z-scores

PSY 3307

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Agenda

- distributions
- z-scores
- using z-scores

Distributions

- raw distribution is where the scores of a distribution are on whatever scale of the variable you are looking at
    - Ex: looking at number of minutes studying for all students in your class/sample
- z-score distribution is a distribution where all the scores are in standard deviation units
- probability distribution is the distribution ranges from 0 to 1 of the likelihood of something happening (0 = not happening, 1 = event will definitely happen)
    - **Probability Density Functions** is a mathematical formula to specify the distributions

z-score

Sample z-score

$$z = \frac{X - \overline{X}}{S}$$

Population z-score

$$z = \frac{X - \overline{X}}{\sigma}$$

Getting raw scores from a z-score

$$X = (z)(S) + \overline{X}$$

z-scores

- **z-score** is a statistic that indicates the distance a score is from the mean when measured in standard deviation units
  - positive values are above the mean
  - negative values are below the mean

z-score Example

- Get a score of 90 to the mean of 60
    - deviate of 30
- Standard deviation of 10
- Now means that the +30 score is 3 SD above the mean
    - 30 score doesn't tell us much
    - 3 sd tells us much more
    - also tells us that the z-score is 3

z-distribution to interpret scores

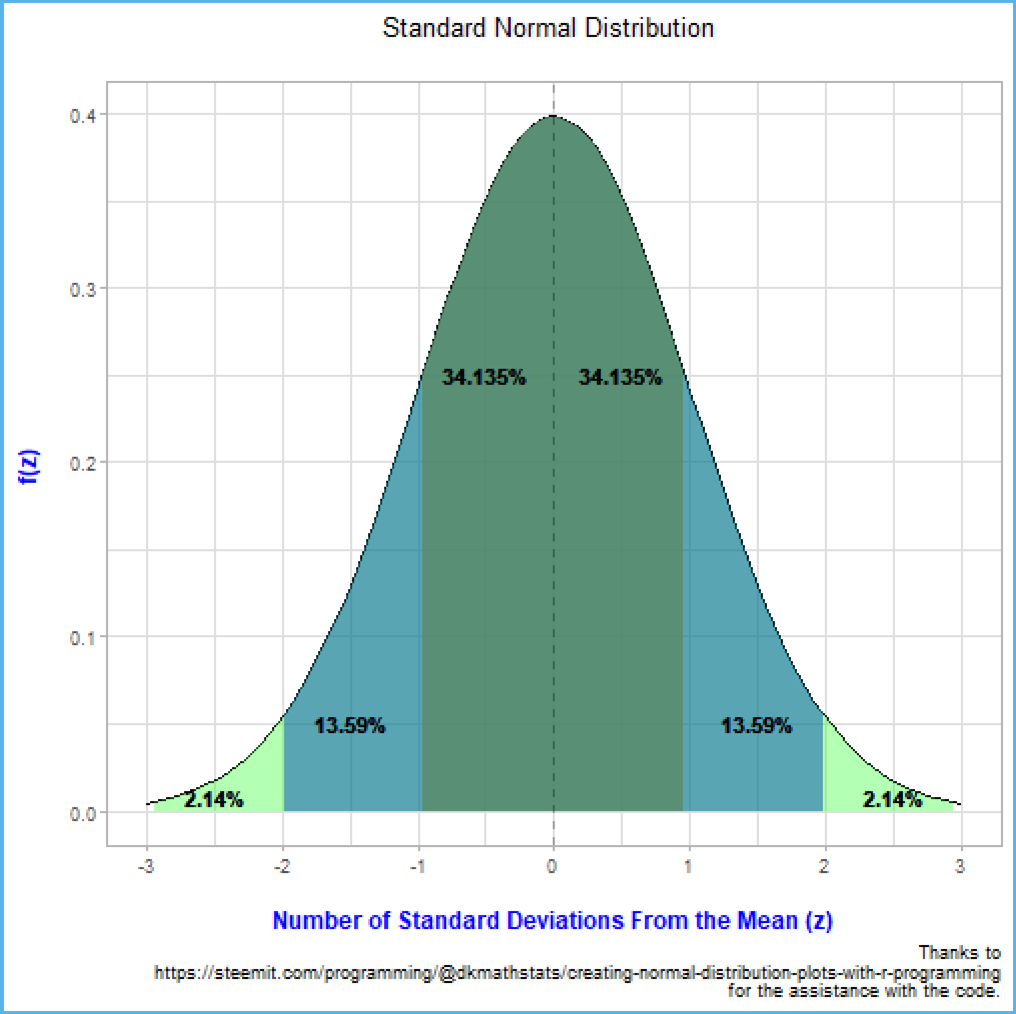
- **z-distribution** is transforming all raw scores into z-scores
- a z-score of 0 is the mean
- the z-distribution also represents that everything has been standardized
    - every variable is now comparable and on the same scale

JP Reminders

- A z-distribution will always have the same shape as the raw score distribution
- the mean is and will always be zero
- the standard deviation is 1, even when the standard deviation of the raw scores is a different value (e.g., 10, 15, 100)
- z-scores that are greater in either the positive or negative direction mean that values are less likely to occur

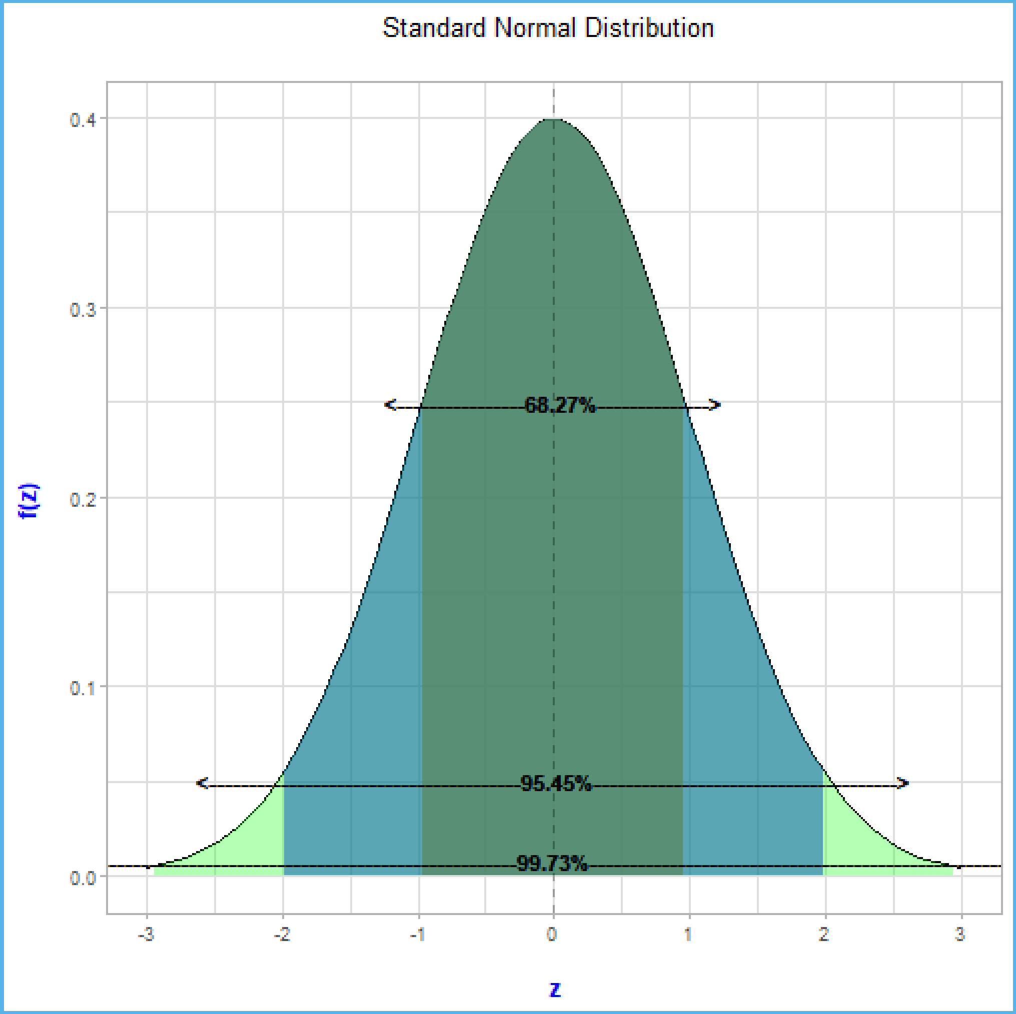
Comparing Apples to Oranges

- When using the z-distribution, every variable is put on the same scale
- often referred to as **standardized scores**


Standard Normal Distribution
0.4
0.3
0.2
0.1
0.0
f(z)
34.135%
34.135%
13.59%
13.59%
2.14%
2.14%
-3
-2
-1
0
1
2
3
Number of Standard Deviations From the Mean (z)
Thanks to
https://steemit.com/programming/@dkmathstats/creating-normal-distribution-plots-with-r-programming
for the assistance with the code.

Standard Normal Curve

- **Standard Normal Curve** is a perfect normal z-distribution that serves as the model of any approximately normal z-distribution
- Approach is most common with
  - large sample (or population)
  - interval or ratio scores
  - come close to forming a normal distribution
- While z-scores past +-3SD are possible, they occur .0013 of the time, which is why we often only look at +-3SD



Standard Normal Distribution
f(z)
0.4
0.3
0.2
0.1
0.0
68.27%
95.45%
99.73%
-3
-2
-1
0
1
2
3
z

Relative Frequency Using the z-Distribution

- Calculate the z-score for one observation
- Multiply by N
- Answer will be the number of observations between the mean and the z-score you calculated
- Then you can add up the other half to see all the observations that are at or below the score that the one observation had

Example

```r
z_score = -1

N = 100

#
.3413*100
```

```
## [1] 34.13
```

```r
50 - 34.13
```

```
## [1] 15.87
```

```r
#15.87% of scores were at or below the z-score we calculated
```

Using the z-Table

- In your Book at the end, you'll see the z-distribution
- There are two ways to show this
    - Area between mean and the z-score along with area beyond z in tail
    - z-scores with the tenths position on the column and the hundredths position on the top row

Using z-Scores to Describe Sample Means

- **Sampling Distribution of Means** is the frequency distribution showing all possible sample means when samples are drawn from a population
- Every time a population is sampled from, they will always be slightly different, with some means being higher and some being lower
- **Central Limit Theorem** statistical concept that defines the mean, standard deviation, and shape of a sampling distribution
    - Always follows some rules
    - A sampling distribution is always an approximately normal distribution
    - mean of sampling distribution equals mean of underlying raw score population used to create the sampling distribution
        - \mu is the mean of the means
    - standard deviation of the sampling distribution is related to the standard deviation of the raw score population

Central Limit Theorem

- The use of this concept is that we can assume that our sample is representative of the population without having to sample the whole population