Math for Econ I

Problem Set 1*

Exercise 1

(i) Prove that $\mathrm{cl}A = \mathrm{int}(A) \cup \mathrm{bdry}(A)$.

(ii) Let (X, d) be a metric space and $(x_m), (y_m) \in X^\infty$. Show that if $x_m \to x$ and $y_m \to y$ then $d(x_m, y_m) \to d(x, y)$.

Exercise 2

Prove that if X is compact in metric space (X, d) then X is separable.

Exercise 3

Prove (l^∞, d_∞) is complete.

Exercise 4

Show that (X, d) is a compact metric space if and only if for every sequence of closed subset of X such that $\bigcap F_n = \emptyset$ there is a finite subcollection $\{F_{n_1}, \dots, F_{n_K}\}$ such that $\bigcap_{k=1}^K F_{n_k} = \emptyset$.

Exercise 5

A metric space (X, d) is complete if and only if every decreasing sequence $F_1 \supset F_2 \supset F_3 \dots$ of nonempty closed sets with $\mathrm{diam}F_k \to 0$ is such that $\bigcap_{k \geq 1} F_k$ is a singleton.[1]

Exercise 6

Show that (l^∞, d_∞) is not separable.

Exercise 7

Prove all the following statements:

(i) Every sequentially compact metric space is complete.

*Due by **October Wed 30th, 7pm**.

[1] $\mathrm{diam}A = \sup_{a,b \in A} d(a, b)$

(ii) Every compact metric space is complete. In showing this, do not use the known equivalence between compactness and sequential compactness.[2]

[2]You may want to follow the following hints:

a. Let (x_m) be a Cauchy sequence in a compact metric space (X, d). Argue that, for every $\varepsilon > 0$, there exists a $y \in X$ s.t. $B(y, \varepsilon)$ contains all but finitely many elements of (x_m).

b. Use the Finite Intersection Property to show that (x_m) has a limit point.