Juan Camilo Perdomo Soto CC: 1081393093

Parcial 1.

A. Determinar la distancia entre las dos señales

$$d^2(X_1,X_2) = \bar{P}_{X_1-X_2} = \lim_{T\to\infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt$$

$$X_1(t) = A e^{jw_0 t}$$

$$X_2(t) = B e^{j5w_0 t}$$

$$\bar{P}_{X_1-X_2} = \lim_{T\to\infty} \frac{1}{T} \int_T |x_1(t) - X_2(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_T (x_1(t) - X_2(t))(X_1(t) - X_2(t))^* dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_T (X_1(t) - X_2(t))(x_1(t)^* - X_2(t)^*) dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_T X_1(t)X_1(t)^* - X_1(t)X_2(t)^* - X_1(t)^* X_2(t) + X_2(t)X_2(t)^* dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_T |X_1(t)|^2 dt - 2X_1(t)X_2(t)^* + |X_2(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\Big(\underbrace{\int_T |X_1(t)|^2 dt}_{1} - 2 \underbrace{\int_T X_1(t)X_2(t)^* dt}_{2} + \underbrace{\int_T |X_2(t)|^2 dt}_{3}$$

1 $$\int_0^T |X_1(t)|^2 dt = \int_0^T |Ae^{jw_0 t}|^2 dt = \int_0^T |A|^2 |e^{jw_0t}|^2 dt$$

$$= A^2 \int_0^T (e^{jw_0t} \cdot e^{-jw_0t}) dt$$

$$= A^2 \int_0^T e^0 dt$$

$$= A^2 \int_0^T 1 dt$$

$$= A^2(T-0) = A^2 T$$

3 $$\int_T |X_2(t)|^2 dt = \int_0^T |Be^{j5w_0t}|^2 dt$$

$$= \int_0^T |B|^2 |e^{j5w_0t}|^2 dt$$

$$= B^2 \int_0^T (e^{j5w_0t} \cdot e^{-j5w_0t}) dt$$

$$= B^2 \int_0^T e^0 dt$$

$$= B^2 \int_0^T 1\, dt$$

$$= B^2(T-0)$$

$$= B^2 T$$

2 $$2\int_T X_1(t) X_2(t)^* dt = 2\int_0^T X_1(t)^* X_2(t) dt$$

$$= 2\int_0^T Ae^{-jw_0t} Be^{j5w_0t} dt$$

$$= 2AB \int_0^T e^{j4w_0t} dt \qquad \boxed{\begin{array}{l} u = j4w_0t \\ du = jw_0t \\ dt = \frac{du}{j4w_0} \end{array}}$$

$$= \frac{2AB}{j4w_0} \int_0^T e^u du$$

$$= \frac{2AB}{j4w_0} e^u \Big|_0^T$$

$$= \frac{2AB}{j4w_0} \left(e^{j4w_0T} - e^{j4w_0 \cdot 0} \right)$$

$$= \frac{2AB}{j4w_0} \left(e^{j4\frac{2\pi}{\cancel{T}}\cancel{T}} - 1 \right)$$

$$= \frac{2AB}{j4w_0} \left(\overset{1}{\cancel{Cos(8\pi)}} + j\overset{0}{\cancel{Sen(8\pi)}} - 1 \right)$$

$$= \frac{2AB}{j4w_0}(1+0-1)$$

$$= \frac{2AB}{j4w_0}(0)$$

$$= 0$$

Entonces:

$$= \lim_{T\to\infty} \frac{1}{T}(A^2T - 0 + B^2T)$$

$$= \lim_{T\to\infty} A^2 - 0 + B^2$$

$$= A^2 + B^2$$

B. $X(t) = 3\cos 1000\pi t + 5\,\text{sen}\, 2000\pi t + 10\cos 11000\pi t$

$F_s = 5\,KHz$

$W_1 = 1000\pi \quad \therefore \quad F_1 = \frac{1000\pi}{2\pi} = 500$

$W_2 = 2000\pi \quad \therefore \quad F_2 = \frac{2000\pi}{2\pi} = 1000$

$W_3 = 11000\pi \quad \therefore \quad F_3 = \frac{11000\pi}{2\pi} = 5500$ No cumple Nyquist

Como no cumple Nyquist, no es apropiado $F_s = 5000\,Hz$, entonces usamos $F_s > 2F_{max}$

Usamos $F_s = 12000\,Hz$

Entonces

Discretizamos con la nueva F_s $\quad t = nT_s = \frac{n}{F_s}$

$$x(t) = 3\cos \frac{1\cancel{000}\pi}{12\cancel{000}} t + 5\,\text{sen}\, \frac{2\cancel{000}}{12\cancel{000}}\pi t + 10\cos \frac{11\cancel{000}}{12\cancel{000}}\pi t$$

$$= 3\cos \frac{1}{12}\pi t + 5\,\text{sen}\, \frac{1}{6}\pi t + 10\cos \frac{11}{12}\pi t$$

$$= 3\cos \frac{\pi}{12} n + 5\,\text{sen}\, \frac{\pi}{6} + 10\cos \frac{11}{12}\pi n$$

$$\Omega_1 = \frac{\pi}{12}\ ; \ \Omega_2 = \frac{\pi}{6}\ ; \ \Omega_3 = \frac{11}{12}\pi$$

$$\boxed{x[n] = 3\cos \frac{\pi}{12} n + 5\,\text{sen}\, \frac{\pi}{6} n + 10\cos \frac{11}{12}\pi n}$$