MATH FOR ECON - PROBLEM SET 2

New York University, a.y. 2013-2014

Exercise 1

Let $X = (0, 1]$ with the usual metric $d(x, y) = |x - y|$. Show that (X, d) is not complete. Let $\tilde{d}(x, y) = \left|\frac{1}{x} - \frac{1}{y}\right|$ for all $x, y \in X$. Show that $\tilde{d}$ is a metric on X that is equivalent to d, and that $(X, \tilde{d})$ is complete.

Exercise 2

Let (X, d) be complete and $\Phi : X \to X$. Show that $d(\Phi(x), \Phi(y)) < d(x, y)$ for all $x, y \in X$, $x \neq y$, is insufficient for the existence of a fixed point of Φ. (An example is enough)

Exercise 3

Let $f : \mathbb{R} \to \mathbb{R}$ be increasing. Show that f can be discontinuous at only countably many points.

Exercise 4

Let $f : X \to X$ be a continuous function and X is a compact metric space. Also, $d(f(x), f(y)) = d(x, y)$. Prove that f is onto.

Exercise 5

Continuous function on compact domains are uniformly continuous.

Due on **September 25th by 7pm** in Jacopo's mailbox (19W4 6th floor) or send to jacopo.perego@nyu.edu.