• Trigonométrica

① $t \in \left[-\frac{1}{2F_0} , \frac{1}{2F_0} \right]$ $\quad \omega_0 = 2\pi F_0$

$$X(t) = |A \operatorname{Sen}(2\pi F_0 t)|^2$$

$$X(t) = A^2 \operatorname{Sen}^2(2\pi F_0 t)$$

$$X(t) = A^2 \left(\frac{1}{2} - \frac{1}{2} \operatorname{Cos}(2\pi 2F_0 t) \right)$$

$$X(t) = \frac{A^2}{2} - \frac{A^2}{2} \operatorname{Cos}(2\pi 2 F_0 t)$$

→ Igualamos

$$a_0 + \sum_{n=-N}^{N} a_n \operatorname{Cos}(n\omega_0 t) = \frac{A^2}{2} - \frac{A^2}{2} \operatorname{Cos}(2\pi 2 F_0 t)$$

$$\rightarrow \boxed{a_0 = \frac{A^2}{2} \; ; \; a_2 = -\frac{A^2}{2}}$$

• Exponencial

$$C_0 = a_0 = \frac{A^2}{2}$$

$$C_n = \frac{a_n - jb_n}{2} = \frac{\frac{A^2}{2}}{2} = \frac{A^2}{4} \quad \forall \; n = \{2, -2\}$$

$$F\{C_0\} = C_0 \delta(f) ; \; F\{C_2 e^{j2t}\} = C_2 \delta(f - 2F_0) ; \; F\{C_2 e^{-j2t}\} = C_{-2} \delta(F + 2F_0)$$

$$\rightarrow X(F) = \frac{A^2}{2} \delta(f) - \frac{A^2}{4} \delta(f - 2F_0) - \frac{A^2}{4} \delta(f + 2F_0)$$

③ Primero definimos cómo queda el espectro

$$Y(\omega) = F\{y(t)\}$$

$$= F\left\{\left(1 + \frac{m(t)}{A_c}\right) c(t)\right\}$$

$$= F\left\{c(t) + \frac{m(t)\,c(t)}{A_c}\right\}$$

$$= \underbrace{F\{c(t)\}}_{} + \frac{1}{A_c} F\{m(t)\,c(t)\} \quad ②$$

$$F\{c(t)\} = F\{A_c \operatorname{Sen}(2\pi F_c t)$$

$$= A_c\, F\left\{\frac{e^{j2\pi F_c t} - e^{-j2\pi F_c t}}{2j}\right\}$$

$$\rightarrow F\{e^{\pm j2\pi F_c t}\} = 2\pi\,\delta(\omega \mp \omega_0)$$

$$\therefore\ C(\omega) = \frac{A_c \pi}{j}\left(\delta(\omega - 2\pi F_c) - \delta(\omega + 2\pi F_c)\right)$$

② $$\frac{1}{A_c} F\{m(t)\,c(t)\} = \frac{1}{A_c} F\{m(t)\, A_c \operatorname{Sen}(2\pi F_c t)$$

$$= F\{m(t) \operatorname{Sen}(2\pi F_c t)\} = F\left\{\frac{m(t)\,e^{2\pi F_c t} - m(t)\,e^{-2\pi F_c t}}{2j}\right\}$$

$$\rightarrow \frac{1}{A_c} F\{m(t)\,c(t)\} = \frac{1}{2j}\left(M(\omega - 2\pi F_c) - M(\omega + 2\pi F_c)\right)$$

$$Y(\omega) = \frac{A_c\pi}{j}\left(\delta(\omega - 2\pi F_c) - \delta(\omega + 2\pi F_c)\right) + \frac{1}{2j}\left(M(\omega - 2\pi F_c) - M(\omega + 2\pi F_c)\right)$$

$$P_x = \frac{1}{T}\int_0^T |x(t)|^2\, dt$$

$$= \frac{1}{T}\int_0^T |A\operatorname{sen}(2\pi F_o t)|^2\, dt$$

$$= \frac{1}{T}\int_0^T A^2 \operatorname{sen}^2(2\pi F_o t)\, dt$$

El valor promedio de $\sin^2(\theta)$ en el periodo completo es $\frac{1}{2}$

$$\rightarrow \quad P_x = \frac{A^2 T}{2T} = \frac{A^2}{2}$$