

$x$	term variable	
$\alpha$	type variable	
$T$	abstract type	
$c$	coercion variable	
$t$	$::=$	term
	$x$	variable
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : \kappa\} \Rightarrow t$	type abstraction
	$\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t$	coercion abstraction
	$t_1 t_2$	application
	$t [\tau]$	type application
	$t \sim[\gamma]$	coercion application
	$t : \tau$	type annotation
	$t \blacktriangleright \gamma$	coercion
	$(t)$	S parenthesis
$v$	$::=$	value
	$\lambda(x : \tau) \Rightarrow t$	abstraction
$\kappa$	$::=$	kind
	$*$	star
	$\kappa_1 \rightarrow \kappa_2$	kind arrow
	$(\kappa)$	S parenthesis
$\tau$	$::=$	type
	$\alpha$	type variable
	$T$	abstract type
	$\tau_1 \rightarrow \tau_2$	S $\equiv (\rightarrow) \tau_1 \tau_2$ where $(\rightarrow) : * \rightarrow * \rightarrow *$
	$\{\tau_1 \sim \tau_2\} \rightarrow \tau_3$	coercion arrow
	$\lambda(\alpha : \kappa), \tau$	operator abstraction
	$\forall(\alpha : \kappa), \tau$	forall
	$\tau_1 \tau_2$	operator application
	$(\tau)$	S parenthesis
$\gamma$	$::=$	coercion proof term
	$c$	variable
	<b>refl</b> $\tau$	reflexivity
	<b>sym</b> $\gamma$	symmetry
	$\gamma_1 \circ \gamma_2$	composition
	$\gamma_1 \rightarrow \gamma_2$	S $\equiv (\rightarrow) \gamma_1 \gamma_2$
	$\{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3$	coercion arrow introduction
	$\lambda(\alpha : \kappa), \gamma$	operator abstraction introduction
	$\forall(\alpha : \kappa), \gamma$	forall introduction
	$\gamma_1 \gamma_2$	application introduction
	<b>left</b> $\gamma$	left elimination
	<b>right</b> $\gamma$	right elimination
	$(\gamma)$	S parenthesis
$\Gamma$	$::=$	typing environment
	$\emptyset$	empty
	$\Gamma, x : \tau$	variable

	$\Gamma, T : \kappa$	abstract type
	$\Gamma, \alpha : \kappa$	type variable
	$\Gamma, c : \tau_1 \sim \tau_2$	coercion variable

$\boxed{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau_3}$     Type substitution

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau] \alpha \triangleright \tau} \text{ SUBST\_VAR1} \\
\\
\frac{}{[\alpha_1 \mapsto \tau] \alpha_2 \triangleright \alpha_2} \text{ SUBST\_VAR2} \\
\\
\frac{}{[\alpha \mapsto \tau] T \triangleright T} \text{ SUBST\_TYPE} \\
\\
\frac{
\begin{array}{c}
[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \\
[\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3 \\
[\alpha \mapsto \tau_1] \tau_4 \triangleright \tau'_4
\end{array}
}{[\alpha \mapsto \tau_1] \{\tau_2 \sim \tau_3\} \rightarrow \tau_4 \triangleright \{\tau'_2 \sim \tau'_3\} \rightarrow \tau'_4} \text{ SUBST\_CARROW} \\
\\
\frac{}{[\alpha \mapsto \tau_1] \lambda(\alpha : \kappa), \tau_2 \triangleright \lambda(\alpha : \kappa), \tau_2} \text{ SUBST\_ABS1} \\
\\
\frac{[\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1] \lambda(\alpha_2 : \kappa), \tau_2 \triangleright \lambda(\alpha_2 : \kappa), \tau'_2} \text{ SUBST\_ABS2} \\
\\
\frac{}{[\alpha \mapsto \tau_1] \forall(\alpha : \kappa), \tau_2 \triangleright \forall(\alpha : \kappa), \tau_2} \text{ SUBST\_FORALL1} \\
\\
\frac{[\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1] \forall(\alpha_2 : \kappa), \tau_2 \triangleright \forall(\alpha_2 : \kappa), \tau'_2} \text{ SUBST\_FORALL2} \\
\\
\frac{
\begin{array}{c}
[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \\
[\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3
\end{array}
}{[\alpha \mapsto \tau_1] \tau_2 \tau_3 \triangleright \tau'_2 \tau'_3} \text{ SUBST\_APP}
\end{array}$$

$\boxed{\Gamma \vdash t : \tau}$     Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \text{ T\_VAR} \\
\\
\frac{
\begin{array}{c}
\Gamma, x : \tau_1 \vdash t : \tau_2 \\
\Gamma \vdash \tau_1 : *
\end{array}
}{\Gamma \vdash \lambda(x : \tau_1) \Rightarrow t : \tau_1 \rightarrow \tau_2} \text{ T\_ABS} \\
\\
\frac{
\begin{array}{c}
\Gamma, \alpha : \kappa \vdash t : \tau \\
\alpha \notin \Gamma
\end{array}
}{\Gamma \vdash \lambda\{\alpha : \kappa\} \Rightarrow t : \forall(\alpha : \kappa), \tau} \text{ T\_TYABS} \\
\\
\frac{
\begin{array}{c}
\Gamma, c : \tau_1 \sim \tau_2 \vdash t : \tau_3 \\
\Gamma \vdash \tau_1 : \kappa \\
\Gamma \vdash \tau_2 : \kappa
\end{array}
}{\Gamma \vdash \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3} \text{ T\_CABS} \\
\\
\frac{
\begin{array}{c}
\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \\
\tau_2 \equiv \tau'_2 \\
\Gamma \vdash t_2 : \tau'_2
\end{array}
}{\Gamma \vdash t_1 t_2 : \tau_1} \text{ T\_APP}
\end{array}$$

$$\frac{\begin{array}{l} \Gamma \vdash t : \forall (\alpha : \kappa), \tau_2 \\ \Gamma \vdash \tau_1 : \kappa \\ [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \end{array}}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T\_TYAPP}$$

$$\frac{\begin{array}{l} \Gamma \vdash t : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \\ \tau_1 \equiv \tau'_1 \\ \tau_2 \equiv \tau'_2 \\ \Gamma \vdash \gamma : \tau'_1 \sim \tau'_2 \end{array}}{\Gamma \vdash t \sim [\gamma] : \tau_3} \quad \text{T\_CAPP}$$

$$\frac{\begin{array}{l} \Gamma \vdash t : \tau_2 \\ \tau_1 \equiv \tau_2 \end{array}}{\Gamma \vdash (t : \tau_1) : \tau_1} \quad \text{T\_ANNOT}$$

$$\frac{\begin{array}{l} \Gamma \vdash \gamma : \tau_1 \sim \tau_2 \\ \tau_1 \equiv \tau'_1 \\ \Gamma \vdash t : \tau'_1 \end{array}}{\Gamma \vdash t \blacktriangleright \gamma : \tau_2} \quad \text{T\_COERCE}$$

$\Gamma \vdash \gamma : \tau_1 \sim \tau_2$     Coercion typing

$$\frac{c : \tau_1 \sim \tau_2 \in \Gamma}{\Gamma \vdash c : \tau_1 \sim \tau_2} \quad \text{C\_VAR}$$

$$\frac{}{\Gamma \vdash \mathbf{refl} \tau : \tau \sim \tau} \quad \text{C\_REFL}$$

$$\frac{\Gamma \vdash \gamma : \tau_2 \sim \tau_1}{\Gamma \vdash \mathbf{sym} \gamma : \tau_1 \sim \tau_2} \quad \text{C\_SYM}$$

$$\frac{\begin{array}{l} \Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2 \\ \tau_2 \equiv \tau'_2 \\ \Gamma \vdash \gamma_2 : \tau'_2 \sim \tau_3 \end{array}}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \quad \text{C\_COMP}$$

$$\frac{\begin{array}{l} \Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \\ \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \\ \Gamma \vdash \gamma_3 : \tau_3 \sim \tau'_3 \\ \Gamma \vdash \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 : * \end{array}}{\Gamma \vdash \{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3 : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \sim \{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3} \quad \text{C\_CARROW}$$

$$\frac{\Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash \lambda(\alpha : \kappa), \gamma : \lambda(\alpha : \kappa), \tau_1 \sim \lambda(\alpha : \kappa), \tau_2} \quad \text{C\_ABS}$$

$$\frac{\begin{array}{l} \Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2 \\ \Gamma \vdash \forall (\alpha : \kappa), \tau_1 : * \end{array}}{\Gamma \vdash \forall (\alpha : \kappa), \gamma : \forall (\alpha : \kappa), \tau_1 \sim \forall (\alpha : \kappa), \tau_2} \quad \text{C\_FORALL}$$

$$\frac{\begin{array}{l} \Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \\ \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \\ \Gamma \vdash \tau_1 \tau_2 : \kappa \end{array}}{\Gamma \vdash \gamma_1 \gamma_2 : \tau_1 \tau_2 \sim \tau'_1 \tau'_2} \quad \text{C\_APP}$$

$$\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau'_1 \rightarrow \tau'_2}{\Gamma \vdash \mathbf{left} \gamma : \tau_1 \sim \tau'_1} \quad \text{C\_LEFT1}$$

$$\begin{array}{c}
\frac{\Gamma \vdash \gamma : \tau_1 \tau_2 \sim \tau'_1 \tau'_2}{\Gamma \vdash \mathbf{left} \gamma : \tau_1 \sim \tau'_1} \quad \text{C\_LEFT2} \\
\frac{\Gamma \vdash \gamma : \tau_1 \rightarrow \tau_2 \sim \tau'_1 \rightarrow \tau'_2}{\Gamma \vdash \mathbf{right} \gamma : \tau_2 \sim \tau'_2} \quad \text{C\_RIGHT1} \\
\frac{\Gamma \vdash \gamma : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \sim \{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3}{\Gamma \vdash \mathbf{right} \gamma : \tau_3 \sim \tau'_3} \quad \text{C\_RIGHT2} \\
\frac{\Gamma \vdash \gamma : \tau_1 \tau_2 \sim \tau'_1 \tau'_2}{\Gamma \vdash \mathbf{right} \gamma : \tau_2 \sim \tau'_2} \quad \text{C\_RIGHT3}
\end{array}$$

$\boxed{\Gamma \vdash \tau : \kappa}$     Kinding

$$\begin{array}{c}
\frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \quad \text{K\_VAR} \\
\frac{T : \kappa \in \Gamma}{\Gamma \vdash T : \kappa} \quad \text{K\_ABSTYPE} \\
\frac{\Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash \lambda(\alpha : \kappa_1), \tau : \kappa_1 \rightarrow \kappa_2} \quad \text{K\_ABS} \\
\frac{\Gamma \vdash \tau_1 : \kappa_2 \rightarrow \kappa_1 \quad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \tau_2 : \kappa_1} \quad \text{K\_APP} \\
\frac{\Gamma \vdash \tau_1 : \kappa \quad \Gamma \vdash \tau_2 : \kappa \quad \Gamma \vdash \tau_3 : *}{\Gamma \vdash \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 : *} \quad \text{K\_CARROW} \\
\frac{\Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash \forall(\alpha : \kappa), \tau : *} \quad \text{K\_FORALL}
\end{array}$$

$\boxed{\tau_1 \equiv \tau_2}$     Type equivalence

$$\begin{array}{c}
\frac{}{\tau \equiv \tau} \quad \text{EQ\_REFL} \\
\frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \quad \text{EQ\_SYMM} \\
\frac{\tau_1 \equiv \tau_2 \quad \tau_2 \equiv \tau_3}{\tau_1 \equiv \tau_3} \quad \text{EQ\_TRANS} \\
\frac{}{\alpha \equiv \alpha} \quad \text{EQ\_VAR} \\
\frac{}{T \equiv T} \quad \text{EQ\_ABSTYPE} \\
\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2 \quad \tau_3 \equiv \tau'_3}{\{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \equiv \{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3} \quad \text{EQ\_CARROW} \\
\frac{\tau_1 \equiv \tau_2}{\forall(\alpha : \kappa), \tau_1 \equiv \forall(\alpha : \kappa), \tau_2} \quad \text{EQ\_FORALL}
\end{array}$$

$$\frac{\tau_1 \equiv \tau_2}{\lambda(\alpha : \kappa), \tau_1 \equiv \lambda(\alpha : \kappa), \tau_2} \quad \text{EQ\_ABS}$$

$$\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2}{\tau_1 \tau_2 \equiv \tau'_1 \tau'_2} \quad \text{EQ\_APP}$$

$$\frac{[\alpha \mapsto \tau_2] \tau_1 \triangleright \tau'_1}{(\lambda(\alpha : \kappa), \tau_1) \tau_2 \equiv \tau'_1} \quad \text{EQ\_APPAbs}$$

$$\boxed{[x \mapsto v] t_1 \triangleright t_2} \quad \text{substitution}$$

$$\frac{}{[x \mapsto v] x \triangleright v} \quad \text{SUBST\_VAR1}$$

$$\frac{}{[x_1 \mapsto v] x_2 \triangleright x_2} \quad \text{SUBST\_VAR2}$$

$$\frac{}{[x \mapsto v] \lambda(x : \tau) \Rightarrow t \triangleright \lambda(x : \tau) \Rightarrow t} \quad \text{SUBST\_ABS1}$$

$$\frac{[x_1 \mapsto v] t_1 \triangleright t_2}{[x_1 \mapsto v] \lambda(x_2 : \tau) \Rightarrow t_1 \triangleright \lambda(x_2 : \tau) \Rightarrow t_2} \quad \text{SUBST\_ABS2}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] \lambda\{\alpha : \kappa\} \Rightarrow t_1 \triangleright \lambda\{\alpha : \kappa\} \Rightarrow t_2} \quad \text{SUBST\_TABS}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_1 \triangleright \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_2} \quad \text{SUBST\_CABS}$$

$$\frac{[x \mapsto v] t_1 \triangleright t'_1 \quad [x \mapsto v] t_2 \triangleright t'_2}{[x \mapsto v] t_1 t_2 \triangleright t'_1 t'_2} \quad \text{SUBST\_APP}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 [\tau] \triangleright t_2 [\tau]} \quad \text{SUBST\_TAPP}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 \sim [\gamma] \triangleright t_2 \sim [\gamma]} \quad \text{SUBST\_CAPP}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 : \tau \triangleright t_2 : \tau} \quad \text{SUBST\_ANNOT}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 \blacktriangleright \gamma \triangleright t_2 \blacktriangleright \gamma} \quad \text{SUBST\_COERCE}$$

$$\boxed{t_1 \longrightarrow t_2} \quad \text{Evaluation}$$

$$\frac{t_2 \longrightarrow t'_2}{t_1 t_2 \longrightarrow t_1 t'_2} \quad \text{E\_APP1}$$

$$\frac{t_1 \longrightarrow t'_1}{t_1 v \longrightarrow t'_1 v} \quad \text{E\_APP2}$$

$$\frac{[x \mapsto v] t \triangleright t' \quad t' \longrightarrow t''}{(\lambda(x : \tau) \Rightarrow t) v \longrightarrow t''} \quad \text{E\_APPAbs}$$

$$\frac{t \longrightarrow t'}{\lambda\{\alpha : \kappa\} \Rightarrow t \longrightarrow t'} \quad \text{E\_TABS}$$

$$\begin{array}{c}
\frac{t \longrightarrow t'}{\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t \longrightarrow t'} \quad \text{E\_CABS} \\
\\
\frac{t \longrightarrow t'}{t [\tau] \longrightarrow t'} \quad \text{E\_TAPP} \\
\\
\frac{t \longrightarrow t'}{t \sim[\gamma] \longrightarrow t'} \quad \text{E\_CAPP} \\
\\
\frac{t \longrightarrow t'}{t : \tau \longrightarrow t'} \quad \text{E\_ANNOT} \\
\\
\frac{t \longrightarrow t'}{t \blacktriangleright \gamma \longrightarrow t'} \quad \text{E\_COERCE}
\end{array}$$

Definition rules: 67 good 0 bad  
 Definition rule clauses: 156 good 0 bad