PSY 3307

Analysis of Variance (ANOVA)

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Agenda

- Review t-tests
- Overview of Analysis of Variance (ANOVA)
  - Quick Review of all the different types of ANOVA
  - One-way ANOVA
  - Two-way ANOVA
  - Factorial ANOVA
  - Repeated-measures ANOVA
  - Mixed-effect ANOVA
- Components of ANOVA
- Performing ANOVA
- Post-hoc Tests (Tukey's HSD Test)
- Effect Size and Eta^2
- JP is Including More

Review t-tests

- Things we need to remember
    - How to read a table (z, t, and now F)
    - Differences between within- and between-designs
    - What is an IV, DV, and conditions

Analysis of Variance (ANOVA)

- **Factor** is just another word for IV
- A **level** is the same thing as a condition (from t-tests) and similar to a t-test, ANOVA has to do with differences between or among sample means
  - however ANOVA does not have restrictions on the number of groups we can test
- k is the symbol for the number of levels in a factor
  - otherwise known as the number of conditions in an IV

$$k = number\ of\ levels\ in\ factor$$

Example of the Absurdity

```
## Warning: Missing column names filled in: 'X1' [1]

##
## -- Column specification ---------------------------------------------
## cols(
##   .default = col_double(),
##   state_fips_code = col_character(),
##   county_fips_code = col_character(),
##   fips_code = col_character(),
##   state_abbreviation = col_character(),
##   county_name = col_character(),
##   reading_scores_aian = col_logical(),
##   math_scores_aian = col_logical(),
##   communicable_disease = col_logical(),
##   cancer_incidence = col_logical(),
##   coronary_heart_disease_hospitalizations = col_logical(),
##   cerebrovascular_disease_hospitalizations = col_logical(),
##   smoking_during_pregnancy = col_logical(),
##   opioid_hospital_visits = col_logical(),
##   alcohol_related_hospitalizations = col_logical(),
##   motor_vehicle_crash_occupancy_rate = col_logical(),
##   on_road_motor_vehicle_crash_related_er_visits = col_logical(),
```

```
aov_find <- aov(adult_smoking ~ state_abbreviation, data = data)
summary(aov_find)
```

```
##                       Df Sum Sq Mean Sq F value              Pr(>F)    
## state_abbreviation    50  2.338 0.04676   80.92 <0.0000000000000002 ***
## Residuals           3142  1.815 0.00058                                
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
tukey_find <- TukeyHSD(aov_find)
tukey_find$`state_abbreviation`[1:10, 1:4]
```

```
##               diff          lwr        upr          p adj
## AK-CA  0.087343654  0.065752343 0.10893497 0.0000000000000
## AL-CA  0.079242299  0.062110931 0.09637367 0.0000000000000
## AR-CA  0.083350172  0.066642931 0.10005741 0.0000000000000
## AZ-CA  0.043470569  0.016330229 0.07061091 0.0000002016392
## CO-CA  0.022238079  0.004924024 0.03955213 0.0003319958281
## CT-CA -0.001099152 -0.035556182 0.03335788 1.0000000000000
## DC-CA  0.038867947 -0.030362089 0.10809798 0.9909882820559
## DE-CA  0.042165472 -0.007583593 0.09191454 0.3079515102058
## FL-CA  0.070667490  0.053536122 0.08779886 0.0000000000000
## GA-CA  0.061787805  0.047121974 0.07645364 0.0000000000000
```

Another Reason Why I Like Regression

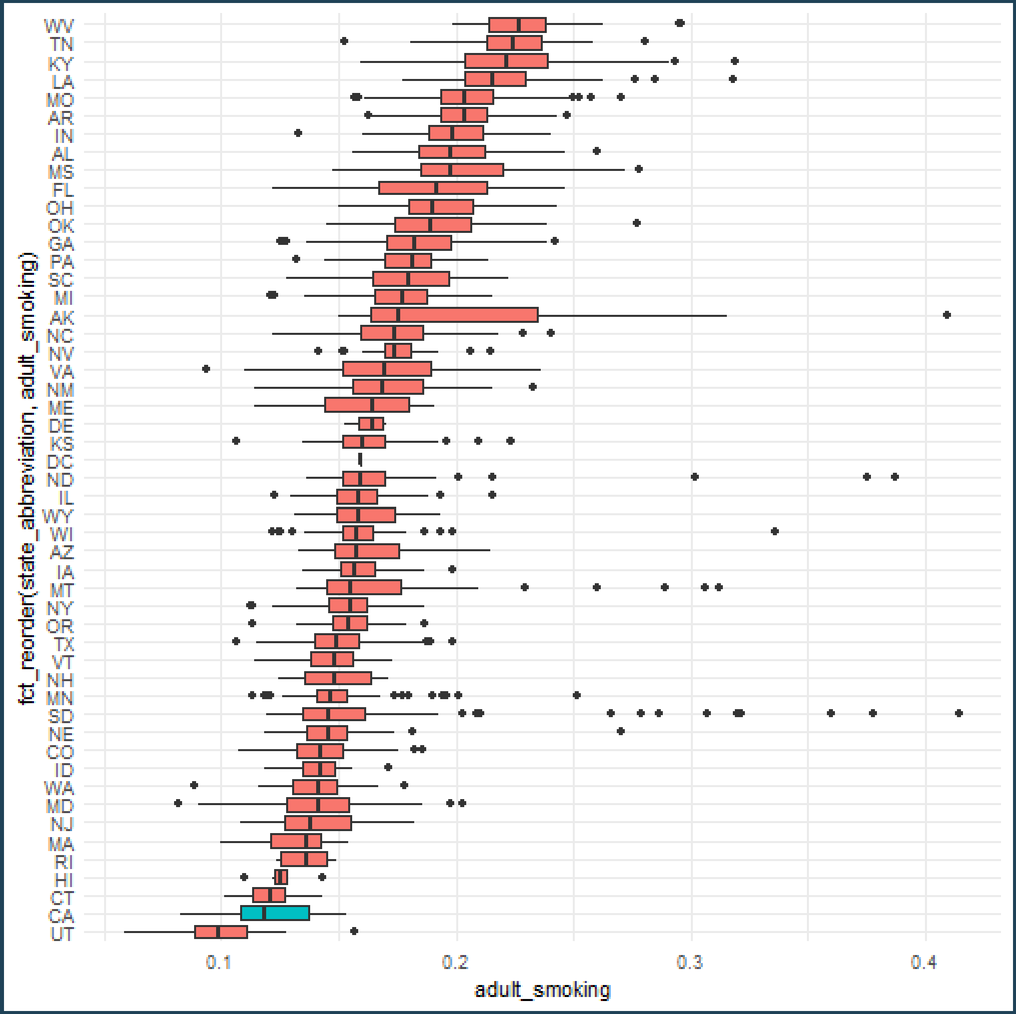
- In a real-life scenario, you would already have a hypothesis where you would be interested in whether or not a state is different from the rest
- Therefore, you'd already have a reference group to compare everyone to and just need to run one test
- Below is the same test, just through a regression framework

```r
lm_find <- lm(adult_smoking ~ state_abbreviation, data = data)
summary(lm_find)
```

```
## 
## Call:
## lm(formula = adult_smoking ~ state_abbreviation, data = data)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -0.07620 -0.01314 -0.00115  0.01065  0.24411 
## 
## Coefficients:
##                       Estimate Std. Error t value             Pr(>|t|)    
## (Intercept)           0.120788   0.003129  38.598 < 0.0000000000000002 ***
## state_abbreviationAK  0.087344   0.005390  16.205 < 0.0000000000000002 ***
## state_abbreviationAL  0.079242   0.004277  18.529 < 0.0000000000000002 ***
## state_abbreviationAR  0.083350   0.004171  19.984 < 0.0000000000000002 ***
## state_abbreviationAZ  0.043471   0.006775   6.416   0.0000000001609179 ***
## state_abbreviationCO  0.022238   0.004322   5.145   0.0000002839612803 ***
## state_abbreviationCT -0.001099   0.008602  -0.128             0.898331    
## state_abbreviationDC  0.038868   0.017283   2.249             0.024584 *  
## state_abbreviationDE  0.042165   0.012419   3.395             0.000694 ***
## state_abbreviationFL  0.070667   0.004277  16.524 < 0.0000000000000002 ***
## state_abbreviationGA  0.061788   0.003661  16.876 < 0.0000000000000002 ***
## state_abbreviationHI  0.004873   0.010300   0.473             0.636138    
## state_abbreviationIA  0.037774   0.003946   9.573 < 0.0000000000000002 ***
## state_abbreviationID  0.020741   0.004757   4.360   0.0000134333660949 ***
```

```r
broom::tidy(lm_find)
```

```
## # A tibble: 51 x 5
##    term                 estimate std.error statistic   p.value
##    <chr>                   <dbl>     <dbl>     <dbl>     <dbl>
##  1 (Intercept)           0.121     0.00313    38.6   4.13e-267
##  2 state_abbreviationAK  0.0873    0.00539    16.2   8.82e- 57
##  3 state_abbreviationAL  0.0792    0.00428    18.5   7.99e- 73
##  4 state_abbreviationAR  0.0834    0.00417    20.0   9.75e- 84
##  5 state_abbreviationAZ  0.0435    0.00678     6.42  1.61e- 10
##  6 state_abbreviationCO  0.0222    0.00432     5.14  2.84e-  7
##  7 state_abbreviationCT -0.00110   0.00860    -0.128 8.98e-  1
##  8 state_abbreviationDC  0.0389    0.0173      2.25  2.46e-  2
##  9 state_abbreviationDE  0.0422    0.0124      3.40  6.94e-  4
## 10 state_abbreviationFL  0.0707    0.00428    16.5   7.02e- 59
## # ... with 41 more rows
```


WV
TN
KY
LA
MO
AR
IN
AL
MS
FL
OH
OK
GA
PA
SC
MI
AK
NC
NV
VA
NM
ME
DE
KS
DC
ND
IL
WY
WI
AZ
IA
MT
NY
OR
TX
VT
NH
MN
SD
NE
CO
ID
WA
MD
NJ
MA
RI
HI
CT
CA
UT
fct_reorder(state_abbreviation, adult_smoking)
0.1
0.2
0.3
0.4
adult_smoking

Breaking Down A One-way ANOVA

- **One-way ANOVA** is when we have an IV that has multiple levels (3+)
  - **NOTE** if you were to go on to SPSS and run a one-way ANOVA with the `sex` variable you would get the same answer
  - Essentially the same test being run
- Similar to a t-test, this also has within- and between-subjects designs
- Now instead of a t-table, we will be using a F-table

We Are Now Working With Modeling

- There's just one problem. We have to work with ANOVA modeling
- **ANOVA** is a parametric procedure for dterming whether significant differences occur in an experiment containing two or more sample means

$$X_{ij} = \mu + \gamma_j + \epsilon_{ij}$$

- mu is the grand mean
- gamma is the specific treatment effect for group j (which group you are interested in looking at)
- epsilon if the error/residual of a specific individual (how much an individual deviates from the group's mean)

Assumptions

- **homogeneity of variance** is the assumption that each population has the same variance

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \ldots$$

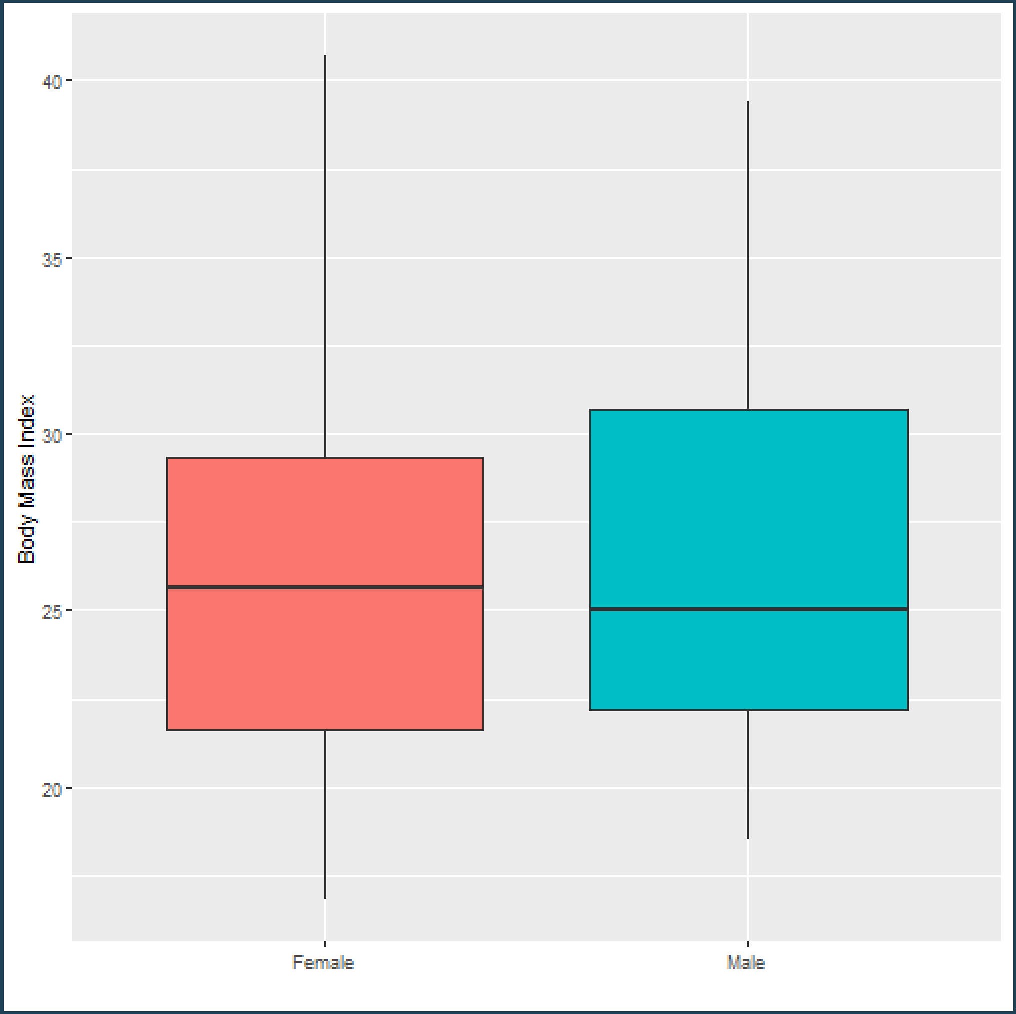
- **error variance** variance unrelated to any treatment differences
- **heterogeneity of variance** is when populations have different variances
- **normality** DV values are normally distributed
- **independence** observations are independent of one another
  - it really is that the residual/error is independent but for now we'll keep it as observations are different from one another
- The n in each level doesn't have to be exactly the same but they should not be drastically different

Controlling Experiment-Wise Error Rate

- I had mentioned this previously that multiple independent-samples t-tests could do the same thing as a one-way ANOVA
- **experiment-wise error rate** is the probability of making a Type I error when comparing all means in an experiment
- with an F-test we are less likely to commit a type I error because we are not running all the tests possible

Example

```
jp <- rio::import(here::here("jp_thesis_1.sav")) %>%
  janitor::clean_names() %>%
  rowid_to_column() %>%
  rename(sex = ccc_gender)
```



Body Mass Index
40
35
30
25
20
Female
Male

```r
bmi_aov <- aov(ccc_bmi ~ factor(sex), data = jp)
summary(bmi_aov)
```

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## factor(sex)   1      3   2.515   0.086  0.769
## Residuals   370  10793  29.169
```

```r
TukeyHSD(bmi_aov)
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = ccc_bmi ~ factor(sex), data = jp)
##
## $`factor(sex)`
##           diff       lwr      upr     p adj
## 2-1 -0.1775135 -1.366163 1.011136 0.7691805
```

```r
t.test(ccc_bmi ~ factor(sex), data = jp, var.equal = TRUE)
```

```
## 
## 	Two Sample t-test
## 
## data:  ccc_bmi by factor(sex)
## t = 0.29366, df = 370, p-value = 0.7692
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.011136  1.366163
## sample estimates:
## mean in group 1 mean in group 2 
##        26.32931        26.15180
```

Steps to Conduct ANOVA

- Hypotheses
  - Similar to the independent-samples t-test, a one-way ANOVA's null hypothesis states that there is no difference between the levels/conditions
  - We just have more levels in a one-way ANOVA.
- our null hypothesis would be

$$H0 : \mu_1 = \mu_2 = \mu_3$$

$$H1 : not\ all\ \mu\ are\ equal$$

- our alternative/research hypothesis is not that all groups would be significantly different from one another
- **JP Note** another option is to state that one group will be significantly different from the other groups

F statistic

- Steps to ANOVA

1. Compute a F-statistic

2. Conduct a post-hoc test

F-statistic

- we compute a F-statistic to see if any means are different
  - Significant F-statistic then there are differences somewhere between the multiple levels
  - Non-significant F-statistic means there are no differences between any levels
- The F-statistic only tells us whether or not a significant difference is found between any of the levels
- F-obtained is compared to a F-critical value to find statistical significance

Post-hoc Tests OR Planned Comparisons

- Post-hoc often refers to after the fact
- Post-hoc tests are often considered after finding a statistically significant F-statistic
- like t-tests when comparing all combinations of each levels to see if there differences between two specific levels
- Planned comparisons are when you are interested in having a specific levels being different from the other levels

Post-hoc Tests

- only look at post-hoc findings if there is a significant F-statistic
- no post-hoc tests are run when you only have two levels

Different Types of Post-hoc Tests

- **Tukey Test or Tukey's HSD (Honestly Significant Difference)**
- Fisher's Least Significant Difference (LSD) Procedure
- Newman-Keuls Test
- Scheffe Test
- Dunnett's Test
- Benjamini-Hochberg Test
- **Bonferroni Test/Correction**
  - whatever alpha you are using, you then divide by the number of tests you conduct
  - alpha = .05, and you run 20 tests
- We'll get to these in the upcoming weeks (if only shortly)

Bonferroni Correction Examples

- Alpha = .05, number of tests = 20 -new p-value should be .0025

```
.05/20
```

```
## [1] 0.0025
```

- alpha = .05, number of tests = 10
  - new p-value = _

Contrasts

- **a priori comparison** or before data is collected
    - also called **contrasts**
- **post hoc comparison** after data has been collected, looked at descriptive statistics, and somtimes...
    - they've looked at their test results
- gets complicated with the type of contrast chosen
    - linear contrasts
    - orthogonal contrasts

Unequal Sample Sizes

- ANOVA is robust to slightly unequal sample sizes
  - like the example we did in SPSS
- **balanced designs** are when each level has the same amount of participants
- formula corrections are needed for unequal samples (see below)
- another option to make sure your levels are all equal in number of observations (n) is to fill in potential missing data
  - old-school methods include changing missing values to the mean or median
  - new-school methods include things like full-information maximum likelihood or multiple imputation (real fancy stuff)

Componenets of ANOVA

$$S_x^2 = \frac{\Sigma(X - \overline{X})^2}{N - 1}$$

$$S_X^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N - 1}$$

$$S_x^2 = \frac{Sum\ of\ Squares\ (SS)}{Degrees\ of\ Freedom\ (df)}$$

Terminology

- Sum of Squares is broken down into three different categories
  - Sum of Squares Total
  - Sum of Squares Between/Treatment
  - Sum of Squares Within/Error

$$SS_{total}$$

$$SS_{bn} = SS_{treat}$$

$$SS_{wn} = SS_{error}$$

- `Sum of squares (SS)`/`degrees of freedom (df)` is equal to `mean square`
- `mean square` is often seen as `MS`
- when referring to ANOVA, `sum of the squared deviations` is called `sum of squares`
- this calculation of sum of squares divided by the degrees of freedom is called `mean square` or `MS`
- so our variance calculation is really our mean square in ANOVA
  - this is because we are calculating the `mean square within groups` and `mean square between groups`

Formulas for Sum of Squares

$$SS_{total} = \Sigma(X_{ij} - \overline{X})^2$$

- This means that every value (i) within each level/group/condition (j) is put into this equation and then subtracting the `grand mean`
- **the grand mean** is the mean of all the observations of each level or the mean of each level's mean

$$SS_{treat} = n\Sigma(\overline{X_j} - \overline{X})^2$$

$$SS_{error} = SS_{total} - SS_{treat}$$

- Sum of squares error is actually the sum of each score within each level minus the mean for that level and then added all together to get the final value

$$SS_{error} = \Sigma(X_{ij} - \overline{X_j})^2$$

$$SS_{wn\ level1} = \Sigma((first\ observation - level\ 1\ mean)^2 + (second\ observation - leve$$

Unequal Sample Size Correction

$$SS_{treat} = n\Sigma(\overline{X_j} - \overline{X})^2$$

will become

$$SS_{treat} = \Sigma[n_j(\overline{X_j} - \overline{X})^2]$$

so this would then include the n for each group/level/condition

Mean Square Within Groups

- **MS within groups** describes the variability in scores within the conditions/levels of an experiment

$$MS_{wn}$$

- we find differences among values in each level/condition and "pool" them together
- **MS within groups** is the "average" variability of scores within each level
- it is essentially a measure of variability of individual scores

Mean Square Between Groups

- **MS between groups** is the differences between the means of each condition/level in a factor/IV

$$MS_{bn}$$

- measure difference between the means by treating them as scores, with an "average" amount they deviate from their mean, which in this case is the overall mean of the experiment

- similar to how scores deviate from a mean, this is a measure of the deviations of sample means from the overall mean

The F-ratio

- MS between groups tells us whether or not the levels differ from one another and support our null/alternative hypotheses
- MS within groups estimates variability of individual scores
- if working with one population, our MS between should equal our MS within
- so if our null is true the MS between should be the same answer as the MS within
- **F-ratio** is a fraction consisting of MS between divided by the MS within or

$$F_{obt} = \frac{MS_{bn}}{MS_{wn}}$$

F-ratio

- while it is unlikely that an exact value of 1 would be a F-obtained value, a value around 1 should be supportive of the null hypothesis
- the larger the F-ratio the more likely that the result is from sampling error or from the IV
- if our F-obtained value is larger than the F-critical value then we reject the null and accept the alternative/research hypothesis

Performing an ANOVA

- before beginning, its important to note that for many of the calculations symbols will be similar like `MS` but it is important to note the subscripts

$$MS_{bn} \neq MS_{wn}$$

$$\begin{bmatrix} Source \\ Between \\ Within \\ Total \end{bmatrix} \begin{bmatrix} Sum\ of\ Squares \\ SS_{bn} \\ SS_{wn} \\ SS_{total} \end{bmatrix} \begin{bmatrix} df \\ df_{bn} \\ df_{wn} \\ df_{total} \end{bmatrix} \begin{bmatrix} MeanSquare \\ MS_{bn} \\ MS_{wn} \\ \end{bmatrix} \begin{bmatrix} F-ratio \\ F_{obt} \\ \\ \end{bmatrix}$$

Computing for a F-obtained value

Steps for getting the F-obtained value

1. Calculate the sum of squares
2. Calculate the degrees of freedom
3. Calculate the mean squares
4. Calculate the F-obtained value

```r
difficulty <- data.frame(easy = c(9, 12, 4, 8, 7),
                         medium = c(4, 6, 8, 2, 10),
                         hard = c(1, 3, 4, 5, 2))
```

```r
easy_sum = 9+12+4+8+7
med_sum = 4+6+8+2+10
hard_sum = 1+3+4+5+2

easy_sum
```

```
## [1] 40
```

```r
med_sum
```

```
## [1] 30
```

```r
hard_sum
```

```
## [1] 15
```

```r
easy_sum2 = 9^2+12^2+4^2+8^2+7^2
med_sum2 = 4^2+6^2+8^2+2^2+10^2
hard_sum2 = 1^2+3^2+4^2+5^2+2^2

easy_sum2
```

```
## [1] 354
```

```r
med_sum2
```

```
## [1] 220
```

```r
hard_sum2
```

```
## [1] 55
```

```r
easy_n = 5
med_n = 5
hard_n = 5

easy_mean = easy_sum/easy_n
med_mean = med_sum/med_n
hard_mean = hard_sum/hard_n

easy_mean
```

```
## [1] 8
```

```r
med_mean
```

```
## [1] 6
```

```r
hard_mean
```

```
## [1] 3
```

Total Sum of all the Values

```
total_sum = easy_sum + med_sum + hard_sum
total_sum
```

```
## [1] 85
```

Total Sum of all the Squared Values

```
total_sum2 = easy_sum2 + med_sum2 + hard_sum2
total_sum2
```

```
## [1] 629
```

Total N

```
total_n = easy_n + med_n + hard_n
total_n
```

```
## [1] 15
```

k or the number of levels/conditions

```r
k = 3
k
```

```
## [1] 3
```

Computing the Sums of Squares Total

$$SS_{total} = \Sigma X^2_{total} - \frac{(\Sigma X_{total})^2}{N}$$

$$SS_{total} = 629 - \frac{(85)^2}{15}$$

```r
85^2
```

```
## [1] 7225
```

$$SS_{total} = 629 - \frac{7225}{15}$$

```r
7225/15
```

```
## [1] 481.6667
```

$$SS_{total} = 629 - 481.67$$

```r
629 - 481.67
```

```
## [1] 147.33
```

$$SS_{total} = 147.33$$

Filling in the Table

$$\begin{bmatrix} Source \\ Between \\ Within \\ Total \end{bmatrix} \begin{bmatrix} Sum\ of\ Squares \\ SS_{bn} \\ SS_{wn} \\ 147.33 \end{bmatrix} \begin{bmatrix} df \\ df_{bn} \\ df_{wn} \\ df_{total} \end{bmatrix} \begin{bmatrix} MeanSquare \\ MS_{bn} \\ MS_{wn} \\ \end{bmatrix} \begin{bmatrix} F-ratio \\ F_{obt} \\ \\ \end{bmatrix}$$

Sums of Squares Between

$$SS_{bn} = \Sigma(\frac{(\Sigma X\ in\ each\ column)^2}{n\ in\ each\ column}) - \frac{(\Sigma X_{total})^2}{N}$$

$$SS_{bn} = \Sigma(\frac{(40)^2}{5} + \frac{(30)^2}{5} + \frac{(15)^2}{5}) - \frac{(85)^2}{15}$$

```r
40^2
```

```
## [1] 1600
```

```r
30^2
```

```
## [1] 900
```

```r
15^2
```

```
## [1] 225
```

```r
85^2
```

```
## [1] 7225
```

$$SS_{bn} = \Sigma(\frac{1600}{5} + \frac{900}{5} + \frac{225}{5}) - \frac{7225}{15}$$

```r
1600/5
```

```
## [1] 320
```

```r
900/5
```

```
## [1] 180
```

```r
225/5
```

```
## [1] 45
```

```r
7225/15
```

```
## [1] 481.6667
```

$$SS_{bn} = (320 + 180 + 45) - 481.67$$

```r
(320 + 180 + 45) - 481.67
```

```
## [1] 63.33
```

$$SS_{bn} = 63.33$$

Filling in the Table

$$\begin{bmatrix} Source \\ Between \\ Within \\ Total \end{bmatrix} \begin{bmatrix} Sum\ of\ Squares \\ 63.33 \\ SS_{wn} \\ 147.33 \end{bmatrix} \begin{bmatrix} df \\ df_{bn} \\ df_{wn} \\ df_{total} \end{bmatrix} \begin{bmatrix} MeanSquare \\ MS_{bn} \\ MS_{wn} \\ \end{bmatrix} \begin{bmatrix} F - ratio \\ F_{obt} \\ \\ \end{bmatrix}$$

Sum of Squares Within Groups

$$SS_{wn} = SS_{total} - SS_{bn}$$

```
147.33 - 63.33
```

```
## [1] 84
```

Using the other way

$$SS_{total} = \Sigma(X_{ij} - \overline{X})^2$$

$$SS_{treat} = n\Sigma(\overline{X_j} - \overline{X})^2$$

$$SS_{error} = SS_{total} - SS_{treat}$$

```r
difficulty
```

```
##   easy medium hard
## 1    9      4    1
## 2   12      6    3
## 3    4      8    4
## 4    8      2    5
## 5    7     10    2
```

```r
grand_mean = (9 + 12 + 4 + 8 + 7 + 4 + 6 + 8 + 2 + 10 + 1 + 3 + 4 + 5 +
grand_mean
```

```
## [1] 5.666667
```

```r
(easy_mean + med_mean + hard_mean)/3
```

```
## [1] 5.666667
```

```r
ss_total = (9 - grand_mean)^2 + (12 - grand_mean)^2 + (4 - grand_mean)^2
  (4 - grand_mean)^2 + (6 - grand_mean)^2 + (8 - grand_mean)^2 + (2 - gr
  (1 - grand_mean)^2 + (3 - grand_mean)^2 + (4 - grand_mean)^2 + (5 - gr

ss_total
```

```
## [1] 147.3333
```

```r
ss_treat = 5*((easy_mean - grand_mean)^2 + (med_mean - grand_mean)^2 + (

ss_treat
```

```
## [1] 63.33333
```

```r
ss_error = ss_total - ss_treat

ss_error
```

```
## [1] 84
```

Filling in the Table

$$\begin{bmatrix} Source \\ Between \\ Within \\ Total \end{bmatrix} \begin{bmatrix} Sum\ of\ Squares \\ 63.33 \\ 84 \\ 147.33 \end{bmatrix} \begin{bmatrix} df \\ df_{bn} \\ df_{wn} \\ df_{total} \end{bmatrix} \begin{bmatrix} MeanSquare \\ MS_{bn} \\ MS_{wn} \\ \end{bmatrix} \begin{bmatrix} F-ratio \\ F_{obt} \\ \\ \end{bmatrix}$$

Calculating degrees of freedom

- df between groups is simply the number of `groups/levels/conditions - 1`

$$df_{bn} = k - 1$$

```r
3 - 1
```

```
## [1] 2
```

- df within groups is `N - k`

```r
15 - 3
```

```
## [1] 12
```

- df total is still `N - 1`

```r
15 - 1
```

```
## [1] 14
```

Filling in the Table

$$\begin{bmatrix} Source \\ Between \\ Within \\ Total \end{bmatrix} \begin{bmatrix} Sum\ of\ Squares \\ 63.33 \\ 84 \\ 147.33 \end{bmatrix} \begin{bmatrix} df \\ 2 \\ 12 \\ 14 \end{bmatrix} \begin{bmatrix} MeanSquare \\ MS_{bn} \\ MS_{wn} \\ \\ \end{bmatrix} \begin{bmatrix} F-ratio \\ F_{obt} \\ \\ \\ \end{bmatrix}$$

Computing the Mean Squares

$$MS_{bn} = \frac{SS_{bn}}{df_{bn}}$$

$$MS_{bn} = \frac{63.33}{2}$$

```r
63.33/2
```

```
## [1] 31.665
```

$$MS_{bn} = 31.67$$

Filling in the Table

$$\begin{bmatrix} Source \\ Between \\ Within \\ Total \end{bmatrix} \begin{bmatrix} Sum\ of\ Squares \\ 63.33 \\ 84 \\ 147.33 \end{bmatrix} \begin{bmatrix} df \\ 2 \\ 12 \\ 14 \end{bmatrix} \begin{bmatrix} MeanSquare \\ 31.67 \\ MS_{wn} \\ \end{bmatrix} \begin{bmatrix} F-ratio \\ F_{obt} \\ \\ \end{bmatrix}$$

Computing the Mean Square within Groups

$$MS_{wn} = \frac{SS_{wn}}{df_{wn}}$$

$$MS_{wn} = \frac{84}{12}$$

```r
84/12
```

```
## [1] 7
```

$$MS_{wn} = 7$$

Filling in the Table

$$\begin{bmatrix} Source \\ Between \\ Within \\ Total \end{bmatrix} \begin{bmatrix} Sum\ of\ Squares \\ 63.33 \\ 84 \\ 147.33 \end{bmatrix} \begin{bmatrix} df \\ 2 \\ 12 \\ 14 \end{bmatrix} \begin{bmatrix} MeanSquare \\ 31.67 \\ 7 \\ \\ \end{bmatrix} \begin{bmatrix} F - ratio \\ F_{obt} \\ \\ \\ \end{bmatrix}$$

Calculating the F-statistic

$$F_{obt} = \frac{MS_{bn}}{MS_{wn}}$$

$$F_{obt} = \frac{31.67}{7}$$

```r
31.67/7
```

```
## [1] 4.524286
```

$$F_{obt} = 4.52$$

Filling in the Table

$$\begin{bmatrix} Source \\ Between \\ Within \\ Total \end{bmatrix} \begin{bmatrix} Sum\ of\ Squares \\ 63.33 \\ 84 \\ 147.33 \end{bmatrix} \begin{bmatrix} df \\ 2 \\ 12 \\ 14 \end{bmatrix} \begin{bmatrix} MeanSquare \\ 31.67 \\ 7 \\ \end{bmatrix} \begin{bmatrix} F-ratio \\ 4.52 \\ \\ \end{bmatrix}$$

Interpreting the F-obtained value

- **F-distribution** is the sampling distribution with values of F to represent when H0 is true and all conditions represent one population (no differences between groups/levels)
- F cannot be less than zero
- There is no limit to how large an F-obtained value can be
- the mean of the F-distribution is 1
- F-distribution shape also depends on the df

F-table

- uses alpha, df within, df between
- line up the df within with the df between and choose the value based on the alpha decided on
- the F-table only tells us one thing, `is there a statistically significant difference between the means of the three groups/conditions/levels`
- in order to see which specific group comparisons are significantly different from one another, we will rely on post-hoc tests or examination of the contrasts

Writing up Findings

- when reporting, you would first include the test that you ran and the context
  - how many groups, your IV, your DV
- you would then report the F statistic
  - F(df between, df within) = F obtained value, p value
- then you would report the post-hoc tests, which would include the group/level means that were statistically significant from one another
- lastly you would report the effect size/eta squared (we'll get to this next week)

Next Class

- We'll talk about:
  - effect sizes
  - calculations for post-hoc tests
  - proportions of variance accounted for
  - slight introduction to what an ANCOVA is

Practice Problems

You are interested in ways your class (N = 30) can increase their happiness. You are testing two methods to improve happiness and one control group. Your groups are: 1 = having plants to take care of (n = 10), 2 = support animal (n = 10), 3 = control (n = 10). You are interested in there is a significant difference between these three groups on your students happiness.

```r
set.seed(101421)

plants <- rnorm(10, mean = 80, sd = 11)
animal <- rnorm(10, mean = 40, sd = 8.4)
control <- rnorm(10, mean = 50, sd = 2.1)

happy <- data.frame(plants,
                    animal,
                    control)

happy
```

```
##       plants   animal  control
## 1   90.71611 49.36230 46.56667
## 2   62.01473 34.80282 48.64966
## 3   71.23168 38.74386 48.14495
## 4   78.82846 43.41543 50.64383
## 5   60.72292 33.21453 49.84474
## 6  107.16287 36.53926 50.35684
## 7   67.26932 32.73007 50.33949
## 8   79.73614 47.87016 49.10797
## 9   84.54266 22.45582 51.92304
## 10  86.23949 31.74746 53.10834
```

You are having a movie marathon of all your favorite spooky movies to watch. You decide to make a competition between you and your friends by seeing whose method of staying away longer will allow you to watch more movies in a weekend. You decide that you and some others (n = 5) are going to drink two pots of coffee each day to stay away all weekend. Another group (n = 5) decide that they are going to drink energy drinks to stay up. Your last group of friends (n = 5) are going to rely on each other to stay up by shaking each other awaken. You are interested in what group will stay up the latest.

```r
set.seed(101421)

coffee <- rnorm(5, mean = 20, sd = 4.5)
energy <- rnorm(5, mean = 43, sd = 1.2)
support <- rnorm(5, mean = 10, sd = .5)

spooky <- data.frame(coffee,
                     energy,
                     support)

spooky
```

```
##     coffee   energy   support
## 1 24.38386 45.96322 10.557279
## 2 12.64239 41.61120  9.690644
## 3 16.41296 42.97121  9.925230
## 4 19.52073 43.49556 10.203300
## 5 12.11392 43.68067  9.596103
```