Regression

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Regression

- The reason I'm excited is because regression is one of the most important, useful, and powerful tools you can learn
- It can pretty much do everything we have covered so far
  - comparing two groups –> t-test
    - it can do this by dummy coding a variable into a reference group and the other group and will compare them
    - the test in the regression is actually a t-test comparing if there was a difference between your group and zero (no change)
  - comparing multiple groups –> ANOVA
    - dummy coding and comparing one group (reference group) to the other groups
  - interactions –> factorial ANOVA
    - it can do that with some dummy coding and then **splitting groups by the second variable (moderator) to then run the analyses separately for reach group (simple effects)**
  - interactions with continuous variables –> regression
  - interactions with continuous and categorical variables –> regression

Difference between Correlation and Regression

- Both focus on relationships
  - regression focused more on the direction of a relationship
    - which is the IV and which is the DV
- correlation only states if the two variables are positively or negatively related
  - "there is a relationship present"
- regression is scale dependent
  - coefficients are expected change on average in your outcome given a one-point/one-unit increase in your predictor
    - "For a one point increase in horsepower, there is a ________ average increase/decrease in MPG"
- a **standardized** regression coefficient in a simple linear regression is the same thing as a correlation coefficient

Example

```
    Pearson's product-moment correlation

data:  mtcars$hp and mtcars$mpg
t = -6.7424, df = 30, p-value = 1.788e-07
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.8852686 -0.5860994
sample estimates:
       cor
-0.7761684
```

Example

```
lm(formula = mpg ~ hp, data = mtcars)
            coef.est coef.se t value Pr(>|t|)
(Intercept) 30.10     1.63   18.42    0.00
hp          -0.07     0.01   -6.74    0.00
---
n = 32, k = 2
residual sd = 3.86, R-Squared = 0.60

Call:
lm(formula = mpg ~ hp, data = mtcars)

Standardized Coefficients::
(Intercept)          hp
  0.0000000  -0.7761684
```

Variety of Regressions

- linear regression
- logistic regression (binary/dichotomous outcome)
- multinomial regression (several levels/categories in outcome)
- spatial regression (takes into account the spatial nature of what you are testing)
- poisson regression (count/ordinal variable as your outcome)
- structural equation modeling (super duper regression... with theory)
- mixed-effect/multi-level/hierarchical modeling (takes into account those **random** variables)
  - nested data: Ex: students within schools within districts within states

Back to Basics

- lets get back to our model equation

$$outcome_i = (b_1 X_i) + error_i$$

$$outcome_i = (b_0 + b_1 X_i) + error_i$$

$$Y_i = (b_0 + b_1 X_i) + \epsilon_i$$

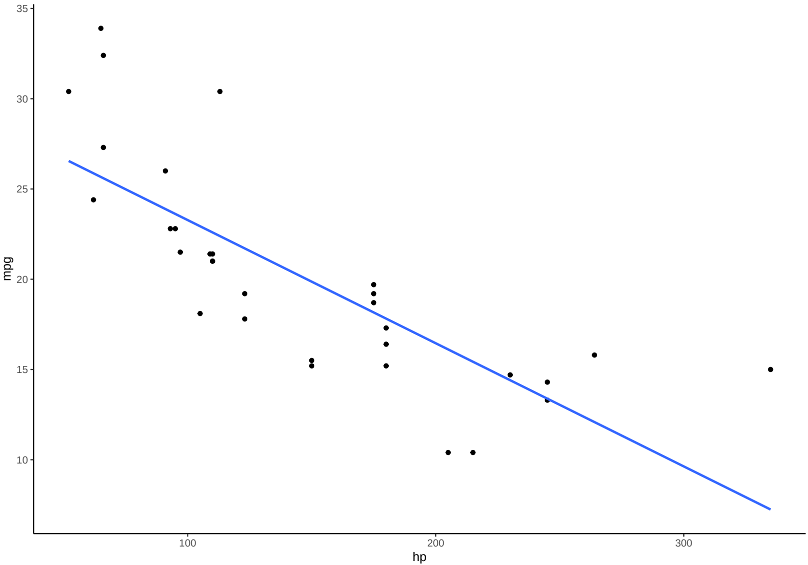
- still looking at our outcome Y_i, our X_i as our predictor, and b_i as the parameter that quantifies the relationship with the outcome
  - the b_i value is also an unstandardized measure of the relationship between X_i and Y_i
- b_0 or our intercept, now tells us the average value of our outcome when the predictor is at **zero**
  - no longer is it the average value of the outcome

Back to Basics

- What is b_1 now in this relationship?

$$outcome_i = mx + b$$

Back to Basics



Back to Basics

```
lm(formula = mpg ~ hp, data = mtcars)
            coef.est coef.se t value Pr(>|t|)
(Intercept) 30.10     1.63   18.42    0.00
hp          -0.07     0.01   -6.74    0.00
---
n = 32, k = 2
residual sd = 3.86, R-Squared = 0.60
```

Back to Basics

- the b values are referred to as regression coefficients
  - can also be callled unstandardized coefficients
- let's look at our example between a car's **horsepower** and **miles per gallon**

Regression Example

$$MPG_i = b_0 + b_1 HP_i + \epsilon_i$$

- with the estimation of our b values above, we would be able to make a prediction about MPG by replacing the value of HP with a new value

$$MPG_i = 30.10 + (-0.07 * 200) + \epsilon_i$$

$$MPG_i = 30.10 + (-14) + \epsilon_i$$

$$MPG_i = 16.1 + \epsilon_i$$

Regression Example

- so if we had a car with 200 HP, then the MPG for the car would be 16.1
  - which is pretty bad for a car
  - this value is now our **predicted value,** or what we predicted from examining a specific value
- the error value ϵ is left in because this prediction is properly not the most accurate

Linear Model with Several Predictors

- the best part about linear regression is when you include all your predictors that are empirically (past literature) and/or theoretically relevant (theory testing)
  - this makes the linear regression –> multiple (linear) regression

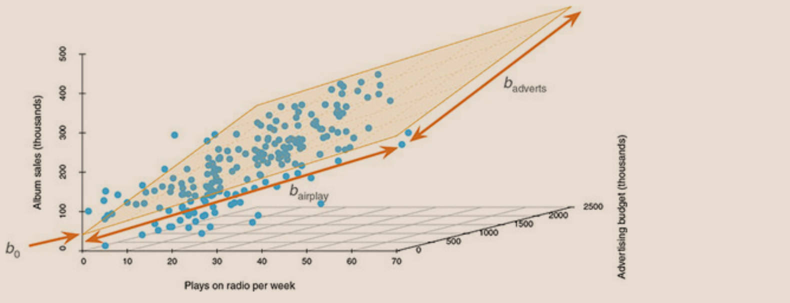
Linear Model with Several Predictors

$$Y_i = (b_0 + b_1 X_{1i} + b_2 X_{2i}) + \epsilon_i$$

$$Y_i = (b_0 + b_1 HP_i + b_2 Disp_i) + \epsilon_i$$

Linear Model with Several Predictors

Figure 9.3 Scatterplot of the relationship between album sales, advertising budget and radio play



Linear Model with Several Predictors

- the new model now has the b values for both the variables included in the model
  - and the intercept
- the difference in distance between the regression plane (orange) and the blue points is the residuals, or the error, of the model
- JP: not the biggest fan of 3D visuals and it is really only helpful for models with 2 variables
  - any more variables and it gets difficult to visualize

Linear Model with Several Predictors

```
lm(formula = mpg ~ hp + disp, data = mtcars)
            coef.est coef.se t value Pr(>|t|)
(Intercept) 30.74     1.33   23.08    0.00
hp          -0.02     0.01   -1.86    0.07
disp        -0.03     0.01   -4.10    0.00
---
n = 32, k = 3
residual sd = 3.13, R-Squared = 0.75
```

Linear Model with Several Predictors

```
lm(formula = mpg ~ hp + disp + as.factor(cyl), data = mtcar
                coef.est coef.se t value Pr(>|t|)
(Intercept)     31.15     1.77   17.63    0.00
hp              -0.02     0.01   -1.49    0.15
disp            -0.03     0.01   -2.50    0.02
as.factor(cyl)6 -4.05     1.69   -2.40    0.02
as.factor(cyl)8 -2.43     3.24   -0.75    0.46
---
n = 32, k = 5
residual sd = 2.89, R-Squared = 0.80
```

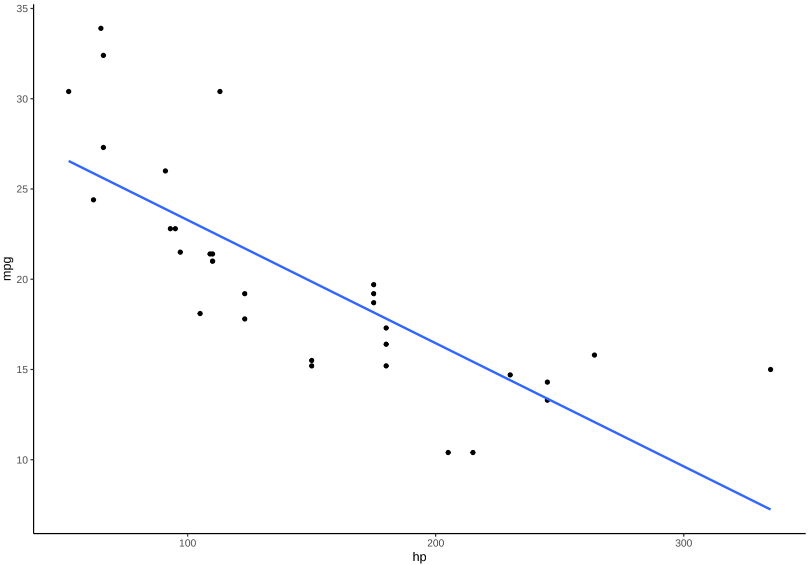
Terminology

- outcome = dependent variable
- predictor = independent variable
- when a model only has one predictor, then it is a **simple linear regression**
- when a model has 2+ predictors, it is a **multiple regression**

Regression Coefficient Values

- in both simple and multiple linear regressions, you have actual scores and predicted scores
  - **actual scores** are the values that are provided by participants in your study
  - **predicted scores** are the values that are predicted by the regression line

Estimating the Model



Estimating the Model

- remember that we estimated the model previously by looking at the deviations between the model and the data collected
- with regressions, we can do the same thing
- the distance between the predicted line and the data points are called residuals
  - they are the same thing as **deviations** when we look at the mean

Estimating the Model

- when we squared the differences between the observed values and the outcome and the predicted values that come from the model, we can get the **total error in the model**

$$total\ error = \sum_{i=1}^{n}(observed_i - model_i)^2$$

- similar to when we assessed the fit of the mean using the variance, we used the sum of squared errors/sum of squares, we can do the same thing with the residuals to get the **sum of squared residuals** or the **residual sum of squares**
  - similar to the ANOVA
  - this is what is left over that our model does not account for/predict

Assessing Goodness of Fit

- **goodness of fit** is a term for how well our model fits the data
  - you can have a model that has significant relationships but it can still have a horrible fit
- in order to assess the goodness of fit, we must first fit a baseline model then compare it to the model with our predictors included
  - we then calculate the SS_R and the new model should have significantly less error within it than the baseline model
  - this assesses the inaccuracy of our regression model
- our baseline model will be the average outcome variable
  - essentially a model with only the intercept included in the model

Assessing Goodness of Fit

- the sum of squared differences or **total sum of squares** (SS_T) represents how well the mean as a model is
- **model sum of squares** is what our model accounts for
  - the improvement our model makes over the baseline model
- R^2 still tells us how much of the variation is “explained’ with our predictor(s)
- still use the mean squares values to get the averages for our model and residual

$$MS_M = \frac{SS_M}{k}$$

$$MS_R = \frac{SS_R}{N - k - 1}$$

- k is now the number of predictors in our model
- N is still the sample size/number of observations

Assessing Goodness of Fit

$$F = \frac{MS_M}{MS_R}$$

- our F test will still be calculated the same way
- we can also use the F test to calculate the significance of our R^2 value

$$F = \frac{(N-k-1)R^2}{k(1-R^2)}$$

Assumptions of the Linear Model

- linearity
  - is the assessment that the relationship between your predictor and outcome is a linear relationship
- independent Errors
  - your residuals should be uncorrelated, often this is referred to as **autocorrelation**
  - Durbin-Watson test can test for this assumption
    - checks to make sure your residuals are not correlated with one another
    - the test has a range of 0-4, with a value of 2 meaning that the residuals are uncorrelated
    - greater than 2 indicates a negative correlation
    - value below 2 indicates a positive correlation
- homoscedasticity
  - the variation of the residual terms should be constant
  - the amount of actual scores should be roughly the same around the regression line

Assumptions of the Linear Model

- normally distributed errors
  - your residuals should show a normal distribution if plotted with a histogram
  - if you have issues with this, you can bootstrap your confidence intervals and ignore this assumption
- predictors are uncorrelated with *external variables*
  - **external variables** are variables that haven't been included in the model and can influence the outcome variable
    - confounding variables
  - variables that are relevant should be included in the model
    - used to control/adjust for these variables and get the best understanding of the unique relationships between your predictor of interest and your outcome

Assumptions of the Linear Model

- variable types
  - make sure your categorical variables are dummy coded to be included in the model
- no perfect **multicollinearity**
  - your predictors that are included in the model should not have perfect (or near perfect correlations) with other predictors in the model
    - threshold is $\sim$.7 or somtimes .9
- non-zero variance
  - make sure there is variation in your variables
  - not everyone has the same score