

The Selective Disclosure of Evidence: An Experiment

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September 2023

INTRODUCTION

Core economic environment:

- Sender has private information about a state of the world.
- Receiver wants to learn the state.
- Sender wants to pretend the state is high.
- Sender can send a message about the state.
- How much **communication** can be achieved?

It depends on whether messages are verifiable or not.

Often **verifiability is partial** and **evidence is noisy**

- Sender has multiple verifiable signals about the state.
- Signals can be selected for disclosure.

e.g. SAT, news about political candidate, oil fields...

Signals have both **intrinsic** and **context-dependent** meanings:

- SAT may have been taken multiple times.
- There are many stories about the candidate.
- There are reports from many geologists.

This Paper

- Considers a **modified communication game**.
 - Sender has noisy signals and can selectively disclose.
- Studies how **information transmission** is affected by:
 - Variations in the number of signals available (**selection**)
 - Changes in sender's communication capacity (**verifiability**)
- Derives a theory based **experimental design** to test the main predictions.
 - Selective disclosure in equilibrium.
 - Withholding information and selectively disclosing possible.
 - Deception (rather than lying) possible.

Overview: Theory

Cheap Talk

e.g., Crawford-Sobel '82

- “Soft” information—Messages are Unverifiable.
- Large frictions in information transmission.

Disclosure

e.g., Milgrom '81

- “Hard” information—Messages are Verifiable.
- No frictions in information transmission (*unravelling*).

Our Framework

- Flexible verifiability.
- Spans cheap-talk and disclosure results.

Overview: Experiments

Cheap Talk

e.g., Cai-Wang '06

- Over-communication (with misaligned preferences).
- Some information transmission.

Disclosure

e.g., Jin-Luca-Martin '20

- Incomplete unravelling (*failure to account for selection*).
- Frictions in information transmission.

Our Framework

- Both over and under communication are possible.
- What dominates?

Disclosure: Jin, Luca and Martin (2022, AEJ: Micro)

Cheap talk: Blume, Lai and Lim (2020, Handbook of Experimental GT)

Partially verifiable disclosure: Penczynski, Koch and Zhang (2021)

Theory: Milgrom (1981, Bell), Fishman and Hagerty (1990, QJE), Di Tillio, Ottaviani and Sorensen (2021, Ecma)

MODEL

The Communication Game—Milgrom (1981)

Sender

1. Privately observes state $\theta \in \Theta$, with:
 - Θ finite.
 - Prior $p \in \Delta(\Theta)$.
2. Given θ , draws N i.i.d. signals, $s_i \in \mathcal{S} \subseteq \mathbb{R}$.
 - An exogenous information structure $f : \Theta \rightarrow \Delta(\mathcal{S})$, MLRP.
 - Notation: $\bar{s} = (s_1, \dots, s_N) \in \mathcal{S}^N$.
3. Can disclose up to K of the N drawn signals:
 - N , the number of available signals.
 - K , the number of reportable signals.

The Communication Game—Milgrom (1981)

Receiver

4. Observes the message.
5. Takes an action ($a \in A$) to maximize the expected payoff.

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Payoffs

6. $u_S(\theta, a) = a$.
7. $u_R(\theta, a) = c - (a - \theta)^2$, where $c \in \mathbb{R}$

Example

Let $\Theta = \{\theta_H, \theta_L\}$, $S = \{A, B, C, D\}$ and f be

State	Signal			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
θ_L	10%	20%	25%	45%
θ_H	45%	25%	20%	10%

Example

Let $\Theta = \{\theta_H, \theta_L\}$, $S = \{A, B, C, D\}$ and f be

State	Signal			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
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θ_H	45%	25%	20%	10%

Let $N = 3$ and $\theta = \theta_L$.

Example

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Assume signals are $\{B, D, D\}$.

Example

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Let $N = 3$ and $\theta = \theta_L$.

Assume signals are $\{B, D, D\}$.

If $K = 1$

Sender can send message from $\{\emptyset, B, D\}$.

Example

Let $\Theta = \{\theta_H, \theta_L\}$, $S = \{A, B, C, D\}$ and f be

State	Signal			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
θ_L	10%	20%	25%	45%
θ_H	45%	25%	20%	10%

Let $N = 3$ and $\theta = \theta_L$.

Assume signals are $\{B, D, D\}$.

If $K = 3$

Sender can send message from $\{\emptyset, B, D, BD, BDD\}$.

Role of K and N

When $K = N$, information is **fully verifiable**.

- Can disclose all signals \rightsquigarrow unraveling \rightsquigarrow no frictions

When $K < N$, information is **partially verifiable**.

- Can't disclose all signals \rightsquigarrow unraveling is unfeasible.
- Scope for imitation via **selective disclosure**.
- Messages verifiable, but selection \rightarrow meaning **context dependent**.

Hybrid framework b/w cheap-talk games and disclosure games.

Proposition

Milgrom (1981)

Fix any (N, K) , there exists a Sequential Equilibrium with maximal selective disclosure: Sender reports the K most favorable signals.

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Observable Implications

Sender

- $\uparrow K$: disclosed signals increases.
- $\uparrow N$: most favorable signal sent more often.

Receiver

- $\uparrow N$: most favorable signals become less persuasive.

Equilibrium: Refinements

Unlike classic disclosure games, the sequential equilibrium outcome is **not unique** when $K < N$.

- Off-path beliefs can support other equilibrium outcome.
- Refinements for signalling games (e.g., Cho-Kreps '87, Banks-Sobel '87) have no force here.
- Refinements for cheap talk games: Farrel (1993)'s **Neologism Proofness**.

» Example

Proposition

The equilibrium with maximal selective disclosure is Neologism Proof.

Equilibrium: Uniqueness

Proposition

The equilibrium with maximal selective disclosure is Neologism Proof.

Neologism Proofness delivers outcome uniqueness

An equilibrium (σ, μ) induces an outcome $x : S^N \rightarrow A$,

$$x(\bar{s}) = \sum_{\bar{s}'} \mu(\bar{s}' | \sigma(\bar{s})) \mathbb{E}(\theta | \bar{s}') \quad \forall \bar{s}.$$

Equilibrium: Uniqueness

Proposition

The equilibrium with maximal selective disclosure is Neologism Proof.

Proposition

Let (σ^*, μ^*) be the equilibrium with maximal selective disclosure and (σ, μ) be any other Neologism Proof equilibrium. Let x^* and x their respective outcomes. Then, $x^* = x$.

» Example

Main Outcome Variable

We study the effects of changing (N, K) .

Our main outcome of interest is **equilibrium informativeness**.

- How effectively the receiver learns the state θ .

Informativeness can be measured in several ways:

- Correlation between θ and a .
- **Receiver's expected payoff.**

Increasing K (Verifiability)

Fix $N \geq 1$.

Proposition

Equilibrium informativeness increases in K .

Intuition

- Easier to send messages that others cannot imitate.
 \Rightarrow Less pooling.
 \Rightarrow More information transmitted.

Increasing N (Selection)

Fix $K \geq 1$.

Proposition

When $N \rightarrow \infty$, equilibrium informativeness converges to zero.

Intuition

- When N grows, “highest” message available to every θ .
 \Rightarrow All types pool.

Increase in N (for small N) generates two contrasting effects:

1. **Information Effect**

- Sender has more evidence to prove her type.
- Selection contains information about undisclosed signals: “bad” messages are more informative.

2. **Selection Effect**

- Sender is more selective, making “higher” signals less informative.

(1) + (2) \implies informativeness may be non monotonic in N

EXPERIMENT

Experimental Design

- Two urns: Yellow (low) and Red (high).
- Four balls: A, B, C, or D.
- f is

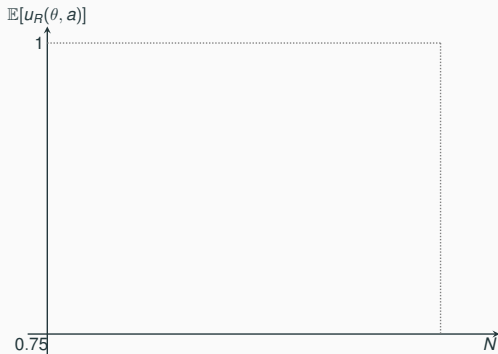
State	Signal			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
Yellow (θ_L)	10%	20%	25%	45%
Red (θ_H)	45%	25%	20%	10%

Experimental Design: Treatments

	N=1	N=3	N=10	N=50
$K = 1$	<i>i</i>	<i>.</i>	<i>ii</i>	<i>iii</i>
$K = 3$	<i>.</i>	<i>iv</i>	<i>v</i>	<i>vi</i>

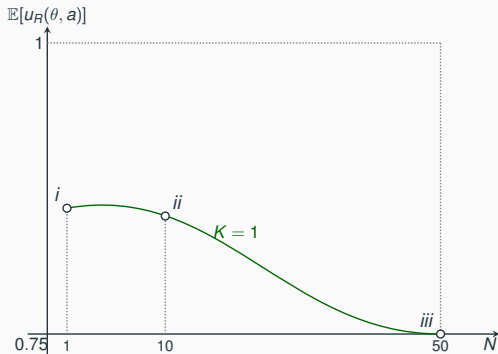
Experimental Design: Treatments

	N=1	N=3	N=10	N=50
$K = 1$	<i>i</i>	<i>.</i>	<i>ii</i>	<i>iii</i>
$K = 3$	<i>.</i>	<i>iv</i>	<i>v</i>	<i>vi</i>



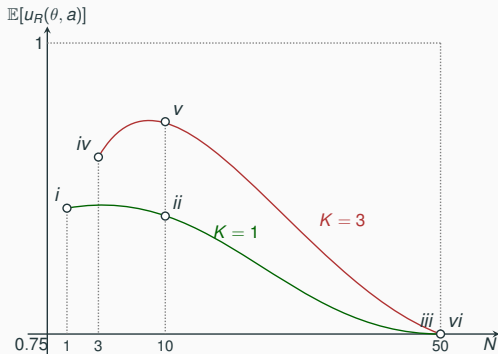
Experimental Design: Treatments

	N=1	N=3	N=10	N=50
$K = 1$	<i>i</i>	\cdot	<i>ii</i>	<i>iii</i>
$K = 3$	\cdot	<i>iv</i>	<i>v</i>	<i>vi</i>

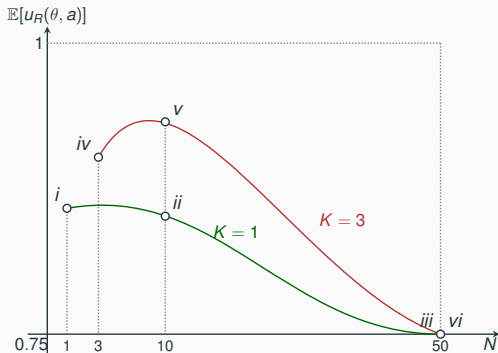


Experimental Design: Treatments

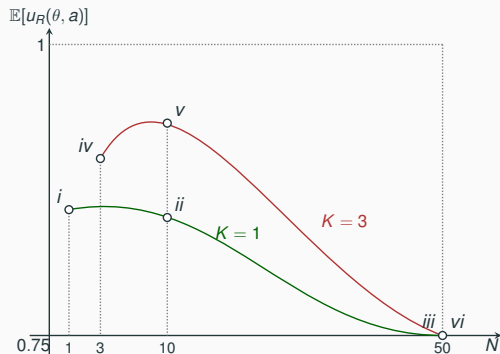
	N=1	N=3	N=10	N=50
$K = 1$	<i>i</i>	.	<i>ii</i>	<i>iii</i>
$K = 3$.	<i>iv</i>	<i>v</i>	<i>vi</i>



Experimental Design: Testable Implications



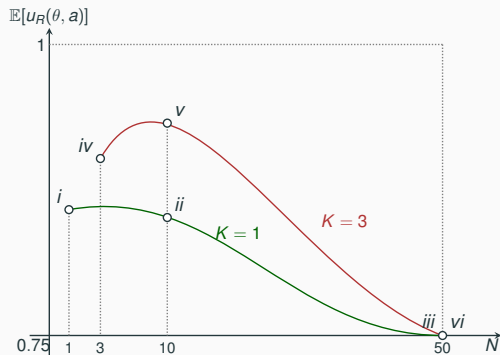
Experimental Design: Testable Implications



Test 1. If $K = N$, informativeness increases with N (more info)

$$iv > i$$

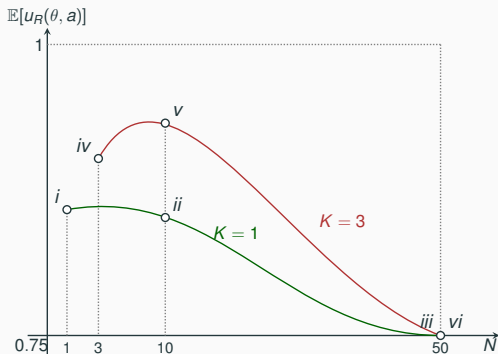
Experimental Design: Testable Implications



Test 2. $\uparrow K$: informativeness increases (more verifiability)

$$v > ii$$

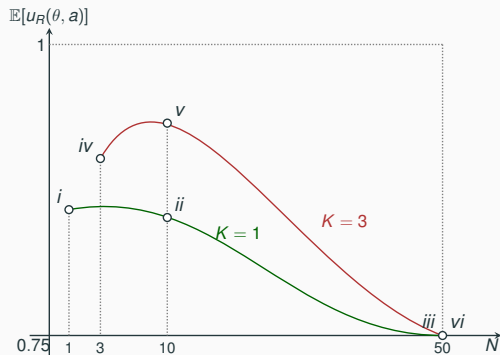
Experimental Design: Testable Implications



Test 3. $\uparrow N$: informativeness decreases (selection effect)

$$ii > iii \quad v > vi$$

Experimental Design: Testable Implications



Test 4. $\uparrow N$: informativeness increases (information effect)

$$v > iv$$

Experimental Design: Sender Interface

Round 7 of 30: Communication Stage

You are the Sender

Reminder:

If the secret Urn is **Yellow**, this is the probability of each ball being drawn:

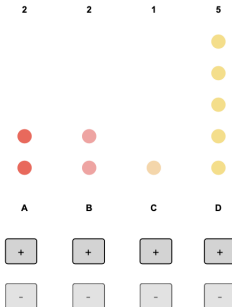
A	B	C	D
10 %	20 %	25 %	45 %

If the secret Urn is **Red**, this is the probability of each ball being drawn:

A	B	C	D
45 %	25 %	20 %	10 %

The secret Urn is **Yellow**

Available Balls



Your message to the Receiver is:



Send

Experimental Design: Sender Interface

Round 7 of 30: Communication Stage

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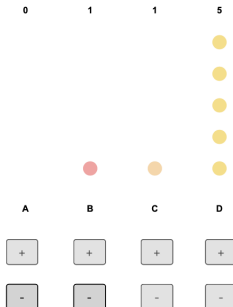
A	B	C	D
10 %	20 %	25 %	45 %

If the secret Urn is **Red**, this is the probability of each ball being drawn:

A	B	C	D
45 %	25 %	20 %	10 %

The secret Urn is **Yellow**

Available Balls



Your message to the Receiver is:

A

A

B

Send

Experimental Design: Receiver Interface

Round 7 of 30: Guessing Stage

You are the Receiver

Reminder:

If the secret Urn is **Yellow**, this is the probability of each ball being drawn:

A	B	C	D
10 %	20 %	25 %	45 %

If the secret Urn is **Red**, this is the probability of each ball being drawn:

A	B	C	D
45 %	25 %	20 %	10 %

The Sender had 10 available balls and they could disclose 3 or less. This is the Sender's message:



Use the slider below to indicate your Guess of how likely it is that the secret Urn is Red.

Your Guess: 10



Submit

Experimental Design: Summary

Round 7 of 30: Summary

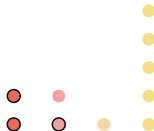
You are the Sender

Sender's Summary

The secret Urn was **Yellow**

Available Balls

2 2 1 5



A B C D

Sender's Message:



Receiver's Summary

The Sender had 10 available balls and they could disclose 3 or less. This is the Sender's message:



Use the slider below to indicate your Guess of how likely it is that the secret Urn is Red.

Guess: 10



Experimental Design: History

Round 7 of 30: History

You are the Sender

Round	Secret Urn	Message	Guess
7	Yellow	A A B	10
6	Red	A A ○	77
5	Red	A B B	77
4	Red	A A A	97
3	Red	A A ○	87
2	Yellow	C C ○	52
1	Red	○ ○ ○	0

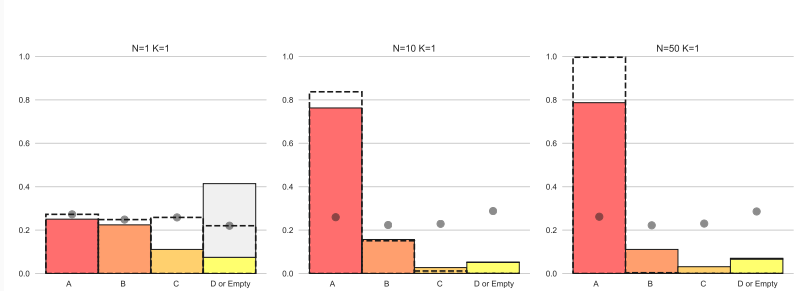
Next

RESULTS

SENDER'S AGGREGATE BEHAVIOR

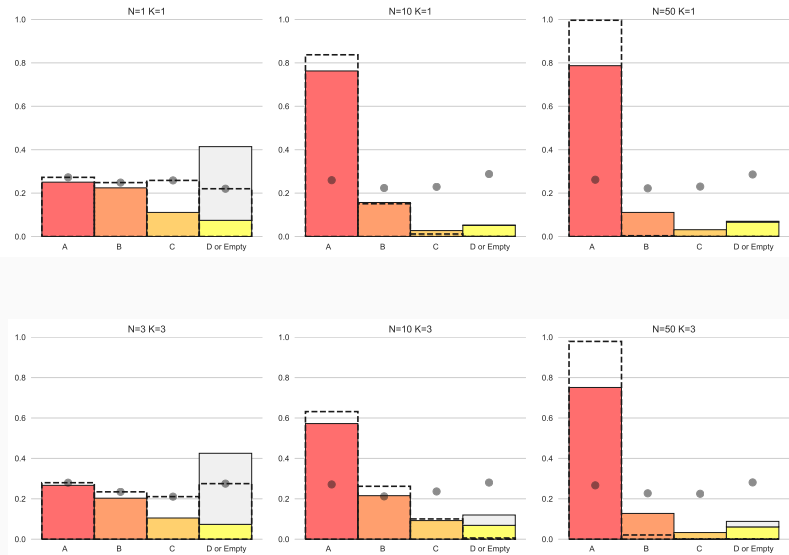
Sender's Disclosure Choices

Signals in Sender's Message: Observed Distribution (Bars) vs Theoretical Distribution (Dashed Bars) vs Random Distribution (Dots)



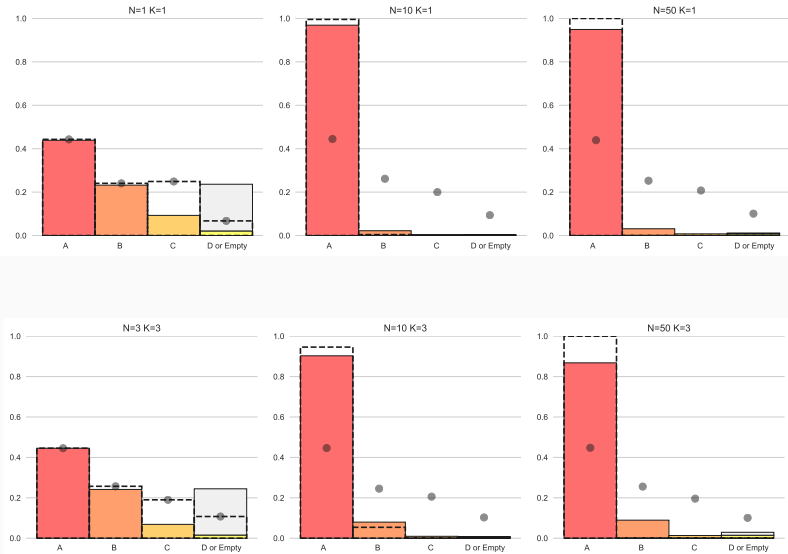
Sender's Disclosure Choices

Signals in Sender's Message: Observed Distribution (Bars) vs Theoretical Distribution (Dashed Bars) vs Random Distribution (Dots)



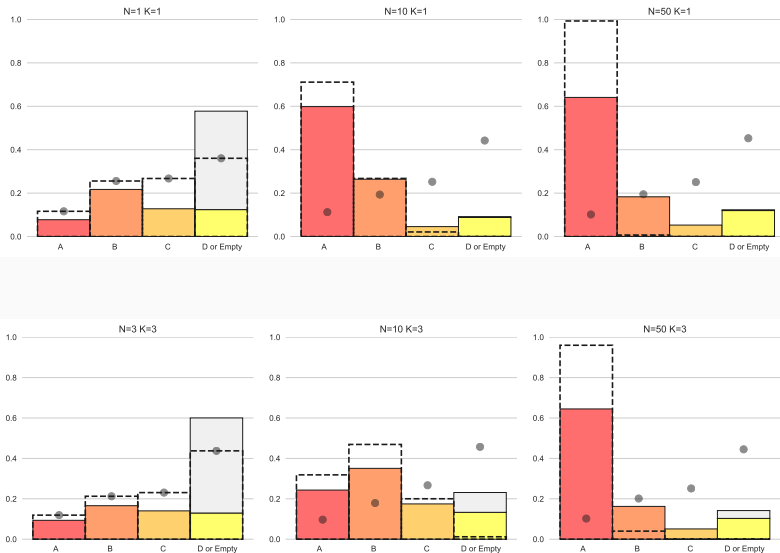
High Type Sender's Disclosure Choices

Signals in Sender's Message | H: Observed Distribution (Bars) vs Theoretical Distribution (Dashed Bars) vs Random Distribution (Dots)

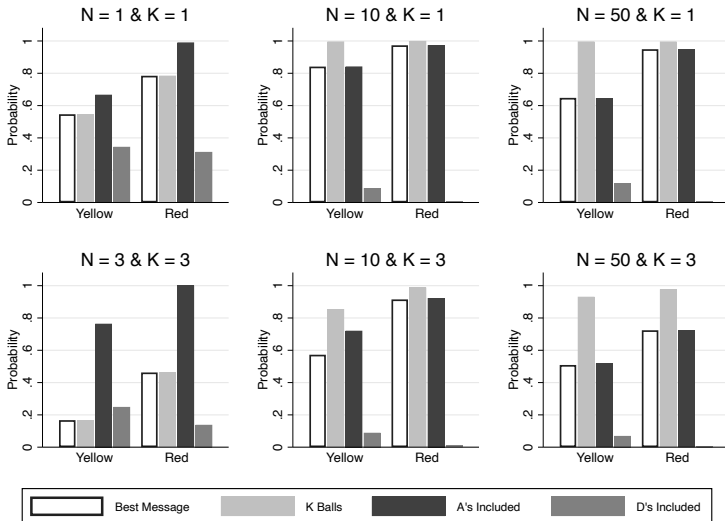


Low Type Sender's Disclosure Choices

Signals in Sender's Message | L: Observed Distribution (Bars) vs Theoretical Distribution (Dashed Bars) vs Random Distribution (Dots)



Sender's Messages



Result 1 (Senders)

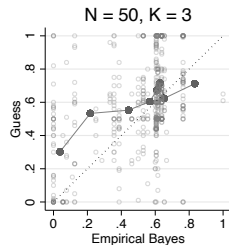
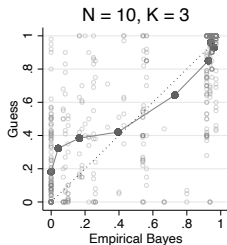
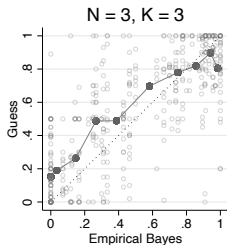
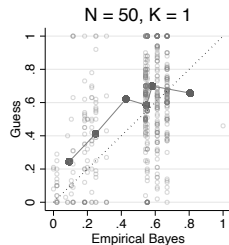
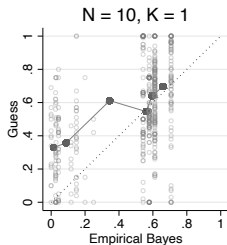
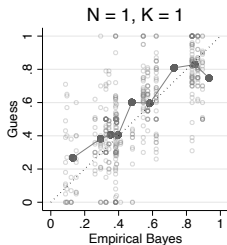
Result: The distribution of balls in messages is “close” to equilibrium.

Main Deviation:

- Not disclosing bad balls when $N = K$.
- Not disclosing good balls as often as predicted when the type of the sender is low.
- Overall, more information transmitted than predicted.

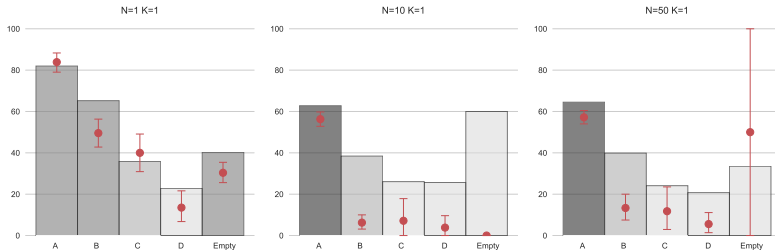
RECEIVER'S AGGREGATE BEHAVIOR

Receiver's Updating



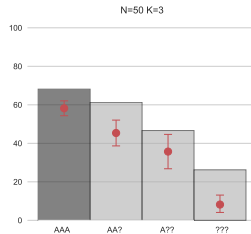
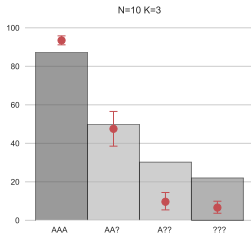
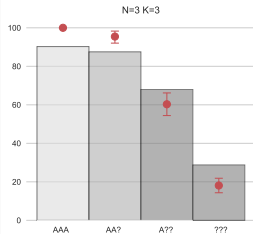
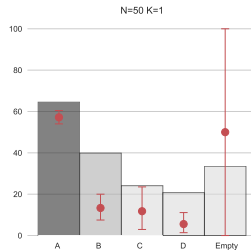
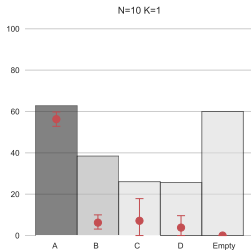
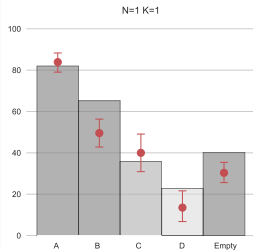
Receiver's Beliefs

Receiver's Elicited Beliefs (Bars) vs Empirical Beliefs (Red Dots)



Receiver's Beliefs

Receiver's Elicited Beliefs (Bars) vs Empirical Beliefs (Red Dots)



Result 2 (Receivers)

Result: Receivers overestimate the probability of an high type sender when it is less likely, more so when selection is more acute.

SENDER'S HETEROGENEITY

Challenge

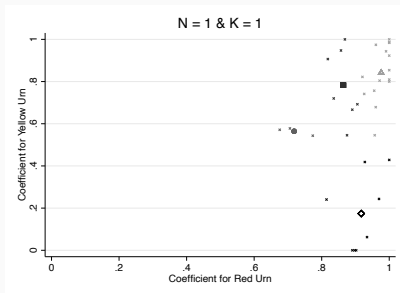
- Large number of urn / balls / message combinations.
- Specific behavior of interest varies across treatments.
 - Number of balls sent ($K = 1$ vs $K = 3$).
 - Balls sent vs balls available ($N = K$ vs $N > K$).

→ Precludes a unified approach using those variables.

Solution

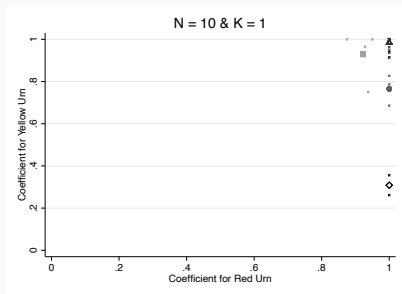
- Transform balls and messages to numbers ($B^\#$ and $M^\#$).
- Regress $M^\#$ on $B^\#|_{\text{yellow urn}}$ and $B^\#|_{\text{red urn}}$.
- Cluster the coefficient estimates.
- Describe behavior along key dimensions of interest.

Heterogeneity: Senders



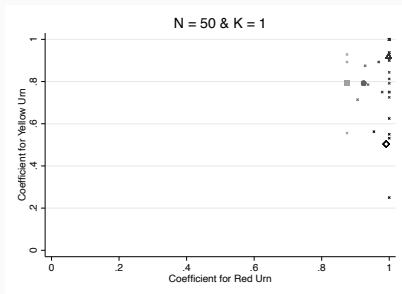
Cluster	Obs (33)	Urn	K	A	D
Triangle	15	Red	0.91	1	0.38
		Yellow	0.64	1	0.27
Square	7	Red	0.73	1	0.25
		Yellow	0.51	1	0.21
Circle	3	Red	0.5	0.92	n/a
		Yellow	0.54	0.67	0.49
Diamond	8	Red	0.71	1	0.20
		Yellow	0.30	0	0.46

Heterogeneity: Senders



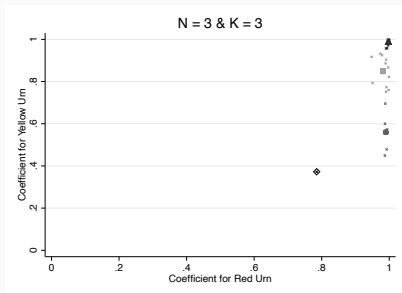
Cluster	Obs (34)	Urn	K	A	D
Triangle	24	Red	1	1	0
		Yellow	1	0.97	0.02
Square	4	Red	1	0.81	0.08
		Yellow	1	0.88	0.07
Circle	4	Red	1	1	0
		Yellow	1	0.46	0.14
Diamond	2	Red	1	1	0
		Yellow	1	0	0.89

Heterogeneity: Senders



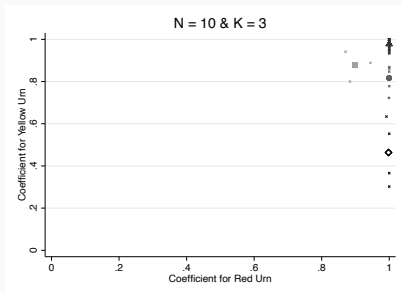
Cluster	Obs (36)	Urn	K	A	D
Triangle	25	Red	1	0.99	0
		Yellow	1	0.74	0.03
Square	3	Red	1	0.04	0.82
		Yellow	1	0	0.51
Circle	3	Red	1	0.78	0
		Yellow	1	0.63	0.18
Diamond	5	Red	1	0.96	0
		Yellow	0.95	0.26	0.46

Heterogeneity: Senders



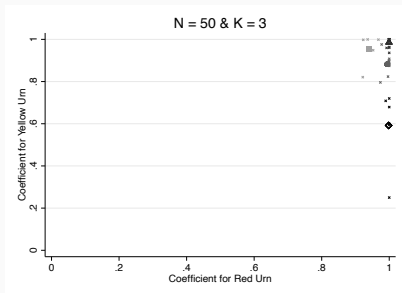
Cluster	Obs (29)	Urn	K	A	D
Triangle	12	Red	0.64	1	0.23
		Yellow	0.25	1	0.17
Square	11	Red	0.34	1	0.05
		Yellow	0.12	0.80	0.18
Circle	5	Red	0.26	1	0
		Yellow	0.12	0	0.80
Diamond	1	Red	0	1	0
		Yellow	0	0.50	0

Heterogeneity: Senders



Cluster	Obs (33)	Urn	K	A	D
Triangle	19	Red	0.99	0.99	0
		Yellow	0.88	0.96	0.01
Square	3	Red	1	0.46	0.17
		Yellow	1	0.43	0.04
Circle	7	Red	1	0.94	0
		Yellow	0.74	0.66	0.10
Diamond	4	Red	0.92	0.83	0
		Yellow	0.76	0.28	0.43

Heterogeneity: Senders



Cluster	Obs (35)	Urn	K	A	D
Triangle	15	Red	1	0.88	0
		Yellow	0.94	0.80	0
Square	5	Red	0.89	0.17	0
		Yellow	0.87	0.32	0
Circle	9	Red	0.97	0.70	0
		Yellow	0.94	0.31	0.04
Diamond	6	Red	1	0.86	0.03
		Yellow	0.95	0.31	0.41

Heterogeneity: Senders

Equilibrium type (55%)

- Most common.
- $N > K$: Mostly report best balls independently of the type.
- $N = K$: Disclose fewer than K balls.

Deception Averse Type (15%)

- A's reported more often when the type is high.
- D's reported more often when the type is low.
- $N = K$: Disclose fewer than K balls.

Others (30%)

- Similar to *equilibrium types* when the type is high.
- Report A's less but do not report D's when the type is low.
- Some low rates of A's when the type is high [confusion].

RECEIVER'S HETEROGENEITY

Challenge

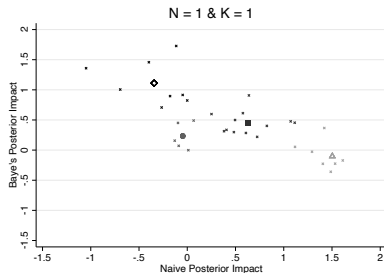
- Large number of messages.
- Different messages across treatments.
- Some messages have very few observations.

→ Precludes a unified approach using that variable.

Solution

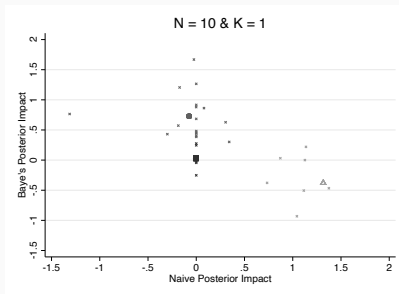
- Compute *equilibrium update* following each message.
- Compute the update of someone who ignores selection: *naive update*.
- Regress guesses on a constant (α) and the equilibrium and naive updates.
- Cluster the coefficient estimates.
- Describe behavior along key dimensions of interest.

Heterogeneity: Receivers



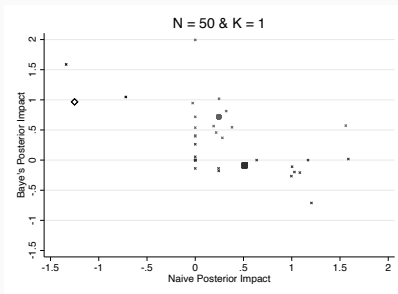
Cluster	Obs (33)	A	B	\emptyset	C
Diamond $\alpha = 0.23$	8	0.87	0.67	0.23	0.47
Circle $\alpha = 0.39$	5	0.56	0.49	0.41	0.37
Square $\alpha = 0.02$	12	0.86	0.73	0.41	0.38
Triangle $\alpha = -0.23$	8	0.90	0.67	0.51	0.23

Heterogeneity: Receivers



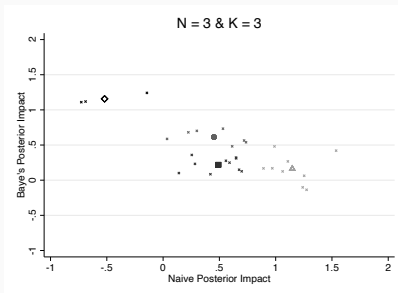
Cluster	Obs (29)	A	B	\emptyset	D
Diamond $\alpha = 4.20$	1	0.60*	0.23*	0.60*	n/a
Circle $\alpha = 0.28$	17	0.66	0.26	n/a	0.11
Square $\alpha = 0.56$	8	0.58	0.60	n/a	0.60
Triangle $\alpha = -0.23$	8	0.62	0.52	n/a	0.11

Heterogeneity: Receivers



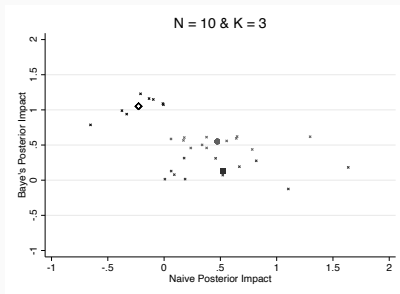
Cluster	Obs (34)	A	B	\emptyset	D
Diamond $\alpha = 0.89$	3	0.35	0.17	0.21*	0.75
Circle $\alpha = 0.15$	13	0.71	0.29	0.46*	0.11
Square $\alpha = 0.26$	18	0.63	0.53	n/a	0.19
Triangle $\alpha = -1.15$	2	0.69	0.41	n/a	n/a

Heterogeneity: Receivers



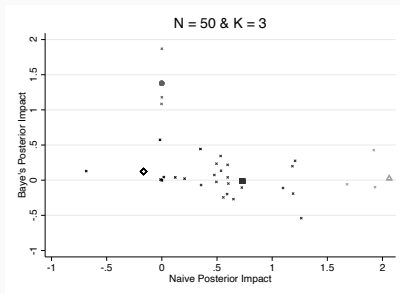
Cluster	Obs (33)	AAA	AAB	AA	AB
Diamond $\alpha = 0.35$	3	0.79*	0.90	0.82	0.50
Circle $\alpha = 0.02$	7	0.96	0.90	0.96	0.85
Square $\alpha = 0.13$	10	0.85	0.81	0.72	0.71
Triangle $\alpha = -0.24$	9	1	0.97	0.96	0.88

Heterogeneity: Receivers



Cluster	Obs (36)	AAA	AAB	AA	ABB
Diamond $\alpha = 0.19$	8	0.95	0.11	0.02	0.03
Circle $\alpha = -0.07$	13	0.89	0.70	0.24	0.26
Square $\alpha = 0.10$	11	0.74	0.70	n/a	0.61
Triangle $\alpha = -3.98$	1	1*	0.54*	n/a	0.02*

Heterogeneity: Receivers



Cluster	Obs (35)	AAA	AAB	AA	DDD
Diamond $\alpha = 0.64$	10	0.54	0.49	0.33	0.32
Circle $\alpha = 0.11$	3	0.84	0.01*	n/a	0.07
Square $\alpha = -0.04$	18	0.67	0.69	0.57	0.12
Triangle $\alpha = -1.16$	4	0.89	0.80	0.91*	n/a

Heterogeneity: Receivers

- Variation in updating strategies.
 - Extent they account for selection.
- Being closer to equilibrium \nrightarrow higher payoffs.
- However, in many treatments, groups better at accounting for selection is among the highest.
- With $N = 50$, few differences in payoffs.

Senders

- The majority:
 - Select the better balls to send.
 - Behave similarly for both urns.
- Some convey more information by conditioning on the type.

→ More information transmitted than predicted.

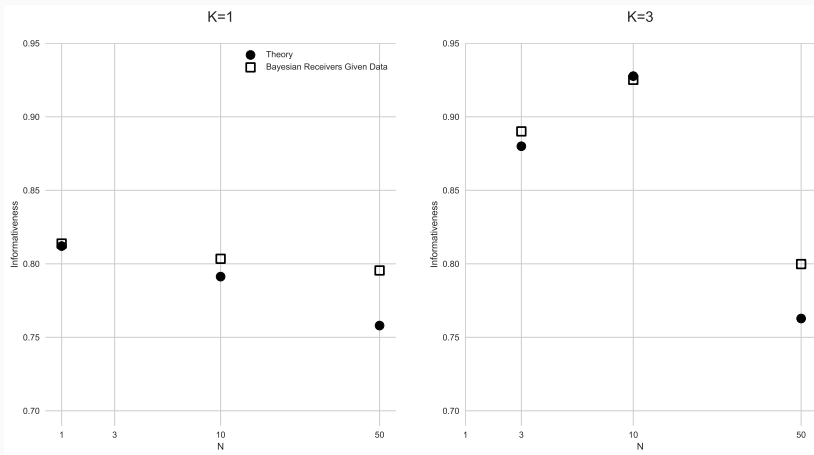
Receivers

- Many do not fully account for selection.
- Some are not very responsive.

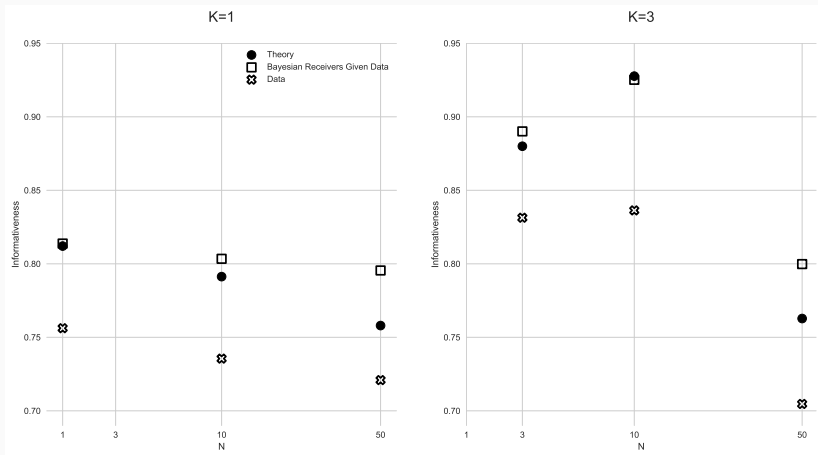
→ Less information received than predicted.

INFORMATIVENESS

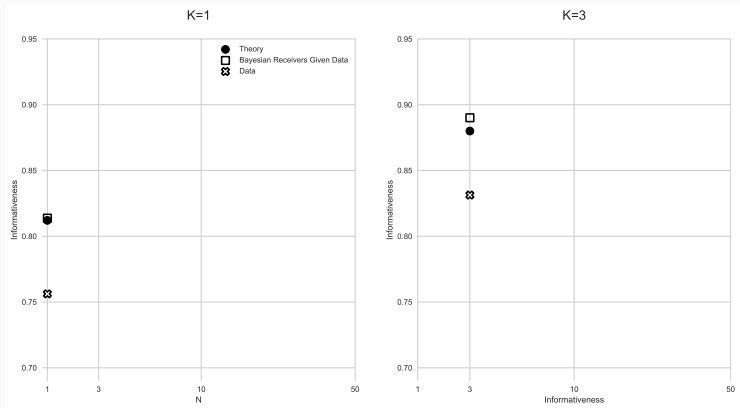
Information Transmission: Bayesian Receivers (Given Data)



Information Transmission



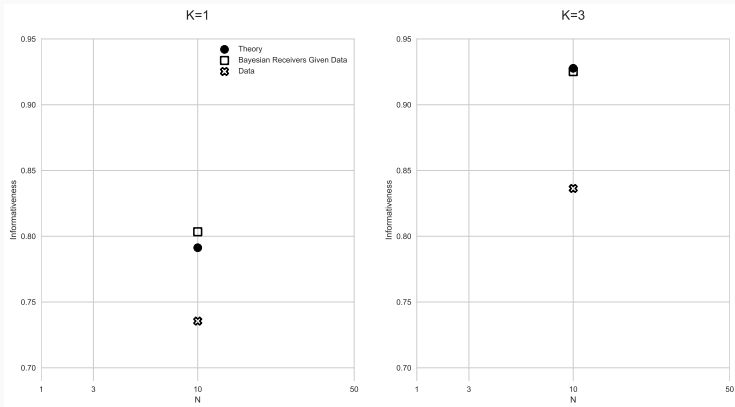
Test 1: More Information



Test 1. If $K = N$, informativeness increases with N (more info)

- Data: $p - value = 0.00$.
- Bayesian receivers given data: $p - value = 0.00$.

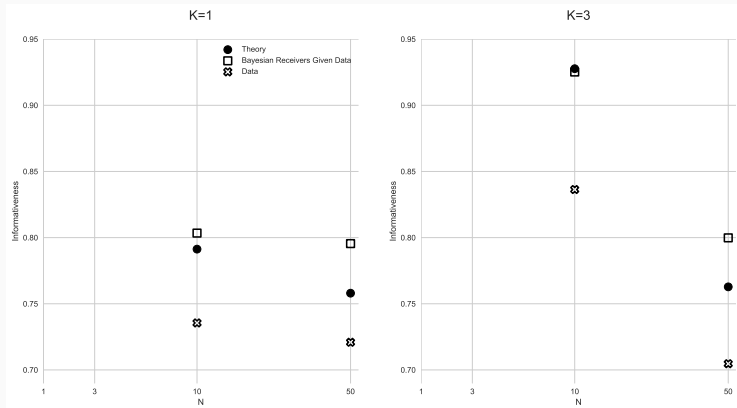
Test 2: More Verifiability



Test 2. $\uparrow K$: informativeness increases (more verifiability)

- Data: $p - value = 0.00$.
- Bayesian receivers given data: $p - value = 0.00$.

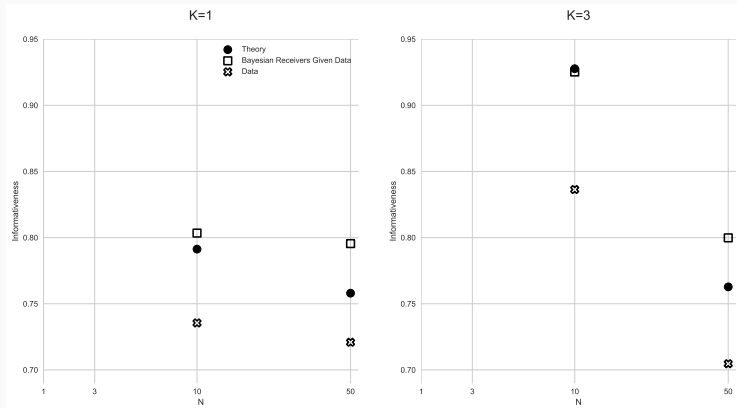
Test 3: Selection Effect



Test 3. $\uparrow N$: informativeness decreases (selection effect)

- Data ($K = 1$): $p - value = 0.43$.
- Bayesian receivers given data ($K = 1$): $p - value = 0.64$.

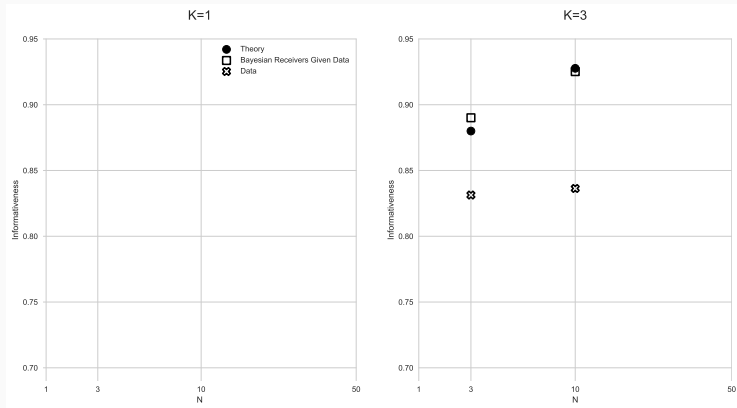
Test 3: Selection Effect



Test 3. $\uparrow N$: informativeness decreases (selection effect)

- Data ($K = 3$): $p - value = 0.00$.
- Bayesian receivers given data ($K = 3$): $p - value = 0.00$.

Test 4: Information Selection



Test 4. $\uparrow N$: informativeness increases (information effect)

- Data: $p - value = 0.81$.
- Bayesian receivers given data: $p - value = 0.04$.

Result 3 (Informativeness)

Result: Changes in K and N moves informativeness in the directions predicted by the theory in most cases.

- $N = K \uparrow \implies$ informativeness \uparrow .
→ More information.
- Fix N : $K \uparrow \implies$ informativeness \uparrow .
→ More verifiability.
- Fix K : $N \uparrow \implies$ informativeness never \uparrow .
→ Selection effect \geq Information effect.

Model:

- Selective disclosure in equilibrium.
- Spans cheap talk and disclosure models.
- Studies role of information and selection effect.

Experimental results:

- Important selective disclosure (predicted and otherwise).
- Some deception aversion.
- Receivers have difficulty accounting for selection.
- Less information transmission than predicted.

APPENDIX

Some Notation: Strategies and Beliefs

Denote \mathcal{M} the space of all messages

Sender's Strategy

pure and θ -independent

- $\sigma : S^N \rightarrow \mathcal{M}$ s.t. $\sigma(\bar{s}) \in M(\bar{s})$, for all \bar{s}

where $M(\bar{s})$ is the space of available messages given \bar{s}

Receiver's Beliefs and Strategy

- $\mu : \mathcal{M} \rightarrow \Delta(S^N)$
- $a : \mathcal{M} \rightarrow \Delta(A)$

Given μ , receiver's optimal strategy given by

$$a(m) = \mathbb{E}(\theta|m) = \sum_{\bar{s}} \mu(\bar{s}|m) \mathbb{E}(\theta|\bar{s}) \quad \forall m$$

Sequential Equilibrium

A **Sequential Equilibrium** is a pair (σ^*, μ^*) s.t.

1. For all $\bar{s} \in S^N$, $\sigma^*(\bar{s}) \in M(\bar{s})$ and

$$\sum_{\bar{s}'} \mu^*(\bar{s}' | \sigma^*(\bar{s})) \mathbb{E}(\theta | \bar{s}') \geq \sum_{\bar{s}'} \mu^*(\bar{s}' | m') \mathbb{E}(\theta | \bar{s}') \quad m' \in M(\bar{s})$$

2. For all m , $\text{supp } \mu^*(\cdot | m) \subseteq C(m) = \{\bar{s} \in S^N : m \in M(\bar{s})\}$. In particular, if $m \in \sigma^*(S^N)$,

$$\mu^*(\bar{s} | m) = q(\bar{s} | \sigma^{*-1}(m)) \quad \forall \bar{s}$$

where $q(\bar{s}) = \sum_{\theta} p(\theta) f(\bar{s} | \theta)$

Equilibrium Multiplicity

$\Theta = \{0, 1\}$ and $p(1) = \frac{1}{2}$. $N = 2$ and $K = 1$.

$S = \{A, B\}$, $f(A|\theta_H) = 1$ and $f(A|\theta_L) = \frac{1}{2}$.

θ		\bar{s}	$M(\bar{s})$	$\sigma^*(\bar{s})$
1	----->	(A, A)	$\{\emptyset, A\}$	A
0	----->	(A, B)	$\{\emptyset, A, B\}$	A
	----->	(B, B)	$\{\emptyset, B\}$	B

$$\mathbb{E}[\theta|m = A] = \frac{4}{7} \text{ and } \mathbb{E}[\theta|m = B] = \mathbb{E}[\theta|m = \emptyset] = 0 \implies$$

No incentive to deviate

Equilibrium Multiplicity

$\Theta = \{0, 1\}$ and $p(1) = \frac{1}{2}$. $N = 2$ and $K = 1$.

$S = \{A, B\}$, $f(A|\theta_H) = 1$ and $f(A|\theta_L) = \frac{1}{2}$.

θ		\bar{s}		$M(\bar{s})$	$\sigma^*(\bar{s})$
1	----->	(A, A)		$\{\emptyset, A\}$	\emptyset
0	----->	(A, B)		$\{\emptyset, A, B\}$	\emptyset
	----->	(B, B)		$\{\emptyset, B\}$	\emptyset

$$\mathbb{E}[\theta|m = \emptyset] = \frac{1}{2} \text{ and } \mathbb{E}[\theta|m = A] = \mathbb{E}[\theta|m = B] = 0 \implies$$

No incentive to deviate

Neologism Proof Equilibrium

A **neologism** is a pair (m, C) , $C \subseteq \{\bar{s} \in S^N : m \in M(\bar{s})\}$

Literal meaning of $(m, C) \rightsquigarrow$ “My type \bar{s} belongs to C ”

A neologism (m, C) is **credible** relative to equilibrium (σ^*, μ^*) if

1. $\sum_{\bar{s}'} q(\bar{s}'|C) \mathbb{E}(\theta|\bar{s}') > \sum_{\bar{s}'} \mu^*(\bar{s}'|\sigma^*(\bar{s})) \mathbb{E}(\theta|\bar{s}')$ for all $\bar{s} \in C$
2. $\sum_{\bar{s}'} q(\bar{s}'|C) \mathbb{E}(\theta|\bar{s}') \leq \sum_{\bar{s}'} \mu^*(\bar{s}'|\sigma^*(\bar{s})) \mathbb{E}(\theta|\bar{s}')$ for all $\bar{s} \notin C$

The equilibrium is **Neologism Proof** if no neologism is credible.

Back to the Example

θ		\bar{s}	$M(\bar{s})$	$\sigma^*(\bar{s})$
1	----->	(A, A)	$\{\emptyset, A\}$	\emptyset
	----->			
0	----->	(A, B)	$\{\emptyset, A, B\}$	\emptyset
	----->			
	----->	(B, B)	$\{\emptyset, B\}$	\emptyset

$$m = A \text{ and } C = \{(A, A), (A, B)\} \implies$$

$$\mathbb{E}[\theta | m = A] = \frac{4}{7} > \mathbb{E}[\theta | m = \emptyset] = \frac{1}{2}$$

Credible neologism \implies no Neologism Proof equilibrium