The Three Body Problem Arenstorf orbit

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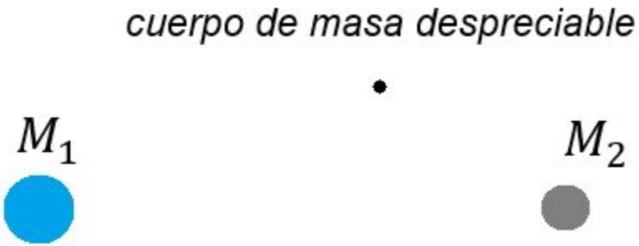
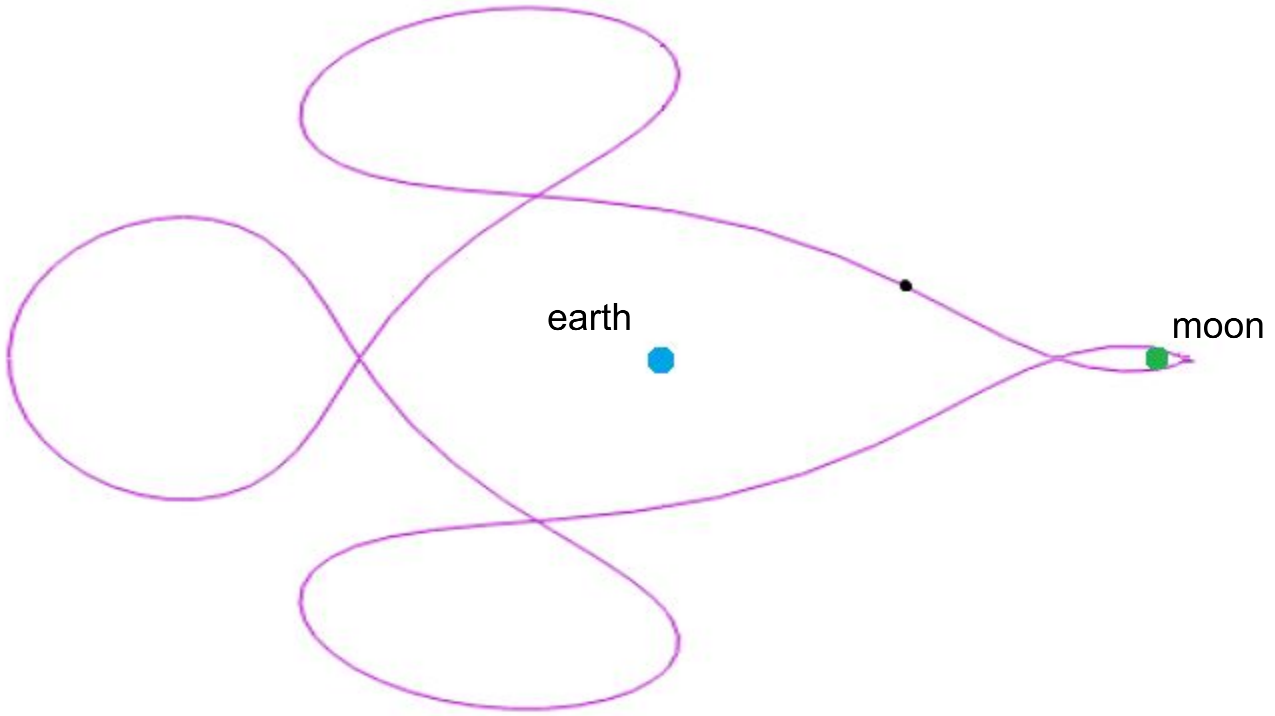


$$\frac{M_1}{M_2} = \mu \quad \mu : (1 - \mu)$$


earth
moon

Ecuaciones de movimiento

$$\ddot{x} = x + 2\dot{y} - \frac{(1-\mu)(x+\mu)}{r^3} - \frac{\mu(x-1+\mu)}{s^3},$$

$$\ddot{y} = y - 2\dot{x} - \frac{(1-\mu)y}{r^3} - \frac{\mu y}{s^3},$$

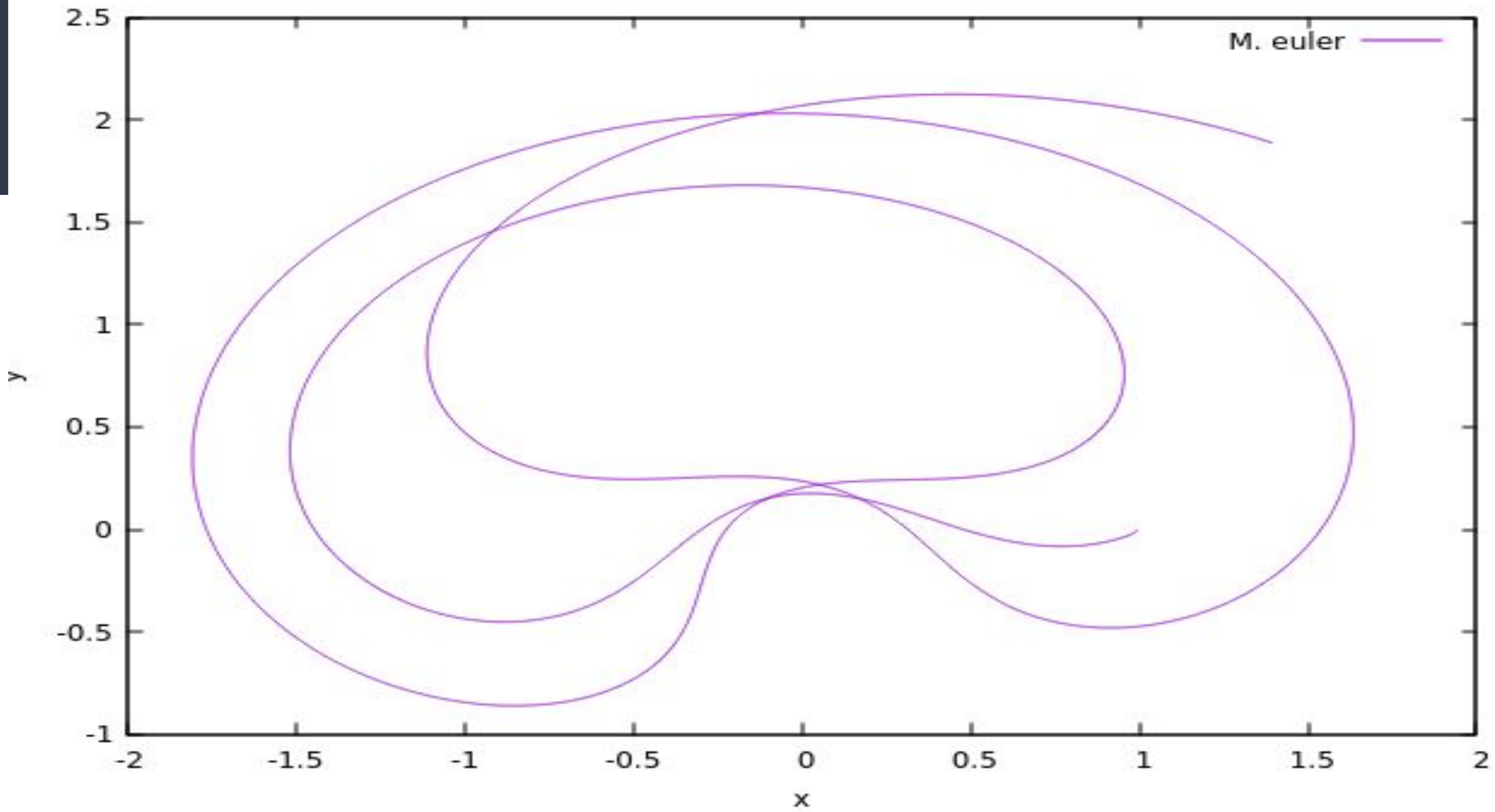
$$\ddot{z} = -\frac{(1-\mu)z}{r^3} - \frac{\mu z}{s^3},$$

$$r = \sqrt{(x+\mu)^2 + y^2 + z^2}, \qquad s = \sqrt{(x-1+\mu)^2 + y^2 + z^2}.$$

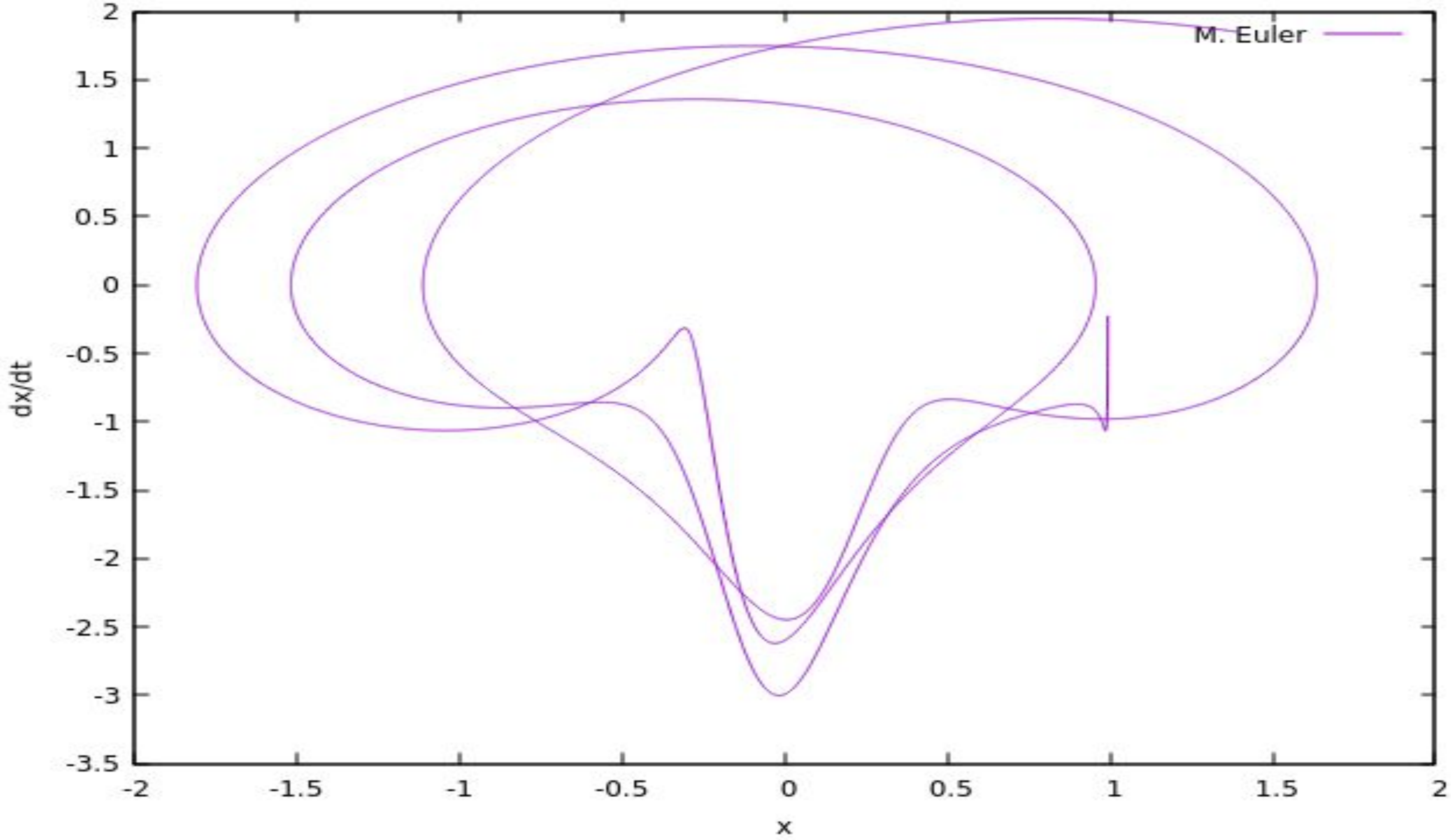
Método de Euler

$$\frac{d\mathbf{y}(t)}{dt} \simeq \frac{\mathbf{y}(t_{n+1}) - \mathbf{y}(t_n)}{h} = \mathbf{f}(t, \mathbf{y}),$$

$$\mathbf{y}_{n+1} \simeq \mathbf{y}_n + h\mathbf{f}(t_n, \mathbf{y}_n),$$



M. euler
2.5
2
1.5
1
0.5
0
-0.5
-1
y
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x



M. Euler
dx/dt
x
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Runge-Kutta de 4 orden

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4),$$

$$\mathbf{k}_1 = h\mathbf{f}(t_n, \mathbf{y}_n),$$

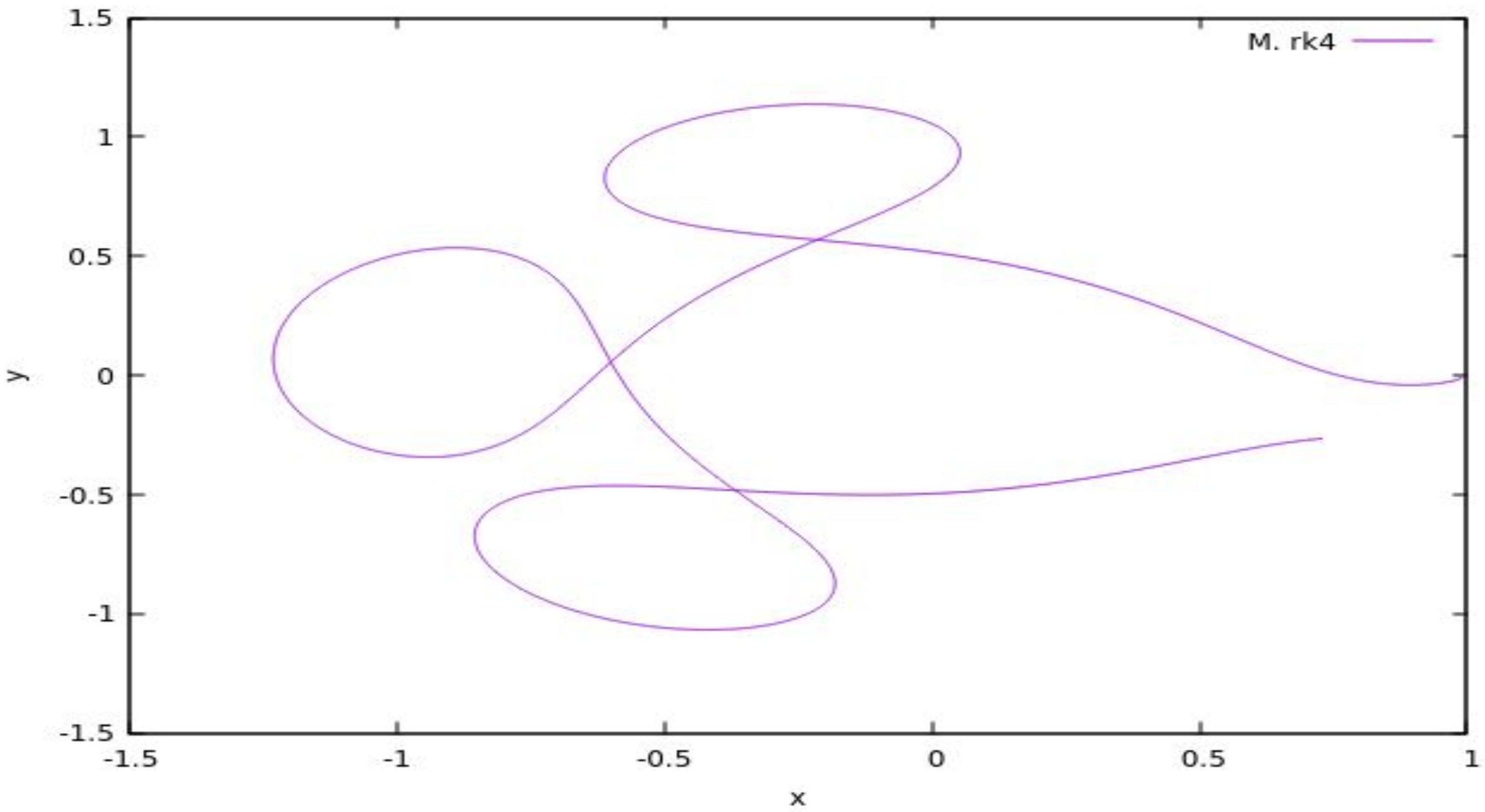
$$\mathbf{k}_2 = h\mathbf{f}\left(t_n + \frac{h}{2}, \mathbf{y}_n + \frac{\mathbf{k}_1}{2}\right),$$

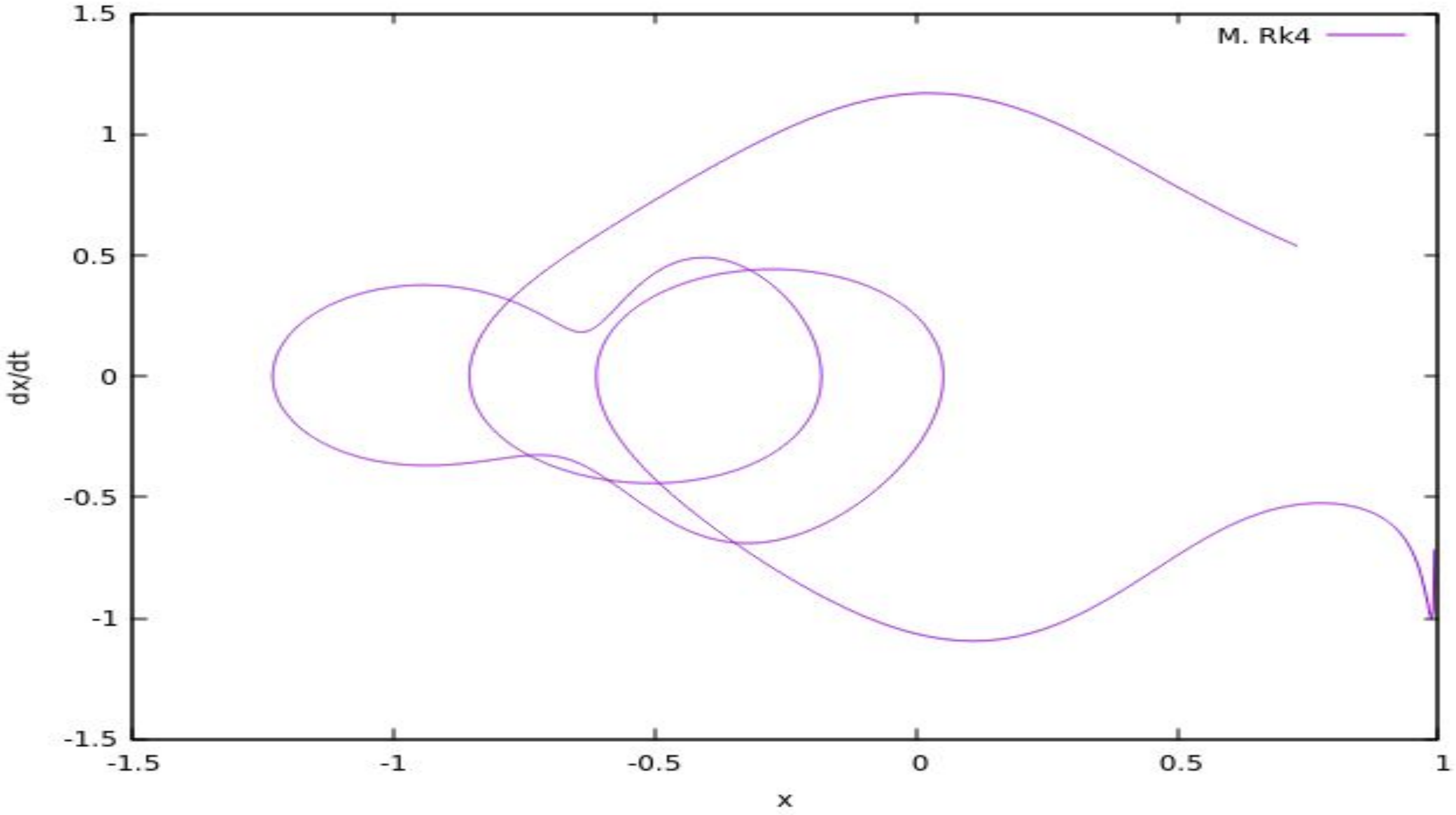
$$\mathbf{k}_3 = h\mathbf{f}\left(t_n + \frac{h}{2}, \mathbf{y}_n + \frac{\mathbf{k}_2}{2}\right),$$

$$\mathbf{k}_4 = h\mathbf{f}(t_n + h, \mathbf{y}_n + \mathbf{k}_3).$$

Matriz de Butcher

$$\begin{array}{r|rrrr}
0 & & & & \\
1/2 & 1/2 & & & \\
1/2 & 0 & 1/2 & & \\
1 & 0 & 0 & 1 & \\
\hline
 & 1/6 & 2/6 & 2/6 & 1/6
\end{array}$$


M. rk4
1.5
1
0.5
0
-0.5
-1
-1.5
y
-1.5
-1
-0.5
0
0.5
1
x


M. Rk4
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
dx/dt
x

Método Dormand–Prince 5(4)

$$
\begin{aligned}
k_1 &= hf(t_k, y_k),\\
k_2 &= hf(t_k + \frac{1}{5}h, y_k + \frac{1}{5}k_1),\\
k_3 &= hf(t_k + \frac{3}{10}h, y_k + \frac{3}{40}k_1 + \frac{9}{40}k_2),\\
k_4 &= hf(t_k + \frac{4}{5}h, y_k + \frac{44}{45}k_1 - \frac{56}{15}k_2 + \frac{32}{9}k_3),\\
k_5 &= hf(t_k + \frac{8}{9}h, y_k + \frac{19372}{6561}k_1 - \frac{25360}{2187}k_2 + \frac{64448}{6561}k_3 - \frac{212}{729}k_4),\\
k_6 &= hf(t_k + h, y_k + \frac{9017}{3168}k_1 - \frac{355}{33}k_2 - \frac{46732}{5247}k_3 + \frac{49}{176}k_4 - \frac{5103}{18656}k_5),\\
k_7 &= hf(t_k + h, y_k + \frac{35}{384}k_1 + \frac{500}{1113}k_3 + \frac{125}{192}k_4 - \frac{2187}{6784}k_5 + \frac{11}{84}k_6).
\end{aligned}
$$

Runge-Kutta 4

$$y_{k+1} = y_k + \frac{35}{384}k_1 + \frac{500}{1113}k_3 + \frac{125}{192}k_4 - \frac{2187}{6784}k_5 + \frac{11}{84}k_6.$$

Runge-Kutta 5

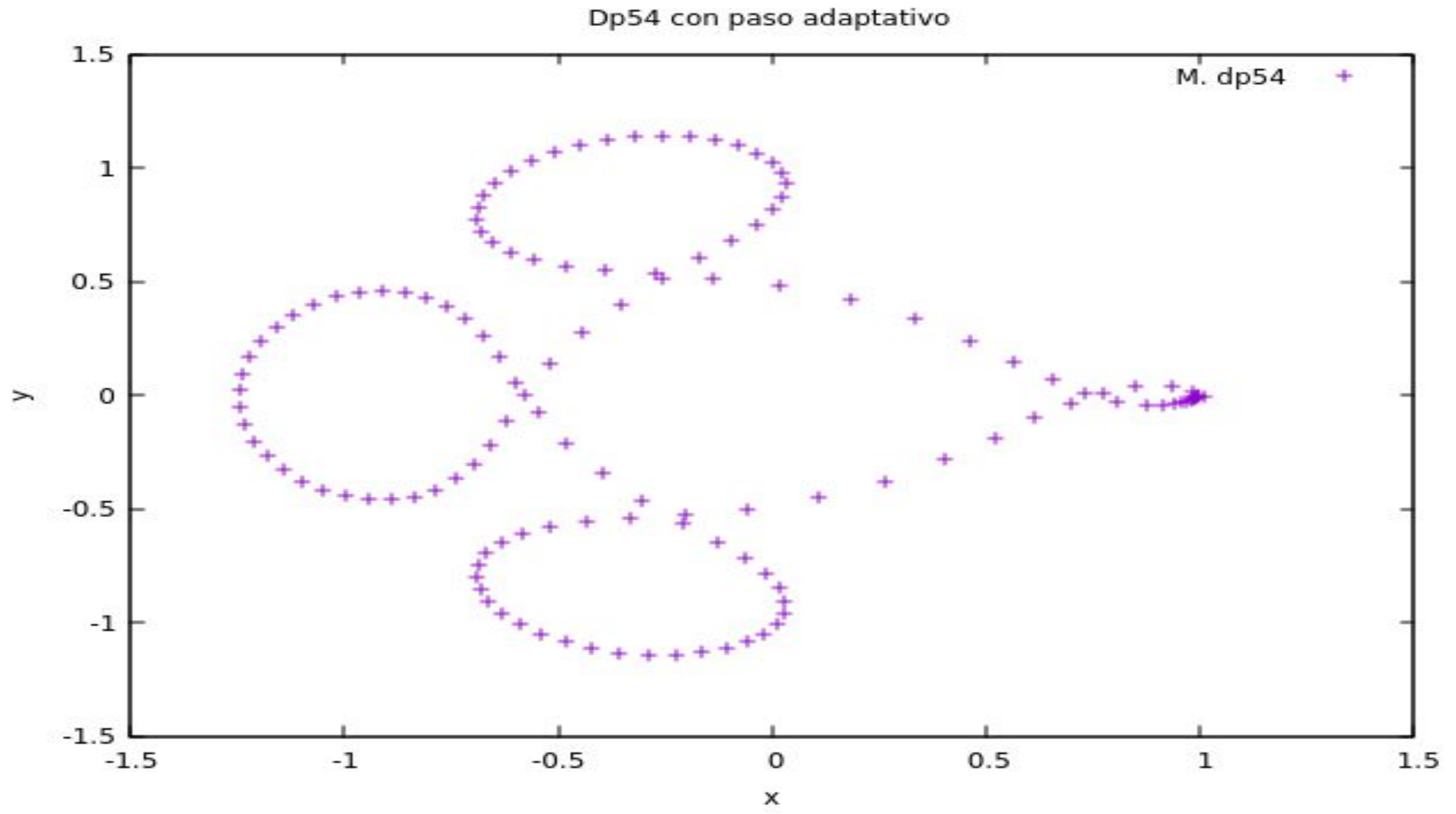
$$z_{k+1} = y_k + \frac{5179}{57600}k_1 + \frac{7571}{16695}k_3 + \frac{393}{640}k_4 - \frac{92097}{339200}k_5 + \frac{187}{2100}k_6 + \frac{1}{40}k_7.$$

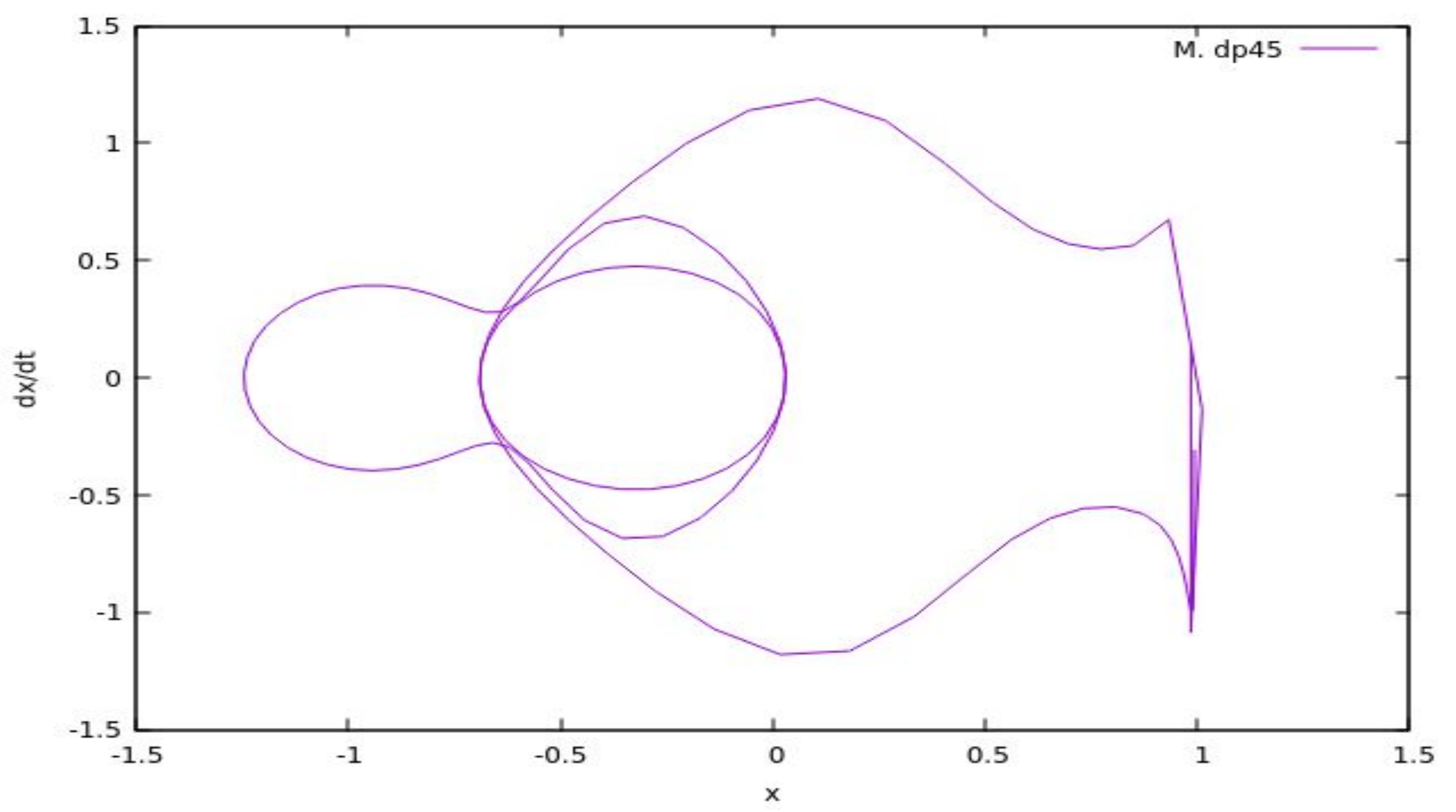
Paso adaptativo

$$|z_{k+1} - y_{k+1}| = |\frac{71}{57600}k_1 - \frac{71}{16695}k_3 + \frac{71}{1920}k_4 - \frac{17253}{339200}k_5 + \frac{22}{525}k_6 - \frac{1}{40}k_7|.$$

```
double dtnew(const double p, const double v, const double dt)
{
  double hmin=0.0001;
  double hmax=0.137;
  double sprom = (saux(p,dt) + saux(v,dt))*0.5;
  if(dt*sprom <= hmin){
    return hmin;
  }
  if(dt*sprom >= hmax){
    return hmax;
    }
  else{
    return dt*sprom;
  }
}
```

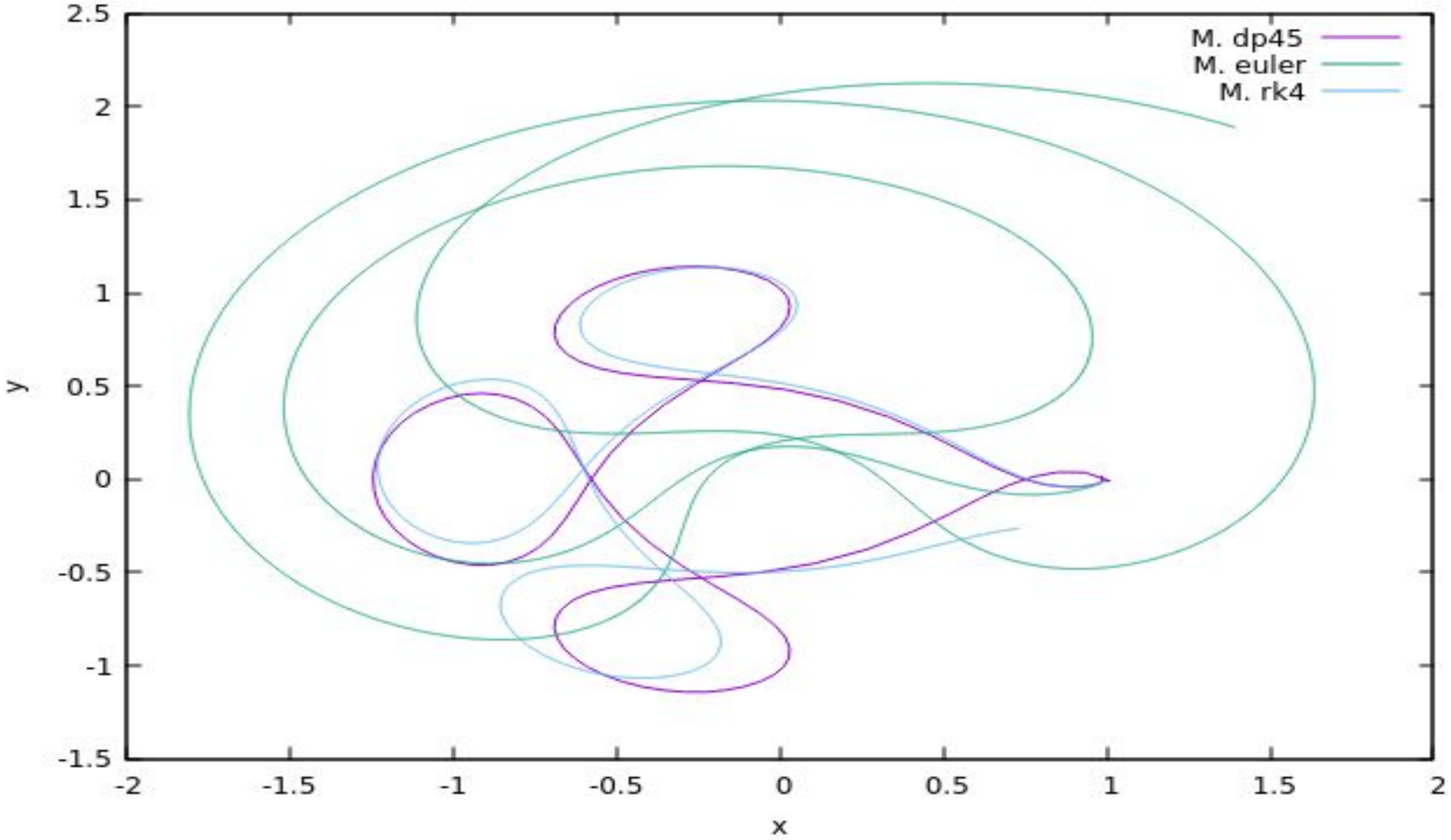
$$\begin{aligned} s &= \left(\frac{\epsilon \mathrm{dt}}{2|z_{k+1} - y_{k+1}|}\right)^{\frac{1}{5}} \\ \mathrm{dt}_{opt} &= s\mathrm{dt} \end{aligned}$$


Dp54 con paso adaptativo
M. dp54
1.5
1
0.5
0
-0.5
-1
-1.5
y
-1.5
-1
-0.5
0
0.5
1
1.5
x


M. dp45
1.5
1
0.5
0
-0.5
-1
-1.5
dx/dt
-1.5
-1
-0.5
0
0.5
1
1.5
x

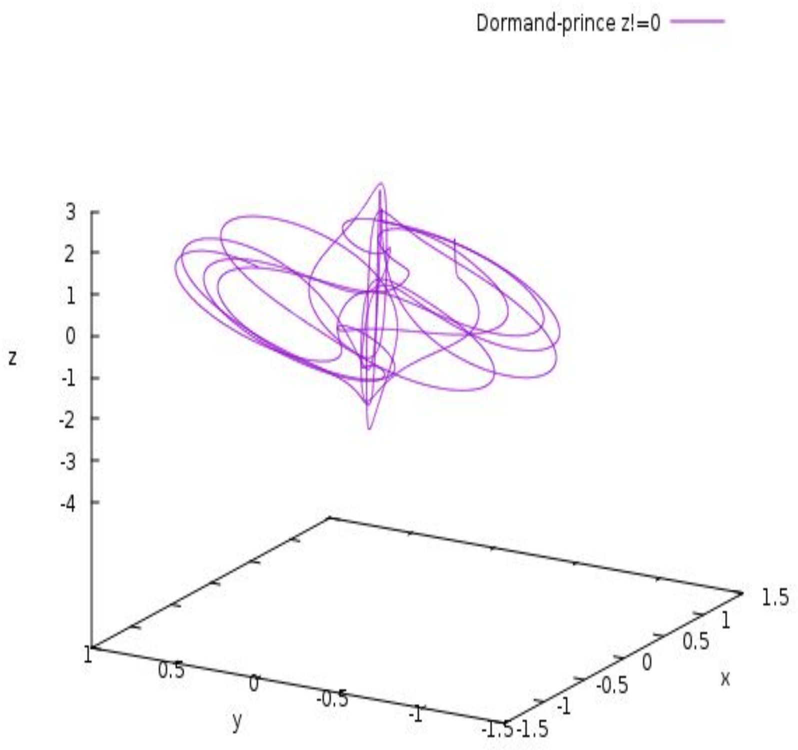
Cantidad de iteraciones con cada método para obtener una órbita completa.

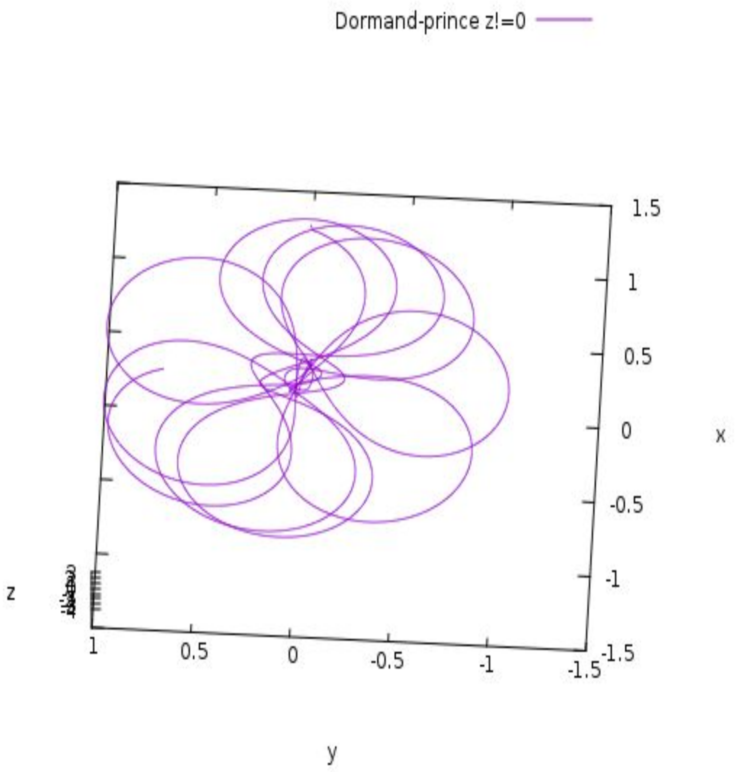
M. Euler , 24.000 iteraciones
M. Rk4   , 5.977 iteraciones
M. Dp5(4), 134 iteraciones

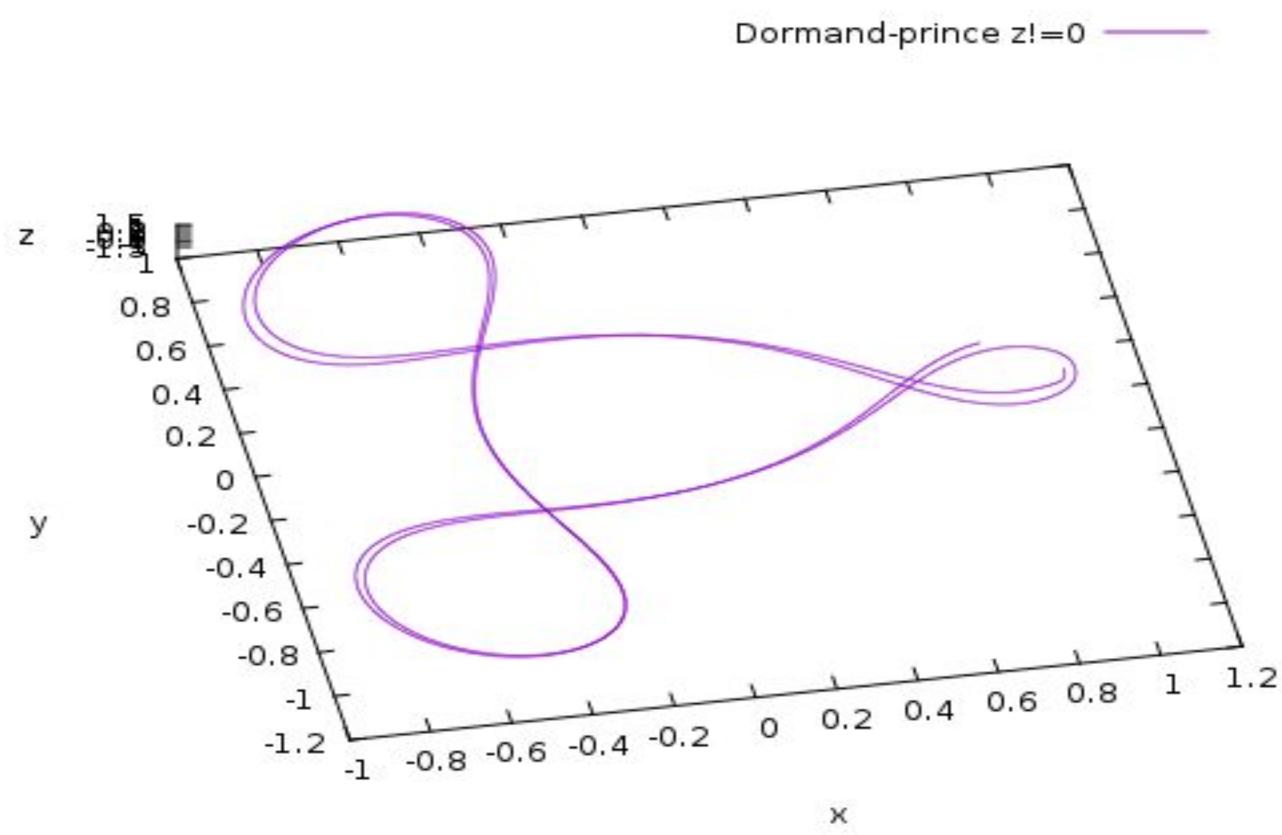


M. dp45
M. euler
M. rk4
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x
y

Órbitas más generales







Dormand-prince z!=0
z
y
x
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2

Bibliografía

- Simon Sirca and Martin Horvat. Computational Methods for Physicists. Springer Berlin Heidelberg, 2012.
- Toshinori Kimura, On Dormand-Prince Method, 2009.
- logotipo NASA. https://commons.wikimedia.org/wiki/File:NASA_logo.svg;