Math for Econ I

Problem Set 5*

Exercise 1

Let $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$.

(a) Show that one and only one of the following two system has a solution:

$$(I) \quad Ax \geq b, \quad \text{s.t } x \geq 0 \qquad \text{and} \qquad (II) \quad \lambda^T A \leq 0, \quad \text{s.t } \lambda \geq 0 \text{ and } \lambda^T b = 1.$$

(b) Show that one and only one of the following two system has a solution:

$$(I) \quad Ax = b \qquad \text{and} \qquad (II) \quad \lambda^T A = 0, \quad \text{s.t } \lambda^T b = 1.$$

Exercise 2

(a) Show that if $\partial f(x) \neq \emptyset$ then $\partial f(x)$ is closed and convex.

(b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be s.t. $f(x_1, x_2) = |x_1 + x_2|$. Show that f is convex and compute $\partial f(0,0)$.

Exercise 3

(a) Assume that $C \subseteq \mathbb{R}^n$ is convex, $f : C \to \bar{\mathbb{R}}$ is convex, and $g : \mathbb{R} \to \mathbb{R}$ is convex and increasing. Show that $g \circ f : C \to \mathbb{R}$ is convex.

(b) Show that the function $f(x) = (1 + \|x\|^2)^{\frac{p}{2}}$, where $p \geq 1$, is convex on $\mathbb{R}^n$. Here, $\|x\|$ simply denotes the Euclidean norm of x.

Exercise 4

Let $\emptyset \neq O \subset \mathbb{R}^n$ be convex. Show that if $\varphi \in \mathbb{R}^O$ is convex then $\varphi_{|S}$ is Lipschitz continuous for any compact subset S of $\mathrm{ri}(O)$.[1]

*Due by **November, Mon 25th**, in class or in my mailbox before noon.

[1] Recall that $\varphi_{|S}$ is just the restriction of φ on S, that is $\varphi_{|S} : S \to \mathbb{R}$ s.t. $\varphi_{|S}(x) = \varphi(x)$ for all $x \in S$.