```
term variable
\boldsymbol{x}
      type variable
\alpha
T
      type constructor
      coercion variable
c
t
           ::=
                                                        _{\text{term}}
                                                            variable
                   \lambda(x:\tau) \Rightarrow t
                                                            abstraction
                   \lambda\{\alpha:\kappa\}\Rightarrow t
                                                            type abstraction
                   \lambda\{c: \tau_1 \sim \tau_2\} \Rightarrow t
                                                            coercion abstraction
                                                            application
                    t [\tau]
                                                            type application
                    t \sim [\gamma]
                                                            coercion application
                                                            type annotation
                                                            coercion
                                                        kind
\kappa
                                                            star
                                                            kind arrow
                   \kappa_1 \to \kappa_2
                                                        type
                                                            type variable
                                                            type constructor
                                                            \equiv (\rightarrow) \ \tau_1 \ \tau_2
                    \{\tau_1 \sim \tau_2\} \to \tau_3  \lambda(\alpha:\kappa), \tau 
                                                            coercion arrow
                                                            operator abstraction
                   \forall (\alpha : \kappa), \tau
                                                            universal quantification
                                                            operator application
                                                        coercion proof term
           ::=
                                                            variable
                   \operatorname{\mathbf{refl}} \tau
                                                            reflexivity
                                                            symmetry
                   \operatorname{sym} \gamma
                                                            composition
                   \gamma_1 \circ \gamma_2
                                                             \equiv (\rightarrow) \gamma_1 \gamma_2 \text{ where } (\rightarrow) : \text{refl } (\rightarrow)
                    \gamma_1 \rightarrow \gamma_2
                   \{\gamma_1 \sim \gamma_2\} \to \gamma_3
                                                            coercion arrow introduction
                   \lambda(\alpha:\kappa),\gamma
                                                            operator abstraction introduction
                   \forall (\alpha : \kappa), \gamma
                                                            universal quantification introduction
                                                            application introduction
                   \gamma_1 \gamma_2
                   left \gamma
                                                            left elimination
                   \mathbf{right} \, \gamma
                                                            right elimination
Γ
                                                         typing environment
                   Ø
                                                            empty
                   \Gamma, x : \tau
                                                            variable
                   \Gamma, T : \kappa
                                                            type constructor
                                                            type variable
                                                            coercion variable
                                                        typed value
tv
                   \lambda(x:\tau) \Rightarrow t
\lambda\{\alpha:\kappa\} \Rightarrow tv
                                                            abstraction
```

type abstraction

Initial environment:  $\Gamma = \emptyset,$   $(\rightarrow): * \rightarrow * \rightarrow *$ 

# $\boxed{ [\alpha \mapsto \tau_1] \ \tau_2 \triangleright \tau_3 } \quad \text{Type substitution}$

#### $|\Gamma \vdash t : \tau|$ Typing rules

$$\frac{x:\tau\in\Gamma}{\Gamma\vdash x:\tau}\quad \text{T-VAR}$$

$$\frac{\Gamma,x:\tau_1\vdash t:\tau_2}{\Gamma\vdash\tau_1:*}$$

$$\frac{\Gamma\vdash(\lambda(x:\tau_1)\Rightarrow t):\tau_1\to\tau_2}{\Gamma\vdash(\lambda(x:\tau_1)\Rightarrow t):\tau_1\to\tau_2}\quad \text{T-Abs}$$

$$\frac{\Gamma,\alpha:\kappa\vdash t:\tau}{\alpha\notin\Gamma}$$

$$\frac{\alpha\notin\Gamma}{\Gamma\vdash(\lambda\{\alpha:\kappa\}\Rightarrow t):\forall\,(\alpha:\kappa),\tau}\quad \text{T-TYABS}$$

$$\frac{\Gamma,c:\tau_1\sim\tau_2\vdash t:\tau_3}{\Gamma\vdash\tau_1:\kappa}$$

$$\frac{\Gamma\vdash\tau_1:\kappa}{\Gamma\vdash\tau_2:\kappa}$$

$$\frac{\Gamma\vdash\lambda\{c:\tau_1\sim\tau_2\}\Rightarrow t:\{\tau_1\sim\tau_2\}\to\tau_3}{\Gamma\vdash\lambda\{c:\tau_1\sim\tau_2\}\Rightarrow t:\{\tau_1\sim\tau_2\}\to\tau_3}\quad \text{T-CABS}$$

$$\begin{array}{c} \Gamma \vdash t_1 : \tau_2 \to \tau_1 \\ \tau_2 \equiv \tau_2' \\ \hline \Gamma \vdash t_2 : \tau_2' \\ \hline \Gamma \vdash t_1 t_2 : \tau_1 \end{array} \quad \text{T-APP} \\ \hline \Gamma \vdash t : \forall (\alpha : \kappa), \tau_2 \\ \hline \Gamma \vdash \tau_1 : \kappa \\ \hline \left[ \alpha \mapsto \tau_1 \right] \tau_2 \rhd \tau_2' \\ \hline \Gamma \vdash t : \left[ \tau_1 \right] : \tau_2' \end{array} \quad \text{T-TYAPP} \\ \hline \Gamma \vdash t : \left\{ \tau_1 \sim \tau_2 \right\} \to \tau_3 \\ \hline \Gamma \vdash \gamma : \tau_1' \sim \tau_2' \\ \hline \tau_1 \equiv \tau_1' \\ \hline \tau_2 \equiv \tau_2' \\ \hline \hline \Gamma \vdash t : \tau_2 \\ \hline \Gamma \vdash (t : \tau_1) : \tau_1 \end{array} \quad \text{T-CAPP} \\ \hline \Gamma \vdash \gamma : \tau_1 \sim \tau_2 \\ \hline \Gamma \vdash t : \tau_1' \\ \hline \Gamma \vdash t : \tau_1' \\ \hline \Gamma \vdash t : \tau_1' \\ \hline \Gamma \vdash (t \triangleright \gamma) : \tau_2 \end{array} \quad \text{T-COERCE} \\ \hline \end{array}$$

## $\Gamma \vdash \gamma : \tau_1 \sim \tau_2$ Coercion typing

$$\frac{c:\tau_1 \sim \tau_2 \in \Gamma}{\Gamma \vdash c:\tau_1 \sim \tau_2} \quad \text{C_VAR}$$

$$\frac{\Gamma \vdash \text{reff } \tau : \tau \sim \tau}{\Gamma \vdash \text{reff } \tau : \tau \sim \tau} \quad \text{C_Refl}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2}{\Gamma \vdash \text{sym} \gamma : \tau_1 \sim \tau_2} \quad \text{C_SYM}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2}{\Gamma \vdash \gamma_2 : \tau_2' \sim \tau_3} \quad \text{C_COMP}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \gamma_2 : \tau_2 \sim \tau_2'} \quad \text{C_TOMP}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \gamma_2 : \tau_2 \sim \tau_2'} \quad \text{C_TOMP}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2} \quad \text{C_TOMP}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \{\tau_1 \sim \tau_2\} \to \tau_3 : *} \quad \text{C_TOMP}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2}{\Gamma \vdash \{\tau_1 \sim \tau_2\} \to \tau_3 \sim \{\tau_1' \sim \tau_2'\} \to \tau_3'} \quad \text{C_TOARROW}$$

$$\frac{\Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash \lambda(\alpha : \kappa), \gamma : \lambda(\alpha : \kappa), \tau_1 \sim \lambda(\alpha : \kappa), \tau_2} \quad \text{C_TORALL}$$

$$\frac{\Gamma, \alpha : \kappa \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash \forall (\alpha : \kappa), \gamma : \forall (\alpha : \kappa), \tau_1 \sim \forall (\alpha : \kappa), \tau_2} \quad \text{C_TORALL}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \gamma_2 : \tau_2 \sim \tau_2'} \quad \text{C_TORALL}$$

$$\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_1'}{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2'} \quad \text{C_TORALL}$$

$$\frac{\Gamma \vdash \gamma : \tau_{1} \rightarrow \tau_{2} \sim \tau_{1}' \rightarrow \tau_{2}'}{\Gamma \vdash \mathbf{left} \gamma : \tau_{1} \sim \tau_{1}'} \quad \text{C-Left1}$$

$$\frac{\Gamma \vdash \gamma : \tau_{1} \tau_{2} \sim \tau_{1}' \tau_{2}'}{\Gamma \vdash \mathbf{left} \gamma : \tau_{1} \sim \tau_{1}'} \quad \text{C-Left2}$$

$$\frac{\Gamma \vdash \gamma : \tau_{1} \rightarrow \tau_{2} \sim \tau_{1}' \rightarrow \tau_{2}'}{\Gamma \vdash \mathbf{right} \gamma : \tau_{2} \sim \tau_{2}'} \quad \text{C-Right1}$$

$$\frac{\Gamma \vdash \gamma : \{\tau_{1} \sim \tau_{2}\} \rightarrow \tau_{3} \sim \{\tau_{1}' \sim \tau_{2}'\} \rightarrow \tau_{3}'}{\Gamma \vdash \mathbf{right} \gamma : \tau_{3} \sim \tau_{3}'} \quad \text{C-Right2}$$

$$\frac{\Gamma \vdash \gamma : \tau_{1} \tau_{2} \sim \tau_{1}' \tau_{2}'}{\Gamma \vdash \mathbf{right} \gamma : \tau_{2} \sim \tau_{2}'} \quad \text{C-Right3}$$

### $\Gamma \vdash \tau : \kappa$ Kinding rules

$$\frac{\alpha:\kappa\in\Gamma}{\Gamma\vdash\alpha:\kappa}\quad \text{K_VAR}$$
 
$$\frac{T:\kappa\in\Gamma}{\Gamma\vdash T:\kappa}\quad \text{K_TYPECONSTR}$$
 
$$\frac{\Gamma,\alpha:\kappa_1\vdash\tau:\kappa_2}{\Gamma\vdash(\lambda(\alpha:\kappa_1),\tau):\kappa_1\to\kappa_2}\quad \text{K_ABS}$$
 
$$\frac{\Gamma\vdash\tau_1:\kappa_2\to\kappa_1}{\Gamma\vdash\tau_2:\kappa_2}\quad \text{K_APP}$$
 
$$\frac{\Gamma\vdash\tau_1:\kappa}{\Gamma\vdash\tau_1:\kappa}\quad \text{K_APP}$$
 
$$\frac{\Gamma\vdash\tau_1:\kappa}{\Gamma\vdash\tau_2:\kappa}\quad \text{K_APP}$$
 
$$\frac{\Gamma\vdash\tau_1:\kappa}{\Gamma\vdash\tau_2:\kappa}\quad \text{K_CARROW}$$
 
$$\frac{\Gamma\vdash\tau_3:*}{\Gamma\vdash(\{\tau_1\sim\tau_2\}\to\tau_3):*}\quad \text{K_CARROW}$$
 
$$\frac{\Gamma,\alpha:\kappa\vdash\tau:*}{\Gamma\vdash(\forall(\alpha:\kappa),\tau):*}\quad \text{K_FORALL}$$

#### $\tau_1 \equiv \tau_2$ Type equivalence

$$\frac{\tau_{1} \equiv \tau_{2}}{\forall (\alpha : \kappa), \tau_{1} \equiv \forall (\alpha : \kappa), \tau_{2}} \quad \text{EQ\_FORALL}$$

$$\frac{\tau_{1} \equiv \tau_{2}}{\lambda(\alpha : \kappa), \tau_{1} \equiv \lambda(\alpha : \kappa), \tau_{2}} \quad \text{EQ\_ABS}$$

$$\frac{\tau_{1} \equiv \tau'_{1}}{\tau_{2} \equiv \tau'_{2}}$$

$$\frac{\tau_{2} \equiv \tau'_{2}}{\tau_{1} \tau_{2} \equiv \tau'_{1} \tau'_{2}} \quad \text{EQ\_APP}$$

$$\frac{[\alpha \mapsto \tau_{2}] \ \tau_{1} \rhd \tau'_{1}}{(\lambda(\alpha : \kappa), \tau_{1}) \ \tau_{2} \equiv \tau'_{1}} \quad \text{EQ\_APPABS}$$

 $[x \mapsto tv] \ t_1 \rhd t_2$  substitution

 $[\alpha \mapsto \tau] \ t_1 \rhd t_2$ 

substitution of type variable in term

$$\frac{\left[\alpha \mapsto \tau\right] \; x \rhd x}{\left[\alpha \mapsto \tau_1\right] \; \tau_2 \rhd \tau_2'}$$
 
$$\frac{\left[\alpha \mapsto \tau_1\right] \; t_1 \rhd t_2}{\left[\alpha \mapsto \tau_1\right] \; \left(\lambda(x : \tau_2) \Rightarrow t_1\right) \rhd \left(\lambda(x : \tau_2') \Rightarrow t_2\right)} \quad \text{TtSubst\_Abs}$$

$$[\alpha_1 \mapsto \tau] \ t_1 \mapsto t_2$$

$$[\alpha_1 \mapsto \tau] \ (\lambda\{\alpha_2 : \kappa\} \Rightarrow t_1\} \triangleright (\lambda\{\alpha_2 : \kappa\} \Rightarrow t_2)$$

$$[\alpha \mapsto \tau_1] \ \tau_2 \triangleright \tau_2'$$

$$[\alpha \mapsto \tau_1] \ \tau_3 \triangleright \tau_3'$$

$$[\alpha \mapsto \tau_1] \ t_1 \triangleright t_2$$

$$[\alpha \mapsto \tau_1] \ t_1 \triangleright t_2$$

$$[\alpha \mapsto \tau_1] \ t_1 \triangleright t_2'$$

$$[\alpha \mapsto \tau_1] \ t_1 \triangleright t_2 \land \tau_2'$$

$$[\alpha \mapsto \tau_1] \ t_1 \vdash t_2 \land \tau_2'$$

$$[\alpha \mapsto \tau_1] \ t_1 \vdash t_2 \land \tau_2'$$

$$[\alpha \mapsto \tau_1] \ t_1 \vdash t_2 \land \tau_2'$$

$$[\alpha \mapsto \tau_1] \ t_1 \triangleright t_2$$

$$[\alpha \mapsto \tau_1] \ t_2 \triangleright t_2'$$

$$[\alpha \mapsto \tau_1] \ t_2 \vdash \tau_2'$$

$$[\alpha \mapsto \tau_1] \ t_2 \vdash \tau$$

## <<no parses (char 6): [c\*\*\* |-> C] @ t |> t' >>

$$t' \longrightarrow t \iota$$

$$\frac{(\lambda\{c:\tau_1 \sim \tau_2\} \Rightarrow t) \sim [\gamma] \longrightarrow tv}{t \longrightarrow tv}$$

$$\frac{t \longrightarrow tv}{(t:\tau) \longrightarrow (tv:\tau)} \quad \text{E\_Annot}$$

$$\frac{t \longrightarrow tv}{(t \blacktriangleright \gamma) \longrightarrow (tv \blacktriangleright \gamma)} \quad \text{E\_COERCE}$$

 $tv \longrightarrow v$  type erasure

Definition rules: 80 good 3 bad Definition rule clauses: 200 good 3 bad