MICROECONOMICS II.I – PROBLEM SET 2

New York University, a.y. 2012-2013

Exercise 1

Consider the two following definitions.

Definition 1. A two-person game $G = \langle \{1, 2\}, (S_i, u_i)_{i\in\{1,2\}} \rangle$ is **constant sum** if for some $c \in \mathbb{R}$,

$$u_1(s) + u_2(s) = c \text{ for each } s \in S_1 \times S_2.$$

Definition 2. Two Nash equilibria of G, say (m_1, m_2) and (m'_1, m'_2), are

(i) **Equivalent**: if $u_i(m) = u_i(m')$ for $i = 1, 2$, and

(ii) **Interchangeable**: if (m_1, m'_2) and (m'_1, m_2) are both Nash equilibria of G.

Now answer the following two questions:

(a) Prove that any two Nash equilibria of a two-person constant-sum game G are equivalent and interchangeable.

(b) Is the result from part (a) also true for three-person constant-sum games (where $\sum_{i=1}^{3} u_i(s) = c$ for all $s \in S$)?

Exercise 2

Modify the standard symmetric linear Cournot model with $p = a - bQ$ (where Q is market output) by letting marginal cost for a firm equal c times that firm's output (c is the same for all n firms).

(a) Argue that only pure Nash equilibria can exist.

(b) Argue that all equilibria are symmetric.

(c) Each firm's best reply function shows the firm's optimal output level as a function of what it believes other firms are putting on the market. Compare the best reply function here to that in the standard model with constant marginal cost. Give an intuition based on marginal analysis for the difference.

(c) How much does each firm produce in Nash equilibrium? What happens to output per firm, aggregate output and aggregate profits as n grows large?

Due Feb 19th by 10am in Jacopo's mailbox (19W4 6th floor).

Exercise 3

There are n people living in separate apartments overlooking an icy road. An old lady (not one of the retired pirates) has fallen on the road and needs to be taken to the hospital quickly. For each $i = 1, ..., n$, the occupant of the i^{th} apartment gains two utils if an ambulance arrives to collect the old lady, but loses one util if he gets up to call 9-1-1 himself. If he were the only person living there, he would make the call for a net gain of one util. But he's not. Treat this as a simultaneous game with each of the n players having two pure strategies.

(a) If $n = 2$, find the symmetric equilibrium of the game. Are there asymmetric equilibria? Explain.

(b) As n increases, how does the old lady's expected welfare change in the symmetric equilibrium?

(c) Suppose instead that the ambulance will be sent only if calls are received from at least *two* different apartments (so it takes at least two "good Samaritans" to get help for the old lady). Again, this is a simultaneous game with two pure strategies for each player.

- (i) When $n = 3$, in the symmetric equilibrium (ignoring the one in pure strategies), how likely is the ambulance to arrive?
- (ii) Prove that for sufficiently large n, there is *no* chance of the ambulance arriving in any symmetric equilibrium! It may be useful to remember that for x close to 0, the n^{th} power of $(1 - x)$ is well approximated by $(1 - nx)$.

Exercise 4. Optional, not graded

This exercise is optional and covers correlated equilibrium, discussed in the third TA session. Consider the relationship between Nash and correlated equilibrium payoffs:

(a) Can there be a correlated equilibrium where a player gets less than her lowest Nash equilibrium payoff? Explain or give an example. (hint: Is it possible in a two players game?)

(b) (*Harder*) Can there be a correlated equilibrium where every player gets less than her lowest Nash payoff?