PSY 3307

Two Sample t-test Pt2

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Agenda

- Review terms
  - independent samples t-test, homogeneity of variance, pooled variance, standard error of the difference
- Between & Within Designs
- One Tailed Independent Samples t-test
- Paired Sample t-test
- Performing a Paired Sample t-test
- Effect Size

One Sample t-test

- used when we are confident that the direction of the relationship
- if you think one group will have a higher score/value then you would make that statement in your alternative hypothesis

Steps for one-tailed test

- Variable(Groups) = Intervention(Got Intervention/Control)
- Outcome = Fruit & Vegetable Intake

1. decide which sample/group score you think will be larger
    - I think that the intervention group will eat more fruits and vegetables
2. decide which condition/group to subtract from the other
    - `intervention group - control group`
3. decide whether the difference will be positive or negative
    - a positive value should be returned from the previous equation

1. create hypotheses
    - The intervention group will eat the same or less fruits and vegetables than the control group
    - H0: mu(intervention group) - mu(control group) ≤ 0
    - The intervention group will eat more fruits and vegetables than the control group
    - H1: mu(intervention group) - mu(control group) > 0

1. locate regions of rejection
    - since we are interested in a positive difference in our hypothesis, we'll only be looking at the upper tail of the distribution
2. conduct your independent samples t-test
    - make sure your groups/samples are subtracted the same way as your hypotheses
    - `intervention group - control group`

Paired Samples t-test

- within-subjects/groups design
- also called related-samples t-test
  - each participant gets measured in each condition
- an example would be an intervention for weight loss where everyone's weight is measured before (first time point to measure) the intervention and after (second time point to measure)
- **matched-samples design** is when a participant in one condition is matched with a participant in the other condition
- **repeated-measures design** is when each participant gets measured twice
  - can be more for more rigorous statistics, but not the ones we'll learn about in this class

Why we use paired-samples t-tests

- pretest/posttest design
  - before everyone gets the intervention/experiment (pretest or first time point) and then after the intervention/experiment (posttest or second time point)

- we subtract every person's before/after score from the before/score score to get a `difference` score
- you can subtract whatever score you want from the other (before - after or after - before)
- **mean differences** the mean of the differences between the paired scores
  - repesented as dbar
- add all the values of differences for before/after to get a sum and then divide by the number of participants (each person has a before and after score or the number of difference scores)

$$Sample\ mean\ difference = \overline{D}$$

$$population\ mean\ difference = \mu_D$$

- now because we have a sample mean from a sample, we can now perform a one-sample t-test

Hypotheses

- our null hypothesis is that there will be no change in scores on both occasions so there should be a difference of zero

$$H_0 : \mu_D = 0$$

- our alternative hypothesis is that either our before or after scores should be higher
  - our hypothesis supports either a positive or negative change

$$H_1 : \mu_D \neq 0$$

- similar to the independent samples t-test, our sampling distribution is of mean differences

Steps to paired-samples t-test

Calculate the estimated variance of the difference scores

$$s_D^2 = \frac{\Sigma D^2 - \frac{(\Sigma D)^2}{N}}{N - 1}$$

Note the notation changes from X to D because we are working with differences in scores and not means

Calculate the standard error of the mean differences

$$S_{\overline{D}} = \sqrt{\frac{S_D^2}{N}}$$

Find the obtained t-value

$$t_{obt} = \frac{\overline{D} - \mu_D}{S_{\overline{D}}}$$

The population difference will be zero unless you are testing for a nonzero difference

Calculate the degrees of freedom

$$df = N - 1$$

Calculate effect size

$$d = \frac{\overline{D}}{\sqrt{S_D^2}}$$

OR

$$r_{pb}^2 = \sqrt{\frac{(t_{obt})^2}{(t_{obt})^2 + df}}$$

Discuss Significance

- Is the difference between time point 1 and time point 2 statistically significant?

Example

```
set.seed(100521)
t1 = rnorm(10, mean = 10, sd = 1.5)

t2 = rnorm(10, mean = 5.8, sd = 4)

df <- data.frame(participant = 1:10,
                 t1 = round(t1, 2),
                 t2 = round(t2, 2),
                 score_difference = c("_____________", "_____________",
                                    "_____________", "_____________", "
                 score_difference_squared = c("_____________", "________
                                    "_____________", "_____________", "
```

```r
df$t2
```

```
##  [1] 12.75 10.54  7.51  4.32  9.95  3.64  6.45  3.40  3.43 13.77
```

```r
df$t1
```

```
##  [1]  8.32 13.96  8.17  8.40  8.90  7.58  9.67 11.51 10.13 11.49
```

```r
df2 <- df %>%
  mutate(score_difference = t2 - t1,
         score_difference_squared = score_difference^2)

df2 %>%
  summarize(mean = mean(score_difference),
            variance = sd(score_difference)^2,
            sd = sd(score_difference))
```

```
##     mean variance       sd
## 1 -2.237 15.69073 3.961153
```

```r
12.75 - 8.32
```

```
## [1] 4.43
```

```r
10.54 - 13.96
```

```
## [1] -3.42
```

```r
7.51 - 8.17
```

```
## [1] -0.66
```

```r
4.32 - 8.40
```

```
## [1] -4.08
```

```r
9.95 - 8.90
```

```
## [1] 1.05
```

```r
3.64 - 7.58
```

```
## [1] -3.94
```

```
df <- data.frame(participant = 1:10,
  t1 = round(t1, 2),
                 t2 = round(t2, 2),
                 score_difference = c(4.43, -3.42, -.66, -4.08, 1.05, -3
                 score_difference_squared = c("_____________", "________
                                               "_____________", "_____________", "
```

```r
df$score_difference
```

```
##  [1]  4.43 -3.42 -0.66 -4.08  1.05 -3.94 -3.22 -8.11 -6.70  2.28
```

```r
(4.43)^2
```

```
## [1] 19.6249
```

```r
(-3.42)^2
```

```
## [1] 11.6964
```

```r
(-.66)^2
```

```
## [1] 0.4356
```

```r
(-4.08)^2
```

```
## [1] 16.6464
```

```r
(1.05)^2
```

```
## [1] 1.1025
```

```r
(-3.94)^2
```

```
## [1] 15.5236
```

```r
df <- data.frame(participant = 1:10,
  t1 = round(t1, 2),
                 t2 = round(t2, 2),
                 score_difference = c(4.43, -3.42, -.66, -4.08, 1.05, -3
                 score_difference_squared = c(19.62, 11.70, .44, 16.64,

df
```

```
##    participant    t1    t2 score_difference score_difference_squared
## 1            1  8.32 12.75             4.43                    19.62
## 2            2 13.96 10.54            -3.42                    11.70
## 3            3  8.17  7.51            -0.66                     0.44
## 4            4  8.40  4.32            -4.08                    16.64
## 5            5  8.90  9.95             1.05                     1.10
## 6            6  7.58  3.64            -3.94                    15.52
## 7            7  9.67  6.45            -3.22                    10.37
## 8            8 11.51  3.40            -8.11                    65.77
## 9            9 10.13  3.43            -6.70                    44.89
## 10          10 11.49 13.77             2.28                     5.20
```

```r
df$score_difference
```

```
##  [1]  4.43 -3.42 -0.66 -4.08  1.05 -3.94 -3.22 -8.11 -6.70  2.28
```

```r
4.43 +-3.42 +-0.66 +-4.08  +1.05 +-3.94 +-3.22 +-8.11 +-6.70  +2.28
```

```
## [1] -22.37
```

```r
# sum difference is -22.37

-22.37/10
```

```
## [1] -2.237
```

```r
# mean difference is -2.24
```

Calculate the estimated variance of the difference scores

$$s_D^2 = \frac{\Sigma D^2 - \frac{(\Sigma D)^2}{N}}{N - 1}$$

$$s_D^2 = \frac{\Sigma D^2 - \frac{(-22.37)^2}{10}}{10 - 1}$$

```
19.62+ 11.70+ .44+ 16.64+ 1.10+ 15.52+ 10.37+ 65.77+ 44.89+ 5.20
```

```
## [1] 191.25
```

$$s_D^2 = \frac{191.25 - \frac{(-22.37)^2}{10}}{10 - 1}$$

```r
(-22.37)^2
```

```
## [1] 500.4169
```

$$s_D^2 = \frac{191.25 - \frac{500.42}{10}}{10 - 1}$$

```r
500.42/10
```

```
## [1] 50.042
```

$$s_D^2 = \frac{191.25 - 50.04}{9}$$

```r
191.25 - 50.04
```

```
## [1] 141.21
```

$$s_D^2 = \frac{141.21}{9}$$

```r
141.21/9
```

```
## [1] 15.69
```

$$s_D^2 = 15.69$$

Calculate the standard error of the mean differences

$$S_{\overline{D}} = \sqrt{\frac{S_D^2}{N}}$$

$$S_{\overline{D}} = \sqrt{\frac{15.69}{10}}$$

```r
15.69/10
```

```
## [1] 1.569
```

$$S_{\overline{D}} = \sqrt{1.57}$$

```r
sqrt(1.57)
```

```
## [1] 1.252996
```

$$S_{\overline{D}} = 1.25$$

Find the obtained t-value

$$t_{obt} = \frac{\overline{D} - \mu_D}{S_{\overline{D}}}$$

$$t_{obt} = \frac{-2.24 - 0}{1.25}$$

```r
-2.24 - 0
```

```
## [1] -2.24
```

$$t_{obt} = \frac{-2.24}{1.25}$$

```r
-2.24/1.25
```

```
## [1] -1.792
```

$$t_{obt} = -1.79$$

Calculate the degrees of freedom

$$df = N - 1$$

```r
10 - 1
```

```
## [1] 9
```

$$df = 9$$

Calculate effect size

$$d = \frac{\overline{D}}{\sqrt{S_D^2}}$$

```r
mean(df$score_difference)
```

```
## [1] -2.237
```

```r
sd(df$score_difference)^2
```

```
## [1] 15.69073
```

```r
-2.24
```

```
## [1] -2.24
```

```r
15.69
```

```
## [1] 15.69
```

$$d = \frac{-2.24}{\sqrt{15.69}}$$

```r
sqrt(15.69)
```

```
## [1] 3.96106
```

```r
sd(df$score_difference)
```

```
## [1] 3.961153
```

$$d = \frac{-2.24}{3.96}$$

```r
-2.24/3.96
```

```
## [1] -0.5656566
```

$$d = -.57$$

so really this means

$$d = .57$$

$$r_{pb}^2 = \sqrt{\frac{(t_{obt})^2}{(t_{obt})^2 + df}}$$

$$r_{pb}^2 = \sqrt{\frac{(-1.79)^2}{(-1.79)^2 + df}}$$

```r
(-1.79)^2
```

```
## [1] 3.2041
```

$$r^2_{pb} = \sqrt{\frac{3.20}{3.20 + 9}}$$

```r
3.20 + 9
```

```
## [1] 12.2
```

$$r_{pb}^2 = \sqrt{\frac{3.20}{12.20}}$$

```r
3.2/12.2
```

```
## [1] 0.2622951
```

$$r^2_{pb} = \sqrt{.26}$$

```r
sqrt(.26)
```

```
## [1] 0.509902
```

$$r_{pb}^2 = .51$$

Discuss Significance

t-obtained value of -1.79

t-critical value of

One-tailed paired-samples t-test

- choose whether you think your `after` score would be lower/higher than the `before` score
- have your hypotheses reflect that
- if you think after scores should be higher than before (learning intervention) then the difference scores should be positive
  - because you subtracted before scores from after scores

$$H_a : \mu_D > 0$$

$$H_0 : \mu_D \leq 0$$

Reporting

- report in a similar style to all other t-test
  - t(df) = t value, p value
- you'll also report the means of the before and after scores
  - you won't report the difference between the two means

Effect Sizes

- **effect size** is the amount of influence that changing the conditions of the IV has on the DV
- **cohen's d** is a measure of effect size in two-sample studies that reflects the magnitude of difference
  - small = .2
  - medium = .5
  - large = .8
- larger effect size means stronger the strength of the association/relationship between the IV and DV

Effect Size using Proportion of Variance Accounted For

- used to determine how consistently scores change
- **proportion of variance accounted for** is the proportion of differences in DV scores assocaited with changing the conditions of the IV
  - effect size using **squared point-biserial correlation coefficient**, which indicates the proprotion of variance in DV scores that is accounted fro by IV variable in two-sample studies, are below
  - small = .09
  - moderate = .10-.25
  - large = .25

Practice Time

```
set.seed(093021)

mistakes_made_tutor = rnorm(10, mean = 1.5, sd = 1.4)
mistakes_made_control = rnorm(8, mean = 4.1, sd = 1)

mistakes_made_tutor
```

```
##  [1] 3.0495964 0.7931460 2.1999939 0.4106114 1.3852964 2.2035563 1.8161635
##  [8] 2.4894226 3.1436843 1.0698578
```

```
mistakes_made_control
```

```
## [1] 5.347698 3.962054 5.072715 3.985207 4.945898 2.984944 5.004625 5.405361
```

```r
set.seed(093021)

translating_native_speaker = rnorm(9, mean = 20, sd = 4.7)
translating_non_native = rnorm(14, mean = 10, sd = .99)

translating_native_speaker
```

```
## [1] 25.20222 17.62699 22.34998 16.34277 19.61492 22.36194 21.06141 23.32163
## [9] 25.51808
```

```r
translating_non_native
```

```
##  [1]  9.695828 11.235221  9.863433 10.962988  9.886355 10.837439  8.896095
##  [8] 10.895579 11.292308  9.102475 12.603552  9.039360 10.388593  9.487347
```

```r
set.seed(093021)

first_gen_bmi = rnorm(6, mean = 22, sd = 2.2)
second_gen_bmi = rnorm(9, mean = 28, sd = 5)

first_gen_bmi
```

```
## [1] 24.43508 20.88923 23.09999 20.28810 21.81975 23.10559
```

```r
second_gen_bmi
```

```
## [1] 29.12916 31.53365 33.87030 26.46378 34.23849 27.31027 32.86358 27.42604
## [9] 32.22949
```

```r
set.seed(100521)
t1 = rnorm(10, mean = 15, sd = 2.7)

t2 = rnorm(10, mean = 12, sd = 2.1)

df1 <- data.frame(participant = 1:10,
                  t1 = round(t1, 2),
                  t2 = round(t2, 2),
                  score_difference = c("_____________", "_____________",
                                       "_____________", "_____________", "
                  score_difference_squared = c("_____________", "________
                                       "_____________", "_____________", "

df1$t2
```

```
##  [1] 15.65 14.49 12.90 11.22 14.18 10.86 12.34 10.74 10.76 16.19
```

```r
df1$t1
```

```
##  [1] 11.98 22.12 11.71 12.11 13.02 10.65 14.40 17.73 15.23 17.69
```

```r
set.seed(100521)
t1 = rnorm(10, mean = 8, sd = 1.2)

t2 = rnorm(10, mean = 10.5, sd = 4.8)

df11 <- data.frame(participant = 1:10,
                 t1 = round(t1, 2),
                 t2 = round(t2, 2),
                 score_difference = c("_____________", "_____________",
                                     "_____________", "_____________", '
                 score_difference_squared = c("_____________", "________
                                     "_____________", "_____________", '

df11$t2
```

```
## [1] 18.83 16.19 12.55  8.72 15.48  7.90 11.28  7.62  7.66 20.07
```

```r
df11$t1
```

```
## [1]  6.66 11.17  6.54  6.72  7.12  6.06  7.73  9.21  8.10  9.19
```

```r
set.seed(100521)
t1 = rnorm(10, mean = 15, sd = 5.6)

t2 = rnorm(10, mean = 4.3, sd = 5)

df111 <- data.frame(participant = 1:10,
                 t1 = round(t1, 2),
                 t2 = round(t2, 2),
                 score_difference = c("_____________", "_____________",
                                     "_____________", "_____________", "
                 score_difference_squared = c("_____________", "_______
                                     "_____________", "_____________", "

df111$t2
```

```
##  [1] 12.98 10.22  6.43  2.45  9.49  1.60  5.11  1.30  1.34 14.27
```

```r
df111$t1
```

```
##  [1]  8.73 29.78  8.18  9.01 10.89  5.97 13.76 20.65 15.49 20.57
```