Verifiability in Communication
A (Prospective) Experimental Analysis

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September 2022

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very preliminary

Overview

An experimental study on the role of **verifiability in communication**

- Communication is classic problem in economics
- Verifiability (or lack thereof) is a core ingredient in our theories of communication

This Paper:

- Flexible framework to introduce rich variations in verifiability
- Develop novel comparative statics that inform experimental design
- Test main qualitative prediction of theory against observed subjects' behavior

model

We build on the communication model by Milgrom (1981):

1. Sender privately observes the state θ:
   - $\theta \in \Theta$ with common prior $p \in \Delta(\Theta)$

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1. Sender privately observes the state θ:
   - $\theta \in \Theta$ with common prior $p \in \Delta(\Theta)$
2. Given θ, Sender draws $N \in \mathbb{N}$ signals from:
   - An information structure $f : \Theta \to \Delta(\Omega)$
   - N conditionally independent draws from $f(\cdot|\theta)$

   Notation: $\bar{\omega} = (\omega_1, \ldots, \omega_N) \in \Omega^N$ sender's "*type*"

We build on the communication model by Milgrom (1981):

3. Sender verifiably discloses at most K of her N signals
   - Given $\bar{\omega}$, sender chooses $m \in M(\bar{\omega})$:

$$M(\bar{\omega}) := \Big\{ m \in \Omega^k \mid k \leq K \text{ and } \exists \text{ injective}$$
$$\rho : \{1, \ldots, k\} \to \{1, \ldots, N\} \text{ s.t. } m = (\omega_{\rho(1)}, \ldots, \omega_{\rho(k)}) \Big\} \cup \{\varnothing\}.$$

A Sender-Receiver Game

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4. Receiver observes m, takes an action, payoff realizes
   - Receiver's action $a \in A$ and payoffs are:

$$u_S(\theta, a) = a \qquad u_R(\theta, a) = -(a - \theta)^2$$

SUMMARY

Summary: Three parameters of interests

- N, the number of verifiable signals
- K, the number of reportable signals
- f, the information structure

Assumptions:

- Θ and Ω finite subsets of $\mathbb{R}$; $A = \mathbb{R}$
- f satisfies **MLR property**: For $\theta' > \theta$, $\dfrac{f(\omega|\theta')}{f(\omega|\theta)}$ strictly increasing in ω

Applications: News Media; VC financing; etc.

Interpretations $K < N$: institutional norm, attention cost

Theory Literature

Closest papers that feature partially verifiable information, $\bar{\omega} \notin M(\bar{\omega})$

The Basic Setting:

- Milgrom (1981, Bell), example to showcase MLRP
- Fishman and Hagerty (1990, QJE), optimal amount of discretion

Mechanism-Design Approach:

- Glazer and Rubinstein (2004, Ecma) – Receiver's Verification, $K = 1$
- Glazer and Rubinstein (2006, TE) – Sender's verification

Richer Settings: Uknown N or Endogenous K

- Shin (2003, Ecma)
- Dziuda (2011, JET)

example

A Simple Example

- Binary state $\Theta = \{\theta_L, \theta_H\}$ and binary signals $\Omega = \{0, 1\}$
- "Conclusive bad news": $f(\omega = 1|\theta_H) = 1$ and $f(\omega = 1|\theta_L) \in (0, 1)$
- $N = 3$, $K = 1$

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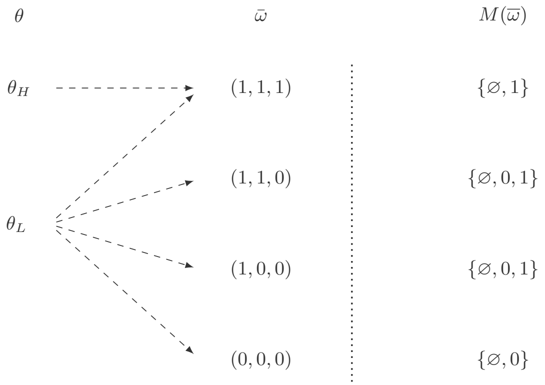

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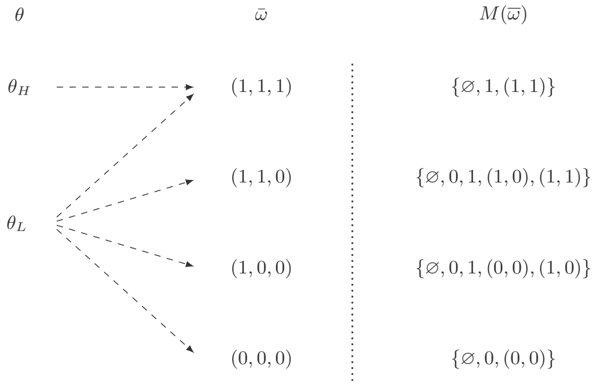

Discussion

A model of **partially verifiable** information:

- When $K = N$, $\bar{\omega} \in M(\bar{\omega})$, ubiquitous assumption $\rightsquigarrow$ unravelling
- When $K < N$, $\bar{\omega} \notin M(\bar{\omega})$, Sender can only prove so much about herself: scope for imitation via **selective disclosure**

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Hybrid framework btw cheap-talk games and disclosure games

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Hybrid framework btw **cheap-talk** games and **disclosure** games

Changing K and N affects degree to which information is verifiable

- $N \uparrow$, Sender draws more verifiable signals about her type
- $K \uparrow$, Sender can report more signals to receiver

Verifiability (or lack thereof) is a fundamental ingredient of communication

Changing K and N is tool to introduce variation in degree of verifiability

Generates rich and asymmetric comparative statics, which inform our experimental design

Test qualitative predictions against observed behavior

Question overlooked by experimental literature:

- **Disclosure.** Failure of unraveling and explanations; e.g., Jin, Luca, Martin (2021)
- **Cheap Talk.** Lying aversion and overcommunication; e.g., Chen-Wang (2006)
- Methodologically closest to Frechette, Lizzeri, Perego (2021)

equilibrium

Solution Concept: Sequential Equilibrium

details

Proposition (Existence) Milgrom (1981)

For all N and $0 \leq K \leq N$, a sequential equilibrium exists where sender reports the K **most favorable** signals in $\bar{\omega}$.

Two predictions about sender's behavior:

1. Sender discloses K signals
2. When $K < N$, sender discloses most favorable signals

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Proposition (Existence) Milgrom (1981)

For all N and $0 \leq K \leq N$, a sequential equilibrium exists where sender reports the K **most favorable** signals in $\bar{\omega}$.

Two predictions about sender's behavior:

1. Sender discloses K signals "quantity"
2. When $K < N$, sender discloses most favorable signals "quality"

Back to Example

- Binary state $\Theta = \{\theta_L, \theta_H\}$ and binary signals $\Omega = \{0, 1\}$
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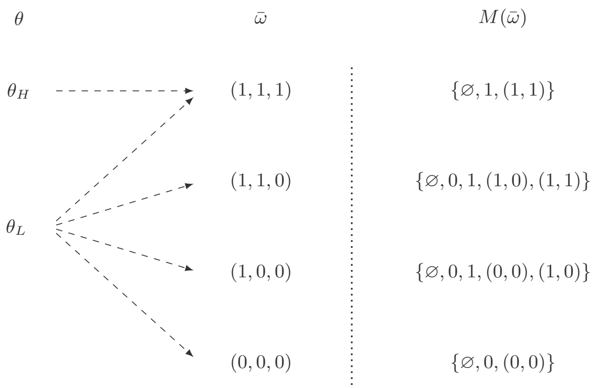

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θ	$\bar{\omega}$	$M(\bar{\omega})$	$\sigma^*(\bar{\omega})$
θ_H	$(1, 1, 1)$	$\{\varnothing, 1, (1, 1)\}$	$(1, 1)$
	$(1, 1, 0)$	$\{\varnothing, 0, 1, (1, 0), (1, 1)\}$	$(1, 1)$
θ_L	$(1, 0, 0)$	$\{\varnothing, 0, 1, (0, 0), (1, 0)\}$	$(1, 0)$
	$(0, 0, 0)$	$\{\varnothing, 0, (0, 0)\}$	$(0, 0)$

Equilibrium Multiplicity

Unlike classic disclosure games, SE outcome not unique when $K < N$

Off-path beliefs can support other equilibrium outcome

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Off-path beliefs can support other equilibrium outcome

$$N = 3, K = 1$$

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θ_H	$(1,1,1)$	$\{\varnothing, 1\}$	$\varnothing$
	$(1,1,0)$	$\{\varnothing, 0, 1\}$	$\varnothing$
θ_L	$(1,0,0)$	$\{\varnothing, 0, 1\}$	$\varnothing$
	$(0,0,0)$	$\{\varnothing, 0\}$	$\varnothing$

Multiplicity is a serious (albeit common) challenge for experiments

- Data will provide guidance as to which equilibrium is played

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- Refinements for signalling games (e.g., Cho-Kreps ’87, Banks-Sobel ’87) have no force here

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The selective-disclosure outcome is the only one that survives certain refinements:

- Refinements for signalling games (e.g., Cho-Kreps '87, Banks-Sobel '87) have no force here
- Refinements for cheap talks: Farrel (1993)'s **Neologism Proofness**, Matthews, Okuno-Fujiwara, Postelwite (1991), and some weaker versions

Go to Credible Neologism

Remark (Existence, again)

Our equilibrium is Neologism Proof.

Moreover, Neologism Proofness delivers outcome uniqueness

An equilibrium (σ, μ) induces an outcome $x : \Omega^N \to A$,

$$x(\bar{\omega}) = \sum_{\bar{\omega}'} \mu(\bar{\omega}'|\sigma(\bar{\omega}))\mathbb{E}(\theta|\bar{\omega}') \qquad \forall \bar{\omega}.$$

Proposition (Uniqueness)

Let (σ^*, μ^*) be our equilibrium and (σ, μ) be any other NP equilibrium. Let x^* and x their respective outcomes. Then, $x^* = x$.

verifiability and communication

INCREASING K

How does an increase in K affect information transmission?

- $M(\bar{\omega})$ becomes larger $\Rightarrow$ Easier to send messages that others cannot imitate

Proposition

Fix N and f and $K < K' \leq N$. The equilibrium under K' is Blackwell more informative than under K.

Proof shows that equilibrium partition $\{\sigma^{*-1}(m)\}_{m \in \sigma^*(\Omega^N)} \subseteq \Omega^N$ becomes finer as K increases

Back to Example: Increasing K

- Binary state $\Theta = \{\theta_L, \theta_H\}$ and binary signals $\Omega = \{0, 1\}$
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θ	$\bar{\omega}$	$M(\bar{\omega})$	$\sigma^*(\bar{\omega})$
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	$(1,1,0)$	$\{\varnothing, 0, 1\}$	1
θ_L	$(1,0,0)$	$\{\varnothing, 0, 1\}$	1
	$(0,0,0)$	$\{\varnothing, 0\}$	0

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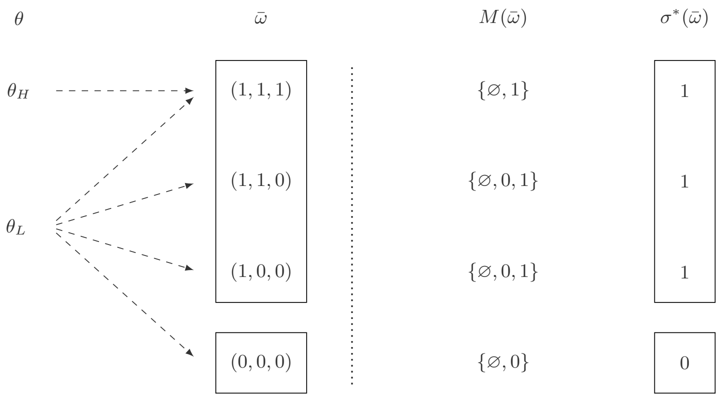

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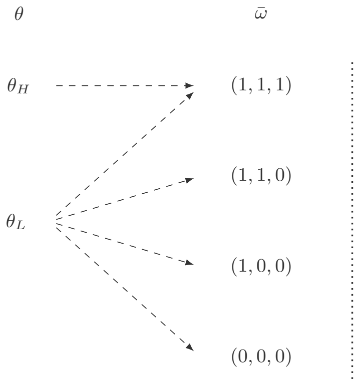

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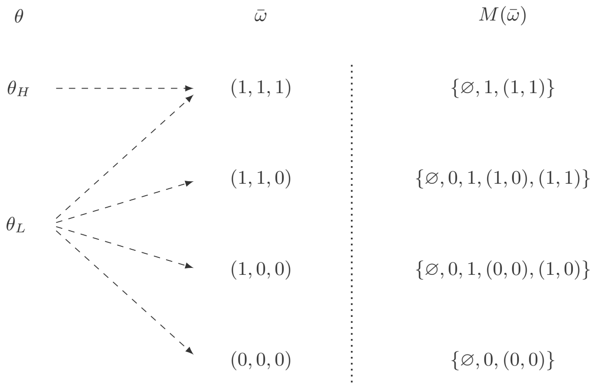

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How does an increase in N affect information transmission?

A nontrivial tradeoff:

+ Higher N $\rightsquigarrow$ sender is endowed with more verifiable signals
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Increasing N

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− Higher $N \rightsquigarrow$ sender can be more selective re signals to disclose

Today:

- Results for two special (but interesting) cases
- A conjecture for the general case

Case 1: Increasing N is Bad

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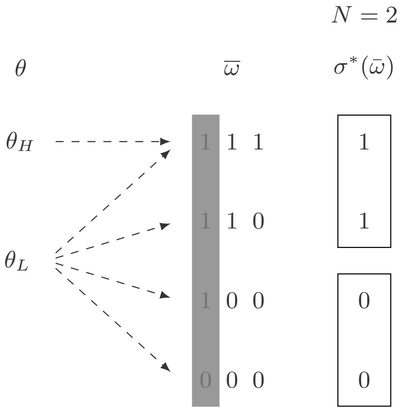

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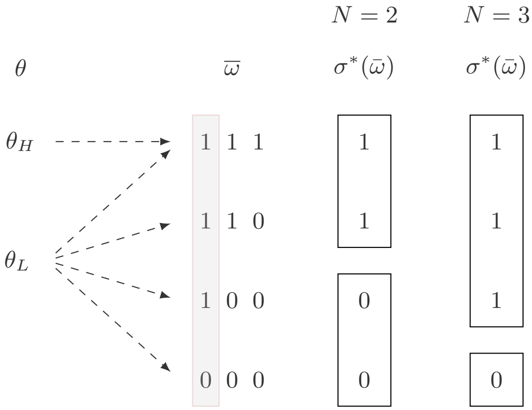

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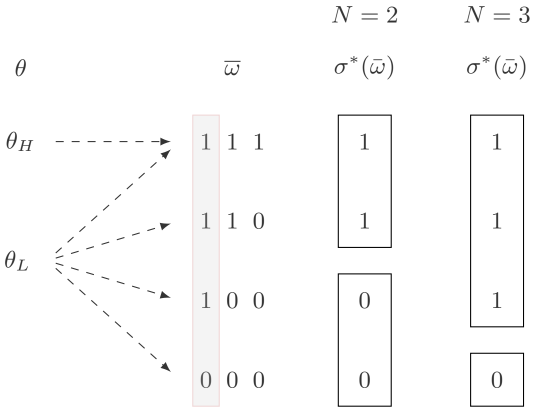

$N \uparrow \rightsquigarrow$ more likely that θ_L is able to imitate $\theta_H \rightsquigarrow$ informativeness $\downarrow$

Case 2: Increasing N is Good

- Binary state $\Theta = \{\theta_L, \theta_H\}$ and binary signals $\Omega = \{0, 1\}$, $K = 1$
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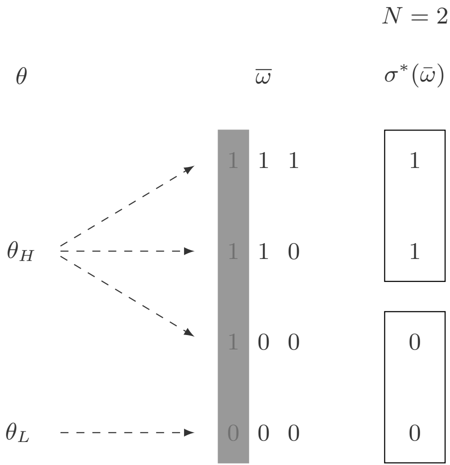

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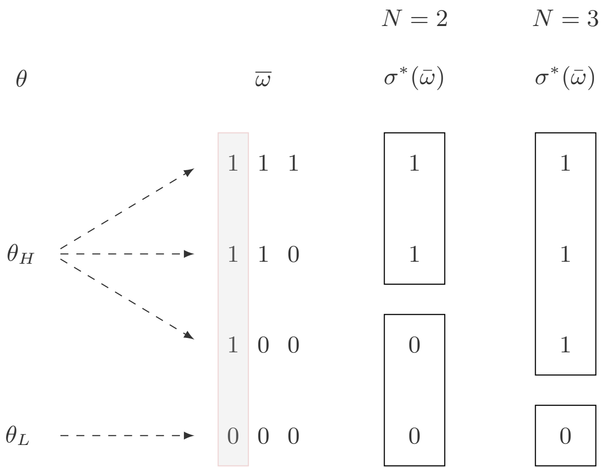

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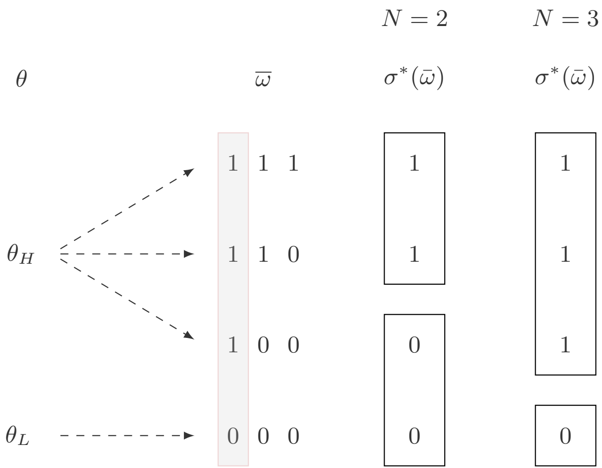

$N \uparrow \rightsquigarrow$ less likely that θ_L is able to imitate θ_H $\rightsquigarrow$ informativeness $\uparrow$

When f is conclusive, we obtain the following comparative statics:

Proposition

Let Θ and Ω be binary. Fix any K. As N increases, the receiver's payoff

- Increases if f has conclusive good news
- Decreases if f has conclusive bad news

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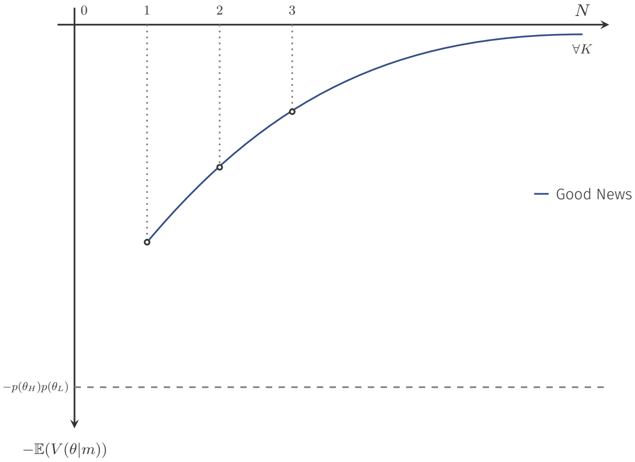

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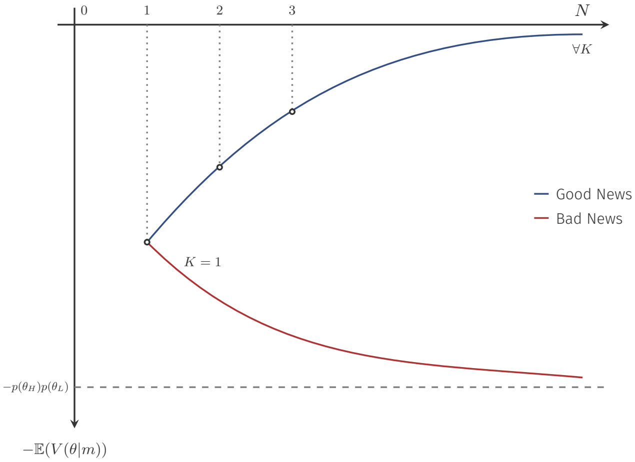

Comparative Statics

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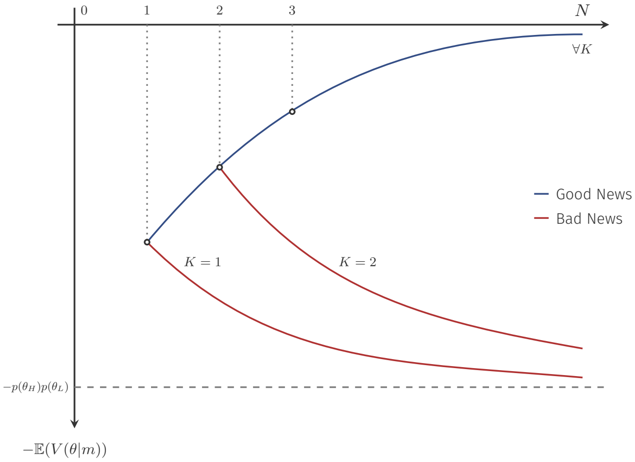

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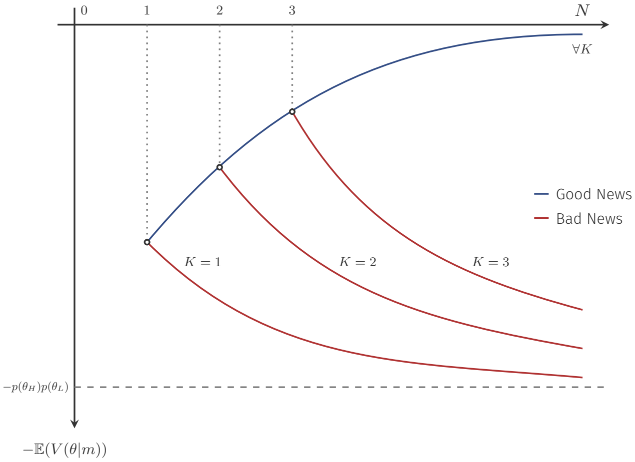

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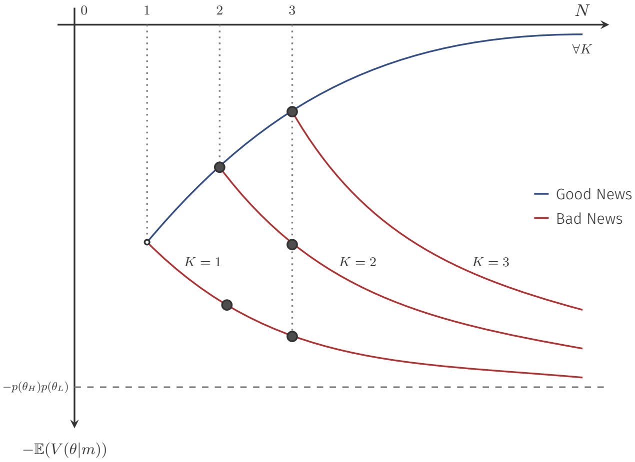

Experimental Design: Discussion of pros and cons

What About the General Case?

Towards a more general result for the effects of changing N:

Binary state θ $\quad + \quad$ Binary signals ω $\quad + \quad$ Conclusive good/bad news f

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~~Binary state θ~~	$+$	~~Binary signals ω~~	$+$	~~Conclusive good/bad news f~~
Generic state θ		Generic signals ω		Generic f

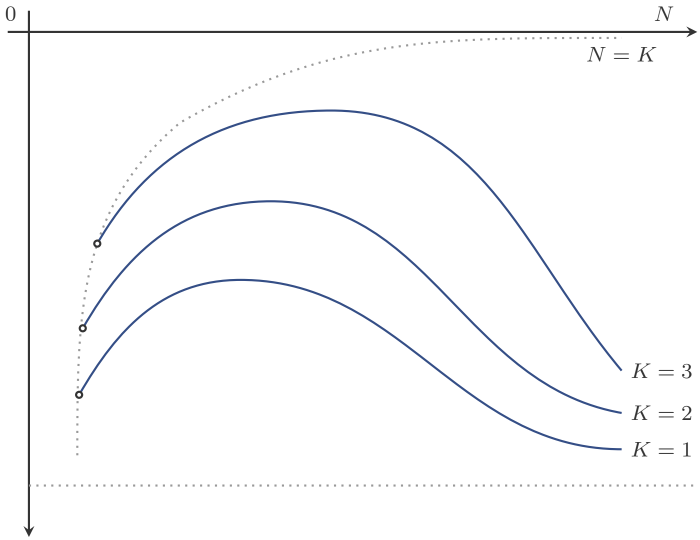
What About the General Case?

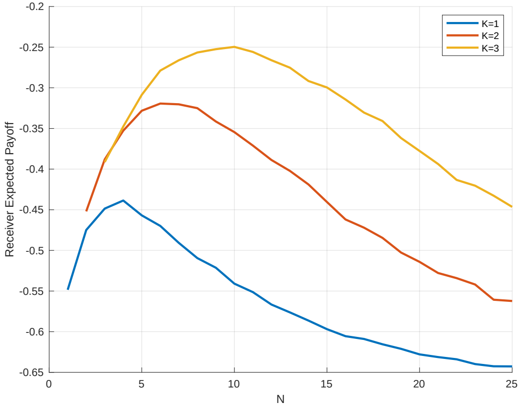
Towards a more general result for the effects of changing N:

~~Binary state θ~~	$+$	~~Binary signals ω~~	$+$	~~Conclusive good/bad news f~~
Generic state θ		Generic signals ω		Generic f

Conjecture

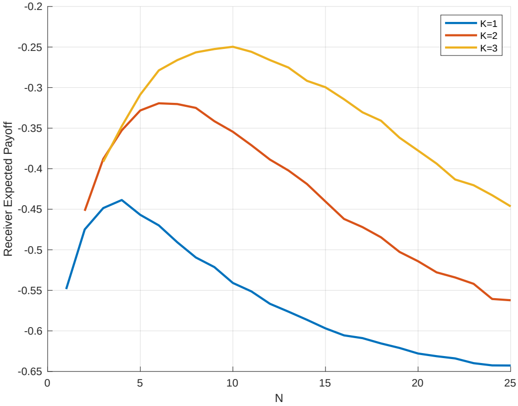
Fix K and f. There exists $N^\star \in \mathbb{N} \cup \{\infty\}$ such that informativeness increases up until $N^\star$ and decreases afterwards


0
N
$N = K$
$K = 3$
$K = 2$
$K = 1$



K=1
K=2
K=3
Receiver Expected Payoff
N
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
-0.55
-0.6
-0.65
0
5
10
15
20
25

Experimental Design #2



Experimental Design: Discussion of pros and cons

Other Directions

The framework is conducive to analyze more questions:

- Partial verifiability enables study of preference alignement:

  Verifiability helps info transmission when preferences misaligned

  Verifiability may hurt info transmission when preferences aligned

- Costly N

- Ex-ante disclosure

summing up

Summary

Verifiability is a key ingredient in communication

Flexible framework to introduce rich variations in verifiability

Stark comparative statics inform our experimental design

Test main qualitative predictions of the theory against observed subjects behavior (contrast with literature)

- If confirmed, this is empirical validation for a core component of our theories of communication
- If not, it indicates something off in our theories

thank you

appendix

Denote $\mathcal{M} = \bigcup_{\bar{\omega} \in \Omega^N} M(\bar{\omega})$ the space of all messages

Sender's Strategy

pure and θ-independent

- $\sigma : \Omega^N \to \mathcal{M}$ s.t. $\sigma(\bar{\omega}) \in M(\bar{\omega})$, for all $\bar{\omega}$

Receiver's Beliefs and Strategy

- $\mu : \mathcal{M} \to \Delta(\Omega^N)$
- $a : \mathcal{M} \to \Delta(A)$

Given μ, receiver's optimal strategy given by

$$a(m) := \arg\max_a \mathbb{E}(-(a-\theta)^2|m) = \mathbb{E}(\theta|m)$$

Denote $\mathcal{M} = \bigcup_{\bar{\omega} \in \Omega^N} M(\bar{\omega})$ the space of all messages

Sender's Strategy

pure and θ-independent

- $\sigma : \Omega^N \to \mathcal{M}$ s.t. $\sigma(\bar{\omega}) \in M(\bar{\omega})$, for all $\bar{\omega}$

Receiver's Beliefs and Strategy

- $\mu : \mathcal{M} \to \Delta(\Omega^N)$
- $a : \mathcal{M} \to \Delta(A)$

Given μ, receiver's optimal strategy given by

$$a(m) := \arg\max_a \mathbb{E}(-(a-\theta)^2 | m) = \mathbb{E}(\theta|m) = \sum_{\bar{\omega}} \mu(\bar{\omega}|m)\mathbb{E}(\theta|\bar{\omega}) \qquad \forall m$$

Definition:

A Sequential Equilibrium is a pair (σ^*, μ^*) s.t.

1. For all $\bar{\omega} \in \Omega^N$, $\sigma^*(\bar{\omega}) \in M(\bar{\omega})$ and

$$\sum_{\bar{\omega}'} \mu^*(\bar{\omega}'|\sigma^*(\bar{\omega}))\mathbb{E}(\theta|\bar{\omega}') \geq \sum_{\bar{\omega}'} \mu^*(\bar{\omega}'|m')\mathbb{E}(\theta|\bar{\omega}') \qquad m' \in M(\bar{\omega})$$

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Total Prob: $q(\bar{\omega}) = \sum_\theta p(\theta) f(\bar{\omega}|\theta)$; Conditional Prob: $q(\bar{\omega}|K)$

We refine off-path beliefs via **Neologism Proofness** (Farrel, 1993)

A **neologism** is a a pair (m, C) such that $C \subseteq \tilde{C}(m)$.

Literal meaning of (m, C) $\rightsquigarrow$ "*My type $\bar{\omega}$ belongs to C*"

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Definition

A neologism (m, C) is **credible** relative to equilibrium (σ^*, μ^*) if

$$(i) \quad \sum_{\bar{\omega}'} q(\bar{\omega}'|C)\mathbb{E}(\theta|\bar{\omega}') > \sum_{\bar{\omega}'} \mu^*(\bar{\omega}'|\sigma^*(\bar{\omega}))\mathbb{E}(\theta|\bar{\omega}') \text{ for all } \bar{\omega} \in C,$$

$$(ii) \quad \sum_{\bar{\omega}'} q(\bar{\omega}'|C)\mathbb{E}(\theta|\bar{\omega}') \leq \sum_{\bar{\omega}'} \mu^*(\bar{\omega}'|\sigma^*(\bar{\omega}))\mathbb{E}(\theta|\bar{\omega}') \text{ for all } \bar{\omega} \notin C,$$

The equilibrium is **neologism proof** if no neologism is credible

Remark (Existence, again)

Our equilibrium is Neologism Proof.

Moreover, Neologism Proofness delivers outcome uniqueness

An equilibrium (σ, μ) induces an outcome $x : \Omega^N \to A$,

$$x(\bar{\omega}) = \sum_{\bar{\omega}'} \mu(\bar{\omega}' | \sigma(\bar{\omega})) \mathbb{E}(\theta | \bar{\omega}') \qquad \forall \bar{\omega}.$$

Proposition (Uniqueness)

Let (σ^*, μ^*) be our equilibrium and (σ, μ) be any other NP equilibrium. Let x^* and x their respective outcomes. Then, $x^* = x$.

In lab implementation, we may not need all these neologisms in:

$$\{(m, C) : m \in \mathcal{M}, C \subseteq \tilde{C}(m)\}$$

When Ω is binary, it is sufficient to consider these neologisms:

If m is off-path its literal meaning is "*my highest k signals are m*"

Proposition

If Ω is binary, weaker refinement guarantees outcome uniqueness.