

$x$  term variable  
 $\alpha$  type variable  
 $c$  coercion variable  
 $i$  index metavariable

$t ::=$   
 $| x$  variable  
 $| \lambda(x : \tau) \Rightarrow t$  abstraction  
 $| \lambda\{\alpha : *\} \Rightarrow t$  type abstraction  
 $| \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t$  coercion abstraction  
 $| t_1 t_2$  application  
 $| t [\tau]$  type application  
 $| t \sim[\gamma]$  coercion application  
 $| t \blacktriangleright \gamma$  coercion

$\tau ::=$   
 $| \alpha$  type variable  
 $| \tau_1 \rightarrow \tau_2$  arrow  
 $| \{\tau_1 \sim \tau_2\} \rightarrow \tau_3$  coercion arrow  
 $| \forall(\alpha : *), \tau$  universal quantification

$\gamma ::=$   
 $| c$  variable  
 $| \mathbf{refl} \tau$  reflexivity  
 $| \mathbf{sym} \gamma$  symmetry  
 $| \gamma_1 \circ \gamma_2$  composition  
 $| \gamma_1 \rightarrow \gamma_2$  arrow introduction  
 $| \{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3$  coercion arrow introduction  
 $| \forall(\alpha : *), \gamma$  universal quantification introduction  
 $| \gamma @ \tau$  instantiation (quantification elimination)  
 $| \mathbf{elim}_i \gamma$  generalized elimination

$\Gamma ::=$   
 $| \emptyset$  empty  
 $| \Gamma, x : \tau$  variable  
 $| \Gamma, \alpha : *$  type variable  
 $| \Gamma, c : \tau_1 \sim \tau_2$  coercion variable

$v ::=$   
 $| \lambda(x : \tau) \Rightarrow t$  abstraction  
 $| \lambda\{\alpha : *\} \Rightarrow v$  type abstraction  
 $| \lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow v$  coercion abstraction

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Initial environment:  $\Gamma = \emptyset$

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$\Gamma \vdash t : \tau$  Typing rules

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{T\_VAR}$$

$$\begin{array}{c}
\frac{\Gamma, x : \tau_1 \vdash t : \tau_2 \quad \Gamma \vdash \tau_1 : *}{\Gamma \vdash (\lambda(x : \tau_1) \Rightarrow t) : \tau_1 \rightarrow \tau_2} \quad \text{T\_ABS} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash t : \tau}{\Gamma \vdash (\lambda\{\alpha : *\} \Rightarrow t) : \forall(\alpha : *), \tau} \quad \text{T\_TYABS} \\
\\
\frac{\Gamma, c : \tau_1 \sim \tau_2 \vdash t : \tau_3 \quad \Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : *}{\Gamma \vdash (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3} \quad \text{T\_CABS} \\
\\
\frac{\Gamma \vdash t_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash t_2 : \tau_2}{\Gamma \vdash t_1 t_2 : \tau_1} \quad \text{T\_APP} \\
\\
\frac{\Gamma \vdash t : \forall(\alpha : *), \tau_2 \quad \Gamma \vdash \tau_1 : * \quad [\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{\Gamma \vdash t [\tau_1] : \tau'_2} \quad \text{T\_TYAPP} \\
\\
\frac{\Gamma \vdash t : \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 \quad \Gamma \vdash \gamma : \tau_1 \sim \tau_2}{\Gamma \vdash t \sim[\gamma] : \tau_3} \quad \text{T\_CAPP} \\
\\
\frac{\Gamma \vdash \gamma : \tau_1 \sim \tau_2 \quad \Gamma \vdash t : \tau_1}{\Gamma \vdash (t \blacktriangleright \gamma) : \tau_2} \quad \text{T\_COERCE}
\end{array}$$

$$\boxed{\Gamma \vdash \gamma : \tau_1 \sim \tau_2}$$

Coercion typing

$$\begin{array}{c}
\frac{c : \tau_1 \sim \tau_2 \in \Gamma}{\Gamma \vdash c : \tau_1 \sim \tau_2} \quad \text{C\_VAR} \\
\\
\frac{\Gamma \vdash \tau : *}{\Gamma \vdash \mathbf{refl} \tau : \tau \sim \tau} \quad \text{C\_REFL} \\
\\
\frac{\Gamma \vdash \gamma : \tau_2 \sim \tau_1}{\Gamma \vdash \mathbf{sym} \gamma : \tau_1 \sim \tau_2} \quad \text{C\_SYM} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau_2 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau_3}{\Gamma \vdash \gamma_1 \circ \gamma_2 : \tau_1 \sim \tau_3} \quad \text{C\_COMP} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \tau_1 \rightarrow \tau_2 : *}{\Gamma \vdash (\gamma_1 \rightarrow \gamma_2) : (\tau_1 \rightarrow \tau_2) \sim (\tau'_1 \rightarrow \tau'_2)} \quad \text{C\_ARROW} \\
\\
\frac{\Gamma \vdash \gamma_1 : \tau_1 \sim \tau'_1 \quad \Gamma \vdash \gamma_2 : \tau_2 \sim \tau'_2 \quad \Gamma \vdash \gamma_3 : \tau_3 \sim \tau'_3 \quad \Gamma \vdash \{\tau_1 \sim \tau_2\} \rightarrow \tau_3 : *}{\Gamma \vdash (\{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3) : (\{\tau_1 \sim \tau_2\} \rightarrow \tau_3) \sim (\{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3)} \quad \text{C\_CARROW} \\
\\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash \gamma : \tau_1 \sim \tau_2 \quad \Gamma \vdash \forall(\alpha : *), \tau_1 : *}{\Gamma \vdash (\forall(\alpha : *), \gamma) : (\forall(\alpha : *), \tau_1) \sim (\forall(\alpha : *), \tau_2)} \quad \text{C\_FORALL} \\
\\
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \gamma : (\forall(\alpha_1 : *), \tau_2) \sim (\forall(\alpha_2 : *), \tau_3) \quad [\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha_2 \mapsto \tau_1] \tau_3 \triangleright \tau'_3}{\Gamma \vdash \gamma @ \tau_1 : \tau'_2 \sim \tau'_3} \quad \text{C\_INST} \\
\\
\frac{i \in \{1, 2\} \quad \Gamma \vdash \gamma : (\tau_1 \rightarrow \tau_2) \sim (\tau'_1 \rightarrow \tau'_2)}{\Gamma \vdash \mathbf{elim}_i \gamma : \tau_i \sim \tau'_i} \quad \text{C\_ELIMARROW} \\
\\
\frac{i \in \{1, 2, 3\} \quad \Gamma \vdash \gamma : (\{\tau_1 \sim \tau_2\} \rightarrow \tau_3) \sim (\{\tau'_1 \sim \tau'_2\} \rightarrow \tau'_3)}{\Gamma \vdash \mathbf{elim}_i \gamma : \tau_i \sim \tau'_i} \quad \text{C\_ELIMCARROW}
\end{array}$$

$$\boxed{\Gamma \vdash \tau : *}$$

Type  $\tau$  is well formed

$$\frac{\alpha : * \in \Gamma}{\Gamma \vdash \alpha : *} \quad \text{K\_VAR}$$

$$\begin{array}{c}
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : *}{\Gamma \vdash (\tau_1 \rightarrow \tau_2) : *} \quad \text{K\_ARROW} \\
\frac{\Gamma \vdash \tau_1 : * \quad \Gamma \vdash \tau_2 : * \quad \Gamma \vdash \tau_3 : *}{\Gamma \vdash (\{\tau_1 \sim \tau_2\} \rightarrow \tau_3) : *} \quad \text{K\_CARROW} \\
\frac{\alpha \notin \Gamma \quad \Gamma, \alpha : * \vdash \tau : *}{\Gamma \vdash (\forall (\alpha : *), \tau) : *} \quad \text{K\_FORALL}
\end{array}$$

$t \longrightarrow t'$  Operational semantics

$$\begin{array}{c}
\frac{t_2 \longrightarrow t'_2}{t_1 t_2 \longrightarrow t_1 t'_2} \quad \text{E\_APP1} \\
\frac{t \longrightarrow t'}{t v \longrightarrow t' v} \quad \text{E\_APP2} \\
\frac{[x \mapsto v] t \triangleright t'}{(\lambda(x : \tau) \Rightarrow t) v \longrightarrow t'} \quad \text{E\_APPAbs} \\
\frac{t \longrightarrow t'}{(\lambda\{\alpha : *\} \Rightarrow t) \longrightarrow (\lambda\{\alpha : *\} \Rightarrow t')} \quad \text{E\_TABS} \\
\frac{t \longrightarrow t'}{t [\tau] \longrightarrow t' [\tau]} \quad \text{E\_TAPP} \\
\frac{[\alpha \mapsto \tau] v \triangleright v'}{(\lambda\{\alpha : *\} \Rightarrow v) [\tau] \longrightarrow v'} \quad \text{E\_TAPPAbs} \\
\frac{t \longrightarrow t'}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \longrightarrow (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t')} \quad \text{E\_CABS} \\
\frac{t \longrightarrow t'}{t \sim [\gamma] \longrightarrow t' \sim [\gamma]} \quad \text{E\_CAPP} \\
\frac{[c \mapsto \gamma] t \triangleright t'}{(\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \sim [\gamma] \longrightarrow t'} \quad \text{E\_CAPPAbs} \\
\frac{t \longrightarrow t'}{(t \blacktriangleright \gamma) \longrightarrow t'} \quad \text{E\_COERCE}
\end{array}$$

$[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau_3$  Type substitution

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau] \alpha \triangleright \tau} \quad \text{SUBST\_VAR1} \\
\frac{\alpha_1 \neq \alpha_2}{[\alpha_1 \mapsto \tau] \alpha_2 \triangleright \alpha_2} \quad \text{SUBST\_VAR2} \\
\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3}{[\alpha \mapsto \tau_1] (\tau_2 \rightarrow \tau_3) \triangleright \tau'_2 \rightarrow \tau'_3} \quad \text{SUBST\_ARROW} \\
\frac{[\alpha \mapsto \tau_1] \tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1] \tau_3 \triangleright \tau'_3 \quad [\alpha \mapsto \tau_1] \tau_4 \triangleright \tau'_4}{[\alpha \mapsto \tau_1] (\{\tau_2 \sim \tau_3\} \rightarrow \tau_4) \triangleright (\{\tau'_2 \sim \tau'_3\} \rightarrow \tau'_4)} \quad \text{SUBST\_CARROW} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau_1] \tau_2 \triangleright \tau'_2}{[\alpha_1 \mapsto \tau_1] (\forall (\alpha_2 : *), \tau_2) \triangleright (\forall (\alpha_2 : *), \tau'_2)} \quad \text{SUBST\_FORALL}
\end{array}$$

$[x \mapsto t] t_1 \triangleright t_2$  substitution

$$\begin{array}{c}
\frac{}{[x \mapsto t]x \triangleright t} \text{ SUBST\_VAR1} \\
\\
\frac{x_1 \neq x_2}{[x_1 \mapsto t]x_2 \triangleright x_2} \text{ SUBST\_VAR2} \\
\\
\frac{}{[x \mapsto t_1](\lambda(x : \tau) \Rightarrow t_2) \triangleright (\lambda(x : \tau) \Rightarrow t_2)} \text{ SUBST\_ABS1} \\
\\
\frac{x_1 \neq x_2 \quad x_2 \notin fv(t_1) \quad [x_1 \mapsto t_1]t_2 \triangleright t'_2}{[x_1 \mapsto t_1](\lambda(x_2 : \tau) \Rightarrow t_2) \triangleright (\lambda(x_2 : \tau) \Rightarrow t'_2)} \text{ SUBST\_ABS2} \\
\\
\frac{x_1 \neq x_2 \quad x_3 \notin fv(t_1, t_2) \quad [x_2 \mapsto x_3]t_2 \triangleright t'_2 \quad [x_1 \mapsto t_1]t'_2 \triangleright t''_2}{[x_1 \mapsto t_1](\lambda(x_2 : \tau) \Rightarrow t_2) \triangleright (\lambda(x_3 : \tau) \Rightarrow t''_2)} \text{ SUBST\_ABS3} \\
\\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2}{[x \mapsto t_1](\lambda\{\alpha : *\} \Rightarrow t_2) \triangleright (\lambda\{\alpha : *\} \Rightarrow t'_2)} \text{ SUBST\_TABS} \\
\\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2}{[x \mapsto t_1](\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t_2) \triangleright (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t'_2)} \text{ SUBST\_CABS} \\
\\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2 \quad [x \mapsto t_1]t_3 \triangleright t'_3}{[x \mapsto t_1](t_2 t_3) \triangleright t'_2 t'_3} \text{ SUBST\_APP} \\
\\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2}{[x \mapsto t_1](t_2 [\tau]) \triangleright (t'_2 [\tau])} \text{ SUBST\_TAPP} \\
\\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2}{[x \mapsto t_1](t_2 \sim [\gamma]) \triangleright (t'_2 \sim [\gamma])} \text{ SUBST\_CAPP} \\
\\
\frac{[x \mapsto t_1]t_2 \triangleright t'_2}{[x \mapsto t_1](t_2 \blacktriangleright \gamma) \triangleright (t'_2 \blacktriangleright \gamma)} \text{ SUBST\_COERCE}
\end{array}$$

$\boxed{[\alpha \mapsto \tau]t_1 \triangleright t_2}$  substitution of type variable in term

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau]x \triangleright x} \text{ TT\_SUBST\_VAR} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda(x : \tau_2) \Rightarrow t_1) \triangleright (\lambda(x : \tau'_2) \Rightarrow t_2)} \text{ TT\_SUBST\_ABS} \\
\\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]t_1 \triangleright t_2}{[\alpha_1 \mapsto \tau](\lambda\{\alpha_2 : *\} \Rightarrow t_1) \triangleright (\lambda\{\alpha_2 : *\} \Rightarrow t_2)} \text{ TT\_SUBST\_TABS} \\
\\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2 \quad [\alpha \mapsto \tau_1]\tau_3 \triangleright \tau'_3 \quad [\alpha \mapsto \tau_1]t_1 \triangleright t_2}{[\alpha \mapsto \tau_1](\lambda\{c : \tau_2 \sim \tau_3\} \Rightarrow t_1) \triangleright (\lambda\{c : \tau'_2 \sim \tau'_3\} \Rightarrow t_2)} \text{ TT\_SUBST\_CABS} \\
\\
\frac{[\alpha \mapsto \tau]t_1 \triangleright t'_1 \quad [\alpha \mapsto \tau]t_2 \triangleright t'_2}{[\alpha \mapsto \tau](t_1 t_2) \triangleright t'_1 t'_2} \text{ TT\_SUBST\_APP} \\
\\
\frac{[\alpha \mapsto \tau_1]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](t_1 [\tau_2]) \triangleright (t_2 [\tau'_2])} \text{ TT\_SUBST\_TAPP} \\
\\
\frac{[\alpha \mapsto \tau]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](t_1 \sim [\gamma_1]) \triangleright (t_2 \sim [\gamma_2])} \text{ TT\_SUBST\_CAPP} \\
\\
\frac{[\alpha \mapsto \tau]t_1 \triangleright t_2 \quad [\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](t_1 \blacktriangleright \gamma_1) \triangleright (t_2 \blacktriangleright \gamma_2)} \text{ TT\_SUBST\_COERCE}
\end{array}$$

$\boxed{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}$ 

substitution of type variable in coercion term

$$\begin{array}{c}
\frac{}{[\alpha \mapsto \tau]c \triangleright c} \text{ ACSUBST\_VAR} \\
\frac{[\alpha \mapsto \tau_1]\tau_2 \triangleright \tau'_2}{[\alpha \mapsto \tau_1](\mathbf{refl} \tau_2) \triangleright \mathbf{refl} \tau'_2} \text{ ACSUBST\_REFL} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](\mathbf{sym} \gamma_1) \triangleright \mathbf{sym} \gamma_2} \text{ ACSUBST\_SYM} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2}{[\alpha \mapsto \tau](\gamma_1 \circ \gamma_2) \triangleright \gamma'_1 \circ \gamma'_2} \text{ ACSUBST\_COMP} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2}{[\alpha \mapsto \tau](\gamma_1 \rightarrow \gamma_2) \triangleright \gamma'_1 \rightarrow \gamma'_2} \text{ ACSUBST\_ARROW} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma'_1 \quad [\alpha \mapsto \tau]\gamma_2 \triangleright \gamma'_2 \quad [\alpha \mapsto \tau]\gamma_3 \triangleright \gamma'_3}{[\alpha \mapsto \tau](\{\gamma_1 \sim \gamma_2\} \rightarrow \gamma_3) \triangleright (\{\gamma'_1 \sim \gamma'_2\} \rightarrow \gamma'_3)} \text{ ACSUBST\_CARROW} \\
\frac{\alpha_1 \neq \alpha_2 \quad [\alpha_1 \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha_1 \mapsto \tau](\forall(\alpha_2 : *), \gamma_1) \triangleright (\forall(\alpha_2 : *), \gamma_2)} \text{ ACSUBST\_FORALL} \\
\frac{[\alpha \mapsto \tau_1]\gamma_1 \triangleright \gamma_2 \quad [\alpha \mapsto \tau_1]\tau_2 \triangleright \tau_3}{[\alpha \mapsto \tau_1](\gamma_1 @ \tau_2) \triangleright \gamma_2 @ \tau_3} \text{ ACSUBST\_INST} \\
\frac{[\alpha \mapsto \tau]\gamma_1 \triangleright \gamma_2}{[\alpha \mapsto \tau](\mathbf{elim}_i \gamma_1) \triangleright \mathbf{elim}_i \gamma_2} \text{ ACSUBST\_ELIM}
\end{array}$$

 $\boxed{[c \mapsto \gamma]t_1 \triangleright t_2}$ 

substitution of coercion variable in term

$$\begin{array}{c}
\frac{}{[c \mapsto \gamma]x \triangleright x} \text{ CTSUBST\_VAR} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](\lambda(x : \tau) \Rightarrow t_1) \triangleright (\lambda(x : \tau) \Rightarrow t_2)} \text{ CTSUBST\_ABS} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](\lambda\{\alpha : *\} \Rightarrow t_1) \triangleright (\lambda\{\alpha : *\} \Rightarrow t_2)} \text{ CTSUBST\_TABS} \\
\frac{}{[c \mapsto \gamma](\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t) \triangleright (\lambda\{c : \tau_1 \sim \tau_2\} \Rightarrow t)} \text{ CTSUBST\_CABS1} \\
\frac{c_1 \neq c_2 \quad [c_1 \mapsto \gamma]t_1 \triangleright t_2}{[c_1 \mapsto \gamma](\lambda\{c_2 : \tau_1 \sim \tau_2\} \Rightarrow t_1) \triangleright (\lambda\{c_2 : \tau_1 \sim \tau_2\} \Rightarrow t_2)} \text{ CTSUBST\_CABS2} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t'_1 \quad [c \mapsto \gamma]t_2 \triangleright t'_2}{[c \mapsto \gamma](t_1 t_2) \triangleright t'_1 t'_2} \text{ CTSUBST\_APP} \\
\frac{[c \mapsto \gamma]t_1 \triangleright t_2}{[c \mapsto \gamma](t_1 [\tau]) \triangleright (t_2 [\tau])} \text{ CTSUBST\_TAPP} \\
\frac{[c \mapsto \gamma_1]t_1 \triangleright t_2 \quad [c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2}{[c \mapsto \gamma_1](t_1 \sim [\gamma_2]) \triangleright (t_2 \sim [\gamma'_2])} \text{ CTSUBST\_CAPP} \\
\frac{[c \mapsto \gamma_1]t_1 \triangleright t_2 \quad [c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2}{[c \mapsto \gamma_1](t_1 \blacktriangleright \gamma_2) \triangleright (t_2 \blacktriangleright \gamma'_2)} \text{ CTSUBST\_COERCE}
\end{array}$$

 $\boxed{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}$ 

substitution of coercion variable in coercion term

$$\begin{array}{c}
\frac{}{[c \mapsto \gamma]c \triangleright \gamma} \quad \text{CCSUBST\_VAR1} \\
\\
\frac{c_1 \neq c_2}{[c_1 \mapsto \gamma]c_2 \triangleright c_2} \quad \text{CCSUBST\_VAR2} \\
\\
\frac{}{[c \mapsto \gamma](\mathbf{refl} \tau) \triangleright \mathbf{refl} \tau} \quad \text{CCSUBST\_REFL} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\mathbf{sym} \gamma_2) \triangleright \mathbf{sym} \gamma_3} \quad \text{CCSUBST\_SYM} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3}{[c \mapsto \gamma_1](\gamma_2 \circ \gamma_3) \triangleright \gamma'_2 \circ \gamma'_3} \quad \text{CCSUBST\_COMP} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3}{[c \mapsto \gamma_1](\gamma_2 \rightarrow \gamma_3) \triangleright \gamma'_2 \rightarrow \gamma'_3} \quad \text{CCSUBST\_ARROW} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma'_2 \quad [c \mapsto \gamma_1]\gamma_3 \triangleright \gamma'_3 \quad [c \mapsto \gamma_1]\gamma_4 \triangleright \gamma'_4}{[c \mapsto \gamma_1](\{\gamma_2 \sim \gamma_3\} \rightarrow \gamma_4) \triangleright (\{\gamma'_2 \sim \gamma'_3\} \rightarrow \gamma'_4)} \quad \text{CCSUBST\_CARROW} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\forall (\alpha : *), \gamma_2) \triangleright (\forall (\alpha : *), \gamma_3)} \quad \text{CCSUBST\_FORALL} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\gamma_2 @ \tau) \triangleright \gamma_3 @ \tau} \quad \text{CCSUBST\_INST} \\
\\
\frac{[c \mapsto \gamma_1]\gamma_2 \triangleright \gamma_3}{[c \mapsto \gamma_1](\mathbf{elim}_i \gamma_2) \triangleright \mathbf{elim}_i \gamma_3} \quad \text{CCSUBST\_ELIM}
\end{array}$$

Definition rules: 84 good 0 bad  
 Definition rule clauses: 162 good 0 bad