

<i>termvar</i> , $x$	term variable
<i>variant</i> , $V$	variant
<i>typvar</i> , $X$	type variable
<i>exc</i> , <i>Exc</i>	exception
$n$	
$m$	

<i>program</i>	$::=$     <i>top program</i>	program nothing a piece of the program
<i>top</i>	$::=$   <b>let</b> <i>x params</i> = <i>t</i>   <b>let rec</b> <i>x params</i> = <i>abs</i>   <b>type alias</b> <i>X</i> = <i>T</i>   <b>type</b> <i>X variantArgs</i> = <i>V</i> <sub>1</sub> <i>tyList</i> <sub>1</sub>   ..   <i>V</i> <sub><i>n</i></sub> <i>tyList</i> <sub><i>n</i></sub>   <b>exception</b> <i>Exc tyList</i>	toplevel construct let binding recursive let binding type alias variants exception
<i>variantArgs</i>	$::=$   ( <i>X</i> <sub>1</sub> : <i>K</i> <sub>1</sub> ) .. ( <i>X</i> <sub><i>n</i></sub> : <i>K</i> <sub><i>n</i></sub> )	
<i>tyList</i>	$::=$   <i>T</i> <sub>1</sub> .. <i>T</i> <sub><i>n</i></sub>	
<i>t</i>	$::=$   <i>x</i>   <i>V</i>   $\lambda(x : T) \rightarrow t$   $\lambda(X : K) \rightarrow t$   $\lambda params \rightarrow t$   <i>t t'</i>   <i>t</i> [ <i>T</i> ]   <b>let</b> <i>x params</i> = <i>t</i> <sub>1</sub> <b>in</b> <i>t</i> <sub>2</sub>   <b>let rec</b> <i>x params</i> = <i>abs</i> <b>in</b> <i>t</i>   <b>match</b> <i>t</i> <b>with</b> <i>p</i> <sub>1</sub> $\rightarrow$ <i>t</i> <sub>1</sub>   ..   <i>p</i> <sub><i>n</i></sub> $\rightarrow$ <i>t</i> <sub><i>n</i></sub> <b>end</b>   <i>t</i> : <i>annot</i>   <b>fail</b> [ <i>T</i> ] <i>Exc t</i> <sub>1</sub> .. <i>t</i> <sub><i>n</i></sub>   <b>try</b> <i>t</i> <b>with</b> <i>pe</i> <sub>1</sub> $\rightarrow$ <i>t</i> <sub>1</sub>   ..   <i>pe</i> <sub><i>n</i></sub> $\rightarrow$ <i>t</i> <sub><i>n</i></sub> <b>end</b>   <i>t</i> ; <i>t'</i>   ( <i>t</i> )   <b>failure</b> <i>exnval</i>   <b>TConstr</b> <i>V v</i> <sub>1</sub> .. <i>v</i> <sub><i>n</i></sub>	term variable type constructors abstraction type abstraction S application type application let binding recursive let binding pattern matching type annotation fail try S == let _ : Unit = t in t' S M M
<i>v</i>	$::=$   <b>TConstr</b> <i>V v</i> <sub>1</sub> .. <i>v</i> <sub><i>n</i></sub>   $\lambda(x : T) \rightarrow t$   $\lambda valueParams \rightarrow v$   <b>let rec</b> <i>x params</i> = <i>valAbs</i> <sub>1</sub> <b>in</b> <i>valAbs</i> <sub>2</sub>	value M type constructors abstraction S recursive let binding
<i>valAbs</i>	$::=$   $\lambda(x : T) \rightarrow t$	abstraction
<i>abs</i>	$::=$   $\lambda(x : T) \rightarrow t$	lambda abstractions abstraction

		$\lambda(X : K) \rightarrow abs$		type abstraction
		<b>let</b> $x params = t$ <b>in</b> $abs$		let binding
		<b>let rec</b> $x params = abs_1$ <b>in</b> $abs_2$		recursive let binding
		$abs : annot$		type annotation
$eff$	$::=$			effect
		$effelm_1, .., effelm_n$		
		$eff_1 \cup eff_2 \cup .. \cup eff_n$	M	
		$eff_1 \setminus [exn]$	M	
		$(eff)$	S	
$effelm$	$::=$			effects elements
		$X$		effect
		<b>IO</b>		IO effect
		<b>Exn</b> $[exn]$		exception
$exn$	$::=$			exceptions
		$Exc_1   ..   Exc_n$		
$K$	$::=$			kinds
		$*$		star
		$\varphi$		the effect kind
		$K \rightarrow K'$		kind arrow
$T$	$::=$			type
		$X$		variable
		$[eff]$		effects
		<b>Unit</b>		Unit type (contained in the module opened by default)
		$T \rightarrow T'$	S	$== T -[]- \dot{\leftarrow} T'$
		$T - [eff] - > T'$		function
		$\lambda(X : K), T$		operator abstraction
		$\forall (X : K), T$		forall
		$\forall tyParams, T$	S	
		$T T'$		operator application
		$(T)$	S	
		$[X \mapsto T] T'$	M	
		$T_1 \rightarrow .. \rightarrow T_n \rightarrow T$	M	
		$T T_1 .. T_n$	M	
$p$	$::=$			pattern
		$V p_1 .. p_n$		variant
		$x$		wildcard variable
$pe$	$::=$			try pattern
		$Exc x_1 .. x_n$		Exception pattern
$lambda$	$::=$			lambda parameters

		$(x : T)$		value
		$tyLambda$		type
$valueLambda$	::=			
		$(x : T)$		value
$tyLambda$	::=			
		$(X : K)$		type
		$X$	S	
$valueParams$	::=			
		$valueLambda_1 .. valueLambda_n$	S	
$tyParams$	::=			
		$tyLambda_1 .. tyLambda_n$	S	
$exnval$	::=			runtime value of exceptions
		$Exc\ v_1 .. v_n$		
$annot$	::=			
		$T$	S	
		$[[eff]]\ T$	S	
$params$	::=			
		$lambda_1 .. lambda_n$	S	
		$lambda_1 .. lambda_n : annot$	S	
$\Gamma$	::=			type environment
		$\emptyset$		empty
		$\Gamma, x : T$		vars
		$\Gamma, x_1 : T_1, .., x_n : T_n$	S	vars
		$\Gamma, V : T$	S	type constructors (contained in the above value)
		$\Gamma, X : K$		tvars
		$\Gamma, X_1 : K_1, .., X_n : K_n$	S	tvars
		$\Gamma, X : \{ V_1\ tyList_1 .. V_n\ tyList_n \}$		variants
		$\Gamma, Exc\ tyList$		exceptions
		$\Gamma_1 \cup .. \cup \Gamma_n$	M	
$\Delta$	::=			runtime environment
		$\emptyset$		empty
		$\Delta, \{ x_1 \leftarrow v_1 .. x_n \leftarrow v_n \}$		vars
		$\Delta, \{ V_1 \leftarrow v_1 .. V_n \leftarrow v_n \}$		variant vars
		$\Delta_1 \cup .. \cup \Delta_n$	M	
$terminals$	::=			
		$\lambda$		
		$\backslash$		

		$\longrightarrow$	
		$\rightarrow$	
		$\vdash$	
		$\mapsto$	
		$\in$	
		$\equiv$	
		$\varphi$	
		$\forall$	
		$\cup$	
		$\triangleright$	
		$\leftarrow$	
		$\&$	
		$\emptyset$	
		$\notin$	
<i>formula</i>	$::=$		
		<i>judgement</i>	
		<i>formula</i> <sub>1</sub> .. <i>formula</i> <sub>n</sub>	
		<b>not</b> ( <i>formula</i> )	
		$x : T \in \Gamma$	
		$X : K \in \Gamma$	
		$V : T \in \Gamma$	
		$T : \{Variant\} \in \Gamma$	
		<i>Exc tyList</i> $\in \Gamma$	
		<b>set</b> ( <i>eff</i> ) = <b>set</b> ( <i>eff'</i> )	
		<b>set</b> ( <i>exn</i> ) = <b>set</b> ( <i>exn'</i> )	
		$V \in Variant \triangleright T_1 .. T_n$	
		<i>RetVar</i> = <i>RetVar'</i>	
		$\Gamma = \Gamma'$	
		$\{x \leftarrow v\} \in \Delta$	
		$\{V \leftarrow v\} \in \Delta$	
		( <i>formula</i> ) <b>after applications</b>	
		<i>effelm</i> $\notin$ <i>eff</i>	
<i>patterns</i>	$::=$		
		$p_1 .. p_n$	
<i>VArgs</i>	$::=$		
		$T_1 .. T_n$	
<i>Variant</i>	$::=$		
		$V_1 VArgs_1 .. V_n VArgs_n$	
		$Variant \setminus V$	M
<i>RetVar</i>	$::=$		
		$X X_1 .. X_n$	

$JGammaValidity$	$::=$   $\Gamma \vdash \mathbf{ok}$	Typing environment validity
$JProgram$	$::=$   $\Gamma \vdash program \triangleright \Gamma'$	Program typing
$JTopType$	$::=$   $\Gamma \vdash top \triangleright \Gamma'$	Toplevel typing
$JTypeDecl$	$::=$   $RetVar \ \& \ \Gamma \vdash V \ tyList \triangleright \Gamma'$	Type declaration
$Jtype$	$::=$   $\Gamma \vdash t : [[eff]] T$	Typing
$Jkind$	$::=$   $\Gamma \vdash T : K$	Kinding
$JEff$	$::=$   $\Gamma \vdash eff : \varphi$	Effects typing
$JEffElm$	$::=$   $\Gamma \vdash effelm : \varphi$	Effects elements typing
$JPatternsTyping$	$::=$   $Variant \ \& \ \Gamma \vdash patterns : T \triangleright \Gamma_1 .. \Gamma_n$	Patterns matching typing
$JPatternTyping$	$::=$   $Variant \ \& \ \Gamma \vdash p : T \triangleright \Gamma'$	Pattern matching typing
$JExnPatternTyping$	$::=$   $\Gamma \vdash pe \triangleright Exc \ \& \ \Gamma'$	Exception pattern matching typing
$Jequiv$	$::=$   $T \equiv T'$	Type equivalence
$JEffEquiv$	$::=$   $eff \equiv eff'$	Effects equivalence
$JEffElmEquiv$	$::=$   $effelm \equiv effelm'$	Effect element equivalence
$JTopOp$	$::=$   $\Delta \vdash program \longrightarrow \Delta' \vdash program'$	Toplevel evaluation
$JVarCreationOp$	$::=$   $V \ tyList \triangleright v$	Variants creation

$Jop$	$::=$ $\mid \Delta \vdash t \longrightarrow \Delta' \vdash t'$	Evaluation
$JExnMatches$	$::=$ $\mid exnval \textbf{ matches } pe \triangleright \Delta$	Exception pattern matching with substitution creation
$JMatches$	$::=$ $\mid v \textbf{ matches } p \triangleright \Delta$	Pattern matching with substitution creation
$judgement$	$::=$ $\mid JGammaValidity$ $\mid JProgram$ $\mid JTopType$ $\mid JTypeDecl$ $\mid Jtype$ $\mid Jkind$ $\mid JEff$ $\mid JEffElm$ $\mid JPatternsTyping$ $\mid JPatternTyping$ $\mid JExnPatternTyping$ $\mid Jequiv$ $\mid JEffEquiv$ $\mid JEffElmEquiv$ $\mid JTopOp$ $\mid JVarCreationOp$ $\mid Jop$ $\mid JExnMatches$ $\mid JMatches$	
$user\_syntax$	$::=$ $\mid termvar$ $\mid variant$ $\mid typvar$ $\mid exc$ $\mid n$ $\mid m$ $\mid program$ $\mid top$ $\mid variantArgs$ $\mid tyList$ $\mid t$ $\mid v$ $\mid valAbs$ $\mid abs$ $\mid eff$ $\mid effelm$	

$exn$   
 $K$   
 $T$   
 $p$   
 $pe$   
 $lambda$   
 $valueLambda$   
 $tyLambda$   
 $valueParams$   
 $tyParams$   
 $exnval$   
 $annot$   
 $params$   
 $\Gamma$   
 $\Delta$   
 $terminals$   
 $formula$   
 $patterns$   
 $VArgs$   
 $Variant$   
 $RetVar$

$\boxed{\Gamma \vdash \mathbf{ok}}$  Typing environment validity

$$\begin{array}{c}
\overline{\emptyset \vdash \mathbf{ok}} \quad \text{GAMMA\_VALID\_EMPTY} \\
\frac{\Gamma \vdash \mathbf{ok}}{\Gamma, x : T \vdash \mathbf{ok}} \quad \text{GAMMA\_VALID\_VARS}
\end{array}$$

$\boxed{\Gamma \vdash \text{program} \triangleright \Gamma'}$  Program typing

$$\begin{array}{c}
\overline{\Gamma \vdash \triangleright \Gamma} \quad \text{PROG\_EMPTY} \\
\frac{\Gamma \vdash \text{top} \triangleright \Gamma' \quad \Gamma' \vdash \text{program} \triangleright \Gamma''}{\Gamma \vdash \text{top program} \triangleright \Gamma''} \quad \text{PROG\_PROGRAM}
\end{array}$$

$\boxed{\Gamma \vdash \text{top} \triangleright \Gamma'}$  Toplevel typing

$$\begin{array}{c}
\frac{\Gamma \vdash t : [[]] T}{\Gamma \vdash \mathbf{let} x = t \triangleright \Gamma, x : T} \quad \text{TOP\_LET} \\
\frac{\Gamma, x : T \vdash \text{abs} : [[]] T}{\Gamma \vdash \mathbf{let rec} x : T = \text{abs} \triangleright \Gamma, x : T} \quad \text{TOP\_LETREC} \\
\frac{\Gamma \vdash T : K}{\Gamma \vdash \mathbf{type alias} X = T \triangleright \Gamma, X : K} \quad \text{TOP\_TYPEALIAS} \\
\begin{array}{l}
RetVar = X X_1 .. X_n \\
\Gamma' = \Gamma, X : K, X_1 : K_1, .., X_n : K_n \\
RetVar \ \& \ \Gamma' \vdash V_1 \text{ tyList}_1 \triangleright \Gamma_1 \quad .. \quad RetVar \ \& \ \Gamma' \vdash V_n \text{ tyList}_n \triangleright \Gamma_n
\end{array} \\
\hline
\Gamma \vdash \mathbf{type} X (X_1 : K_1)(X_n : K_n) = V_1 \text{ tyList}_1 | .. | V_n \text{ tyList}_n \triangleright \Gamma \cup \Gamma_1 \cup .. \cup \Gamma_n, X : \{ V_1 \text{ tyList}_1 .. V_n \text{ tyList}_n \} \quad \text{TOP\_TYPE} \\
\frac{\Gamma \vdash T_1 : K_1 \quad .. \quad \Gamma \vdash T_n : K_n}{\Gamma \vdash \mathbf{exception} Exc T_1 .. T_n \triangleright \Gamma, Exc T_1 .. T_n} \quad \text{TOP\_EXCEPTION}
\end{array}$$



$\boxed{RetVar \ \& \ \Gamma \vdash V \ tyList \triangleright \Gamma'}$     Type declaration

$$\frac{\Gamma \vdash T_1 : K_1 \quad \dots \quad \Gamma \vdash T_n : K_n}{X \ X_1 \dots X_n \ \& \ \Gamma \vdash V \ T_1 \dots T_n \triangleright \emptyset, V : T_1 \rightarrow \dots \rightarrow T_n \rightarrow X \ X_1 \dots X_n} \quad \text{TyDECL\_DECL}$$

$\boxed{\Gamma \vdash t : [[eff]] T}$     Typing

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : [[]] T} \quad \text{T\_VAR}$$

$$\frac{V : T \in \Gamma}{\Gamma \vdash V : [[]] T} \quad \text{T\_VARIANT}$$

$$\frac{\Gamma, x_1 : T_1 \vdash t : [[eff]] T \quad \Gamma \vdash T_1 : *}{\Gamma \vdash \lambda(x_1 : T_1) \rightarrow t : [[]] T_1 - [eff] - > T} \quad \text{T\_ABS}$$

$$\frac{\Gamma \vdash t : [[eff_1]] T_1 - [eff_2] - > T_2 \quad \Gamma \vdash t' : [[eff_3]] T_1}{\Gamma \vdash t t' : [[eff_1 \cup eff_2 \cup eff_3]] T_2} \quad \text{T\_APP}$$

$$\frac{\Gamma, X : K \vdash t : [[eff]] T \quad \mathbf{IO} \notin eff}{\Gamma \vdash \lambda(X : K) \rightarrow t : [[eff]] \forall (X : K), T} \quad \text{T\_TABS}$$

$$\frac{\Gamma \vdash t : [[eff]] \forall (X : K), T_2 \quad \Gamma \vdash T_1 : K}{\Gamma \vdash t[T_1] : [[eff]] [X \mapsto T_1] T_2} \quad \text{T\_TAPP}$$

$$\frac{\Gamma \vdash t : [[eff]] X \quad X \equiv X' \quad \Gamma \vdash X' : *}{\Gamma \vdash t : [[eff]] X'} \quad \text{T\_EQ}$$

$$\frac{\Gamma, x : T_1 \vdash t_2 : [[eff_2]] T_2}{\Gamma \vdash \mathbf{let} \ x = (t_1 : [[eff_1]] T_1) \ \mathbf{in} \ t_2 : [[eff_1 \cup eff_2]] T_2} \quad \text{T\_LET}$$

$$\frac{\Gamma, x : T_1 \vdash \mathbf{abs} : [[eff_1]] T_1 \quad \Gamma, x : T_1 \vdash t_2 : [[eff_2]] T_2}{\Gamma \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathbf{abs} : [[eff_1]] T_1 \ \mathbf{in} \ t_2 : [[eff_1 \cup eff_2]] T_2} \quad \text{T\_LETREC}$$

$$\frac{\Gamma \cup \Gamma_1 \vdash t_1 : [[eff_1]] T_2 \quad \dots \quad \Gamma \cup \Gamma_n \vdash t_n : [[eff_n]] T_2 \quad T_1 : \{Variant\} \in \Gamma \quad Variant \ \& \ \Gamma \vdash p_1 \dots p_n : T_1 \triangleright \Gamma_1 \dots \Gamma_n \quad \Gamma \vdash t : [[eff]] T_1}{\Gamma \vdash \mathbf{match} \ t \ \mathbf{with} \ p_1 \rightarrow t_1 \mid \dots \mid p_n \rightarrow t_n \ \mathbf{end} : [[eff \cup eff_1 \cup \dots \cup eff_n]] T_2} \quad \text{T\_MATCH}$$

$$\frac{\Gamma \vdash t : [[eff]] T \quad \Gamma \vdash T : * \quad \Gamma \vdash eff : \varphi}{\Gamma \vdash (t : [[eff]] T) : [[eff]] T} \quad \text{T\_ANNOT}$$

$$\frac{\Gamma \vdash t_1 : [[eff_1]] T_1 \quad \dots \quad \Gamma \vdash t_n : [[eff_n]] T_n \quad \Gamma \vdash T : * \quad Exc \ T_1 \dots T_n \in \Gamma}{\Gamma \vdash \mathbf{fail} \ [T] Exc \ t_1 \dots t_n : [[\mathbf{Exn} \ [Exc] \cup eff_1 \cup \dots \cup eff_n]] T} \quad \text{T\_FAIL}$$

$$\begin{array}{c}
\Gamma_1 \vdash t_1 : [[eff_1]] T \quad \dots \quad \Gamma_n \vdash t_n : [[eff_n]] T \\
\Gamma \vdash pe_1 \triangleright Exc_1 \ \& \ \Gamma_1 \quad \dots \quad \Gamma \vdash pe_n \triangleright Exc_n \ \& \ \Gamma_n \\
\Gamma \vdash t : [[eff]] T \\
\hline
\Gamma \vdash \mathbf{try} \ t \ \mathbf{with} \ pe_1 \rightarrow t_1 \mid \dots \mid pe_n \rightarrow t_n \ \mathbf{end} : [((eff \setminus [Exc_1] \mid \dots \mid [Exc_n]) \cup eff_1 \cup \dots \cup eff_n)] T
\end{array} \quad \text{T\_TRY}$$

$\boxed{\Gamma \vdash T : K}$     Kinding

$$\begin{array}{c}
\frac{X : K \in \Gamma}{\Gamma \vdash X : K} \quad \text{K\_TVAR} \\
\frac{\Gamma \vdash eff : \varphi}{\Gamma \vdash [eff] : \varphi} \quad \text{K\_EFF} \\
\frac{\Gamma, X : K_1 \vdash T : K_2}{\Gamma \vdash \lambda(X : K_1), T : K_1 \rightarrow K_2} \quad \text{K\_ABS} \\
\frac{\Gamma \vdash T_1 : K_{11} \rightarrow K_{12} \quad \Gamma \vdash T_2 : K_{11}}{\Gamma \vdash T_1 \ T_2 : K_{12}} \quad \text{K\_APP} \\
\frac{\Gamma \vdash T_1 : * \quad \Gamma \vdash eff : \varphi \quad \Gamma \vdash T_2 : *}{\Gamma \vdash T_1 - [eff] - > T_2 : *} \quad \text{K\_ARROW} \\
\frac{\Gamma, X : K_1 \vdash T_2 : *}{\Gamma \vdash \forall (X : K_1), T_2 : *} \quad \text{K\_ALL}
\end{array}$$

$\boxed{\Gamma \vdash eff : \varphi}$     Effects typing

$$\frac{\Gamma \vdash effelm_1 : \varphi \quad \dots \quad \Gamma \vdash effelm_n : \varphi}{\Gamma \vdash effelm_1, \dots, effelm_n : \varphi} \quad \text{EFF\_EFF}$$

$\boxed{\Gamma \vdash effelm : \varphi}$     Effects elements typing

$$\begin{array}{c}
\frac{X : \varphi \in \Gamma}{\Gamma \vdash X : \varphi} \quad \text{EFFELM\_EFF} \\
\frac{}{\Gamma \vdash \mathbf{IO} : \varphi} \quad \text{EFFELM\_IO} \\
\frac{Exc_1 \ tyList_1 \in \Gamma \quad \dots \quad Exc_n \ tyList_n \in \Gamma}{\Gamma \vdash \mathbf{Exn} [Exc_1] \mid \dots \mid [Exc_n] : \varphi} \quad \text{EFFELM\_EXN}
\end{array}$$

$\boxed{Variant \ \& \ \Gamma \vdash patterns : T \triangleright \Gamma_1 \dots \Gamma_n}$     Patterns matching typing

$$\frac{Variant \ \& \ \Gamma \vdash p_1 : T \triangleright \Gamma_1 \quad \dots \quad Variant \ \& \ \Gamma \vdash p_n : T \triangleright \Gamma_n}{Variant \ \& \ \Gamma \vdash p_1 \dots p_n : T \triangleright \Gamma_1 \dots \Gamma_n} \quad \text{PsTy\_PATTERNS}$$

$\boxed{Variant \ \& \ \Gamma \vdash p : T \triangleright \Gamma'}$     Pattern matching typing

$$\frac{
\begin{array}{c}
V \in Variant \triangleright T_1 \dots T_n \\
T_1 : \{Variant_1\} \in \Gamma \quad \dots \quad T_n : \{Variant_n\} \in \Gamma \\
Variant_1 \ \& \ \Gamma \vdash p_1 : T_1 \triangleright \Gamma_1 \quad \dots \quad Variant_n \ \& \ \Gamma \vdash p_n : T_n \triangleright \Gamma_n
\end{array}
}{Variant \ \& \ \Gamma \vdash V \ p_1 \dots p_n : T \triangleright \Gamma_1 \cup \dots \cup \Gamma_n} \quad \text{PTY\_VARIANT}$$

$$\frac{}{Variant \ \& \ \Gamma \vdash x : T \triangleright \emptyset, x : T} \quad \text{PTY\_WILDCARD}$$

$\boxed{\Gamma \vdash pe \triangleright Exc \ \& \ \Gamma'}$     Exception pattern matching typing

$$\frac{Exc\ T_1 \dots T_n \in \Gamma}{\Gamma \vdash Exc\ x_1 \dots x_n \triangleright Exc\ \& \Gamma, x_1 : T_1, \dots, x_n : T_n} \text{PETY\_EXC}$$

$$\boxed{T \equiv T'} \quad \text{Type equivalence}$$

$$\frac{}{T \equiv T} \text{Q\_REFL}$$

$$\frac{T \equiv T'}{T' \equiv T} \text{Q\_SYMM}$$

$$\frac{T_1 \equiv T_2 \quad T_2 \equiv T_3}{T_1 \equiv T_3} \text{Q\_TRANS}$$

$$\frac{T_{11} \equiv T_{21} \quad eff_1 \equiv eff_2 \quad T_{12} \equiv T_{22}}{T_{11} - [eff_1] - > T_{12} \equiv T_{21} - [eff_2] - > T_{22}} \text{Q\_ARROW}$$

$$\frac{T_1 \equiv T_2}{\forall (X : K), T_1 \equiv \forall (X : K), T_2} \text{Q\_ALL}$$

$$\frac{T_1 \equiv T_2}{\lambda (X : K), T_1 \equiv \lambda (X : K), T_2} \text{Q\_ABS}$$

$$\frac{T_{11} \equiv T_{21} \quad T_{12} \equiv T_{22}}{T_{11}\ T_{12} \equiv T_{21}\ T_{22}} \text{Q\_APP}$$

$$\frac{}{(\lambda (X : K), T_{11})\ T_{12} \equiv [X \mapsto T_{12}]\ T_{11}} \text{Q\_APPAbs}$$

$$\boxed{eff \equiv eff'} \quad \text{Effects equivalence}$$

$$\frac{}{eff \equiv eff} \text{EFFEQ\_REFL}$$

$$\frac{\mathbf{set}\ (effelm_1, \dots, effelm_n) = \mathbf{set}\ (effelm'_1, \dots, effelm'_n)}{effelm_1, \dots, effelm_n \equiv effelm'_1, \dots, effelm'_n} \text{EFFEQ\_EQ}$$

$$\boxed{effelm \equiv effelm'} \quad \text{Effect element equivalence}$$

$$\frac{}{effelm \equiv effelm} \text{EFFELMEQ\_REFL}$$

$$\frac{effelm \equiv effelm'}{effelm' \equiv effelm} \text{EFFELMEQ\_SYMM}$$

$$\frac{effelm_1 \equiv effelm_2 \quad effelm_2 \equiv effelm_3}{effelm_1 \equiv effelm_3} \text{EFFELMEQ\_TRANS}$$

$$\frac{\mathbf{set}\ (Exc_1 | \dots | Exc_n) = \mathbf{set}\ (Exc'_1 | \dots | Exc'_n)}{\mathbf{Exn}\ [Exc_1 | \dots | Exc_n] \equiv \mathbf{Exn}\ [Exc'_1 | \dots | Exc'_n]} \text{EFFELMEQ\_EXNEQ}$$

$$\boxed{\Delta \vdash \text{program} \longrightarrow \Delta' \vdash \text{program}'} \quad \text{Toplevel evaluation}$$

$$\frac{\Delta \vdash t \longrightarrow \Delta \vdash t'}{\Delta \vdash \mathbf{let}\ x = t\ \text{program} \longrightarrow \Delta \vdash \mathbf{let}\ x = t'\ \text{program}} \text{TOPE\_LET1}$$

$$\begin{array}{c}
\frac{}{\Delta \vdash \mathbf{let} \ x = v \ \mathit{program} \longrightarrow \Delta, \{x \leftarrow v\} \vdash \mathit{program}} \quad \text{TOPE\_LET2} \\
\frac{\Delta \vdash \mathit{abs} \longrightarrow \Delta \vdash \mathit{abs}'}{\Delta \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathit{abs} \ \mathit{program} \longrightarrow \Delta \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathit{abs}' \ \mathit{program}} \quad \text{TOPE\_LETREC1} \\
\frac{}{\Delta \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathit{valAbs} \ \mathit{program} \longrightarrow \Delta, \{x \leftarrow \mathbf{let} \ \mathbf{rec} \ x = \mathit{valAbs} \ \mathbf{in} \ \mathit{valAbs}\} \vdash \mathit{program}} \quad \text{TOPE\_LETREC2} \\
\frac{}{\Delta \vdash \mathbf{type} \ \mathbf{alias} \ X = T \ \mathit{program} \longrightarrow \Delta \vdash \mathit{program}} \quad \text{TOPE\_TYPEALIAS} \\
\frac{V_1 \ \mathit{tyList}_1 \triangleright v_1 \quad \dots \quad V_n \ \mathit{tyList}_n \triangleright v_n}{\Delta \vdash \mathbf{type} \ X \ \mathit{variantArgs} = V_1 \ \mathit{tyList}_1 | \dots | V_n \ \mathit{tyList}_n \ \mathit{program} \longrightarrow \Delta, \{V_1 \leftarrow v_1 \dots V_n \leftarrow v_n\} \vdash \mathit{program}} \quad \text{TOPE\_T} \\
\frac{}{\Delta \vdash \mathbf{exception} \ \mathit{Exc} \ \mathit{tyList} \ \mathit{program} \longrightarrow \Delta \vdash \mathit{program}} \quad \text{TOPE\_EXCEPTION} \\
\boxed{V \ \mathit{tyList} \triangleright v} \quad \text{Variants creation} \\
\frac{(\{x_1 \leftarrow v_1\} \in \Delta) \ \mathbf{after} \ \mathbf{applications} \quad \dots \quad (\{x_n \leftarrow v_n\} \in \Delta) \ \mathbf{after} \ \mathbf{applications}}{V \ T_1 \dots T_n \triangleright \lambda(x_1 : T_1) \dots (x_n : T_n) \rightarrow \mathbf{TConstr} \ V \ v_1 \dots v_n} \quad \text{VARCREATION\_CREATE} \\
\boxed{\Delta \vdash t \longrightarrow \Delta' \vdash t'} \quad \text{Evaluation} \\
\\
\frac{\frac{\{x \leftarrow v\} \in \Delta}{\Delta \vdash x \longrightarrow \Delta \vdash v}}{\Delta \vdash x \longrightarrow \Delta \vdash v} \quad \text{E\_VAR} \\
\frac{\frac{\{V \leftarrow v\} \in \Delta}{\Delta \vdash V \longrightarrow \Delta \vdash v}}{\Delta \vdash V \longrightarrow \Delta \vdash v} \quad \text{E\_VARIANT} \\
\\
\frac{}{\Delta \vdash (\mathbf{failure} \ \mathit{exnval}) \ t \longrightarrow \Delta \vdash \mathbf{failure} \ \mathit{exnval}} \quad \text{E\_APP1FAILURE} \\
\frac{}{\Delta \vdash v \ (\mathbf{failure} \ \mathit{exnval}) \longrightarrow \Delta \vdash \mathbf{failure} \ \mathit{exnval}} \quad \text{E\_APP2FAILURE} \\
\\
\frac{\frac{\Delta \vdash t_1 \longrightarrow \Delta \vdash t'_1}{\Delta \vdash t_1 \ t \longrightarrow \Delta \vdash t'_1 \ t}}{\Delta \vdash t_1 \ t \longrightarrow \Delta \vdash t'_1 \ t} \quad \text{E\_APP1} \\
\frac{\frac{\Delta \vdash t_1 \longrightarrow \Delta \vdash t'_1}{\Delta \vdash v \ t_1 \longrightarrow \Delta \vdash v \ t'_1}}{\Delta \vdash v \ t_1 \longrightarrow \Delta \vdash v \ t'_1} \quad \text{E\_APP2} \\
\\
\frac{}{\Delta \vdash (\lambda(x : T) \rightarrow t_{12}) \ v_2 \longrightarrow \Delta, \{x \leftarrow v_2\} \vdash t_{12}} \quad \text{E\_APPAbs} \\
\frac{}{\Delta \vdash \lambda(X : K) \rightarrow t \longrightarrow \Delta \vdash t} \quad \text{E\_TABS} \\
\frac{}{\Delta \vdash t[T] \longrightarrow \Delta \vdash t} \quad \text{E\_TAPP} \\
\\
\frac{}{\Delta \vdash \mathbf{let} \ x = \mathbf{failure} \ \mathit{exnval} \ \mathbf{in} \ t_2 \longrightarrow \Delta \vdash \mathbf{failure} \ \mathit{exnval}} \quad \text{E\_LETFAILURE} \\
\frac{\frac{\Delta \vdash t_1 \longrightarrow \Delta \vdash t'_1}{\Delta \vdash \mathbf{let} \ x = t_1 \ \mathbf{in} \ t_2 \longrightarrow \Delta \vdash \mathbf{let} \ x = t'_1 \ \mathbf{in} \ t_2}}{\Delta \vdash \mathbf{let} \ x = t_1 \ \mathbf{in} \ t_2 \longrightarrow \Delta \vdash \mathbf{let} \ x = t'_1 \ \mathbf{in} \ t_2} \quad \text{E\_LET1} \\
\frac{}{\Delta \vdash \mathbf{let} \ x = v \ \mathbf{in} \ t \longrightarrow \Delta, \{x \leftarrow v\} \vdash t} \quad \text{E\_LET2} \\
\frac{\frac{\Delta \vdash \mathit{abs} \longrightarrow \Delta \vdash \mathit{abs}'}{\Delta \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathit{abs} \ \mathbf{in} \ t \longrightarrow \Delta \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathit{abs}' \ \mathbf{in} \ t}}{\Delta \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathit{abs} \ \mathbf{in} \ t \longrightarrow \Delta \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathit{abs}' \ \mathbf{in} \ t} \quad \text{E\_LETREC1} \\
\frac{}{\Delta \vdash \mathbf{let} \ \mathbf{rec} \ x = \mathit{valAbs} \ \mathbf{in} \ t \longrightarrow \Delta, \{x \leftarrow \mathbf{let} \ \mathbf{rec} \ x = \mathit{valAbs} \ \mathbf{in} \ \mathit{valAbs}\} \vdash t} \quad \text{E\_LETREC2} \\
\frac{}{\Delta \vdash \mathbf{match} \ \mathbf{failure} \ \mathit{exnval} \ \mathbf{with} \ p_1 \rightarrow t_1 | \dots | p_n \rightarrow t_n \ \mathbf{end} \longrightarrow \Delta \vdash \mathbf{failure} \ \mathit{exnval}} \quad \text{E\_MATCHFAILURE}
\end{array}$$

$\frac{\Delta \vdash t \longrightarrow \Delta \vdash t'}{\Delta \vdash \mathbf{match} \ t \ \mathbf{with} \ p_1 \rightarrow t_1 \mid \dots \mid p_n \rightarrow t_n \ \mathbf{end} \longrightarrow \Delta \vdash \mathbf{match} \ t' \ \mathbf{with} \ p_1 \rightarrow t_1 \mid \dots \mid p_n \rightarrow t_n \ \mathbf{end}}$	E_MATCH
$\frac{v \ \mathbf{matches} \ p_1 \triangleright \Delta'}{\Delta \vdash \mathbf{match} \ v \ \mathbf{with} \ p_1 \rightarrow t_1 \mid \dots \mid p_n \rightarrow t_n \ \mathbf{end} \longrightarrow \Delta \cup \Delta' \vdash t_1}$	E_MATCHFOUND
$\frac{\mathbf{not} \ (v \ \mathbf{matches} \ p_1 \triangleright \Delta')}{\Delta \vdash \mathbf{match} \ v \ \mathbf{with} \ p_1 \rightarrow t_1 \mid p_2 \rightarrow t_2 \mid \dots \mid p_n \rightarrow t_n \ \mathbf{end} \longrightarrow \Delta \vdash \mathbf{match} \ v \ \mathbf{with} \ p_2 \rightarrow t_2 \mid \dots \mid p_n \rightarrow t_n \ \mathbf{end}}$	E_MATCHS
$\frac{\Delta \vdash t \longrightarrow \Delta \vdash t'}{\Delta \vdash (t : [[\mathit{eff}]] \ T) \longrightarrow \Delta \vdash t'} \quad \text{E\_ANNOT}$	
$\frac{\Delta \vdash t \longrightarrow \Delta \vdash t'}{\Delta \vdash \mathbf{fail} \ [T] \mathit{Exc} \ v_1 \dots v_n \ t \ t_1 \dots t_n \longrightarrow \Delta \vdash \mathbf{fail} \ [T] \mathit{Exc} \ v_1 \dots v_n \ t' \ t_1 \dots t_n} \quad \text{E\_FAILUREARGS}$	
$\frac{}{\Delta \vdash \mathbf{fail} \ [T] \mathit{Exc} \ v_1 \dots v_n \longrightarrow \Delta \vdash \mathbf{failure} \ \mathit{Exc} \ v_1 \dots v_n} \quad \text{E\_FAILURE}$	
$\frac{\Delta \vdash t \longrightarrow \Delta \vdash t'}{\Delta \vdash \mathbf{try} \ t \ \mathbf{with} \ p_{e1} \rightarrow t_1 \mid \dots \mid p_{e_n} \rightarrow t_n \ \mathbf{end} \longrightarrow \Delta \vdash \mathbf{try} \ t' \ \mathbf{with} \ p_{e1} \rightarrow t_1 \mid \dots \mid p_{e_n} \rightarrow t_n \ \mathbf{end}}$	E_TRY
$\frac{}{\Delta \vdash \mathbf{try} \ v \ \mathbf{with} \ p_{e1} \rightarrow t_1 \mid \dots \mid p_{e_n} \rightarrow t_n \ \mathbf{end} \longrightarrow \Delta \vdash v} \quad \text{E\_TRYNOFAILURE}$	
$\frac{\mathbf{not} \ (\mathit{exnval} \ \mathbf{matches} \ p_{e1} \triangleright \Delta')}{\Delta \vdash \mathbf{try} \ \mathbf{failure} \ \mathit{exnval} \ \mathbf{with} \ p_{e1} \rightarrow t_1 \ \mathbf{end} \longrightarrow \Delta \vdash \mathbf{failure} \ \mathit{exnval}} \quad \text{E\_TRYNOTFOUND}$	
$\frac{\mathit{exnval} \ \mathbf{matches} \ p_{e1} \triangleright \Delta'}{\Delta \vdash \mathbf{try} \ \mathbf{failure} \ \mathit{exnval} \ \mathbf{with} \ p_{e1} \rightarrow t_1 \mid \dots \mid p_{e_n} \rightarrow t_n \ \mathbf{end} \longrightarrow \Delta \cup \Delta' \vdash t_1} \quad \text{E\_TRYFOUND}$	
$\frac{\mathbf{not} \ (\mathit{exnval} \ \mathbf{matches} \ p_{e1} \triangleright \Delta')}{\Delta \vdash \mathbf{try} \ \mathbf{failure} \ \mathit{exnval} \ \mathbf{with} \ p_{e1} \rightarrow t_1 \mid p_{e2} \rightarrow t_2 \mid \dots \mid p_{e_n} \rightarrow t_n \ \mathbf{end} \longrightarrow \Delta \vdash \mathbf{try} \ \mathbf{failure} \ \mathit{exnval} \ \mathbf{with} \ p_{e2} \rightarrow t_2 \mid \dots \mid p_{e_n} \rightarrow t_n \ \mathbf{end}}$	
<div style="border: 1px solid black; padding: 2px; display: inline-block;"><math>\mathit{exnval} \ \mathbf{matches} \ p_e \triangleright \Delta</math></div> Exception pattern matching with substitution creation	
$\frac{}{\mathit{Exc} \ v_1 \dots v_n \ \mathbf{matches} \ \mathit{Exc} \ x_1 \dots x_n \triangleright \emptyset, \{x_1 \leftarrow v_1 \dots x_n \leftarrow v_n\}} \quad \text{EXNMATCHES\_MATCHES}$	
<div style="border: 1px solid black; padding: 2px; display: inline-block;"><math>v \ \mathbf{matches} \ p \triangleright \Delta</math></div> Pattern matching with substitution creation	
$\frac{}{v \ \mathbf{matches} \ x \triangleright \emptyset, \{x \leftarrow v\}} \quad \text{MATCHES\_ANY}$	
$\frac{v_1 \ \mathbf{matches} \ p_1 \triangleright \Delta_1 \quad \dots \quad v_n \ \mathbf{matches} \ p_n \triangleright \Delta_n}{\mathbf{TConstr} \ V \ v_1 \dots v_n \ \mathbf{matches} \ V \ p_1 \dots p_n \triangleright \Delta_1 \cup \dots \cup \Delta_n} \quad \text{MATCHES\_MATCHES}$	

Definition rules: 88 good 0 bad

Definition rule clauses: 180 good 0 bad