

### Homework 3

Due Date : Oct 4, 2016

1. Write down the PDF of a chi-squared random variable with  $v$  degrees of freedom. Then, for the special cases  $v = 2$  and  $v = 4$ , obtain the CDFs in the simplest form.
2. Let  $X$  be a binomial random variable with parameters  $n$  and  $p$ . Show that

$$E\left[\frac{1}{X+1}\right] = \frac{1 - (1-p)^{n+1}}{(n+1)p}.$$

3. Let  $X = 1$  denote the event that a student passes IMEN366 and  $X = 0$  denote the event that he/she fails IMEN366. Likewise let  $Y = 1$  denote the event that a student passes IMEN472 and  $Y = 0$  denote the event that he/she fails IMEN472. The joint PMF of  $X$  and  $Y$  is given by  $P[X = 0, Y = 0] = 0.15$ ,  $P[X = 0, Y = 1] = 0.05$ ,  $P[X = 1, Y = 0] = 0.1$ , and  $P[X = 1, Y = 1] = c$ . Compute the following
  - (a) What is the value of  $c$ ?
  - (b) What is the probability that the student passes IMEN472?
  - (c) Given a student passes IMEN472, what is the probability he or she passed IMEN366?
4. The time to manufacture a part on a particular machine is random and according to an Erlang distribution with mean 12 minutes and standard deviation 6 minutes. What is the probability that it would take more than 15 minutes to manufacture a part on this machine?
5. If  $E[X] = 1$  and  $\text{Var}[X] = 5$  for a random variable  $X$ , find
  - (a)  $E[(2 + X)^2]$
  - (b)  $\text{Var}[4 + 3X]$