PSY 3307

Two Sample t-test

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Agenda

- Review terms
  - one-sample t test, t-distribution, df, confidence intervals
- Understanding a two-sample study
- Between & Within Designs
- Independent-samples t-test
- Performing an independent samples t-test
- Effect Size

Review

- One-sample t-test
- t-distribution
- df
- confidence intervals

Between & Within Designs

- Experiments can be broken down into two different types of designs
- **Between-subject/group** design is when you are interested in comparing two (for now) or more groups on an outcome variable
- **Within-subject/group** design is when you have the same participants but you test them twice (either with two different variables or two different time points)

Two tests we are talking about

- **independent samples t-test** is when there are two groups of participants are separated into two different conditions to compare based on that condition
    - comparing the physical activity levels (DV) of sexes (Condition 1 = Male, Condition 2 = Female)
    - parametric test
- **paired-samples t-test** is when there are two experimental conditions that the same participants take part in
    - interested in two variables in the same sample of participants
    - can be the same variable and two different time points
    - bmi levels before an experiment and after the experiment for all participants
    - parametric test

Independent Samples t-test

- JP note: probably the most often used t-test
- because it is a parametric test, it has assumptions
- Assumptions are
    - DV is normally distributed interval/ratio scores
    - populations have homogeneous variance
    - not a true assumption but something important to note is that your groups should be equal in n (condition) size

Homogeneity of Variance

- **homogeneity of variance** is when the variances of the populations represented in a study have "equal" variances
- in order to test that the variances are equal, we can look at it through visuals
  - however, a better option is to use the Levene's test

Independent samples t-test

- hypotheses are now focused on the differences between the two groups/conditions

$$H0 : \mu_1 - \mu_2 = 0$$

H0: There will be no difference in DV scores between group 1 and group 2.

- both samples/groups represent the population

$$H1 : \mu_1 - \mu_2 \neq 0$$

H1: There will be differences in DV scores between group 1 and group 2.

- the groups represent a different population or don't represent the current population

t-distribution for independent samples t-test

- we are interested in the difference between our group/sample means
- we have two samples from one raw score population
- **sampling distribution of differences between means** show all differences between two means that occur when random samples are drawn from a population of scores
- the mean of the sampling distribution is zero because both sample means will equal the population mean of the raw score population

Independent samples t-test

- determines the probability of obtaining our difference between our means when H0 is true
- Term changes
    - N is now the full sample size
    - n is the size of each group/sample
    - so for each group/sample, we have an n

Performing the indepdendent samples t-test

$$s_x^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N-1}$$

```r
male_scores = c(4, 6, 2, 3, 5, 1, 2, 4, 3, 5)
female_scores = c(4, 6, 9, 6, 5, 8, 2, 5, 3, 7)

male_scores
```

```
##  [1] 4 6 2 3 5 1 2 4 3 5
```

```r
female_scores
```

```
##  [1] 4 6 9 6 5 8 2 5 3 7
```

First we'll calculate the variance

```r
# male sum
4+6+2+3+5+1+2+4+3+5
```

```
## [1] 35
```

```r
# sum is 35
```

$$s_{x_1}^2 = \frac{\Sigma X^2 - \frac{(35)^2}{N}}{N-1}$$

```r
# female sum
4+6+9+6+5+8+2+5+3+7
```

```
## [1] 55
```

```r
# sum 55
```

$$s^2_{x_2} = \frac{\Sigma X^2 - \frac{(55)^2}{N}}{N-1}$$

```r
35/10
```

```
## [1] 3.5
```

```r
# male mean 3.5
```

```r
55/10
```

```
## [1] 5.5
```

```r
# female mean 5.5
```

```r
# male sum of squared Xs
4^2+6^2+2^2+3^2+5^2+1^2+2^2+4^2+3^2+5^2
```

```
## [1] 145
```

```r
# 145
```

$$s_{x_1}^2 = \frac{145 - \frac{(35)^2}{10}}{10 - 1}$$

```r
# female sum of squared Xs
4^2+6^2+9^2+6^2+5^2+8^2+2^2+5^2+3^2+7^2
```

```
## [1] 345
```

```r
# female 345
```

$$s_{x_2}^2 = \frac{345 - \frac{(55)^2}{10}}{10 - 1}$$

```r
# male sum of X squared and divided by N
35^2
```

```
## [1] 1225
```

```r
1225/10
```

```
## [1] 122.5
```

```r
# 122.5
```

$$s_{x_1}^2 = \frac{145 - \frac{1225}{10}}{10 - 1}$$

```r
# female sum of X squared and divided by N
55^2
```

```
## [1] 3025
```

```r
3025/10
```

```
## [1] 302.5
```

```r
# 302.5
```

$$s_{x_2}^2 = \frac{345 - \frac{302.5}{10}}{10 - 1}$$

```r
# male variance calculations
(145 - 122.5)/(10-1)
```

```
## [1] 2.5
```

```r
# variance is 2.5
```

$$ \;s^2{x{1}} = \frac{145 - 122.5}{10 - 1}$$

```r
# female variance calculations
(345 - 302.5)/(10 - 1)
```

```
## [1] 4.722222
```

```r
# variance is 4.72
```

$$ \;s^2{x{2}} = \frac{345 - 302.5}{10 - 1}$$

```r
sd(male_scores)^2
```

```
## [1] 2.5
```

```r
sd(female_scores)^2
```

```
## [1] 4.722222
```

New Terms

- **pooled variance** is the weighted average variance of the groups'/samples' variances in a independent samples t-test
- **standard error of the difference** is the estimated standard deviation of the sampling distribution of differences between the means

Now we can calculate the pooled variance n1 = 10 n2 = 10 variance of group 1 = 2.5 variance of group 2 = 4.72

$$S^2_{pool} = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

```r
# start with the numerator
(10 - 1)*2.5 + (10 - 1)*4.72
```

```
## [1] 64.98
```

```r
# numerator is 64.98

# denominator
(10 - 1) + (10 - 1)
```

```
## [1] 18
```

```r
# denominator is 18
```

$$S^2_{pool} = \frac{(10-1)2.5 + (10-1)4.72}{(10-1)+(10-1)}$$

```r
9*2.5 + 9*4.72
```

```
## [1] 64.98
```

```r
# 64.98

9+9
```

```
## [1] 18
```

```r
# 18
```

$$S^2_{pool} = \frac{(9)2.5 + (9)4.72}{9 + 9}$$

```r
64.98/18
```

```
## [1] 3.61
```

```r
# pooled variance is 3.61
```

$$S^2_{pool} = \frac{64.98}{18}$$

Let's calculate for the standard error of the difference

$$S_{\overline{X_1} - \overline{X_2}} = \sqrt{(S^2_{pool})(\frac{1}{n_1} + \frac{1}{n_2})}$$

```r
1/10
```

```
## [1] 0.1
```

```r
3.61*(.1 + .1)
```

```
## [1] 0.722
```

```r
sqrt(.72)
```

```
## [1] 0.8485281
```

$$S_{\overline{X_1}-\overline{X_2}} = \sqrt{(3.61)(\frac{1}{10} + \frac{1}{10})}$$

$$S_{\overline{X_1}-\overline{X_2}} = \sqrt{(3.61)(.1 + .1)}$$

```r
3.61*(.1+.1)
```

```
## [1] 0.722
```

```r
# se is .72
```

$$S_{\overline{X_1}-\overline{X_2}} = \sqrt{.72}$$

```r
sqrt(.72)
```

```
## [1] 0.8485281
```

```r
# se of the difference is .85
```

Now we can calculate the independent samples t-test obtained value

Note the population mean 1 minus the population mean 2 is what is specified in the null hypothesis, so it will be zero

```r
((3.5 - 5.5) - 0)/.85
```

```
## [1] -2.352941
```

```r
# t obtained value is -2.35
```

$$t_{obt} = \frac{(3.5 - 5.5) - 0}{.85}$$

Let's now calculate the degrees of freedom

$$df = (n_1 - 1) + (n_2 - 1)$$

```r
(10 - 1) + (10 - 1)
```

```
## [1] 18
```

```r
# df is 18

# t critical is +-2.101
```

$$df = (10 - 1) + (10 - 1)$$

So we get a value of -2.35 and the t-critical value is -2.101

Is there a statistically significant difference between the two groups?

-2.35 > -2.101

Now let's get confidence intervals

```r
# group 1 mean = 3.5
# group 2 mean = 5.5
# t critical value is 2.101
# n1 = 10
# n2 = 10
# variance of group 1 = 2.5
# variance of group 2 = 4.72

# lower
(3.5 - 5.5) - 2.101 * sqrt((2.5/10) + (4.72/10))
```

```
## [1] -3.785232
```

```r
# -3.79

# upper
(3.5 - 5.5) + 2.101 * sqrt((2.5/10) + (4.72/10))
```

```
## [1] -0.214768
```

```r
# -.21
```

Effect Sizes

- Reminder: r effect sizes are .1 = small, .3 = medium, .5 = large
- Reminder: cohen's d effect sizes are .2 = small, .5 = medium, .8 = large
- these are both measures of the strength of a relationship
  - better than simply using p value alone
- cohen's d can never be negative so the value you get is the absolute value (.e.g., its always positive)

$$\hat{d} = \frac{(\overline{X_1} - \overline{X_2})}{S_2}$$

```r
(3.5 - 5.5)/sqrt(4.72)
```

```
## [1] -0.9205746
```

```r
# each step below
3.5 - 5.5
```

```
## [1] -2
```

```r
-2/sqrt(4.72)
```

```
## [1] -0.9205746
```

```r
# cohen's d is .92 or the number of standard deviations between the mear
```

$$\hat{d} = \frac{(3.5 - 5.5)}{\sqrt{4.72}}$$

```r
(-2.78)^2
```

```
## [1] 7.7284
```

```r
(-2.78)^2 + 18
```

```
## [1] 25.7284
```

```r
sqrt(7.73/25.73)
```

```
## [1] 0.5481127
```

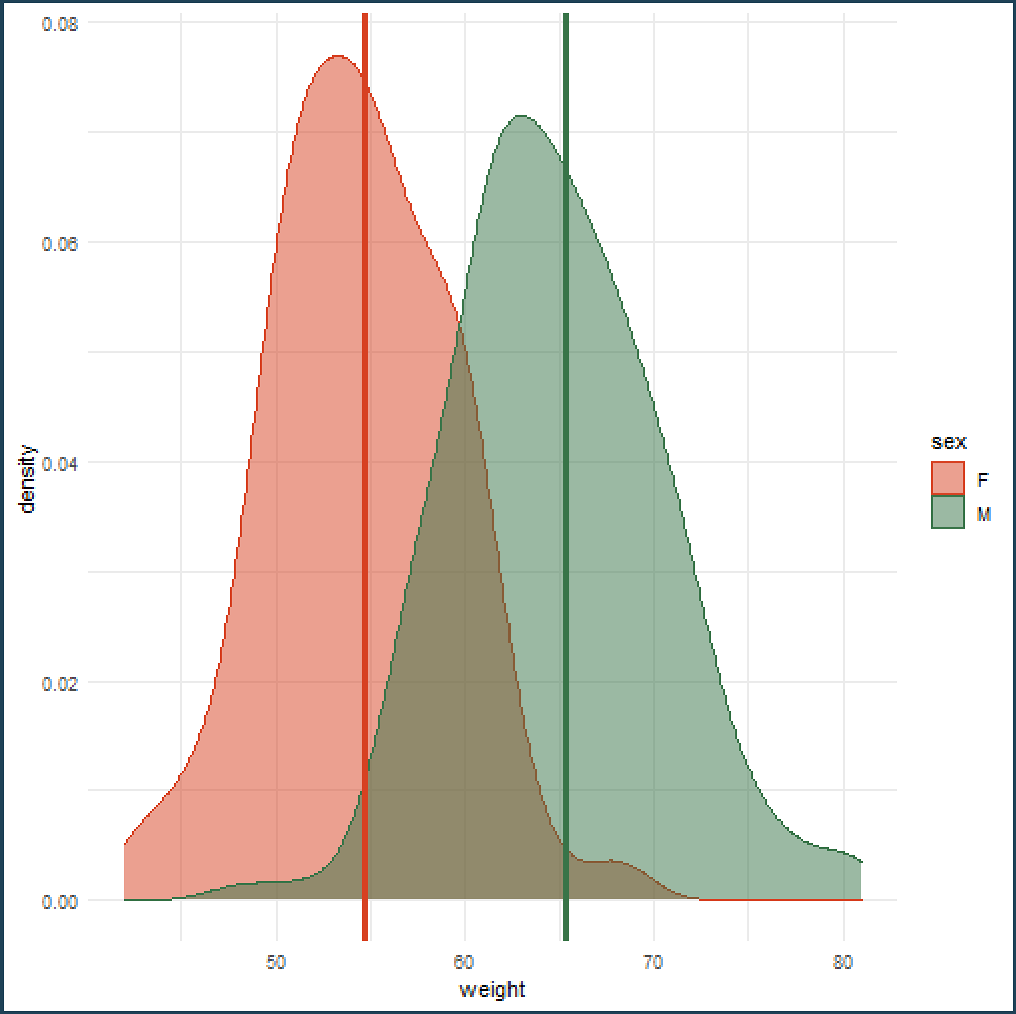
```r
# r value of .55
```

Steps for independent samples t-test

1. Get the means of both groups/samples
2. Get the variances of both groups/samples
3. Get the group/sample sizes (n)
4. Get the pooled variance by getting the groups'/samples' variances averaged
5. Get the standard error of the differences
6. Calculate the t-obtained value
7. Get the degrees of freedom
8. Calculate the confidence intervals
9. Get the effect size

Independent samples t-test
Example

```
##
##  Descriptive statistics by group
## group: F
##    vars   n  mean   sd median trimmed  mad min max range skew kurtosis   se
## X1    1 200 54.52 4.83     54   54.52 4.45  42  69    27 0.07     0.02 0.34
## ------------------------------------------------------------
## group: M
##    vars   n  mean   sd median trimmed  mad min max range skew kurtosis   se
## X1    1 200 64.87 5.44   64.5   64.71 5.19  48  81    33 0.25     0.28 0.38
```

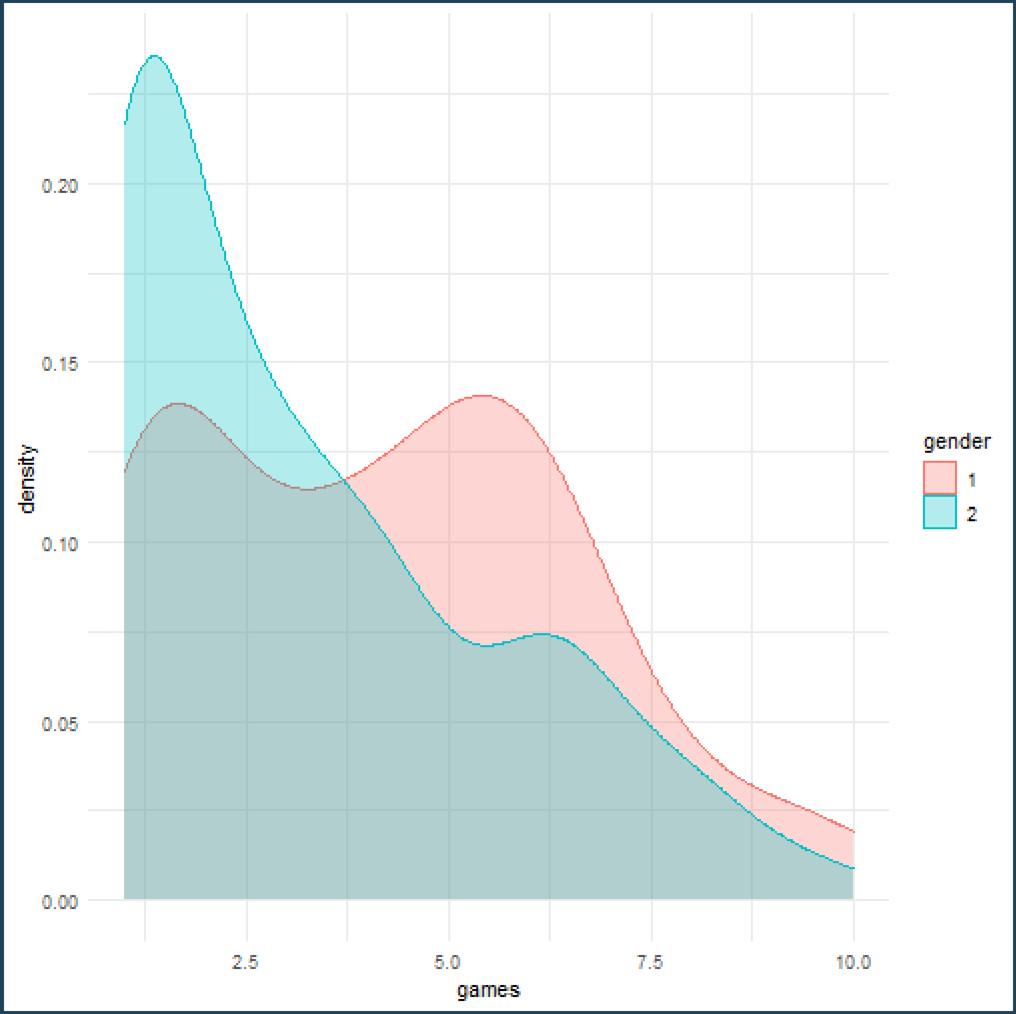


0.08
0.06
0.04
0.02
0.00
density
50
60
70
80
weight
sex
F
M

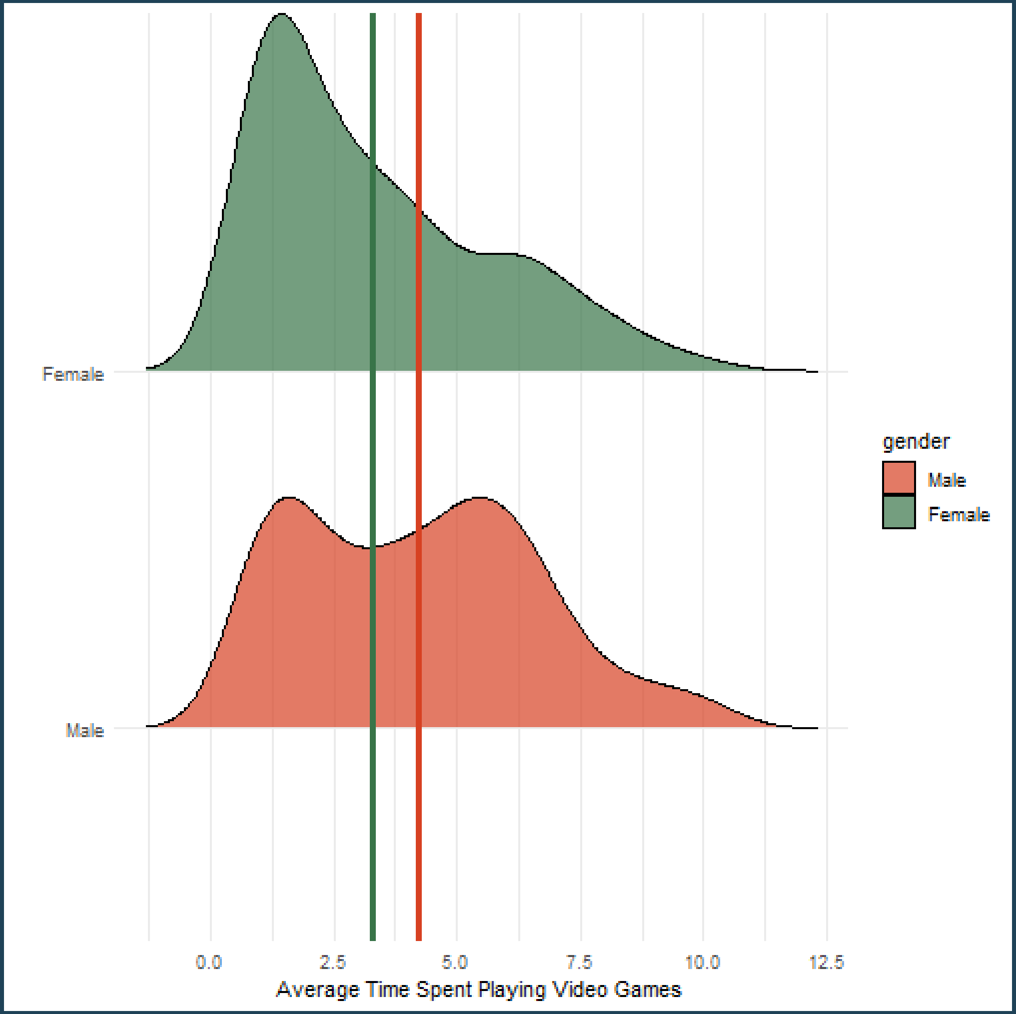
```
## Levene's Test for Homogeneity of Variance (center = median)
##        Df F value Pr(>F)
## group   1   1.972  0.161
##       398

##
##     Two Sample t-test
##
## data:  weight by sex
## t = -20.116, df = 398, p-value < 0.00000000000000022
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -11.361529  -9.338471
## sample estimates:
## mean in group F mean in group M
##           54.52           64.87
```

Real life independent samples t-test

```
## # A tibble: 2 x 2
##   gender games
##   <fct>  <dbl>
## 1 1       4.23
## 2 2       3.30
```


0.20
0.15
0.10
0.05
0.00
density
2.5
5.0
7.5
10.0
games
gender
1
2

```
## Picking joint bandwidth of 0.763
```



```
## Levene's Test for Homogeneity of Variance (center = median)
##        Df F value Pr(>F)
## group   1  1.3319 0.2492
##       370

##
##     Two Sample t-test
##
## data:  games by gender
## t = 3.5171, df = 370, p-value = 0.0004906
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.4066502 1.4379978
## sample estimates:
## mean in group 1 mean in group 2
##        4.227011        3.304688
```

```
##    vars   n mean   sd median trimmed  mad min max range skew kurtosis   se
## X1    1 372 3.59 2.38      3    3.34 2.97   1  10     9 0.69    -0.52 0.12
```

Practice Time

```r
set.seed(093021)

mistakes_made_tutor = rnorm(10, mean = 1.5, sd = 1.4)
mistakes_made_control = rnorm(8, mean = 4.1, sd = 1)

mistakes_made_tutor
```

```
##  [1] 3.0495964 0.7931460 2.1999939 0.4106114 1.3852964 2.2035563 1.8161635
##  [8] 2.4894226 3.1436843 1.0698578
```

```r
mistakes_made_control
```

```
## [1] 5.347698 3.962054 5.072715 3.985207 4.945898 2.984944 5.004625 5.405361
```

```r
set.seed(093021)

translating_native_speaker = rnorm(9, mean = 20, sd = 4.7)
translating_non_native = rnorm(14, mean = 10, sd = .99)

translating_native_speaker
```

```
## [1] 25.20222 17.62699 22.34998 16.34277 19.61492 22.36194 21.06141 23.32163
## [9] 25.51808
```

```r
translating_non_native
```

```
##  [1]  9.695828 11.235221  9.863433 10.962988  9.886355 10.837439  8.896095
##  [8] 10.895579 11.292308  9.102475 12.603552  9.039360 10.388593  9.487347
```

```r
set.seed(093021)

first_gen_bmi = rnorm(6, mean = 22, sd = 2.2)
second_gen_bmi = rnorm(9, mean = 28, sd = 5)

first_gen_bmi
```

```
## [1] 24.43508 20.88923 23.09999 20.28810 21.81975 23.10559
```

```r
second_gen_bmi
```

```
## [1] 29.12916 31.53365 33.87030 26.46378 34.23849 27.31027 32.86358 27.42604
## [9] 32.22949
```