Media Competition and Social Disagreement

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Abstract

We study the competitive provision and endogenous acquisition of political information. Our main result identifies a natural equilibrium channel through which a more competitive market decreases the efficiency of policy outcomes. A critical insight we put forward is that competition among information providers leads to informational specialization: firms provide relatively less information on issues that are of common interest and relatively more information on issues on which agents' preferences are heterogeneous. This enables agents to acquire information about different aspects of the policy, specifically, those that are particularly important to them. This leads to an increase in social disagreement, which has negative welfare implications. We establish that, in large enough societies, competition makes every agent worse off by decreasing the utility that she derives from the policy outcome. Furthermore, we show that this decline cannot be compensated by the decrease in prices resulting from competition.

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1. Introduction

We study the competitive provision and endogenous acquisition of political information. Our interest is motivated by a growing public debate on the consequences of a fast-changing media landscape and information consumption habits on our democracies.[1] The political-economy literature still lacks a comprehensive understanding of how competition affects the strategic incentives of information providers in this market and its possible consequences on the political process (Strömberg, 2015). We contribute to filling this gap by presenting a simple model in which nonpartisan information providers compete to sell information to agents before they cast a vote. Our analysis leads to three novel conclusions. First, we show that competition leads to informational specialization. The critical insight we put forward is that competition forces information providers to become relatively less informative on issues that are of common interest and therefore are particularly important from a social perspective. Second, we analyze the downstream effects of such specialization and show that, while agents become better informed on an individual level, competition amplifies social disagreement. Third, we highlight the social welfare implications of increased disagreement. Specifically, we establish that in societies that are large enough, competition makes every agent worse off by decreasing the utility that she derives from the policy outcome.

In our model, a finite number of firms compete to provide information to a finite number of Bayesian agents about a newly proposed policy with uncertain prospects. Whether the new policy is implemented to replace a known status quo depends on its approval rate. The policy features a vertical component, which is a *valence* aspect on which agents' preferences are identical, and two horizontal components, which are *ideological* aspects on which preferences are heterogeneous. Each firm sells a signal about the policy but faces a constraint on how informative such a signal can be on the different components. That is, being more informative about one of these components requires the firm to be less informative about the others. To illustrate, imagine a new healthcare bill is under discussion, the details of which are not yet fully known to the public. The bill potentially affects many dimensions of social life, and voters might evaluate these dimensions differently. For example, the new bill could promote an increase in the overall quality of health care (vertical component), expand the budget deficit (horizontal component), and induce more redistribution via increasing the share of the population covered (horizontal component). Voters acquire information from the media before they approve or disapprove the policy. A larger consensus increases the probability that the bill is ultimately implemented. The news outlets com-

[1] Pew Research Center (2016), Sunstein (2017), and Nichols (2017) provide a comprehensive description of the media market and how it dramatically changed in the last decade.

pete for profits by allocating their limited resources (journalists, airtime, etc.) to a possibly different mix of these policy components and by setting prices.[2]

The equilibrium of our model demonstrates how competition among information providers induces informational specialization. While all agents want to learn about the policy, different agents care about different aspects of it. To maximize profits, firms sell information that is valuable for a diverse set of agents. They can do so by being informative about aspects of the policy that are of common interest, i.e., valence. However, as the market becomes more competitive, the effectiveness of such a "generalist" approach declines; different firms target different agents, providing signals tailored to the specific needs of those agents.

The equilibrium analysis leads to novel insights. We find that competition creates a broader spectrum of informational options, enabling agents to acquire information that is better aligned with their needs. Also, the market does not overspecialize. Indeed, as the number of competing firms grows large, the equilibrium converges to a *Daily Me* paradigm, a situation in which each agent finds an information provider that perfectly meets her unique informational needs (Sunstein, 2001). Furthermore, competition decreases the price associated with such information. Thus, competition benefits agents by enabling them to be better informed at lower prices. These results conform to the classic view that sees the market for news as a "marketplace of ideas" that promotes knowledge and the discovery of truth (see, Posner, 1986).

We then use our model to study the effects of competition on welfare, which extend well beyond the information supplied by firms. The market for political news differs from other markets partly because of the externalities that information acquisition imposes on the political process. Such a process, by definition, aggregates the opinions of agents who are potentially in conflict with each other. Because of this, information has both direct and indirect welfare effects. The *direct* effect measures how the information that an agent personally acquires enables her to sway the policy outcome in the direction of her own preferences. The *indirect* effect, instead, measures how the information acquired by others is used to sway the policy outcome toward their preferences. Agents, who try to maximize their own impact on the political process, acquire information based on its *direct* value. Therefore, a competitive market specializes to meet such demand. However, as firms specialize, agents learn about increasingly different aspects of the policy. Thus, their opinions diverge, leading to an increase in social disagreement. This generates a decline in the *indirect* value of information, capturing the externality agents impose on each other. Our main result demonstrates that, in large enough societies, this externality becomes critical. That is, the

[2]As another example, consider the social-distancing policies adopted to slow the spread of COVID-19. These policies have both public health and economic components, which are weighted differently by different people. News outlets may differ in which components they emphasize (see, Bursztyn et al., 2020; Simonov et al., 2020).

utility that each agent derives from the policy outcome decreases with competition. Moreover, the decrease in prices resulting from competition does not compensate for this decline.

Finally, we discuss how the main insights of the paper extend beyond our simplifying assumptions by comparing two notable market structures: monopoly and perfect competition. This further highlights the importance of two key features of our model: the heterogeneity in agents' preferences and the constraints on how much they can learn about the policy. The interaction of these features leads to information specialization, which plays a critical role in the inefficiency identified in this paper.

The rest of the paper is organized as follows. The next subsection reviews the related literature and discusses the empirical implications of our work. Section 2 introduces the model, while Section 3 characterizes its equilibrium. Our main results are presented in Section 4, and Section 5 discusses their extensions. All proofs are relegated to the Appendix.

1.1. Related Literature and Empirical Implications

Our paper contributes to the burgeoning literature on the political economy of mass media (see, Anderson et al., 2015; Prat and Strömberg, 2013). Specifically, we contribute to the branch of this literature that studies the effects of the endogenous provision of information and its externalities on the political process. One robust finding of this literature is that when information providers are partisans—namely, they are interested in persuading the public to take a certain action—competition generally brings about better social outcomes. Intuitively, competition forces firms to better align with what consumers demand, thus reducing their inherent biases. Results along this line are reflected in the works of Baron (2006), Chan and Suen (2009), and Anderson and McLaren (2012).[3] Similarly, Duggan and Martinelli (2011) find that slanting is an equilibrium outcome in a richer model that allows for electoral competition, but otherwise abstracts away the problem of competitive information provision. While not modeling competition, the works of Alonso and Câmara (2016) and Bandyopadhyay et al. (2020) also belong to this strand of the literature. Instead, a general treatment of competition among biased senders is discussed in Gentzkow and Kamenica (2016). Our work differs from these papers as we assume that information providers are non-partisans and compete for profits. Chan and Suen (2008) consider a model with features that can be mapped back to our setup. Their primary interest, however, is to study the effects of exogenously located firms on electoral competition. They show that a new entrant

[3] The welfare-increasing effects of competition are also illustrated in Besley and Prat (2006), Corneo (2006), and Gehlbach and Sonin (2014), although for reasons orthogonal from those discussed here, namely the potential risks of media capture by the government.

increases the probability that parties will choose the policy favored by the median voter, thereby increasing social welfare. In an extension, they also endogenize competition, but the only industry structure they can feasibly analyze (a duopoly) typically leads to higher welfare. Closer to our work, Chen and Suen (2019) study a competition model in which biased media firms compete for the scarce attention of readers, finding that an increase in competition leads to an increase in the overall informativeness of the industry. Similarly, results consistent with the idea that competition is welfare-increasing are discussed in Burke (2008), Gentzkow and Shapiro (2006), and Gentzkow et al. (2014). Sobbrio (2014) does not analyze the social welfare implications of media competition, but shows that competition can lead to specialization. Galperti and Trevino (2020) study a model of endogenous provision and acquisition of information and show how competition for attention can lead to a homogeneous supply of information, even when consumers would value accessing heterogeneous sources. Overall, when consumers are rational, the evidence is stacked in favor of the welfare-increasing effects of media competition.

Our paper contributes to this literature by developing a full-fledged competition model that illustrates a novel and natural channel through which competition can be welfare-decreasing. While not analyzing the competitive provision of information, Ali et al. (2018) study the interaction between private information and distributive conflicts in the context of a voting game. More specifically, they provide necessary and sufficient conditions under which the strategic interactions among agents can preclude a policy that is both ex-ante and ex-post optimal from being implemented. This is due to a form of adverse selection when information is scarce—an effect that is markedly distinct from the inefficiency we highlight in this paper. Departing from the assumption of rationality when processing information, Mullainathan and Shleifer (2005) consider a model in which heterogeneous consumers derive psychological utility from having their prior views confirmed by the information they receive. Their main result demonstrates how firms specialize in response to competition by slanting news toward the beliefs of their readers, resulting in a less informed electorate. In contrast, our model shows that firm specialization comes at the expense of information about valence (the vertical dimension) and studies of how this channel affects voting and, ultimately, social welfare. In a related model with behavioral preferences for confirmation, Bernhardt et al. (2008) study the welfare implications of competition, showing that competition increases the probability the society will make mistakes in policy selection. Bordalo et al. (2016) analyze a model in which two firms compete for the attention of a group of "salient thinkers" by strategically setting the quality and the price of the product they sell. They show how distortions in consumers' perceptions can explain the equilibrium degree of commoditization of some markets. Strömberg (2004) shows that media outlets have incentives to invest more in the coverage of issues that are important for groups that are valuable to advertisers, thus inducing a policy bias. Relatedly, Matějka and Tabellini (2020) study policy selection when voters are rationally inattentive. They find that divisive issues attract the most attention from voters and that this can create inefficiencies in public good provision.

Industrial Organization. Our paper also relates to a large body of literature on spatial competition.[4] As in Salop (1979), we use a circular setup to tractably model competition with an *arbitrary* number of firms. As in Lederer and Hurter (1986), Hamilton et al. (1991), and Vogel (2011), we use spatial price discrimination to avoid well-known technical issues related to equilibrium existence when both prices and locations are chosen endogenously (see, D'Aspremont et al., 1979). As in Vogel (2008), we allow firms to differentiate in *both* vertical and horizontal features of the product space. We contribute to this literature in three ways. First, we explicitly model equilibrium interactions between vertical and horizontal competition. With this, we can show that competition leads firms to disinvest from vertical features—which are beneficial to all consumers—in favor of horizontal features—which are beneficial only to a niche segment. Second, we study the consequences of specialization in a context in which private consumption generates social externalities. This feature is common to many markets, well beyond our political-economy application. Finally, our firms sell information and, to account for this, we build Bayesian foundations into a spatial competition model.[5] For these reasons, our model can be used to study other aspects of media competition (e.g., implications on turnout, campaign spending, or candidate selection), or more generally, consequences of competition in other information markets (Bergemann and Bonatti, 2019).

Empirical Implications. Our paper also relates to the large body of empirical literature that studies the effects of media on political outcomes. We contribute to this literature with several predictions that have empirical content. First, we predict that agents with different ideological preferences will be differentially informed. Using a large-scale incentivized survey, Angelucci and Prat (2020) show that subjects with different party affiliations are informed about different political facts. Second, we predict that such differences are caused by the fact that these agents acquire information from different media. Using an "audience-based" approach, the literature has established that media outlets can be reliably ranked in terms of the ideological preferences of their audience (e.g., Bakshy et al., 2015; Gentzkow and Shapiro, 2011; Zhou et al., 2011). Third, we predict that media outlets differentiate by emphasizing different issues (e.g., civil rights, healthcare, labor issues). Early evidence for this type of differentiation relies on specific examples: Puglisi (2011) identifies a Democratic bias—more coverage of issues championed by Democrats—for

[4]See Anderson et al. (1992) for a review of such literature.

[5]Technically, the Bayesian value of information becomes a "transportation" cost, which is neither convex nor concave in the distance between the firm and the agent.

the *New York Times*. Larcinese et al. (2011) and Puglisi and Snyder (2011) study selective coverage of some economic issues and political scandals, respectively. Using an alternative approach, Chopra et al. (2020) provide evidence that people expect newspapers to selectively choose which issues to report on. However, testing our specific predictions on how firms differentiate requires adopting a more direct "content-based" approach, which involves a comprehensive analysis of the content provided by the outlets. Incidentally, the research frontier is moving in this direction (see Gentzkow et al., 2019).[6] Budak et al. (2016) recruit human subjects to classify political articles according to topic and ideological position. Cagé et al. (2019) use a topic-detection algorithm to identify the set of news stories in online media. Angelucci et al. (2020) use machine-learning techniques to identify changes in content production of local newspapers in the 1950ies. Most closely connected to our prediction, Nimark and Pitschner (2019) and Chahrour et al. (2019) use machine-learning techniques to document significant differences in the coverage of political and economic issues among media outlets.

The most substantive and novel testable predictions of our paper, however, concern the effects of increased competition. First, we predict that an increase in media competition should be associated with a decrease in the relative provision of information on the vertical dimensions, such as valence. Extensive empirical literature studies the effects of the introduction of the radio (e.g., Stromberg, 2004), the television (e.g., Gentzkow, 2006), and internet broadband (e.g., Campante et al., 2018; Falck et al., 2014; Gavazza et al., 2019; Miner, 2015) on political outcomes such as public spending and political participation. Gentzkow et al. (2011), Drago et al. (2014), and Cagé (2020) employ a more direct approach and study the impact of entry in the local newspaper industry. Of these papers, Cagé (2020) is the only one that can discern the effects of competition on the content supplied by firms. She documents a negative relationship between media competition and information quality, a vertical dimension. Moreover, the results suggest that increased competition affects the relative coverage of different issues. Second, we predict that increased specialization, resulting from media competition, generates higher social disagreement. Similar concerns have been raised in public discourse. For example, Sunstein (2001) has argued that convergence to a daily-me paradigm could lead individuals to isolate themselves from the larger public debate, making it harder for people to come together on common issues.Although the literature on political polarization and social media is growing (e.g., Campante and Hojman, 2013; Iyengar et al., 2019; Prior, 2013), the only implicit test for this prediction comes from Allcott et al. (2020). In a large-scale field experiment, they show that social media usage causes a significant increase in social disagreement, measured as polarization in the strength of political

[6] Groseclose and Milyo (2005), Baum and Groeling (2008), Ho et al. (2008), Gentzkow and Shapiro (2010), and Martin and Yurukoglu (2017) are among the first papers in this field to use these techniques.

preferences. Although this study does not explicitly control for increased competition, it is plausible that social media facilitates informational specialization.

2. Model

This section introduces the model and discusses its main assumptions. We model the interaction among a group of firms and agents: firms choose what information to produce about an uncertain policy and its price for each agent; Agents choose which firm to acquire information from and whether to approve the policy, thereby affecting its chances of being implemented.

We now formally introduce the components of the model. There are $N \geq 1$ identical firms and $I \geq 1$ heterogeneous and Bayesian agents. We denote a typical firm by n, a typical agent by i, and her payoff-type by θ_i. Firms and agents interact over three consecutive stages.

In the first stage, before observing the agents' types, firms compete to provide agents with information about an uncertain policy $\omega = (\omega_0, \omega_1, \omega_2)$, whose three components are identically distributed as independent standard normals. Specifically, firm n chooses an *editorial strategy* $b_n = (b_{n,0}, b_{n,1}, b_{n,2})$, subject to the constraint that $\|b_n\| \leq 1$.[7]

In the second stage, agents' types $(\theta_1, \ldots, \theta_I)$ and firms' editorial strategies are publicly observed. Firm n sets a price $p_n(\theta_i)$ for each agent.

In the last stage, each agent chooses at most one firm to acquire information from. If agent i chooses firm n, she pays the price $p_n(\theta_i)$ and privately observes a signal realization $s_i(\omega, b_n) = b_n \cdot \omega + \varepsilon_i$. The error term ε_i is independent across firms and agents and is distributed as a standard normal.

Finally, conditional on the observed signal, the agent approves or disapproves the policy. The policy is implemented with a probability equal to its approval rate, which is the fraction of agents who approved the policy. If the policy is implemented, agent i earns a payoff $u(\omega, \theta_i)$, which depends on the realized policy and her type. Otherwise, the status quo prevails, whose payoff for the agent we normalize to zero.

Payoffs. The agent's payoff $u(\omega, \theta_i)$ is meant to capture the impact of a policy ω that features both "vertical" and "horizontal" components. We assume that the agent's type is $\theta_i = (\theta_{i,0}, \theta_{i,1}, \theta_{i,2})$ and let $u(\omega, \theta_i) = \theta_i \cdot \omega$. Agents have identical preferences over ω_0—the "vertical" component of the policy. To this purpose, we set $\theta_{i,0} = 1$ for all i. By contrast, agents have heterogeneous preferences on the remaining "horizontal" components, ω_1 and ω_2. We conveniently set $\theta_{i,1}^2 + \theta_{i,2}^2 = 1$ and assume that, subject to this constraint, the agent's type θ_i is independently

[7] We denote by $\|\cdot\|$ the ℓ_2 norm of a vector.

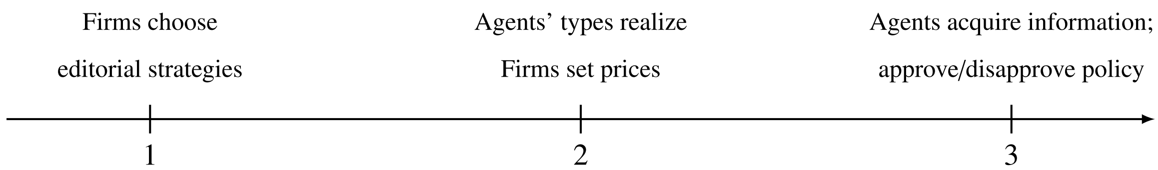

Figure 1: Timeline

drawn from the uniform distribution F.

Agents maximize their expected payoff, which depends on the implemented policy and the price paid for information. Firms maximize expected profits, which depend on the agents they serve and the price they pay. The solution concept is Perfect Bayesian Equilibrium, which we characterize in the next section.

2.1. Discussion

We pause for a brief discussion of our main assumptions. A central feature of our model is that it captures the equilibrium interactions between vertical and horizontal competition. To do so, we assume that agents' preferences are heterogeneous on a rich policy space and we impose constraints on the firms' supply. Both these ingredients are empirically plausible and, indeed, very common in the industrial-organization literature (e.g., Tirole, 1988). They are both critical to our results. Absent the former, all agents would demand the same information. Absent the latter, all firms would supply the same information—a fully revealing signal. While there are several different ways in which such features could be introduced, our modeling choices provide tractability and allow for a transparent depiction of the main forces behind our results.

On the agents' side, we introduce heterogeneity by assuming that θ_i is uniformly distributed on the *two* ideological components.[8] The symmetry of this setup is just a modeling tool that allows us to characterize the equilibrium of the game for an arbitrary number of firms. Moreover, it enables us to think of information provision as a location problem on a disk, in the spirit of Salop (1979). In Section 5.1, we relax the uniformity assumption and allow for a larger class of type distributions. On the firms' side, instead, we assume that firms are constrained in the resources—journalists, pages, airtime, etc.—they can allocate on the components of the policy

[8] Pew Research Center (2017) provides evidence on the differences in voters' agendas. Consistently, multidimensional preferences with similar characteristics are common in this literature. See for example Groseclose (2001), Eyster and Kittsteiner (2007), Carillo and Castanheira (2008), Ashworth and de Mesquita (2009), Stone and Simas (2010), Dragu and Fan (2016), Aragones et al. (2015), and Yuksel (2020) for applications in the context of party competition, and Alesina et al. (1999), Lizzeri and Persico (2005), and Fernandez and Levy (2008) for applications to public goods.

(see, Chan and Suen, 2008; Gentzkow et al., 2015; Strömberg, 2015). Each component of the editorial strategy b_n can be interpreted as the firm's emphasis on the corresponding component of the policy. More emphasis translates into a signal that is more informative about such component. This substitutability generates the tradeoff that is at the heart of our model. In Section 5.2, we study a model of multimedia in which agents can acquire information from multiple firms.

Two other assumptions in our model are worth further discussion. First, the finite number of agents guarantees that information has *instrumental* value. Agents acquire information because it allows them to better sway the policy outcome in the direction of their preferences. Second, we assume that the policy is implemented probabilistically, as a function of the approval rate (e.g., Banks and Duggan, 2004; Patty, 2007). This eliminates the scope for learning about the policy from pivotal reasoning and reduces the complexity of the agents' problem, thus enabling us to focus on the most novel aspect of the model—the competitive supply of information.

Finally, prices in our model do not necessarily need to represent monetary transfers from agents to firms. Alternatively, they can be interpreted as advertising revenues. In this interpretation, firms compete for the agents' attention, which increases with the value of the information they acquire but decreases with the intensity of advertisements they observe. As in Lederer and Hurter (1986) and Hamilton et al. (1991), we use spatial price discrimination to avoid well-known technical issues related to equilibrium existence when both prices and locations are chosen endogenously (see, D'Aspremont et al., 1979). While forms of price discrimination are common in the market for news, such an assumption is arguably even more reasonable when prices are interpreted as resulting from advertisement, which is increasingly targeted (Athey and Gans, 2010).

3. Equilibrium

This section analyzes the equilibrium of our game. First, we establish equilibrium existence by solving the game via backward induction. Second, we illustrate how to transform the firm's problem into an equivalent location problem on a disk. This is not only analytically convenient but also provides a useful spatial interpretation for firms' equilibrium behavior. Building on this spatial interpretation, we conclude by discussing the uniqueness of the equilibrium.

3.1. Existence and Characterization

3.1.1. The Agent's Problem: Information Acquisition and Approval

We begin by characterizing equilibrium behavior in the last stage of the game. In this stage, agents choose which information to acquire and, conditional on what they learn, whether to approve the policy. To this purpose, fix an agent's type θ_i and suppose that she acquires information from firm n, whose editorial strategy is b_n. Her equilibrium *approval strategy* is relatively straightforward, as it only depends on the realized signal and not on the strategies of other agents.

Lemma 1 (Approval)**.** *Conditional on a signal realization $\bar{s}_i = s(\omega, b_n)$, type θ_i approves the policy if and only if $\mathbb{E}_\omega(u(\omega, \theta_i)|\bar{s}_i) \geq 0$.*

The agent's approval strategy affects the outcome of the game, as it impacts the probability with which the policy is ultimately implemented. Nonetheless, her equilibrium behavior is simple and abstracts from pivotal reasoning. Specifically, she computes the expectation of $u(\omega, \theta_i)$ conditional on $\bar{s}_i$ and approves the policy if and only if it leads to a payoff that is higher than the status quo.[9] That is, the agent behaves as if she were voting expressively (Brennan and Buchanan, 1984). Lemma 1 follows from the fact that the policy is implemented probabilistically, as a function of the approval rate. Because of this, each agent impacts the policy outcome equally—changing the implementation probability by $1/I$—regardless of the decisions of others. This eliminates the scope for learning about the policy by engaging in pivotal reasoning.

Next, we characterize the *information acquisition strategy* of the agent. Since information allows the agent to better sway the policy outcome in the direction of her preferences, she attaches an instrumental value to the information she acquires. Abstracting from prices, this *value of information* is defined as the difference between her expected equilibrium payoff associated with observing a signal from a firm n and the one associated with observing no signal whatsoever. Characterizing this value is a key step for the equilibrium analysis of the game.

Lemma 2 (Value of Information)**.** *The value of firm n's information for an agent of type θ_i is*

$$v(b_n|\theta_i) = \frac{|\theta_i \cdot b_n|}{I\sqrt{2\pi(1 + ||b_n||^2)}}.$$

Lemma 2 computes the value of information from firm n for type θ_i. It establishes that such value is independent of the information acquired by other agents and, hence, the editorial strategies of firms other than n. The value of information has several intuitive properties. First, it is decreasing in the number of agents I. This captures the fact that the larger a society is, the smaller

[9] We assume, without loss of generality, that the agent approves the policy when she is indifferent.

the marginal impact that an agent has on the policy outcome, thus decreasing the instrumental value that such an agent assigns to information. Second, the value of information is increasing in $|\theta_i \cdot b_n|$, which corresponds to the statistical correlation between the agent's payoff $u(\omega, \theta_i)$ and the signal $s_i(\omega, b_n)$.[10] We will return to this point shortly in Section 3.2.

In equilibrium, type θ_i chooses the firm that provides the highest value of information net of its price. More formally, given a profile of editorial strategies and prices, $(b_n, p_n(\theta_i))_{n=1}^N$, type θ_i acquires information from firm n only if $v(b_n|\theta_i) - p_n(\theta_i) \geq v(b_m|\theta_i) - p_m(\theta_i)$, for all m.

3.1.2. The Firm's Problem: Prices and Editorial Strategies

We now turn to the analysis of firms' equilibrium behavior. We begin with the second stage of the game, where firms set prices after observing each other's editorial strategies $b = (b_n)_{n=1}^N$ and the agents' types $(\theta_1, \ldots, \theta_I)$. Each firm maximizes profits, which are affected by the prices it charges to its readers. Firms set a price for each type θ_i and, hence, compete *à la* Bertrand for each potential reader (see, Lederer and Hurter, 1986).

To provide intuition on what prices prevail in equilibrium, let us consider a simple example in which two firms compete for type θ_i. Suppose that their editorial strategies b_1 and b_2, chosen in the first stage, are such that $v(b_1|\theta_i) > v(b_2|\theta_i)$. The most competitive price that firm 2 can set is $p_2(\theta_i) = 0$. Even in this case, firm 1 can nonetheless "win" the agent by simply setting a price $p_1(\theta_i) < v(b_1|\theta_i) - v(b_2|\theta_i)$. Therefore, in equilibrium, firm 1 must win the agent and earn a profit equal to $v(b_1|\theta_i) - v(b_2|\theta_i)$.

This reasoning easily generalizes to $N > 2$. Fix a profile of editorial strategies $b = (b_n)_{n=1}^N$. In equilibrium, firm n wins type θ_i only if $v(b_n|\theta_i) \geq \max_{m \neq n} v(b_m|\theta_i)$, in which case she earns a profit of $v(b_n|\theta_i) - \max_{m \neq n} v(b_m|\theta_i) \geq 0$. Conversely, if $v(b_n|\theta_i) < \max_{m \neq n} v(b_m|\theta_i)$, the firm loses type θ_i and earns no profit. Conveniently, the equilibrium profit that firm n accrues from type θ_i is uniquely pinned down by $\max_m v(b_m|\theta_i) - \max_{m \neq n} v(b_m|\theta_i) \geq 0$.[11] Therefore, firm n's total profit is $\sum_{i=1}^I \max_m v(b_m|\theta_i) - \max_{m \neq n} v(b_m|\theta_i)$, which depends only on the profile of editorial strategies b.

Finally, we analyze the first stage of the game. Firms choose their editorial strategies before observing the agents' types, which are identically and independently distributed according to distribution F. Given the analysis above, for any profile of editorial strategies (b_n, b_{-n}), firm n's expected profit is

$$\Pi_n(b_n, b_{-n}) = I \mathbb{E}_{\theta_i}\Big(\max_m v(b_m|\theta_i) - \max_{m \neq n} v(b_m|\theta_i)\Big). \tag{1}$$

In the first stage of the game, firms play a one-shot complete-information game with payoffs

[10] Furthermore, note that $\mathbb{V}(\epsilon_i) = 1$ and $||b_n|| \leq 1$ are normalizations.

[11] This formula is valid even $N = 1$. In that case, $\{m \neq 1\} = \emptyset$ and we define $\max_{m \neq 1} v(b_m|\theta_i) = 0$.

defined by Π_n. The next result establishes the existence of a pure-strategy Nash equilibrium in this game.

Theorem 1 (Existence)**.** *A pure-strategy equilibrium $(b_n)_{n=1}^N$ exists.*

By construction, the profit function Π_n incorporates the equilibrium behavior of firms and agents in the subsequent stages of the game. By backward induction, a Nash Equilibrium of the first-stage game corresponds to a PBE in the grand game, and the equilibrium strategies for the subsequent stages, both on and off the equilibrium path, are defined as described above. Conversely, any PBE of the grand game must induce a Nash Equilibrium in the first-stage game.

3.2. Information Provision as a Location Problem

This subsection discusses the equilibrium and its properties. To this purpose, we transform the firm's problem in the first stage of the game—which consists of choosing an editorial strategy—into an equivalent location problem on a disk. This transformation is both conceptually and analytically convenient, as it facilitates the interpretation of the equilibrium and allows us to characterize its uniqueness. We do so by transforming the agent's type θ_i and the firm's editorial strategy b_n into polar coordinates.

Remark 1. *Let $T = [-\pi, \pi]$.*

- *For all θ_i, a unique $t_i \in T$ exists such that $\theta_i = (1, \cos(t_i), \sin(t_i))$.*
- *For all b_n such that $\|b_n\| = 1$, a unique pair $(x_n, t_n) \in [0, 1] \times T$ exists such that $b_n = (\sqrt{x_n}, \sqrt{1 - x_n}\cos(t_n), \sqrt{1 - x_n}\sin(t_n)))$.*

In light of the equivalence of Remark 1, we abuse terminology and notation in the remainder of the paper and refer to t_i as the agent's *type* and to the pair (x_n, t_n) as the firm's *editorial strategy*.[12]

Interpretation. In this equivalent formulation of the model, each agent's type t_i is a *location* on a circle and it is drawn uniformly from the set T. The closer two types t_i and t_j are to each other, the higher the correlation in their preferences for the policy, namely $u(\omega, t_i)$ and $u(\omega, t_j)$. In this sense, the arc distance between any two types on the circle represents their *ideological* distance.[13]

A firm's editorial strategy, in this formulation, is equivalent to choosing the pair (x_n, t_n), which

[12]Note that in Remark 1, we focus on editorial strategies for which the constraint $\|b_n\| \leq 1$ binds. These are the only strategies that firms use in equilibrium as shown in Lemma A1 (Appendix A).

[13]When this correlation is high, for example, if one agent benefits from a policy, the other is likely to benefit as well. In this sense, they are ideologically similar. A large empirical literature measures polarization using the bliss-point distance as a proxy for ideological distance (Downs, 1957). Our model adds to this literature by showing that two agents can be ideologically different even when their respective "bliss points" are the same. This happens when they trade off the components of the policy in different ways.

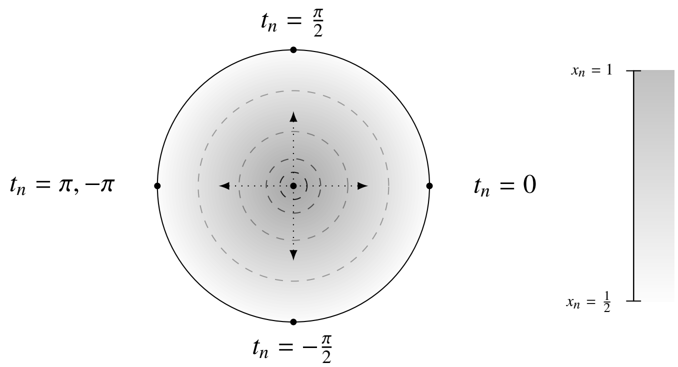

FIGURE 2: Mapping the firm's problem into a location choice

has the following interpretation: $x_n \in [0, 1]$ captures how *generalist* the firm is, as it measures the relative informativeness of the firm's signal about the valence versus the ideological components; $t_n \in T$ is the firm's *target* type, who evaluates the different ideological components ω_1 and ω_2 of the policy in a way that perfectly matches the corresponding relative weights in the signal designed by the firm.

Graphically, each editorial strategy (x_n, t_n) corresponds to a location on a disk, as illustrated in Figure 2. In contrast to the familiar Salop (1979) model of product differentiation, firms can locate in the interior of the disk. For example, a firm could locate at the center of the disk by setting $x_n = 1$, which corresponds to choosing a maximally generalist editorial strategy. Such a firm would offer a signal that is informative only about the valence component. When $x_n < 1$, the firm specializes by locating away from the center, in the direction indicated by t_n. Such a firm would be offering a signal that is also informative about the two ideological components, which are weighted in a way that is perfectly aligned with the ideological preference of type t_n.

The Value of Information, Revisited. The transformation into polar coordinates also simplifies the expression of the value of information derived in Lemma 2. We can show that it is strictly dominated for firm n to choose an editorial strategy (x_n, t_n) such that $x_n < 1/2$, irrespective of t_n. Therefore, without loss of generality, we restrict attention to editorial strategies that satisfy $x_n \geq 1/2$. In light of this, the value of information can be written as:

$$v((x_n, t_n) \mid t_i) = \lambda\Big(\sqrt{x_n} + \sqrt{1 - x_n}\cos(t_i - t_n)\Big), \tag{2}$$

where $\lambda = \frac{1}{2I\sqrt{\pi}}$.[14] This expression is not only more tractable than that of Lemma 2, but it is also easier to interpret. Net of the scaling factor λ, the value of information is the sum of two terms. The first term, $\sqrt{x_n}$, refers to the valence component of the policy. This term is independent of the agent's type t_i and is increasing in x_n—how generalist the firm is. Intuitively, since all agents care

[14] These claims are shown in Lemma A2.

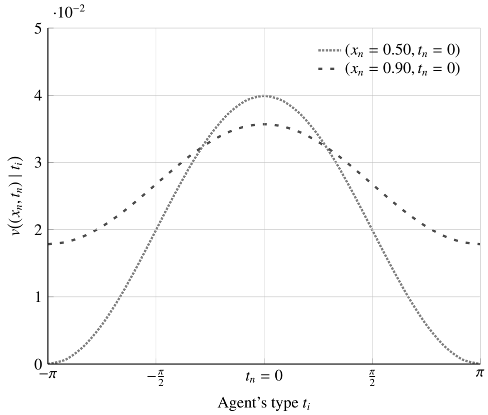

FIGURE 3: The Value of Information Induced by Two Editorial Strategies (if $I = 10$)

about the valence component, a signal that is more informative about it (higher x_n) will benefit *all* agents, irrespective of their types. The second term, $\sqrt{1 - x_n} \cos(t_i - t_n)$, refers to the ideological components of the policy. This term is decreasing in x_n and depends on the agent's type t_i as well as the firm's target t_n. The lower x_n, the more specialized the firm's editorial strategy is and the more informative it is about a specific mixture of ideological issues. This mixture is determined by t_n and its value depends on $\cos(t_i - t_n)$, which represents the correlation in how agent t_i and target t_n evaluate the ideological dimensions of the policy. Intuitively, the closer the agent is to the firm's target, the higher the value she attaches to its information.

Equation 2 clarifies the tradeoff that firms face when choosing their editorial strategies. By being more generalist, a firm generates higher values even for types that are far away from its chosen target. By being more specialized, instead, the firm generates higher value only for types who are ideologically close to its target. Figure 3 illustrates this tradeoff by considering two strategies, both of which target type $t_n = 0$. The dotted gray line has a low x_n and, hence, it is highly specialized. It creates high value for the targeted type and the types nearby, but it creates a low value for agents that are farther away. The dashed dark line has a high x_n and, hence, it is more generalist. The value it induces is relatively flatter. By being informative about the valence component, it generates value for all agents, even those who are ideologically distant from the targeted agent t_n.

What is the *first-best* editorial strategy (x_n, t_n) for an agent of type t_i? From Equation 2, it is easy to see that firm n maximizes the value of information for type t_i by directly targeting this type—that is, $t_n = t_i$—and assigning equal weight to valence and ideology—that is, $x_n = 1/2$. By doing so, the firm induces a signal that is maximally correlated (given the firm's constraints) with this type's payoff $u(\omega, t_i)$. Let us denote such first-best value by $\bar{\mathcal{V}} = v((1/2, t_i)|t_i)$ and observe that it is independent of t_i. Note that any editorial strategy with $x_n < 1/2$ would be overly specialized on ideology, even for the targeted type t_n. For this reason, in Figure 2, $x_n = 1/2$ corresponds to the outer border of the disk: we can think of agents as lying on this border, as each one of its points represents the optimal editorial strategy for some type of agent.

Equilibrium Uniqueness. Finally, the transformation into polar coordinates discussed in this section is convenient for characterizing the equilibrium of the game, as we do in the next result.

Theorem 2 (Uniqueness)**.** *Fix $N \geq 1$. There is a unique $x^\star(N) \in [1/2, 1]$ such that, for all equilibria $(x_n, t_n)_{n=1}^N$, $x_n = x^\star(N)$ and $|t_n - t_m| \geq 2\pi/N$, for all firms n and m.*

In equilibrium, all firms are equally specialized and the degree of specialization, $1 - x^\star(N)$, is uniquely pinned down by N. Graphically, this means that firms locate equidistantly from the center of the disk. Moreover, firms' editorial strategies satisfy $|t_n - t_m| \geq 2\pi/N$ for all n and m.[15] Graphically, this means that firms are evenly spread out on an inner circles of the disk (e.g., the dashed circles in Figure 2). Clearly, due to the symmetry in the first-stage game and the uniformity of the type distribution, any relabeling of firms' names or rotation in their locations also constitutes an equilibrium. Nonetheless, this multiplicity is not important for our main results. From an ex-ante perspective, that is, before agents' types realize, all economically relevant outcomes are uniquely pinned down by $x^\star(N)$ in equilibrium—for example, market share, profits, value of information, and the agent's welfare.

4. Competition, Disagreement, and Welfare

We exploit the convenient equilibrium characterization discussed in Section 3 to analyze how firms' and agents' equilibrium behavior change as the market for news becomes more competitive. We divide our analysis into four parts. We study how an increase in competition affects: (1) the kind of information that firms supply in equilibrium; (2) the value and the price of information; (3) the distribution of agents' opinions; and (4) the welfare of the agents.

4.1. Competition and the Supply of Information

We begin by characterizing the effects of competition on *firms*' equilibrium behavior and, in particular, on the information they supply. We study the effects of competition by comparing equilibria as the number of firms in the market increases. We show that, as the market becomes more competitive, a firm's optimal response is to specialize. Importantly, this "informational" specialization takes a particular form: firms specialize by providing relatively less information

[15] We write $t_n + t_m$ (resp. $t_n - t_m$) to indicate the modular addition (resp. subtraction) on the circle T. For example, if $t_n = \pi/2$ and $t_m = 3\pi/2$, $t_n + t_m = 0 \in T$.

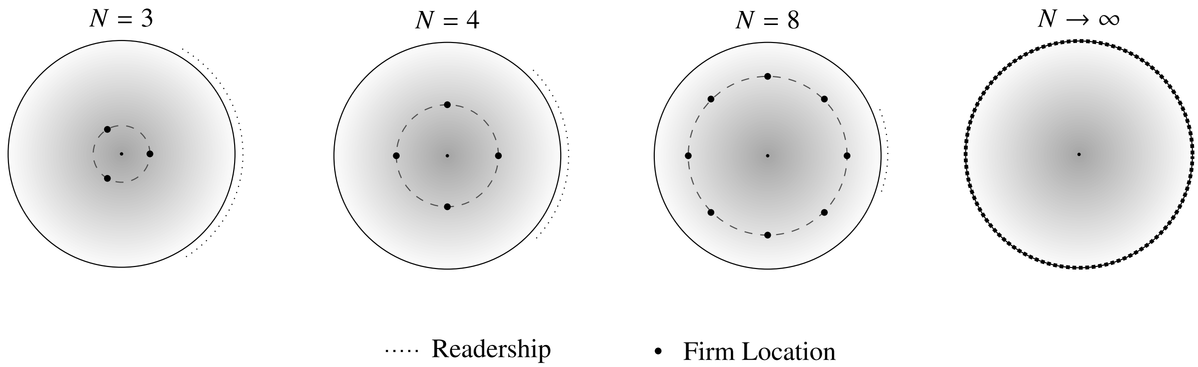

FIGURE 4: Equilibrium in the Information-Provision Stage

on the valence component, which is the common-interest component in agents' preferences, and relatively more information on the ideological components.

Proposition 1. *The equilibrium $x^\star(N)$ is strictly decreasing in N. That is, as competition increases, firms specialize by becoming less informative about the valence component of the policy.*

As the market for news becomes more competitive, it becomes increasingly harder for each firm to compete for types that are farther from its target. Indeed, in equilibrium, the firm's expected readership is an arc of length $2\pi/N$ centered around the firm's target type t_n. As N increases, the firm's readership shrinks and, thus, it becomes increasingly homogeneous from an ideological point of view. Expecting to face a more homogeneous set of readers, the firm reacts by further specializing—x_n decreases—and thus provides relatively more information on the ideological components of the policy. Graphically, as N increases, firms locate farther from the center of the disk, as Figure 4 illustrates.

The equilibrium mechanism underlying Proposition 1 can be understood as an information-theoretic counterpart to the more standard idea of product differentiation. Differentiation is a ubiquitous feature of competition games with heterogeneous consumers. However, how do firms differentiate when they sell information? Our result shows that they achieve this by increasing the relative informativeness of private-interest components at the expense of common-interest ones. More broadly, our model captures the equilibrium interactions between vertical and horizontal competition, allowing us to highlight an important effect that has implications beyond the political economy setting studied in this paper. As competition increases, firms disinvest from vertical features—which are beneficial to all consumers—and instead focus on horizontal features—which are beneficial only to a niche segment.

4.2. Competition and the Value of Information

The previous section illustrated how the equilibrium supply of information changes as competition increases. What are the consequences of this change on the agents? In this section, we highlight the positive effects. We focus attention on two main equilibrium objects: the value of information and its price.

We begin with the *ex-ante* perspective of an agent whose type is yet to realize. More precisely, fix an arbitrary equilibrium with N firms and, in such equilibrium, let $n(t_i)$ denote the firm from which t_i acquires information. Given this, let $\mathcal{V}(N) = \mathbb{E}_{t_i}(v((x^\star(N), t^\star_{n(t_i)})|t_i))$ be the expected value for the information that agent i acquires in equilibrium. Similarly, let $\mathcal{P}(N) = \mathbb{E}_{t_i}(p^\star_{n(t_i)}(t_i))$ be its expected price. Note that $\mathcal{V}(N)$ and $\mathcal{P}(N)$ are uniquely pinned down as a function of N—via $x^\star(N)$—and thus do not depend on other features of the equilibrium. We establish the following results.[16]

Proposition 2.

(*a*) *$\mathcal{V}$ is strictly increasing in N. That is, as competition increases, each agent expects to acquire information that is more valuable to her.*

(*b*) *$\mathcal{P}$ is strictly decreasing in N. That is, as competition increases, each agent expects to pay less for the information she acquires.*

The first part in this result speaks to the classic view that sees the market for news as a "marketplace of ideas," which promotes knowledge and the discovery of truth (Posner, 1986). Competition pushes firms to provide information that is increasingly catered to the specific informational needs of each agent. With such information, each agent can better sway the policy outcome in the direction of her own preferences and, for this reason, she attaches a higher value to it. Furthermore, while each agent obtains better information from the market, she expects to pay a lower price, as established in the second part of the Proposition 2. As a consequence, industry profits decline.

When the number of firms tends to infinity, the market becomes perfectly competitive. In this limit, we show that, conditional on her type, each agent acquires her first-best signal, thus achieving the highest possible value $\bar{\mathcal{V}}$. Moreover, she pays a price of zero for it. More precisely, fix t_i and an arbitrary equilibrium with N firms. Let $\mathcal{V}(N|t_i) = v((x^\star(N), t^\star_{n(t_i)})|t_i)$ be the value for the information that type t_i acquires in equilibrium. Similarly, let $\mathcal{P}(N|t_i) = p^\star_{n(t_i)}(t_i)$ be its equilibrium price. While $\mathcal{V}(N|t_i)$ and $\mathcal{P}(N|t_i)$ depend on the specific equilibrium that we fixed,

[16] Conceptually similar results hold when we condition on a specific type, as demonstrated by Remark B3. Taking an ex-ante perspective allows us to abstract away from equilibrium multiplicity due to rotation in firms' locations.

their respective limits do not.

Remark 2 (Daily Me)**.** *Fix a type t_i. Type t_i's equilibrium value of information converges to the first best,* $\lim_{N\to\infty} \mathcal{V}(N|t_i) = \bar{\mathcal{V}}$. *Moreover, type t_i's equilibrium price converges to zero,* $\lim_{N\to\infty} \mathcal{P}(N|t_i) = 0$.

We refer to this limit result as the Daily Me paradigm, a situation in which every consumer in a perfectly competitive market can find an information structure that is exactly tailored to her specific informational needs (Sunstein, 2001). That is, as the market becomes more competitive, the equilibrium value of information converges to the highest possible value for each agent, while its price converges to zero.[17] Graphically, as $N \to \infty$, firms occupy the whole circumference of the disk, where each point on this circumference represents the optimal editorial strategy for some type of agent (Figure 4).

These results show that the equilibrium force that pushes firms to specialize is, indeed, *demand*-driven. As the number of firms grows, each firm serves a progressively smaller set of agents and provides them with an information structure that is increasingly better suited to their specific needs, thus increasing their value of information. Moreover, competition does not lead to *over*-specialization. As competition increases, so does the value agents attach to the information they acquire. Incidentally, this explains why no firm has an incentive to deviate back toward the center of the disk by choosing a generalist editorial strategy: since agents consider the signal they acquire in equilibrium to be under specialized relative to their first-best, an editorial strategy that is highly generalist cannot be enticing for them.

4.3. Competition and Social Disagreement

We established that a more competitive market enables agents to learn more effectively about the components of the policy they care about at lower prices. While this is intuitively good at the individual level, it has social repercussions. Indeed, information is not a standard product: its private consumption generates social externalities that arise because of the policy-approval stage, which aggregates agents' preferences. In this section, we begin exploring the effects of competition in light of such externalities. The most apparent one, perhaps, results from agents becoming more informed about increasingly different aspects of the policy—the different mixtures of ω_1 and ω_2—at the expense of the valence component ω_0. Consequently, agents' opinions on the policy become increasingly uncorrelated, thus increasing social disagreement.

More precisely, fix an arbitrary equilibrium with N firms and suppose that, in such equilib-

[17] Note that $\mathcal{V}(N|t_i)$ and $\mathcal{P}(N|t_i)$ are the interim versions of $\mathcal{V}(N)$ and $\mathcal{P}(N)$, respectively. Therefore, in light of Remark 2, Proposition 2 implies that $\lim_{N\to\infty} \mathcal{V}(N) = \bar{\mathcal{V}}$ and $\lim_{N\to\infty} \mathcal{P}(N) = 0$.

rium, type t_i acquires information from firm n, thus observing the realization of signal $s_i^\star(\omega) = s(\omega, (x^\star(N), t_n^\star))$. Conditional on such a signal, let $z_i(t_i) = \mathbb{E}_\omega(u(\omega, t_i)|s_i^\star(\omega))$ be the expected payoff that type t_i associates with the implementation of the policy. We refer to $z_i(t_i)$ as type t_i's equilibrium *opinion* about the policy. As shown in Lemma 1, such a type approves the policy if she has a positive opinion about the policy and disapproves otherwise. A society in which agents' opinions are highly correlated is a society in which agreement is high. Thus, we define *social agreement* as the expected correlation in the opinions of two agents, i and j, denoted $\mathcal{S}(N) = \mathbb{E}_{t_i,t_j}(\text{Corr}(z_i(t_i), z_j(t_j)))$. Intuitively, a society features high social agreement if it is relatively common to find agents whose opinions about the policy are highly correlated.

Proposition 3. *$\mathcal{S}$ is strictly decreasing in N. That is, social agreement decreases with competition.*

The intuition for this result is simple and it is best conveyed by looking at an extreme example. Consider agents i and j with $t_i = 0$ and $t_j = \pi/2$: the former cares about ω_1, while the latter cares about ω_2. When competition is low, equilibrium editorial strategies are more generalist—$x^\star(N)$ is high. That is, even if these two agents acquire information from different firms, their signals are highly informative about the common valence component ω_0, about which they both care. Consequently, their opinions $z_i(t_i)$ and $z_j(t_j)$ are highly correlated. When N grows large, $x^\star(N)$ decreases, and both agents can find information that is increasingly tailored to their specific needs. In particular, agent i can learn relatively more about ω_1, while agent j can learn relatively more about ω_2. As a consequence, their opinions depend relatively less on the common component ω_0, and relatively more on ω_1 and ω_2, which are independent aspects of the policy. Hence, their opinions become less correlated.

Proposition 3 summarizes an important aspect of the equilibrium mechanism. It is perhaps unsurprising to see that agents are more likely to disagree, provided that they receive more information about ω_1 and ω_2. The subtlety is that there is, in principle, a multitude of ways in which competition could affect the supply of information. Our model demonstrates that, due to the natural interplay between agents' incentives to learn and firms' incentives to maximize profits, competition pushes firms to provide relatively more information precisely about those dimensions on which agents disagree more. This gives rise to a social inefficiency that we document in the next section.

4.4. Competition and its Welfare Consequences

In this section, we conclude our analysis of the effects of competition by studying how increased disagreement ultimately affects agents' welfare. Our main shows that, in large enough societies, competition strictly decreases the expected welfare of the agents. To this purpose, fix an equilibrium of the game with N firms. Denote by $a_i^\star(\omega, t_i)$ the approval decision of type t_i conditional on the information that she receives in equilibrium. This random variable takes a value of 1 if the agent approves the policy, and zero otherwise. The equilibrium approval rate is then $A^\star(\omega, t) = \frac{1}{I}\sum_i a_i^\star(\omega, t_i)$, namely, the fraction of agents who approve the policy. By assumption, this also corresponds to the probability that the society implements policy ω. Using this, the expected welfare of an agent is $\mathcal{U}(N) = \mathbb{E}_{\omega,t}(A^\star(\omega, t)u(\omega, t_i) - p^\star(t_i))$. This expression captures *both* the utility that the agent expects to receive from the implemented policy and the disutility associated with the price that she expects to pay for the information she will acquire in equilibrium. The following result characterizes the effects of competition on the agent's welfare.

Proposition 4. *There exists $\bar{I}$ such that, for all societies with $I > \bar{I}$, $\mathcal{U}$ is strictly decreasing in N. That is, as competition increases, an agent's expected welfare decreases.*

Competition has an overall negative effect on an agent's welfare, despite the positive effects previously highlighted in Proposition 2. To provide intuition for this result, it is useful to decompose the agent i's welfare $\mathcal{U}(N)$ and consider separately agent i's own impact on the policy outcome and the impact of all other agents. Specifically, we have

$$\mathcal{U}(N) = \mathcal{V}(N) + \mathcal{G}(N) - \mathcal{P}(N).^{18} \tag{3}$$

The first term $\mathcal{V}(N)$ is the expected value of information, which we introduced in Section 4.2. This can equivalently be rewritten as $\frac{1}{I}\mathbb{E}_{\omega,t_i}(a_i^\star(\omega, t_i)u(\omega, t_i))$, which is the impact of agent i's *own* approval decision on her utility. The second term is $\mathcal{G}(N) = \frac{1}{I}\mathbb{E}_{\omega,t}(\sum_{j\neq i} a_j^\star(\omega, t_j)u(\omega, t_i))$, which is the impact that *others'* approval decisions have on agent i's utility. The last term, $\mathcal{P}(N)$, is also familiar from Section 4.2 and denotes the expected price that agent i pays for the information she acquires.

This decomposition reveals the key features of the equilibrium mechanism highlighted in this paper. Information has both direct and indirect effects on an agent's welfare. The direct effect is captured by $\mathcal{V}(N)$, which measures how an agent values the information that *she* personally acquires to sway the policy outcome in the direction of her own preferences. The indirect effect is captured by $\mathcal{G}(N)$, which measures how an agent values the information that *other agents* acquire to sway the policy outcome in the direction of their preferences. All agents try to maximize their own impact on the political process and, thus, acquire information based on its direct value. The

[18] This decomposition follows from the definition of welfare $\mathcal{U}(N) = \mathbb{E}_{\omega,t}(A^\star(\omega, t)u(\omega, t_i) - p^\star(t_i))$ and the fact that the approval rate $A^\star(\omega, t)$ can be written as the sum of i's approval decision, $\frac{1}{I}a_i^\star(\omega, t_i)$, and those of all the other agents, $\frac{1}{I}\sum_{j\neq i} a_j^\star(\omega, t_j)$. See the proof of Proposition 4 for more details.

profit-seeking firms specialize to meet such demand. From this perspective, it is not surprising that $\mathcal{V}(N)$ is strictly increasing in N, as shown in Proposition 2. However, as firms specialize, agents learn about increasingly different aspects of the policy, leading to an increase in social disagreement (Proposition 3). As a consequence, the information that other agents acquire becomes increasingly less valuable to agent i, as their approval decisions are less likely to benefit her. This decreases $\mathcal{G}(N)$—the indirect value of information—captures the externality that agents impose on each other.

When the society is large enough, specifically when $I \geq 3$, the overall effect is negative and $\mathcal{V}(N)+\mathcal{G}(N)$ decreases.[19] That is, as competition increases, the total—direct and indirect—value of the information supplied by the market decreases. It is not surprising that this overall effect depends on the size of the society. In larger societies, agent i's own approval decision is less consequential for the final outcome relative to the approval decisions of others. Thus, the increase in $\mathcal{V}$ cannot compensate for the decline in $\mathcal{G}$ resulting from competition. More importantly, the reader may wonder if the negative effect highlighted above could be compensated for by the fact that competition also lower prices. Proposition 4 shows that when I is sufficiently large,[20] the decrease in prices is unable to compensate for the loss of utility generated by the informational externality.

In conclusion, Proposition 4 highlights how competition in the market for political news can have very different consequences than in other, more traditional markets. Our model illustrates how political information differs from other types of products. Political information has value because it allows agents to influence electoral outcomes in a way that aligns with their own personal preferences. However, by definition, electoral outcomes represent collective decisions that have consequences for all members of the society. This implies that individual information-acquisition strategies have social externalities on others, which are exacerbated by the increase in competition.

Complete-Information Benchmark. We conclude this section by highlighting a final result that further illustrates the inefficiency captured by Proposition 4. To do so, we focus on two special sets of policies ω, for which either $u(\omega, t_i) > 0$ for all t_i or $u(\omega, t_i) < 0$ for all t_i. Let us denote them by Ω^+ and Ω^-, respectively. The policies in these sets are special in that, if the society could perfectly learn ω, agents would unanimously agree on its approval, if $\omega \in \Omega^+$, or disapproval, if $\omega \in \Omega^-$. Recall that $A^\star(\omega, t)$ is the equilibrium approval rate conditional on policy ω and the profile of agents' types t. It also corresponds to the probability that the society implements policy

[19]This is shown in Corollary 1. In passing, note that $\mathcal{V}(N) + \mathcal{G}(N)$ corresponds to *social* welfare (i.e., agents and firms). This is because prices simply transfer resources from agents to firms.

[20]Specifically, when I is larger than $\bar{I} = 3(1 + 2\pi)$.

ω. Hence, when $\omega \in \Omega^+$, the policy is "correctly" implemented with probability $A^\star(\omega)$; equivalently, when $\omega \in \Omega^-$, the policy is "correctly" implemented with probability $1 - A^\star(\omega)$. From an ex-ante perspective, the probability the society correctly implements policies in Ω^+ (resp. Ω^-) is given by $\mathbb{E}_\omega(A^\star(\omega, t)|\omega \in \Omega^+)$ (resp. $\mathbb{E}_\omega(1 - A^\star(\omega, t)|\omega \in \Omega^-)$). The next result shows that these terms are decreasing in N, irrespective of the equilibrium that is played by firms and agents.

Remark 3. *The probability that a policy in Ω^+ or Ω^- is correctly implemented by the society is strictly decreasing in N.*

This result allows us to formalize the following idea. In our model, ignorance is not bliss. Rather, there is plenty of scope for information to play a positive role. For example, it could allow agents to identify policies that are uncontroversially good for them. However, the market does not provide such information to the agents. On the contrary, as competition increases, the society is less likely to correctly implement even this class of policies about which there would be full agreement under the complete-information benchmark. This result points to the pervasiveness of the inefficiency in the policy selection that is highlighted by the mechanism of this paper.

5. Extensions

In this section, we discuss the robustness of our main results to some of the simplifying assumptions of our model. This exercise allows us to better appreciate the role of two key ingredients: the heterogeneity in agents' preferences and the constraints on how much they can learn about the policy.

5.1. Preference Heterogeneity

In our baseline model, we assumed that agents' types are uniformly distributed on the circle, an ubiquitous assumption in the industrial-organization literature on spatial competition (see Section 1.1). This endows the model with symmetry: firms spread out evenly and this allows us to pin down the equilibrium behavior for *all* levels of competition, irrespective of the number of firms N. Thanks to this, we can clearly demonstrate the mechanism that leads firms to change their editorial strategies as N increases and how such a change affects the value of information, disagreement, and social welfare. In this section, we drop this distributional assumption and consider a more general class of distributions over the agents' types. These distributions are symmetric around some "median" type t^m, and their density is bounded away from zero.

Definition 1. *The distribution F is* regular *if its density satisfies the following properties: (1) a* *type $t^m \in T$ exists such that for any $\delta > 0$, $f(t^m + \delta) = f(t^m - \delta)$; (2) there exists a $C > 0$ such that $f(t_i) > C > 0$ for all t_i.*

Regular distributions allow for a richer kind of heterogeneity in agents' ideological preferences. In doing so, we go beyond the stark distinction between valence and ideology in our baseline model. For example, a regular distribution F could have a "political center," with most of its mass around type t^m, thus restoring the familiar right-left interpretation of ideology that is not possible with the uniform distribution. Unfortunately, moving beyond the uniform distribution means losing tractability: it is no longer feasible to solve for the equilibrium at *all* N. Nonetheless, we can show that the main insights of the paper still hold by comparing two notable cases: the monopoly case, where $N = 1$, and the perfect competition case, where $N \to \infty$.

Proposition 5. *Fix a regular distribution F.*

a. *(Existence) An equilibrium exists for all $N \geq 1$ and $I \geq 1$.*
b. *(Daily Me) Fix any t_i. As $N \to \infty$, the equilibrium value of information for type t_i, $\mathcal{V}(N|t_i)$, converges to the first-best value $\bar{\mathcal{V}}$.*
c. *(Inefficiency) There exists $\bar{I}$ such that, for all societies with $I > \bar{I}$ agents, the agent's welfare is higher in a monopoly than under perfect competition, i.e. $\mathcal{U}(1) > \lim_{N\to\infty} \mathcal{U}(N)$.*

There are three results. First, we establish the existence of an equilibrium with an arbitrary number of firms. Such an equilibrium involves possibly mixed editorial strategies in the first stage of the game. Second, we demonstrate that, as the market becomes perfectly competitive, every agent can acquire information that is perfectly tailored to her specific needs (the daily-me paradigm). As competition increases, profits decline and firms find it optimal to target even the least populated niches of the market. Third, we show that competition decreases agents' welfare relative to the monopoly benchmark. That is, the inefficiency highlighted by Proposition 4 remains present under this broader class of distributions.

This latter result clarifies the key role that preference heterogeneity plays in our model. Recall from the previous section that the agent's welfare $\mathcal{U}$ can be decomposed into three terms: the direct value of information $\mathcal{V}$, the indirect value of information $\mathcal{G}$, and its associated price $\mathcal{P}$. Just like in our baseline model, when the type distribution is regular, $\mathcal{V}$ converges to the daily-me value, while $\mathcal{P}$ converges to zero. What is perhaps more striking is that, even within this broader class of distributions, specialization by firms leads to a decline in the indirect value of information. That is, $\mathcal{G}$ declines when going from a monopoly to perfect competition.[21] More impor-

[21]The highest value for $\mathcal{G}$ is achieved when the signal induces a policy outcome that maximizes the utility of an arbitrary agent. The monopolist, due to the lack of competition, captures the whole market and thus, shares this same goal (see Remark B1).

tantly, the proof of Proposition 5 shows that this decline in $\mathcal{G}$ is more pronounced—hence, the overall inefficiency is higher—when the heterogeneity in agents' preferences is higher. Formally, the decline in $\mathcal{G}$ is a function of $\beta_F := \mathbb{E}_{t_i}(\cos(t_i - t^m)) \in [0, 1)$, which is the expected correlation between the ideological preferences of an arbitrary agent and the median type t^m. This statistic of F is a measure of the degree of homogeneity in the agents' preferences. When F is uniform, as it is in our baseline model, $\beta_F = 0$ and the society is maximally heterogeneous. In contrast, when F approaches a degenerate distribution centered around t^m, $\beta_F \to 1$ and the society is maximally homogeneous. We show that $\mathcal{G}(1) - \lim_N \mathcal{G}(N)$ decreases in β_F. This implies that the lower the value of β_F, the more heterogeneous society becomes and larger the welfare decline as we transition from the monopoly to perfect competition. From this perspective, we note that our baseline model constitutes a useful extreme benchmark, as it provides the most acute demonstration of the inefficiency associated with competition.

5.2. Multimedia

In our model, agents can acquire information from at most one firm (or "single homing"). To understand the implications of this assumption, it is useful to distinguish between two different ways in which competition can affect the market for news. On the one hand, competition could impact *what kind* of information agents acquire in equilibrium. For example, a more competitive market could allow agents to find information that is better tailored to their needs. This is the channel that we have emphasized in this paper. On the other hand, competition could also impact *how much* information agents acquire. For example, a more competitive market could cause agents to spend more time on the news. Multimedia consumption (or "multi-homing") could have implications for both these channels. We briefly sketch two extensions that address the two channels separately.

The first extension is faithful to the main exercise of the paper. In Appendix B.2, we let agents acquire information from multiple firms while maintaining the baseline assumption on how much they can learn. More specifically, we assume that each agent is endowed with a unit of time, which she can allocate among the N firms. The agent then observes a signal that is a mixture of the firms' editorial strategies, with weights determined by the agent's allocation. By dividing her time on different products, the agent can "construct" new signals that are not directly supplied by the market but are nonetheless better tailored to her own needs. To maintain tractability, we simplify the pricing stage by making a reduced-form assumption on how editorial strategies map into firms' profits. With this setup, we prove Proposition 6, which generalizes the results of Section 5.1. In particular, we show that a perfectly competitive market decreases agents' welfare relative to the monopoly benchmark.

The second extension relaxes the constraint on how much agents can learn. In Appendix C.1, we allow the precision of the signal received by an agent to exogenously increase with the number of firms in the market. This dependence could be the result of firms investing more in news production as competition intensifies, or it could be due to agents spending more time acquiring information from one or multiple sources. We do not mean to suggest that such changes are plausible;[22] our goal is merely to study the limits of our results in the presence of such changes. Proposition 7 in this appendix shows that the results of Section 5.1 can be extended to settings where competition leads to some increase in the signal's precision. However, our result reverses when the increase in precision is excessively large. For example, if agents become fully informed in the limit, the perfectly competitive market can make agents better off. This result is useful as it demonstrates once again that there is plenty of scope for information to play a positive role in our model (see also end of Section 4.4). In doing so, it highlights that the main inefficiency identified in our paper ultimately stems from the tradeoffs that firms and agents face, when choosing which aspects of the policy to emphasize and what kind of information to acquire, respectively. These tradeoffs imply that a competitive market leads to informational specialization, which results in agents becoming less informed about components of the policy that are of common interest.

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A. Proofs

A.1. Proofs for Section 3

A.1.1. Equilibrium Characterization

Proof of Lemma 1. Fix an arbitrary profile of editorial strategies $(b_1, \ldots, b_N)$ and types $(\theta_1, \ldots, \theta_I)$. Fix agent i and an arbitrary signal realization $\bar{s}_i \in \mathbb{R}$. Let $a_j(\omega, \theta_j)$ for $j \neq i$ be agent j's approval strategy. Denote by $A_{-i}(\omega, \theta_{-i}) = I^{-1} \sum_{j \neq i} a_j(\omega, \theta_j)$ the approval rate excluding i. If i approves, the policy is implemented with probability $A_{-i}(\omega, \theta_{-i}) + 1/I$. If i disapproves, instead, the policy is implemented with probability $A_{-i}(\omega, \theta_{-i})$. When the policy is implemented, the agent earns $u(\omega, \theta_i)$ and zero otherwise. Therefore, the value of agent i's problem is:

$$\begin{aligned} & \max\Big\{\mathbb{E}_\omega\Big(A_{-i}(\omega, \theta_{-i})u(\omega, \theta_i)\Big|s_i(\omega, b_n) = \bar{s}_i\Big), \mathbb{E}_\omega\Big((A_{-i}(\omega, \theta_{-i}) + 1/I)u(\omega, \theta_i)\Big|s_i(\omega, b_n) = \bar{s}_i\Big)\Big\} \\ = \; & \mathbb{E}_\omega\Big(A_{-i}(\omega, \theta_{-i})u(\omega, \theta_i)\Big|s_i(\omega, b_n) = \bar{s}_i\Big) + I^{-1} \max\Big\{0, \mathbb{E}_\omega\Big(u(\omega, \theta_i)\Big|s_i(\omega, b_n) = \bar{s}_i\Big)\Big\}, \end{aligned}$$

where the second line exploits the linearity of the operator $\mathbb{E}_\omega$. Therefore, the agent approves the policy if and only if $\mathbb{E}_\omega(u(\omega, \theta_i)|s_i(\omega, b_n) = \bar{s}_i) \geq 0$. □

Proof of Lemma 2. Fix agent i of type θ_i. Consider an arbitrary profile of approval strategies for agents other than i, $a_j(\omega, \theta_j)$ for $j \neq i$. Denote by $A_{-i}(\omega, \theta_{-i}) = I^{-1} \sum_{j \neq i} a_j(\omega, \theta_j)$ the resulting approval rate excluding i. First, we compute the expected utility if type θ_i does not receive any information:

$$\begin{aligned} & \max\Big\{\mathbb{E}_\omega\Big(A_{-i}(\omega, \theta_{-i})u(\omega, \theta_i)\Big), \mathbb{E}_\omega\Big((A_{-i}(\omega, \theta_{-i}) + 1/I)u(\omega, \theta_i)\Big)\Big\} \\ = \; & \mathbb{E}_\omega(A_{-i}(\omega, \theta_{-i})u(\omega, \theta_i)) + I^{-1} \max\{0, \mathbb{E}_\omega(u(\omega, \theta_i))\} \\ = \; & \mathbb{E}_\omega(A_{-i}(\omega, \theta_{-i})u(\omega, \theta_i)) \end{aligned}$$

The last equality holds because $\mathbb{E}_\omega(u(\omega, \theta_i)) = 0$, since $\mathbb{E}_\omega \omega_k = 0$ for $k \in \{0, 1, 2\}$.

Second, we compute type θ_i's expected utility when she observes the signal induced by b_n:

$$\begin{aligned} & \mathbb{E}_{\bar{s}_i}\Big(\max\Big\{\mathbb{E}_\omega\Big(A_{-i}(\omega, \theta_{-i})u(\omega, \theta_i)\Big|s(\omega, b_n) = \bar{s}_i\Big), \mathbb{E}_\omega\Big((A_{-i}(\omega, \theta_{-i}) + 1/I)u(\omega, \theta_i)\Big|s(\omega, b_n) = \bar{s}_i\Big)\Big\}\Big) \\ = \; & \mathbb{E}_{\bar{s}_i}(\mathbb{E}_\omega(A_{-i}(\omega, \theta_{-i})u(\omega, \theta_i)\big|s(\omega, b_n) = \bar{s}_i)) + I^{-1}\mathbb{E}_{\bar{s}_i}(\max\{0, \mathbb{E}_\omega(u(\omega, \theta_i)\big|s(\omega, b_n) = \bar{s}_i)\}) \end{aligned}$$

In the second line, let us separately analyze the two components of the sum. By the law of iterated expectations, the first component is

$$\mathbb{E}_{\bar{s}_i}(\mathbb{E}_\omega(A_{-i}(\omega,\theta_{-i})u(\omega,\theta_i)\big|s(\omega,b_n)=\bar{s}_i)) = \mathbb{E}_\omega(A_{-i}(\omega,\theta_{-i})u(\omega,\theta_i))$$

For the second component, let us note that $u(\omega,\theta_i)\sim\mathcal{N}(0,||\theta_i||^2)$ and $s_i(\omega,b_n)=b_n\cdot\omega+\varepsilon_i\sim\mathcal{N}(0,1+||b_n||^2)$. By the properties of conditional expectations under normal distributions, we have that:

$$\begin{aligned}\mathbb{E}_\omega(u(\omega,\theta_i)\big|s(\omega,b_n)=\bar{s}_i) &= \frac{\theta_i\cdot b_n}{||\theta_i||\sqrt{1+||b_n||^2}}\frac{||\theta_i||}{\sqrt{1+||b_n||^2}}\bar{s}_i \\ &= \frac{\theta_i\cdot b_n}{1+||b_n||^2}\bar{s}_i\sim\mathcal{N}\Big(0,\frac{(\theta_i\cdot b_n)^2}{1+||b_n||^2}\Big)\end{aligned} \tag{A1}$$

That is, the interim expectation $\mathbb{E}_\omega(u(\omega,\theta_i)\big|s(\omega,b_n)=\bar{s}_i)$ is itself a random variable that is normally distributed. Therefore,

$$\begin{aligned}\mathbb{E}_{\bar{s}_i}(\max\{0,\mathbb{E}_\omega(u(\omega,\theta_i)|s(\omega,b_n)=\bar{s}_i)\}) &= \tfrac{1}{2}\mathbb{E}_{\bar{s}_i}\Big(\big|\mathbb{E}_\omega(u(\omega,\theta_i)|s(\omega,b_n)=\bar{s}_i)\big|\Big) \\ &= \tfrac{1}{2}\tfrac{\sqrt{2}}{\sqrt{\pi}}\sqrt{\tfrac{(\theta_i\cdot b_n)^2}{1+||b_n||^2}} \\ &= \tfrac{|\theta_i\cdot b_n|}{\sqrt{2\pi(1+||b_n||^2)}}\end{aligned}$$

where the second equality uses the formula for the expectation of the absolute value of a normal distribution with mean zero and variance as defined above. Therefore, the value of information induced by b_n for an agent of type θ_i is

$$\begin{aligned}v(b_n|\theta_i) &= \mathbb{E}_\omega(A_{-i}(\omega,\theta_{-i})u(\omega,\theta_i)) + \frac{|\theta_i\cdot b_n|}{I\sqrt{2\pi(1+||b_n||^2)}} - \mathbb{E}_\omega(A_{-i}(\omega,\theta_{-i})u(\omega,\theta_i)) \\ &= \frac{|\theta_i\cdot b_n|}{I\sqrt{2\pi(1+||b_n||^2)}}.\end{aligned}$$

This concludes the proof. □

Lemma A1. *It is never optimal for a firm to choose a strategy b_n such that $||b_n||<1$.*

Proof of Lemma A1. Let b_n be such that $||b_n||<1$. We show that there exists a b'_n such that, for all θ_i, $v(b'_n|\theta_i)>v(b_n|\theta_i)$. Define $c=1/||b_n||>1$ and $b'_n=cb_n$. Notice that $||b'_n||=1$ and $|\theta_i b'_n|=c|\theta_i b_n|$. By Lemma 2,

$$\begin{aligned}v(b_n|\theta_i) &= \frac{|\theta_i\cdot b_n|}{I\sqrt{2\pi}}\frac{1}{\sqrt{1+||b_n||^2}} \\ &< \frac{|\theta_i\cdot b_n|}{I\sqrt{2\pi}}\frac{1}{\sqrt{||b_n||^2+||b_n||^2}} = \frac{|\theta_i\cdot b_n|}{I\sqrt{2\pi}}\frac{1}{\sqrt{2}||b_n||} \\ &= \frac{|\theta_i\cdot b_n|}{I\sqrt{2\pi}}\frac{c}{\sqrt{2}} = \frac{|\theta_i\cdot b'_n|}{2I\sqrt{\pi}} = v(b'_n|\theta_i).\end{aligned}$$

Therefore, since θ_i was arbitrary, editorial strategy b_n with $||b_n||<1$ is strictly dominated. □

Proof of Remark 1. Fix θ_i. By assumption, $||\theta_i||=2$ and $\theta_{i,0}=1$. That is $\theta_{i,1}^2+\theta_{i,2}^2=1$ and $(\theta_{i,1},\theta_{i,2})$ is a point on the unit circle. Thus, there exists a unique $t_i\in T=[-\pi,\pi]$ such that $(\theta_{i,1},\theta_{i,2})=$

$(\cos(t_i), \sin(t_i))$. Therefore, $u(\omega, \theta_i) = \omega_0 + \omega_1\theta_{i,1} + \omega_2\theta_{i,2} = \omega_0 + \omega_1 \cos(t_i) + \omega_2 \sin(t_i)$. Now fix an arbitrary b_n. Clearly,

$$s_i(\omega, b_n) = \omega_0 b_{n,0} + \sqrt{||b_n||^2 - b_{n,0}^2}\Big(\omega_1 \frac{b_{n,1}}{\sqrt{||b_n||^2 - b_{n,0}^2}} + \omega_2 \frac{b_{n,2}}{\sqrt{||b_n||^2 - b_{n,0}^2}}\Big) + \varepsilon_i.$$

Moreover, $\frac{b_{n,1}^2}{||b_n||^2 - b_{n,0}^2} + \frac{b_{n,2}^2}{||b_n||^2 - b_{n,0}^2} = 1$. Therefore, $\Big(\frac{b_{n,1}}{\sqrt{||b_n||^2 - b_{n,0}^2}}, \frac{b_{n,2}}{\sqrt{||b_n||^2 - b_{n,0}^2}}\Big)$ is a point on the unit circle and equals $(\cos(t_n), \sin(t_n))$ for a unique $t_n \in T$. Letting $x_n = b_{n,0}^2 \in [0, ||b_n||^2]$, we have

$$s_i(\omega, b_n) \quad = \quad \sqrt{x_n}\omega_0 + \sqrt{||b_n||^2 - x_n}\Big(\omega_1 \cos(t_n) + \omega_2 \sin(t_n)\Big) + \varepsilon_i.$$

Setting $||b_n|| = 1$ concludes the proof. □

Lemma A2. *It is never optimal for a firm to choose an editorial strategy (x_n, t_n) with $x_n < 1/2$. Moreover, setting $\lambda = \frac{1}{2I\sqrt{\pi}}$, the* value of information (x_n, t_n) *when $x_n \geq 1/2$ can be written as:*

$$v\big((x_n, t_n) \mid t_i\big) = \lambda\Big(\sqrt{x_n} + \sqrt{1 - x_n}\cos(t_i - t_n)\Big).$$

Proof of Lemma A2. Fix an arbitrary strategy b_n. By Remark 1 and its proof, notice that

$$|b_n \cdot \theta_i| = \Big|\sqrt{x_n} + \sqrt{1 - x_n}\cos(t_i - t_n)\Big|.$$

To establish this equality, we used the angle-addition trigonometric identity $\cos(t_n)\cos(t_i) + \sin(t_n)\sin(t_n) = \cos(t_i - t_n)$. It is straightforward to see that for all (x_n, t_n) with $x_n \geq 1/2$, $\sqrt{x_n} + \sqrt{1 - x_n}\cos(t_i - t_n) \geq 0$ for all t_i and t_n. Therefore, whenever $x_n \geq 1/2$, the value of information simplifies to

$$v\big((x_n, t_n) \mid t_i\big) = \frac{1}{2I\sqrt{\pi}}\Big(\sqrt{x_n} + \sqrt{1 - x_n}\cos(t_i - t_n)\Big).$$

Now fix (x_n, t_n) with $x_n < 1/2$. We want to show that there is a feasible (x_n', t_n') such that $v((x_n', t_n')|t_i) \geq v(x_n, t_n)|t_i)$, for all t_i. To see this let $x_n' = 1 - x_n > 1/2$ and $t_n' = t_n$. We need to show that

$$\sqrt{1 - x_n} + \sqrt{x_n}\cos(t_i - t_n) \geq \Big|\sqrt{x_n} + \sqrt{1 - x_n}\cos(t_i - t_n)\Big|.$$

Let us first consider the case when the argument in the absolute value is positive. Then,

$$\begin{gathered}\sqrt{1 - x_n} + \sqrt{x_n}\cos(t_i - t_n) \geq \sqrt{x_n} + \sqrt{1 - x_n}\cos(t_i - t_n) \\ (\sqrt{1 - x_n} - \sqrt{x_n})(1 - \cos(t_i - t_n)) \geq 0\end{gathered}$$

Note that $\sqrt{1 - x_n} - \sqrt{x_n} \geq 0$, since $x_n < 1/2$. Therefore, the above inequality holds for all t_i and t_n. Next, we consider the case when the argument in the absolute value is negative. In such case,

$$\begin{gathered}\sqrt{1 - x_n} + \sqrt{x_n}\cos(t_i - t_n) \geq -\sqrt{x_n} - \sqrt{1 - x_n}\cos(t_i - t_n) \\ (\sqrt{1 - x_n} + \sqrt{x_n})(1 + \cos(t_i - t_n)) \geq 0,\end{gathered}$$

which trivially holds for all t_i and t_n. Since $v((x_n, t_n)|t_i) \leq v((1 - x_n, t_n)|t_i)$ for all t_i, the firm can weakly increase its profit by deviating from (x_n, t_n) to $(1 - x_n, t_n)$. It is therefore with no loss of generality to focus on editorial strategies (x_n, t_n) that have $x_n \geq 1/2$. □

A.1.2. Proof of Theorem 1

It is convenient to prove the statement of the theorem using the polar-coordinate transformation illustrated in Remark 1. Recall that the circle is defined on $T = [\pi, \pi]$ and that we use a modular convention: For example, $t_n = 2\pi$ and $t'_n = 0$ do indicate the same type.

Proof of Theorem 1. If $N = 1$, the profit of the (monopolist) firm when choosing editorial strategy (x_1, t_1) is

$$I\int_{-\pi}^{\pi} v((x_1,t_1)|t_i)dF(t_i) = I\int_{-\pi}^{\pi} v((x_1,0)|t_i)dF(t_i) = I\sqrt{x_1} + I\sqrt{1-x_1}\int_{-\pi}^{\pi}\cos(t_i)dF(t_i) = I\sqrt{x_1}.$$

The last inequality follows from the fact that F is the cdf of the uniform distribution on $T = [-\pi, \pi]$. Therefore, $x_1 = x^\star(1) = 1$ maximizes the monopolist's profit.

Now let $N \geq 2$. We divide proof in two parts. First, we establish the existence and uniqueness of $x^\star \in [1/2, 1]$ such that, if the N firms are evenly spread on the circle, no firm n would want to unilaterally deviate by choosing a $x_n \neq x^\star$. Second, we establish that if all firms $n' \neq n$ choose $x_{n'} = x^\star$ and are evenly spread on the circle located, firm n does not have incentives to deviate away from $(x^\star, t_n^\star)$ to a different strategy (x_n, t_n).

Part 1. Consider a candidate profile of strategies $(x_n, t_n)_{n=1}^N$. Suppose that firms' locations $(t_n)_{n=1}^N$ are evenly spread on the circle and, without loss of generality, let $t_n = 0$. Moreover, let $x_{n'} = \bar{x} \in [1/2, 1]$ for all $n' \neq n$. Define $V((x_{n'}, t_{n'})_{n'\neq n}|t_i) = \max\{v((x_{n'}, t_{n'})|t_i) : n' \neq n\}$. Let $R_n := \{t_i | v((x_n, t_n)|t_i) \geq V((x_{n'}, t_{n'})_{n'\neq n}|t_i)\}$ be the *readership* of firm n, namely, the set of types for whom firm n generates a weakly higher value than the competition. Thus, the profit for firm n, defined in Equation (1), can be rewritten as:

$$\begin{aligned}\Pi\Big((x_n, t_n = 0), (x_{n'}, t_{n'})_{n'\neq n}\Big) &= I\int_{-\pi}^{\pi} \max\{0, v((x_n,t_n)|t_i) - V((x_{n'}, t_{n'})_{n'\neq n}|t_i)\}dF(t_i)\\ &= I\int_{R_n} v((x_n,t_n)|t_i) - V((x_{n'}, t_{n'})_{n'\neq n}|t_i)dF(t_i)\end{aligned}$$

Notice that readership R_n is equal to the union of intervals $R_n = \cup_{k=1}^K [\bar{t}_l^k, \bar{t}_r^k]$. We guess and later verify that K is finite (indeed, equal to 1). We refer to $\bar{t}_l^k$ and $\bar{t}_r^k$ as *threshold* types. Thus,

$$\Pi\Big((x_n, t_n = 0), (x_{n'}, t_{n'})_{n'\neq n}\Big) = I\sum_{k=1}^{K}\int_{\bar{t}_l^k}^{\bar{t}_r^k} v((x_n,t_n)|t_i) - V((x_{n'}, t_{n'})_{n'\neq n}|t_i)dF(t_i) \tag{A2}$$

The derivative of such function with respect to x_n is given by

$$\Pi_{x_n}\Big((x_n, t_n = 0), (x_{n'}, t_{n'})_{n'\neq n}\Big) = I\sum_{k=1}^{K}\frac{d}{dx_n}\int_{\bar{t}_l^k}^{\bar{t}_r^k} v((x_n,t_n)|t_i) - V((x_{n'}, t_{n'})_{n'\neq n}|t_i)dF(t_i)$$

Importantly, for each k,

$$\begin{aligned}\frac{d}{dx_n}\int_{\bar{t}_l^k}^{\bar{t}_r^k} v((x_n,t_n)|t_i) - V((x_{n'}, t_{n'})_{n'\neq n}|t_i)dF(t_i) &= \int_{\bar{t}_l^k}^{\bar{t}_r^k}\frac{d}{dx_n}v((x_n,t_n)|t_i)dF(t_i)\\ &= \frac{1}{2\pi}\Big(\frac{1}{2\sqrt{x_n}}(\bar{t}_r^k - \bar{t}_l^k) - \frac{1}{2\sqrt{1-x_n}}\Big(\sin(\bar{t}_r^k) - \sin(\bar{t}_l^k)\Big)\Big).\end{aligned}$$

The first equality holds because, by definition of each threshold type $\bar{t}_z^k$ for $z \in \{l, r\}$, $v((x_n, t_n)|\bar{t}_z^k) - V((x_{n'}, t_{n'})_{n' \neq n}|\bar{t}_z^k) = 0$. Therefore, all terms with $d\bar{t}_z^k/dx_n$ cancel. Thus,

$$\Pi_{x_n}\Big((x_n, t_n = 0), (x_{n'}, t_{n'})_{n' \neq n}\Big) = \frac{I}{2\pi}\left(\frac{1}{2\sqrt{x_n}}\sum_{k=1}^{K}(\bar{t}_r^k - \bar{t}_l^k) - \frac{1}{2\sqrt{1 - x_n}}\sum_{k=1}^{K}\Big(\sin(\bar{t}_r^k) - \sin(\bar{t}_l^k)\Big)\right). \tag{A3}$$

Setting this derivative equal to zero gives the following equilibrium condition:

$$\sqrt{\frac{1 - x_n}{x_n}} = \frac{\sum_{k=1}^{K}\Big(\sin(\bar{t}_r^k) - \sin(\bar{t}_l^k)\Big)}{\sum_{k=1}^{K}(\bar{t}_r^k - \bar{t}_l^k)}.$$

When $x_n = \bar{x}$, $K = 1$, that is, readership is single connected interval. To see this, note that a necessary condition for $K > 1$ is that $v((x_n, t_n = 0)|3\pi/N) > v((\bar{x}, 2\pi/N)|3\pi/N)$ or, equivalently, $v((x_n, t_n = 0)|3\pi/N) > v((\bar{x}, 0)|\pi/N)$. This is ruled out by Lemma B7 and the fact that $v((x_n, t_n = 0)|3\pi/N) < v((x_n, t_n = 0)|2\pi/N)$ in this range. Therefore, the equilibrium condition above simplifies to:

$$\sqrt{\frac{1 - \bar{x}}{\bar{x}}} = \frac{\sin(\bar{t}_r) - \sin(\bar{t}_l)}{\bar{t}_r - \bar{t}_l} = \frac{\sin(\bar{t}_r)}{\bar{t}_r} = \frac{\sin(\pi/N)}{\pi/N}. \tag{A4}$$

In the equation above, we dropped the index $k = 1$ for notational simplicity. In the second equality, we used the fact that, since $(t_n)_{n=1}^N$ are equidistant, $\bar{t}_l = -\bar{t}_r$. Finally, in the last equality, we used the fact that, since $x_n = \bar{x}$, the threshold type $\bar{t}_r$ is π/N. It is immediate to see that this equation has a unique solution $\bar{x} = x^\star \in (1/2, 1)$.

Part 2. To verify that $(x^\star, t_n)_{n=1}^N$ is indeed an equilibrium, we need to make sure there is no profitable deviation (x'_n, t'_n) for firm n, provided that every other firm follows $(x^\star, t_{n'})_{n' \neq n}$. Our strategy is to show that $\Pi_{t_n}((x'_n, t'_n), (x^\star, t_{n'})_{n' \neq n}) < 0$ for arbitrary (x'_n, t'_n) with $x'_n \in [1/2, 1]$ and $t'_n \in (0, 2\pi/N)$. Note that a deviation in the opposite direction, $t'_n \in (-2\pi/N, 0)$, would lead to a derivation that is identical to the one below. Therefore, we omit this case. The derivative of the profit function, as expressed in (A2), is

$$\Pi_{t_n}\Big((x'_n, t'_n), (x^\star, t_{n'})_{n' \neq n}\Big) = I\sum_{k=1}^{K}\frac{d}{dt_n}\int_{\bar{t}_l^k}^{\bar{t}_r^k} v((x'_n, t'_n)|t_i) - V((x^\star, t_{n'})_{n' \neq n}|t_i)dF(t_i)$$

As in Part 1, for each k, the derivative in t_n simplifies and we get:

$$\begin{aligned} \tfrac{d}{dt_n}\int_{\bar{t}_l^k}^{\bar{t}_r^k} v((x'_n, t'_n)|t_i) - V((x^\star, t_{n'})_{n' \neq n}|t_i)dF(t_i) &= \int_{\bar{t}_l^k}^{\bar{t}_r^k} \tfrac{d}{dt_n}v((x'_n, t'_n)|t_i)dF(t_i) \\ &= -\tfrac{1}{2\pi}\sqrt{1 - x'_n}\Big(\cos(\bar{t}_r^k - t'_n) - \cos(\bar{t}_l^k - t'_n)\Big). \end{aligned}$$

Therefore,

$$\Pi_{t_n} < 0 \quad \Longleftrightarrow \quad \sum_{k=1}^{K}\Big(\cos(\bar{t}_r^k - t'_n) - \cos(\bar{t}_l^k - t'_n)\Big) > 0. \tag{A5}$$

Let us first consider the case when $K = 1$, i.e. the readership is a single interval. To simplify notation, we drop the index $k = 1$ and denote the readership interval $R_n = [\bar{t}_l, \bar{t}_r]$. By definition, $\bar{t}_l \leq \bar{t}_r$ and $t_n \in [\bar{t}_l, \bar{t}_r]$. Moreover, $\bar{t}_l \geq -2\pi/N$. To see this, note that, if $\bar{t}_l < -2\pi/N$, we would need $v((x'_n, t'_n)| - 2\pi/N) > v((x^\star, -2\pi/N)| - 2\pi/N)$. However, this is not possible due to Lemma B7 and the fact that $v((x'_n, t'_n)| - 2\pi/N) \leq v((x'_n, 0)| - 2\pi/N) = v((x'_n, 0)|2\pi/N)$ and $v((x^\star, -2\pi/N)| - 2\pi/N) = v((x^\star, 0)|0)$.

We consider three different cases, according to which values $\bar{t}_l$ and $\bar{t}_r$ take.

1. Suppose $\bar{t}_l \geq 0$. Note that this implies $\bar{t}_l \leq 2\pi/N$. If this was not the case, $t_n' \geq 2\pi/N$, a contradiction. Let us assume by contradiction that $\cos(\bar{t}_r - t_n') \leq \cos(\bar{t}_l - t_n')$. This is equivalent to assuming that $v((x_n', t_n')|\bar{t}_l) \geq v((x_n', t_n')|\bar{t}_r)$. Define $\hat{t} = 2\pi/N + 2\pi/N - \bar{t}_l$. By construction, type $\hat{t}$ is located as far to the right of $2\pi/N$ as t_l is to the left of $2\pi/N$. Because $v((x^\star, 2\pi/N)|t_i)$ is symmetric around $2\pi/N$, we have $v(x^\star, 2\pi/N|\bar{t}_l) = v(x^\star, 2\pi/N|\hat{t})$. Since, by assumption, $v((x^\star, 2\pi/N)|\hat{t}) \geq v((x_n', t_n')|\bar{t}_r)$, it must be that $v((x_n', t_n')|\hat{t}) \geq v((x^\star, 2\pi/N)|\hat{t})$. Therefore, we have:

$$v((x_n', t_n')|\bar{t}_l) \leq v((x_n', t_n')|\hat{t}) \quad \Rightarrow \quad \cos(\bar{t}_l - t_n') \leq \cos(\hat{t} - t_n'),$$

   Note that $\bar{t}_l - t_n' \leq 0$ and $\hat{t} - t_n' \geq 0$. Thus, $\bar{t}_l - t_n' \leq -\hat{t} + t_n'$, or

$$t_n \geq \frac{\hat{t} + t_l}{2} = \frac{2\pi/N + 2\pi/N - t_l + t_l}{2} = 2\pi/N,$$

   which contradicts our initial assumption that $t_n < 2\pi/N$.

2. We now suppose that $\bar{t}_l \leq 0$ and $\bar{t}_r \leq 2\pi/N$. This necessarily implies that $v((x^\star, 2\pi/N)|\bar{t}_r) = v((x_n', t_n')|\bar{t}_r)$ and $v((x^\star, -2\pi/N)|\bar{t}_l) = v((x_n', t_n')|\bar{t}_l)$. As before, let us assume by contradiction that $\cos(\bar{t}_r - t_n') \leq \cos(\bar{t}_l - t_n')$. This is equivalent to assuming that $v((x_n', t_n')|\bar{t}_l) \geq v((x_n', t_n')|\bar{t}_r)$. Since $v((x_n', t_n')|t_i)$ is symmetric in t_i relative to t_n', this means that $t_n' \leq \frac{1}{2}(\bar{t}_r + \bar{t}_l)$. By assumption, we also have that $v((x^\star, -2\pi/N)|\bar{t}_l) \geq v((x^\star, 2\pi/N)|\bar{t}_r)$, which implies $\cos(\bar{t}_l + 2\pi/N) \geq \cos(\bar{t}_r - 2\pi/N)$, which requires $|\bar{t}_l + 2\pi/N| < |\bar{t}_r - 2\pi/N|$. By assumption, $\bar{t}_l + 2\pi/N \geq 0$ and $\bar{t}_r - 2\pi/N < 0$. Therefore, we have that $-\bar{t}_l - 2\pi/N \geq \bar{t}_r - 2\pi/N$, hence $\bar{t}_r + \bar{t}_l \leq 0$. Since $t_n' \leq \frac{1}{2}(\bar{t}_r + \bar{t}_l)$, we have $t_n' \leq 0$, a contradiction.

3. Finally, suppose that $\bar{t}_l \leq 0$ and $\bar{t}_r \geq 2\pi/N$. As before, suppose by contradiction that $v((x_n', t_n')|\bar{t}_l) \geq v((x_n', t_n')|\bar{t}_r)$. This implies that $v((x^\star, -2\pi/N)|\bar{t}_l) \geq v((x^\star, 2\pi/N)|\bar{t}_r)$. This holds irrespective of whether $v((x^\star, 2\pi/N)|\bar{t}_r) = v((x_n', t_n')|\bar{t}_r)$ or $v((x^\star, 2\pi/N)|\bar{t}_r) < v((x_n', t_n')|\bar{t}_r)$. Therefore, $\cos(\bar{t}_l + 2\pi/N) \geq \cos(\bar{t}_r - 2\pi/N)$. By assumption, $\bar{t}_l + 2\pi/N \geq 0$ and $\bar{t}_r - 2\pi/N \geq 0$. Therefore, $\bar{t}_l + 2\pi/N \leq \bar{t}_r - 2\pi/N$ and, hence, $\bar{t}_r - \bar{t}_l \geq 4\pi/N$. Moreover, note that $v((x_n', t_n')|\bar{t}_r)$ is bounded below by $v((x^\star, 2\pi/N)|3\pi/N)$ or, equivalently, by $v((x^\star, 0)|\pi/N)$. A necessary condition for $\bar{t}_r - \bar{t}_l \geq 4\pi/N$ and $v((x_n', t_n')|\bar{t}_r) \geq v((x^\star, 0)|\pi/N)$ to be jointly true is that there exists a $x_n' \in [1/2, 1]$ such that $v((x_n', 0)|2\pi/N) \geq v((x^\star, 0)|\pi/N)$. By Lemma B7, this is not possible, hence we have a contradiction.

Therefore, we showed that, when $K = 1$, $\Pi_{t_n} < 0$. Hence, the arbitrary deviation (x_n', t_n') can not be a profitable one.

To conclude the proof, we analyze the case $K > 1$. In this case, deviation (x_n', t_n') generates a readership with K disconnected intervals. Note that $K > 1$ is possible only if $N > 2$. Therefore, let $N \geq 3$ for the remainder of the proof. Consider an arbitrary deviation (x_n', t_n') with $t_n' \in (0, 2\pi/N)$. We begin by noting that firm n cannot win over type $t = -3\pi/N$. That is,

$$v((x_n', t_n')| - 3\pi/N) \leq v((x_n', 0)| - 3\pi/N) \leq v((x^\star, -2\pi/N)| - 3\pi/N).$$

This is equivalent to showing that, for all $x_n' \in [1/2, 1]$, $v((x_n', 0)|3\pi/N) \leq v((x^\star, 0)|\pi/N)$, which immediately follows from Lemma B7. Next, we show that firm n cannot win over type $t = 5\pi/N$

either (this type exists only if $N \geq 5$. That is,

$$v((x'_n, t_n)|5\pi/N) \leq v((x'_n, 2\pi/N)|5\pi/N) \leq v((x^\star, 4\pi/N)|5\pi/N)$$

This is equivalent to showing that, for all $x'_n \in [1/2, 1]$, $v((x'_n, 0)|3\pi/N) \leq v((x^\star, 0)|\pi/N)$. Again, this immediately follows from Lemma B7. This means that the only possible multi-interval case to consider is the one where $K = 2$. In such case, there are exactly two intervals, which we shall denote $[\bar{t}^1_l, \bar{t}^1_r]$ and $[\bar{t}^2_l, \bar{t}^2_r]$. Moreover, it is easy to see that, in this case, $\bar{t}^1_l \leq 0$ and $\bar{t}^1_r \in [0, 2\pi/N]$ and that $\bar{t}^2_l \geq 2\pi/N$ and $\bar{t}^2_r \geq 3\pi/N$. By equation A5, we need to show that:

$$\cos(\bar{t}^1_r - t'_n) - \cos(\bar{t}^1_l - t'_n) + \cos(\bar{t}^2_r - t'_n) - \cos(\bar{t}^2_l - t'_n) > 0.$$

We first show that $\cos(\bar{t}^2_r - t'_n) \geq \cos(\bar{t}^1_l - t'_n)$. Suppose not. Then, $v((x'_n, t'_n)|\bar{t}^1_l) > v((x'_n, t'_n)|\bar{t}^2_r)$. Note that $v((x'_n, t'_n)|\bar{t}^2_r)$ is bounded below by $v((x^\star, 2\pi/N)|3\pi/N) = v((x^\star, 0)|\pi/N)$. This implies that $\bar{t}^2_r - \bar{t}^1_l > 4\pi/N$. This is possible only if there exists a x'_n such that $v((x'_n, 0)|2\pi/N) > v((x^\star, 0)|\pi/N)$. However, Lemma B7 shows that this is not possible and, therefore, we must have $\cos(\bar{t}^2_r - t'_n) \geq \cos(\bar{t}^1_l - t'_n)$.

We now show that $\cos(\bar{t}^1_r - t'_n) - \cos(\bar{t}^2_l - t'_n) > 0$. There are two cases to consider. If $t_n \leq \bar{t}^1_r$, then $\bar{t}^1_r - t'_n < \bar{t}^2_l - t'_n$ which immediately implies $\cos(\bar{t}^1_r - t'_n) - \cos(\bar{t}^2_l - t'_n) > 0$. Therefore, suppose instead that $t'_n > \bar{t}^1_r$. Recall that, by definition of $\bar{t}^1_r$, $v((x'_n, t'_n)|\bar{t}^1_r) = v((x^\star, 2\pi/N)|\bar{t}^1_r)$. Define $\hat{t} = t'_n + (t'_n - \bar{t}^1_r)$ and $\tilde{t} = 2\pi/N + (2\pi/N - \bar{t}^1_r)$. Since $t'_n < 2\pi/N$, we have $\tilde{t} > \hat{t}$. By the symmetry of the value function, $v((x'_n, t'_n)|\bar{t}^1_r) = v((x'_n, t'_n)|\hat{t})$ and $v((x^\star, 2\pi/N)|\bar{t}^1_r) = v((x^\star, 2\pi/N)|\tilde{t})$. Therefore, since $\tilde{t} > \hat{t}$, $v((x^\star, 2\pi/N)|\hat{t}) > v((x^\star, 2\pi/N)|\tilde{t}) = v((x'_n, t'_n)|\hat{t})$. This implies that $\hat{t} < \bar{t}^2_l$, hence $v((x'_n, t'_n)|\bar{t}^1_r) > v((x'_n, t'_n)|\bar{t}^2_l)$. We conclude that $\cos(\bar{t}^1_r - t'_n) > \cos(\bar{t}^2_l - t'_n)$. □

A.1.3. Proof of Theorem 2

Lemma A3. *Let $(x_n, t_n)_{n=1}^N$ be a pure-strategy equilibrium. For all n, readership R_n is an interval on the circle.*

Proof. If $N = 1$ there is nothing to prove. Let $N > 1$ and $(x_n, t_n)_{n=1}^N$ be a pure-strategy equilibrium. Without loss of generality, let the firms' labels be such that $x_1 \leq x_2 \leq \ldots \leq x_N$. We divide the proof in three steps. In the first step we establish that R_1 must be an interval on the circle. In the second step, we let $N \geq 2$ and assume that $x_N < 1$. We establish that, if all R_m are intervals on the circle for $m < n$, then R_n is an interval on the circle as well. In the final step, we prove that, when $N \geq 2$, for $(x_n, t_n)_{n=1}^N$ to be an equilibrium, it must be that $x_N < 1$.

Step 1. We establish that firm 1's readership R_1 is an interval on the circle. Without loss of generality, let us normalize locations in $(x_n, t_n)_{n=1}^N$ such that $t_1 = 0$. By definition of readership, $R_1 = \{t \in [-\pi, \pi] | v((x_1, t_1 = 0)|t) \geq V((x_n, t_n)_{n \neq 1}|t)\}$.[23] For each n, define $R_{1,n} = \{t \in [-\pi, \pi] | v((x_1, t_1 = 0)|t) \geq v((x_n, t_n)|t)\}$ and note that $R_1 = \cap_{n \neq 1} R_{1,n}$. Fix an arbitrary $n \neq 1$. Since $x_1 \leq x_n$, the set $R_{1,n}$ is an interval by Lemma B8. Therefore, R_1 is the intersection of finitely many intervals in $[-\pi, \pi]$. Hence, it is an interval.

[23]See definitions at the beginning of the Proof of Theorem 1, Part 1.

Step 2. Fix $1 < n \leq N$ and suppose that for all firms $m < n$, R_m is an interval on the circle. Note that, when $N = 2$, firm 1's readership being an interval on the circle implies that firm 2's readership, the complement of R_1, is an interval on the circle as well. Therefore, let $N \geq 3$. In the proof of this step, we will assume $x_N < 1$, a result that we will establish in the next and final step.

By way of contradiction, suppose that firm n's readership R_n is the union of at least two disconnected intervals on the circle denoted $[\underline{a}, \bar{a}]$ and $[\underline{b}, \bar{b}]$. Without loss of generality, let us normalize locations in $(x_n, t_n)_{n=1}^{N}$ such that $t_n = 0$. Moreover, it is without loss to take $\bar{a} < \underline{b}$ such that $R_n \cap (\bar{a}, \underline{b}) = \emptyset$. Note that it must be that $\underline{a} \geq -\pi$ and $\bar{b} \leq \pi$, with at least one inequality strict. If this was not the case, that is, if both $\underline{a} = -\pi$ and $\bar{b} = \pi$, $[\underline{a}, \bar{a}] \cup [\underline{b}, \bar{b}]$ would represent a single interval on the circle $[-\pi, \pi]$, a contradiction. Since $x_n \leq x_{n'}$ for all $n' \geq n$, Lemma B8 implies that $\bar{R} = \cap_{n' \geq n} R_{n,n'}$ is an interval. Moreover, $[\underline{a}, \bar{a}] \cup [\underline{b}, \bar{b}] \subseteq \bar{R}$. Therefore, types in $(\bar{a}, \underline{b})$ belong to the readership of firms in $M \subseteq \{1, \ldots, n-1\}$. By the inductive assumption, $\{R_m\}_{m \in M}$ are non-overlapping intervals.[24]

Suppose $M = \{m\}$ is a singleton and, therefore, $R_m = [\bar{a}, \underline{b}]$. There are three cases to consider, depending on the location of $\bar{a}$ and $\underline{b}$ relative to $t_n = 0$.

- Suppose that $\underline{b} \leq t_n = 0$. Since $x_n \leq x_N < 1$, $v((x_n, t_n)|t)$ is strictly increasing for all $-\pi < t < t_n$. Therefore, $v((x_m, t_m)|\bar{a}) = v((x_n, t_n)|\bar{a}) < v((x_n, t_n)|\underline{b}) = v((x_m, t_m)|\underline{b})$, where the equalities follow from the definition of threshold types. However, by equation (A5), $v((x_m, t_m)|\bar{a}) < v((x_m, t_m)|\underline{b})$ implies that $\Pi_{t_m} < 0$. Therefore, firm m has a profitable deviation in t_m. A contradiction.
- Suppose, instead, that $\bar{a} \geq t_n = 0$. Since $x_n \leq x_N < 1$, $v((x_n, t_n)|t)$ is strictly decreasing for all $t_n < t < \pi$. Therefore, $v((x_m, t_m)|\bar{a}) = v((x_n, t_n)|\bar{a}) > v((x_n, t_n)|\underline{b}) = v((x_m, t_m)|\underline{b})$. Therefore, by equation (A5), $\Pi_{t_m} > 0$. Hence, firm m has a profitable deviation. A contradiction.
- Finally, suppose that $\bar{a} < 0 < \underline{b}$. Equilibrium requires that $v((x_m, t_m)|\bar{a}) = v((x_n, t_n)|\bar{a}) = v((x_n, t_n)|\underline{b}) = v((x_m, t_m)|\underline{b})$. This implies that $t_m = t_n = 0$. While $\Pi_{t_m} = 0$, profits for firm m are at a local minimum. Suppose firm m deviates to $t'_m = t_m + dt_m$. Such deviation would strictly increase $v((x_m, t'_m)|\bar{a})$ (since $v((x_n, t_n)|t)$ is strictly increasing at $t = \bar{a}$) and strictly decreases $v((x_m, t'_m)|\underline{b})$ (since $v((x_n, t_n)|t)$ is decreasing at $t = \underline{b}$). Therefore, by equation (A5), this implies $\Pi_{t_m} > 0$. Hence, firm m has a profitable deviation. A contradiction.

Suppose M is not a singleton. Denote by m_A and m_B the two firms whose readerships are at opposite extremes of the interval $(\bar{a}, \underline{b})$. Since $m_A, m_B \in M$, R_{m_A} and R_{m_B} are disjoint intervals in $(\bar{a}, \underline{b})$. Therefore, there must be $\bar{a}' \leq \underline{b}'$ such that $R_{m_a} = [\bar{a}, \bar{a}']$ and $R_{m_b} = [\underline{b}', \underline{b}]$. There are two cases to consider, depending on the location of $\bar{a}'$ relative to $t_n = 0$.

- Suppose $\bar{a}' \leq t_n = 0$. Since $x_n \leq x_N < 1$, $v((x_n, t_n)|t)$ is strictly increasing for all $-\pi < t < t_n$. Therefore, $v((x_{m_A}, t_{m_A})|\bar{a}) = v((x_n, t_n)|\bar{a}) < v((x_n, t_n)|\bar{a}') \leq v((x_{m_A}, t_{m_A})|\bar{a}')$. The last inequality comes about because if firm m_A does not share type $\bar{a}'$ with firm n, it must be sharing $\bar{a}'$ with some firm in M yielding a value higher than $v((x_n, t_n)|\bar{a}')$. However, by equation (A5), $v((x_{m_A}, t_{m_A})|\bar{a}) < v((x_{m_A}, t_{m_A})|\bar{a}')$ implies that $\Pi_{t_{m_A}} < 0$. Therefore, firm m_A has a profitable

[24] Whether two intervals R_m and $R_{m'}$ overlap at an end point is a matter of convention. This has not bearing on firms' behavior because a threshold type—namely the type who is at the boundary of a readership interval—yields a profit of 0 to the firm from which she acquires information.

deviation. A contradiction.

– Conversely, suppose that $\bar{a}' > t_n = 0$. Therefore, $\underline{b}' \geq \bar{a}' > 0$. Since $x_n \leq x_N < 1$, $v((x_n, t_n)|t)$ is strictly decreasing for all $t_n < t < \pi$. Therefore, $v((x_{m_B}, t_{m_B})|\underline{b}') \geq v((x_n, t_n)|\underline{b}') > v((x_n, t_n)|\underline{b}) = v((x_{m_B}, t_{m_B})|\underline{b})$. The first inequality comes about because if firm m_B does not share type $\underline{b}'$ with firm n, it must be sharing $\underline{b}'$ with some firm in M yielding a value higher than $v((x_n, t_n)|\underline{b}')$. However, by equation (A5), $v((x_{m_B}, t_{m_B})|\underline{b}) < v((x_{m_B}, t_{m_B})|\underline{b}')$ implies that $\Pi_{t_{m_A}} > 0$. Therefore, firm m_A has a profitable deviation. A contradiction.

Therefore, R_n must be an interval, which concludes the inductive step. By induction, R_n is an interval for all $1 \leq n \leq N$.

Step 3. We are left to establish that $x_N < 1$ and, a fortiori, $x_n < 1$ for all $n \leq N$.[25] To see this, suppose not, i.e. $x_N = 1$. By the inductive assumption $R_n = [\underline{t}_n, \bar{t}_n]$ is an interval on the circle for all $n < N$. Since $x_N = 1$, $v((x_n, t_n)|\bar{t}_n) = 1$ for all n. For $n < N$, denote $\delta_n = \bar{t}_n - t_n$. Equation (A5) requires that $t_n - \underline{t}_n = \delta_n$, as well. Therefore, $\delta_n > 0$, otherwise firm n would make zero profits. Using equation (A4), the value generated for the threshold type with distance to δ_n to the target, $G(\delta_n)$, must satisfy

$$G(\delta_n) = \frac{2\delta_n + \sin(2\delta_n)}{2\sqrt{\delta_n^2 + \sin^2(\delta_n)}} = 1.$$

Note that $\lim_{\delta_n \to 0^+} G(\delta_n) = \sqrt{2} > 1$ and $G(\pi/2) < 1$. Moreover, by Lemma B9, $G(\delta_n)$ strictly decreasing for all $\delta_n \in (0, \pi/2)$. Therefore, $G(\delta_n) = 1$ admits at most one solution in such interval. It is easy to verify that $\delta = \frac{\pi}{2} - \frac{\sqrt{3}}{5}$ is such solution, which is independent of n. That is, all firms $n < N$ have a readership of 2δ. If $N > 3$, $(N-1)2\delta > 2\pi$, hence firm N would make zero profits, a contradiction since $(x_n, t_n)_{n=1}^N$ is an equilibrium. Thus, the only non-trivial case to consider is $N = 3$. Without loss of generality, let $t_1 < t_2$ and $t_3 = \frac{1}{2}(t_1 + t_2) = 0$. Thus, $t_2 = -t_1$. Moreover, we can let $t_2 \geq \pi/2$ (if this was not the case, firm 3 could deviate to $t_3' = \pi$ and the argument that follows would hold). Finally, since $t_2 + \delta \leq \pi$, let $t_2 \leq \pi - \delta = \pi/2 + \sqrt{3}/5$. Therefore, $t_2 \in [\pi/2, \pi/2 + \sqrt{3}/5]$. Now consider a deviation x_3' that is arbitrarily close to 1. If $(x_n, t_n)_{n=1}^N$ is indeed an equilibrium, such deviation should be weakly unprofitable. We will show instead it is strictly profitable. Let $R_2 = [\underline{t}, \bar{t}]$ the new readership for firm 2 that such deviation induces. By symmetry, $R_1 = [-\bar{t}, -\underline{t}]$. Consider the derivative with respect to x_3 of firm 3's profit function evaluated at x_3'. Equation (A3) gives

$$\Pi_{x_3}\Big|_{x_3'} = \frac{1}{2\pi}\left(\frac{1}{2\sqrt{x_3'}}(2\underline{t} + 2(\pi - \bar{t})) - \frac{1}{2\sqrt{1 - x_3'}}\Big(2\sin(\underline{t}) - 2\sin(\bar{t})\Big)\right).$$

The first term is bounded above by $\sqrt{2}\pi$. The second term grows unboundedly to either plus or minus infinity, as x_3' is closer to 1. Its sign is equal to the sign of $\sin(\bar{t}) - \sin(\underline{t})$. Note that $t_2 \geq \pi/2$ by assumption, and $\bar{t} = t_2 + \delta$ and $\underline{t} = t_2 - \delta$ with both $\underline{t}, \bar{t} \in [0, \pi]$. Therefore, $\sin(\bar{t}) - \sin(\underline{t}) < 0$ implying that that the derivative Π_{x_3} is strictly negative when evaluated at a x_3' that is sufficiently close to 1. Therefore, a small deviation that marginally decreases $x_3 = 1$ would be profitable for the firm. □

[25]Note that, if $x_n = 1$ and $n < N$, at least two firms, n and N, have $x_n = x_N = 1$. Therefore, they make zero profits, an immediate contradiction

Proof of Theorem 2. Let $N \geq 1$ and consider an arbitrary pure-strategy equilibrium $(x_n, t_n)_{n=1}^N$. By Lemma A3, the readership of firm n is an interval R_n on the circle. By equation (A5), this implies that each firm is located at the midpoint of its readership interval, i.e. $R_n = [t_n - \delta_n, t_n + \delta_n]$ for some $\delta_n > 0$. Moreover, it implies that firm n sets $x_n = x^\star(\delta_n)$, the only value of x_n that solves equation (A4), given $\bar{t}_{n,r} = \delta_n$ and $\bar{t}_{n,l} = -\delta_n$. We want to show that $\delta_n = \delta = \pi/N$ for all n and, thus, $x_n = x^\star$. That is, $(x_n, t_n)_{n=1}^N$ is the equilibrium characterized by Theorem 1.

Suppose that, in this equilibrium, there are two firms, 1 and 2, such that $\delta_1 > \delta_2$. Without loss of generality, let $0 = t_1 < t_2$ and suppose these firms are adjacent. That is, type $\bar{t} = t_1 + \delta_1 = t_2 - \delta_2$ is their threshold type. This implies that $v((x_1, t_1)|\bar{t}) = v((x_2, t_2)|\bar{t})$. Since $\bar{t} - t_n = \delta_n$ and $x_n = x^\star(\delta_n)$ can be expressed in terms of δ_n only, we can write $v((x_n, t_n)|\bar{t})$ as a function of δ_n:

$$v((x_1, t_1)|\bar{t}) = \frac{2\delta_1 + \sin(2\delta_1)}{2\sqrt{\delta_1^2 + \sin^2(\delta_1)}} \quad \text{and} \quad v((x_2, t_2)|\bar{t}) = \frac{2\delta_2 + \sin(2\delta_2)}{2\sqrt{\delta_2^2 + \sin^2(\delta_2)}}.$$

Suppose $\delta_1 \leq \pi/2$ and, a fortiori, $\delta_2 \leq \pi/2$. Lemma B9 shows that this function is strictly decreasing in the interval $(0, \pi/2)$. Therefore $v((x_1, t_1)|\bar{t}) = v((x_2, t_2)|\bar{t})$ if and only if $\delta_1 = \delta_2$, a contradiction.

Suppose instead $\delta_1 > \pi/2$. Since the size of the market is 2π, $\delta_2 \leq \frac{2\pi - 2\delta_1}{2} = \pi - \delta_1 < \pi/2$. We will show that, in this case, $v((x_1, t_1)|\bar{t}) < v((x_2, t_2)|\bar{t})$, a contradiction. Note that,

$$v((x_2, t_2)|\bar{t}) = \frac{2\delta_2 + \sin(2\delta_2)}{2\sqrt{\delta_2^2 + \sin^2(\delta_2)}} \geq \frac{2(\pi - \delta_1) + \sin(2(\pi - \delta_1))}{2\sqrt{(\pi - \delta_1)^2 + \sin^2(\pi - \delta_1)}},$$

since $0 < \delta_2 \leq \pi - \delta_1 < \pi/2$ and Lemma B9. Put $y = \pi - \delta_1$ and $\delta_1 = \pi - y$. Thus, we need to show that

$$\frac{2y + \sin(2y)}{2\sqrt{y^2 + \sin^2(y)}} > \frac{2(\pi - y) + \sin(2(\pi - y))}{2\sqrt{(\pi - y)^2 + \sin^2(\pi - y)}}$$

Use $\sin(2(\pi - y)) = -\sin(2y)$ and $\sin^2(\pi - y) = \sin^2(y)$ and simplify to obtain

$$\frac{(2y + \sin(2y))^2}{y^2 + \sin^2(y)} > \frac{(2(\pi - y) - \sin(2y))^2}{(\pi - y)^2 + \sin^2(y)}$$

or, equivalently,

$$(2y + \sin(2y))^2((\pi - y)^2 + \sin^2(y)) > (2(\pi - y) - \sin(2y))^2(y^2 + \sin^2(y))$$

Simplifying, we obtain,

$$\begin{aligned} 4y^2\sin^2(y) + 4y\sin(2y)(\pi - y)^2 + 4y\sin(2y)\sin^2(y) + \sin^2(2y)(\pi - y)^2 \quad &> \\ 4(\pi - y)^2\sin^2(y) - 4(\pi - y)\sin(2y)(y)^2 - 4(\pi - y)\sin(2y)\sin^2(y) + \sin^2(2y)y^2 & \end{aligned}$$

Looking at the last term on both sides, note that $\sin^2(2y)(\pi - y)^2 > \sin^2(2y)y^2$, since $y \in (0, \pi/2)$. Looking at the second-to-last term on both sides, note that $4y\sin(2y)\sin^2(y) > -4(\pi - y)\sin(2y)\sin^2(y)$. Therefore, it is enough to show that, in the interval $y \in (0, \pi/2)$

$$G(y) = y^2\sin^2(y) + y\sin(2y)(\pi - y)^2 - (\pi - y)^2\sin^2(y) + (\pi - y)\sin(2y)y^2 > 0.$$

Note that $G(0) = 0$ and $G(\pi/2) = 0$. Moreover, if $y \in (0, \pi/4)$

$$G'(y) = \sin^2(y)2\pi + 2\cos(2y)(y(\pi - y)^2 + (\pi - y)y^2) > 0.$$

Therefore, $G(y)$ is strictly positive for all $y \in (0, \pi/4)$. Moreover, if $y \in (\pi/4, \pi/2)$,

$$G''(y) = \sin(2y)2\pi + 2\cos(2y)((\pi - y)^2 - y^2) - 4\sin(2y)(y(\pi - y)^2 + (\pi - y)y^2) < 0.$$

This implies that, when $G'(y)$ turns negative, it remains negative. Since $G(\pi/2) = 0$, this implies that $G(y)$ cannot cross zero before $\pi/2$.

Therefore, $G(y) > 0$ for all $y \in (0, \pi/2)$. This implies that $v((x_1, t_1)|\bar{t}) < v((x_2, t_2)|\bar{t})$, a contradiction. $\square$

A.2. Proofs for Section 4

Proof of Proposition 1. Let $(x_n^\star, t_n^\star)_{n=1}^N$ be an equilibirum with N firms. From equation A4 in the proof of Theorem 1, we have that $x_n^\star = x^\star$ satisfies

$$\sqrt{\frac{1 - x^\star}{x^\star}} = \frac{\sin(\pi/N)}{\pi/N}.$$

This implies a one-to-one relationship between $x^\star$ and N, which we denote by $x^\star(N)$. With a change of variable $\delta = \pi/N$, let

$$x^\star(\delta) = \frac{\delta^2}{\delta^2 + \sin^2(\delta)}. \tag{A6}$$

It is enough to show that $x^\star(\delta)$ is strictly increasing in δ, for all $\delta \in (0, \pi)$. Note that

$$\frac{d}{d\delta}x^\star(\delta) = \frac{2\delta(\delta^2 + \sin^2(\delta)) - \delta^2(2\delta + 2\sin(\delta)\cos(\delta))}{(\delta^2 + \sin^2(\delta))^2}.$$

We need to show that, for all $\delta \in (0, \pi)$, $\delta\sin(\delta)(\sin(\delta) - \delta\cos(\delta)) > 0$. Note that, $\delta\sin(\delta) > 0$ for all $\delta \in (0, \pi)$. Therefore, it is enough to show that $G(\delta) = \sin(\delta) - \delta\cos(\delta) > 0$. Since $G(0) = 0$, $G'(\delta) = \cos(\delta) - \cos(\delta) + \delta\sin(\delta) = \delta\sin(\delta) > 0$ for all $\delta \in (0, \pi)$ implies $G(\delta) > 0$. We conclude that $x^\star(\delta)$ is strictly increasing in δ for all $\delta \in (0, \pi)$ or, equivalently, $x^\star(N)$ is strictly decreasing in N for all $N > 1$. $\square$

Proof of Proposition 2.

Part (a). Fix $N \geq 1$. We begin by computing the $\mathcal{V}^\star(N)$ for an arbitrary pure-strategy equilibrium of the game with N firms. Let $(x^\star(N), t_n^\star)_{n=1}^N$ be the equilibrium profile of editorial strategies and $r^\star(t_i) \in \{1, \ldots, N\}$ be the equilibrium information-acquisition strategy for type t_i. Then, $\mathcal{V}^\star(N) = \mathbb{E}_{t_i}(v((x^\star(N), t^\star_{r^\star(t_i)})|t_i)$, where the expectation is taken over t_i, which is uniformly distributed on $T =$

$[-\pi,\pi]$. In equilibrium, we know that $r^\star(t_i)=n$ only if $t_i\in[t_n^\star-\pi/N, t_n^\star+\pi/N]$. Therefore,

$$
\begin{aligned}
\mathcal{V}^\star(N) &= \sum_{n=1}^{N}\int_{t_n^\star-\frac{\pi}{N}}^{t_n^\star+\frac{\pi}{N}} v(x^\star(N),t_n^\star)|t_i)\frac{1}{2\pi}dt_i \\
&= N\int_{-\frac{\pi}{N}}^{\frac{\pi}{N}} v(x^\star(N),0)|t_i)\frac{1}{2\pi}dt_i \\
&= \lambda\frac{N}{\pi}\Big(\sqrt{x^\star(N)}\frac{\pi}{N}+\sqrt{1-x^\star(N)}\sin(\pi/N)\Big)
\end{aligned}
$$

The second equality obtains because, thanks to the symmetry in the equilibrium editorial strategies, we can normalize the location of a firm to 0. By substituting the equilibrium value of $x^\star(N)$ (see Equation A4) in the expression above, we obtain

$$
\begin{aligned}
\mathcal{V}^\star(N) &= \lambda\frac{N}{\pi}\sqrt{(\pi/N)^2+\sin^2(\pi/N)} \\
&= \frac{\lambda}{\sqrt{x^\star(N)}}.
\end{aligned}
$$

The last equality holds by definition of $x^\star(N)$. We are left to show that $\mathcal{V}^\star(N)$ is strictly increasing in N. This follows from Proposition 1, since $x^\star(N)$ is strictly decreasing in N.

Part (b). Fix $N\geq 1$. We begin by computing $\mathcal{P}(N)$ for an arbitrary pure-strategy equilibrium of the game with N firms. Later, we will show that it is strictly decreasing in N. Let $(x^\star(N),t_n^\star)_{n=1}^N$ be an equilibrium profile of editorial strategies, $(p_n^\star)_{n=1}^N$ be the equilibrium prices, and $r^\star(t_i)\in\{1,\ldots,N\}$ be the equilibrium information-acquisition strategy for a type t_i. We have that $\mathcal{P}(N)=I\cdot\mathbb{E}_{t_i}(p^\star_{r^\star(t_i)}(t_i)|t_i)$. Indeed, $\mathbb{E}_{t_i}(p^\star_{r^\star(t_i)}(t_i)|t_i)$ is the industry profit generated by one of the I agents. In equilibrium, $r^\star(t_i)=n$ only if $t_i\in[t_n^\star-\pi/N,t_n^\star+\pi/N]$. Moreover, if $t_i\in[t_n^\star-\pi/N,t_n^\star+\pi/N]$, $p_n^\star(t_i)=v((x^\star(N),t_n^\star)|t_i)-\max\{v((x^\star(N),t_m^\star)|t_i)|m\neq n\}$. If $N=1$, it is immediate to see that $\mathcal{P}(1)=I\int_{-\pi}^{\pi}v((1,0)|t_i)\frac{1}{2\pi}dt_i=\frac{1}{2\sqrt{\pi}}$. If $N>1$, we can write:

$$
\begin{aligned}
\mathcal{P}(N) &= 2I\sum_{n=1}^{N}\int_{t_n^\star}^{t_n^\star+\frac{\pi}{N}} p_n^\star(t_i)\frac{1}{2\pi}dt_i \\
&= \frac{2IN}{2\pi}\int_0^{\frac{\pi}{N}} v(x^\star(N),0)|t_i)-v(x^\star(N),2\pi/N)|t_i)dt_i \\
&= \frac{1}{2\sqrt{\pi}}\sqrt{1-x^\star(N)}\frac{2\sin(\frac{\pi}{N})-\sin(\frac{2\pi}{N})}{\pi/N} \\
&= \frac{1}{\sqrt{\pi}}\frac{1-x^\star(N)}{\sqrt{x^\star(N)}}\Big(1-\cos(\pi/N)\Big).
\end{aligned}
$$

The last equality obtains by using Equation (A4). We want to show that $\mathcal{P}(N)$ is strictly decreasing in N. With a change of variable, let $\delta=\pi/N\in(0,\pi]$. Using Equation (A6), we can rewrite $\mathcal{P}(N)$ as

$$
\mathcal{P}(\delta)=\frac{1}{\sqrt{\pi}}\frac{\sin^2(\delta)}{\delta}\frac{1-\cos(\delta)}{\sqrt{\delta^2+\sin^2(\delta)}}.
$$

Using the expression above, it is immediate to compute $\mathcal{P}(\frac{\pi}{2})$ and verify that it is strictly smaller than $\mathcal{P}(\pi)$. Similarly, we can directly verify that $\mathcal{P}(\frac{\pi}{4})<\mathcal{P}(\frac{\pi}{3})<\mathcal{P}(\frac{\pi}{2})$. We are left to show that $\mathcal{P}(\delta)$ is strictly increasing for all $\delta \in (0, \pi/4]$. Note that $\mathcal{P}(\delta) > 0$ for all $\delta \in (0, \pi/4]$. Therefore, it is enough to show that $(\mathcal{P}(\delta))^2$ is strictly increasing. Dropping the constant and replacing variable δ with x, let

$$G(x) = \frac{\sin^4(x)(1-\cos(x))^2}{x^2(\sin^2(x)+x^2)}.$$

Since $\lim_{x\to 0} G(x) = 0$, it is enough to show that $G'(x) > 0$ for all $x \in (0, \pi/4]$. The sign of $G'(x)$ is determined by the sign of its numerator, which is:

$$\begin{aligned}&\Big(4\sin^3(x)\cos(x)(1-\cos(x))^2 + \sin^5(x)2(1-\cos(x))\Big)\Big(x^2(\sin^2(x)+x^2)\Big) - \\ &\Big(\sin^4(x)(1-\cos(x))^2\Big)\Big(2x(\sin^2(x)+x^2) + x^2(2x + 2\sin(x)\cos(x))\Big)\end{aligned}$$

Dividing everything by $2\sin^3(x)(1-\cos(x))x$, which is strictly positive for $x \in (0, \pi/4]$, we obtain

$$\begin{aligned}&\Big(2\cos(x)(1-\cos(x)) + \sin^2(x)\Big)\Big(x(\sin^2(x)+x^2)\Big) - \\ &(\sin(x)(1-\cos(x)))\Big(2x^2 + \sin^2(x) + x\sin(x)\cos(x))\Big) \\ = \;&(1-\cos(x))\,(3\cos(x)+1))\Big(x(\sin^2(x)+x^2)\Big) - \\ &(\sin(x)(1-\cos(x)))\Big(2x^2 + \sin^2(x) + x\sin(x)\cos(x))\Big).\end{aligned}$$

The equality holds since $\sin^2(x) = 1 - \cos^2(x) = (1-\cos(x))(1+\cos(x))$. We can further divide the last expression by $1 - \cos(x) > 0$ to obtain:

$$\begin{aligned}&(1+3\cos(x))\Big(x\sin^2(x) + x^3\Big) - \sin(x)\Big(2x^2 + \sin^2(x) + x\sin(x)\cos(x))\Big) \\ = \;& x\sin^2(x) + 2\cos(x)x\sin^2(x) + (1+3\cos(x))\,x^3 - \sin(x)2x^2 - \sin^3(x) \\ > \;& x\sin^2(x) + 2\cos(x)x\sin^2(x) + 3\cos(x)x^3 - \sin(x)2x^2 \\ > \;& 3\cos(x)x^3 - \sin(x)2x^2 \\ > \;& 0\end{aligned}$$

The first inequality holds because $x > \sin(x)$ if $x \in (0, \pi/4]$. Similarly, the second inequality holds because $x\sin^2(x) \geq 0$ and $2\cos(x)x\sin^2(x) > 0$ in the same range. The last inequality holds because $3\cos(x)x^3 - \sin(x)2x^2 > 0$ if and only if $3\cos(x) - 2\frac{\sin(x)}{x} > 0$. Since $x > \sin(x)$, $3\cos(x) - 2\frac{\sin(x)}{x} > 3\cos(x) - 2 > 0$, which holds true for $\delta \in (0, \pi/4]$. Therefore, $G'(x) > 0$ for all $x \in (0, \pi/4]$. Hence $G(x)$ is strictly increasing in x and, equivalently, $\mathcal{P}(\delta)$ is strictly increasing in $\delta \in (0, \pi/4]$. □

Proof of Remark 2. Fix an arbitrary type t_i. The value of information for type t_i at an arbitrary equilibrium $(x^\star(N), t_n^\star)_{n=1}^N$ is bounded below by $\underline{v}_N = \lambda(\sqrt{x^\star(N)} + \sqrt{1 - x^\star(N)}\cos(\pi/N))$ and it is bounded above by $\hat{v}_N = \lambda(\sqrt{x^\star(N)} + \sqrt{1 - x^\star(N)})$. Equation A4 implies that $\lim_{N\to\infty} x^\star(N) = 1/2$. Therefore, $\lim_{N\to\infty} \underline{v}_N = \lim_{N\to\infty} \hat{v}_N = \lambda\sqrt{2}$. This implies that $v((x^\star(N), t^\star_{r^\star(t_i)})|t_i)$ converges to $\lambda\sqrt{2}$ as $N \to \infty$. Finally, note that $\lambda\sqrt{2}$ is the first-best value of information of an agent of type t_i, namely $\max_{(x_n, t_n)} v((x_n, t_n)|t_i) = \lambda\sqrt{2}$. □

Proof of Proposition 3. Fix $N \geq 1$ and let $(x^\star(N), t_n^\star)_n^N$ be the equilibrium profile of editorial strategies. Consider two agents t_i and t_j and suppose that, in equilibrium, acquire information from firm n and m, respectively. Denote $s_i = s_i(\omega, (x^\star(N), t_n^\star)$ the signal that agent i receives, $z_i = \mathbb{E}_\omega(u(\omega, t_i)|s_i)$ her expected utility, and $v_i = v((x^\star(N), t_n^\star)|t_i)$. Using Equation A1 and Remark 1, we have that

$$z_i = \frac{v_i}{\lambda}\Big(\sqrt{x^\star(N)}\omega_0 + \sqrt{1 - x^\star(N)}(\omega_1\cos(t_n^\star) + \omega_2\sin(t_n^\star)) + \varepsilon_i\Big) \sim \mathcal{N}(0, 2v_i^2/\lambda^2).$$

Therefore, the correlation between z_i and z_j is given by

$$\begin{aligned}
\rho_{z_i,z_j} &= \frac{\lambda^2 \mathrm{Cov}(z_i, z_j)}{2v_i v_j} \\
&= \frac{1}{2v_i v_j \lambda^2} v_i v_j \lambda^2 \Big(x^\star(N) + (1 - x^\star(N))(\cos(t_n^\star)\cos(t_m^\star)) + \sin(t_n^\star)\sin(t_m^\star)) \Big) \\
&= \frac{1}{2}\Big(x^\star(N) + (1 - x^\star(N))\cos(t_n^\star - t_m^\star) \Big)
\end{aligned}$$

Next, we let the type t_i of agent i be uniformly drawn from the type space T. In this case, the firm n that agent i chooses is random. However, since in equilibrium firms are spread out evenly, agent i is equally likely to choose any of the N firms. Therefore,

$$\begin{aligned}
\mathbb{E}_{t_i,t_j}\rho_{z_i,z_j} &= \frac{1}{2}x^\star(N) + (1 - x^\star(N))\mathbb{E}_{t_j}\mathbb{E}_{t_i}\Big(\cos(t_n^\star - t_m^\star) \Big) \\
&= \frac{1}{2}x^\star(N) + (1 - x^\star(N))\frac{1}{2N^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\Big(\cos(t_n^\star - t_m^\star) \Big).
\end{aligned}$$

We are going to show that $\sum_{n=1}^{N}\sum_{m=1}^{N}\Big(\cos(t_n^\star - t_m^\star) \Big) = 0$. Without loss of generality, we can normalize the location of firm $n = 1$ to be $t_1^\star = 0$. As a consequence, $t_n^\star = \frac{2\pi(n-1)}{N}$, for all n. Therefore, letting $\delta = 2\pi/N$,

$$\begin{aligned}
\sum_{n=1}^{N}\sum_{m=1}^{N}\Big(\cos(t_n^\star - t_m^\star) \Big) &= \sum_{n=0}^{N-1}\sum_{m=0}^{N-1}\cos\Big((n-1)\delta - (m-1)\delta \Big) \\
&= \sum_{n=1}^{N}\sum_{m=1}^{N}\cos\Big((n-m)\delta \Big) \\
&= \sum_{n=1}^{N}\sum_{k=n-N}^{n-1}\cos(k\delta) \\
&= \sum_{n=1}^{N}\Big(\sum_{k=n-N}^{0}\cos(k\delta) + \sum_{k=1}^{n-1}\cos(k\delta) \Big)
\end{aligned}.$$

The second-to-last equality follows from the substitution $k = n - m$. The lower and upper indexes of the summation are substituted accordingly. The highest possible m is N and leads to the lower index $n - N$. The lowest possible m is 1 and leads to the highest index $n - 1$. The last equality follows from splitting in two the summation $\sum_{k=n-N}^{n-1}\cos(k\delta)$. This separates the terms with $k \leq 0$ and $k > 0$.

Using $N\delta = 2\pi$ and $\cos(y) = \cos(y + 2\pi)$ for all $y \in \mathbb{R}$, we have

$$\sum_{k=n-N}^{0}\cos(k\delta) = \sum_{k=n-N}^{0}\cos(k\delta + N\delta) = \sum_{k=n-N}^{0}\cos((k+N)\delta) = \sum_{k=n}^{N}\cos(k\delta).$$

Thus,

$$\begin{aligned}
\sum_{n=1}^{N}\sum_{m=1}^{N}\Big(\cos(t_n^\star - t_m^\star)\Big) &= \sum_{n=1}^{N}\Big(\sum_{k=n-N}^{0}\cos(k\delta) + \sum_{k=1}^{n-1}\cos(k\delta)\Big)\\
&= \sum_{n=1}^{N}\Big(\sum_{k=n}^{N}\cos(k\delta) + \sum_{k=1}^{n-1}\cos(k\delta)\Big)\\
&= \sum_{n=1}^{N}\sum_{k=1}^{N}\cos(k\delta)\\
&= N\sum_{k=1}^{N}\cos(k\delta)\\
&= N\Big(-\frac{1}{2} + \frac{\sin((N+\frac{1}{2})\frac{2\pi}{N})}{2\sin(\frac{\pi}{N})}\Big)\\
&= N\Big(-\frac{1}{2} + \frac{\sin(\frac{\pi}{N}))}{2\sin(\frac{\pi}{N})}\Big) = 0.
\end{aligned}$$

The third-to-last row follows from the Lagrange's trigonometric identity. The last row, instead, uses $\sin(y + 2\pi) = \sin(y)$ for all $y \in \mathbb{R}$.

Therefore, we conclude that $\mathbb{E}_{t_i,t_j}\rho_{z_i,z_j} = \dfrac{1}{2}x^\star(N)$, which is strictly decreasing in N by Proposition 1. ◻

Proof of Proposition 4. Fix arbitrary I and N. Let $(x^\star(N), t_n^\star, p_n^\star)_{n=1}^N$ be the equilibrium profile of editorial strategies and prices. Let $r^\star(t_i) \in \{1, \ldots, N\}$ be the equilibrium information-acquisition strategy for a type t_i. Our first goal is to compute the agent's expected welfare $\mathcal{U}(N)$. Fix a realization of agents' types $t = (t_1, \ldots, t_I)$. We have that:

$$\mathcal{U}(N|t) = \mathbb{E}_\omega(A_{-i}^\star(\omega, t_{-i})u(\omega, t_i)) + v((x^\star(N), t_n^\star)|t_i) - p_n^\star(t_i).$$

Therefore, the agent's expected welfare is

$$\mathcal{U}(N) = \mathbb{E}_{(t_1,\ldots,t_I)}\Big(\mathbb{E}_\omega(A_{-i}^\star(\omega, t_{-i})u(\omega, t_i)) + v((x^\star(N), t_{r^\star(t_i)}^\star)|t_i) - p_{r^\star(t_i)}^\star(t_i)\Big).$$

Note that the first term corresponds to the *indirect* value of information, $\mathcal{G}(N)$, the second term to the *direct* value of information, $\mathcal{V}^\star(N)$, and the last term is the expected price, $\mathcal{P}(N)$. To compute $\mathcal{U}(N)$, we proceed in steps. First, by the proof of Proposition 2.(a), we have that

$$\mathcal{V}(N) = \mathbb{E}_{(t_1,\ldots,t_I)}\Big(v((x^\star(N), t_{r^\star(t_i)}^\star)|t_i)\Big) = \mathbb{E}_{t_i}(v((x^\star(N), t_{r^\star(t_i)}^\star)|t_i)) = \mathcal{V}^\star(N) = \frac{\lambda}{\sqrt{x^\star(N)}}.$$

Second, we focus on $\mathcal{P}$. When $N = 1$, note that $\frac{\mathcal{P}(1)}{I} = \lambda$. If $N \geq 2$, instead, from the proof of Proposition 2.(b), we know that:

$$\mathbb{E}_{(t_1,\ldots,t_I)}(p_{r^\star(t_i)}^\star(t_i)) = \mathbb{E}_{t_i}(p_{r^\star(t_i)}^\star(t_i)) = \frac{\mathcal{P}(N)}{I} = 2\lambda\frac{1 - x^\star(N)}{\sqrt{x^\star(N)}}\Big(1 - \cos(\pi/N)\Big).$$

Lastly, we focus on the firm term of $\mathcal{U}(N)$. We have

$$\begin{aligned}\mathcal{G}(N) &= \mathbb{E}_{(t_1,\ldots,t_I)}\Big(\mathbb{E}_\omega(A^\star_{-i}(\omega,t_{-i})u(\omega,t_i))\Big)\\ &= \mathbb{E}_{t_{-i}}\Big(\mathbb{E}_\omega(A^\star_{-i}(\omega,t_{-i})\mathbb{E}_{t_i}u(\omega,t_i))\Big)\\ &= \mathbb{E}_{t_{-i}}\Big(\mathbb{E}_\omega(A^\star_{-i}(\omega,t_{-i})\omega_0)\Big)\\ &= \mathbb{E}_{\omega_0}\Big(\omega_0\mathbb{E}_{t_{-i}}(\mathbb{E}_{(\omega_1,\omega_2)}(A^\star_{-i}(\omega,t_{-i})))\Big),\end{aligned} \tag{A7}$$

where the second equality holds since $\mathbb{E}_{t_i}u(\omega,t_i) = \omega_0 + \frac{\omega_1}{2\pi}\int_{-\pi}^{\pi}\cos(t_i)dt_i + \frac{\omega_2}{2\pi}\int_{-\pi}^{\pi}\sin(t_i)dt_i = \omega_0$. Next, let us focus on the two inner-most expectations in the last expression. Recall that $A^\star_{-i}(\omega,t_{-i})$ is defined as $\frac{1}{I}\sum_{j\neq i}a^\star_j(\omega,t_j)$, where $a^\star_j(\omega,t_j)$ is the equilibrium approval decision of agent j.By Lemma B10, we have

$$\begin{aligned}\mathbb{E}_{\omega_1,\omega_2,t_{-i}}A^\star_{-i}((\omega_0,\omega_1,\omega_2),t_{-i})) &= \mathbb{E}_{\omega_1,\omega_2,t_{-i}}\frac{1}{I}\sum_{j\neq i}a^\star_j(\omega,t_j) = \frac{1}{I}\sum_{j\neq i}\mathbb{E}_{\omega_1,\omega_2,t_{-i}}a^\star_j(\omega,t_j)\\ &= \frac{1}{I}\sum_{j\neq i}\bar{a}_j(\omega_0) = \frac{I-1}{I}\Phi\Big(\frac{\sqrt{x^\star(N)}}{\sqrt{2-x^\star(N)}}\omega_0\Big).\end{aligned} \tag{A8}$$

Putting equations (A7) and (A8) together, we obtain

$$\mathcal{G}(N) = \frac{I-1}{I}\mathbb{E}_{\omega_0}\Big(\omega_0\Phi\Big(\frac{\sqrt{x^\star(N)}}{\sqrt{2-x^\star(N)}}\omega_0\Big)\Big).$$

Next, we use the integral identity $\int_{\mathbb{R}} y\Phi(\gamma y)\phi(y)dy = \frac{\gamma}{\sqrt{2\pi(1+\gamma^2)}}$ (see Patel and Read, 1996) and let $y = \omega_0$ and $\gamma = \sqrt{x^\star}(N)/\sqrt{2-x^\star}(N)$ to obtain

$$\mathcal{G}(N) = \frac{I-1}{2I\sqrt{\pi}}\sqrt{x^\star(N)} = (I-1)\lambda\sqrt{x^\star(N)}.$$

Therefore, we established that, if $N \geq 2$,

$$\mathcal{U}(N) = \lambda\Big((I-1)\sqrt{x^\star(N)} + \frac{1}{\sqrt{x^\star(N)}} - 2\frac{1-x^\star(N)}{\sqrt{x^\star(N)}}(1-\cos(\pi/N))\Big) \tag{A9}$$

whereas $\mathcal{U}(1) = \lambda(I-1)$ when $N = 1$.

The second part of the proof consists of showing that, when letting $\bar{I} = 3(1+2\pi)$ and $I > \bar{I}$, $\mathcal{U}(N)$ is strictly decreasing in N. It can be directly verified that $\mathcal{U}(2) < \mathcal{U}(1)$ when $I > 4 = \bar{I}$. Therefore, the rest of the proof focuses on the case $N \geq 2$. To this purpose, let us ignore the constant term λ in $\mathcal{U}(N)$, let $\delta = \pi/N$, and let us write x in place of $x^\star(N)$, thus leaving the dependence on δ implicit. Our goal is to show that

$$G(\delta) = (I-1)\sqrt{x} + \frac{1}{\sqrt{x}} - 2\frac{1-x}{\sqrt{x}}(1-\cos(\delta))$$

is strictly *increasing* in $\delta \in (0, \pi/2]$. Taking the derivative with respect to δ, we obtain

$$\begin{aligned}G'(\delta) &= \frac{1}{2}x^{-3/2}((I-1)x-1)x' + 2(1-\cos(\delta))\frac{x'}{x}\frac{1+x}{2\sqrt{x}} - 2\frac{1-x}{\sqrt{x}}\sin(\delta)\\ &\geq \frac{1}{2}x^{-3/2}((I-1)x-1)x' - 2\frac{1-x}{\sqrt{x}}\sin(\delta)\\ &= 2\frac{1-x}{\sqrt{x}}\Big(\frac{1}{4(1-x)}\frac{x'}{x}((I-1)x-1) - \sin(\delta)\Big).\end{aligned}$$

The inequality holds since the middle term in the first expression is positive (by Proposition 1, x is strictly increasing in δ and, thus, $x' > 0$). Therefore, it is sufficient to show that

$$\frac{1}{4(1-x)}\frac{x'}{x}((I-1)x-1)-\sin(\delta)>0.$$

Note that

$$x' = \frac{d}{d\delta}\left(\frac{\delta^2}{\delta^2+\sin^2(\delta)}\right) = \frac{2\delta(\sin^2(\delta)-\delta\sin(\delta)\cos(\delta))}{(\delta^2+\sin^2(\delta))^2} = \frac{2x}{\delta}\frac{(\sin^2(\delta)-\delta\sin(\delta)\cos(\delta))}{\delta^2+\sin^2(\delta)}.$$

By substituting x' into the previous inequality and letting $C = \frac{(I-1)x-1}{2(1-x)}$, we obtain

$$\frac{\sin(\delta)}{\delta^2+\sin^2(\delta)}\Big(C(\sin(\delta)-\delta\cos(\delta))-\delta^3-\delta\sin^2(\delta)\Big)>0.$$

Since $\frac{\sin(\delta)}{\delta^2+\sin^2(\delta)} > 0$ for all $\delta \in (0,\pi]$, the proof is complete if we show that

$$F(\delta) = C(\sin(\delta)-\delta\cos(\delta))-\delta^3-\delta\sin^2(\delta)>0.$$

To this purpose note that $F(0) = 0$. Moreover,

$$\begin{aligned} F'(\delta) &= \delta C\sin(\delta)-3\delta^2-\sin^2(\delta)-2\delta\sin(\delta)\cos(\delta) \\ &= \delta^2\Big(C\frac{\sin(\delta)}{\delta}-3-(\frac{\sin(\delta)}{\delta})^2-2\cos(\delta)\frac{\sin(\delta)}{\delta}\Big) \\ &\geq \delta^2\Big(\frac{2C}{\pi}-6\Big) \\ &\geq \delta^2\frac{I-3(1+2\pi)}{\pi}>0. \end{aligned}$$

The first inequality holds because $\cos(\delta) \leq 1$ and $\frac{\sin(\delta)}{\delta} \in [\frac{2}{\pi}, 1]$ for all $\delta \in (0, \pi/2]$. The second-to-last inequality holds instead because C is bounded below by $C \geq \frac{(I-3)}{2}$ (since $x \geq 1/2$). The last inequality holds because, by assumption, $I > \bar{I} = 3(1+2\pi)$. Therefore, $F'(\delta) > 0$ for all $\delta \in (0, \pi/2]$ and, thus, $F(\delta) > 0$. This implies that $G'(\delta) > 0$ and, thus, that $G(\delta)$ is strictly increasing for all $\delta \in (0, \pi/2]$. Hence, we conclude that $\mathcal{U}(N)$ is strictly decreasing in N for all $N \geq 1$. □

Corollary 1. *For $I \geq 3$, $\mathcal{V}(N) + \mathcal{G}(N)$ is decreasing in N.*

Proof. As shown in the proof of Proposition 4, $\mathcal{V}(N) + \mathcal{G}(N) = \lambda(\frac{1}{\sqrt{x^\star(N)}} + (I-1)\sqrt{x^\star(N)})$. The sign of the derivative with respect to N is determined by $\left(\frac{-1}{x^\star(N)} + (I-N)\right)\frac{dx^\star(N)}{N}$. Since $x^\star(N) > 0.5$, the first term in the parentheses is positive whenever $I \geq 3$. Thus, the result follows from Proposition 1 which establishes $x^\star(N)$ to be decreasing in N. □

Online Appendix

B. Additional Material

B.1. Proof of Proposition 5

The proof of Proposition 5 is divided in five Lemmas, structured as follows: Lemma B1 proves claim (a) in the Proposition; Lemmas B2 proves claim (b); Lemmas B3 and B4 are interim results that we use in the proof of Lemma B5; Finally, Lemma B5 proves claim (c).

Lemma B1 (Existence)**.** *Let f be regular, $N \geq 1$, and $I \geq 1$. An equilibirum of the game exists.*

Proof. We first establish that an equilibrium of the game exists. As in Section 3, we solve the game by backward induction. In the last stage of the game, each agent t_i observe the *realized* profile of (pure) editorial strategies and prices $(x_n, t_n, p_n)_{n=1}^N$. The agents' equilibrium strategies are determined by Lemmas 1 and 2. These results are independent of the distribution f and, thus, they equally apply to the case under consideration. In the second stage of the game, each firm observe the *realized* profile of (pure) editorial strategies and the vector of realized types $(t_1, \ldots, t_I)$ and choose a price $p_n(t_i)$ for each type. Since firms observe types and can set discriminatory prices, the equilibrium profile of prices is independent of the distribution f. As for the uniform case, given the *realized* profile of (pure) editorial strategies, the prevailing equilibrium price for firm n is $\max\{0, v((x_n, t_n)|t) - V((x_{n'}, t_{n'})_{n' \neq n}|t)\}$. Therefore, the expected profit for firm n is:

$$\Pi_n\Big((x_n, t_n)_{n=1}^N\Big) = I \int_{-\pi}^{\pi} \max\{0, v((x_n, t_n)|t) - V((x_{n'}, t_{n'})_{n' \neq n}|t)\} dF(t). \tag{B1}$$

Next, we argue that, for all n, $\Pi_n((x_n, t_n)_{n=1}^N)$ is continuous in $(x_n, t_n)_{n=1}^N$. To see this, let us consider an arbitrary sequence of editorial-strategy profiles $((x_n^k, t_n^k)_{n=1}^N)_k \subset ([1/2, 1] \times T)^N$ converging to $(x_n, t_n)_{n=1}^N$ as $k \to \infty$. We want to show that $\lim_{k\to\infty} \Pi_n((x_n^k, t_n^k)_{n=1}^N) = \Pi_n((x_n, t_n)_{n=1}^N)$. Clearly, the set $([1/2, 1] \times T)^N$ is compact. Moreover, $0 \leq \max\{0, v((x_n^k, t_n^k)|t) - V((x_{n'}^k, t_{n'}^k)_{n' \neq n}|t)\} \leq \lambda\sqrt{2}$ for all k and t. We have

$$\begin{aligned}
\lim_{k\to\infty} \Pi_n((x_n^k, t_n^k)_{n=1}^N) &= I \int_{-\pi}^{\pi} \lim_{k\to\infty} \max\{0, v((x_n^k, t_n^k)|t) - V((x_{n'}^k, t_{n'}^k)_{n' \neq n}|t)\} dF(t) \\
&= I \int_{-\pi}^{\pi} \max\{0, \lim_{k\to\infty} v((x_n^k, t_n^k)|t) - V((x_{n'}^k, t_{n'}^k)_{n' \neq n}|t)\} dF(t) \\
&= I \int_{-\pi}^{\pi} \max\{0, v((x_n, t_n)|t) - V((x_{n'}, t_{n'})_{n' \neq n}|t)\} dF(t) \\
&= \Pi_n((x_n, t_n)_{n=1}^N).
\end{aligned}$$

The first equality follows from the Dominated Convergence Theorem. The second equality holds as the max operator is continuous. The third equality follows because $v((x_n^k, t_n^k)|t) = \lambda(\sqrt{x_n^k} +$

$\sqrt{1-x_n^k}\cos(t-t_n^k))$ is continuous in (x_n^k, t_n^k) for all n. Therefore, $v((x_n,t_n)|t) - V((x_{n'},t_{n'})_{n'\neq n}|t)$ is continuous. Therefore, for all n, the strategy space is compact and the payoff function $\Pi_n\Big((x_n,t_n)_{n=1}^N\Big)$ is continuous. By Glicksberg's theorem, the first stage of the game admits a Nash Equilibrium in mixed editorial strategies. Therefore, by backward induction, the game admits an equilibrium. □

Lemma B2 (Daily Me I). *Let F be regular and $I \geq 1$. Fix an arbitrary sequence of equilibiria, one for each N. Denote by $\mathcal{V}(N|t_i)$ the expected value of information for type t_i in the equilibrium with N firms. Then, $\lim_{N\to\infty}\mathcal{V}(N|t_i) = \bar{\mathcal{V}}$ for all t_i.*

Proof Fix $\delta > 0$ and let $\xi_1 = \frac{\delta}{2\lambda}$. Let $\bar{\mathcal{V}} = \max_{(x_n,t_n)} v((x_n,t_n|t_i) = \lambda\sqrt{2}$. This is the highest possible value that $v((x_n,t_n|t_i)$ can achieve and it is independent of t_i. We show that there exists $\bar{N}$ such that, for all $N > \bar{N}$ and any equilibrium profile of possibly mixed editorial strategies $\chi \in (\Delta([1/2,1]\times T))^N$ we have $\mathcal{V}(N|t_i) = \mathbb{E}_\chi(\max_n\{v(x_n,t_n|t_i)\}) > \bar{\mathcal{V}} - \delta$ for all $t_i \in T$. Suppose not. That is suppose that, for all N, there is an equilibrium profile of possibly mixed editorial strategies χ and a type $\bar{t}_i$ such that $\mathbb{E}_\chi(\max_n\{v(x_n,t_n|t_i)\}) \leq \bar{\mathcal{V}} - \delta$. This implies that, for all $t_j \in [\bar{t}_i - \xi_1, \bar{t}_i + \xi_1]$, $\mathbb{E}_\chi(\max_n\{v(x_n,t_n|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$. To see this, suppose, by way of contradiction, that $\mathbb{E}_\chi(\max_n\{v(x_n,t_n|t_j)\}) > \bar{\mathcal{V}} - \frac{\delta}{2}$. Denote by $n(t_j)$ the random variable that, for each realization of χ, indicates the firm from which t_j acquires information. Note that, for all $t_n \in T$, $\cos(\bar{t}_i - t_n) \geq \cos(t_j - t_n) - \xi_1$, since $\frac{d}{dt}\cos(t-t_n) \leq 1$. We have that,

$$\begin{aligned}
\mathbb{E}_\chi(\max_n\{v((x_n,t_n)|t_i)\}) &\geq \mathbb{E}_\chi(v((x_{n(t_j)}, t_{n(t_j)})|t_i)) \\
&= \lambda\mathbb{E}_\chi(\sqrt{x_{n(t_j)}} + \sqrt{1-x_{n(t_j)}}\cos(\bar{t}_i - t_{n(t_j)})) \\
&\geq \lambda\mathbb{E}_\chi(\sqrt{x_{n(t_j)}} + \sqrt{1-x_{n(t_j)}}(\cos(t_j - t_{n(t_j)}) - \xi_1) \\
&\geq \lambda\mathbb{E}_\chi(\sqrt{x_{n(t_j)}} + \sqrt{1-x_{n(t_j)}}\cos(t_j - t_{n(t_j)})) - \lambda\xi_1 \\
&\geq \mathbb{E}_\chi(\max_n\{v(x_n,t_n|t_j)\}) - \lambda\xi_1 \\
&> \bar{\mathcal{V}} - \tfrac{\delta}{2} - \lambda\xi_1 \\
&= \bar{\mathcal{V}} - \delta.
\end{aligned}$$

The first inequality holds as, in the right-hand side, agent $\bar{t}_i$ chooses the firm $n(t_j)$ that is optimal for t_j. The second inequality holds since $\cos(\bar{t}_i - t_n) \geq \cos(t_j - t_n) - \xi_1$ for all t_n. In summary, this contradicts our assumption that $\mathbb{E}_\chi(\max_n\{v(x_n,t_n|t_i)\}) \leq \bar{\mathcal{V}} - \delta$. Therefore, it must be that $\mathbb{E}_\chi(\max_n\{v(x_n,t_n|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$.

Note that, by continuity of $v((x_n,t_n)|t_j)$ in t_j, there exists $\xi_2 > 0$ such that for all $t_j \in [\bar{t}_i - \xi_2, \bar{t}_i + \xi_2]$ such that $v((1/2,\bar{t}_i)|t_j) \geq \bar{\mathcal{V}} - \frac{\delta}{4}$. Moreover, such ξ_2 is independent of N. Let $\xi = \min\{\xi_1,\xi_2\}$, which in turn is independent of N. We have established that for all $t_j \in [\bar{t}_i - \xi, \bar{t}_i + \xi]$,

$$\mathbb{E}_\chi(\max_{n'\neq n}\{v(x_{n'},t_{n'}|t_j)\}) \leq \mathbb{E}_\chi(\max_n\{v(x_n,t_n|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2} < \bar{\mathcal{V}} - \frac{\delta}{4} \leq v((1/2,\bar{t}_i)|t_j), \tag{B2}$$

Consider an arbitrary firm n who deviates from its equilibrium editorial strategy (χ_n) in favor of the pure strategy ($x_n = 1/2, t_n = \bar{t}_i$). Its expected profits are

$$
\begin{aligned}
\Pi_n((x_n,t_n),(\chi_{n'})_{n'\neq n}) &= I\int_{-\pi}^{\pi}\mathbb{E}_{\chi}\Big(\max\{v((x_n,t_n)|t_j)-V((x_{n'},t_{n'})_{n'\neq n}|t_j),0\}\Big)dF(t_j)\\
&\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}\mathbb{E}_{\chi}\Big(\max\{v((x_n,t_n)|t_j)-V((x_{n'},t_{n'})_{n'\neq n}|t_j),0\}\Big)dF(t_j)\\
&\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}\mathbb{E}_{\chi}\Big(v((x_n,t_n)|t_j)-V((x_{n'},t_{n'})_{n'\neq n}|t_j)\Big)dF(t_j)\\
&= I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}v((x_n,t_n)|t_j)-\mathbb{E}_{\chi}(V((x_{n'},t_{n'})_{n'\neq n}|t_j))dF(t_j)\\
&= I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}v((x_n,t_n)|t_j)-\mathbb{E}_{\chi}(\max_{n'\neq n}\{v(x_{n'},t_{n'}|t_j)\})dF(t_j)\\
&\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}\Big(\bar{\mathcal{V}}-\frac{\delta}{4}-\bar{\mathcal{V}}+\frac{\delta}{2}\Big)f(t_j)dt_j\\
&\geq \frac{IC\delta\xi}{2}.
\end{aligned}
$$

The first inequality holds since the integrand function is everywhere positive. The second inequality holds by monotonicity of the operator $\mathbb{E}_{\chi}$. The second-to-last inequality obtains as a consequence of Equation (B2). The last inequality, instead, obtains because $f(t_j) \geq C > 0$ for all t_j. We established that firm n can secure an expected profit of at least $\frac{IC\delta\xi}{2}$ by deviating to ($x_n = 1/2, t_n = \bar{t}_i$). This lower bound is strictly positive and independent of N. To conclude the proof, note that that the industry profits are bounded above by $I\bar{\mathcal{V}}$. Therefore, when N firms are competing, there is at least one firm, which we denote by n, whose expected equilibrium profits is $\Pi_n(\chi) \leq I\bar{\mathcal{V}}/N$. When N is large, $\frac{IC\delta\xi}{2} > I\bar{\mathcal{V}}/N$ and firm n as a strictly profitable deviation from its equilibrium editorial strategy χ_n in the first stage of the game. Therefore χ is not an equilibrium, a contradiction. □

Lemma B3 (Daily Me II). *Let F be regular and $I \geq 1$. For any t_i, denote by $(x^N_{n(t_i)}, t^N_{n(t_i)})$ the random variable specifying the information structure that agent t_i acquires in an equilibrium with N firms. Then, $(x^N_{n(t_i)}, t^N_{n(t_i)}) \to (1/2, t_i)$ in probability as $N \to \infty$.*

Proof. Fix t_i, $\epsilon > 0$, and a sequence of equilibria. For any N, denote by $(x^N_{n(t_i)}, t^N_{n(t_i)})$ the random variable specifying the information structure that agent t_i acquires in equilibrium. We want to show that, for all $\delta > 0$, there exists $\bar{N}$ such that for all $N > \bar{N}$, $Pr(||(x^N_{n(t_i)}, t^N_{n(t_i)})-(1/2,t_i)|| > \epsilon) < \delta$. Suppose not. Then, there is $\delta > 0$ such that for all $\bar{N}$ there is $N > \bar{N}$ such that $Pr(||(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2,t_i)|| > \epsilon) \geq \delta$. Let (x_n, t_n) be a realization of $(x^N_{n(t_i)}, t^N_{n(t_i)})$ such that $||(x_n,t_n) - (1/2,t_i)|| > \epsilon$. That is, $\sqrt{(x_n - 1/2)^2 + (t_n - t_i)^2} > \epsilon$. This implies that

$$\max\{|x_n - 1/2|, |t_n - t_i|\} > \frac{\epsilon}{\sqrt{2}}.$$

Consider the difference $\bar{\mathcal{V}} - v((x_n,t_n)|t_i) = \lambda(\sqrt{2} - (\sqrt{x_n} + \sqrt{1-x_n}\cos(t_n - t_i)))$. Suppose $|t_n - t_i| >$

$\frac{\epsilon}{\sqrt{2}}$. Then,

$$\bar{\mathcal{V}} - v((x_n,t_n)|t_i) \geq \frac{\lambda}{\sqrt{2}}(1-\cos(t_n-t_i)) > \frac{\lambda}{\sqrt{2}}(1-\cos(\frac{\epsilon}{\sqrt{2}})) =: K_1(\epsilon) > 0.$$

Conversely, suppose that $|x_n - 1/2| > \frac{\epsilon}{\sqrt{2}}$. Then,

$$\bar{\mathcal{V}} - v((x_n,t_n)|t_i) \geq \lambda(\sqrt{2}-\sqrt{x_n}-\sqrt{1-x_n}) > \lambda\Big(\sqrt{2}-\frac{1}{2}(\sqrt{1+\epsilon\sqrt{2}}+\sqrt{1-\epsilon\sqrt{2}})\Big) =: K_2(\epsilon) > 0$$

Let $K(\epsilon) = \min\{K_1(\epsilon), K_2(\epsilon)\}$. We established that, for all realizations of the random variable $(x^N_{n(t_i)}, t^N_{n(t_i)})$ that satisfy $\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon$, we have $\bar{\mathcal{V}} - v((x_n,t_n)|t_i) > K(\epsilon) > 0$. This implies that

$$Pr\Big(\bar{\mathcal{V}} - v((x^N_{n(t_i)}, t^N_{n(t_i)}) \mid t_i) > K(\epsilon)\Big) \geq \delta.$$

Since δ and ϵ are independent of N, we conclude that $\mathcal{V}(N|t_i) = \mathbb{E}_\chi(v((x^N_{n(t_i)}, t^N_{n(t_i)}) \mid t_i))$ does not converge to $\bar{\mathcal{V}}$, a contradiction of Lemma B2. □

Lemma B4. *Let F be regular and $I \geq 1$. For any sequence of equilibria indexed by N,*

$$\mathcal{U}(N) \to \frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\frac{1}{\sqrt{2}}u_j(\omega,t_j))u_i(\omega,t_i)\Big) + \bar{\mathcal{V}}.$$

Proof. Fix $N \geq 1$. Let χ be an equilibrium profile of (possibly mixed) editorial strategies. Denote by $(x_{n(t_i)}, t_{n(t_i)})$ the equilibrium random variable which specifies the information structure that is chosen by agent t_i among those that are offered by the N firms. As shown in the proof of Lemma 2 and Proposition 4, the agent's expected welfare can be written as follows:

$$\mathcal{U}(N) = \mathbb{E}_\chi\mathbb{E}_{(t_1,\dots,t_I)}\Big(\mathbb{E}_\omega(A_{-i}(\omega,t_{-i})u(\omega,t_i)) + v((x_{n(t_i)}, t_{n(t_i)}|t_i) - p_{n(t_i)}(t_i)\Big). \tag{B3}$$

We begin by showing that

$$\lim_{N\to\infty}\mathbb{E}_\chi\mathbb{E}_{t_i}\Big(v((x_{n(t_i)}, t_{n(t_i)}|t_i) - p_{n(t_i)}(t_i)\Big) = \bar{\mathcal{V}}. \tag{B4}$$

To see this, fix t_i. We want to show that $\lim_{N\to\infty}\mathbb{E}_\chi(p_{n(t_i)}(t_i)) = 0$. For each N, recall that $p_{n(t_i)}(t_i) = v((x_{n(t_i)}, t_{n(t_j)}) - \max_{m\neq n(t_i)} v((x_m,t_m)|t_i)$. Thanks to Lemma B2, it is enough to show that

$$\lim_{N\to\infty}\mathbb{E}_\chi(\max_{m\neq n(t_i)} v((x_m,t_m)|t_i)) = \bar{\mathcal{V}}.$$

In Lemma B3, we established that, for all t_i, $(x_{n(t_i)}, t_{n(t_i)}) \to (1/2, t_i)$ in probability. This implies that, for any $t_j \neq t_i$, as N goes to infinity, $Pr(n(t_i) = n(t_j)) \to 0$. This implies that the value generated for type t_i by the firm acquired by t_j should be a lower bound for $\max_{m\neq n(t_i)} v((x_m,t_m)|t_i)$ in the limit. Formally,

$$\lim_{N\to\infty} Pr\left(v((x_{n(t_j)}, t_{n(t_j)})|t_i) \leq \max_{n\neq n(t_i)} v((x_n,t_n)|t_i)\right) = 1.$$

By the Continuous Mapping Theorem, the fact that $(x_{n(t_j)}, t_{n(t_j)})$ converges to $(1/2, t_j)$ in probability implies that $v((x_{n(t_j)}, t_{n(t_j)})|t_i) \to v((1/2, t_j)|t_i)$ in probability. Now fix any $\epsilon > 0$ and $\delta > 0$. There exists t_j close enough to t_i such that $\bar{\mathcal{V}} - v((1/2, t_j)|t_i) < \epsilon$. Therefore,

$$Pr\left(\bar{\mathcal{V}} - \max_{n \neq n(t_i)} v((x_n, t_n)|t_i) < \epsilon\right) > 1 - \delta.$$

That is, $\max_{n \neq n(t_i)} v((x_n, t_n)|t_i)$ converges in probability to $\bar{\mathcal{V}}$. Since $|v|$ is bounded, this implies that $\mathbb{E}_\chi(\max_{n \neq n(t_i)} v((x_n, t_n)|t_i))$ converges to $\bar{\mathcal{V}}$. Together with Lemma B2, this shows that, for any t_i, $\lim_{N\to\infty} \mathbb{E}_\chi(v((x_{n(t_i)}, t_{n(t_i)}|t_i) - p_{n(t_i)}(t_i)) = \bar{\mathcal{V}}$. Since t_i was arbitrary and its distribution is independent of χ, Equation B4 holds.

We are left to show that the first term in Equation B3 converges to

$$\frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}(\Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))u_i(\omega, t_i)).$$

To this purpose, recall that

$$A_{-i}(\omega, t_{-i}) = \frac{1}{I}\sum_{j\neq i} a_j(\omega, t_j) = \frac{1}{I}\sum_{j\neq i} \Phi\Big(\sqrt{x_{n(t_j)}}\omega_0 + \sqrt{1 - x_{n(t_j)}}\big(\cos(t_{n(t_j)})\omega_1 + \sin(t_{n(t_j)})\omega_2\big)\Big).$$

Moreover, note that χ, ω and $(t_1, \ldots, t_I)$ are mutually independent random variables. Therefore, by swapping the order of integration and defining $U_i(\omega) = \mathbb{E}_{t_i}u(\omega, t_i)$ to simplify notation, we obtain

$$\begin{aligned}
\lim_{N\to\infty} \mathbb{E}_\chi\mathbb{E}_{(t_1,\ldots,t_I)}\Big(\mathbb{E}_\omega(A_{-i}(\omega, t_{-i})u(\omega, t_i))\Big) &= \tfrac{1}{I}\lim_{N\to\infty} \mathbb{E}_\omega\Big(\textstyle\sum_{j\neq i}\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)\mathbb{E}_{t_i}u(\omega, t_i)\Big)\\
&= \tfrac{(I-1)}{I}\lim_{N\to\infty} \mathbb{E}_\omega\Big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)\mathbb{E}_{t_i}u(\omega, t_i)\Big)\\
&= \tfrac{(I-1)}{I}\lim_{N\to\infty} \mathbb{E}_\omega\Big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)U_i(\omega)\Big).
\end{aligned}$$

Fix (ω, t_j). By Lemma B3, the random variable $(x_{n(t_i)}, t_{n(t_i)})$ converges in probability to the constant $(\frac{1}{2}, t_j)$. Since $\Phi(\cdot)$ is continuous,

$$\Phi\Big(\sqrt{x_{n(t_j)}}\omega_0 + \sqrt{1 - x_{n(t_j)}}\big(\cos(t_{n(t_j)})\omega_1 + \sin(t_{n(t_j)})\omega_2\big)\Big) \to \Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))$$

in probability, by the continuous mapping theorem. Moreover, since $\Phi(\cdot) \in [0, 1]$, convergence in probability implies convergence in expectation. That is,

$$\lim_{N\to\infty} \mathbb{E}_\chi a(\omega, t_i) = \Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j)).$$

Moreover, since $a_j(\omega, t_j) \leq 1$, for all χ, $U_i(\omega)\mathbb{E}_\chi a_j(\omega, t_j) \leq U_i(\omega)$, and $\mathbb{E}_\omega U_i(\omega) \in \mathbb{R}$. Therefore, by the Dominated Convergence Theorem,

$$\begin{aligned}
\lim_{N\to\infty} \mathbb{E}_\chi\mathbb{E}_{(t_1,\ldots,t_I)}\Big(\mathbb{E}_\omega(A_{-i}(\omega, t_{-i})u(\omega, t_i))\Big) &= \tfrac{(I-1)}{I}\lim_{N\to\infty} \mathbb{E}_\omega\Big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)U_i(\omega)\Big)\\
&= \tfrac{(I-1)}{I}\mathbb{E}_\omega\Big(\mathbb{E}_{t_j}\lim_{N\to\infty} \mathbb{E}_\chi a_j(\omega, t_j)U_i(\omega)\Big)\\
&= \tfrac{(I-1)}{I}\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\tfrac{1}{\sqrt{2}}u_j(\omega, t_j))u_i(\omega, t_i)\Big)
\end{aligned}$$

which concludes the proof. $\square$

Lemma B5. *Let F be regular. There exists $\bar{I}$ such that, for all $I > \bar{I}$, the agent's expected welfare is higher under monopoly than perfect competition. That is, $\mathcal{U}(1) > \lim_{N\to\infty} \mathcal{U}(N)$.*

Proof. We first compute $\mathcal{U}(1)$ and then compute $\lim_{N\to\infty} \mathcal{U}(N)$.

(Monopoly, $N = 1$). Fix f and let $N = 1$. The monopolistic firm chooses $(x^\star, t^\star)$ to maximize $I \int_{-\pi}^{\pi} v((x^\star, t^\star)|t_i) f(t_i) dt_i = \lambda I \int_{-\pi}^{\pi} \sqrt{x^\star} + \sqrt{1 - x^\star} \cos(t^\star - t_i) f(t_i) dt_i$. The first-order condition with respect to t implies $-\int \sin(t^\star - t_i) f(t_i) dt_i = 0$. By symmetry of f around t^m, the first-order condition is met at $t^\star \in \{t^m, t^m + \pi \pmod{\pi}\} \subset [-\pi, \pi]$. The second order condition with respect to t implies $-\int \cos(t^\star - t_i) f(t_i) dt_i \leq 0$. Since $\cos(t + \pi) = -\cos(t)$, we have that either $\int \cos(t^m - t_i) f(t_i) dt_i \geq 0$ or $\int \cos(t^m + \pi - t_i) f(t_i) dt_i \geq 0$ (or both). Without loss of generality let t^m be the type at which $\int \cos(t^m - t_i) f(t_i) dt_i \geq 0$. Therefore, the monopolist locates at $t^\star = t^m$. Define $\beta_F = \int \cos(t^m - t_i) f(t_i) dt_i \in [0, 1]$. Given this, we can rewrite the monopoly profits for an arbitrary x as $I(\sqrt{x} + \sqrt{1 - x}\beta_F)$. The first-order condition with respect to x gives $\sqrt{1 - x^\star} = \beta_F \sqrt{x^\star}$ which implies $x^\star = \frac{1}{1+\beta_F^2}$. Hence, we established that the equilibrium editorial strategy chosen by a monopolist is $(\frac{1}{1+\beta_F^2}, t^m)$.

We now compute $\mathcal{U}(1)$. We begin by establishing that:

$$\mathbb{E}_{t_i}(u(\omega, t_i)) = \omega_0 + \beta_F\big(\cos(t^m)\omega_1 + \sin(t^m)\omega_2\big). \tag{B5}$$

To see this, notice that

$$\begin{aligned}
\mathbb{E}_{t_i}(u(\omega, t_i)) &= \omega_0 + \textstyle\int_{-\pi}^{\pi} (\cos(t_i)\omega_1 + \sin(t_i)\omega_2) f(t_i) dt_i \\
&= \omega_0 + \Big(\cos(t^m) \textstyle\int \cos(t_i - t^m) f(t_i) dt_i - \sin(t^m) \int \sin(t_i - t^m) f(t_i) dt_i\Big)\omega_1 \\
&\quad + \Big(\sin(t^m) \textstyle\int \cos(t_i - t^m) f(t_i) dt_i - \cos(t^m) \int \sin(t_i - t^m) f(t_i) dt_i\Big)\omega_2 \\
&= \omega_0 + \beta_F \left(\cos(t^m)\omega_1 + \sin(t^m)\omega_2\right)
\end{aligned}$$

In the first equality, we used $t_i = t^m + (t_i - t^m)$ and the following two trigonometric identities: $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ and $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$. In the second equality, we used that f is symmetric around t_m (implying $\int \sin(t^m - t) f(t) dt = 0$) and the definition of β_F.

Equation B3 characterizes the expected welfare for a typical agent. When $N = 1$, $p^\star(t_i) = v((x^\star, t^\star)|t_i)$ for all t_i. That is, the monopolist extracts all surplus from each type. Therefore, the last two terms of Equation B3. Thus, using the mutual independence between ω and $(t_1, \ldots, t_I)$, we have

$$\begin{aligned}
\mathcal{U}(1) &= \mathbb{E}_{(t_1,\ldots,t_I)}\Big(\mathbb{E}_\omega(A^\star_{-i}(\omega, t_{-i}) u(\omega, t_i))\Big) \\
&= \mathbb{E}_\omega\Big(\mathbb{E}_{t_{-i}}(A^\star_{-i}(\omega, t_{-i}))\mathbb{E}_{t_i}(u(\omega, t_i))\Big) \\
&= \mathbb{E}_\omega\Big(\mathbb{E}_{t_{-i}}(A^\star_{-i}(\omega, t_{-i}))(\omega_0 + \beta_F(\cos(t^m)\omega_1 + \sin(t^m)\omega_2))\Big) \\
&= \mathbb{E}_\omega\Big(\tfrac{I-1}{I}\mathbb{E}_{t_j}(a^\star(\omega, t_j))(\omega_0 + \beta_F(\cos(t^m)\omega_1 + \sin(t^m)\omega_2))\Big) \\
&= \mathbb{E}_\omega\Big(\tfrac{I-1}{I}\Phi(\sqrt{x^\star}\omega_0 + \sqrt{1 - x^\star}(\cos(t^m)\omega_1 + \sin(t^m)\omega_2))(\omega_0 + \beta_F(\cos(t^m)\omega_1 + \sin(t^m)\omega_2))\Big)
\end{aligned} \tag{B6}$$

Next, denote $y = \cos(t^m)\omega_1 + \sin(t^m)\omega_2 \sim \mathcal{N}(0,1)$, $a = \sqrt{x^\star}$, and $b = \sqrt{1-x^\star}$. We have

$$\begin{aligned}
\mathcal{U}(1) &= \tfrac{I-1}{I}\mathbb{E}_{\omega_0}\Big(\mathbb{E}_y\left(\Phi(a\omega_0 + by)(\omega_0 + \beta_F y)\right)\Big) \\
&= \tfrac{I-1}{I}\mathbb{E}_{\omega_0}\mathbb{E}_y\left(\Phi(a\omega_0 + by)(\omega_0 + \beta_F y)\right) \\
&= \tfrac{I-1}{I}\mathbb{E}_{\omega_0}\Big(\omega_0\mathbb{E}_y\left(\Phi(a\omega_0 + by)\right) + \beta_F\mathbb{E}_y\left(y\Phi(a\omega_0 + by)\right)\Big) \\
&= \tfrac{I-1}{I}\mathbb{E}_{\omega_0}\Big(\omega_0\Phi\left(\tfrac{a\omega_0}{\sqrt{1+b^2}}\right) + \tfrac{\beta_F b}{\sqrt{1+b^2}}\phi\left(\tfrac{a\omega_0}{\sqrt{1+b^2}}\right)\Big)
\end{aligned}$$

The last line makes use of the properties of the normal distribution. Specifically, since $y \sim \mathcal{N}(0,1)$, we have that, for all $\alpha, \gamma \in \mathbb{R}$, $\mathbb{E}_y(\Phi(\alpha + \gamma y)) = \Phi(\frac{\alpha}{\sqrt{1+\gamma^2}})$ and $\mathbb{E}_y(y\Phi(\alpha + \gamma y)) = \frac{\gamma}{\sqrt{1+\gamma^2}}\phi(\frac{\alpha}{\sqrt{1+\gamma^2}})$ (for both, see Patel and Read, 1996).

Finally, we integrate with respect to w_0. Define $\tilde{b} = \frac{a}{\sqrt{1+b^2}}$. We have:

$$\begin{aligned}
\mathcal{U}(1) &= \tfrac{I-1}{I}\mathbb{E}_{\omega_0}\Big(\omega_0\Phi(\tilde{b}\omega_0) + \tfrac{\beta_F b}{\sqrt{1+b^2}}\phi(\tilde{b}\omega_0)\Big) \\
&= \tfrac{I-1}{I}\Big(\tfrac{\tilde{b}}{\sqrt{1+\tilde{b}^2}}\phi(0) + \tfrac{\beta_F b}{\sqrt{1+b^2}}\tfrac{1}{\sqrt{1+\tilde{b}^2}}\phi(0)\Big) \\
&= \tfrac{I-1}{I2\sqrt{\pi}}\left(\sqrt{x^\star} + \beta_F\sqrt{1-x^\star}\right) \\
&= (I-1)\lambda\sqrt{1+\beta_F^2}
\end{aligned}$$

In the second line, we used once again the integral properties of the normal distributed listed before. In the third line, we substituted the definitions of $\tilde{b}$, b, and a and used the fact that $\phi(0) = 1/\sqrt{2\pi}$. In the last line, we used $x^\star = \frac{1}{1+\beta_F^2}$ and $\lambda = \frac{1}{2I\sqrt{\pi}}$. In passing, note that $\beta_F = 0$ if f is uniform. In such case, the value of $\mathcal{U}(1)$ matches the one computed in the Proof of Proposition 4.

(Perfect Competition, $N = \infty$). Lemma B4 showed that, for any sequence of equilibria indexed by N,

$$\lim_{N\to\infty}\mathcal{U}(N) = \frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\frac{1}{\sqrt{2}}u_j(\omega,t_j))u_i(\omega,t_i)\Big) + \bar{\mathcal{V}}.$$

We begin by focusing on the first term of the right-hand side. Note that

$$\begin{aligned}
&\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\tfrac{1}{\sqrt{2}}u_j(\omega,t_j))u_i(\omega,t_i)\Big) \\
=\;& \mathbb{E}_{\omega,t_j}\Big(\Phi(\tfrac{1}{\sqrt{2}}u_j(\omega,t_j))\mathbb{E}_{t_i}u_i(\omega,t_i)\Big) \\
=\;& \mathbb{E}_{\omega,t_j}\Big(\Phi(\tfrac{1}{\sqrt{2}}(\omega_0 + \cos(t_j)\omega_1 + \sin(t_j)\omega_2))(\omega_0 + \beta_F(\cos(t^m)\omega_1 + \sin(t^m)\omega_2))\Big),
\end{aligned}$$

where we used Equation B5 and the fact that (ω, t_i, t_j) are independent.

Fix any t_j and consider first the expectation with respect to ω. To simplify notation, let us write $t_j = t$ and $a = b = 1/\sqrt{2}$. Then,

$$\begin{aligned}
&\mathbb{E}_\omega\Big(\Phi(a\omega_0 + b(\cos(t)\omega_1 + \sin(t)\omega_2)))(\omega_0 + \beta_F(\cos(t^m)\omega_1 + \sin(t^m)\omega_2))\Big) \\
=\;& \mathbb{E}_\omega(\omega_0\Phi(a\omega_0 + b(\cos(t)\omega_1 + \sin(t)\omega_2))))+ \\
&\beta_F\cos(t^m)\mathbb{E}_\omega(\Phi(a\omega_0 + b(\cos(t)\omega_1 + \sin(t)\omega_2)))\omega_1)+ \\
&\beta_F\sin(t^m)\mathbb{E}_\omega(\Phi(a\omega_0 + b(\cos(t)\omega_1 + \sin(t)\omega_2)))\omega_2)\omega_2))
\end{aligned}$$

We compute this expectation term by term. We begin with the first term. Let $y_t = \cos(t_j)\omega_1 + \sin(t_j)\omega_2$ and note that $y_t \sim \mathcal{N}(0,1)$. Define $\tilde{b} = \frac{a}{\sqrt{1+b^2}}$. Then, using the independence of $(\omega_0, \omega_1, \omega_2)$, we have

$$\begin{aligned}
\mathbb{E}_\omega(\omega_0\Phi(a\omega_0 + b(\cos(t)\omega_1 + \sin(t)\omega_2)))) &= \mathbb{E}_{\omega_0}(\omega_0\mathbb{E}_{y_t}\Phi(a\omega_0 + by_t)) \\
&= \mathbb{E}_{\omega_0}(\omega_0\Phi(\tfrac{a}{\sqrt{1+b^2}}\omega_0)) \\
&= \tfrac{\tilde{b}}{\sqrt{1+\tilde{b}^2}}\phi(0) \\
&= \tfrac{1}{2}\phi(0).
\end{aligned}$$

We now focus on the second term. We first integrate ω_1, then ω_2, and finally ω_0. As we have done before, we use the integral identity $\mathbb{E}_z(z\Phi(\alpha + \gamma z)) = \frac{\gamma}{\sqrt{1+\gamma^2}}\phi(\frac{\alpha}{\sqrt{1+\gamma^2}})$ for all $\alpha, \gamma \in \mathbb{R}$ and $z \sim \mathcal{N}(0,1)$. Moreover, we use a new integral identity that gives us $\mathbb{E}(\phi(\alpha+\gamma z)) = \frac{1}{\sqrt{1+\gamma^2}}\phi(\frac{\alpha}{\sqrt{1+\gamma^2}})$ (see Patel and Read, 1996). We obtain:

$$\begin{aligned}
&\mathbb{E}_\omega(\Phi(a\omega_0 + b(\cos(t)\omega_1 + \sin(t)\omega_2)))\omega_1) \\
= \;& \mathbb{E}_{\omega_0,\omega_2}(\mathbb{E}_{\omega_1}\Phi((a\omega_0 + b\sin(t)\omega_2) + b\cos(t)\omega_1)))\omega_1) \\
= \;& \tfrac{b\cos(t)}{\sqrt{1+b^2\cos^2(t)}}\mathbb{E}_{\omega_0,\omega_2}\phi(\tfrac{a\omega_0+b\sin(t)\omega_2}{\sqrt{1+b^2\cos^2(t)}}) \\
= \;& \tfrac{b\cos(t)}{\sqrt{1+b^2}}\mathbb{E}_{\omega_0}\phi(\tfrac{a\omega_0}{\sqrt{1+b^2}}) \\
= \;& \tfrac{b\cos(t)}{\sqrt{1+b^2}}\tfrac{1}{\sqrt{1+\tilde{b}^2}}\phi(0) \\
= \;& \tfrac{1}{2}\cos(t)\phi(0).
\end{aligned}$$

We now focus on the last term. We first integrate ω_2, then ω_1, and finally ω_0. Otherwise, the steps and properties we follow are identical to those from the second term.

$$\begin{aligned}
&\mathbb{E}_\omega(\Phi(a\omega_0 + b(\cos(t)\omega_1 + \sin(t)\omega_2)))\omega_2) \\
= \;& \mathbb{E}_{\omega_0,\omega_1}(\mathbb{E}_{\omega_2}\Phi((a\omega_0 + b\sin(t)\omega_2) + b\cos(t)\omega_1)))\omega_2) \\
= \;& \tfrac{b\sin(t)}{\sqrt{1+b^2\sin^2(t)}}\mathbb{E}_{\omega_0,\omega_1}\phi(\tfrac{a\omega_0+b\cos(t)\omega_1}{\sqrt{1+b^2\sin^2(t)}}) \\
= \;& \tfrac{b\sin(t)}{\sqrt{1+b^2}}\mathbb{E}_{\omega_0}\phi(\tfrac{a\omega_0}{\sqrt{1+b^2}}) \\
= \;& \tfrac{b\sin(t)}{\sqrt{1+b^2}}\tfrac{1}{\sqrt{1+\tilde{b}^2}}\phi(0) \\
= \;& \tfrac{1}{2}\sin(t)\phi(0).
\end{aligned}$$

Putting all together, we have that

$$\begin{aligned}
\lim_{N\to\infty}\mathcal{U}(N) &= \tfrac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\tfrac{1}{\sqrt{2}}u_j(\omega,t_j))u_i(\omega,t_i)\Big) + \bar{\mathcal{V}} \\
&= \tfrac{I-1}{I}\mathbb{E}_{t_j}(\tfrac{1}{2}\phi(0) + \tfrac{1}{2}\beta_F\phi(0)(\cos(t_j)\cos(t^m) + \sin(t_j)\sin(t^m)) + \bar{\mathcal{V}} \\
&= \tfrac{I-1}{I}\tfrac{1}{2}\phi(0)\mathbb{E}_{t_j}(1 + \beta_F\cos(t_j - t^m)) + \bar{\mathcal{V}} \\
&= \tfrac{I-1}{I}\tfrac{1}{2}\tfrac{1}{\sqrt{2\pi}}(1 + \beta_F^2) + \lambda\sqrt{2} \\
&= \lambda(I-1)\tfrac{1}{\sqrt{2}}(1 + \beta_F^2) + \lambda\sqrt{2}.
\end{aligned}$$

For the fourth equality, we use the definition β_F. Moreover, we used the fact that $\bar{\mathcal{V}} = \lambda\sqrt{2}$ and $\phi(0) = \frac{1}{\sqrt{2\pi}}$. In passing, note that $\beta_F = 0$ if f is uniform. In such case, the value of $\lim_{N\to\infty}\mathcal{U}(N)$ matches the one computed in Equation A9.

(Comparsion Between Monopoly and Perfect Competition). We established that:

$$\mathcal{U}(1) - \lim_{N\to\infty} \mathcal{U}(N) = \lambda(I-1)\sqrt{1+\beta_F^2}\Big(1 - \sqrt{\frac{1+\beta_F^2}{2}}\Big) - \lambda\sqrt{2}$$

Note that for all non-degenerate distributions F, $\beta_F \in [0, 1)$. Therefore, $1 > \sqrt{\frac{1+\beta_F^2}{2}}$. That is, for any distribution F, there exists a $\bar{I}$ such that, for all $I > \bar{I}$, $\mathcal{U}(1) > \lim_{N\to\infty} \mathcal{U}(N)$. That is, the expected welfare of a typical agent is higher when $N = 1$ than when $N \to \infty$. □

Remark B1. *Fix a regular F. For all N, the equilibrium editorial strategy of the monopolist maximizes $\mathcal{G}(N)$.*

Proof. Fix N. Let $(x_{n(i)}, t_{n(i)})$ denote the editorial strategy associated with the signal acquired by type t_i and $a^\star(\omega, t_i)$ denote the optimal approval decision for type t_i given the signal induced by $(x_{n(i)}, t_{n(i)})$.

$$\begin{aligned}
\mathcal{G}(N) &= \mathbb{E}_{(t_1,\ldots,t_I)}\Big(\mathbb{E}_\omega(A^\star_{-i}(\omega, t_{-i})u(\omega, t_i))\Big) \\
&= \mathbb{E}_\omega\Big(\tfrac{I-1}{I}\mathbb{E}_{t_j}(a^\star(\omega, t_j))(\omega_0 + \beta_F(\cos(t^m)\omega_1 + \sin(t^m)\omega_2))\Big) \\
&= \mathbb{E}_\omega\Big(\tfrac{I-1}{I}\mathbb{E}_{t_j}(\Phi\Big(\sqrt{x} + \sqrt{1-x}(\cos(t)\omega_1 + \sin(t)\omega_2)\Big))(\omega_0 + \beta_F(\cos(t^m)\omega_1 + \sin(t^m)\omega_2))\Big) \\
&= \tfrac{I-1}{I}\tfrac{1}{\sqrt{2}}\phi(0)\Big(\sqrt{x} + \sqrt{1-x}\beta_F\cos(t - t^m)\Big)
\end{aligned}$$

The second line is established Equation (B6) from the proof of Lemma B5. Suppose that each agent's approval decision depends on the sign of the signal she receives (to be confirmed below). If everyone follows the signal, then the optimal solution involves providing the same information structure to all agents. That is, the solution is independent of N. Therefore, the approval probability can written as a function of ω and a single editorial strategy (x, t). The last line follows from implementing the same steps of the second part (i.e. Perfect Competition, $N \to \infty$) of the proof of Lemma B5. Given this derivation, it is immediate to see that $\mathcal{G}(N)$ is maximized when $t = t^m$ and $x = (1 + \beta_F^2)^{-1}$. As shown in the first part (i.e. Monopoly, $N = 1$) of the proof of Lemma B5, this coincides with the equilibrium editorial strategy of the monopolist. To conclude, note that $x > 1/2$, since $\beta_F < 1$, which implies the signal induced is positively correlated with $u(\omega, t_i)$ for any t_i, confirming that all types would indeed vote according to the sign of the signal.

B.2. A Model of Multimedia

In the baseline version of the model, we assumed that agents can acquire at most one signal. This section discusses an extension of our main result to the case when agents can simultaneously acquire information from multiple firms. One obvious effect of increasing the number of competing firms—for example from $N = 1$ to $N = 2$—is that agents can acquire more signals. This can in principle affect the results of the paper. Indeed, if agents can process an unlimited number of signals at no cost and the price of these signals converges to zero with N, then agents could learn the state as the market becomes perfectly competitive.

While extending the main result to the multimedia case, we maintain the assumption that agents are constrained in how many signals they can acquire or process. In particular, we assume that each agent is endowed with a unit of time that she can divide among N firms. That is, agent i chooses α subject to $\sum_n \alpha_n \leq 1$ with $\alpha_n \in [0,1]$ for all n. α_n represents the fraction of time that the agent spends on the signal supplied by firm n. It is convenient to model firms' editorial strategies using the vector notation introduced in Section 2. In particular, firm n chooses $b_n \in \mathbb{R}^3$ such that $||b_n|| \leq 1$. Fix a profile of editorial strategies $(b_1, \ldots, b_N)$ and suppose that agent i's information-acquisition strategy is α. We assume that agent i observes the realization from a mixture signal characterized by b_α, where $b_{\alpha,k} = \sum_{n=1}^N \alpha_n b_{n,k}$ for $k = \{0,1,2\}$.

$$s_i(\omega, b|\alpha) = (\sum_{n=1}^{N} \alpha_n b_n) \cdot \omega + \varepsilon_i = b_\alpha \cdot \omega + \varepsilon_i. \tag{B7}$$

Note that when α is degenerate, this reduces to our baseline model. Moreover, the value of information given α, denoted by $v(b_\alpha|t_i)$ is still characterized by Lemma 2.[26] Since ε_i does not scale with N, this formulation preserves a key feature of the baseline model, namely that the agent is constrained in how much she can learn about the state. At the same time, the ability to mix among multiple signals allows an agent to "construct" signals that are better tailored to her own needs.[27] Nonetheless, the best mixture that type t_i can acquire is $b_{t_i} := \frac{1}{\sqrt{2}}(1, \cos(t_i), \sin(t_i))$, leading to the same first-best value $\bar{\mathcal{V}}$.

The main challenge in such a model is to determine how profits of the firms are linked to the value of information created for each agent and the competition in the market. To make the model tractable, we make a reduced-form assumption on how a firm's profit from an agent depends on the *surplus* generated by the firm for the agent, that is, the difference between the agent's first-best value and the second-best value she could have obtained in the absence of this firm. We assume that firm n's revenue from agent t_i is

$$p_n^\star(t_i|b) = \frac{1}{N}\Big(\max_{\alpha} v(b_\alpha|t_i) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_i)\Big). \tag{B8}$$

When α is degenerate, that is, agent i acquires information from a single firm, then firm n's revenue is the same as in our baseline model, net of weight $\frac{1}{N}$.[28] Overall, a profile of editorial strategies b induces profits for firm n that are $\Pi_n(b_n, b_{-n}) = \int_T p_n^\star(t_i|b)dF(t_i)$.

[26] Lemma 2 uses the notation of θ_i—instead of t_i as we do in this section—to denote an agent's type. Remark 1 establishes how one variable can be transformed into the other. For each t_i, there is an equivalent $\theta_i = (1, \cos(t_i), \sin(t_i))$.

[27] For example, suppose that $t_i = \pi/4$, $b_1 = (0,1,0)$, and $b_2 = (0,0,1)$. Fix $\alpha_i(1) = \alpha_i(2) = \frac{1}{2}$. Then $v(b_{\alpha_i}|t_i) > v(b_1|t_i) = v(b_2|t_i)$. That is, the agent does strictly better by mixing than by acquiring a single signal.

[28] Any weighting vector $(w_i(1|b), \ldots, w_i(N|b))$ that possibly depends on i and b in a continuous manner would generate the same results.

Agents choose α to maximize $v(b_\alpha|t_i) - \sum_{n:\alpha_n>0} p_n^\star(t_i|b)$. Note that the solution of the agent's maximization problem depends on $\max_\alpha v(b_\alpha|t_i)$ via $p_n^\star(t_i|b)$. Remark B2, which we present after the proof of the main result of this section, shows that if $\hat{\alpha}_i \in \arg\max_\alpha v(b_\alpha|t_i)$, then it also solves the agent's maximization problem. Therefore, just like in the baseline model, we can interpret $p_n^\star(t_i|b)$ as a *price* that the agent has to pay to firm n to acquire its signal.

The next result shows that Proposition 5 extends to the multimedia model.

Proposition 6 (Multimedia)**.** *Fix any regular distribution F.*

a. (Existence) An equilibrium exists for any $N \geq 1$ and $I \geq 1$.

b. (Daily-me) Fix any t_i. As $N \to \infty$, the equilibrium expected value of information for type t_i, $\mathcal{V}(N|t_i)$, converges in probability to the first-best value $\bar{\mathcal{V}}$.

c. (Inefficiency) There exists $\bar{I}$ such that, for all $I > \bar{I}$, the agent's welfare in the multimedia model is higher under the monopoly than perfect competition, i.e. $\mathcal{U}(1) > \lim_{N\to\infty} \mathcal{U}(N)$.

Proof of Proposition 6. This proof is divided in five steps that closely follow and leverage on the proofs of Lemma B1 to B5.

Step 1. *(Equilibrium Existence.)* Fix N and I. The timeline in the multimedia model is as in Figure 1, with the difference that prices are being set exogenously as a function of the chosen profile of editorial strategies b. Using backward induction, we argue that an equilibrium of the game exists. Fix an arbitrary profile of *information-acquisition* strategies $(\alpha_1, \ldots, \alpha_I)$. Then, Lemma 1 still characterizes the agents' equilibrium *approval* decisions. Now consider an arbitrary profile of editorial strategies b and the profile prices $(p_n^\star(t_i|b))_n$ that ensues. Agent t_i's equilibrium *information-acquisition* strategy consists of choosing α to maximize $v(b_\alpha|t_i) - \sum_{n:\alpha_n>0} p_n^\star(t_i|b)$. Note that $v(b_\alpha|t_i) - \sum_{n:\alpha_n>0} p_n^\star(t_i|b)$ is continuous in $\alpha \in \mathbb{R}^N$ and that $\{\alpha \,|\, \alpha_n \in [0,1], \sum \alpha_n \leq 1\}$ is compact. Therefore, the agent's problem admits a solution. In the first stage of the game, firms simultaneously choose b_n. Their payoff function is $\Pi_n(b_n, b_{-n}) = \int_T p_n^\star(t_i|b)dF(t_i) = \frac{1}{N}\int_T \max_\alpha v(b_\alpha|t_i) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_i)dF(t_i)$. By the theorem of the maximum, $\max_\alpha v(b_\alpha|t_i)$ is continuous is b. By a similar argument, one can show that $\max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_i)$ is also continuous is b. Therefore, $\Pi_n(b_n, b_{-n})$ is continuous in b for all n. As in Lemma B1, we invoke Glicksberg's theorem to argue that, in the first stage of the game, a Nash Equilibrium exists in (possibly mixed) editorial strategies. By backward induction, we have shown that the game admits an equilibrium.

Step 2. *(Convergence of $\mathbb{E}_\chi(\max_\alpha v(b_\alpha|t_i))$.)* Fix $\delta > 0$ and let $\xi_1 = \frac{\delta}{2\underline{\lambda}}$ where $\underline{\lambda} = \frac{1}{2I\sqrt{\pi}}$. We show that there exists $\bar{N}$ such that, for all $N > \bar{N}$ and any equilibrium profile of possibly mixed editorial strategies χ, we have $\mathbb{E}_\chi(\max_\alpha v(b_\alpha|t_i)) > \bar{\mathcal{V}} - \delta$ for all $t_i \in T$. Suppose not. That is suppose that, for all N, there is an equilibrium profile of possibly mixed editorial strategies χ and a type $\bar{t}_i$ such that $\mathbb{E}_\chi(\max_\alpha v(b_\alpha|t_i)) \leq \bar{\mathcal{V}} - \delta$. This implies that, for all $t_j \in [\bar{t}_i - \xi_1, \bar{t}_i + \xi_1]$, $\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$. To see this, suppose, by way of contradiction, that $\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_j)\}) > \bar{\mathcal{V}} - \frac{\delta}{2}$. Denote by $\alpha(t_i)$ the random variable that, for each realization of χ, indicates the information acquisition strategy of type t_i. That is, $\alpha(t_i) \in \arg\max_\alpha v(b_\alpha|t_i)$. Note that there exists a $t_{\alpha(t_j)} \in T$ and $y_{\alpha(t_j)} \in [0,1]$ and $\lambda_{\alpha(t_j)}$ such that $b_{\alpha(t_j)} = (b_{\alpha(t_j),0}, \sqrt{y_{\alpha(t_j)}}\cos(t_{\alpha(t_j)}), \sqrt{y_{\alpha(t_j)}}\sin(t_{\alpha(t_j)}))$, $y_{\alpha(t_j)} = b^2_{\alpha(t_j),1} + b^2_{\alpha(t_j),2}$ and $\lambda_{\alpha(t_j)} = \frac{1}{I\sqrt{2\pi(1+\|b_{\alpha(t_j)}\|^2)}}$. Note that, for all $t_{\alpha(t_j)} \in T$, $\cos(\bar{t}_i - t_{\alpha(t_j)}) \geq \cos(t_j - t_{\alpha(t_j)}) - \xi_1$, since $\frac{d}{dt}\cos(t - t_{\alpha(t_j)}) \leq 1$. We have that,

$$\begin{aligned}
\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_i)\}) &\geq \mathbb{E}_\chi(v(b_{\alpha(t_j)}|t_i)) \\
&= \lambda_{\alpha(t_j)}\mathbb{E}_\chi(b_{\alpha(t_j),0} + \sqrt{y_{\alpha(t_j)}}\cos(\bar{t}_i - t_{\alpha(t_j)})) \\
&\geq \lambda_{\alpha(t_j)}\mathbb{E}_\chi(b_{\alpha(t_j),0} + \sqrt{y_{\alpha(t_j)}}(\cos(t_j - t_{\alpha(t_j)}) - \xi_1) \\
&\geq \lambda_{\alpha(t_j)}\mathbb{E}_\chi(b_{\alpha(t_j),0} + \sqrt{y_{\alpha(t_j)}}\cos(t_j - t_{\alpha(t_j)})) - \lambda_{\alpha(t_j)}\xi_1 \\
&\geq \mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_j)\}) - \lambda_{\alpha(t_j)}\xi_1 \\
&> \bar{\mathcal{V}} - \tfrac{\delta}{2} - \lambda_{\alpha(t_j)}\xi_1 \\
&\geq \bar{\mathcal{V}} - \delta.
\end{aligned}$$

The first inequality holds as, in the right-hand side, agent $\bar{t}_i$ chooses the information acquisition strategy $\alpha(t_j)$ that is optimal for t_j. The second inequality holds since $\cos(\bar{t}_i - t_{\alpha(t_j)}) \geq \cos(t_j - t_{\alpha(t_j)}) - \xi_1$ for all $t_{\alpha(t_j)}$ and $\sqrt{y_{\alpha(t_j)}} < 1$. The last inequality uses that $\lambda_{\alpha(t_j)}/\underline{\lambda} \geq 1$. In summary, this contradicts our assumption that $\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_i)\}) \leq \bar{\mathcal{V}} - \delta$. Therefore, it must be that $\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$.

Note that, by continuity of $v(b_{\bar{t}_i}|t_j)$ in t_j, there exists $\xi_2 > 0$ such that for all $t_j \in [\bar{t}_i - \xi_2, \bar{t}_i + \xi_2]$ such that $v(b_{\bar{t}_i}|t_j) \geq \bar{\mathcal{V}} - \frac{\delta}{4}$. Moreover, such ξ_2 is independent of N. Let $\xi = \min\{\xi_1, \xi_2\}$, which in turn is independent of N. We have established that for all $t_j \in [\bar{t}_i - \xi, \bar{t}_i + \xi]$,

$$\mathbb{E}_\chi(\max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j)) \leq \mathbb{E}_\chi(\max_\alpha v(b_\alpha|t_j)) \leq \bar{\mathcal{V}} - \frac{\delta}{2} < \bar{\mathcal{V}} - \frac{\delta}{4} \leq v(b_{\bar{t}_i}|t_j), \tag{B9}$$

Now fix an arbitrary firm n who deviates from its equilibrium editorial strategy χ_n in favor of the pure editorial strategy $b_{\bar{t}_i}$. Its expected profits are

$$\begin{aligned}
\Pi_n(b_{\bar{t}_i}, (\chi_{n'})_{n'\neq n}) &= \frac{I}{N}\int_{-\pi}^{\pi} \mathbb{E}_\chi\Big(\max\{v(b_{\bar{t}_i}|t_j) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j), 0\}\Big) dF(t_j) \\
&\geq \frac{I}{N}\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \mathbb{E}_\chi\Big(\max\{v(b_{\bar{t}_i}|t_j) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j), 0\}\Big) dF(t_j) \\
&\geq \frac{I}{N}\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \mathbb{E}_\chi\Big(v(b_{\bar{t}_i}|t_j) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j)\Big) dF(t_j) \\
&= \frac{I}{N}\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} v(b_{\bar{t}_i}|t_j) - \mathbb{E}_\chi(\max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j)) dF(t_j) \\
&\geq \frac{I}{N}\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \Big(\bar{\mathcal{V}} - \frac{\delta}{4} - \bar{\mathcal{V}} + \frac{\delta}{2}\Big) f(t_j) dt_j \\
&\geq \frac{IC\delta\xi}{2N}.
\end{aligned}$$

The first inequality holds since the integrand function is everywhere positive. The second inequality holds by monotonicity of the operator $\mathbb{E}_\chi$. The second-to-last inequality obtains as a consequence of Equation (B9). The last inequality, instead, obtains because $f(t_j) \geq C > 0$ for all t_j. We established that firm n can secure an expected profit of at least $\frac{IC\delta\xi}{2N}$ by deviating to $b_{\bar{t}_i}$. This lower bound is strictly positive and decreasing in N at rate $1/N$. To conclude the proof, note that by Lemma B6 the maximum amount paid for information by any agent can at most be $\bar{\mathcal{V}}/N$. This implies that the industry profits are bounded above by $I\bar{\mathcal{V}}/N$. Therefore, when N firms are competing, there is at least one firm, which we denote by n, whose expected equilibrium profits is $\Pi_n(\chi) \leq I\bar{\mathcal{V}}/N^2$. When N is large, $\frac{IC\delta\xi}{2N} > \frac{I\bar{\mathcal{V}}}{N^2}$ and firm n as a strictly profitable deviation from its equilibrium editorial strategy χ_n in the first stage of the game. A contradiction.

Step 3. *(Convergence of $b^N_{\alpha(t_i)}$.)* Building on the previous argument, we now show that $b^N_{\alpha(t_i)} \to b_{t_i}$ in probability as $N \to \infty$. More formally, fix t_i, $\epsilon > 0$, and a sequence of equilibria. For any N, denote by $b^N_{\alpha(t_i)}$ the random variable specifying the information structure that agent t_i acquires in equilibrium. As above, let $b_{t_i} := \frac{1}{\sqrt{2}}(1, \cos(t_i), \sin(t_i))$ be type t_i's first-best, i.e. $v(b_{t_i}|t_i) = \bar{\mathcal{V}}$. We want to show that, for all $\delta > 0$, there exists $\bar{N}$ such that for all $N > \bar{N}$, $Pr(\|b^N_{\alpha(t_i)} - b_{t_i}\| > \epsilon) < \delta$. Suppose not. Then, there is $\delta > 0$ such that for all $\bar{N}$ there is $N > \bar{N}$ such that $Pr(\|b^N_{\alpha(t_i)} - b_{t_i}\| > \epsilon) \geq \delta$. Consider any particular realization b_i of the random variable $b^N_{\alpha(t_i)}$ for which $\|b_i - b_{t_i}\| > \epsilon$. Then, there exists $K(\epsilon) > 0$ such that $\bar{\mathcal{V}} - v((x_n, t_n)|t) \geq K(\epsilon)$ (See Lemma B3 for more details on $K(\epsilon)$). Thus, $Pr(\|b^N_{\alpha(t_i)} - b_{t_i}\| > \epsilon) \geq \delta$ implies that $Pr\Big(\bar{\mathcal{V}} - v(b_{\alpha(t_i)}|t_i) \geq K(\epsilon)\Big) \geq \delta$. Since δ and ϵ were fixed independently of N, we conclude that $\mathbb{E}(v(b_{\alpha(t_i)}|t_i))$ does not converge to $\bar{\mathcal{V}}$, a contradiction of Step 2 above.

Step 4. *(Convergence of $\mathcal{U}(N)$).* Fix $N \geq 1$. Let $B_n = \{b_n \in \mathbb{R} : \|b_n\| = 1\}$. Let $\chi \in \prod_n \Delta(B_n)$ a (possibly mixed) equilibrium profile of editorial strategy. Denote by $b^N_{\alpha(t_i)}$ the equilibrium random variable which specifies the signal that is acquired by type t_i, possibly by mixing those that are offered by the N firms. As usual, denote by $A_{-i}(\omega, t_{-i})$ the equilibrium approval rate excluding agent i. Following the proof of Lemma B4, an agent's welfare can be written as:

$$\mathcal{U}(N) = \mathbb{E}_\chi \mathbb{E}_{(t_1,\ldots,t_I)}\Big(\mathbb{E}_\omega(A_{-i}(\omega, t_{-i})u(\omega, t_i)) + v(b^N_{\alpha(t_i)}|t_i) - \sum_{n|\alpha_n(t_i)>0} p^\star_n(t_i|b^N)\Big).$$

To compute the limit of $\mathcal{U}(N)$, we split the above expression in three parts. Fix an arbitrary t_i. First, note that by Lemma B6, the total amount paid for information by an agent is at most $\bar{\mathcal{V}}/N$. Therefore, $\lim_N \sum_{n|\alpha_n(t_i)>0} p^\star_n(t_i|b^N) = 0$. Second, as shown in Step 3, $b^N_{\alpha(t_i)} \to b_{t_i}$ in probability as $N \to \infty$, where b_{t_i} is the first-best information structure for agent t_i. By the continuous mapping theorem, $v(b^N_{\alpha(t_i)}|t_i) \to \bar{\mathcal{V}} = \lambda\sqrt{2}$ in probability. Since $|v|$ is bounded, convergence in probability implies convergence in expectation. Together with the first step, we have that, for any t_i, $\lim_{N\to\infty} \mathbb{E}_\chi(v(b_{\alpha(t_i)}|t_i) - \sum_{n|\alpha_n(t_i)>0} p^\star_n(t_i|b^N)) = \bar{\mathcal{V}}$. Since t_i was arbitrary and is independent of χ, we have that $\lim_{N\to\infty} \mathbb{E}_\chi \mathbb{E}_{t_i}\Big(v(b_{\alpha(t_i)}|t_i) - \sum_{n|\alpha_n(t_i)>0} p^\star_n(t_i|b^N)\Big) = \bar{\mathcal{V}}$. The third and final step consists of computing the limit of $\mathbb{E}_{\chi,t,\omega}(A_{-i}(\omega, t_{-i})u(\omega, t_i))$. To this purpose, note that χ, ω and $(t_1, \ldots, t_I)$ are mutually independent random variables. Therefore, by swapping the order of integration and defining $U_i(\omega) = \mathbb{E}_{t_i} u(\omega, t_i)$ to simplify notation, we obtain

$$\begin{aligned}
\lim_{N\to\infty} \mathbb{E}_\chi \mathbb{E}_{(t_1,\dots,t_I)}\Big(\mathbb{E}_\omega(A_{-i}(\omega, t_{-i})u(\omega, t_i))\Big) &= \tfrac{1}{I} \lim_{N\to\infty} \mathbb{E}_\omega\Big(\textstyle\sum_{j\neq i} \mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)\mathbb{E}_{t_i} u(\omega, t_i)\Big) \\
&= \tfrac{(I-1)}{I} \lim_{N\to\infty} \mathbb{E}_\omega\Big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)\mathbb{E}_{t_i} u(\omega, t_i)\Big) \\
&= \tfrac{(I-1)}{I} \lim_{N\to\infty} \mathbb{E}_\omega\Big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)U_i(\omega)\Big).
\end{aligned}$$

Recall that $a_j(\omega, t_j) = \Phi(b^N_{\alpha(t_j)}\omega)$. Once again, as shown in Step 3, $b^N_{\alpha(t_i)} \to b_{t_j}$ in probability as $N \to \infty$. By the continuous mapping theorem, $\Phi(b^N_{\alpha(t_j)}\omega) \to \Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))$ in probability. Moreover, since Φ is bounded, convergence in probability implies convergence in expectation, $\lim_{N\to\infty} \mathbb{E}_\chi a(\omega, t_i) = \Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))$. Following one-to-one the last few steps in the proof of Lemma B4, we conclude that

$$\lim_{N\to\infty} \mathbb{E}_\chi \mathbb{E}_{(t_1,\dots,t_I)}\Big(\mathbb{E}_\omega(A_{-i}(\omega, t_{-i})u(\omega, t_i))\Big) = \frac{(I-1)}{I}\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))u_i(\omega, t_i)\Big).$$

Therefore, $\mathcal{U}(N) \to \frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))u_i(\omega, t_i)\Big) + \bar{\mathcal{V}}$.

Step 5. (*Monopoly vs Perfect Competition*) When $N = 1$, the multimedia model is identical to the baseline model: all agents acquire information from a single firm. Therefore, $\mathcal{U}(1) = (I - 1)\lambda\sqrt{1 + \beta_F}$, as computed as in Lemma B5. The same proof shows $\frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))u_i(\omega, t_i)\Big) + \bar{\mathcal{V}} = \lambda(I-1)\frac{1}{\sqrt{2}}(1+\beta_F) + \lambda\sqrt{2}$. Therefore, by Step 3, we have that $\lim_N \mathcal{U}(N) = \lambda(I-1)\frac{1}{\sqrt{2}}(1 + \beta_F) + \lambda\sqrt{2}$. In summary, we argued that both when $N = 1$ and $N \to \infty$, the baseline model and the multimedia model generate identical expected utilities. Given this, the statement of Proposition 6 follows directly from Proposition 5.c. □

Remark B2. *Fix t_i and a profile of editorial strategies b. Let $\hat{\alpha} \in \arg\max_\alpha v(b_\alpha|t_i)$. Then,*

$$\hat{\alpha} \in \arg\max_\alpha \left(v(b_\alpha|t_i) - \sum_{n|\alpha_n>0} p^\star_n(t_i|b^N) \right)$$

Proof of Remark B2. Suppose the statement is not true. That is, there is $\tilde{\alpha} \neq \hat{\alpha}$ such that

$$v(b_{\tilde{\alpha}}|t_i) - \sum_{n|\tilde{\alpha}_n>0} p^\star_n(t_i|b^N) > v(b_{\hat{\alpha}}|t_i) - \sum_{n:\hat{\alpha}_n>0} p^\star_n(t_i|b).$$

Let $\hat{N} = \{n : \hat{\alpha}_n > 0\} \setminus \{n : \tilde{\alpha}_n > 0\}$. Note that $\hat{N} \neq \emptyset$. Indeed, given that $\hat{\alpha} \in \arg\max_\alpha v(b_\alpha|t_i)$, $\hat{N} = \emptyset$ would contradict the definition of $\tilde{\alpha}$. Thus, we can rewrite the inequality above as:

$$v(b_{\hat{\alpha}}|t_i) - \max_{\alpha\,|\,\alpha_n=0\ \forall n\in\hat{N}} v(b_\alpha|t_i) < \sum_{n\in\hat{N}} p^\star_n(t_i|b).$$

However, a contradiction is reached by noting that

$$\begin{aligned}\sum_{n\in\hat{N}} p_n^\star(t_i|b) &= \sum_{n'\in\hat{N}} \frac{1}{N}\Big(v(b_{\hat{\alpha}}|t_i) - \max_{\alpha\,|\,\alpha_{n'}=0} v(b_\alpha|t_i)\Big)\\ &\leq \sum_{n'\in\hat{N}} \frac{1}{N}\Big(v(b_{\hat{\alpha}}|t_i) - \max_{\alpha\,|\,\alpha_n=0\ \forall n\in\hat{N}} v(b_\alpha|t_i)\Big)\\ &= \Big(v(b_{\hat{\alpha}_i}|t_i) - \max_{\alpha_i\,|\,\alpha_i(n)=0\ \forall n\in\hat{N}} v(b_\alpha|t_i)\Big)\Big(\sum_{n'\in\hat{N}} \frac{1}{N}\Big)\\ &\leq v(b_{\hat{\alpha}}|t_i) - \max_{\alpha\,|\,\alpha_n=0\ \forall n\in\hat{N}} v(b_\alpha|t_i).\end{aligned}$$

The first inequality holds since, for all $n' \in \hat{N}$, $\max_\alpha\{v(b_\alpha|t_i)|\alpha_n = 0, \forall n \in \hat{N}\} \leq \max_\alpha\{v(b_\alpha|t_i)|\alpha_{n'} = 0\}$. The last inequality holds since $\sum_{n'\in\hat{N}} \frac{1}{N} \leq 1$. □

Lemma B6. *For any t_i and b, $\sum\limits_n p^\star(t_i|b) \leq \frac{\bar{\mathcal{V}}}{N}$*

Proof of Lemma B6. We switch to notation θ_i to denote an agent's type. By Lemma 2, $v(b_{\alpha(\theta_i)}|\theta_i) = \lambda_{\alpha(\theta_i)}|\theta_i \cdot b_{\alpha(\theta_i)}|$ where $\lambda_{\alpha(\theta_i)} = \frac{1}{I\sqrt{2\pi(1+\|b_{\alpha(\theta_i)}\|^2)}}$. We establish below that for all n with $\alpha_n(\theta_i) > 0$, the sign of $\theta_i \cdot b_n$ is the same as the sign of $\theta_i \cdot b_{\alpha(\theta_i)}$. This allows us to move the absolute value sign inside as follows: $v(b_{\alpha(\theta_i)}|\theta_i) = \lambda_{\alpha(\theta_i)} \sum\limits_n \alpha_n(\theta_i)|\theta_i \cdot b_n|$.

Assume $\theta_i \cdot b_{\alpha(\theta_i)} > 0$. Now we show that for any n with $\alpha_n(\theta_i) > 0$, $\theta_i \cdot b_n \geq 0$. The proof for the other case, $\theta_i \cdot b_{\alpha(\theta_i)} < 0$, follows a symmetric argument and, thus, it is not replicated here. Assume for contradiction that there exists an n' with $\alpha_{n'}(\theta_i) > 0$, but $\theta_i \cdot b_{n'} < 0$. Consider $\tilde{\alpha}$ such that $\tilde{\alpha}_n = \alpha_n(\theta_i)$ for all $n \neq n'$ with $\tilde{\alpha}_{n'} = 0$. Note that $|\theta_i \cdot b_{\tilde{\alpha}}| = \left|\sum\limits_{n\neq n'} \alpha_n(\theta_i)(\theta_i \cdot b_n)\right| = \theta_i \cdot b_{\alpha(\theta_i)} + \alpha_{n'}(\theta_i)|\theta_i \cdot b_{n'}| > |\theta_i \cdot b_{\alpha(\theta_i)}|$. Also note that $\|b_{\tilde{\alpha}}\| < \|b_{\alpha(\theta_i)}\|$ implying $\lambda_{\tilde{\alpha}} > \lambda_{\alpha(\theta_i)}$. Therefore, we have $v(b_{\tilde{\alpha}}|\theta_i) > v(b_{\alpha(\theta_i)}|\theta_i)$, contradicting $\alpha(\theta_i) \in \arg\max_\alpha v(b_\alpha|\theta_i)$.

Using similar techniques, we can also show that $\max\limits_{\alpha:\alpha_n=0} v(b_\alpha|t_i) > \lambda_{\alpha(\theta_i)} \sum\limits_{n'\neq n} \alpha_{n'}(\theta_i)|\theta_i \cdot b_{n'}|$. This implies that $p_n^\star(\theta_i|b) = \max_\alpha v(b_\alpha|\theta_i) - \max_{\alpha:\alpha_n=0} v(b_\alpha|\theta_i) \leq \lambda_{\alpha_i}\alpha_n(\theta_i)|\theta_i \cdot b_n|$. We conclude the proof by summing over n:

$$\sum_n p_n^\star(\theta_i|b) = \frac{1}{N}\sum_n \Big(\max_\alpha v(b_\alpha|\theta_i) - \max_{\alpha:\alpha_n=0} v(b_\alpha|\theta_i)\Big) \leq \frac{1}{N}\sum_n \lambda_{\alpha(\theta_i)}\alpha_n(\theta_i)|\theta_i \cdot b_n| = \frac{1}{N}\max_\alpha v(b_\alpha|\theta_i) \leq \frac{\bar{\mathcal{V}}}{N}$$

□

B.3. Remaining Proofs

Lemma B7 (Inequality I)**.** *Let $x^\star = 1/(1 + (\sin(\pi/N)/\pi/N)^2)$ and $x_n \in [1/2, 1]$. For all $N \geq 3$, $v((x_n, 0)|2\pi/N) < v((x^\star, 0)|\pi/N)$.*

Proof of Lemma B7. We begin by noting that

$$\sqrt{x_n} + \sqrt{1 - x_n}\cos(2\pi/N) \leq \max_{x_n\in[1/2,1]} v((x_n, 0)|2\pi/N) = \begin{cases} 1 & \text{if } N \leq 4 \\ \sqrt{1 + \cos^2(2\pi/N)} & \text{if } N \geq 5 \end{cases}$$

Moreover, by substituting the definition of $x^\star$ in $v((x^\star, 0)|\pi/N)$ we obtain

$$\sqrt{x^\star} + \sqrt{1 - x^\star}\cos(\pi/N) = \frac{2\pi/N + \sin(2\pi/N)}{2\sqrt{(\pi/N)^2 + \sin^2(\pi/N)}}$$

Let $N \in \{3, 4\}$. In such case, it is enough to show that

$$1 < \frac{2\pi/N + \sin(2\pi/N)}{2\sqrt{(\pi/N)^2 + \sin^2(\pi/N)}}.$$

Rearranging and simplifying, $2\pi/N\cos(\pi/N) - \sin^3(\pi/N) > 0$. It is straightforward to verify that this holds when $N \in \{3, 4\}$. Thus, let $N \geq 5$. In such case, it is enough to show that

$$\sqrt{1 + \cos^2(2\pi/N)} < \frac{2\pi/N + \sin(2\pi/N)}{2\sqrt{(\pi/N)^2 + \sin^2(\pi/N)}}.$$

Denoting $y = \pi/N \in (0, \pi/5]$ and simplifying the above inequality, we obtain

$$G(y) = \frac{1}{4}\sin^2(2y) + y\sin(2y) - \sin^2(y) - \cos^2(2y)(y^2 + \sin^2(y)) > 0$$

Note that $G(0) = 0$. To conclude the proof, we will show that $G'(y) > 0$ for all $y \in (0, \pi/5]$. Note that

$$\begin{aligned} G'(y) &= \sin(2y)\cos(2y) + \sin(2y) + 2y\cos(2y) - 2\sin(y)\cos(y) + \\ &\quad 4\cos(2y)\sin(2y)(y^2 + \sin^2(y)) - \cos^2(2y)(2y + 2\sin(y)\cos(y)) \end{aligned}$$

Since $2\sin(y)\cos(y) = \sin(2y)$, the second and fourth term cancel. Moreover,

$$\begin{aligned} G'(y) &= \sin(2y)\cos(2y) + 2y\cos(2y) + \\ &\quad 4\cos(2y)\sin(2y)(y^2 + \sin^2(y)) - \cos^2(2y)(2y + \sin(2y)) \\ &> \sin(2y)\cos(2y) + 2y\cos(2y) + \\ &\quad 4\cos(2y)\sin(2y)(y^2 + \sin^2(y)) - \cos(2y)(2y + \sin(2y)) \\ &= 4\cos(2y)\sin(2y)(y^2 + \sin^2(y)) \\ &> 0. \end{aligned}$$

The first inequality follows from the fact that, since $\cos(2y) \in (0, 1)$, $\cos(2y) > \cos^2(2y)$. The last inequality follows trivially, as all terms of the expression are strictly positive. □

Lemma B8. *For all $t_2 \in [-\pi, \pi]$ and $x_1 \leq x_2$, the set $\{t \in [-\pi, \pi] | v((x_1, t_1 = 0)|t) \geq v((x_2, t_2)|t)\}$ is an interval in $[-\pi, \pi]$.*

Proof. Let $\Gamma(t) = v((x_1, 0)|t) - v((x_2, t_2)|t)$. If $x_1 = x_2$ and $t_1 = t_2$, $\{\Gamma(t) \geq 0\} = [-\pi, \pi]$ and the claim follows. If $x_1 < x_2$ and $t_1 = t_2$, instead, we have that

$$\{\Gamma(t) \geq 0\} = \{t : \cos(t) \geq \frac{\sqrt{x_2} - \sqrt{x_1}}{\sqrt{1 - x_1} - \sqrt{1 - x_2}} > 0\}.$$

It is immediate to see this is an interval in $[-\pi, \pi]$. Therefore, let $t_2 \neq t_1 = 0$ and $1/2 \leq x_1 \leq x_2$. Suppose $t_2 > 0$. It is immediate to see that $\Gamma(0) > 0$ and $\Gamma(\pi) = \Gamma(-\pi) < 0$. Consider the interval $[0, \pi]$. By continuity of $\Gamma(t)$, there exists at least one $\bar{t} \in (0, \pi)$ such that $\Gamma(\bar{t}) = 0$. We want to show that there is only one such $\bar{t}$. Note that, if $t \in (0, \pi/2]$, the derivative of Γ is

$$\Gamma'(t) = -\sqrt{1 - x_1}\sin(t) + \sqrt{1 - x_2}\sin(t - t_2) < 0.$$

Indeed, $\sqrt{1 - x_1} \geq \sqrt{1 - x_2}$ and $\sin(t) > \sin(t - t_2)$ if $t \in (0, \pi/2]$ (note that while $\sin(t)$ is necessarily positive, $\sin(t - t_2)$ is either negative or positive but smaller than $\sin(t)$). For a similar argument, note that, if $t \in [\pi/2, \pi)$, the second derivative of Γ is

$$\Gamma''(t) = -\sqrt{1 - x_1}\cos(t) + \sqrt{1 - x_2}\cos(t - t_2) > 0.$$

Therefore, $\Gamma'(t)$ is strictly increasing in $[\pi/2, \pi)$ and, hence, it is single-crossing. Since $\Gamma(\pi) < 0$, this implies that $\bar{t}$ is the unique type in $[0, \pi]$ such that $\Gamma(\bar{t}) = 0$.

We now apply a parallel argument for the interval $[-\pi, 0]$. By continuity, there exists at least one $\underline{t} \in (-\pi, 0)$ such that $\Gamma(\underline{t}) = 0$. We need to establish its uniqueness. Note that, if $t \in (-\pi, -\pi/2]$, $\Gamma'(t) > 0$. Similarly, if $t \in [-\pi/2, 0)$, $\Gamma''(t) < 0$. Following the argument made above, we can conclude that there exists a unique $\underline{t} \in [-\pi, 0]$ such that $\Gamma(\underline{t}) = 0$.

Therefore, since $\Gamma(0) > 0$, we have that $\{\Gamma(t) \geq 0\} = [\underline{t}, \bar{t}]$. We omit the discussion of the case $t_2 < 0$ as follows trivially from the argument above. □

Lemma B9. *The function* $G(\delta) = \dfrac{2\delta + \sin(2\delta)}{2\sqrt{\delta^2 + \sin^2(\delta)}}$ *is strictly decreasing in* $\delta \in (0, \pi/2)$.

Proof of Lemma B9. Note that

$$G'(\delta) = \frac{2 + 2\cos(2\delta)}{2\sqrt{\delta^2 + \sin^2(\delta)}} - \frac{(2\delta + \sin(2\delta))(2\delta + 2\sin(\delta)\cos(\delta))}{4(\delta^2 + \sin^2(\delta))^{3/2}}$$

We want to show that $G'(\delta) < 0$ for $\delta \in (0, \pi/2)$. Multiplying both sides by $(\delta^2 + \sin^2(\delta))^{3/2}$ and using $\sin(2\delta) = 2\sin(\delta)\cos(\delta)$, we get that the sign of $G'(\delta)$ is equal to the sign of

$$\begin{aligned}
& (1 + \cos(2\delta))(\delta^2 + \sin^2(\delta)) - (\delta + \sin(\delta)\cos(\delta))^2 \\
= \; & \delta^2 + \sin^2(\delta) + \cos(2\delta)\delta^2 + \cos(2\delta)\sin^2(\delta) - \delta^2 - 2\delta\sin(\delta)\cos(\delta) - \sin^2(\delta)\cos^2(\delta) \\
= \; & \sin^2(\delta) + \cos(2\delta)\delta^2 + \cos(2\delta)\sin^2(\delta) - \delta\sin(2\delta) - \sin^2(\delta)\cos^2(\delta) \\
= \; & \sin^2(\delta) + \cos(2\delta)\delta^2 - \sin^4(\delta) - \delta\sin(2\delta) \\
< \; & \sin^2(\delta) + \cos(2\delta)\delta^2 - \delta\sin(2\delta) \\
= \; & H(\delta)
\end{aligned}$$

where we used the identity $\cos(2\delta)\sin^2(\delta) = \sin^2(\delta)\cos^2(\delta) - \sin^4(\delta)$ for the second-to-last equality and the fact that $\sin^4(\delta) > 0$ for $\delta \in (0, \pi/2)$ for the last inequality. Note that $H(0) = 0$ and, for all $\delta \in (0, \pi/2)$,

$$\begin{aligned} H'(\delta) &= 2\sin(\delta)\cos(\delta) - 2\sin(2\delta)\delta^2 + \cos(2\delta)2\delta - 2\delta\cos(2\delta) - \sin(2\delta) \\ &= \sin(2\delta) - 2\sin(2\delta)\delta^2 - \sin(2\delta) \\ &= -2\sin(2\delta)\delta^2 \\ &< 0 \end{aligned}$$

Therefore, $H(\delta) < 0$ and, hence, $G'(\delta) < 0$ for all $\delta \in (0, \pi/2)$. □

Remark B3. *Fix an arbitrary sequence of equilibria $((x_n^N, t_n^N)_{n=1}^N)_{N\in\mathbb{N}}$ and a type t_i. There exists a subsequence (N_k) such that the equilibrium value of information for agent t_i is strictly increasing in k.*

Proof. Fix $((x_n^N, t_n^N)_{n=1}^N)_{N\in\mathbb{N}}$ and a type t_i. Let $v_{t_i,N}$ be the equilibrium value of information for type t_i when N firms are competing. The proof of Proposition 2 Part (b) shows that the sequence $(v_{t_i,N})_N$ is converging to $\lambda\sqrt{2}$. Therefore, it admits a monotone subsequence. Since $v_{t_i,N} \leq \lambda\sqrt{2}$ for all N, such subsequence must be increasing. □

Lemma B10. *Fix N and let $(x^\star(N), t_n^\star)_{n=1}^N$ be an equilibrium profile of editorial strategies. For all ω_0,*

$$\bar{a}_i(\omega_0) := \mathbb{E}_{\omega_1,\omega_2,t_i}\Big(a_i((\omega_0,\omega_1,\omega_2),t_i)\Big) = \Phi\Big(\frac{\sqrt{x^\star(N)}}{\sqrt{2 - x^\star(N)}}\omega_0\Big).$$

Proof of Lemma B10. Fix N and an equilibrium profile of editorial strategies $(x^\star(N), t_n^\star)_{n=1}^N$. Suppose that, in this equilibrium, agent t_i acquires information from firm n. Conditional on a signal realization $\bar{s} = s(\omega, (x^\star(N), t_n^\star))$, the agent's equilibrium approval strategy is characterized in Lemma 1 and depends on $\mathbb{E}_\omega(u(\omega, t_i)|\bar{s})$. Using Equation A1 and Remark 1, we have that

$$\mathbb{E}_\omega(u(\omega,t_i)|\bar{s}) = \frac{v((x^\star(N), t_n^\star)|t_i)}{\lambda}\Big(\sqrt{x^\star(N)}\omega_0 + \sqrt{1 - x^\star(N)}(\omega_1\cos(t_n^\star) + \omega_2\sin(t_n^\star)) + \varepsilon_i\Big).$$

Since, in equilibrium, $\frac{v((x^\star(N),t_n^\star)|t_i)}{\lambda} > 0$, the agent approves if and only if the signal she observes is positive. Since $\varepsilon_i \sim \mathcal{N}(0,1)$, the probability that agent t_i approves policy ω before ε_i realizes is given by $\bar{a}_i^\star(\omega, t_i) = \Phi\Big(\sqrt{x^\star(N)}\omega_0 + \sqrt{1 - x^\star(N)}(\cos(t_n^\star)\omega_1 + \sin(t_n^\star)\omega_2)\Big)$, where Φ denotes the CDF of the standard normal distribution. Thanks to the symmetry in the equilibrium location (Theorem 2) and the uniformity of the distribution of t_i, we have that

$$\mathbb{E}_{t_i}\Big(a_i(\omega,t_i)\Big) = \frac{1}{N}\sum_n \Phi\Big(\sqrt{x^\star(N)}\omega_0 + \sqrt{1 - x^\star(N)}(\cos(t_n^\star)\omega_1 + \sin(t_n^\star)\omega_2)\Big)$$

We need to compute the expectation of the expression above with respect to ω_1 and ω_2. Since ω_1 and ω_2 are independent, we do so in two separate steps. For both steps, we use the identity $\int_\mathbb{R}\Phi(\alpha + \gamma y)d\Phi(y) = \Phi(\alpha/\sqrt{1+\gamma^2})$ (see, Patel and Read, 1996). We begin by integrating with respect to ω_2. Let $y = \omega_2$ and, for each n, let $\alpha_n = \sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}\cos(t_n^\star)\omega_1$ and $\gamma_n = \sqrt{1-x^\star}\sin(t_n^\star)$. Using the integral identity, we obtain

$$\mathbb{E}_{\omega_2,t_i}\Big(a_i(\omega,t_i)\Big) = \frac{1}{N}\sum_n \Phi\Big(\frac{\alpha_n}{\sqrt{1+\gamma_n^2}}\Big) = \frac{1}{N}\sum_n \Phi\Big(\frac{\sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}\cos(t_n^\star)\omega_1}{\sqrt{1+(1-x^\star)\sin^2(t_n^\star)}}\Big).$$

Next, we integrate the above with respect to ω_1. Let $y = \omega_1$ and

$$\alpha_n' = \frac{\sqrt{x^\star}\omega_0}{\sqrt{1+(1-x^\star)\sin^2(t_n^\star)}} \qquad \gamma_n' = \frac{\sqrt{1-x^\star}\cos(t_n^\star)}{\sqrt{1+(1-x^\star)\sin^2(t_n^\star)}}.$$

Using again the integral identity, we obtain

$$\mathbb{E}_{\omega_1,\omega_2,t_i}\Big(a_i(\omega,t_i)\Big) = \frac{1}{N}\sum_n \Phi\Big(\frac{\alpha_n'}{\sqrt{1+\gamma_n'^2}}\Big) = \Phi\Big(\frac{\sqrt{x^\star(N)}}{\sqrt{2-x^\star(N)}}\omega_0\Big),$$

where we used the fact that $\cos^2(t_n^\star) + \sin^2(t_n^\star) = 1$, for all n. □

Proof of Remark 3. Fix $I \geq 1$ and $N \geq 1$. Let $(x^\star, t_n^\star)_{n=1}^N$ be the equilibrium profile of editorial strategies. Let $A^\star(\omega)$ be the equilibrium rate of approval. By assumption, it is equal to the probability that the society implements ω. We want to show that the total probability of implementing a policy in Ω^+, namely $\int_{\Omega^+} A^\star(\omega)\phi(\omega)d\omega$, decreases in N. To do so, we divide the proof in three steps. First, we partition the set of policies Ω^+. Second, we compute the integral restricting attention on an arbitrary cell of such partition. Third, we show that such integral is decreasing in N.

Step 1. For any $\omega_0 \in \mathbb{R}$ and $K \geq 0$, define the set of policies

$$\Omega^\circ(\omega_0, K) = \Big\{\tilde\omega \in \Omega : \tilde\omega_0 = \omega_0 \text{ and } \sqrt{\tilde\omega_1^2 + \tilde\omega_2^2} = K\Big\}.$$

Fix $\tilde\omega \in \Omega^\circ(\omega_0, K)$. Note that, by letting $t_{\tilde\omega} = \arctan(\tilde\omega_2/\tilde\omega_1) \in [-\pi, \pi]$, we can write $u(\tilde\omega, t_i) = \omega_0 + K\cos(t_i - t_{\tilde\omega})$. Moreover, all $\tilde\omega \in \Omega^\circ(\omega_0, K)$ are equally likely. To see this, note that $\Pr(\tilde\omega) = \phi(\tilde\omega_0)\phi(\tilde\omega_1)\phi(\tilde\omega_2) = \phi(\omega_0)\frac{1}{2\pi}e^{\frac{K^2}{2}}$, which only depends on (ω_0, K). Therefore, $t_{\tilde\omega}$ is uniformly distributed in $[-\pi, \pi]$.

Clearly, $(\omega_0', K') \neq (\omega_0'', K'')$, $\Omega^\circ(\omega_0', K') \cap \Omega^\circ(\omega_0'', K'') = \emptyset$. Moreover, let $C = \{(\omega_0, K) \in \mathbb{R}^2 \mid \omega_0 > K \geq 0\}$. We have that $\Omega^+ = \bigcup_{(\omega_0,K)\in C} \Omega^\circ(\omega_0, K)$. Too see this, let first $\omega \in \Omega^+$. Define $K = \sqrt{\omega_1 + \omega_2} \geq 0$. There is $t_\omega \in [-\pi, \pi]$ such that $u(\omega, t_i) = \omega_0 + K\cos(t_\omega - t_i) > 0$ for all t_i. Moreover, there is $\bar t_i \in [-\pi, \pi]$ such that $\cos(\bar t_i - t_\omega) = -1$. Therefore, $u(\omega, \bar t_i) = \omega_0 - K > 0$. Therefore, $(\omega_0, K) \in C$ and, thus, $\omega \in \Omega^\circ(\omega_0, K)$. Conversely, suppose $\hat\omega \in \Omega^\circ(\omega_0, K)$ for some $\omega_0 > K$. Since for all t_i $\cos(t_{\hat\omega} - t_i) \geq -1$, we have $0 < \omega_0 - K \leq \hat\omega - K\cos(t_{\hat\omega} - t_i) = u(\hat\omega, t_i)$ for all t_i. Therefore, $\hat\omega \in \Omega^+$. We conclude that $\{\Omega^\circ(\omega_0, K)\}_{(\omega_0,K)\in C}$ partitions Ω^+.

Step 2. Next, assume $\omega_0 > K \geq 0$, and focus on the set of policies $\Omega^\circ(\omega_0, K) \subset \Omega^+$. We want to show that the total probability of implementing these policies, namely $\frac{1}{2\pi}\int_{\Omega^\circ(\omega_0,K)} A^\star(\omega)d\omega$, decreases in N. We have that:

$$\begin{aligned}
\frac{1}{2\pi}\int_{\Omega^\circ(\omega_0,K)} A^\star(\omega)d\omega &= \frac{1}{2\pi I}\sum_i \int_{\Omega^\circ(\omega_0,K)} \bar{a}_i^\star(\omega, t_i)d\omega \\
&= \frac{1}{2\pi I}\sum_i \int_{-\pi}^{\pi} \Phi(\sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}K\cos(t^\star_{n_i^\star} - t_\omega))dt_\omega \\
&= \frac{1}{2\pi I}\sum_i \int_0^{2\pi} \Phi(\sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}K\cos(y))dy \\
&= \frac{1}{2\pi}\int_0^{\pi} \Phi(\sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}K\cos(y)) + \\
&\quad \Phi(\sqrt{x^\star}\omega_0 - \sqrt{1-x^\star}K\cos(y))dy
\end{aligned}$$

The first and second equalities follows from the definition of approval rate and the proof of Lemma B10. In the second equality, we use notation $n_i^\star$ to indicate the firm from which agent i acquires information in equilibrium. To obtain the third equality, we used $y = t^\star_{n_i^\star} - t_{\tilde{\omega}}$ and the fact that for any l and u such that $u = l + 2\pi$, $\int_l^u -\cos(y)dy = \int_0^{2\pi}\cos(y)dy$. Finally, to obtain the fourth equality, we used $\cos(y+\pi) = -\cos(y)$.

Step 3. In order to show that $\frac{1}{2\pi}\int_{\Omega^\circ(\omega_0,K)} A^\star(\omega)d\omega$ is decreasing in N, it is sufficient to show that, for all $y \in [0,\pi]$, $\Phi(\sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}K\cos(y)) + \Phi(\sqrt{x^\star}\omega_0 - \sqrt{1-x^\star}K\cos(y))$ is decreasing in N. To this purpose, fix $y \in [0,\pi]$. For notational convenience, let $\alpha = \sqrt{x^\star}$ and $\beta = \sqrt{1-x^\star}K$. We want to show that

$$\frac{d}{dN}\Big(\Phi(\alpha\omega_0 + \beta\cos(y)) + \Phi(\alpha\omega_0 - \beta\cos(y))\Big) < 0. \tag{B10}$$

This derivative is equal to:

$$\Big(\phi(\alpha\omega_0 + \beta\cos(y)) + \phi(\alpha\omega_0 - \beta\cos(y))\Big)\omega_0\alpha' + \Big(\phi(\alpha\omega_0 + \beta\cos(y)) - \phi(\alpha\omega_0 - \beta\cos(y))\Big)\cos(y)\beta'$$

We show that both terms of these derivatives are negative. Let us start from the first term. By assumption $\omega_0 > 0$, since $\omega_0 - K > 0$ and $K \geq 0$. Moreover, the probability density function $\phi(\cdot)$ is everywhere strictly positive. Finally, by Proposition 1, $\alpha' < 0$. Therefore, the first term is strictly negative. Next, we analyze the second term of the derivative. Suppose $\cos(y) \geq 0$. Then since $\omega_0 > 0$, $\alpha\omega_0 + \beta\cos(y) \geq \alpha\omega_0 - \beta\cos(y)$. Moreover, $\alpha\omega_0 > 0$. This implies that $\phi(\alpha\omega_0 + \beta\cos(y)) - \phi(\alpha\omega_0 - \beta\cos(y)) \leq 0$. Conversely, suppose $\cos(y) \leq 0$. Then $\alpha\omega_0 + \beta\cos(y) \leq \alpha\omega_0 - \beta\cos(y)$. Since $\alpha\omega_0 > 0$, this implies that $\phi(\alpha\omega_0 + \beta\cos(y)) - \phi(\alpha\omega_0 - \beta\cos(y)) \geq 0$. In summary, we showed that

$$\Big(\phi(\alpha\omega_0 + \beta\cos(y)) - \phi(\alpha\omega_0 - \beta\cos(y))\Big)\cos(y) \leq 0.$$

Since $\beta' > 0$ (Proposition 1), this implies that the second term of the derivative is weakly negative. We conclude that the derivative in Equation B10 is strictly negative, as we wanted to show. Since y was chosen arbitrary, this implies that $\frac{1}{2\pi}\int_{\Omega^\circ(\omega_0,K)} A^\star(\omega)d\omega$ is decreasing in N. Moreover, since $\Omega^\circ(\omega_0, K)$ is an arbitrary cell in the partition of Ω^+, we conclude that $\int_{\Omega^+} A^\star(\omega)\phi(\omega)d\omega$ is decreasing in N.

A similar argument can be made to prove that $\int_{\Omega^-} A^\star(\omega)\phi(\omega)d\omega$ is increasing in N. The only differences being that, in Step 1, we define $C' = \{(\omega_0, K) \in \mathbb{R}^2 \mid \omega_0 + K < 0, K \geq 0\}$ and, in Step 3, we use the fact that $\omega_0 < 0$. $\square$

C. Additional Extensions

C.1. Variance of Signals and Constraints on Learning

Throughout the paper, we assumed that if agent i acquires information from firm n, she privately observes a signal realization $s_i(\omega, b_n) = b_n \cdot \omega + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, 1)$. In particular, we assumed that ϵ_i has a variance of 1 and, more importantly, that it does not depend on N, the number of firms. This implies that agents are constrained in how much they can learn about the policy from the media and that this constraint is independent of the competitiveness of the market. This is in line with an interpretation of the model where the error ϵ_i is borne by the agent. For example, it arises because she has a limited time to allocate to learning about the policy. In our extension to multimedia (Appendix B.2), we retained this assumption. Agents are still endowed with a unit of time but they can split it freely across multiple firms in the market. By doing so, agents can "construct" signals that are better tailored to their own needs, even when these are not directly supplied by the market.

The goal of this paper has been to demonstrate how a competitive market can affect welfare simply by changing what information agents consume. However, competition may affect not only what information is supplied by firms but also how much agents can learn from them. Here we provide two such examples. First, the level of competitiveness in the market could have an impact on how much firms invest in generating information. This could be modeled by endogenizing supply side constraints on $||b_n||$, which is equivalent to a change in σ_N. A priori, it is not clear whether a more competitive market could lead firms to invest more or less. Second, in a more competitive market, agents may receive *multiple* signals from different firms. This could be modeled as a decline in σ_N, as N increases. Note how this is different from the multimedia model of Section B.2, where agents get a *single* signal by mixing the editorial strategies of different firms.

Without adding more structure to the model, we investigate how the results presented in Section 5 are affected when σ_N is allowed to change as a function of N. In particular, we consider a decreasing sequence of σ_N and show a counterpart of our most general result of the paper, namely

Proposition 5.

We begin by fixing I, N, and let $\epsilon_i \sim \mathcal{N}(0, \sigma_N)$, for some $\sigma_N > 0$. For a fixed N, it is not surprising to see that $\sigma_N \neq 1$ only rescales the main equilibrium objects of Section 3. In particular, fix an editorial strategy b_n and a type θ_i. Following Lemma 2, the value of information b_n for type θ_i is

$$v(b_n|\theta_i) = \frac{|\theta_i \cdot b_n|}{I\sqrt{2\pi(\sigma_N^2 + ||b_n||^2)}}.$$

Intuitively, a lower σ_N increases the value that θ_i attaches to the information from firm n. Similarly, we can follow Lemma A2 and transform b_n and θ_i into polar coordinates to obtain the value $v_{\sigma_N}((x_n, t_n) \mid t_i) = \lambda_{\sigma_N}\Big(\sqrt{x_n} + \sqrt{1 - x_n}\cos(t_i - t_n)\Big)$, where $\lambda_{\sigma_N} = \frac{1}{I\sqrt{2\pi(1+\sigma_N^2)}}$.

Instead of replicating all results in the paper, we focus attention on Proposition 5 to demonstrate the impact that a N-varying σ_N has on the main takeaways of the paper.

Proposition 7. *Fix a regular F. Let $(\sigma_N)_N$ be a decreasing sequence with $\sigma_N \to \sigma_\infty > 0$.*

a. (Existence) An equilibrium exists for any $N \geq 1$ and $I \geq 1$.

b. (Daily Me) Fix any t_i. As $N \to \infty$, the equilibrium value of information for type t_i, $\mathcal{V}(N|t_i)$, converges in probability to the first-best value $\bar{\mathcal{V}}_\infty := \lambda_{\sigma_\infty}\sqrt{2}$.

c. (Inefficiency) There exists $\bar{I}$ and $\bar{\sigma} \geq 0$ such that

(i) If $\sigma_\infty > \bar{\sigma}$ and $I > \bar{I}$, $\mathcal{U}(1) > \lim_{N\to\infty} \mathcal{U}(N)$.

(ii) If $\sigma_\infty \leq \bar{\sigma}$ and for any I, $\mathcal{U}(1) \leq \lim_{N\to\infty} \mathcal{U}(N)$.

Part a and b are qualitatively identical to their counterparts in Proposition 5. Part c shows that the inefficiency associated with competition remains even when agents can learn more, in the sense of lower σ, in a more competitive market. It also shows that if σ_N decreases too much, the inefficiency disappears. Part $c.(ii)$ should be considered as a sanity check. It further highlights that information can play a positive role in our model (see discussion at the end of Section 4.4). For example, this result shows that, for any distribution of preferences F, if the society converges to the complete information benchmark, i.e. if $\lim_{N\to\infty} \sigma_N = 0$, then the agents are indeed better off. That is, in this limit, the expected welfare of the typical agent is higher under perfect competition relative to a monopoly. This provides an important benchmark, illustrating how there is plenty of scope in the model for information to play a positive role. The main inefficiency identified in the model is not due to competition moving society closer to the complete information benchmark; it arises because of the tradeoffs firms face in terms of which aspects of the policy to emphasize in their editorial strategies. As competition increases, firms specialize by shifting emphasis from common-interest to private-interest components of the policy.

Proof of Proposition 7. The proof is divided in five steps that closely follow Lemmas B1 to B5. In the interest of space, we omit the proofs that are identical up to a rescaling. We focus attention on the steps of the proof that are affected by the dependence of σ_N on N.

Step 1. (Existence.) In this step, we fix N. A such, the proof of Lemma B1 applies identically.

Step 2. (*Daily Me I*) This follows the proof of Lemma B2. Fix $\delta > 0$ and let $\xi_1 = \frac{\delta}{2\lambda_{\sigma_\infty}}$. Let $\bar{\mathcal{V}}_{\sigma_\infty} = \max_{(x_n,t_n)} v_{\sigma_\infty}((x_n, t_n|t_i) = \lambda_{\sigma_\infty}\sqrt{2}$. This is the highest possible value that $v_{\sigma_\infty}((x_n, t_n|t_i)$ can achieve and, like in the baseline model, it is independent of t_i. We show that there exists $\bar{N}$ such that, for all $N > \bar{N}$ and any equilibrium profile of possibly mixed editorial strategies χ we have $\mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_i)\}) > \bar{\mathcal{V}}_{\sigma_\infty} - \delta$ for all $t_i \in T$. Suppose not. That is suppose that, for all N, there is an equilibrium profile of possibly mixed editorial strategies χ and a type $\bar{t}_i$ such that $\mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_i)\}) \le \bar{\mathcal{V}}_{\sigma_\infty} - \delta$. This implies that, for all $t_j \in [\bar{t}_i - \xi_1, \bar{t}_i + \xi_1]$, $\mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_j)\}) \le \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{2}$. To see this, suppose, by way of contradiction, that $\mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_j)\}) > \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{2}$. Denote by $n(t_j)$ the random variable that, for each realization of χ, indicates the firm from which t_j acquires information. Note that, for all $t_n \in T$, $\cos(\bar{t}_i - t_n) \ge \cos(t_j - t_n) - \xi_1$, since $\frac{d}{dt}\cos(t - t_n) \le 1$. We have that,

$$
\begin{aligned}
\mathbb{E}_\chi(\max_n\{v_{\sigma_N}((x_n, t_n)|t_i)\}) &\ge \mathbb{E}_\chi(v_{\sigma_N}((x_{n(t_j)}, t_{n(t_j)})|t_i)) \\
&= \lambda_{\sigma_N}\mathbb{E}_\chi(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}\cos(\bar{t}_i - t_{n(t_j)})) \\
&\ge \lambda_{\sigma_N}\mathbb{E}_\chi(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}(\cos(t_j - t_{n(t_j)}) - \xi_1) \\
&\ge \lambda_{\sigma_N}\mathbb{E}_\chi(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}\cos(t_j - t_{n(t_j)})) - \lambda_{\sigma_N}\xi_1 \\
&\ge \mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_j)\}) - \lambda\xi_1 \\
&> \bar{\mathcal{V}}_{\sigma_\infty} - \tfrac{\delta}{2} - \lambda_{\sigma_N}\xi_1 \\
&> \bar{\mathcal{V}}_{\sigma_\infty} - \tfrac{\delta}{2} - \lambda_{\sigma_\infty}\xi_1 \\
&= \bar{\mathcal{V}}_{\sigma_\infty} - \delta.
\end{aligned}
$$

The first inequality holds as, in the right-hand side, agent $\bar{t}_i$ chooses the firm $n(t_j)$ that is optimal for t_j. The second inequality holds since $\cos(\bar{t}_i - t_n) \ge \cos(t_j - t_n) - \xi_1$ for all t_n. The last inequality holds because $\lambda_{\sigma_N} < \lambda_{\sigma_\infty}$, since σ_N is decreasing. In summary, this contradicts our assumption that $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_i)\}) \le \bar{\mathcal{V}} - \delta$. Therefore, it must be that $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_j)\}) \le \bar{\mathcal{V}} - \frac{\delta}{2}$.

Fix $\bar{N}$. Note that, by continuity of $v_{\sigma_N}((x_n, t_n)|t_j)$ in t_j, there exists $\xi_2^{\bar{N}} > 0$ such that for all $t_j \in [\bar{t}_i - \xi_2^{\bar{N}}, \bar{t}_i + \xi_2^{\bar{N}}]$ such that $v_{\sigma_N}((1/2, \bar{t}_i)|t_j) \ge \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{4}$. Since σ_N is decreasing, this holds for all for all $N > \bar{N}$. Hence, $\xi_2^{\bar{N}}$ is independent of N as it increases to infinity. Let $\xi = \min\{\xi_1, \xi_2^{\bar{N}}\}$, which in turn is independent of N. We have established that for all $t_j \in [\bar{t}_i - \xi, \bar{t}_i + \xi]$,

$$
\mathbb{E}_\chi(\max_{n' \neq n}\{v_{\sigma_N}(x_{n'}, t_{n'}|t_j)\}) \le \mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_j)\}) \le \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{2} < \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{4} \le v_{\sigma_N}((1/2, \bar{t}_i)|t_j), \tag{C1}
$$

Consider an arbitrary firm n who deviates from its equilibrium editorial strategy (χ_n) in favor of the pure strategy $(x_n = 1/2, t_n = \bar{t}_i)$. Its expected profits are

$$\begin{aligned}
\Pi_n((x_n,t_n),(\chi_{n'})_{n'\neq n}) &= I\int_{-\pi}^{\pi}\mathbb{E}_\chi\Big(\max\{v_{\sigma_N}((x_n,t_n)|t_j) - V_{\sigma_N}((x_{n'},t_{n'})_{n'\neq n}|t_j),0\}\Big)dF(t_j)\\
&\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}\mathbb{E}_\chi\Big(\max\{v_{\sigma_N}((x_n,t_n)|t_j) - V_{\sigma_N}((x_{n'},t_{n'})_{n'\neq n}|t_j),0\}\Big)dF(t_j)\\
&\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}\mathbb{E}_\chi\Big(v_{\sigma_N}((x_n,t_n)|t_j) - V_{\sigma_N}((x_{n'},t_{n'})_{n'\neq n}|t_j)\Big)dF(t_j)\\
&= I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}v_{\sigma_N}((x_n,t_n)|t_j) - \mathbb{E}_\chi(V_{\sigma_N}((x_{n'},t_{n'})_{n'\neq n}|t_j))dF(t_j)\\
&= I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}v_{\sigma_N}((x_n,t_n)|t_j) - \mathbb{E}_\chi(\max_{n'\neq n}\{v_{\sigma_N}(x_{n'},t_{n'}|t_j)\})dF(t_j)\\
&\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi}\Big(\bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{4} - \bar{\mathcal{V}}_{\sigma_\infty} + \frac{\delta}{2}\Big)f(t_j)dt_j\\
&\geq \frac{IC\delta\xi}{2}.
\end{aligned}$$

Note that that the industry profits are bounded above by $I\bar{\mathcal{V}}_{\sigma_\infty}$. Then, an identical argument as in the last paragraph of Lemma B2 applies here.

Step 3 (*Daily Me II*) This follows the proof of Lemma B3. Fix t_i, $\epsilon > 0$, and a sequence of equilibria. For any N, denote by $(x^N_{n(t_i)}, t^N_{n(t_i)})$ the random variable specifying the information structure that agent t_i acquires in equilibrium. We want to show that, for all $\delta > 0$, there exists $\bar{N}$ such that for all $N > \bar{N}$, $Pr(\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon) < \delta$. Suppose not. Then, there is $\delta > 0$ such that for all $\bar{N}$ there is $N > \bar{N}$ such that $Pr(\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon) \geq \delta$. Let (x_n, t_n) be a realization of $(x^N_{n(t_i)}, t^N_{n(t_i)})$ such that $\|(x_n, t_n) - (1/2, t_i)\| > \epsilon$. That is, $\sqrt{(x_n - 1/2)^2 + (t_n - t_i)^2} > \epsilon$. This implies that

$$\max\{|x_n - 1/2|, |t_n - t_i|\} > \frac{\epsilon}{\sqrt{2}}.$$

Consider the difference $\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}((x_n,t_n)|t_i) = \lambda_{\sigma_\infty}\sqrt{2} - \lambda_{\sigma_N}(\sqrt{x_n} + \sqrt{1-x_n}\cos(t_n - t_i)))$. Suppose $|t_n - t_i| > \frac{\epsilon}{\sqrt{2}}$. Then, since $\lambda_{\sigma_\infty} > \lambda_{\sigma_N}$,

$$\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}((x_n,t_n)|t_i) \geq \frac{\lambda_{\sigma_\infty}}{\sqrt{2}}(1-\cos(t_n - t_i)) > \frac{\lambda_{\sigma_\infty}}{\sqrt{2}}(1-\cos(\frac{\epsilon}{\sqrt{2}})) =: K_1(\epsilon) > 0.$$

Conversely, suppose that $|x_n - 1/2| > \frac{\epsilon}{\sqrt{2}}$. Then,

$$\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}((x_n,t_n)|t_i) \geq \lambda_{\sigma_\infty}(\sqrt{2} - \sqrt{x_n} - \sqrt{1-x_n}) > \lambda_{\sigma_\infty}\Big(\sqrt{2} - \frac{1}{2}(\sqrt{1+\epsilon\sqrt{2}} + \sqrt{1-\epsilon\sqrt{2}})\Big) =: K_2(\epsilon) > 0$$

Let $K(\epsilon) = \min\{K_1(\epsilon), K_2(\epsilon)\}$. We established that, for all realizations of the random variable $(x^N_{n(t_i)}, t^N_{n(t_i)})$ that satisfy $\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon$, we have $\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}((x_n,t_n)|t_i) > K(\epsilon) > 0$. This implies that

$$Pr\Big(\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}((x^N_{n(t_i)}, t^N_{n(t_i)}) \mid t_i) > K(\epsilon)\Big) \geq \delta.$$

Since δ and ϵ are independent of N, we conclude that $\mathbb{E}(v_{\sigma_N}((x^N_{n(t_i)}, t^N_{n(t_i)}) \mid t_i))$ does not converge to $\bar{\mathcal{V}}_{\sigma_\infty}$, a contradiction.

Step 4 (*Convergence of* $\mathcal{U}(N)$). In light of the previous two steps, the proof of Step 4 follows the proof of Lemma B4.

Step 5 (*Monopoly vs Competition*). Following Lemma B5, we compute $\mathcal{U}(1)$ and $\lim_{N\to\infty}\mathcal{U}(N)$ taking into account the role of σ_N. In both cases, details associated with the steps used for derivation can be found in the original proof. First, let $N = 1$. We have that:

$$\begin{aligned}\mathcal{U}(1) &= \tfrac{I-1}{I}\mathbb{E}_{\omega_0}\Big(\omega_0\Phi(\tilde{b}\omega_0) + \tfrac{\beta_F b}{\sqrt{1+b^2}}\phi(\tilde{b}\omega_0)\Big) \\ &= \tfrac{I-1}{I}\Big(\tfrac{\tilde{b}}{\sqrt{1+\tilde{b}^2}}\phi(0) + \tfrac{\beta_F b}{\sqrt{1+b^2}}\tfrac{1}{\sqrt{1+\tilde{b}^2}}\phi(0)\Big) \\ &= \tfrac{I-1}{I\sqrt{2}\sqrt{1+\sigma_1^2}\sqrt{\pi}}\Big(\sqrt{x^\star} + \beta_F\sqrt{1-x^\star}\Big) \\ &= (I-1)\lambda_{\sigma_1}\sqrt{1+\beta_F^2}\end{aligned} \tag{C2}$$

Second, let $N \to \infty$. We have that:

$$\begin{aligned}\lim_{N\to\infty}\mathcal{U}(N) &= \tfrac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\Big(\Phi(\tfrac{1}{\sqrt{2}\sigma_\infty}u_j(\omega,t_j))u_i(\omega,t_i)\Big) + \bar{\mathcal{V}} \\ &= \tfrac{I-1}{I}\mathbb{E}_{t_j}(\tfrac{1}{\sqrt{2}\sqrt{1+\sigma_\infty{}^2}}\phi(0) + \tfrac{1}{\sqrt{2}\sqrt{1+\sigma_\infty{}^2}}\beta_F\phi(0)(\cos(t_j)\cos(t^m) + \sin(t_j)\sin(t^m)) + \bar{\mathcal{V}} \\ &= \tfrac{I-1}{I}\tfrac{1}{\sqrt{2}\sqrt{1+\sigma_\infty{}^2}}\phi(0)\mathbb{E}_{t_j}(1 + \beta_F\cos(t_j - t^m)) + \bar{\mathcal{V}} \\ &= \tfrac{I-1}{I}\tfrac{1}{\sqrt{2}\sqrt{1+\sigma_\infty{}^2}}\tfrac{1}{\sqrt{2\pi}}(1+\beta_F^2) + \lambda_{\sigma_\infty}\sqrt{2} \\ &= \lambda_{\sigma_\infty}(I-1)\tfrac{1}{\sqrt{2}}(1+\beta_F^2) + \lambda_{\sigma_\infty}\sqrt{2}.\end{aligned} \tag{C3}$$

By Equations C2 and C3:

$$\begin{aligned}\mathcal{U}(1) - \lim_{N\to\infty}\mathcal{U}(N) &= \lambda_{\sigma_1}(I-1)\sqrt{1+\beta_F^2} - \lambda_{\sigma_\infty}(I-1)\frac{1}{\sqrt{2}}(1+\beta_F^2) - \lambda_{\sigma_\infty}\sqrt{2} \\ &= \lambda_{\sigma_1}(I-1)\sqrt{1+\beta_F^2}\left(1 - \frac{\lambda_{\sigma_\infty}}{\lambda_{\sigma_1}\sqrt{2}}\sqrt{1+\beta_F^2}\right) - \lambda_{\sigma_1}\sqrt{2}\end{aligned}$$

Note that for all non-degenerate distributions F there is an associated $\beta_F \in [0,1)$. Note that $\frac{\lambda_{\sigma_\infty}}{\lambda_{\sigma_1}\sqrt{2}}\sqrt{1+\beta_F^2} = \left(\frac{\sqrt{1+\sigma_1^2}}{\sqrt{1+\sigma_\infty^2}\sqrt{2}}\right)\sqrt{1+\beta_F^2}$. For any β_F, there exist a $\bar{\sigma} \geq 0$ such that the sign of $1 - \left(\frac{\sqrt{1+\sigma_1^2}}{\sqrt{1+\sigma_\infty^2}\sqrt{2}}\right)\sqrt{1+\beta_F^2}$ is positive whenever $\sigma_\infty < \bar{\sigma}$ and negative otherwise. Part (*ii*) follows directly from the sign being negative for both terms. Part (*i*) relies on showing that the first term is increasing in I, as we have done in the proof of Proposition 5. □

C.2. Policy Implementation Rule

Throughout the paper, we assume that the policy is implemented with a probability that is equal to its approval rate. This implementation rule, combined with a finite number of agents, implies that information has instrumental value for the agents, as their approval decisions directly affect the policy outcome. At the same time, this simple implementation rule eliminates the scope for pivotal reasoning to learn about the policy. As discussed in Section 3.1.1, this substantially reduces the complexity of the agent's problem, while enabling us to focus attention on the most novel aspect of the model: the competitive supply of information.

In general, the probability that a policy is implemented or that a candidate is elected can be a nonlinear function of the behavior of individual agents within a society. The political science and political economy literature study conditions under which maximizing vote share is equivalent to maximizing the probability of winning (see, Banks and Duggan, 2004; McKelvey and Patty, 2006; Patty, 2005, 2007). One of these is the presence of aggregate uncertainty, for example, when voting decisions are influenced by independent random perturbations. Under appropriate distributional assumptions, this aggregate uncertainty generates linearity. Below, we discuss a simple extension of our baseline model in which the policy is implemented according to the majority rule. In this extension, there is aggregate noise due to behavior of "non-policy" voters, whose vote is a uniform random variable that is independent of the actual policy ω. In light of this, the policy obtains a simple majority with a probability that is proportional to its approval rate among the "policy" voters, as in our baseline model.

More formally, let there be I agents to whom we refer to as policy voters. These voters acquire information and vote as described in Section 2. A group of $\tilde{I}$ agents, to whom we refer to as nonpolicy voters, also participates in the collective decision. For simplicity, assume that $\tilde{I} \geq I$ and $\tilde{I} + I$ is an odd number. Non-policy voters are not affected by the policy outcome (e.g., for all such voters, $\theta_i = (0, 0, 0)$ and, thus, $u(\omega, t_i) = 0$). Their vote is determined by other factors that are independent of the policy. Specifically, we assume that the approval rate among non-policy voters, denoted $\tilde{A}(\omega)$, is distributed according to the uniform distribution on the interval $[0, 1]$. Finally, suppose that the policy is implemented if it receives a simple majority of all the votes.

Remark C1. *Under simple majority, the probability that the policy is implemented is a linear function of the approval rate among policy voters, namely $A(\omega, \theta)$.*

Proof of Remark C1. Fix t and suppose that the approval rate of the policy voters is $A(\omega, t)$. Under a simple majority, the policy is implemented if $IA(\omega, t) + \tilde{I}\tilde{A}(\omega) > \frac{I+\tilde{I}}{2}$. Since $\tilde{A}(\omega) \sim$ Unif$[0, 1]$, the probability that ω is implemented is equal to $Pr(\tilde{A}(\omega) > \frac{I+\tilde{I}-2IA(\omega,t)}{2\tilde{I}}) = \frac{\tilde{I}-I}{2\tilde{I}} + \frac{I}{\tilde{I}}A(\omega, t)$, which is a linear function of the approval rate of policy voters. □