Paired-Samples t-test

Repeated Measures

- with a paired-samples t-test, we are interested in the sampling distribution of difference scores
  - JP: My mistake, I told you that it was for all t-tests last week
- since we are looking at pairs of scores for the same participants, we want the difference between the two scores

Paired-Samples t-test Equation

- when you want to test different conditions for all participants in a within-subjects design, you'd use a **paired-samples t-test**
  - Ex: reduce salt intake –> blood pressure –> blood pressure medication –> blood pressure

$$t = \frac{\overline{D} - \mu_D}{\sigma_{\overline{D}}}$$

$$t = \frac{\overline{D}}{\sigma_{\overline{D}}}$$

Paired-Samples t-test Equation

- this equation compares the mean difference between our samples to the difference that we would expect to find between population means divided by the standard error of the difference
- with the assumption that the differences between the population means is consistent with the null hypothesis, the difference between the two population means should be zero
- due to sampling variation, it is possible to get differences between the condition means that are large but it will happen infrequently

Paired-Samples t-test Equation

- the standard deviation of the sampling distribution of differences between means is the **standard error of differences**
  - small standard error suggests difference between the means is close to the population mean
  - large standard error suggests that differences between means will be more spread out/distant

Paired-Samples t-test Equation

- we should expect that the differences between means would be centered around zero

$$t = \frac{\overline{D}}{\sigma_{\overline{D}}} = \frac{\overline{D}}{\frac{S_D}{\sqrt{N}}}$$

Example

```r
data <- data.frame(participant = 1:5,
                   t1 = c(10, 8, 7, 9, 10),
                   t2 = c(5, 4, 2, 5, 3))
data
```

```
  participant t1 t2
1           1 10  5
2           2  8  4
3           3  7  2
4           4  9  5
5           5 10  3
```

Paired-Samples t-test Equation

- First, we must calculate the difference between every participants' first time point and second time point

```
10 - 5
```

```
[1] 5
```

```
8 - 4
```

```
[1] 4
```

```
7 - 2
```

```
[1] 5
```

```
9 - 5
```

```
[1] 4
```

```
10 - 3
```

Paired-Samples t-test Equation

```r
data$difference <- data$t1 - data$t2
data
```

```
  participant t1 t2 difference
1           1 10  5          5
2           2  8  4          4
3           3  7  2          5
4           4  9  5          4
5           5 10  3          7
```

Paired-Samples t-test Equation

- then we'll get the average of the difference scores

```
(5 + 4 + 5 + 4 + 7)/5
```

```
[1] 5
```

- now, we have an average score of 5, so we now need our sum of squares, df, variance/standard deviation to get a standard error of the differences

Paired-Samples t-test Equation

```
(5 - 5)^2 + (4 - 5)^2 + (5 - 5)^2 + (4 - 5)^2 + (7 - 5)^2
```

```
[1] 6
```

- sum of squares is 6
- df is 5 - 1= 4

Paired-Samples t-test Equation

```
6/4
```

```
[1] 1.5
```

- variance is 1.5

Paired-Samples t-test Equation

```
sqrt(1.5)
```

```
[1] 1.224745
```

- standard deviation is 1.22

Paired-Samples t-test Equation

- now we can fill in the rest of our formula

$$t = \frac{5}{\frac{1.22}{\sqrt{5}}}$$

Paired-Samples t-test Equation

```r
sqrt(5)
```

```
[1] 2.236068
```

$$t = \frac{5}{\frac{1.22}{2.24}}$$

Paired-Samples t-test Equation

```r
1.22/2.24
```

```
[1] 0.5446429
```

$$t = \frac{5}{0.54}$$

Paired-Samples t-test Equation

```r
5/.54
```

```
[1] 9.259259
```

$$t = 9.26$$

Paired-Samples t-test Equation

- could also use the variance with slight changes

$$t = \frac{5}{\sqrt{\frac{1.5}{\sqrt{5}}}}$$

Effect Sizes

- when your t-obtained value passes the t-critical value, you have a significant finding
  - ***this does not tell you the size of the relationship***
- you can convert your t-test into an r value (used for correlation, is standardized, we'll get there soon)

$$r = \sqrt{\frac{t^2}{t^2 + df}}$$

Effect Sizes

- for correlations
  - $.1 = \text{small}$
  - $.3 = \text{moderate}$
  - $.5 = \text{large}$

Example - Independent Samples

- from the last set of slides
- t = -.93
- df = 10

$$r = \sqrt{\frac{-.93^2}{-.93^2 + 10}}$$

Example - Independent Samples

```
-.93*-.93
```

```
[1] 0.8649
```

$$r = \sqrt{\frac{.86}{.86 + 10}}$$

Example - Independent Samples

```r
.86+10
```

```
[1] 10.86
```

$$r = \sqrt{\frac{.86}{10.86}}$$

Example - Independent Samples

```r
.86/10.86
```

```
[1] 0.07918969
```

$$r = \sqrt{.08}$$

Example - Independent Samples

```r
sqrt(.08)
```

```
[1] 0.2828427
```

$$r = .28$$

Example - Paired Samples

- t = 9.26
- df = 4

$$r = \sqrt{\frac{-9.26^2}{-9.26^2 + 4}}$$

Example - Paired Samples

```r
(-9.26)^2
```

```
[1] 85.7476
```

$$r = \sqrt{\frac{85.75}{85.75 + 4}}$$

Example - Paired Samples

```r
95.75+4
```

```
[1] 99.75
```

$$r = \sqrt{\frac{85.75}{89.75}}$$

Example - Paired Samples

```r
85.75/89.75
```

```
[1] 0.9554318
```

$$r = \sqrt{.96}$$

Example - Paired Samples

```r
sqrt(.96)
```

```
[1] 0.9797959
```

$$r = .98$$

Effect Sizes

- instead, you could calculate cohen's d, using the two means and the standard deviation of the control group

$$\hat{d} = \frac{\overline{X}_1 - \overline{X}_2}{S_2}$$

Example - Independent Samples

- X1 $= 4.5$
- X2 $= 5.5$
- S2 $= 1.87$

$$\hat{d} = \frac{4.5 - 5.5}{1.87}$$

Example - Independent Samples

```
4.5-5.5
```

```
[1] -1
```

$$\hat{d} = \frac{-1}{1.87}$$

Example - Independent Samples

```r
-1/1.87
```

```
[1] -0.5347594
```

$$\hat{d} = -.53$$

Example - Independent Samples

$$\hat{d} = .53$$

Example - Paired Samples

$$\hat{d} = \frac{\overline{X}_1 - \overline{X}_2}{S_2}$$

- We didn't calculate the means for both conditions
  - now we have to calculate the means for both conditions

Example - Paired Samples

```r
(10+8+7+9+10)/5 #t1
```

```
[1] 8.8
```

```r
(5+4+2+5+3)/5 #t2
```

```
[1] 3.8
```

Example - Paired Samples

- we also now need our sd for both conditions

```
# t1
(10 - 8.8)^2 + (8 - 8.8)^2 + (7 - 8.8)^2 + (9 - 8.8)^2 + (
```

```
[1] 6.8
```

```
6.8/4
```

```
[1] 1.7
```

```
sqrt(1.7)
```

```
[1] 1.30384
```

Example - Paired Samples

```r
# t2
(5 - 3.8)^2 + (4 - 3.8)^2 + (2 - 3.8)^2 + (5 - 3.8)^2 + (3
```

```
[1] 6.8
```

```r
6.8/4
```

```
[1] 1.7
```

```r
sqrt(1.7)
```

```
[1] 1.30384
```

$$\hat{d} = \frac{8.8 - 3.8}{1.30}$$

Example - Paired Samples

```r
8.8-3.8
```

```
[1] 5
```

$$\hat{d} = \frac{5}{1.30}$$

Example - Paired Samples

```r
5/1.3
```

```
[1] 3.846154
```

$$\hat{d} = 3.85$$

Independent Samples t-test in SPSS

- Steps
  - Test assumptions of your outcome by the two groups in your IV
  - Analyze –> Compare Means –> Independent Samples t-test
    - Test Variable = Outcome (must be continuous)
    - Grouping Variable = IV (must only have two groups)
  - *Defining Groups* is used to create two groups, these should correspond to the values of your IV
  - you can then get confidence intervals and bootstrapping if you'd like
  -
    - if you don't trust your noramlity, you could bootstrap your t-test to get bootstrapped confidence intervals
    - it will re-estimate your standard error using the number of samples you asked for
- don't worry about reading the Bayesian test of two independent means (I'm not going to test you on that)

Paired Samples t-test in SPSS

- Steps
  - Test assumptions of your outcomes
  - Analyze –> Compare Means –> Paired-Samples t-test
    - We put both conditions/pairs on the same row for *Pair 1*
    - you can put more pairs of variables underneath and run all your paired t-tests all at once
  - you can then get confidence intervals and bootstrapping if you'd like
    - if you don't trust your normality, you could bootstrap your t-test to get bootstrapped confidence intervals
    - it will re-estimate your standard error using the number of samples you asked for