

|          |  |  |
|----------|--|--|
| $x$      | term variable                              |  |
| $\alpha$ | type variable                              |  |
| $T$      | abstract type                              |  |
| $t$      | $::=$                                      | term   |
|          | $x$  | variable   |
|          | $\lambda(x : \tau) \Rightarrow t$          | abstraction  |
|          | $\lambda\{\alpha : \kappa\} \Rightarrow t$ | type abstraction   |
|          | $t_1 t_2$                                  | application  |
|          | $t [\tau]$                                 | type application   |
|          | $t : \tau$                                 | type annotation  |
|          | $(t)$                                      | S parenthesis  |
| $v$      | $::=$                                      | value  |
|          | $\lambda(x : \tau) \Rightarrow t$          | abstraction  |
| $\kappa$ | $::=$                                      | kind   |
|          | $*$  | star   |
|          | $\kappa_1 \rightarrow \kappa_2$            | kind arrow   |
|          | $(\kappa)$                                 | S parenthesis  |
| $\tau$   | $::=$                                      | type   |
|          | $\alpha$                                   | type variable  |
|          | $T$  | abstract type  |
|          | $\tau_1 \rightarrow \tau_2$                | S $\equiv (\rightarrow) \tau_1 \tau_2$ where $(\rightarrow) : * \rightarrow * \rightarrow *$ |
|          | $\lambda(\alpha : \kappa), \tau$           | operator abstraction   |
|          | $\forall(\alpha : \kappa), \tau$           | universal quantification   |
|          | $\tau_1 \tau_2$                            | operator application   |
|          | $(\tau)$                                   | S parenthesis  |
| $\Gamma$ | $::=$                                      | typing environment   |
|          | $\emptyset$                                | empty  |
|          | $\Gamma, x : \tau$                         | variable   |
|          | $\Gamma, T : \kappa$                       | abstract type  |
|          | $\Gamma, \alpha : \kappa$                  | type variable  |

$[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau_3$     Type substitution

|   |                |
|---|----------------|
| $\frac{}{[\alpha \mapsto \tau] \alpha \triangleright \tau}$   | SUBSTT_VAR1    |
| $\frac{}{[\alpha_1 \mapsto \tau] \alpha_2 \triangleright \alpha_2}$   | SUBSTT_VAR2    |
| $\frac{}{[\alpha \mapsto \tau] T \triangleright T}$   | SUBSTT_TYPE    |
| $\frac{}{[\alpha \mapsto \tau_1] \lambda(\alpha : \kappa), \tau_2 \triangleright \lambda(\alpha : \kappa), \tau_2}$   | SUBSTT_ABS1    |
| $\frac{[\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1] \lambda(\alpha_2 : \kappa), \tau_2 \triangleright \lambda(\alpha_2 : \kappa), \tau'_2}$ | SUBSTT_ABS2    |
| $\frac{}{[\alpha \mapsto \tau_1] \forall(\alpha : \kappa), \tau_2 \triangleright \forall(\alpha : \kappa), \tau_2}$   | SUBSTT_FORALL1 |
| $\frac{[\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1] \forall(\alpha_2 : \kappa), \tau_2 \triangleright \forall(\alpha_2 : \kappa), \tau'_2}$ | SUBSTT_FORALL2 |

$$\frac{\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1] \tau_2 \tau_3 \triangleright \tau'_2 \tau'_3}}{\text{SUBST\_APP}}$$

$\boxed{\Gamma \vdash t : \tau}$  Typing rules

$$\begin{array}{c} \frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T\_VAR} \\[10pt] \frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash \lambda(x : \tau_1) \Rightarrow t : \tau_1 \rightarrow \tau_2} \quad \text{T\_ABS} \\[10pt] \frac{\Gamma, \alpha : \kappa \vdash t : \tau \quad \alpha \notin \Gamma}{\Gamma \vdash \lambda\{\alpha : \kappa\} \Rightarrow t : \forall(\alpha : \kappa), \tau} \quad \text{T\_TYABS} \\[10pt] \frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \tau_2 \equiv \tau'_2 \quad \Gamma \vdash t_2 : \tau'_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T\_APP} \\[10pt] \frac{\Gamma \vdash t : \forall(\alpha : \kappa), \tau_2 \quad \Gamma \vdash \tau_1 : \kappa \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T\_TYAPP} \\[10pt] \frac{\Gamma \vdash t : \tau_2 \quad \tau_1 \equiv \tau_2}{\Gamma \vdash (t : \tau_1) : \tau_1} \quad \text{T\_ANNOT}$$

$\boxed{\Gamma \vdash \tau : \kappa}$  Kinding

$$\begin{array}{c} \frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \quad \text{K\_VAR} \\[10pt] \frac{T : \kappa \in \Gamma}{\Gamma \vdash T : \kappa} \quad \text{K\_ABSTYPE} \\[10pt] \frac{\Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash \lambda(\alpha : \kappa_1), \tau : \kappa_1 \rightarrow \kappa_2} \quad \text{K\_ABS} \\[10pt] \frac{\Gamma \vdash \tau_1 : \kappa_2 \rightarrow \kappa_1 \quad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \tau_2 : \kappa_1} \quad \text{K\_APP} \\[10pt] \frac{\Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash \forall(\alpha : \kappa), \tau : *} \quad \text{K\_FORALL}\end{array}$$

$\boxed{\tau_1 \equiv \tau_2}$  Type equivalence

$$\begin{array}{c} \frac{}{\tau \equiv \tau} \quad \text{EQ\_REFL} \\[10pt] \frac{\tau_2 \equiv \tau_1}{\tau_1 \equiv \tau_2} \quad \text{EQ\_SYMM} \\[10pt] \frac{\tau_1 \equiv \tau_2 \quad \tau_2 \equiv \tau_3}{\tau_1 \equiv \tau_3} \quad \text{EQ\_TRANS}\end{array}$$

$$\frac{}{\alpha \equiv \alpha} \text{EQ\_VAR}$$

$$\frac{}{T \equiv T} \text{EQ\_ABSTYPE}$$

$$\frac{\tau_1 \equiv \tau_2}{\forall (\alpha : \kappa), \tau_1 \equiv \forall (\alpha : \kappa), \tau_2} \text{EQ\_FORALL}$$

$$\frac{\tau_1 \equiv \tau_2}{\lambda(\alpha : \kappa), \tau_1 \equiv \lambda(\alpha : \kappa), \tau_2} \text{EQ\_ABS}$$

$$\frac{\tau_1 \equiv \tau'_1 \quad \tau_2 \equiv \tau'_2}{\tau_1 \tau_2 \equiv \tau'_1 \tau'_2} \text{EQ\_APP}$$

$$\frac{[\alpha \mapsto \tau_2] \tau_1 \triangleright \tau'_1}{(\lambda(\alpha : \kappa), \tau_1) \tau_2 \equiv \tau'_1} \text{EQ\_APPAbs}$$

$$\boxed{[x \mapsto v] t_1 \triangleright t_2} \quad \text{substitution}$$

$$\frac{}{[x \mapsto v] x \triangleright v} \text{SUBST\_VAR1}$$

$$\frac{}{[x_1 \mapsto v] x_2 \triangleright x_2} \text{SUBST\_VAR2}$$

$$\frac{}{[x \mapsto v] \lambda(x : \tau) \Rightarrow t \triangleright \lambda(x : \tau) \Rightarrow t} \text{SUBST\_ABS1}$$

$$\frac{[x_1 \mapsto v] t_1 \triangleright t_2}{[x_1 \mapsto v] \lambda(x_2 : \tau) \Rightarrow t_1 \triangleright \lambda(x_2 : \tau) \Rightarrow t_2} \text{SUBST\_ABS2}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] \lambda\{\alpha : \kappa\} \Rightarrow t_1 \triangleright \lambda\{\alpha : \kappa\} \Rightarrow t_2} \text{SUBST\_TABS}$$

$$\frac{[x \mapsto v] t_1 \triangleright t'_1 \quad [x \mapsto v] t_2 \triangleright t'_2}{[x \mapsto v] t_1 t_2 \triangleright t'_1 t'_2} \text{SUBST\_APP}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 [\tau] \triangleright t_2 [\tau]} \text{SUBST\_TAPP}$$

$$\frac{[x \mapsto v] t_1 \triangleright t_2}{[x \mapsto v] t_1 : \tau \triangleright t_2 : \tau} \text{SUBST\_ANNOT}$$

$$\boxed{t_1 \longrightarrow t_2} \quad \text{Evaluation}$$

$$\frac{t_2 \longrightarrow t'_2}{t_1 t_2 \longrightarrow t_1 t'_2} \text{E\_APP1}$$

$$\frac{t_1 \longrightarrow t'_1}{t_1 v \longrightarrow t'_1 v} \text{E\_APP2}$$

$$\frac{[x \mapsto v] t \triangleright t' \quad t' \longrightarrow t''}{(\lambda(x : \tau) \Rightarrow t) v \longrightarrow t''} \text{E\_APPAbs}$$

$$\frac{t \longrightarrow t'}{\lambda\{\alpha : \kappa\} \Rightarrow t \longrightarrow t'} \text{E\_TABS}$$

$$\frac{t \longrightarrow t'}{t \ [\tau] \longrightarrow t'} \quad \text{E\_TAPP}$$

$$\frac{t \longrightarrow t'}{t : \tau \longrightarrow t'} \quad \text{E\_ANNOT}$$

Definition rules: 42 good 0 bad  
Definition rule clauses: 86 good 0 bad