rightarrow t) \longrightarrow (\lambda\{\alpha\} \Rightarrow tv)} \quad \textsc{E\_TAbs}$$

$$\frac{t \longrightarrow tv}{t\ [\tau] \longrightarrow tv\ [\tau]} \quad \textsc{E\_TApp}$$

$$\frac{[\alpha \mapsto \tau]t \rhd t' \qquad t' \longrightarrow tv}{(\lambda\{\alpha\} \Rightarrow t)\ [\tau] \longrightarrow tv} \quad \textsc{E\_TAppAbs}$$

$$\frac{t \longrightarrow tv}{(t : \tau) \longrightarrow (tv : \tau)} \quad \textsc{E\_Annot}$$

$\boxed{tv \longrightarrow v}$ type erasure

$$\frac{}{(\lambda(x : \tau) \Rightarrow t) \longrightarrow (\lambda x \Rightarrow t)} \quad \textsc{Erase\_Abs}$$

$$\frac{tv \longrightarrow v}{(\lambda\{\alpha\} \Rightarrow tv) \longrightarrow v} \quad \textsc{Erase\_TAbs}$$

$$\frac{tv \longrightarrow v}{(tv\ [\tau]) \longrightarrow v} \quad \textsc{Erase\_TApp}$$

$$\frac{tv \longrightarrow v}{(tv : \tau) \longrightarrow v} \quad \textsc{Erase\_Annot}$$

$\boxed{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_3}$ Type substitution

$$\frac{}{[\alpha \mapsto \tau]\alpha \rhd \tau} \quad \textsc{SubstT\_Var1}$$

$$\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau]\alpha_2 \rhd \alpha_2} \quad \textsc{SubstT\_Var2}$$

$$\frac{}{[\alpha \mapsto \tau]T \rhd T} \quad \textsc{SubstT\_Type}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha \mapsto \tau_1]\tau_3 \rhd \tau_3'}{[\alpha \mapsto \tau_1]\tau_2 \to \tau_3 \rhd \tau_2' \to \tau_3'} \quad \textsc{SubstT\_Arrow}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha_1 \mapsto \tau_1](\forall\, \alpha_2, \tau_2) \rhd (\forall\, \alpha_2, \tau_2')} \quad \textsc{SubstT\_Forall}$$

$\boxed{[x \mapsto tv]t_1 \rhd t_2}$ substitution

$$\frac{}{[x \mapsto tv]x \rhd tv} \quad \textsc{Subst\_Var1}$$

$$\frac{x_1 \neq x_2}{[x_1 \mapsto tv]x_2 \rhd x_2} \quad \textsc{Subst\_Var2}$$

$$\frac{}{[x \mapsto tv](\lambda(x : \tau) \Rightarrow t) \rhd (\lambda(x : \tau) \Rightarrow t)} \quad \textsc{Subst\_Abs1}$$

$$\frac{x_1 \neq x_2 \qquad [x_1 \mapsto tv]t_1 \rhd t_2}{[x_1 \mapsto tv](\lambda(x_2 : \tau) \Rightarrow t_1) \rhd (\lambda(x_2 : \tau) \Rightarrow t_2)} \quad \textsc{Subst\_Abs2}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](\lambda\{\alpha\} \Rightarrow t_1) \rhd (\lambda\{\alpha\} \Rightarrow t_2)} \quad \textsc{Subst\_TAbs}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_1' \qquad [x \mapsto tv]t_2 \rhd t_2'}{[x \mapsto tv](t_1\ t_2) \rhd t_1'\ t_2'} \quad \textsc{Subst\_App}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](t_1\ [\tau]) \rhd (t_2\ [\tau])} \quad \textsc{Subst\_TApp}$$

$$\frac{[x \mapsto tv]t_1 \rhd t_2}{[x \mapsto tv](t_1 : \tau) \rhd (t_2 : \tau)} \quad \textsc{Subst\_Annot}$$

$\boxed{[\alpha \mapsto \tau]t_1 \rhd t_2}$ substitution of type variable in term

$$\frac{}{[\alpha \mapsto \tau]x \rhd x} \quad \textsc{TtSubst\_Var}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2' \qquad [\alpha \mapsto \tau_1]t_1 \rhd t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \rhd (\lambda(x : \tau_2') \Rightarrow t_2)} \quad \textsc{TtSubst\_Abs}$$

$$\frac{\alpha_1 \neq \alpha_2 \qquad [\alpha_1 \mapsto \tau]t_1 \rhd t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2\} \Rightarrow t_1) \rhd (\lambda\{\alpha_2\} \Rightarrow t_2)} \quad \textsc{TtSubst\_TAbs}$$

$$\frac{[\alpha \mapsto \tau]t_1 \rhd t_1' \qquad [\alpha \mapsto \tau]t_2 \rhd t_2'}{[\alpha \mapsto \tau](t_1\ t_2) \rhd t_1'\ t_2'} \quad \textsc{TtSubst\_App}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \rhd t_2 \qquad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha \mapsto \tau_1](t_1\ [\tau_2]) \rhd (t_2\ [\tau_2'])} \quad \textsc{TtSubst\_TApp}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \rhd t_2 \qquad [\alpha \mapsto \tau_1]\tau_2 \rhd \tau_2'}{[\alpha \mapsto \tau_1](t_1 : \tau_2) \rhd (t_2 : \tau_2')} \quad \textsc{TtSubst\_Annot}$$

```
Definition rules:        40 good    0 bad
Definition rule clauses: 74 good    0 bad
```