# The Selective Disclosure of Evidence An Experiment

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**Motivation** introduction

In many settings, agents communicate by disclosing selected evidence:

- Journalists select which facts to report in their articles
- Managers select which results to discuss in performance reports
- Job candidates select which achievements to list on their CVs

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A pervasive force in communication, e.g., a principal source of news media bias

("filtering," Gentzkow et al. '14)

This Paper introduction

An experimental study on selective disclosure

We build on a small theoretical literature on selective disclosure of noisy evidence Milgrom ('81, Bell), Fishman and Hagerty ('90, QJE), Di Tillio et al ('21, Ecma)

Our model generates rich comparative statics in N and K:

- Which evidence do senders disclose?
- How much information do they transmit to receivers?
- Do receivers account the selection in the evidence they see?

These comparative statics inform a novel experimental design, and provide a rigorous test of the theory

Our data corroborates the key qualitative predictions of the theory

 Validation of selective disclosure as a force in communication that is behaviorally descriptive

We document two novel *quantitative* deviations from the theory:

- A form of deception aversion in senders leads to overcommunication
- Evidence of selection neglect in a strategic setting

**Policy implications:** Mandating disclosure can be ineffective (and possibly detrimental) when selection opportunities are large

### **Related Literature: Theory**

Classic disclosure models focus on **rich** evidence (e.g., Grossman, '81; Milgrom, '81; Jovanovic, '82; Okuno-Fujiwara et al., '90)

Senders can verifiably disclose their type → unravelling results

We focus on settings where evidence is **not rich** and, thus, unravelling does not occur

Fishman and Hagerty ('90, QJE), Di Tillio et al ('21, Ecma)

 This enables nontrivial comparative statistics, which are instrumental for testing the theory

Related but less connected settings:

Glazer and Rubinstein ('04, Ecma; '06, TE), Shin ('03, Ecma), Dziuda ('11, JET), Haghtalab et al. ('21), Gao ('23)

## **Related Literature: Empirics**

Disclosure settings with verifiable and "rich" evidence Forsythe et al ('89, RAND),

Jin and Leslie ('03, QJE), Jin, Luca and Martin ('22, AEJ: Micro)

Settings with unverifiable evidence, i.e., cheap talk Cai and Wang ('06, GEB)

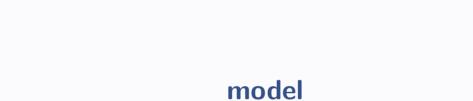
Recent and related settings with selective disclosure

Degan, Li, Xie ('23, CJE), Penczynski, Koch, Zhang ('23)

Methodologically: (close to Frechette, Lizzeri and Perego (2022, Ecma))

Exploit rich comparative statics to test underlying forces in the theory

- 1. Model
- 2. Equilibrium and Testable Predictions
- 3. Experimental Design
- 4. Results



from Milgrom (1981)

Sender privately observes the state  $\theta \in \Theta$ :

-  $\Theta$  finite and ordered,  $p \in \Delta(\Theta)$  common prior

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Receiver observes the message and takes an action  $a \in A$ 

Given state  $\theta$  and action a,

- Receiver's payoff is 
$$u(\theta, a) = -(a - \theta)^2$$

- Sender's payoff is 
$$v(\theta, a) = a$$

higher actions preferred

wants to guess the state

No message can verifiably reveal  $\theta \leadsto \text{failure of richness}$  (Okuno-Fujiwara et al., '90)

Sender does not choose N, i.e., available evidence is  ${\bf exogenous}$ 

If K = N, the sender can disclose all her available evidence if so she wants

If K < N, sender can cherry pick which evidence to disclose

 $-\ K < N$  captures exogenous communication constraints

Changes in  ${\cal K}$  and  ${\cal N}$  generate rich testable predictions, which we use as a test of the theory



Our analysis focuses on pure-strategy PBEs (as in Okuno-Fujiwara et al., '90, Restud)

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We refine the equilibrium set using **neologism proofness**, Farrel ('93, *GEB*) adapted to our setting with verifiable information

Under this refinement, our game admits a unique equilibrium outcome

More formally, we focus on a natural class of sender's strategies:

#### **Definition**

A sender's strategy is  ${\it maximally selective}$  if, given the available signals, she discloses the K-highest ones.

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#### Proposition 1

There exists a PBE in which the sender plays a maximally selective strategy  $\ensuremath{\mathsf{PBE}}$ 

(Milgrom 1981)

Moreover, the outcome it induces is unique in the class of neologism-proof PBEs

Main outcome of interest is the informativeness the equilibrium strategies

I.e., how effectively sender and receiver communicate

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We measure informativeness as the  ${\bf correlation}$  btw  $\theta$  and a, denoted by

$$\mathcal{I} = Corr( heta,a)$$
 as in Lizzeri, Frechette, Perego ('22, Ecma)

### **Main Comparative Statics**

Rich predictions regarding how informativeness changes in  ${\cal K}$  and  ${\cal N}$ 

### **Proposition 2**

Fixing N, Equilibrium informativeness increases in  ${\cal K}$ 

Fixing K, equilibrium informativeness can increase for small N but eventually decreases to zero as  $N\to\infty$ 

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That is, giving the sender more discretion can make the receiver's better off see also Fishman and Hagerty ('90, QJE), Di Tillio, Ottaviani and Sorensen ('21, Ecma)

Suppose 
$$\Theta=\{\theta_L,\theta_H\}$$
,  $p(\theta_H)=\frac{1}{2}$ ,  $S=\{A,B\}$ ,  $K=1$ 

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$$\begin{array}{cccc} f(s|\theta) & & \text{Signal} \\ \text{State} & & A & B \\ \hline \theta_L & 0 & 1 \\ \hline \theta_H & & \gamma & 1-\gamma \end{array}$$

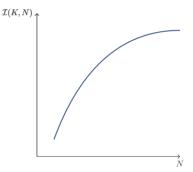
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$$\mathcal{I}(K,N) = \frac{1}{4} - \frac{(1-\gamma)^N}{2(1+(1-\gamma)^N)}$$

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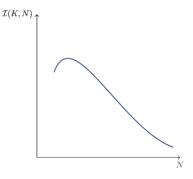
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experiment

### **Experimental Design**

Binary state:  $\theta_L$  and  $\theta_H$ , equal probability

Four possible signals  $S = \{A, B, C, D\}$ 

Information structure f:

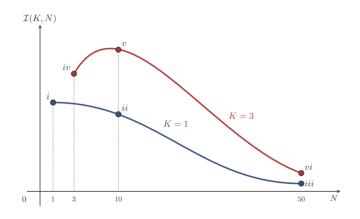
State	Signal			
	A	B	C	D
$\theta_L$	10%	20%	25%	45%
$\theta_H$	45%	25%	20%	10%

Receiver's action  $a \in [0,1]$ , which makes it equivalent to a belief elicitation task implemented using BSR (Hossain and Okui, '13 Restud)  $\overset{\text{Details}}{}$ 

We vary K and N as follows:

	N = 1	N = 3	N = 10	N = 50
K=1	$\checkmark$	•	<b>√</b>	$\checkmark$
K = 3		✓	✓	✓

# **Main Comparative Statics**



# **Experimental Details**

- Undergrad population Columbia and NYU: Spring, Summer, Fall 2023
- 420 subjects, between-subject design
- 6 treatments
- 4 sessions per treatment
- 30 rounds per session, random rematching
- 17.5 subjects per sessions on average
- Average payout \$30 per subject
- Fixed roles

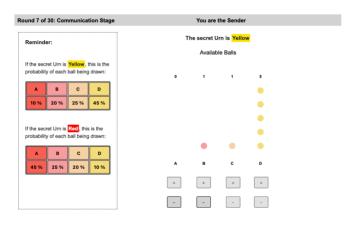


Your message to the Receiver is:



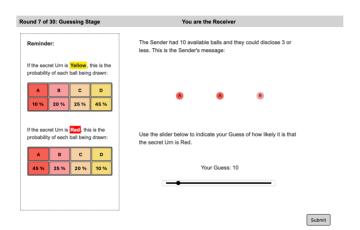


Send



Your message to the Receiver is:





# results

#### Progression of our analysis:

- Which evidence do senders disclose?
- How informative is it?
- How do receivers respond to it?

# result 1

(which evidence is disclosed)

# Which Evidence is Disclosed?

Question 1: Which evidence do senders disclose?

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Theory predicts that:

- If N increases, the evidence disclosed should become **more** favorable
  - sender can be more selective with larger sample
- If K increases, evidence disclosed should become less favorable

held to higher a standard, sender needs to be less selective

### Which Evidence is Disclosed?

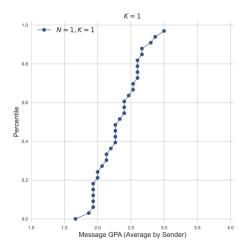
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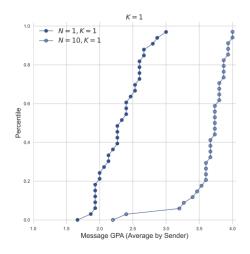
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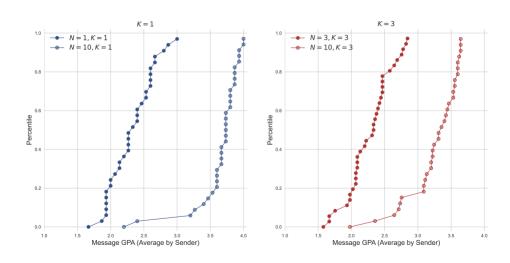
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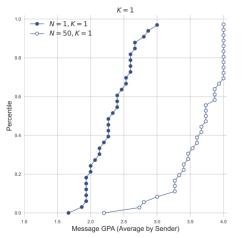
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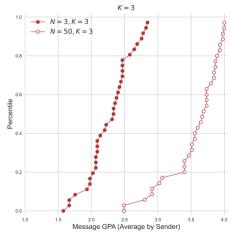
To test this, we compute the **GPA** of each message ( $A\leadsto 4$ ,  $B\leadsto 3$ , etc) and study how it changes in N and K

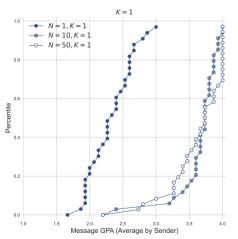


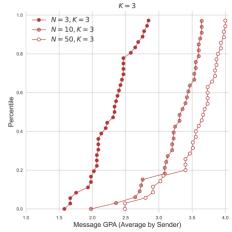


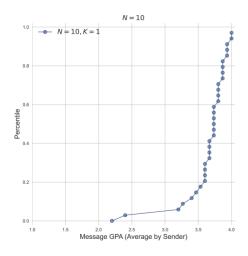


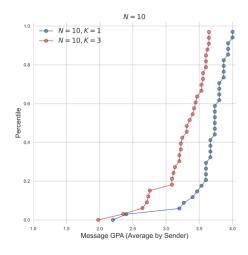


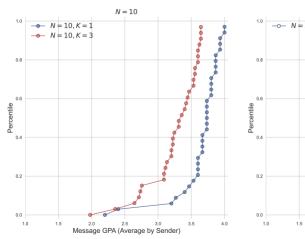


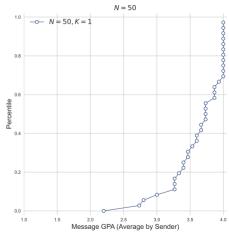


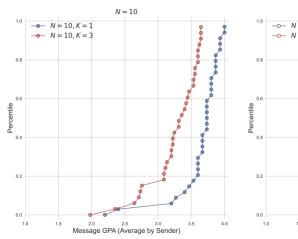


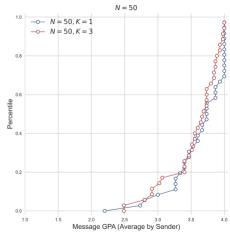












#### Robustness:

- Theoretical predictions
- Average treatment effects, statistical tests
- Raw data

▶ Appendix

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**Result 1.** Senders selectively disclose the available evidence in ways that are consistent with the key qualitative predictions of the theory

Theory is held to a high standard:

- FOSD rankings are a rather demanding test for the theory
- Contrasting signs reduce scope for alternative explanations

# result 2

(informativeness)

Previous result documents that senders engage in selective disclosure:

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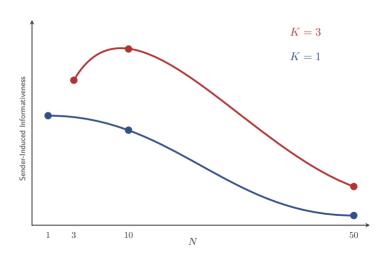
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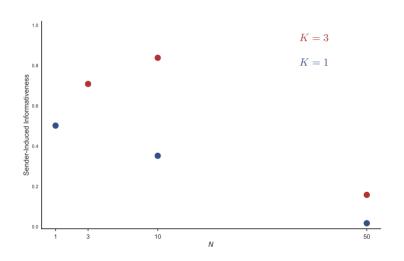
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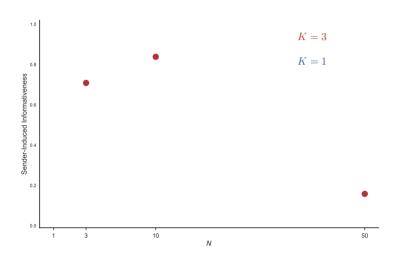
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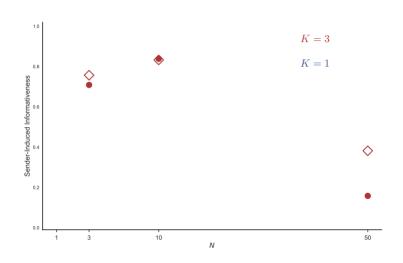
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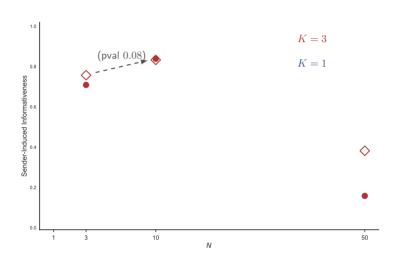
We measure **informativeness** as the correlation between the state  $\theta$  and the guess a induced by the sender' strategy

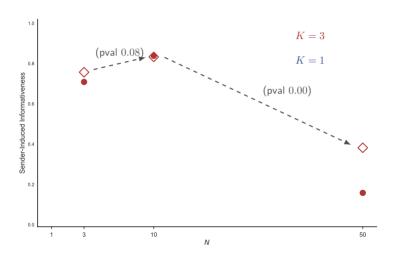


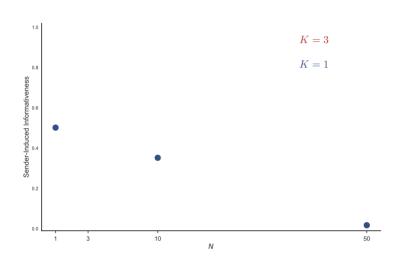


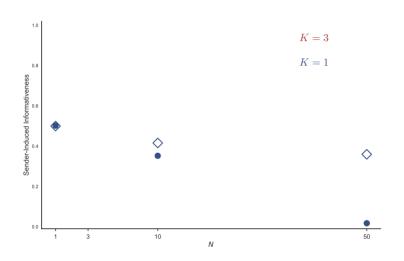


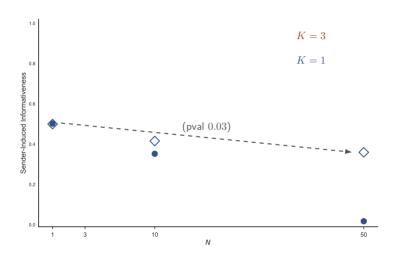


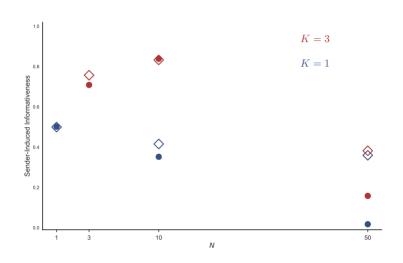


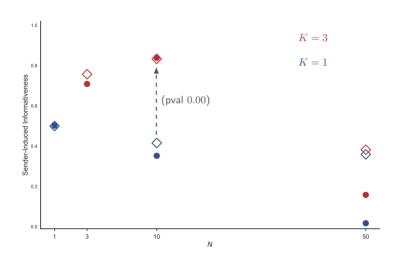


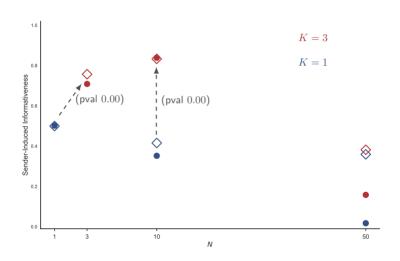


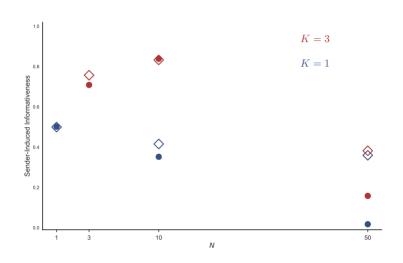












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**Result 2**. Informativeness changes in ways that is consistent with the key qualitative predictions of the theory

Overall, as a force in communication, selective disclosure seems behaviorally descriptive

Yet, our results also reveal substantial quantitative deviations

 Senders transmit (weakly) more information than it is predicted. That is, they overcommunicate

# result 3

(overcommunication)

#### This finding is at odds with existing experimental literature on disclosure

e.g., Forsythe, Isaac and Palfrey ('89, Rand), Jin and Leslie ('03, QJE), Jin, Luca, and Martin ('22, AEJ: Micro), Lizzeri, Frechette, Perego ('22, Ecma)

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- These papers consistently find that senders undercommunicate
- Failure of the "unraveling principle" → Senders fail to disclose evidence when it is sufficiently unfavorable
- They offer empirical support to policies that mandate disclosure in the marketplace

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**Question 3**. Then why do we observe **overcommunication**?

In our setting, full unraveling is not an equilibrium:

₩ Why?

- Informativeness is always predicted to be interior  $\mathcal{I} \in (0,1)$
- In contrast, literature largely focused on an extreme prediction:  $\mathcal{I}=1$

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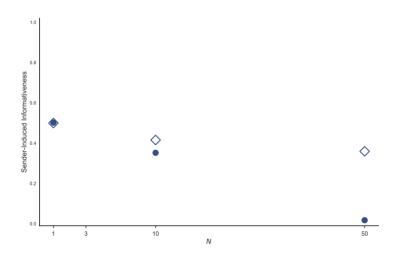


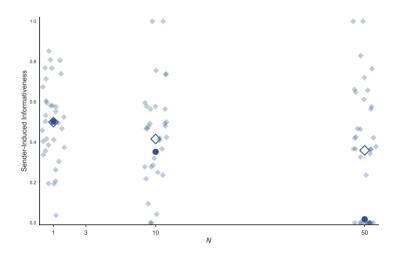
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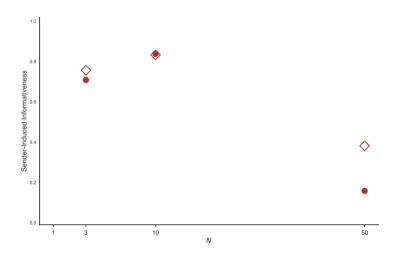
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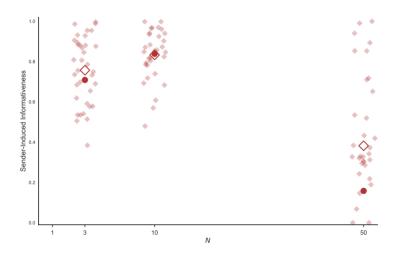
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Our findings suggest that **undercommunication** is not a robust behavioral feature in disclosure games









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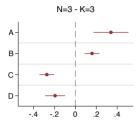
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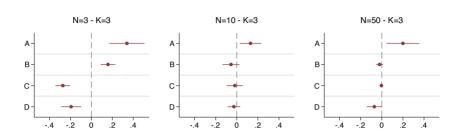
In contrast, we find that some senders adopt state-dependent strategies

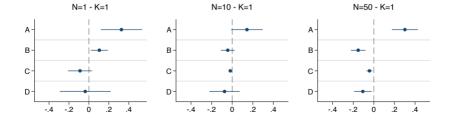
To illustrate, we estimate an OLS regression model:

$$\mathsf{Prob}(s \text{ is disclosed}) = \alpha + \beta_s \cdot \theta + \gamma \cdot X + \varepsilon$$

where X is a set of regressors that controls for senders' available evidence







We find consistent patterns across all treatments:

When the state is low (relative to when is high), senders under-disclose good
 evidence and over-disclose bad evidence

## **Summary of Result 3**

We find consistent patterns across all treatments:

When the state is low (relative to when is high), senders under-disclose good
 evidence and over-disclose had evidence

#### Result 3. Senders exhibit a form of deception aversion

State-dependent behavior generates overcommunication

#### Discussion:

- Senders can't lie in our setting, yet some avoid being deceptive Sobel, '23, JPE
- Never a best response to observed receivers behavior

# result 4

(receivers' beliefs)

#### Do Receivers Account for Selection?

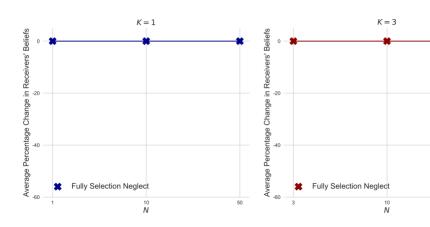
The previous results have established that (modulo overcommunication) the evidence receivers see is **endogenously selected** 

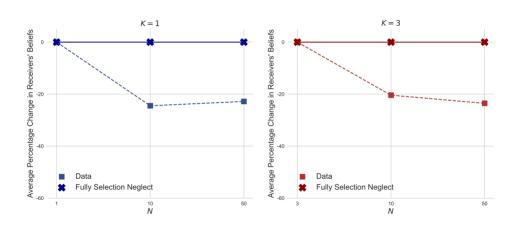
**Question 4**. To what extent do receivers account this selection in their responses?

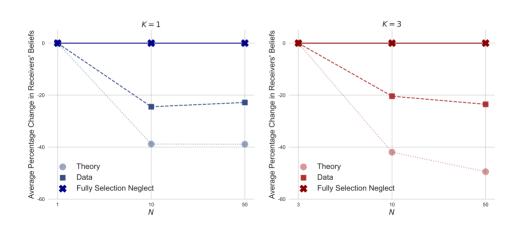
To test this, we exploit the following prediction of the theory:

 $-\,$  Given any message, as N increases, receivers' beliefs about the state being high should decrease

We report the percentage change in receiver' beliefs averaged out across all messages and receivers







**Result 4.** On average, receivers are increasingly skeptical of the evidence they see as it becomes more selected, as predicted by the theory

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Yet, quantitatively, they fail to fully account for selection

First evidence of *selection neglect* in **communication**, a setting where selection arises endogenously, as an equilibrium outcome

Recent literature has documented selection neglect in non-strategic settings, where selection is exogenous

Esponda, Vespa ('18, QE), Enke ('20, QJE), Barron, Huck, Jehiel (2023, AEJ:Micro)

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In progress: Identify receivers' "types"

# result 5

(receivers' accuracy)

#### Do Receivers Account for Selection?

**Question 5**. The previous result focused on beliefs, but how costly are these mistakes in terms of payoffs?

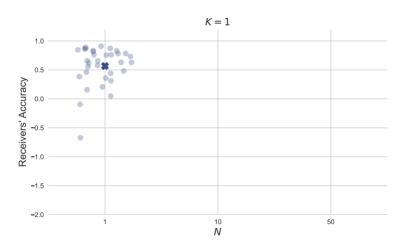
#### Do Receivers Account for Selection?

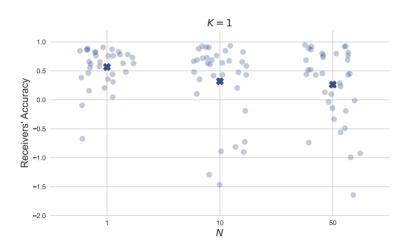
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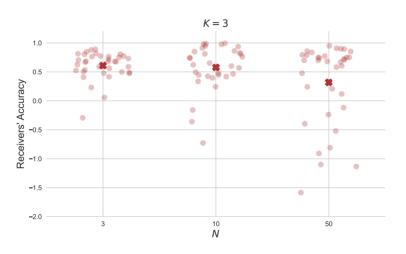
Define **accuracy** as the percentage of the payoff the receiver obtains relative to what a Bayesian would have obtained

We normalize accuracy by the accuracy a receiver would obtain if she behaved at random

The theory predicts accuracy is equal to 1 in all treatments







**Result 5**. Receivers become less accurate as N increases

This is puzzling especially if considering that the receiver' problem becomes "easier" as N increases

In progress: Is it selection neglect that drives this payoff losses?



conclusion

#### **Conclusion**

A comprehensive experimental study on selective disclosure

We exploit comparative statics to inform a novel experimental design

Our data corroborates the key qualitative predictions of the theory

 Validation of selective disclosure as a force in communication that is behaviorally descriptive

We detect two main *quantitative* deviations from the theory:

- A form of deception aversion in senders leads to overcommunicate
- We find evidence of selection neglect in a strategic setting

## The Selective Disclosure of Evidence **An Experiment**

Agata Farina Guillaume Fréchette Alessandro Lizzeri Jacopo Perego NYU NYU

Princeton

Columbia



thank you

# **Appendix**

Question: How informative are senders' strategies?

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- The state  $\theta$
- The receiver's guess a

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- The state  $\theta$
- $-\,$  The  $\,$  receiver's guess of a hypothetical Bayesian receiver,  $a^{B}$

Question: How informative are senders' strategies?

We measure informativeness  $\mathcal{I}(K,N)$  as correlation between:

- The state  $\theta$
- The receiver's guess a guess of a hypothetical Bayesian receiver,  $a^B$

We refer to  $Corr(\theta, a^B)$  as the **sender-induced correlation** 



Theoretically, our setting differs from typical disclosure model because evidence structure is not "rich"

Evidence structure is rich if, for all  $\theta$ , sender can send message that verifiably reveals  $\theta$ 

In our setting, evidence is noisy, and K and N are finite. No message can verifiably reveal  $\boldsymbol{\theta}$ 

Richness drives unraveling results

(Okuno-Fujiwara et al., 1990, Restud)

Strong assumption in many practical settings



## **OLS** Regression Model

We restrict attention to the observations in which s is available:

$$\mathsf{Prob}(s \text{ is disclosed}) = \alpha + \beta_s \cdot \theta + \sum_{s \in S} \gamma_s \cdot \min\{k, \mathsf{av}_s\} + \varepsilon$$

Senders' random effects; Standard errror clustered at the session level

Regressors  $\{\min\{k, \mathsf{av}_s\}\}_{s \in S}$  controls for senders available evidence

Results robust to controlling for set of available messages

▶ Back

### Some Notation: Strategies and Beliefs

Denote  ${\mathcal M}$  the space of all messages

#### Sender's Strategy

$$-\ \sigma:\Omega^N o\mathcal{M}$$
 s.t.  $\sigma(\bar{s})\in M(\bar{s}),$  for all  $\bar{s}$ 

where  $M(\bar{s})$  is the space of available messages given  $\bar{s}$ 

#### Receiver's Beliefs and Strategy

- $-\mu:\mathcal{M}\to\Delta(S^N)$
- $-a: \mathcal{M} \to \Delta(A)$

Given  $\mu$ , receiver's optimal strategy given by

$$a(m) = \mathbb{E}(\theta|m) = \sum_{\bar{s}} \mu(\bar{s}|m) \mathbb{E}(\theta|\bar{s}) \quad \forall m$$

pure and  $\theta$ -independent

## **Sequential Equilibrium**

A **Sequential Equilibrium** is a pair  $(\sigma^*, \mu^*)$  s.t.

1. For all  $\bar{s} \in \Omega^N$ ,  $\sigma^*(\bar{s}) \in M(\bar{s})$  and

$$\sum_{\bar{s}'} \mu^*(\bar{s}'|\sigma^*(\bar{s})) \mathbb{E}(\theta|\bar{s}') \ge \sum_{\bar{s}'} \mu^*(\bar{s}'|m') \mathbb{E}(\theta|\bar{s}') \qquad m' \in M(\bar{s})$$

2. For all m, supp  $\mu^*(\cdot|m)\subseteq C(m)=\{\bar s\in S^N: m\in M(\bar s)\}$ . In particular, if  $m\in\sigma^*(S^N)$ ,

$$\mu^*(\bar{s}|m) = q(\bar{s}|\sigma^{\star^{-1}}(m)) \quad \forall \, \bar{s}$$

where  $q(\bar{s}) = \sum_{\theta} p(\theta) f(\bar{s}|\theta)$ 

## **Multiplicity and Neologism Proofness**

Unlike classic disclosure games, the sequential equilibrium outcome is **not unique** when K < N.

- ▶ Off-path beliefs can support other equilibrium outcome.
- ▶ Refinements for signalling games (e.g., Cho-Kreps '87, Banks-Sobel '87) have no force here.
- ► Refinements for cheap talk games: Farrel (1993)'s **Neologism Proofness**.

## **Multiplicity and Neologism Proofness**

$$\Theta = \{0, 1\}$$
 and  $p(1) = \frac{1}{2}$ .  $N = 2$  and  $K = 1$ .

$$\Omega = \{A, B\}, f(A|\theta_H) = 1 \text{ and } f(A|\theta_L) = \frac{1}{2}.$$

$$\mathbb{E}[\theta|m=A] = \tfrac{4}{7} \text{ and } \mathbb{E}[\theta|m=B] = \mathbb{E}[\theta|m=\varnothing] = 0 \implies$$

No incentive to deviate

## **Multiplicity and Neologism Proofness**

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$$\Omega = \{A, B\}, f(A|\theta_H) = 1 \text{ and } f(A|\theta_L) = \frac{1}{2}.$$

$$\theta \qquad \overline{s} \qquad M(\overline{s}) \qquad \sigma^*(\overline{s})$$

$$1 \qquad (A, A) \qquad \{\emptyset, A\} \qquad \emptyset$$

$$0 \qquad (A, B) \qquad \{\emptyset, A, B\} \qquad \emptyset$$

$$(B, B) \qquad \{\emptyset, B\} \qquad \emptyset$$

$$\mathbb{E}[\theta|m=\varnothing] = \frac{1}{2} \text{ and } \mathbb{E}[\theta|m=A] = \mathbb{E}[\theta|m=B] = 0 \implies$$

No incentive to deviate

## **Neologism Proofness**

A neologism is a pair (m,C),  $C\subseteq \{\bar{s}\in S^N: m\in M(\bar{s})\}$ 

Literal meaning of  $(m,C) \leadsto$  "My type  $\bar{s}$  belongs to C and I can prove it by sending message m "

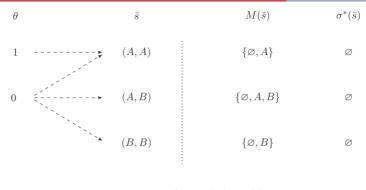
A neologism (m,C) is **credible** relative to equilibrium  $(\sigma^*,\mu^*)$  if

1. 
$$\sum_{\bar{s}'} q(\bar{s}'|C) \mathbb{E}(\theta|\bar{s}') > \sum_{\bar{s}'} \mu^*(\bar{\omega}'|\sigma^*(\bar{s})) \mathbb{E}(\theta|\bar{s}') \text{ for all } \bar{s} \in C$$

$$2. \ \sum_{\bar{s}'} q(\bar{s}'|C) \mathbb{E}(\theta|\bar{s}') \leq \sum_{\bar{s}'} \mu^*(\bar{s}'|\sigma^*(\bar{s})) \mathbb{E}(\theta|\bar{s}') \text{ for all } \bar{s} \notin C$$

The equilibrium is **Neologism Proof** if no neologism is credible.

## **Neologism Proofness**



$$m=A \text{ and } C=\{(A,A),(A,B)\} \implies$$
 
$$\mathbb{E}[\theta|m=A]=\frac{4}{7}>\mathbb{E}[\theta|m=\varnothing]=\frac{1}{2}$$

Since neologism (m,C) is credible, this PBE is not neologism proof equilibrium

## **Neologism Proofness**

#### **Proposition**

The equilibrium with maximal selective disclosure is Neologism Proof.

Neologism Proofness delivers outcome uniqueness

An equilibrium  $(\sigma,\mu)$  induces an outcome  $x:S^N\to A$ ,

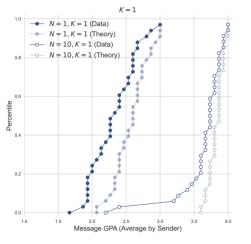
$$x(\bar{s}) = \sum_{\bar{s}'} \mu(\bar{s}' | \sigma(\bar{s})) \mathbb{E}(\theta | \bar{s}') \quad \forall \bar{s}.$$

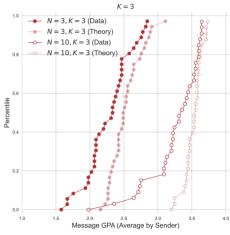
Since  $\Theta$  is binary and  $u(a,\theta)=-(a-\theta)^2$ , the receiver's task is equivalent to eliciting her beliefs via a quadratic scoring rule (QSR)

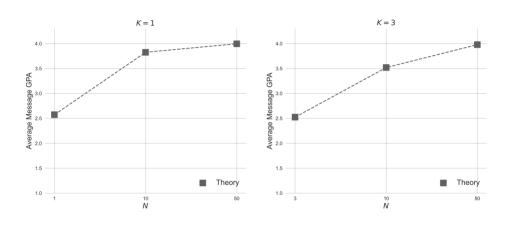
A large literature on belief elicitation has shown that QSR can be biased when subjects are not risk-neutral

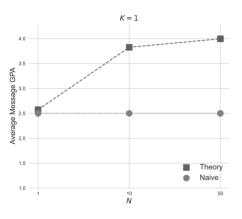
To avoid this issue, we implement a binarized scoring rule  $a\ la$  Hossain and Okui (2013), which is robust to various risk preferences

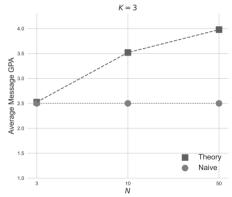


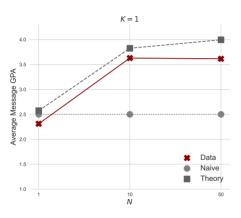


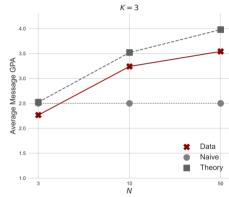


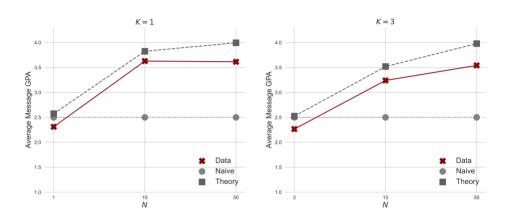






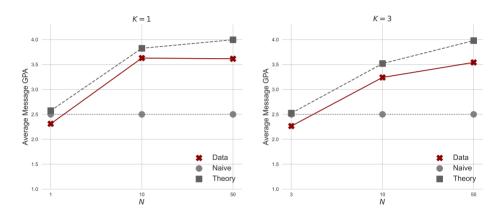




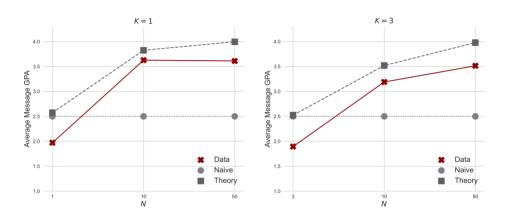


Qualitative predictions are corroborated by the data (pvals  $\sim 0.01)$ 

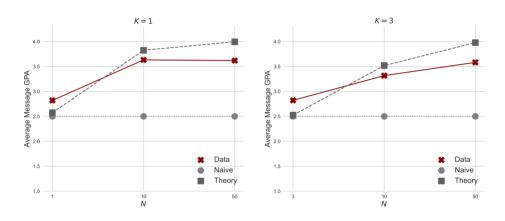
### Which Evidence is Disclosed?



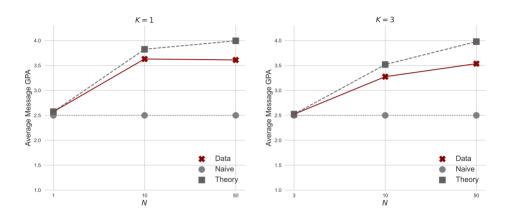




## Alternative GPA: Empty = 2.5

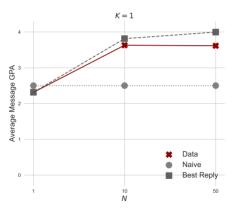


## Alternative GPA: Empty = Avg Undisclosed



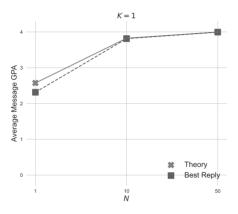
### Which Evidence is Disclosed?

For K=1 we can compare the observed message GPA with the one that would arise from an optimal empirical behavior of the sender:  $\varnothing$  better than C and D



Quantitatively, GPA smaller than theory predicts for N>K (pvals < 0.05)

## **Best Reply vs Theoretical Predictions**



Quantitatively, best reply and theory are different for N < 50 (pvals < 0.01)

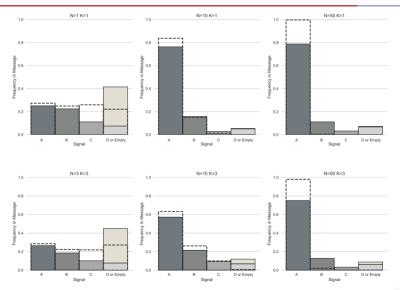
The behavior is the same 74% of the times for N=1, 99% of the times for N=10 and 100% of the times for N=50

### **Empirical Best Response vs Equilibrium**

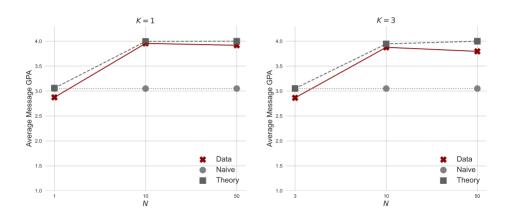
Given observed receivers' behavior, senders' best response coincides with equilibirum strategy

- ▶ 74% of the times in treatment (N = 1, K = 1)
- ▶ 99% of the times in treatment (N = 10, K = 1)
- ▶ 100% of the times in treatment (N = 50, K = 1)



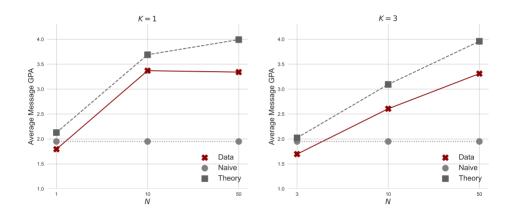


# Which Evidence is Disclosed? High Type



Quantitatively, senders select less than theory predicts (pvals < 0.1)

# Which Evidence is Disclosed? Low Type



Quantitatively, senders select less than theory predicts (pvals < 0.05)

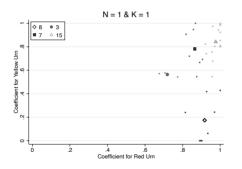


#### Challenge

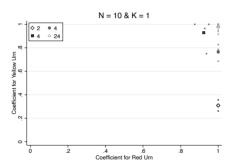
- ► Large number of urn / balls / message combinations
- ► Specific behavior of interest varies across treatments
  - Number of balls sent (K = 1 vs K = 3)
  - ▶ Balls sent vs balls available (N = K vs N > K)
- ightarrow Precludes a unified approach using those variables

#### **Solution**

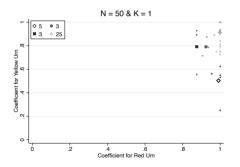
- ▶ Transform balls and messages to numbers  $(B^{\#} \text{ and } M^{\#})$
- ▶ Regress  $M^\#$  on  $B^\#|{\rm yellow}$  urn and  $B^\#|{\rm red}$  urn
- Cluster the coefficient estimates
- Describe behavior along key dimensions of interest



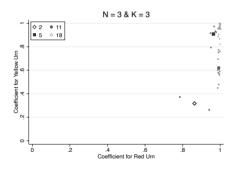
Cluster	Obs	Urn			
	(33)		K	Α	D
Triangle	15				
		Red	0.91	1	0.38
		Yellow	0.64	1	0.27
Square	7				
		Red	0.73	1	0.25
		Yellow	0.51	1	0.21
Circle	3				
		Red	0.5	0.92	n/a
		Yellow	0.54	0.67	0.49
Diamond	8				
		Red	0.71	1	0.20
		Yellow	0.30	0	0.46



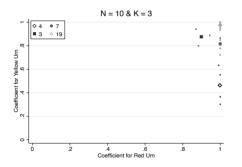
Cluster	Obs	Urn			
	(34)		K	Α	D
Triangle	24				
		Red	1	1	0
		Yellow	1	0.97	0.02
Square	4				
		Red	1	0.81	0.08
		Yellow	1	0.88	0.07
Circle	4				
		Red	1	1	0
		Yellow	1	0.46	0.14
Diamond	2				
		Red	1	1	0
		Yellow	1	0	0.89



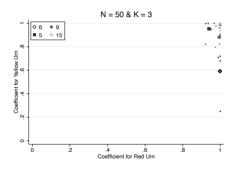
Cluster	Obs	Urn			
	(36)		K	Α	D
Triangle	25				
		Red	1	0.99	0
		Yellow	1	0.74	0.03
Square	3				
		Red	0.96	0.82	0.1
		Yellow	1	0.51	0.15
Circle	3				
		Red	1	0.78	0
		Yellow	1	0.63	0.18
Diamond	5				
		Red	1	0.96	0
		Yellow	0.95	0.26	0.46



Cluster	Obs	Urn			
	(36)		K	Α	D
Triangle	18				
		Red	0.58	1	0.15
		Yellow	0.18	1	0.12
Square	5				
		Red	0.29	1	0
		Yellow	0.10	0.88	0.05
Circle	11				
		Red	0.26	1	0.06
		Yellow	0.15	0.23	0.60
Diamond	2				
		Red	0	1	0
		Yellow	0.06	0.25	0.50



Cluster	Obs	Urn			
	(33)		K	Α	D
Triangle	19				
		Red	0.99	0.99	0
		Yellow	0.88	0.96	0.01
Square	3				
		Red	1	0.46	0.17
		Yellow	1	0.43	0.04
Circle	7				
		Red	1	0.94	0
		Yellow	0.74	0.66	0.10
Diamond	4				
		Red	0.92	0.83	0
		Yellow	0.76	0.28	0.43



Cluster	Obs	Urn			
	(35)		K	Α	D
Triangle	15				
		Red	1	0.88	0
		Yellow	0.94	0.80	0
Square	5				
		Red	0.89	0.17	0
		Yellow	0.87	0.32	0
Circle	9				
		Red	0.97	0.70	0
		Yellow	0.94	0.31	0.04
Diamond	6				
		Red	1	0.86	0.03
		Yellow	0.95	0.31	0.43

#### **Equilibrium type** (56%)

- ► Most common
- ightharpoonup N>K: Mostly report best balls independently of the state
- ightharpoonup N = K: Disclose fewer than K balls

### **Deception Averse Type** (17%)

- ► A's reported more often when the state is high
- D's reported more often when the state is low
- ightharpoonup N = K: Disclose fewer than K balls

### **Others** (27%)

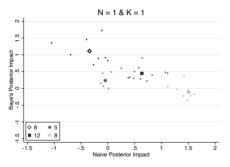
- ► Similar to equilibrium types when the state is high
- ▶ Report A's less but do not report D's when the state is low
- ► Some low rates of A's when the state is high [confusion]

#### Challenge

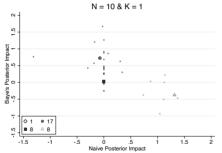
- ► Large number of messages
- ► Different messages across treatments
- ► Some messages have very few observations
- ightarrow Precludes a unified approach using that variable

#### Solution

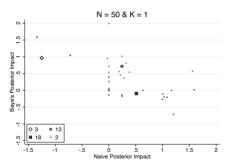
- Compute equilibrium update following each message
- ► Compute the update of someone who ignores selection: naive update
- ightharpoonup Regress guesses on a constant  $(\alpha)$  and the equilibrium and naive updates
- Cluster the coefficient estimates
- ▶ Describe behavior along key dimensions of interest



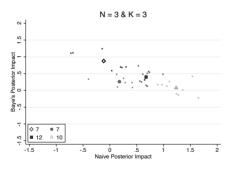
Cluster	Obs (33)	Α	В	Ø	С
Diamond	8				
$\alpha = 0.23$		0.87	0.67	0.23	0.47
Circle	5				
$\alpha = 0.39$		0.56	0.49	0.41	0.37
Square	12				
$\alpha = 0.02$		0.86	0.73	0.41	0.38
Triangle	8				
$\alpha = -0.23$		0.90	0.67	0.51	0.23



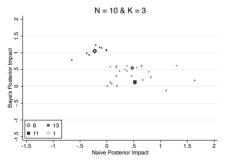
Cluster	Obs (34)	Α	В	Ø	D
Diamond	1				
$\alpha = 4.20$		0.60*	0.23*	0.60*	n/a
Circle	17				
$\alpha = 0.28$		0.66	0.26	n/a	0.11
Square	8				
$\alpha = 0.56$		0.58	0.60	n/a	0.60
Triangle	8				
$\alpha = -0.23$		0.62	0.52	n/a	0.11



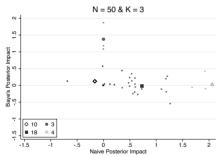
Cluster	Obs (36)	А	В	Ø	D
Diamond	3				
$\alpha = 0.89$		0.35	0.17	0.21*	0.75
Circle	13				
$\alpha = 0.15$		0.71	0.29	0.46*	0.11
Square	18				
$\alpha = 0.26$		0.63	0.53	n/a	0.19
Triangle	2				
$\alpha = -1.15$		0.69	0.41	n/a	n/a



Cluster	Obs (36)	AAA	AAB	AA	АВ
Diamond	7				
$\alpha = 0.20$		0.92*	0.86	0.86	0.62
Circle	7				
$\alpha = 0.30$		0.72	0.66	0.63	0.68
Square	12				
$\alpha = -0.04$		0.88	0.92	0.91	0.86
Triangle	10				
$\alpha = -0.24$		1	0.97	0.96	0.90



Cluster	Obs (33)	AAA	AAB	AA	ABB
Diamond	8				
$\alpha = 0.19$		0.95	0.11	0.02	0.03
Circle	13				
$\alpha = -0.07$		0.89	0.70	0.24	0.26
Square	11				
$\alpha = 0.10$		0.74	0.70	n/a	0.61
Triangle	1				
$\alpha = -3.98$		1*	0.54*	n/a	0.02*



Cluster	Obs				
	(35)	AAA	AAB	AA	DDD
Diamond	10				
$\alpha = 0.64$		0.54	0.49	0.33	0.32
Circle	3				
$\alpha = 0.11$		0.84	0.01*	n/a	0.07
Square	18				
$\alpha = -0.04$		0.67	0.69	0.57	0.12
Triangle	4				
$\alpha = -1.16$		0.89	0.80	0.91*	n/a

- ► Variation in updating strategies
  - Extent they account for selection
- ▶ Being closer to equilibrium → higher payoffs
- ► However, in many treatments, groups better at accounting for selection are among the highest
- ightharpoonup With N=50, few differences in payoffs

### **Summary**

#### **Senders**

- ► The majority:
  - ► Select the better balls to send.
  - ► Behave similarly for both urns.
- ▶ Some convey more information by conditioning on the type.
- $\rightarrow$  More information transmitted than predicted.

#### Receivers

- Many do not fully account for selection.
- Some are not very responsive.
- $\rightarrow$  Less information received than predicted.