

$x$	term variable	
$\alpha$	type variable	
$i, n$	index variables	
$t$	$::=$	term
	$x$	variable
	$\lambda(x : \tau) \Rightarrow t$	abstraction
	$\lambda\{\alpha : \kappa\} \Rightarrow t$	type abstraction
	$t_1 t_2$	application
	$t [\tau]$	type application
$\kappa$	$::=$	kind
	$*$	star
	$!$	effect
	$\kappa_1 \rightarrow \kappa_2$	kind arrow
$\tau$	$::=$	type
	$\alpha$	type variable
	$[\varphi]$	effects type
	$\tau_1 \multimap [\varphi] \tau_2$	$\equiv (\rightarrow) \tau_1 [\varphi] \tau_2$
	$\forall(\alpha : \kappa), \tau$	universal quantification
	$\lambda(\alpha : \kappa), \tau$	operator abstraction
	$\tau_1 \tau_2$	operator application
$\varphi$	$::=$	effects
	$\tau_1, \dots, \tau_n$	effects
$\Gamma$	$::=$	typing environment
	$\emptyset$	empty
	$\Gamma, x : \tau$	variable
	$\Gamma, \alpha : \kappa$	type variable
$v$	$::=$	value
	$\lambda(x : \tau) \Rightarrow t$	abstraction

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Initial environment:  $\Gamma = \emptyset,$   
 $(\rightarrow) : * \rightarrow ! \rightarrow * \rightarrow *$

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$\Gamma \vdash t : [[\varphi]] \tau$     Typing rules

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : [[]] \tau} \quad \text{T\_VAR} \\
\\
\frac{\Gamma, x : \tau_1 \vdash t : [[\varphi]] \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : [[]] \tau_1 \multimap [\varphi] \tau_2} \quad \text{T\_ABS} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : \kappa \vdash t : [[]] \tau}{\Gamma \vdash (\lambda\{\alpha : \kappa\} \Rightarrow t) : [[]] \forall(\alpha : \kappa), \tau} \quad \text{T\_TYABS} \\
\\
\frac{\Gamma \vdash t_1 : [[\varphi_1]] \tau_2 \multimap [\varphi_3] \tau_1 \quad \tau'_2 \prec \tau_2 \quad \Gamma \vdash t_2 : [[\varphi_2]] \tau'_2}{\Gamma \vdash t_1 t_2 : [[\varphi_1 \cup \varphi_2 \cup \varphi_3]] \tau_1} \quad \text{T\_APP}
\end{array}$$

$$\frac{\Gamma \vdash t : [[\varphi]] \quad \forall (\alpha : \kappa), \tau_2 \quad \Gamma \vdash \tau_1 : \kappa \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : [[\varphi]] \quad \tau'_2} \quad \text{T\_TYAPP}$$

$\boxed{\Gamma \vdash \tau : \kappa}$     Kinding rules

$$\begin{array}{c} \frac{\alpha : \kappa \in \Gamma}{\Gamma \vdash \alpha : \kappa} \quad \text{K\_VAR} \\[10pt] \frac{\Gamma \vdash \tau_1 : ! \quad \dots \quad \Gamma \vdash \tau_n : !}{\Gamma \vdash [\tau_1, \dots, \tau_n] : !} \quad \text{K\_EFF} \\[10pt] \frac{\alpha \notin \Gamma \quad \Gamma, \alpha : \kappa_1 \vdash \tau : \kappa_2}{\Gamma \vdash (\lambda(\alpha : \kappa_1), \tau) : \kappa_1 \rightarrow \kappa_2} \quad \text{K\_ABS} \\[10pt] \frac{\Gamma \vdash \tau_1 : \kappa_2 \rightarrow \kappa_1 \quad \Gamma \vdash \tau_2 : \kappa_2}{\Gamma \vdash \tau_1 \tau_2 : \kappa_1} \quad \text{K\_APP} \\[10pt] \frac{\alpha \notin \Gamma \quad \Gamma, \alpha : \kappa \vdash \tau : *}{\Gamma \vdash (\forall (\alpha : \kappa), \tau) : *} \quad \text{K\_FORALL} \end{array}$$

$\boxed{\tau_1 \prec \tau_2}$     Subtyping relation

$$\begin{array}{c} \frac{}{\tau \prec \tau} \quad \text{TSUB\_REFL} \\[10pt] \frac{\tau_1 \prec \tau_2 \quad \tau_2 \prec \tau_3}{\tau_1 \prec \tau_3} \quad \text{TSUB\_TRANS} \\[10pt] \frac{}{\alpha \prec \alpha} \quad \text{TSUB\_VAR} \\[10pt] \frac{\varphi_1 \subseteq \varphi_2}{[\varphi_1] \prec [\varphi_2]} \quad \text{TSUB\_EFF} \\[10pt] \frac{\tau_1 \prec \tau_2}{(\forall (\alpha : \kappa), \tau_1) \prec (\forall (\alpha : \kappa), \tau_2)} \quad \text{TSUB\_FORALL} \\[10pt] \frac{\tau_1 \prec \tau_2}{(\lambda(\alpha : \kappa), \tau_1) \prec (\lambda(\alpha : \kappa), \tau_2)} \quad \text{TSUB\_ABS} \\[10pt] \frac{\tau_1 \prec \tau'_1 \quad \tau'_2 \prec \tau_2}{\tau_1 \tau_2 \prec \tau'_1 \tau'_2} \quad \text{TSUB\_APP} \\[10pt] \frac{[\alpha \mapsto \tau_2] \tau_1 \triangleright \tau'_1}{(\lambda(\alpha : \kappa), \tau_1) \tau_2 \prec \tau'_1} \quad \text{TSUB\_APPABS} \end{array}$$

$\boxed{\varphi_1 \subseteq \varphi_2}$     Effects subset relation

$$\begin{array}{c} \frac{}{\emptyset \subseteq \varphi} \quad \text{ESUB\_EMPTY} \\[10pt] \frac{\exists (\tau_i \in \varphi_2), \tau \prec \tau_i \quad \varphi_1 \subseteq \varphi_2}{\tau, \varphi_1 \subseteq \varphi_2} \quad \text{ESUB\_EFF} \end{array}$$

$\boxed{t \longrightarrow t'}$     Operational semantics

$$\begin{array}{c} \frac{t_2 \longrightarrow t'_2}{t_1 t_2 \longrightarrow t_1 t'_2} \quad \text{E\_APP1} \\[10pt] \frac{t \longrightarrow t'}{t v \longrightarrow t' v} \quad \text{E\_APP2} \end{array}$$

$$\frac{[x \mapsto v]t \triangleright t'}{(\lambda(x : \tau) \Rightarrow t) v \longrightarrow t'} \quad \text{E\_APPABS}$$

$$\frac{t \longrightarrow t'}{(\lambda\{\alpha : \kappa\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : \kappa\} \Rightarrow t')} \quad \text{E\_TABS}$$

$$\frac{t \longrightarrow t'}{t [\tau] \longrightarrow t' [\tau]} \quad \text{E\_TAPP}$$

$$\frac{[\alpha \mapsto \tau]v \triangleright v'}{(\lambda\{\alpha : \kappa\} \Rightarrow v) [\tau] \longrightarrow v'} \quad \text{E\_TAPPABS}$$

$$\boxed{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3}$$

Type substitution

$$\overline{[\alpha \mapsto \tau]\alpha \triangleright \tau} \quad \text{SUBSTT\_VAR1}$$

$$\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau]\alpha_2 \triangleright \alpha_2} \quad \text{SUBSTT\_VAR2}$$

$$\frac{[\alpha \mapsto \tau]\varphi_1 \triangleright \varphi_2}{[\alpha \mapsto \tau][\varphi_1] \triangleright [\varphi_2]} \quad \text{SUBSTT\_EFF}$$

$$\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\lambda(\alpha_2 : \kappa), \tau_2) \triangleright (\lambda(\alpha_2 : \kappa), \tau'_2)} \quad \text{SUBSTT\_ABS}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1](\tau_2 \tau_3) \triangleright \tau'_2 \tau'_3} \quad \text{SUBSTT\_APP}$$

$$\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1](\forall(\alpha_2 : \kappa), \tau_2) \triangleright (\forall(\alpha_2 : \kappa), \tau'_2)} \quad \text{SUBSTT\_FORALL}$$

$$\boxed{[x \mapsto v]t_1 \triangleright t_2}$$

substitution

$$\overline{[x \mapsto v]x \triangleright v} \quad \text{SUBST\_VAR1}$$

$$\frac{x_1 \neq x_2}{[x_1 \mapsto v]x_2 \triangleright x_2} \quad \text{SUBST\_VAR2}$$

$$\overline{[x \mapsto v](\lambda(x : \tau) \Rightarrow t) \triangleright (\lambda(x : \tau) \Rightarrow t)} \quad \text{SUBST\_ABS1}$$

$$\frac{x_1 \neq x_2 \quad [x_1 \mapsto v]t_1 \triangleright t_2}{[x_1 \mapsto v](\lambda(x_2 : \tau) \Rightarrow t_1) \triangleright (\lambda(x_2 : \tau) \Rightarrow t_2)} \quad \text{SUBST\_ABS2}$$

$$\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](\lambda\{\alpha : \kappa\} \Rightarrow t_1) \triangleright (\lambda\{\alpha : \kappa\} \Rightarrow t_2)} \quad \text{SUBST\_TABS}$$

$$\frac{[x \mapsto v]t_1 \triangleright t'_1 \quad [x \mapsto v]t_2 \triangleright t'_2}{[x \mapsto v](t_1 t_2) \triangleright t'_1 t'_2} \quad \text{SUBST\_APP}$$

$$\frac{[x \mapsto v]t_1 \triangleright t_2}{[x \mapsto v](t_1 [\tau]) \triangleright (t_2 [\tau])} \quad \text{SUBST\_TAPP}$$

$$\boxed{[\alpha \mapsto \tau]t_1 \triangleright t_2}$$

substitution of type variable in term

$$\overline{[\alpha \mapsto \tau]x \triangleright x} \quad \text{TTSUBST\_VAR}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \triangleright (\lambda(x : \tau'_2) \Rightarrow t_2)} \quad \text{TtSUBST\_ABS}$$

$$\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]t_1 \triangleright t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : \kappa\} \Rightarrow t_1) \triangleright (\lambda\{\alpha_2 : \kappa\} \Rightarrow t_2)} \quad \text{TtSUBST\_TABS}$$

$$\frac{[\alpha \mapsto \tau]t_1 \triangleright t'_1 \quad [\alpha \mapsto \tau]t_2 \triangleright t'_2}{[\alpha \mapsto \tau](t_1 \ t_2) \triangleright t'_1 \ t'_2} \quad \text{TtSUBST\_APP}$$

$$\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 \ [\tau_2]) \triangleright (t_2 \ [\tau'_2])} \quad \text{TtSUBST\_TAPP}$$

$[\alpha \mapsto \tau]\varphi_1 \triangleright \varphi_2$  substitution of type variable in effects

$$\overline{[\alpha \mapsto \tau]\emptyset \triangleright \emptyset} \quad \text{ESUBST\_EMPTY}$$

$$\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\varphi \triangleright \varphi'}{[\alpha \mapsto \tau_1]\tau_2, \varphi \triangleright \tau'_2, \varphi'} \quad \text{ESUBST\_EFF}$$

Definition rules: 46 good 0 bad

Definition rule clauses: 84 good 0 bad