MICROECONOMICS II.I – PROBLEM SET 4

New York University, a.y. 2012-2013

Exercise 1

The 100 residents of a small town are electing a mayor by secret ballot, and everyone is required to vote. There are three candidates: a Democrat (ranked first in strict reference for 60 voters), a Republican (ranked first in the other 40 voters' strict preference orderings) and a woman who has declared that she subscribes to Darwin's theory of evolution. For that reason, the town is unanimous that she is the worst candidate. The candidate with the highest number of votes wins the election. If two candidates are tied for the first place then the winner is decided by the toss of a coin. Candidates are not residents of the town and thus do not vote.[1]

(*a*). Consider everyone in the town voting for the evolutionist. Is this a Nash Equilibrium, and is it trembling hand perfect?

(*b*). Consider instead everyone in the town voting for the Republican. Is this a Nash Equilibrium, and is it trembling hand perfect?

Exercise 2: Costly Voting

The 100 residents of a small town are electing a mayor. There are two candidates: (*i*) a Democrat (strictly preferred by 70 voters) and (*ii*) a Republican (strictly preferred by 30 voters). Each person is 10 utils happier if her favored candidate is elected, but loses 1 util if she indeed votes (there is a cost to voting). Voting is NOT compulsory. The candidate with the highest number of votes wins the election. If there is a tie then the winner is decided by the toss of a coin.

(*a*). Voting is by secret ballot. No one can see how many others are voting. Find a Nash Equilibrium in pure strategies or prove that there isn't one.

(*b*). Suppose INSTEAD that voting is done at a public meeting, by roll call. Attendance at the meeting is compulsory. When your name is called, you can pass (no fee is charged) or you can declare your vote out loud (and be charged the fee). The order in which names will be called out is common knowledge before the calling starts. Using subgame perfect equilibrium as the solution concept, answer two questions:

i. Does the order in which names are to be called out affect who is elected?

ii. Does the order in which names are to be called out affect how many people vote? Can we Pareto rank the equilibria?

Due on March 4th by 7pm in Jacopo's mailbox (19W4 6th floor).

[1] For (a) and (b) above your explanation should be convincing but do not need to provide the exact sequence of trembles.

Exercise 3

Recall that in Hotelling's simultaneous location choice problem with $N = 3$ (as done in class), there is no equilibrium in pure strategies. Consider instead the sequential location problem for three players: 1 chooses a location, 2 observes this and chooses her location and 3 observes both previous choices and chooses her location. In this game assume that a player who chooses the same location as an earlier player can get the limit of nearby payoffs from the right or the left, whichever is more beneficial. Show that this game of perfect information has multiple subgame perfect equilibria with different payoffs for the third player.

Exercise 4

Each of two risk-neutral firms playing a discrete Bertrand game can choose its price to be any integer between 0 and 100, inclusive. Marginal cost is 0.5 for each firm and capacities are unlimited. Consumers buy $q(p) = 100 - p$ units, where p is the lower of the two prices. If the prices are the same, demand is split evenly between the two firms. What is the set of rationalizable prices for Firm 1? Explain. Are any non degenerate mixed strategies rationalizable?