Media Competition and Social Disagreement

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Abstract

We study the competitive provision and endogenous acquisition of political information. Our main result identifies a natural equilibrium channel through which a more competitive market for information increases social disagreement. A critical insight we put forward is that competition among information providers leads to a particular kind of informational specialization: firms provide relatively less information on issues that are of common interest and relatively more information on issues along which agents' preferences are more heterogeneous. This enables agents to find information providers that are better aligned with their preferences. While agents become better informed on an individual level, the social value of the information provided in equilibrium decreases, thereby decreasing the probability that the society will implement socially optimal policies.

1. Introduction

We present a simple model to study the competitive provision and endogenous acquisition of political information. Our primary goal is to illustrate the effects of increased competition in the market for news at three distinct levels: the kind of information that firms supply, the distribution of opinions in the society, and the efficiency of voting outcomes. Our interest is motivated by a growing public debate on the consequences of a fast-changing media landscape and information consumption habits on our democracies.[1] So far, the literature on the political economy of mass media has not evaluated competition in the context of a full-fledged equilibrium model in which an arbitrary number of non-partisan media firms compete for the attention of rational voters. This paper fills this gap, presenting a particularly simple model – the analysis of which leads to two main conclusions: First, we show that competition leads to *content* specialization, even when consumers are rational and news sources are unbiased. The novel insight we put forward is that competition forces information providers to become relatively less informative on issues that are particularly important from a social point of view. Second, we analyze the downstream effects of such specialization and show that, while agents becomes better informed on an individual level, competition amplifies social disagreement. Next, we highlight the social welfare implications of increased disagreement. Previous results in this literature have emphasized inefficiencies associated with supply- or demand-side frictions in the media market. By contrast, we illustrate a natural channel through which increased competition systematically decreases the probability that the society will be able to successfully implement socially optimal policies.

In our model, a finite number of firms compete to provide information to a continuum of Bayesian agents about a newly proposed policy with uncertain prospects. Whether the new policy is implemented to replace the known status quo depends on its approval rate. The policy contains a vertical component, *valence* issue, along which preferences are identical, and two horizontal components, *ideological* issues, along which preferences are heterogeneous. Firms generate signals about these components, but in doing so face a budget constraint on how informative these signals can be. Specifically, being more precise about one of these three components requires the firm to be less precise about the other two. To fix ideas, imagine a new health care bill is under discussion, the details of which are not yet fully known by the public. The bill potentially affects many dimensions of social life, and voters might evaluate these dimensions differently. For example, the new bill could promote an increase in the overall quality of health care (vertical dimension), expand the budget deficit (horizontal), and

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[1]Pew Research Center (2016), Sunstein (2017), and Nichols (2017) provide a comprehensive description of the media market and how it changed in the last two decades.

induce more redistribution via increasing the share of the population covered (horizontal). Voters gather information from the media and voice their opinions. A larger public consensus increases the probability that the bill will successfully go through Congress. Media compete for the attention of the public and allocate their limited resources (journalists, airtime etc.) to a possibly different mix of these policy dimensions in order to maximize their readership.

The equilibrium of our model demonstrates how competition among information providers affects content specialization. While all agents want to learn about the underlying state of nature (the details of the policy), different agents would like to learn about different aspects of it. But since firms compete for readership, they have an incentive to generate information that is simultaneously valuable for agents of different types. They can do so by being informative about dimensions of common interest, i.e. valence issues. However, as the market becomes more competitive, the effectiveness of such a generalist approach declines; different firms target agents of different types, providing signals tailored to those specific agents' informational needs. This particular interaction between producers and consumers of information does not seem special to our setup. Rather, this interaction appears to be a generic feature of the competitive provision of information by profit maximizing firms to a heterogeneous Bayesian audience. The equilibrium analysis leads to novel insights. First, competition creates social value in the sense that it makes each agent better informed about her own evaluation of the policy. However, agents become more informed on increasingly different aspects of the state space: those aspects that they specifically care about. Second, the market never overspecializes. As the number of firms in the market grows to infinity, the equilibrium converges to a *daily-me* paradigm, a situation in which each agent finds a news source perfectly designed to meet her unique informational needs. Third, because agents become better informed about different dimensions of the state space, they disagree more and therefore the probability that they will collectively implement policies that are socially optimal decreases.

The equilibrium mechanism behind our main results can be explained as the interaction between two distinct parts. The first part exploits a simple idea. Suppose a group of agents individually choose between two options: a safe option, ensuring a payoff of zero, and a risky one, yielding an uncertain payoff that depends on both a common component and a private one. An informative signal about the common (private) component will increase (decrease) the probability that agents will choose the same option. A similar force is behind our main result: valence acts as a common component, while ideology *can* act as a private component. While simple, this mechanism is incomplete. In this paper, we illustrate why, when information is competitively supplied, firms react to a shrinking market share by providing less information precisely on the common component and not in other ways. This

brings us to the second part of our mechanism. In our model, specialization comes about because information on the vertical component (i.e. valence) is, by definition, equally useful to all agents. This is not the case for information on the horizontal components (ideology). As agents have heterogeneous preferences on these two horizontal components, there is scope for firms to specialize in different mixtures of these dimensions. The interactions between these two parts of the mechanism generate an equilibrium channel through which an increase in competition leads to an increase in disagreement among the population – a result that is at the heart of our contribution.

We then use our model as a benchmark to study the effects of media competition on social welfare. From an ex ante point of view, we find that competition – via specialization – creates a larger spectrum of informational options for agents, enabling them to select news sources that are better aligned with their needs. In this sense, competition increases the ex ante social welfare, as it makes agents individually better informed. This result conforms to the classic view that sees the market for news as a "marketplace of ideas," promoting knowledge and the discovery of truth.[2] More generally, it aligns with previous results in this literature that we will review below. However, the welfare effects of media competition extend well beyond the individual information acquisition stage. The market for political news differs from other markets partly because it has an indirect effect on welfare through information externalities imposed on the policy process (Prat (Forthcoming)). Consistent with this view, our model predicts that, while agents become individually more informed, their opinions diverge as they become informed on increasingly different aspects of the uncertain policy. This effect triggers the increase in social disagreement documented above and it is responsible of potentially undesirable social outcomes. These inefficiencies originates from heterogeneity in agents' preferences. The political process, by definition, aggregates the opinions of agents who are potentially in conflict with each other. We show that, for any non-degenerate distribution of ideological preferences, this ex post aggregation creates a wedge between the *social* and the *individual* value of information. That is, competition necessarily results in a supply of information on the valence issue that is inefficiently low from a social perspective. As a consequence, we show that a more competitive market for news decreases the probability that society will be able to discern between socially "good" and "bad" policies.

The rest of the paper is organized as follows. The next subsection reviews the related literature and discusses the empirical implications of our work. Section 2 introduces our model and discusses its main assumptions. In Section 3, we solve for the equilibrium of the information provision game for an arbitrary number of competing firms. Moreover, we establish how competition affects the equilibrium supply of information. Our main results

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[2]See Posner (1986) for a wide-ranging introduction to this classic view.

are presented in Section 4, where we discuss how competition affects the value of information, social disagreement and welfare. Sections 5 and 6 discuss important extensions of our model and offer some concluding remarks. All proofs are relegated to the Appendix.

1.1.   Related Literature and Empirical Implications

Our paper contributes to the burgeoning literature on the political economy of mass media.[3] Specifically, we contribute to the branch of this literature that studies the effects of the endogenous provision of information and its externalities on the political process. One robust finding of this literature is that when information providers are partisans – namely, they are interested in persuading the public to take a certain action – competition generally brings about better social outcomes. Intuitively, competition forces firms to better align with what consumers demand, thus reducing their inherent biases. Results along this line are reflected in the works of Baron (2006), Chan and Suen (2009), and Anderson and McLaren (2012).[4] Similarly, Duggan and Martinelli (2011) find that slanting is an equilibrium outcome in a richer model that allows for electoral competition, but otherwise abstracts away the problem of competitive information provision. While not modeling competition, the works of Alonso and Câmara (2016) and Bandyopadhyay et al. (2017) also belong to this strand of the literature. Instead, a general treatment of competition among biased senders is discussed in Gentzkow and Kamenica (2017). Our work differs from these papers; we assume that information providers are non-partisans and they compete for consumers' attention to maximize advertisement revenues. Chan and Suen (2008) consider a model with features that can be mapped back to our setup. Their primary interest, however, is to study the effects of exogenously located firms on electoral competition. They show that a new entrant increases the probability that parties will choose the policy favored by the median voter, thereby increasing social welfare. In an extension, they also endogenize competition, but the only industry structure they can feasibly analyze (a duopoly) typically leads to higher welfare. Closer to our work, Chen and Suen (2018) study a competition model in which a number of biased media firms compete for the scarce attention of readers, finding that an increase in competition leads to an increase in the overall informativeness of the industry. Similarly, results consistent with the idea that competition is welfare-increasing are discussed in Burke (2008), Gentzkow and Shapiro (2006), and Gentzkow et al. (2014). Sobbrio (2014) does

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[3]See Prat and Strömberg (2013) and Gentzkow et al. (2015) for recent and comprehensive reviews of the literature.

[4]The welfare-increasing effects of competition are also illustrated in Besley and Prat (2006), Corneo (2006) and Gehlbach and Sonin (2014), although for orthogonal reasons from those discussed here, namely the potential risks of media capture by the government.

not analyze the social welfare implications of media competition, but shows that competition can lead to specialization. Galperti and Trevino (2018) study a model of endogenous provision and acquisition of information and show how competition for attention can lead to a homogeneous supply of information, even when consumers would value accessing heterogeneous sources. Overall, when consumers are rational, evidence is stacked in favor of the welfare-increasing effects of media competition. Our paper contributes to this literature by developing a full-fledged competition model that illustrates a novel and natural channel through which competition can be welfare-decreasing. While not analyzing the competitive provision of information, Ali et al. (2017) study the interaction between private information and distributive conflicts in the context of a voting game. More specifically, they provide necessary and sufficient conditions under which the strategic interactions among agents can preclude a policy that is both ex ante and ex post optimal from being implemented. This is due to a form of adverse selection when information is scarce, an effect that is markedly distinct from the inefficiency we highlight it this paper. Departing from the assumption of rationality when processing information, Mullainathan and Shleifer (2005) consider a model in which heterogeneous consumers derive psychological utility from their prior views being confirmed by new observations. Consistent with the findings discussed above, they also find that more competition leads to specialization and a decrease in prices. In a related model with agents having behavioral preferences for confirmation, Bernhardt et al. (2008) study the welfare implications of competition, showing that competition increases the probability the society will make mistakes in policy selection. Bordalo et al. (2016) analyze a model in which two firms compete for the attention of group of "salient thinkers" by strategically setting the quality and the price of the product they sell. They show how distortions in consumers' perception can explain the equilibrium degree of commoditization of some markets. Relatedly, Matějka and Tabellini (2017) study policy selection when voters are rationally inattentive. Complementing our results, they find that divisive issues attract the most attention by voters, and that this can create inefficiencies in public good provision.

Empirical Implications. Our paper also relates to the large empirical literature that specifically studies the effects of media competition on political participation and electoral outcomes (Stromberg (2004), Gentzkow (2006), Stone and Simas (2010), Gentzkow et al. (2011), Falck et al. (2014), Drago et al. (2014), Miner (2015), Cagè (2017), Gavazza et al. (Forthcoming), Campante et al. (Forthcoming)).[5] Our paper contributes to this literature with three distinct empirical predictions. First, it predicts that stronger media competition leads to more *content* specialization. To date, a great deal of attention in the empirical

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[5]While not explicitly focusing on media competition, DellaVigna and Kaplan (2007) and, more recently, Martin and Yurukoglu (2017) study the effect of biased news on voting behavior. Relatedly, Boxell et al. (2017) study the relationship between social media use and polarization.

literature was dedicated to media ideological biases. Our paper shows that, even in the absence of such biases, firms can specialize their products by creating *content* that, while not ideologically slanted, targets audiences with different preferences. The analysis of news' content involves non-trivial technical challenges. However, novel empirical methods that exploit machine learning techniques to analyze textual data have provided initial evidence that aligns with content specialization, as formalized by our model. Angelucci et al. (2018) explore the content production of local newspapers in the 50ies, as they started competing with television. They find evidence of content specialization, taking the form of an increased emphasis on local news, as opposed to national ones. Similarly, Nimark and Pitschner (2018) provide a comprehensive and recent account of the extent to which American newspapers specialize in the production of their content. Finally, Martin and Yurukoglu (2017) find evidence of content specialization for major cable outlets and provide evidence of correlation with competition. While more research on this topic is needed, the use of these novel techniques is expected to grow in the field (Gentzkow et al. (Forthcoming)). Second, our model predicts that an increase in media competition is correlated with an increase in disagreement even among rational agents. There is a growing literature analyzing polarization in public opinion. A number of papers have investigated this channel (Prior (2013), Campante and Hojman (2013)), but evidence is mixed and more research is needed. Finally, our model predicts that content specialization will develop at the expense of information about valence issues, dimensions of the policy space along which agents have particularly homogeneous preferences.

2. Model

A society with a unit mass of Bayesian agents evaluates an uncertain policy $\theta := (\theta_0, \theta_1, \theta_2) \in \mathbb{R}^3$, the components of which are believed to be mutually independent and identically distributed as standard normals. Agents have heterogeneous preferences about the policy θ. Specifically, an agent of type $t \in T := [-\pi, \pi]$, drawn from a uniform distribution F, evaluates policy θ according to the utility function

$$u(\theta, t) := \lambda\theta_0 + (1-\lambda)\theta_t^{id} \qquad \text{with} \qquad \theta_t^{id} := \cos(t)\theta_1 + \sin(t)\theta_2. \tag{1}$$

We refer to dimension θ_0 – along which agents' preferences are perfectly aligned – as the *valence* component of policy θ and to dimensions θ_1 and θ_2 – along which agents' preferences are type-dependent – as its *ideological* components.[6] The parameter $\lambda \in (0,1)$ denotes the importance of the valence component relative to ideological ones.

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[6]In the political science literature, this distinction goes back to Downs (1957) and Stokes (1963).

A set N with $n \in \mathbb{N}$ firms provides information about θ by committing to an information structure. To do so, each firm $i \in N$ chooses a vector of weights $b_i \in \mathbb{R}^3$ that satisfies the budget constraint $\|b_i\| \leq 1$, where $\|\cdot\|$ is the Euclidean norm. A choice of a b_i induces an information structure $s_i(\theta) := (s_i^v(\theta), s_i^{id}(\theta))$, composed of signals $s_i^v(\theta) := b_{i,0}\theta_0 + \varepsilon$, informative about the valence component, and $s_i^{id}(\theta) := b_{i,1}\theta_1 + b_{i,2}\theta_2 + \varepsilon$, informative about the ideological ones. The error term ε is assumed to be standard normal and independent across both firms and agents. A firm's profits increase with its readership, which the measure of agents $S \subset T$ who acquire information from it.

Each agent selects an information structure and costlessly receives a signal realization $s_i := (s_i^v, s_i^{id})$.[7] Given the signal realization, the agent evaluates the unknown policy θ relative to a known status, the utility of which is normalized at zero. We assume that agents vote sincerely and receive utility $\mathbb{E}(u(\theta, t)|s_i)$ if they approve the policy and zero otherwise. Let $z(\theta, t)$ be the random variable describing type t's approval behavior of the policy conditional on θ.[8] We assume that policy θ is implemented with a probability that is strictly increasing in $\Gamma(\theta) := \mathbb{E}(\int_T z(\theta, t)dF(t))$, the expected *approval rate* of policy θ. Figure 1 depicts the timeline of the game. The solution concept we use is Perfect Bayesian Equilibrium (PBE).

Each firm $i \in N$ commits   Nature determines θ
to an information structure   and signals are realized

<div align="center">

0   1   2   3

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Each agent chooses which   Agents express opinions,
information structure to consume   collective decision is implemented
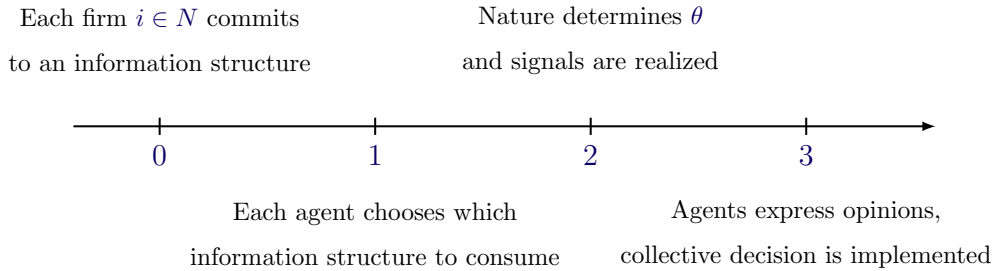
FIGURE 1: Timeline of the game

2.1. Discussion of the Model

We pause for a discussion of our model and the implications of our assumptions. As detailed in the following sections, a key aspect of the equilibrium mechanism is how competition affects firms' incentives to specialize. Non-price competition among firms leads to product specialization in markets with two features (Tirole (1988)): firms are constrained in their supply and there is heterogeneity in consumers' demand. There are several different ways

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[7]In Section 5, we show that our results extend to the case where each agent can choose *finitely* many firms.

[8]Whenever it is not a source of confusion, we abuse notation and don't explicitly condition z on the received information.

in which such constraints and heterogeneity can be introduced and modeled. The choices we have made serve two main purposes: they provide enough tractability to solve for the equilibrium with an arbitrary number of firms, and they allow for a particularly transparent and clean depiction of the main forces at play. We should, however, emphasize that, as discussed below, the key forces driving our results are more general than the model we present.

Signals. Firms solve a problem of finite-resource allocation by trading off informativeness on different dimensions of the state space. Without this type of substitutability imposed by the budget constraint and the signal structure, firms would always choose to reveal the state perfectly, and the competition problem would become uninteresting. More importantly, this assumption captures aspects of the real world that are ubiquitous: firms face both supply-side constraints (in the number of journalists they can employ, the number of pages they can fill, their allotted airtime, etc.) and demand-side ones (e.g. consumers' attention). Also, just like in the work of Duggan and Martinelli (2011), when providing information on the ideological dimensions of the policy, our firms reduce a two-dimensional state (θ_1, θ_2) to a one-dimensional signal. While immaterial for our main results, this assumption proves to be extremely convenient, as it allows us to think of the firm's problem as a location on a *disk*.[9]

Preferences. Despite their Bayesian nature, agents in our model may assign different valuations to the same information structure. This is because of the heterogeneity in their preferences. Heterogeneity in voter preferences is a well-documented phenomenon that has been studied in different contexts. Of course, there are several different ways to model such heterogeneity, all leading to the similar conclusion that different types of agents assign different values to the same information structure. Our preferences are designed to capture heterogeneity in a tractable way, i.e. via a one-dimensional type $t \in T$, while retaining a the following key feature: agents can disagree both on which issues are important to them (their "agenda", so to speak) and on how each issue in their agenda should be addressed (their "slant").[10] An agent's type t simultaneously captures both the relative weight she puts on

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[9]Other kinds of budget constraints have been discussed in the literature. For example, while considering a one-dimensional state space, Chan and Suen (2008) restrict their news sources to binary signals. This choice generates trade-offs that are qualitatively similar, as signals cannot be equally informative to all agents. More generally, constraints can be introduced via a cost function that imposes a cost on precision.

[10]Pew Research Center (January, 2017) provides plenty of evidence on the differences in voters agendas. Consistently, multidimensional preferences with similar characteristics are common in this literature. See for example Groseclose (2001), Eyster and Kittsteiner (2007), Carillo and Castanheira (2008), Ashworth and de Mesquita (2009), Dragu and Fan (2015), Aragones et al. (2015), and Yuksel (2015) for applications in the context of party competition, and Alesina et al. (1999), Lizzeri and Persico (2005), and Fernandez and Levy (2008) for applications to public goods.

different issues and her position on each of these issues. For example, given our preference specification in Equation 1, type $t = \pi/4$ prefers higher realizations of both θ_1 and θ_2 and attaches equal weight to both dimensions (Figure 2). By contrast, type $t = -\pi/4$ prefers higher realizations of θ_1 and lower realizations of θ_2, but still attaches equal weight to both issues. This way of modeling heterogeneity has other desirable features. First, the distance $|t - t'|$ between two types on the circumference provides us with a simple measure of the degree of correlation between their preferences. This feature allows us to map the competition game among firms into a spatial problem. Second, these preferences give us a natural normalization in which all agents, despite their heterogeneity, ex ante dislike uncertainty in the same way.[11] Finally, λ measures how important the valence dimension is relative to the ideological ones and thus can be interpreted as an ex ante measure of polarization.

Distribution over types. We assume that types are uniformly distributed over T. While important for characterizing the equilibrium in closed-form, this assumption is not particularly problematic from a conceptual point of view. This is because of the following two important reasons. First, content specialization is a robust consequence of competition that generalizes beyond the uniform distribution. In our model, irrespective of the details on the type-distribution, each agent receives their most-preferred information structure as the number of firms in the market goes to infinity (see Proposition 4 and Remark 3). Second, a key feature in our welfare result will be that the *social* value of information on the valence component exceeds its *individual* value. Remark 4 shows that is true under any non-degenerate type-distribution. Finally, conditional on θ, our model can always be mapped into a more standard, one-dimensional model in which there is heterogeneity in the evaluation of the policy. The uniformity of F guarantees that the distribution is symmetric around the median evaluation, which also corresponds to $\lambda\theta_0$. We return to these points in the final discussion in Section 6.

Readership. Firms do not compete on prices, rather on readership. This assumption seems sensible for at least three reasons. First, the largest share of revenues in the media industry is generated through advertising, which mostly depends on readership. Whenever positive, the price for political news is nevertheless often negligible. Second, whenever present, price competition in the media industry is often highly regulated.[12] Third, price competition would provide an additional incentive for product differentiation, thereby exacerbating disagreement in the society (Section 4). In this sense, our result is perhaps surprising in that,

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[11]Indeed, the variance $\mathbb{V}\big(u(\theta, t)\big)$ is independent of t. To see this, notice that $\theta_1 \cos(t) + \theta_2 \sin(t) \sim \mathcal{N}(0, 1)$ for all t.

[12]A major example of this is the Newspaper Preservation Act of 1970 in the United States exempting competing newspapers from certain provisions of antitrust laws.
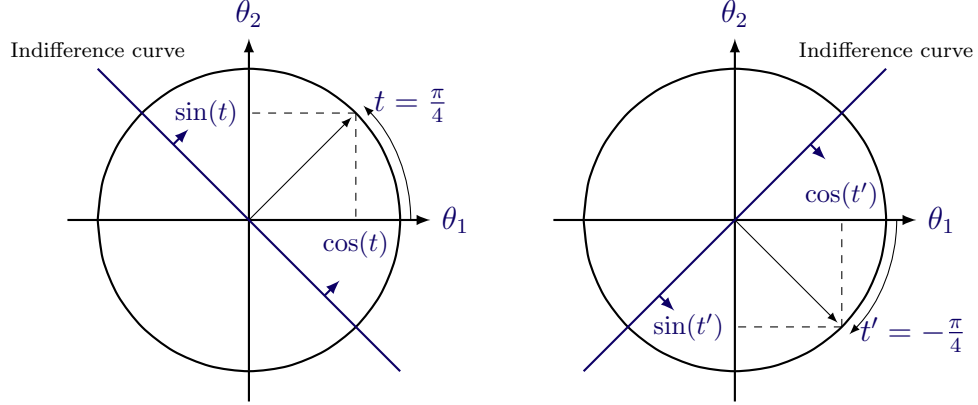
FIGURE 2: The ideological trade-offs between (θ_2, θ_3) for different types $t \in T$.

even in the absence of price competition, incentives for differentiation are strong enough to produce negative welfare implications.

Sincere voting. Agents express either a favorable or an unfavorable opinion about the policy and receive direct utility from such activity. One can think of this as voting for or against a policy in a referendum or a political challenger who takes positions on several issues. We put aside the question of why people express their preferences and vote. Indeed, in a model with a continuum of voters, no individual has an impact on the election outcome. A direct utility from honest voting (perhaps arising from a sense of civic responsibility) is the most straightforward and possibly most realistic assumption in this context. In Section 6, we discuss the robustness of our main results to strategic voting.

3. Equilibrium

This section is devoted to the analysis of the equilibrium of our game and is divided into three parts. We begin by reducing the firm's problem to a location on a disc. Then, we solve the agents' information acquisition problem and characterize its properties. Finally, we solve for the equilibrium of the game.

3.1. Problem of the Firm: Consumer-Targeting

Firm $i \in N$ chooses a vector of weights $b_i \in \mathbb{R}^3$ to maximize readership, while respecting the budget constraint $\|b_i\| \leq 1$. In this subsection, we reduce this problem to a *consumer-targeting* problem on the unit disc, something that will prove to be extremely convenient in the equilibrium analysis. Denote the action set $A_i := T \times [0,1] \in A_i$ with typical element

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$$a_i = (x_i, \tau_i).$$

Remark 1. *Fix b_i s.t. $\|b_i\| = 1$. There exists a unique action $a_i = (x_i, \tau_i) \in A_i$ such that the signals induced by b_i are distributed as $s^v(\theta) \sim \mathcal{N}\left(\theta_0, \frac{1}{\tau_i}\right)$ and $s^{id}(\theta) \sim \mathcal{N}\left(\theta_{x_i}^{id}, \frac{1}{1-\tau_i}\right)$.*

Note that it is without loss of generality to focus attention on b_i such that $\|b_i\| = 1$. When this is not the case, i.e. when $\|b_i\| < 1$, a scalar $c > 1$ exists such that $\|cb_i\| \leq 1$ is still feasible and induces an information structure that Blackwell-dominates the one induced by b_i. By Blackwell's theorem, every type $t \in T$ would assign a higher value to cb_i and therefore the readership of firm i would weakly increase, which would not be consistent with equilibrium behavior. Therefore, the result in Remark 1 allows us to think of the problem of the firm "as if" firm i were choosing (1) a consumer $x_i \in T$ to target and (2) how much to invest in the precision of the signal about the valence component, $\tau_i \in [0,1]$. This representation will allow us to think of the firm's problem as a location on a disc. Like in a standard location model, agents will acquire information from the firm located "closest" to them. However, unlike most location models, the notion of distance will not given exogenously, via a so-called transportation cost. In our model, the notion of distance is a by-product of the fact that agents are Bayesian: it will be measured via the "value of information."

3.2. The Value of Information

In our model, agents behave in a straightforward way: an agent of type t will acquire information from the firm that produces the highest *value of information* conditional on her type t. In this section, our main task is to analytically compute the value of a generic information structure for a generic agent of type t. Consider the information structure induced by $a_i = (x_i, \tau_i)$. After observing the realization of signals $s = (s^v, s^{id})$, agent $t \in T$ can compute the expected utility of policy θ, denoted by $\mathbb{E}(u(\theta,t)|s)$. Given the agent's incentives specified in Section 2, she either approves the policy or rejects it to receive utility $\mathbb{E}(u(\theta,t)|s)$ or zero, respectively. The value of the information structure induced by a_i is therefore the *ex ante* expectation of a such future contingent payoff, namely

$$V(a_i|t) := \mathbb{E}\left(\max\left\{0, \mathbb{E}(u(\theta,t)|s)\right\}\right) \geq 0$$

where the outermost expectation is taken with respect to the possible realization of s defined by a_i and the agent's prior belief on θ. Intuitively, the value of an information structure for a type $t \in T$, i.e. $V(a_i|t)$, measures how much better off she expects to be after receiving the signals, relative to receiving no signals whatsoever.[13] The function $V(a_i|t)$ is the

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[13]Note that the value of receiving no information is $\mathbb{E}\left(\max\left\{0, \mathbb{E}(u(\theta,t))\right\}\right) = 0$.

information-theoretic counterpart of the "transportation cost" in a location model. This function is a key building block of our model and, importantly for our later results, we can express it analytically, as the next proposition shows.

Proposition 1. *The value of information* $a_i = (\tau_i, x_i) \in A_i$ *for an agent of type* $t \in T$ *is*

$$V(a_i|t) = \frac{\sigma(a_i|t)}{\sqrt{2\pi}} \tag{2}$$

with $\sigma^2(a_i|t) := \lambda^2 g(\tau_i) + (1-\lambda)^2 \cos^2(t - x_i) g(1 - \tau_i)$ *and* $g(\tau_i) := \dfrac{\tau_i}{1 + \tau_i}$.

In equilibrium, given a profile of actions $a \in A := \prod_i A_i$, agent t chooses the firm offering the highest value of information $V(a_i|t)$. Formally, given the profile of information structures induced by a, we denote the information acquisition behavior of type t by $r(a, t) \in \Delta(N)$.

Proposition 1 is an important step in the solution of our model, as it provides us with a full characterization of the value of information in our game. The first insight is that $\sigma^2(a_i|t)$, namely the variance of the random variable $\mathbb{E}(u(\theta, t)|s)$, is in a one-to-one relation with the value of information. The intuition is simple: when the variance of interim utility is high, the relative gain of choosing one option versus another, conditional on the signal realizations, is likely to be high as well. This implies that agent t's ex ante expectation of future payoffs is also high. The second insight is that the value of information can be reduced to two components, both with precise economic meaning. These two components will reveal the main trade-off that firms face when designing these information structures.

Remark 2. *The value of information* $V(a_i|t)$ *is determined by two components:*

1. *A* generalist *component* $\lambda^2 g(\tau_i)$, *strictly increasing in* τ_i *and type-independent.*

2. *A* specialist *component* $(1 - \lambda)^2 \cos^2(t - x_i) g(1 - \tau_i)$, *strictly decreasing in* τ_i *and increasing in* $\cos^2(t - x)$.

To understand this result, consider an increase in τ_i. This corresponds to an increase in the precision allocated to the valence signal s^v. Remark 2 shows that an increase in τ_i makes the information structure more *generalist*: since all agents (equally) care about the valence component, an information structure that is more informative on the valence dimension will benefit all agents, irrespective of their type t. By contrast, a decrease in τ_i makes the information more *specialist*. However, there are a multitude of different ways in which an information structure can be specialist, according to which "ideological mixture" it highlights. In Remark 2, this mixture is captured formally by the term $\cos^2(t - x_i)$, which has a precise economic interpretation. It turns out, in fact, that the correlation between the agent t's

preferences and those of the targeted agent x_i is $\mathbb{C}\big(u(\theta, t), u(\theta, x_i)\big) = \lambda^2 + (1 - \lambda^2)\cos(t - x)$. This has two implications for our model. First, the length of the arc between agents t and x_i measures their relative *ideological distance*.[14] Second, the value of information is ultimately affected by $\cos^2(t - x_i)$, the square of the correlation. Therefore, two types with perfectly negatively correlated preferences, i.e. t and $t + \pi$, value information equally, as they agree on the weights they attach to θ_1 and θ_2. This implication is a by-product of the fact that agents are Bayesian and information structures are unbiased. Therefore, from the point of view of the firms, targeting agent x_i is equivalent to targeting agent $x_i + \pi$: therefore, the problem of the firm can be further reduced to choosing a target $x_i \in \tilde{T} := [-\pi/2, \pi/2]$, with no loss of generality.

The trade-off between being generalist vs specialist in the choice of the information structure is depicted in Figure 3. We fix $x_i = 0$ and plot the value of two information structures as a function of t. When τ_i is high, the information structure is more *generalist*. The value associated with this news source is not particularly high, even for the agents that are close to the target x_i, but remains steady even for agents that are far away from it. On the other hand, when τ_i is low, the information structure is more *specialist*. The associated value is high for types that are close to the target x_i, but drops rapidly for voters whose ideological preferences are farther away. The problem of the firm can be visualized as the choice of a location on a disk (Figure 4). Each firm chooses a precision τ_i on the valence component – implying a distance from the center – and a target $x_i \in \tilde{T}$ – implying a particular angle in the circle.[15]

3.3. The Equilibrium Design of Information Structures

After having characterized the information acquisition problem, we proceed with the characterization of the firms problem. When choosing $a_i \in A$, a firm optimally designs its information structures to maximize readership. More specifically, if firm $i \in N$ chooses a_i, it earns profits given by:

$$\Pi_i(a_i, a_{-i}) := \int_T r(a, t)[i] \; dF(t).$$

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[14]Under this measure, two agents can be ideologically similar even when their respective "bliss-points" are far apart. This happens when they trade off different payoff-relevant dimensions in similar ways, and hence their preferences are highly correlated. This way of measuring ideological distance adds to the existing literature on polarization (see Gordon and Landa (2017)), which had mostly focused on bliss-points distances, by offering a more nuanced way of measuring polarization empirically.

[15]For example, choosing $x_i = 0$ (resp. $x_i = \pm\frac{\pi}{2}$) implies that the firm only reports about dimensions (θ_0, θ_1) (resp. (θ_0, θ_2)).
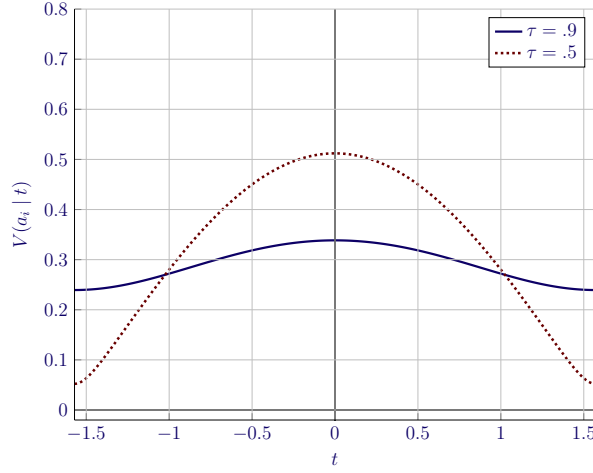
FIGURE 3: The value of information $a_i = (\tau_i, x_i)$ for an agent of type t. On the horizontal axis, $t - x_i$ represents the ideological distance between type t and the targeted type $x_i = 0$
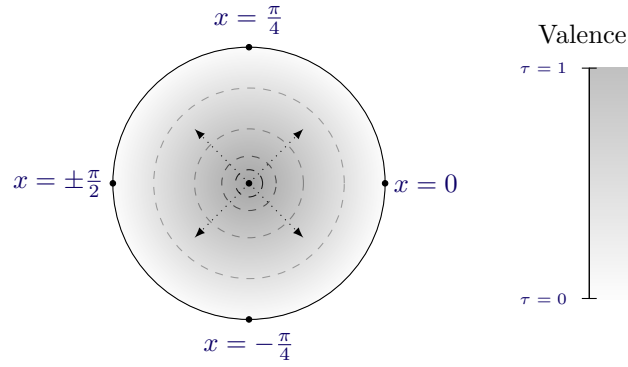
FIGURE 4: Mapping the firm's problem into a location choice

where a_{-i} is the vector of its opponents' choices and $r(a, t)[i]$ is the probability that type t chooses a_i given the profile $a = (a_i, a_{-i})$.

We focus on equilibria in pure strategies, defined as follows.

Definition 1. *A PBE is a triple* $(a^\star, r^\star, z^\star)$ *such that:*

(i) *for all* $i \in N$ *and* $a_i \in A_i$, $\Pi_i(a_i^\star, a_{-i}^\star) \geq \Pi_i(a_i, a_{-i}^\star)$;

(ii) *for all* $t \in T$ *and* $a \in A$, *supp* $r^\star(t, a) \subseteq \arg\max_{i \in N} V(a_i|t)$;

We say that an equilibrium is symmetric *if* $a^\star$ *is such that* $\tau_i = \tau_j$ *for every* $i, j \in N$.

In equilibrium, each firm maximizes its readership relative to the behavior of other firms, agents only consume information structures that yield the highest value and, conditional on the signals received, agents sincerely approve or disapprove the policy θ. Figure 4 provides a

graphical illustration of the concept of symmetric equilibrium. In a symmetric equilibrium, firms are located on the same inner circle ($\tau_i = \tau_j$ for all $i, j \in N$). The next result establishes the existence and generic uniqueness of the equilibrium in our game.

Proposition 2. *A PBE exists and all PBEs are necessarily symmetric. Moreover, there exists $\bar{n} \in \mathbb{N}$ such that, for all $n \geq \bar{n}$, firms target a set of agents equidistant from each other and the equilibrium is unique up to* modular rotations *of the firms' locations.*[16]

The symmetry in our equilibria, as set out in Definition 1, relies heavily on the uniformity of the type-distribution F.[17] We focus most of our attention on the uniform case for two reasons. The first one is tractability: symmetry allows us to provide a full characterization of how firms locate on the circle, and explicitly solve for the locations of an arbitrary number of firms. The second reason is salience: uniformity creates an environment in which the strategic tensions we highlight in this paper are extreme, therefore making the channel behind our results more transparent. While useful, the assumption of uniformity is however inessential from a more qualitative point of view. As we will argue in Section 6 and, in particular, in Proposition 4 and Remark 3, our main insights extend to more general type-distributions. Finally, it is worth noting that, while a continuum of equilibria can be generated by simply rotating the locations of firms, this multiplicity is immaterial for the problem we study. Due to the uniformity of F, it is not the firms' absolute location that matters, but rather their relative distance, which is not affected by modular rotations.

Our next goal is to analyze the equilibrium of the game, as more firms enter the market. In particular, we will study the effects of an increase in competition on the type of information that firms produce and, ultimately, on the agents' behavior.

3.4. Increasing Competition in the Market for News

In this final part of the section, we characterize the effects of competition on *firms'* behavior. The result of our next proposition will serve as the building block for our study of how competition affects *agents'* behavior and, more generally, social welfare. We study the increase in competition by comparing equilibria as the number of firms in the market increases. We show that, as the market becomes more competitive, a firm's optimal response is to specialize. This kind of "informational specialization" takes a specific form: firms specialize by

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[16]More formally, if $(x_i)_{i \in N}$ are the firms' equilibrium locations, any modular rotation $(x_1 \oplus k, \ldots, x_n \oplus k)$ for $k \in \mathbb{R}$ with modulo π still constitutes an equilibrium.

[17]The mere existence of an equilibrium, in fact, is guaranteed by standard existence results, via Glicksberg's Theorem, for any atomless type-distribution F.

providing relatively less information on the common-interest component of agents' preferences, namely the valence component.

Proposition 3. *As competition increases, the precision $\tau^\star(n)$ on the valence component is decreasing. More specifically, there exists a $\bar{n} \in \mathbb{N}$ such that the equilibrium $\tau^\star(n)$ is constant for $n < \bar{n}$, and strictly decreasing otherwise.*

The intuition behind this result is closely linked to Remark 2. As n increases, the market becomes more competitive and firms compete over an increasingly smaller market share. The market share of each firm has a well-defined structure when mapped into the circle of circumference $\tilde{T}$, namely it is a connected set of types (Figure 5). Graphically, this means that the arc of agents serviced by the firms becomes smaller as n increases. This implies that the preferences of the agents in this shrinking arc become increasingly correlated with those of the targeted agent x_i. In other words, the firms provide information to a set of agents whose ideological preferences are increasingly aligned. The best response to a market share that becomes increasingly homogeneous is to decrease τ_i, that is, to disinvest in valence in favor of ideology. In fact, as n increases, the firm's marginal benefit of decreasing τ_i is higher since $\cos^2(t - x_i)$ is higher for all $t \in T$ such that $r(t, a)_i > 0$ (Proposition 1). Figure 5 illustrate graphically these effects of increased competition on the location of firms.
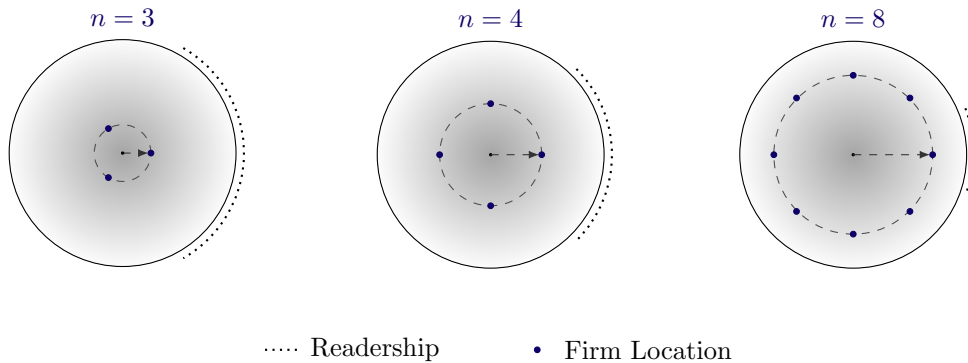
FIGURE 5: The representation of the symmetric equilibrium for several values of n

It is important to note that, as τ_i decreases, the firm actually creates value for the consumers who are ex post still acquiring information from firm i. This is why such disinvestment in valence is the best response for the firm. In this sense, the market does not *over-differentiate*: it never provides "too much" information about the ideological dimensions relative to what the agents demand. We will formalize this observation in the next section (Proposition 4). Finally, we can think of the equilibrium mechanism illustrated in Proposition 3 as an information-theoretic counterpart of the more standard idea of *product differentiation*. Differentiation is a ubiquitous feature of competition games with heterogeneous consumers.

However, it is not entirely obvious how firms that sell information would achieve such differentiation. Proposition 3 suggests an extension of the standard differentiation result to the realm of information structures. Firms differentiate their products by making the information structure they sell increasingly uncorrelated. In our model, this happens by decreasing the precision on θ_0, the common-interest component in agents' preferences, while increasing precision on different combinations of (θ_1, θ_2).

4. Competition, Disagreement and Welfare

The previous section illustrated how the equilibrium supply of information changes as competition increases. In this section, we will investigate the consequences of this result at three distinct levels. First, we study whether agents become more or less informed as competition increases. Second, we study whether or not competition fosters social agreement in agents' opinions about the policy. Third, we study the overall social welfare implications of an increase in competition.

4.1. Competition and the Value of Information

We begin by highlighting the positive effect of competition in the market for news. In the next proposition, we show that competition brings about more information to the agents. This force is manifested in our model in three different ways. First, from an *aggregate* point of view, competition increases the aggregate value of information: the society as a whole becomes better informed. Second, from an *individual* point of view, each and every agent becomes progressively more informed. Finally, as the number of competiting firms grows to infinity, each agent is provided with her first-best information structure. We refer to this limit result as the *daily-me* effect, a situation in which every consumer can find an information structure on the market that is exactly tailored to her needs (Sunstein (2001)).

Proposition 4. *For any sequence of symmetric equilibria* $(a^\star(n))_n \in \mathcal{A}^\infty$:

(a) *The* total *value of information* $\mathcal{V}(a^\star(n)) := \int_T V(a^\star(n) \,|\, t) dF(t)$ *increases in* n*. That is, as competition increases, the society as a whole becomes more informed overall.*

(b) *For any agent* $t \in T$*, there exists a subsequence* $a^\star(n_k)$ *such that* $V(a^\star(n_k) \,|\, t)$ *is increasing in* n_k*. That is, as competition increases, every single agent becomes progressively more informed.*

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(c) (Daily-me effect). *For every agent $t \in T$,* $\lim\limits_{n \to \infty} V(a^\star(n) \,|\, t) = v^\star := \max\limits_{\tau_i} V(\tau_i, x_i = t \,|\, t)$.

The first result shows that the aggregate value of information increases in the number of firms. This result speaks to the classic view that sees the market for news as a "marketplace of ideas," which promotes knowledge and the discovery of truth. The positive effects of competition are, however, even stronger. Indeed, competition not only increases the value of information in the aggregate, but also progressively increases the value of information agent-by-agent. To formalize this idea, the second result in Proposition 4 considers an arbitrary sequence of equilibria and shows that, for any given agent $t \in T$, there always exists a subsequence along which her value of information increases.[18] Finally, as the number of firms in the market grows arbitrarily large, the value of information for each agent $t \in T$ converges to the *same* limit value $v^\star$. This value represents the *first-best* value of information, that is, the value that an agent t would achieve if she could choose on her own τ_i and x_i to maximize her value of information.

These results also shed additional light on the equilibrium force behind our Proposition 3. The force that pushes firms to decrease $\tau^\star$ is, in fact, demand-driven. As the number of firms grows, each firm serves a progressively smaller set of agents and provides them with an information structure that is increasingly better suited to their specific needs, thus increasing their value of information. This result is in sharp contrast with the main result of Mullainathan and Shleifer (2005) and it is a by-product of the fact that our agents are Bayesian. On a similar note, the result in Proposition 4.(c) illustrates more specifically in what sense the market does not *over-differentiate*. When n grows arbitrarily large, firms lower τ only insofar as it makes their target consumers better off. Incidentally, this also provides intuition for why, as n grows to infinity, no firm has an incentive to deviate back to the center of the disk (Figure 5). Indeed, in such case, each agent can find a firm that is located arbitrarily close and that provides an information structure which approximate her own first-best. Proposition 4 points out that the inefficiency identified in this paper is not due to some form of market failure. On the contrary, competition enables agents to learn more effectively. It is crucial to note, however, that they learn about increasingly different aspects of the state space, as they shift focus on increasingly different mixtures of θ_1 and θ_2. This will lead their final opinion about the uncertain policy to become increasingly uncorrelated, as we show in the next subsection.

Before that, however, it is important to remark that the result in Proposition 4(c) does not

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[18]The use of a subsequence is due to the fact that any rotation of the firms' equilibrium locations that affects the value of information of a given agent t is itself an equilibrium of our game. A conceptually equivalent way to capture the same idea is to say that the *worst-case* equilibrium value of information for any agent t is increasing in n.

quite depend on the uniformity assumption on F. More specifically, such result extends in the following way.

Remark 3. *Let F be a continuous and strictly increasing distribution on T and $\left(a^\star(n)\right)_n$ be any sequence of equilibria:*

(c') *For all types $t \in T$, $\lim_{n\to\infty} |x^\star_{r^\star(a^\star(n),t)} - t| = 0$. That is, the distance between type t and the closest firm vanishes in the limit.*

(c'') *(Daily-me effect) For all types $t \in T$, $\lim_{n\to\infty} V(a^\star(n)\,|\,t) = v^\star := \max_{\tau_i} V(\tau_i, x_i = t\,|\,t)$.*

From a qualitative point of view, the important insight from this result is that, as the market converges to perfect competition, the equilibrium precision on valence becomes minimal: that is, extreme specialization is achieved irrespective of the type-distribution. In particular, while it is no longer guaranteed that the sequence $\tau^\star(n)$ is monotone decreasing at each n, it is still the case that such sequence eventually converges to the same limit of Proposition 4(c).

4.2. Competition as the Source of Disagreement

While competition increases the value of information for each agent, different agents become informed about different sub-dimensions of the state space – those that they specifically care about. This change in the supply of information has an effect on agents' approval behavior and, ultimately, on the probability that the policy will be implemented. In this section, we analyze these effects and show that increased competition unequivocally increases disagreement in the society. We define social disagreement as the probability $D_n(\theta_0)$ that, conditional on θ_0 and at an arbitrary equilibrium with n firms, two randomly selected agents $t, t' \sim F$ will disagree on whether or not the policy should be implemented. More formally, fix n and an equilibrium $a^\star(n)$. $z^\star(\theta, t)$ is the equilibrium random variable, describing the voting behavior of agent t as a function of the realized (θ_1, θ_2) and of the idiosyncratic shocks. The approval rate $\Gamma_n(\theta_0) = \mathbb{E}\big(\int_T z(\theta, t)dF(t)\big)$ can be interpreted as the probability that, conditional on θ_0, a randomly selected agent will be in favor of implementing the policy. Therefore, we define disagreement as $D_n(\theta_0) := 2\Gamma_n(\theta_0)(1 - \Gamma_n(\theta_0))$. Intuitively, a society features low (high) disagreement if it is relatively unlikely to find two agents who disagree (agree). Our next result formalizes the idea that more competition leads to higher disagreement.

Proposition 5. *Fix an arbitrary sequence of equilibria $(a^\star(n))_n$. As the number of competing firms n increases, social disagreement $D_n(\theta_0)$ increases. That is, the probability that two*

randomly selected agents will disagree about implementing a policy with valence component θ_0 increases.

To better understand this result, it is instructive to look at the interim expected utility of an agent $t \in T$ after receiving the signal realization s. For concreteness, we focus attention on an equilibrium $(\tau^\star, x^\star)$ in which agent t is targeted by firm i, i.e. $x_i^\star = t$. The agent's interim expected utility is then

$$\mathbb{E}\big(u(\theta, t)|s\big) = \lambda g(\tau^\star) s^v(\theta) + (1 - \lambda) g(1 - \tau^\star) s^{id}(\theta).$$

When $\tau^\star = 1$, agent t's approval decision is entirely based on s^v, which is identically distributed for all agents $t' \in T$, irrespective of the firm from which they are gathering information. Therefore, integrating out the random errors in s^v, the probability that agent t will be in favor of θ is entirely determined by θ_0. As n increases, however, $\tau^\star$ decreases (Proposition 3). A rational agent responds by increasing the weight she puts on signal s^{id}, which is now more informative, at the expense of the weight assigned to s^v, which is now less informative. Crucially, when gathering information from different sources, different types of agents face different distributions of s^{id}. Therefore, their voting decisions assign an increasing weight to signals that are increasingly uncorrelated with one another. For example, when signal s^{id} is negative (positive) for some agent t, the agent requires an increasingly higher (lower), and therefore increasingly less likely, realization of s^v in order to overturn her ideological taste for θ and approve (disapprove) the policy.

4.3. Disagreement and its Welfare Consequences

In this section, we conclude our analysis of the effects of increased competition by studying how disagreement affects social welfare in our model. There are multiple reasons for why increased disagreement could generate social inefficiencies: deliberation time could be longer, parliamentary representation could become increasingly fragmented, the necessity of compromise could reduce the effectiveness of proposed policies, and so on. While our model is silent on all these institutional details, we assumed monotonicity of the implementation rule: namely that a larger public support for a policy translates into a weakly higher probability of implementation.[19] We will study the effects of competition on social welfare using two distinct welfare criteria.

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[19]It is straightforward to provide a natural micro-foundation for this implementation rule. Under a classic majority rule, preferences may be subject to an aggregate interim shock (a political scandal, a terroristic attack, etc.) that sways preferences in favor of or against the policy. In such cases, $\Gamma_n(\theta_0)$ determines the probability that the will of the people will not be overturned by a particular realization of the aggregate shock. See Baron (1994) and Grossman and Helpman (1996) for more details.

Utilitarian Welfare. We begin by assessing the impact of increased competition by measuring the social ex post welfare of implementing a given policy θ, denoted by $W(\theta) := \int_T u(\theta, t) dF(t)$. Since the status quo yields utility that is normalized at zero for all agents, this means that, from a social point of view, it is optimal to implement a policy if and only if $W(\theta) \geq 0$. From this perspective, which differs from our analysis in Section 4.1 where we focused on *individual* values, different kind of information can potentially generate *social* externalities.[20] To understand this point in general terms, we drop our uniformity assumption on F and allow for more general type-distributions. We say that a distribution F on T is *symmetric* around a type $t^\star \in T$ if its density f satisfies $f(t^\star + \delta) = f(t^\star - \delta)$ for all $\delta \geq 0$.[21] In the next result, we analytically compute the utilitarian welfare $W(\theta) := \int_T u(\theta, t) dF(t)$ to establish the non-existence of a fictitious "representative agent" in our society.

Remark 4. *Let F be symmetric around type $t^\star \in T$. Then,*

$$W(\theta) := \int_T u(\theta, t) dF(t) = \lambda \theta_0 + \beta_F (1 - \lambda) \big(\theta_1 \cos(t^\star) + \theta_2 \sin(t^\star) \big),$$

where $\beta_F := \int_T \cos(t) dF(t) \in [0, 1]$.

The result above illustrates an important feature of our model. For a general class of type-distributions, the social welfare associated with implementing the uncertain policy θ, namely $W(\theta)$, is reminiscent of the private utility that agent $t^\star$ would derive from θ, namely $u(\theta, t^\star)$. However, there is an important difference, that is the factor β_F. To understand the implications of this, let's consider a few cases. First, note that $\beta_F = 1$ if and only if F is degenerate on a point $t^\star$. This would mean that the society is perfectly homogeneous: the preferences of the planner are aligned with those of the agents and there is no informational externality whatsoever. Second, if $\beta_F < 1$ or, equivalently, if F is non-degenerate, the society allows for *some* degree of preference-heterogeneity. In all these cases, the planner attaches a strictly higher relative weight to the valence component, compared to any other agent in the society.[22] In a nutshell, no agent in the economy cares about valence as much as the social planner. The valence component has an obvious "superior status" in the eyes of the social planner, because it avoids the kinds of trade-offs among agents that are otherwise inevitable with ideology. Any increase in the vector (θ_1, θ_2) will necessarily make some agents

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[20]Incidentally, this also highlights the sense in which the market for political news is peculiar and differs from other markets. Agents' preferences are aggregated at the voting stage and information is consequential to how people vote. Therefore, changes in information acquisition patterns, purely driven by competitive forces, have the potential to create important social externalities.

[21]Note that given how preferences are represented in our model, type $t + 2\pi$ is identical to $t - 2\pi$ and t. Hence, we can always define a symmetric distribution around any $t^\star$.

[22]For example, when $f = U[0, \pi/2]$, we have that $t^\star = \pi/4$ and $\beta_F = \frac{\pi}{2}$.

better off at the expense of some other agents, and the exact proportions depend on the distribution F. Instead, an increase in θ_0 unambiguously makes everyone better off. This discrepancy creates a wedge between the *social* value of information, formally the Bayesian value of information for a decision maker with utility $W(\theta)$, and the *private* value of information. This discrepancy becomes extreme when F is uniform, our working assumption on the type-distribution. In this case, we have that $\beta_F = \int_T \cos(t) dF(t) = 0$ and the planner cares infinitely more about valence compared to any other agent in the society. Intuitively, ideology is not valued by the planner because the benefits to any one group of agents are offset *exactly* (due to uniformity) by losses to another group. This case is instructive because our result becomes extreme: *any* increase in competition leads to a welfare loss.

Proposition 6. *Fix θ_0 and an arbitrary sequence of equilibria $(a^\star(n))_n$. As the number of competing firms n increases, the probability that the collective decision conditional on θ will match the socially optimal one decreases.*

While valence has a superior social value relative to ideology, an increasingly competitive market generate incentives to disinvest resources away from the valence component. As shown in Section 4.2, competition increases ideological voting and intensifies social disagreement. This implies that the approval decision of each agent t is increasingly uncorrelated with θ_0. However, as we argued above, θ_0 is the only relevant dimension when the distribution of types is uniform. Therefore, the social planner approves a policy θ if and only if $\theta_0 \geq 0$. By Proposition 3, an increasingly competitive market supplies less information about the valence component. Hence, the agents' approval decisions become increasingly uncorrelated with those of the social planner, thereby generating the inefficiency illustrated above.

Complete Information Benchmark. In the last part of this section, we discuss another result that, while complementing the previous one, highlights the inefficiency that pervades our model in a more striking, perhaps even unexpected way. To do so, we focus on a special class of policies, those that Pareto-dominate the status quo under complete information. More formally, one such policy satisfies $u(\theta, t) \geq 0$ for all $t \in T$. Realizations of θ that are Pareto-dominant under complete information are characterized by a particularly large θ_0, relative to (θ_1, θ_2). By construction, the set of such policies is non-empty and has strictly positive measure. These policies are special in that, if the society could perfectly learn the state θ, it would unanimously agree on its approval. We show next that the probability of the society being able to implement even these particularly straightforward class of policies also decreases with competition.

Proposition 7. *Fix a Pareto-dominant policy θ, and for each n assume all equilibria are equally likely. The expected approval rate $\Gamma_n(\theta)$ decreases with n.*

This result illustrates two important features of our model. First, this result shows that it would be misleading to think that the complete information benchmark is the worst of all possible worlds and that "ignorance is bliss." Instead, this result demonstrates that there is plenty of scope for information to play a positive role in our model, but that competition is detrimental even in such case. Second, Proposition 7 illustrates that it would be equally misleading to expect competition to bring about "more information" to the agents, thereby pushing the society "closer" to its complete information benchmark. The result above shows that the exact opposite is true. In fact, along any sequence of equilibria, each agent is offered a sequence of information structures that cannot be Blackwell-ranked. The society does not move closer to its complete information benchmark. Instead, the market supplies increasingly more imprecise information about the valence component. A Bayesian agent rationally reacts to this change by increasing the weight she attaches to her ideology signal, hence even ex post Pareto-dominant policies become harder to implement.

On a more technical note, the result in Proposition 7 is formulated *conditional* on the state θ and, a fortiori, conditional on some ideology pair (θ_1, θ_2). As such, it is no longer true that our equilibrium multiplicity is immaterial, as the exact sequence of equilibria $(a^\star(n))_n$ matters when computing $\Gamma_n(\theta)$. In Proposition 7, we take care of this issue by taking an expectation over the set of equilibria (which are assumed to be uniformly distributed). In Remark 5, we show that we can always construct an actual equilibrium sequence $(a^\star(n))_n$ along which the approval rate declines.

Remark 5. *Fix a Pareto-dominant policy θ. There always exists a sequence of equilibria $(a^\star(n))_n$ along which the approval rate $\Gamma_n(\theta)$ decreases with n.*

Polarization. We conclude this section with a final comparative static, this time with respect to λ. In our model, λ represents the weight agents put on the valence component (See Equation 1). For this reason, one can think of $1 - \lambda$ as a simple measure of ex ante "polarization" in the political preferences of the electorate. Our next result finds that the inefficiency created by competition increases in more polarized societies.

Remark 6. *Fix $n > 1$. As heterogeneity in agents' preferences increases (a decrease in λ), the probability that the collective decision conditional on θ_0 will match the socially optimal one decreases.*

Remark 6 shows that the inefficiency associated with competition is exacerbated by polar-

ization in the distribution of political preferences in the society. As polarization increases, demand for information on ideology increases. In a competitive market, firms respond to this demand by shifting precision from valence to ideology.

5.   Consuming Multiple News Sources

So far, we have maintained the assumption that each agent chooses one and only one information provider. In this section, we relax this assumption and show how our results extend to the case in which agents consume multiple information structures. When agents consume only one product, we have shown that each agent chooses to consume the information provider that is "located" closest to her, that is, the information structure that is mostly correlated with her own preferences. This assumption significantly reduced the complexity of the game played by the firms and thus enabled us to demonstrate the main forces that drive our results in a transparent way. When agents consume multiple news sources, a possibly counteracting force is introduced, namely that as n increases, agents may consume more information. If agents can freely access the products of all n information providers that are competing on the market, then firms do not face a congestion problem. Consequently, no specialization is expected to occur. In fact, in such a case, since *all* agents will consume *all* products, firms won't be actually competing with one another. Yet, it is possibly even more extreme to assume that agents can consume *every* information structure produced by the market, irrespective of n, than it is to assume they can only acquire one. More realistically, agents have limitations on how many signals they can process, due to time or cognitive constraints, opportunity costs, and so on. In this section, we show that our main results generalize to allowing agents to consume an arbitrarily large but finite number of different news sources. That is, we assume there is a cap $\kappa \in \mathbb{N}$ on the number of news sources an agent can consume, and study how competition, i.e. increases in n, affects social outcomes. We focus attention on the interesting cases, namely those in which $n \geq \kappa$ and, therefore, agents have to select the best κ products from the n that are available to them. Since the comparative statics are performed on n, it is convenient to normalize the budget constraint that each firm faces to $1/\kappa$. In this way, agents receive κ signals whose total precision is 1, just as in the previous sections. Our results in Proposition 2 and 3 extend in the following way:

Proposition 8. *Let $n \geq \kappa$. A symmetric equilibrium in which agents consume the closest news sources always exists. Moreover, as the number n of competing firms increases, the precision $\tau^{\star}(n)$ on the valence component is decreasing.*

When agents can consume multiple news sources, we face a technical challenge of carefully defining the readership for each firm. The normalization on the total precision imposes constraints on how much information agents are able to extract from κ news sources. This provides a sufficient condition under which agents always pick the firms that are closest to them. In effect, the optimal learning strategy entails choosing the κ firms that are individually ranked highest (for type t) in terms of the value associated with the information structure they provide.[23] Once this is established, the game played among the firms can be mapped back to the $\kappa = 1$ case we solved before, with adjustments on how market share is defined. For example, if $n = 8$ and $\kappa = 2$ as shown in Figure 6, each firm will cater to a quarter of the market, with neighboring firms serving overlapping shares of the population. However, once these adjustments are made, forces underlying the structure of the symmetric equilibrium are identical to the $\kappa = 1$ case. Each firm will choose its reporting strategy to maximize the value of the signals it provides for its most "extreme" readers – the threshold types that are just indifferent between this firm and another. The only difference will be that each news source will effectively compete over these threshold types with news sources that are κ to the right and to the left.
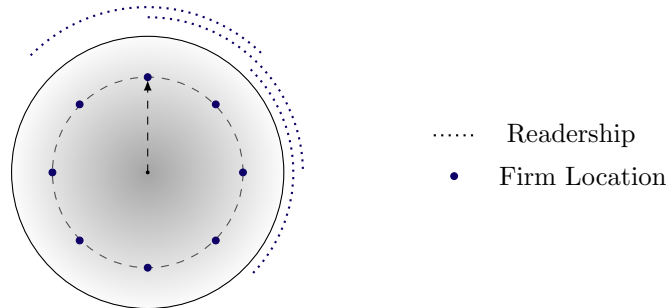

FIGURE 6: The representation of a symmetric equilibrium for $n = 8$ and $\kappa = 2$. Overlapping readership is marked for the three adjacent firms located in the first quadrant.

The competitive tensions that this situation generates are very similar to the $\kappa = 1$ case. In fact, as before, firms will choose the precision of their signal on ideology relative to valence based on the correlation of their readers' preferences on the ideological dimensions. For any κ, as n increases, the market will be segmented into smaller and smaller groups, with increasingly correlated preferences. Consequently, news sources will shift focus to ideological issues, forgoing those consumers that are "far away" and creating more value for those that are close by.

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[23]Without any constraints on how much information can be transmitted with κ news sources, we could encounter situations in which agents choose to consume news sources according to different criteria. In these cases, agents would care about how symmetrically distributed the news sources are around t, more than how informative each news source individually is.

6.   Discussion and Concluding Remarks

Distribution of preferences. Our model highlights how competition leads to informational specialization. A key insight emerging from our model is that firms specialize by providing relatively less information on dimensions that are more of common interest. Specifically, in our model this takes the form of less information about the valence dimension. Our primary goal in the paper has been to demonstrate how this specialization can amplify of social disagreement and to analyze its possible welfare consequences. Of course, the assumptions we made on the structure and the distribution of agents' preferences are only a coarse description of reality, and they intend to capture its most general features. For example, the stark distinction between valence and ideology is just a useful theoretical construct borrowed from the political science literature. In reality, people's preferences may well depend on complex and heterogeneous combinations of a large number of issues. From this perspective, we think of valence as capturing the principal component of such heterogeneity, a statistical dimension along which agents' preferences are maximally correlated, and with respect to which the residual heterogeneity, which we call ideology, is indeed orthogonal. Our simplified way of modeling the policy space aims at capturing this generic feature of the real world, while offering a tractable illustration of the mechanism behind our analysis and improving the transparency of our main results.

The most substantive assumptions we have made are those that lead to firms' specialization following an increase in competition. This result is mainly driven by supply-side budget constraints on the firms, and by demand-side heterogeneity in agents' preferences.[24] This allows us to capture a world in which, due to constraints (on budget, airtime, attention etc.) and heterogeneous tastes, information *needs* to be targeted. Competition leads to specialization, which crucially emphasizes components in voters' preferences that are more heterogeneous. We find this process natural and compelling. From this perspective, it is important to note that, in our model, media firms do not *create* social disagreement out of nowhere. Rather, they uncover and amplify the primitive heterogeneity that is already embedded in the agents' preferences.

Another important simplifying assumption of our model is the uniformity of the type-distribution F. Specifically, this assumption allows us to analytically solve for the equilibrium of our game for any number of competing firms. This allows us to characterize the effects of increased competition for all industry models, spanning the whole space from

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[24]In the absence any budget constraint, all firms would want to fully reveal the state θ. In the absence of heterogeneity in agents' preferences, all firms would want to provide exactly the same information structure. In either case, specialization would not arise.

monopoly to perfect competition. However, as we established in the previous sections, the details of the type-distribution are not essential for comparing monopoly and perfect competition, two qualitatively important benchmarks. In Remark 3, we showed that, as the market becomes arbitrarily large, firms do maximally specialize, irrespective of the details of the type-distribution. Relatedly, in Remark 4, we illustrated that, for any non-degenerate type-distribution, agents necessarily care less about the valence issue than the social planner. Such a wedge drives the perfectly competitive market to over-supply information about the ideological components relative to what the social planner would have prescribed.

Strategic voting. When solving the agent's problem, we assumed sincere voting. This is a common assumption in the political economy literature. This assumption allows us to work with a continuum of voters and to abstract away from the details of the electoral rule. The literature on strategic voting, instead, assumes that voters are motivated by instrumental considerations of how their voting behavior could affect the electoral outcome.[25] Strategic voting could potentially affect our results at two important levels. First, it could affect the way voters vote, given the information they have acquired. Interestingly, Ali et al. (2017) illustrate how the interactions between private information and distributive preferences under strategic voting can lead to a form of inefficiency that, while relying on an economic mechanism that is distinct from the one highlighted in this paper, decreases the probability that an ex post optimal policy is implemented. Second, strategic voting could affect how voters value information and, therefore, it could alter both the demand and the supply of information in this market. While not solving for this explicitly, we believe that the main insight in our paper, namely that competition leads to more disagreement, would also apply to a version of our model in which we consider the majoritarian electoral rule in combination with strategic voting. Under the majority rule, an agent's vote is pivotal when there is maximal disagreement within the society on whether or not the policy should be approved. In our framework, conditioning on such an event would be particularly informative with respect to the common component of agents' preferences. As a result, a strategic voter would put even more weight on signals that are informative on how her preferences differ from those of the rest of the society. Such a channel could reinforce the demand for ideological information and, consequently, provide further incentives for firms' specialization as a result of competition.

Introducing a government-funded news source. Our main results highlight how competition

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[25]Such effects crucially hinge on the mechanism that maps vote shares into electoral outcome. In this paper, we have focused on settings where the *distribution* of votes - and not only who wins the majority - has an impact on agents' welfare. It is easy to see that, in this case, strategic voting moves closer to sincere voting.

in the information market can amplify social disagreement by shifting the focus from valence issues to ideological issues. Our model demonstrates that this shift can be a natural consequence of the contrast between the *social* value and *individual* value of information on these different dimensions. Profit maximizing firms shift the focus of their informational products to cater more to individual demands as the market becomes increasingly segmented. It is interesting to consider what role government-funded news sources that are not affected by competitive forces can play in such an environment. After all, despite the dramatic increase in the number of news sources available to voters, government-funded news sources still exist in most countries.[26] For example, we can consider how our results would be affected by the presence of a non-strategic player (a government-funded news source) that *uniquely* provides information on the valence dimension. While the presence of such a player increases social welfare by moving agents' voting behavior closer to the socially optimal one, our main results on the effects of competition would still go through. An increase in competition would still induce firms to decrease their precision the valence component. Due to the presence of the government-funded news source, there would be less value in providing information to agents on the valence dimension and more value in providing information on the ideological dimensions. The resulting specialization would trigger the increase in social disagreement observed in our model.

This simple example also provides some additional context for one of the assumptions we make on agents' preferences. We assume that the weight λ that agents assign on valence relative to ideology is type-independent.[27] Relaxing this assumption would not alter the main tension in our model, while it would make the analytical solution to the firms' equilibrium strategies significantly more cumbersome. Similar to the example of the government-funded new source, as the number of competing firms increases and the marginal gains from catering to the informational needs of the "moderates" become negligible, *some* firms will find specialization profitable. This effect, while dampened by the presence of the moderates relative to the baseline version of our model, would still trigger the increase in ideological voting and, therefore, as by Proposition 5, an increase in social disagreement.

Our results also suggest that the role played by public news sources can change with the level of competition in the market. Public news sources have historically been founded on principles that emphasize "universal geographic accessibility," "attention to minorities," "contribution to national identity and sense of community," and "distance from vested interests."[28] As acquisition of political news shifts online and the number of news sources

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[26]BBC, PBS/NPR, Deutsche Welle, CBC and VOA are some prominent examples.

[27]Relaxing this assumption in the framework of our model would imply the distribution of types to correspond to a distribution on a disk rather than on a circle.

[28]These highly referenced principles were first stated by the Broadcasting Research Unit in Britain in

simultaneously available to voters dramatically increases, there is arguably less concern over some of the issues addressed above.[29] Nonetheless, our model demonstrates that, as the level of competition in the market increases, public news sources can still play a role in refocusing public discourse on issues that are relevant to the population as a whole.

Conclusion. Our paper contributes to the literature on the political economy of mass media by showing how competition among information providers leads to a particular kind of informational specialization: firms provide relatively less information on issues that are of common interest, and relatively more information on issues along which agents' preferences are more heterogeneous. Our main goal in the paper has been to demonstrate how specialization can amplify social disagreement and illustrate its potential effects on social welfare. Our results rely on a peculiar feature of the market for political news, that is not present in more conventional markets. Agents use political information to form individual opinions on policies. In turn, public opinion affects which policies are implemented. However, when agents acquire information, they fail to take into account the externality that this information will produce on the society. Competition pushes firms to design informational products that match individual demand more closely, and this comes at a social cost. Importantly, the negative welfare consequences highlighted in our model come about even in the absence of behavioral biases or partisan media. In this sense, our paper offers a new, and to some extent more distressing, perspective on how the changing media landscape can impact political outcomes.

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[29]Government funded news sources can also be manipulated and censored more easily. In this discussion, we assume that the public source is unbiased. We refer the reader to Besley and Prat (2006) for a study of competition and media capture.

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A. Proofs

Proof of Remark 1. Fix α_i with $\|\alpha_i\| = 1$ and define $\tau_i := \alpha_{0,i}^2$. The equation $\alpha_{1,i}^2 + \alpha_{2,i}^2 = 1 - \tau_i$ implicitly defines a circle in $(\alpha_{1,i}, \alpha_{2,i})$ that has amplitude $\sqrt{1 - \tau_i}$. Therefore, for any given pair $(\alpha_{1,i}, \alpha_{2,i})$ there exists a unique $x_i \in T$ such that $\alpha_{1,i} := \sqrt{1 - \tau_i} \cos(t)$ and $\alpha_{2,i} := \sqrt{1 - \tau_i} \sin(t)$. To conclude the proof, let s_i and $\tilde{s}_i$ be the information structured induced by α_i and a_i, respectively. Notice that $\tilde{s}_i$ arises as a linear transformation of s_i: $\tilde{s}_i^v = \frac{1}{\sqrt{\tau_i}} s_i^v$ and $\tilde{s}_i^{id} = \frac{1}{\sqrt{1-\tau_i}} s_i^v$. Therefore, the two information structures are equivalent for a Bayesian agent. $\square$

Lemma A1. *Fix an information structure s induced by some $a_i \in A_i$ and a type $t \in T$. The interim expected utility is given by:*

$$\mathbb{E}\big(u(\theta, t)|s\big) = \lambda g(\tau_i) s^v + (1 - \lambda) \cos(t - x_i) g(1 - \tau_i) s^{id}$$

with $g(\tau_i) = \frac{\tau_i}{1 + \tau_i}$.

Proof of Lemma A1. Fix $t \in T$ and $a_i \in A_i$. Recall that $\theta \sim \mathcal{N}(0, I_3)$, where I_3 is the identity matrix. We consider signals defined as $s^v(\theta) = \theta_0 + \frac{1}{\sqrt{\tau_i}} \varepsilon_i^v$ and $s^{id}(\theta) = \theta_1 \cos(x_i) + \theta_2 \sin(t) + \frac{1}{\sqrt{1-\tau_i}} \varepsilon_i^{id}$. We can explicitely compute this interim expectation:

$$\mathbb{E}\big(u(\theta, t)|s\big) = \mathbb{E}\Big(\lambda \theta_0 + (1 - \lambda)\big(\theta_1 \cos(t) + \theta_2 \sin(t)\big)\big|s\Big)$$
$$= \lambda \mathbb{E}\big(\theta_0|s^v\big) + (1 - \lambda) \cos(t) \mathbb{E}\big(\theta_1|s^{id}\big) + (1 - \lambda) \sin(t) \mathbb{E}\big(\theta_2|s^{id}\big).$$

where we used independence of $(\theta_0, \theta_1, \theta_2)$. Computing these conditional expectations and letting $g(\tau_i) := \frac{\tau_i}{1 + \tau_i}$ we get:

$$\mathbb{E}\big(\theta_0|s^v\big) = g(\tau_i) s_{\mathrm{v}} \qquad \mathbb{E}\big(\theta_1|s^{id}\big) = \cos(x_i) g(1 - \tau_i) s_{\mathrm{id}} \qquad \mathbb{E}\big(\theta_2|s^{id}\big) = \sin(x_i) g(1 - \tau_i) s_{\mathrm{id}}.$$

Using the trigonometric identity $\cos(t) \cos(x_i) + \sin(t) \sin(x_i) = \cos(t - x_i)$ and putting everything together concludes the proof. $\square$

Proof of Proposition 1. Let X be a random variable distributed according to $X \sim \mathcal{N}(0, \sigma^2)$. We have that:

$$\mathbb{E}\big(\max\{0, X\}\big) = \frac{1}{2}\mathbb{E}\big(X|X \geq 0\big) = \frac{\sigma}{\sqrt{2\pi}}.$$

Now fix some $a_i \in A_i$ and let $s = (s^v, s^{id})$ be the information structure that is induced by it. Also, let $X := \mathbb{E}(u(\theta, t)|s)$. From Lemma A1, we know that $X = \lambda g(\tau_i) s^v + (1 - \lambda) \cos(t - x_i) g(1 - \tau_i) s^{id}$. That is, X is the sum of independent normally distributed random variables. By definition, we have $s^v \sim \mathcal{N}(0, 1 + \frac{1}{\tau_i})$ and $s^{id} \sim \mathcal{N}(0, 1 + \frac{1}{1 - \tau_i})$, where we used the trigonometric identity $\cos(x_i)^2 + \sin(x_i)^2 = 1$. Therefore, $X \sim \mathcal{N}(0, \sigma^2)$ where

$$\sigma^2 := \lambda^2 g(\tau_i) + (1 - \lambda)^2 \cos^2(t - x_i) g(1 - \tau_i).$$

Therefore, $V(a_i|t) = \frac{\sigma}{\sqrt{2\pi}}$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $\square$

Proof of Proposition 2. When $n = 1$, the firm can choose $\tau_1 = 1$ and select an arbitrary location x_1. In such way, the value produced by $a_1 = (\tau_1, x_1)$ is equal to $V(a_1|t) = \frac{\lambda}{2\sqrt{\pi}} > 0$ for all $t \in T$. Therefore, all agents acquire information from the firm. Profits are maximized, $\Pi_1(a_1) = 1$, and the firm has no incentive to deviate. This equilibrium is trivially symmetric since there is only one firm.

Now let $n \geq 2$. First we construct a candidate symmetric equilibrium and, then, we prove it is indeed a symmetric equilibrium. With no loss of generality, we focus on firm $i = 1$ and normalize its location in the candidate equilibrium strategy profile to $x_1 = 0$. Condition $|x_i - x_j| \geq \pi/n$ for all $i, j \in N$ pins down exactly the location of all other firms in this candidate symmetric equilibrium. Specifically, firm $i \in \{1, \ldots, n\}$ is located at $x_i^\star = \frac{(i-1)\pi}{n}$. In our candidate symmetric equilibrium, all firms but $i = 1$ play a $\tau \in [0, 1]$. Firm $i = 1$ instead plays some, possibly identical, τ_1, and maximize its profit conditional on all other firms playing τ. Denote $t_r \in T$ (resp. $t_l \in T$) to be the unique solution of $V(a_1|t_r) = V(a_2|t_r)$ (resp. $V(a_1|t_l) = V(a_n|t_l)$), namely the type indifferent between acquiring information from either firm 1 and firm 2 (resp. n). The problem of the firm is equivalent to maximizing $t_r - t_l \geq 0$. The first-order condition is $\frac{\partial t_r}{\partial \tau_1} = \frac{\partial t_l}{\partial \tau_1}$. By definition of t_r, we have that

$$\frac{\partial}{\partial \tau_1}\Big(V\big((\tau_1, x_1^\star)|t_r\big) - V\big((\tau^\star, x_2^\star)|t_r\big)\Big)\Big|_{\tau_1 = \tau} = 0$$

This equilibrium condition allows us to retrieve an expression for $\frac{\partial t_r}{\partial \tau_1}$. In fact

$$\frac{\partial t_r}{\partial \tau_1} = \frac{\lambda^2 g'(\tau_1) - (1-\lambda)^2 \cos^2(t_r - x_1)g'(1-\tau_1)}{(1-\lambda)^2\big(g(1-\tau_1)\sin 2(t_r - x_1) + g(1-\tau^\star)\sin 2(x_2^\star - t_r)\big)}$$

Similarly, from $V(\tau_1, x_1^\star|t_l) - V(\tau^\star, x_n^\star|t_l) = 0$, we can retrieve an expression for for $\frac{\partial t_l}{\partial \tau_1}$:

$$\frac{\partial t_l}{\partial \tau_1} = -\frac{\lambda^2 g'(\tau_1) - (1-\lambda)^2 \cos^2(x_1 - t_l)g'(1-\tau_1)}{(1-\lambda)^2\big(g(1-\tau_1)\sin 2(x_1 - t_l) + g(1-\tau^\star)\sin 2(t_l - x_n^\star)\big)}$$

By symmetry, $t_l - x_n^\star = x_2^\star - t_r$ and $t_r - x_1^\star = x_1^\star - t_l$. Therefore, $\lambda^2 g'(\tau_1) = (1-\lambda)^2 \cos^2(t_r - x_1)g'(1-\tau_1)$, or

$$\frac{\lambda^2}{(1-\lambda)^2}\frac{g'(\tau_1)}{g'(1-\tau_1)} = \cos^2(t_r - x_1)$$

Since our candidate equilibrium is symmetric, it must be that $\tau_1 = \tau^\star$ and $t_r - x_1 = \frac{\pi}{2n}$, that is

$$\frac{\lambda^2}{(1-\lambda)^2}\frac{g'(\tau^\star)}{g'(1-\tau^\star)} = \cos^2\left(\frac{\pi}{2n}\right). \qquad\qquad (3)$$

The left-hand side is strictly decreasing in $\tau^\star$ and the right-hand side is constant. Therefore, there exists a unique solution $\tau^\star$ to the equation above that. Together with $x^\star$ defined above, the pair $a^\star = (\tau^\star, x^\star)$ constitute our candidate symmetric equilibrium. Importantly, not only $\tau^\star$ is unique, but it is derived independently of the specific locations $x^\star$ chosen for our candidate equilibrium. That is, once we verify that $a^\star$ is indeed an equilibrium, uniqueness up to modular rotations of

the locations $x^\star$ will follow. Notice that, by construction, we have already shown that at such a candidate symmetric equilibrium firm 1, and a fortiori any firm i, has no incentive to unilaterally deviate to any other $\tau_1' \neq \tau^\star$. To prove, $(\tau^\star, x^\star)$ is an equilibrium it remains to be shown that firm 1 has no profitable deviation neither on x_1 nor on (x_1, τ_1).

In equilibrium, it must be that $\frac{\partial}{\partial x_1} V(\tau_1, x_1|t_r) = \frac{\partial}{\partial x_1} V(\tau^\star, x_2^\star|t_r)$. Therefore, we can derive expressions for $\frac{\partial t_r}{\partial x_1}$. Indeed, we get

$$(1-\lambda)^2 g(1-\tau_1)\sin 2(t_r - x_1)\big(1 - \frac{\partial t_r}{\partial x_1}\big) = (1-\lambda)^2 g(1-\tau_2)\sin 2(x_2 - t_r)\frac{\partial t_r}{\partial x_1}$$

giving us the following expression:

$$\frac{\partial t_r}{\partial x_1} = \frac{g(1-\tau_1)}{g(1-\tau_1) + \psi_r g(1-\tau_2)}, \quad \text{with } \psi_r := \frac{\sin 2(x_2 - t_r)}{\sin 2(t_r - x_1)}.$$

Similarly, we can get

$$\frac{\partial t_l}{\partial x_1} = \frac{g(1-\tau_1)}{g(1-\tau_1) + \psi_l g(1-\tau_n)}, \quad \text{with } \psi_l := \frac{\sin 2(t_l - x_n)}{\sin 2(x_1 - t_l)}.$$

In a symmetric equilibrium, thresholds t_r and t_l fall at the midpoints between locations, namely $t_r = (x_1 + x_2)/2$ and $t_r = (x_1 + x_n)/2$. This implies that $\psi_r = \psi_l = 1$ and therefore $\frac{\partial t_r}{\partial x_1} - \frac{\partial t_l}{\partial x_1} = 0$. Thus, firm 1, and a fortiori any firm i, does not strictly gain by locating itself away from $x_1^\star$.

It remains to be shown that there are no joint deviations in τ_i *and* x_i that could make firm i better off. We do this in the next two claims.

Claim 1. *For all $\tau_1 > \tau^\star$ and locations x_1, $\Pi(a_1', a_{-i}^\star) < \pi/n$.*

Fix $\tau_1 > \tau^\star$ and $x_1 > x_1^\star$ (the case in which $x_1 < x_1^\star$ is symmetric). Consider the type $\tilde{t} := (x_1 + x_2^\star)/2$ which is midway between x_1 and $x_2^\star$. We want to show that $\tilde{t}$ does prefer $i+1$ to i. Notice that since $x_1 > x_1^\star$ and, by Definition 4.1, $x_2^\star - x_1^\star = \pi/2n$, we have that $\tilde{t} - x_1 = x_2^\star - \tilde{t} < \pi/2n$. By construction, $\tau^\star$ is the optimal level of valence for a type t who is $\pi/2n$-away from the information provider. All types that are closer than $\pi/2n$ would prefer less valence. Thus, $\tilde{t}$ strictly prefers firm $i+1$ since, compared with firm i, it offers a lower level of valence, $\tau^\star < \tau_1$. We conclude that $x_2^\star - t_r > t_r - x_1 > 0$, hence $\psi_r > 1$. Now let's consider t_l. If it is such that $t_l - x_n > x_1 - t_l$ then firm i's profits are necessarily less than $\pi/2n$. Thus, the only case we need to consider is the one in which $t_l - x_n < x_1 - t_l$. In this case, $\psi_l < 1$. Summing up, we have that $\psi_r > 1$ and $\psi_l < 1$, implying that $\frac{\partial t_r}{\partial x_1} - \frac{\partial t_l}{\partial x_1} < 0$.

Claim 2. *For all $\tau_1 < \tau^\star$ and all locations x_1, firm 1's profit are smaller than π/n.*

Fix $\tau_1 < \tau^\star$ and $x_1 > x_1^\star$ (the case in which $x_1 < x_1^\star$ is symmetric). There are two subcases to consider here. Either the left threshold type t_l is indifferent between firm 1 and n (as it was in the previous Claim), or it is indifferent between firm 1 and 2. This second case is possible because firm 1 is now deviating to a lower τ_1 than its neighbors. On the other side, the right threshold t_r will always correspond to a type who is indifferent between firm 1 and 2.

Subcase 1: Let's assume t_l is indifferent between 1 and n. A similar argument to the one in the Claim above will show that $t_l - x_n^\star > x_1 - t_l > 0$. In fact the midpoint $\tilde{t} := (x_1 + x_n^\star)/2$ is now more than $\frac{\pi}{2n}$-away from both x_1 and $x_n^\star$. Thus she would prefer more valence than $\tau^\star$. Since $\tau_1 < \tau^\star$, type $\tilde{t}$ prefers x_n. This shows $t_l - x_n^\star > x_1 - t_l > 0$ and implies that $\psi_l > 1$. Now we look at t_r. Once again, either *(a)* firm 1 is conquering more than half of the market between 1 and 2, i.e. $x_2 - t_r < t_r - x_1$ or *(b)* firm 2 does, i.e. $x_2 - t_r > t_r - x_1$. If *(b)* is the case, then firm 1's profits are necessarily less than π/n and we are done. If *(a)* holds, instead, $\psi_r < 1$ and therefore $\frac{\partial t_r}{\partial x_1} - \frac{\partial t_l}{\partial x_1} > 0$. Since $x_1 \in [x_1^\star, x_2^\star]$ was arbitrary, we proved that the derivative of profits is strictly increasing in such region. Thus, firm 1 will keep increasing x_1, getting closer and closer to x_2. Eventually, firm x_1 will locate in the same spot of x_2, but with a lower τ_1. Thus the threshold type t_l will be no longer indifferent between firm 1 and n, but rather with firm 1 and 2. This is Subcase 2, which we analyze next.

Subcase 2: Let's assume t_l is indifferent between 1 and 2. It must be that t_l is closer to 2 than n. If not, t_l should prefer n to 2, a contradiction. Now consider $\tilde{t} = \frac{x_2^\star + x_{i+2}^\star}{2}$, which is the midpoint between firm 2 and $i+2$. Notice that since $x_1 \in [x_1^\star, x_2^\star]$, $\tilde{t} - x_2^\star \geq \tilde{t} - x_i$. Since firm 1, relative to firm 2, is offering lower valence τ_1 and it is weakly farther away to $\tilde{t}$, then such type will prefer firm 2 to 1. Since by construction $\tilde{t} - t_l \leq \pi/n$, firm 1 's profit are lower than π/n.

This shows that the candidate symmetric strategy profile $a^\star = (\tau^\star, x^\star)$ is indeed a symmetric equilibrium of our game.

Now we show non-existence of asymmetric equilibria. We'll make use of the following lemma.

Lemma A2. *Readership for all firms in equilibrium is a connected interval.*

Proof. Assume for contradiction that there exists a firm i such that there exists an interval $[\underline{x}, \bar{x}]$ and $\epsilon > 0$, such that i has no readers in the interval, but it's readership includes $[\underline{x} - \epsilon, \underline{x})$, and $(\bar{x}, \bar{x} + \epsilon]$. There has to be at least two firms targeting voters in $[\underline{x}, \bar{x}]$. (If there was only one firm, it would increase τ.) We can also assume that these firms have readership that is a connected interval. Otherwise we would focus on one of them and relabel i. Wlog assume that x_k is weakly to the left of $\frac{x + \bar{x}}{2}$ Pick the firm in $[\underline{x}, \bar{x}]$ most to the right. Denote it's readership as $[x - \delta_l, x - \delta_r]$. Note that moving x the right without changing τ would increase readership for this firm which contradicts the equilibrium assumption. $\qquad\square$

Now assume for contradiction that an asymmetric equilibrium exists where firms choose different τ values. Then there has to be a firm 0 such that $\tau_0 \leq \tau_i$ for all i and $\tau_i > \tau_0$ for either it's neighbor to the right or to the left. Denote the strategy of this firm as (x_0, τ_0). Our proof strategy is to construct a profitable deviation for either firm 0 or one of it's neighbors.

Denote 0's readership as $[x_0 - \delta_l, x_0 + \delta_r]$. Since τ_0 is chosen in equilibrium, it must be that $\frac{\partial \delta_l}{\partial \tau_0} \leq 0$ or $\frac{\partial \delta_r}{\partial \tau_0} \leq 0$. Otherwise, τ_0 would have been higher. Note that both of these cannot be 0, because that would imply the neighbor with the higher τ to be closer to the threshold type, but then firm 0 would definitely deviate to locate where this firm is located and choose a τ slightly lower than this firm's and increase it's readership.

Hence, we can assume wlog that $\frac{\partial \delta_r}{\partial \tau_0} < 0$. Denote the strategy of the closest firm to the right as (x_1, τ_1). There are two cases to consider. (1) $\tau_0 = \tau_1$; (2) $\tau_0 < \tau_1$. Denote the strategy of the closest firm to the left as (x_{-1}, τ_{-1}). Let $d_{-1} = x_0 - x_{-1}$ and $d_1 = x_1 - x_0$.

Case 1: ($\tau_0 = \tau_1$) Note that by assumption $\tau_{-1} > \tau_0$. Since threshold types $x_0 - \delta_l$ and $x_0 + \delta_r$ are indifferent between firms the following holds.

$$\lambda^2 g(\tau_0) + (1-\lambda)^2 cos^2(\delta_l) g(1-\tau_0) = \lambda^2 g(\tau_{-1}) + (1-\lambda)^2 cos^2(d_{-1} - \delta_l) g(1 - \tau_{-1})$$

$$\lambda^2 g(\tau_0) + (1-\lambda)^2 cos^2(\delta_r) g(1-\tau_0) = \lambda^2 g(\tau_1) + (1-\lambda)^2 cos^2(d_1 - \delta_r) g(1 - \tau_1)$$

Note that moving x_0 for example to the right implies a decrease in d_1 and an equivalent increase in d_{-1}. If we differentiate these equations with respect to d_1 ad d_{-1} accounting for how δ_r and δ_l change respectively, we get

$$\frac{\partial \delta_r}{\partial d_1} = \frac{1}{1 + \frac{cos(\delta_r) sin(\delta_r) g(1-\tau_0)}{cos(d_1 - \delta_r) sin(d_1 - \delta_r) g(1-\tau_1)}}$$

$$\frac{\partial \delta_l}{\partial d_{-1}} = \frac{1}{1 + \frac{cos(\delta_l) sin(\delta_l) g(1-\tau_0)}{cos(d_{-1} - \delta_l) sin(d_{-1} - \delta_l) g(1-\tau_{-1})}}$$

In equilibrium it must be that
$$\frac{\partial \delta_r}{\partial d_1} = \frac{\partial \delta_l}{\partial d_{-1}}$$

which implies

$$\frac{cos(\delta_r) sin(\delta_r) g(1 - \tau_0)}{cos(d_1 - \delta_r) sin(d_1 - \delta_r) g(1 - \tau_1)} = \frac{cos(\delta_l) sin(\delta_l) g(1 - \tau_0)}{cos(d_{-1} - \delta_l) sin(d_{-1} - \delta_l) g(1 - \tau_{-1})}$$

which can be rewritten as

$$\frac{cos(\delta_r) sin(\delta_r)}{cos(d_1 - \delta_r) sin(d_1 - \delta_r)} = \frac{cos(\delta_l) sin(\delta_l) g(1 - \tau_0) g(1 - \tau_1)}{cos(d_{-1} - \delta_l) sin(d_{-1} - \delta_l) g(1 - \tau_{-1})}$$

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Since $\tau_1 = \tau_0 < \tau_{-1}$, it must be that $\frac{g(1-\tau_1)}{g(1-\tau_{-1})} > 1$, but since $\tau_1 = \tau_0$, $\delta_r = d_1 - \delta_r$, which implies $\frac{\cos(\delta_r)\sin(\delta_r)}{\cos(d_1-\delta_r)\sin(d_1-\delta_r)} = 1$. But then it must be that $\frac{\cos(\delta_l)\sin(\delta_l)g(1-\tau_0)}{\cos(d_{-1}-\delta_l)\sin(d_{-1}-\delta_l)} < 1$. This implies $\delta_l < d_{-1} - \delta_l$. This means that firm 0 is getting less than half of $d_1 + d_{-1}$. Consider the following deviation for firm 0: $(x_{-1} + \frac{d_1+d_{-1}}{2}, \tau')$ where τ' is the optimal τ for a type who is $\frac{d_1+d_{-1}}{4}$ far from the firm. This will clearly guarantee readership at least equal to $\frac{d_1+d_{-1}}{2}$ which generates the contradiction.

Case 2: $(\tau_0 < \tau_1)$ We'll replicate the same proof above relabeling firm 1 as firm 0, and 0 as -1. The following condition should still hold.

$$\frac{\cos(\delta_r)\sin(\delta_r)}{\cos(d_1-\delta_r)\sin(d_1-\delta_r)} = \frac{\cos(\delta_l)\sin(\delta_l)g(1-\tau_0)g(1-\tau_1)}{\cos(d_{-1}-\delta_l)\sin(d_{-1}-\delta_l)g(1-\tau_{-1})}$$

$\delta_l < d_{-1} - \delta_l$ and $\tau_1 \geq \tau_{-1}$ imply that the left hand side less than 1, which implies that the right side is also less than 1, and hence $\delta_r < d_1 - \delta_r$, so a similar profitable deviation can be constructed.

$\square$

Lemma A3. *For each type $t \in T$ and sequence of equilibria $(a^\star(n))_{n\in\mathbb{N}} \in A^\infty$ there exist increasing sequences $\underline{V}_t(n)$ and $\bar{V}_t(n)$ such that, for all $n \in \mathbb{N}$, $\underline{V}_t(n) \leq V(a_{i_n(t)}|t) \leq \bar{V}_t(n)$ and $\lim_n \underline{V}_t(n) = \lim_n \bar{V}_t(n) = v^\star := \max_\tau V(\tau, t \,|\, t)$.*

Proof. Let $(a^\star(n))_{n\in\mathbb{N}} \in A^\infty$ be a sequence of symmetric equilibria of the competition game and fix an arbitrary type $t \in T$. In this proof, we shall denote $i_n(t) \in N$ the label of the firm chosen by type t at equilibrium $a^\star(n)$. Because any modular rotation of equilibrium locations $x^\star(n)$ still constitutes an equilibrium at n, there exists two sequences of equilibria $(\bar{a}^\star(n))_n$ and $(\underline{a}^\star(n))_n$, that depend on the type t and have the following two properties: (1) for each $n \in \mathbb{N}$, the uniqueness part in Proposition 2 implies that equilibria $a^\star(n)$, $\bar{a}^\star(n)$ and $\underline{a}^\star(n)$ have the same component $\tau^\star(n)$; (2) for each $n \in \mathbb{N}$, $\bar{a}^\star(n)$ is such that $x_i = t$, for some $i \in N$; (3) for each $n \in \mathbb{N}$, $\underline{a}^\star(n)$ is such that type t is indifferent between i and $i+1$, for some $i \in N$. For each $n \in \mathbb{N}$, denote $\underline{V}_t(n)$ and $\bar{V}_t(n)$ the value of information for type t at equilibrium $\underline{a}^\star(n)$ and $\bar{a}^\star(n)$, respectively. For all $n \in \mathbb{N}$, we have that $\underline{V}_t(n) \leq V(a_{i_n(t)}|t) \leq \bar{V}_t(n)$. To see this, fix $n \in \mathbb{N}$. By construction, in equilibrium $\bar{a}^\star(n)$, $x_i = t$, hence in the expression for $\bar{V}_t(n)$ we have a term, $\cos^2(t-t) = 1 \geq \cos^2(t - x_{i_n(t)})$. Since $\tau^\star(n)$ is the same under both $\bar{a}^\star(n)$ and $a^\star(n)$, we conclude that $V(a_{i_n(t)}|t) \leq \bar{V}_t(n)$. Similarly, by construction of $\underline{a}^\star(n)$, type t is indifferent between two firms and, therefore, $\cos^2(t - x_i) = \cos^2(\frac{\pi}{2n}) \leq \cos^2(t - x^\star_{i_n(t)})$. Since $\tau^\star(n)$ is the same under both $\underline{a}^\star(n)$ and $a^\star(n)$, we conclude that $\underline{V}_t(n) \leq V(a_{i_n(t)}|t)$. Next, we show that sequence $\underline{V}_t(n)$ is increasing in n. Showing that $\underline{V}_t(n)$ is increasing amounts to show that, the value of information for an indifferent type, type $t \in T$ in our construction, is increasing when going from n to $n+1$. Suppose not, that is suppose $\underline{V}_t(n) > \underline{V}_t(n+1)$. Fix a firm i and its location $x_i \in T$ in both n and $n+1$. Denote $\tilde{t}(n)$ and $\tilde{t}(n+1)$ to be the indifferent types to firm i at n and $n+1$. Without loss of generality assume that $\tilde{t}(n), \tilde{t}(n+1) \leq x_i$. This implies that $\tilde{t}(n) \leq \tilde{t}(n+1)$. Notice that the value

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of information is always decreasing in the distance between a type and its firm. This implies that at $\tau^{\star}(n)$, type $\tilde{t}(n+1)$ is better off than at $\tau^{\star}(n+1)$. Since, by definition, $\tilde{t}(n+1)$ is indifferent between i and $i+1$, firm i can deviate from $\tau^{\star}(n+1)$, increasing its precision on valence, and strictly gain from this deviation, a contradiction. Therefore, $\underline{V}_t(n) \leq \underline{V}_t(n+1)$. Next, we show that also the sequence $\bar{V}_t(n)$ is increasing in n. This amounts to show that, as $\tau^{\star}(n)$ decreases, the value of information of the targeted type, i.e. $x_i = t$ is increasing. Suppose this is not the case. That is suppose that $\bar{V}_t(n) > \bar{V}_t(n+1)$. This implies that $V(\tau^{\star}(n), t \mid t) > V(\tau^{\star}(n+1), t \mid t)$. This is can be the case if and only if $\lambda^2 g(\tau^{\star}(n)) + (1-\lambda)^2 g(1-\tau^{\star}(n)) > \lambda^2 g(\tau^{\star}(n+1)) + (1-\lambda)^2 g(1-\tau^{\star}(n+1))$, which can be the case if and only if, $\tau^{\star}(n) \leq 3\lambda + 1 =: \arg\max V(\tau, t \mid t)$. This contradicts the assumption that $a^{\star}(n)$ is an equilibrium, since if $\tau^{\star}(n) \leq 3\lambda + 1$, firm i can deviate to a $\tau' > \tau^{\star}(n)$ and increase the value of information for the indifferent type. Therefore, we conclude that $\bar{V}_t(n)$ is increasing in n. Next, we show that $\lim_n \underline{V}_t(n) = \lim_n \bar{V}_t(n)$. First, notice that both sequence converge to some limit point, since they are increasing and bounded by 1. Second, suppose these two limits point are such that $\lim_n \bar{V}_t(n) - \lim_n \underline{V}_t(n) > \epsilon$. This implies that for arbitrarily large $n \in \mathbb{N}$, $\bar{V}_t(n) - \underline{V}_t(n) > \epsilon$, which, in turn, implies that there exists a $\delta > 0$, independent of n, such that $\cos^2(\pi/2n) < 1 - \delta$. Since n was chosen arbitrarily, we have a contradiction. We conclude that $\epsilon = 0$, that is the sequences have the same limit point, denoted $v^{\star}$. Finally, by solving Equation 3 for τ, we know that as $n \to \infty$, $\tau^{\star}(n) \to 3\lambda + 1$. We conclude that the limit point $v^{\star}$ of the sequences $\underline{V}_t(n)$ and $\bar{V}_t(n)$ is the agent t's preferred τ when $x_i = t$, that is $v^{\star} = \max_{\tau} V(\tau, t \mid t)$. $\square$

Proof of Proposition 4. Consider a sequence of symmetric equilibria $(a^{\star}(n))_{n \in \mathbb{N}} \in A^{\infty}$ and an arbitrary type $t \in T$. For each $n \in \mathbb{N}$, denote $V_t(n)$ the value of information for agent t at equilibrium $a^{\star}(n)$.

(*a.*) Fix $n \in \mathbb{N}$ and consider a firm $i \leq n$. It is without loss of generality to assume that for both n and $n+1$ firm i is located at zero, that is $x_i^{\star}(n) = x_i^{\star}(n+1) = 0$. This is true because modular rotations of equilibrium locations constitute equivalent equilibria (Proposition 2). Thus, firm i provides information to agents of type $t \in T_i(n+1) := [-\frac{\pi}{2n}, \frac{\pi}{2n}]$ at equilibrium $a^{\star}(n)$, and to agents of type $t \in T_i(n+1) := [-\frac{\pi}{2(n+1)}, \frac{\pi}{2(n+1)}]$ at equilibrium $a^{\star}(n)$. Clearly, $T_i(n+1) \subset T_i(n)$. For all types $t \in T_i(n+1)$, we have that $V(\tau^{\star}(n), x_i = 0 \mid t) \leq V(\tau^{\star}(n+1), x_i = 0 \mid t)$. To see that, we compute the solution to the maximization problem $\max_{\tau \in [0,1]} V(\tau, x_i = 0 \mid t = \frac{\pi}{2(n+1)})$. This pins down agent $t = \frac{\pi}{2(n+1)}$'s preferred τ, given that she receives information from firm i, with $x_i = 0$. The first-order condition of this maximization problem is:

$$\frac{\lambda^2}{(1-\lambda)^2} \frac{g'(\tau)}{g'(1-\tau)} = \cos^2\left(\frac{\pi}{2(n+1)}\right).$$

This equation is also the equilibrium condition we have derived in Equation 3 (Proposition 2), evaluated at $n+1$. This implies that agent $t = \frac{\pi}{2(n+1)}$'s preferred τ, given that she receives information from firm i, is precisely $\tau^{\star}(n+1)$, the equilibrium precision on valence when $n+1$. Therefore, agent $t = \frac{\pi}{2(n+1)}$'s value of information, conditional on receiving information from firm i, is maximized at $\tau^{\star}(n+1)$. A fortiori, all agents $t \in T_i(n+1)$, we have that $V(\tau^{\star}(n), x_i = 0 \mid t) \leq$

$V(\tau^\star(n+1), x_i = 0 \,|\, t)$. Now consider the agents that are served by firm i at $a^\star(n)$, but not at $a^\star(n+1)$. These are types in the set $T_i(n) \setminus T_i(n+1)$. Fix $\bar{t}$, one of such types. Notice that $V(\tau^\star(n), x_i = 0 \,|\, \bar{t}) \leq V(\tau^\star(n), x_i = 0 \,|\, t = \frac{\pi}{2(n+1)}) \leq V(\tau^\star(n+1), x_i = 0 \,|\, t')$ for all $t' \in T_i(n+1)$. At $a^\star(n+1)$, type $\bar{t}$ will receive information from a firm $j \neq i$ and, by definition of the symmetric equilibrium, $|\bar{t} - x_j| \leq \frac{\pi}{2n}$. Therefore, there exists some $t \in T_i(n+1)$, such that $V(\tau^\star(n+1), x_i = 0 \,|\, \bar{t}) = V(\tau^\star(n+1), x_i = 0 \,|\, t)$. Therefore, $V(\tau^\star(n+1), x_i = 0 \,|\, \bar{t}) \geq V(\tau^\star(n), x_i = 0 \,|\, \bar{t})$. We conclude that all types $t \in T_i(n)$ are better off at $a^\star(n+1)$ relative to $a^\star(n)$. Since the identity of firm i was chosen arbitrary, this also concludes the proof.

(b.) Fix an arbitrary $n_k \in \mathbb{N}$. We want to show that there is a $n_{k+1} > n_k$ such that $V_t(n_{k+1}) \geq V_t(n_k)$. Suppose not. That is, for all $n > n_k$, $V_t(n) < V_t(n_k)$. By Lemma A3, we have that, for all $n \geq n_k$, $\underline{V}_t(n) \leq V_t(n) < V_t(n_k) \leq \bar{V}(n_k)$. This implies that the sequence $\underline{V}_t(n)$ is bounded away from $\lim_n \bar{V}_t(n)$, a contradiction. Therefore, there exists a $n_{k+1} > n_k$ such that $V_t(n_{k+1}) \geq V_t(n_k)$. By the induction principle, we can construct a subsequence $(V_t(n_k))$ that is increasing.

(c.) By Lemma A3, we have that $\underline{V}_t(n) \leq V_t(n) \leq \bar{V}(n)$ for all $n \in \mathbb{N}$ and that $\lim_n \underline{V}_t(n) = \lim_n \bar{V}(n) = v^\star$. These two facts imply that $\lim_n V_t(n) = v^\star$. $\qquad \square$

Proof of Remark 3: Define Δ_n in the following way. $\min_{t \in T} F(t + \Delta_n) - F(t - \Delta_n) = \frac{1}{n-1}$. Clearly assumption on F imply $\Delta_n > 0$ for all n and $\lim_{n \to \infty} \Delta_n = 0$.

Fix any t. Let δ_n be the closest firm when there are n firms. We show that $\delta_n < 2\Delta_n$. Assume not for contradiction. There has to be at least one firm in equilibrium whose readership is weakly less than $\frac{1}{n}$ in equilibrium. Consider this firm deviating to locate at t and choosing the optimal τ targeting types $t - \Delta_n$ and $t + \Delta_n$. By construction, all types between them would strictly prefer this firm, which means that the firm can guarantee readership that is weakly higher than $\frac{1}{n-1}$ which generates the contradiction. Hence we have shown that farthest firm must be closer than $2\Delta_n$ at any n which is sufficient for the result.

Fix any t. We show that $V(a^\star(n) \ t) > V_n$ for some sequence of $(V_n)_n$ such that $\lim_{n \to \infty} V_n = 0$. Let a^t denote the best information structure for type t such that $V(a^t \,|\, t) = v^\star$. Since a_n is an equilibrium, it must be that no firm wants to deviate to a^t. This means that deviating to a^t cannot guarantee readership for the firm from $[t - \Delta_n, t + \Delta_n]$ (otherwise there would have to be a firm that would deviate as it would guarantee $\frac{1}{n-1}$ readership which would clearly be profitable.) This means that types $t - \Delta_n$ and $t + \Delta_n$ have an information structure that creates value for them more than what a^t would, $V(a^t \,|\, t - \Delta_n)$. Define

$$V_n = \min_{\{a_i \in A_i | V(a_i \,|\, t - \Delta_n) > V(a^t \,|\, t - \Delta_n)\}} V(a_i \,|\, t)$$

Basically, we search over all information structures that create value of at least $V(a^t \,|\, t - \Delta_n)$ for type $t - \Delta_n$, then among those identify the lowest value one for type t. Clearly, $\Delta_n \to 0$, is sufficient to show that $V_n \to V^\star$. $\qquad \square$

Proof of Proposition 3: In the Proof of Proposition 2, we have derived the symmetric solution for $\tau^\star$ as a function of the number of player in the game n:

$$\frac{\lambda^2}{(1-\lambda)^2}\frac{g'(\tau^\star)}{g'(1-\tau^\star)} = \cos^2\left(\frac{\pi}{2n}\right)$$

Notice that the right-hand side is increasing in n, while the left-hand side is strictly decreasing in $\tau^\star$. Therefore, an increase in n is compensated by a decrease in $\tau^\star$. $\qquad\square$

Proof of Proposition 5. Fix an equilibrium where firms locate on $x = (x_1,...x_n)$. Note that given x_1, $x_2 = x_1 + \frac{\pi}{n}$ and so on. Also, since types t and $t+\pi$ choose the same firm, without loss of generality, we'll focus on half the type distribution, $[0,\pi]$. Conditional on θ, aggregate approval rate conditional on θ is

$$\int_0^\pi \Phi\left(\frac{a(\tau)\theta_0 + b(\tau,x_t)(\cos(x_t)\theta_1 + \sin(x_t)\theta_2)}{c(\tau,x_t)}\right) + \Phi\left(\frac{a(\tau)\theta_0 - b(\tau,x_t)(\cos(x_t)\theta_1 + \sin(x_t)\theta_2)}{c(\tau,x_t)}\right)\frac{dt}{\pi}$$

where the two terms refer to expected approval rates for type t and $t+\pi$ with

$$a(\tau) = \lambda g(\tau) \quad b(\tau,x_t) = (1-\lambda)g(\bar\tau-\tau)\cos(x_t-t) \quad c(\tau,x_t) = \sqrt{\frac{\lambda^2 g^2(\tau)}{\tau} + \frac{(1-\lambda)^2 g^2(\bar\tau-\tau)\cos^2(x_t-t)}{\bar\tau-\tau}}$$

Now we look at approval rate conditional only on θ_0 and apply the following identity twice $\int_{\mathbb{R}} \Phi(\alpha + \beta x)d\Phi(x) = \Phi\left(\frac{\alpha}{\sqrt{1+\beta^2}}\right)$.

which simplifies to

$$\int_0^\pi 2\Phi\left(\frac{a(\tau)\theta_0}{\sqrt{c^2(\tau,x_t) + b^2(\tau,x_t)(\sin^2(x_t) + \cos^2(x_t))}}\right)\frac{dt}{\pi} = \int_0^\pi 2\Phi\left(\frac{a(\tau)\theta_0}{\sqrt{c^2(\tau,x_t) + b^2(\tau,x_t)}}\right)\frac{dt}{\pi}$$

It will be useful to change the notation focusing on $\delta = |x_t-t|$. We use $b(\tau,\delta) = (1-\lambda)g(\bar\tau-\tau)\cos(\delta)$ and $c(\tau,\delta) = \sqrt{\frac{\lambda^2 g^2(\tau)}{\tau} + \frac{(1-\lambda)^2 g^2(\bar\tau-\tau)\cos^2(\delta)}{\bar\tau-\tau}}$.

Note that in any equilibrium δ is uniformly distributed between $[0, \frac{\pi}{2n}]$. Thus,

$$\int_0^\pi 2\Phi\left(\frac{a(\tau)\theta_0}{\sqrt{c^2(\tau,x_t) + b^2(\tau,x_t)}}\right)\frac{dt}{\pi} = \int_0^{\frac{\pi}{2n}} 2\Phi\left(\frac{a(\tau)\theta_0}{\sqrt{c^2(\tau,\delta) + b^2(\tau,\delta)}}\right)\frac{2n}{\pi}d\delta$$

$$c^2(\tau,\delta) + b^2(\tau,\delta) = \lambda^2\frac{\tau}{(1+\tau)^2} + (1-\lambda)^2\cos^2(\delta)\frac{(\bar\tau-\tau) + (\bar\tau-\tau)^2}{(1+\bar\tau-\tau)^2}$$

taking the derivate give us

$$\lambda^2\left(\frac{1-\tau}{(1+\tau)^3}\right) - (1-\lambda)^2\cos^2(\delta)\left(\frac{1}{(1+\bar\tau-\tau)^2}\right)$$

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I use $\frac{\lambda^2}{(1-\lambda)^2}\frac{g'(\tau)}{g'(\bar{\tau}-\tau)} = cos^2\left(\frac{\pi}{2n}\right)$ which can be written as $\frac{\lambda^2}{(1-\lambda)^2}\frac{(1+\bar{\tau}-\tau)^2}{(1+\tau)^2} = cos^2\left(\frac{\pi}{2n}\right)$ which comes from the optimality condition of the firms.

$$< \lambda^2\left(\frac{1-\tau}{(1+\tau)^3}\right) - \lambda^2\left(\frac{1}{(1+\tau)^2}\right) = \frac{\lambda^2}{(1+\tau)^2}\left(\frac{1-\tau}{1+\tau}-1\right) < 0$$

So we have shown that $c^2(\tau,\delta) + b^2(\tau,\delta)$ increases when τ decreases. We also know that $a(\tau)$ also decreases when τ decreases. We also know that $c^2(\tau,\delta) + b^2(\tau,\delta)$ is higher for smaller values for δ. As n increases, the limits of the integral shrink. Moreover, τ decreases. Both of these force the approval rate to move closer to one half. $\square$

Proof of Remark 4. Let F be symmetric around $t^\star \in T$. We need to show that

$$\int_T \big(\theta_1\cos(t) + \theta_2\sin(t)\big)dF(t) = \big(\theta_1\cos(t^\star) + \theta_2\sin(t^\star)\big)\int_T\cos(t)dF(t)$$

Consider

$$\theta_1\int_T\cos(t)dF(t) = \theta_1\int_{t^\star-\bar{\delta}}^{t^\star+\bar{\delta}}\cos(t)f(t)dt$$

With a change of variable and using symmetry of F, we have that

$$\int_{t^\star-\bar{\delta}}^{t^\star+\bar{\delta}}\cos(t)f(t)dt = \int_{-\bar{\delta}}^{\bar{\delta}}\cos(t^\star+\delta)f(t^\star+\delta)d\delta = \int_0^{\bar{\delta}}\Big(\cos(t^\star+\delta) + \cos(t^\star-\delta)\Big)f(t^\star+\delta)d\delta$$

Using the trigonometric identity $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$, we can write

$$\cos(t^\star+\delta) + \cos(t^\star-\delta) = \cos(t^\star)\Big(\cos(\delta) + \cos(-\delta)\Big) - \sin(t^\star)\Big(\sin(\delta) + \sin(-\delta)\Big)$$
$$= 2\cos(\delta)\cos(t^\star).$$

We can rewrite the integral above as $\theta_1\cos(t^\star)\int_0^{\bar{\delta}}(2\cos(\delta))f(t^\star+\delta)d\delta$. Finally, using symmetry again and another change of variable we can write $\int_0^{\bar{\delta}}(2\cos(\delta))f(t^\star+\delta)d\delta = \int_T\cos(t)dF(t)$. This gives us the final expression

$$\theta_1\int_T\cos(t)dF(t) = \theta_1\cos(t^\star)\int_T\cos(t)dF(t). \tag{4}$$

Now we consider the second term $\theta_2\int_T\sin(t)dF(t)$. By performing the same manipulations and using the trigonometric identity $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$, we can write that

$$\theta_2\int_T\sin(t)dF(t) = \theta_2\sin(t^\star)\int_T\cos(t)dF(t). \tag{5}$$

Summing equations 4 and 5, we get

$$\int_T\big(\theta_1\cos(t) + \theta_2\sin(t)\big)dF(t) = \big(\theta_1\cos(t^\star) + \theta_2\sin(t^\star)\big)\beta_F$$

where $\beta_F := \int_T \cos(t) dF(t)$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Proof of Proposition 6. This follow from Proposition 5. We have shown that the approval rate moves closer to $1/2$ conditional on θ_0, we also know whether or not it is larger than $1/2$ is determined by the sign of θ_0. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Proof of Proposition 7. Let $\rho(t|\theta, \delta, \tau_n)$ be the probability that type t approves the policy conditional on θ which will depend on the location of the closest new source $\delta = |x_t - t|$ and τ_n. As we have shown in the proof of Proposition 5, we can write

$$\rho(t|\theta, \delta, \tau_n) = \Phi\left(\frac{a(\tau)\theta_0 + b(\tau, \delta)(\cos(t + \delta)\theta_1 + \sin(t + \delta)\theta_2)}{c(\tau, \delta)}\right)$$

with

$$b(\tau, \delta) = (1 - \lambda)g(\bar{\tau} - \tau)\cos(\delta) \qquad c(\tau, \delta) = \sqrt{\frac{\lambda^2 g^2(\tau)}{\tau} + \frac{(1 - \lambda)^2 g^2(\bar{\tau} - \tau)\cos^2(\delta)}{\bar{\tau} - \tau}}$$

We use this to specify the expected approval rate

$$\Gamma(\theta) = \int_0^\pi \int_{\frac{-\pi}{2n}}^{\frac{\pi}{2n}} \rho(t|\theta, \delta, \tau_n) + \rho(t + \pi|\theta, \delta, \tau_n)\frac{nd\delta}{\pi}\frac{dt}{\pi} \qquad (6)$$

Call $f(t) = \cos(t)\theta_1 + \sin(t)\theta_2$.

Using this we can write:

$$
\begin{aligned}
\Gamma(\theta) &= \int_0^\pi \int_{\frac{-\pi}{2n}}^{\frac{\pi}{2n}} \left(\Phi\left(\frac{a(\tau)\theta_0 + b(\tau,\delta)f(t+\delta)}{c(\tau,\delta)}\right) + \Phi\left(\frac{a(\tau)\theta_0 - b(\tau,\delta)f(t+\delta)}{c(\tau,\delta)}\right)\right)\frac{nd\delta}{\pi}\frac{dt}{\pi} \\
&= \int_0^\pi \int_{\frac{-\pi}{2n}}^{\frac{\pi}{2n}} \left(\Phi\left(\frac{a(\tau)\theta_0 + b(\tau,\delta)f(t)}{c(\tau,\delta)}\right) + \Phi\left(\frac{a(\tau)\theta_0 - b(\tau,\delta)f(t)}{c(\tau,\delta)}\right)\right)\frac{nd\delta}{\pi}\frac{dt}{\pi}
\end{aligned} \qquad (7)
$$

The second equality is due to the symmetry. Note that what I've done is basically to collect all terms such that $t' + \delta = t$. This must be uniformly distributed with $t' \in [t - \delta, t + \delta]$.

Claim 3. $\frac{a(\tau)}{c(\tau, \delta)}$ increases with τ and increases with $|\delta|$.

Proof. Let's look at $\left(\frac{a(\tau)}{c(\tau, \delta)}\right)^2$ which gives us

$$\frac{1}{\frac{1}{\tau} + \frac{(1 - \lambda)^2 g^2(\bar{\tau} - \tau)\cos^2(\delta)}{\lambda^2 g^2(\tau)(\bar{\tau} - \tau)}}$$

Using $\frac{\lambda^2}{(1 - \lambda)^2}\frac{(1 + \bar{\tau} - \tau)^2}{(1 + \tau)^2} = \cos^2\left(\frac{\pi}{2n}\right)$ the expression above becomes (setting $\bar{\delta} = \frac{\pi}{2n}$)

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$$\dfrac{1}{\frac{1}{\tau} + \frac{cos^2(\delta)(\bar{\tau}-\tau)^2}{cos^2(\delta)\tau^2(\bar{\tau}-\tau)}}$$

we simplify

$$\dfrac{1}{\frac{1}{\tau} + \frac{cos^2(\delta)(\bar{\tau}-\tau)}{cos^2(\delta)\tau^2}}$$

This is clearly increasing in τ and increases with $|\delta|$ and increases in $\bar{\delta}$. $\qquad\square$

Claim 4. $\frac{b(\tau,\delta)}{c(\tau,\delta)}$ *decreases with τ and decreases with $|\delta|$.*

Proof. Let's look at $\left(\frac{b(\tau,\delta)}{c(\tau,\delta)}\right)^2$ which gives us

$$\dfrac{1}{\frac{\lambda^2 g^2(\tau)}{(1-\lambda)^2 g^2(\bar{\tau}-\tau)cos^2(\delta)\tau} + \frac{1}{\bar{\tau}-\tau}}$$

Using $\frac{\lambda^2}{(1-\lambda)^2}\frac{(1+\bar{\tau}-\tau)^2}{(1+\tau)^2} = cos^2\left(\frac{\pi}{2n}\right)$ the expression above becomes

$$\dfrac{1}{\frac{cos^2(\bar{\delta})\tau}{(\bar{\tau}-\tau)^2 cos^2(\delta)} + \frac{1}{(\bar{\tau}-\tau)}}$$

we see how everything is opposite of the previous case giving us the desired result. $\qquad\square$

Claim 5. *Call $\Gamma(\theta) = \Phi(\alpha\theta_0 + \beta f(t)) + \Phi(\alpha\theta_0 - \beta f(t))$. $\Gamma(\theta)$ moves away from one half as α increases, and moves towards one half as β increases.*

Proof. First note that $\Gamma(\theta) > 0$ whenever $\theta_0 > 0$.

$$\begin{aligned}
\frac{\partial \Gamma(\theta)}{\partial \alpha} &= \theta_0 \left(\phi(\alpha\theta_0 + \beta f(t)) + \phi(\alpha\theta_0 - \beta f(t))\right) \\
\\
\frac{\partial \Gamma(\theta)}{\partial \beta} &= f(t) \left(\phi(\alpha\theta_0 + \beta f(t)) - \phi(\alpha\theta_0 - \beta f(t))\right)
\end{aligned} \tag{8}$$

Note that in the first equation $\phi(\alpha\theta_0 + \beta f(t)) + \phi(\alpha\theta_0 - \beta f(t))$ is always positive, so the sign depends on the sign of θ_0 giving us the desired result with respect to α.

In the second equation, assume that $f(t) > 0$ and $\theta_0 > 0$. Then the expression in the parentheses must be negative. Otherwise if $f(t) < 0$ and $\theta_0 > 0$, then the expression in the parentheses must be positive, but it's multiplied by a negative term. The cases where $\theta_0 < 0$ work the same way. $\quad\square$

Combining the three claims gives us the desired result for any t conditional on θ, which implies that $A(\theta)$ moves towards one half. $\qquad\square$

Proof of Remark 5. Let $\Gamma(\theta|\vec{x_n})$ be the approval rate when the firm locations are fixed by $\vec{x_n}$. In the proof above we have shown that $\Gamma_n(\theta) = \int_{\vec{x_n}} \Gamma(\theta|\vec{x_n}) \frac{n dx_n}{\pi}$ is decreasing with n. Note that $\Gamma(\theta|\vec{x_n})$ changes continuously when we vary firm 1's location in $[0, \frac{\pi}{n}]$. This means that we can always find a $\vec{x_n^\star}$ such that $\Gamma(\theta|\vec{x_n^\star}) = \Gamma_n(\theta)$. Construct a sequence of equilibria $(\vec{x_n})_{n \in \mathbb{N}}$ by setting each $\vec{x_n}$ as described above. By construction approval rate is decreasing along the sequence. $\square$

Proof of Remark 6. Proof of Proposition 3 reveals that $\tau^\star$ is lower when λ is lower. Using this, we can follow proof of Proposition 6 to get to the desired result. $\square$

Proof of Proposition 8. We'll make use of the following lemmas.

Lemma A4. *For any two news sources on the same side of t, if the agent is consuming the farthest one, then she must be consuming the one closer as well.*

Proof. Fix the learning strategy used by an agent of type t. Since we are focusing on symmetric equilibria where all news sources provide the same precision of signals on valence vs. ideology, we can focus on learning from the signals associated with ideology. Any learning strategy consists of two parts. Set of κ chosen news sources and a vector $\omega = (\omega_1, \omega_2, ...\omega_\kappa)$ which specifies how the signals from news sources are used in calculating the expected $f(\theta_{id}, t)$. Namely, $\mathbb{E}(f(\theta_{id}, t)|s) = \sum \omega_i s$ where linearity follows from the fact that the signals are normally distributed. Assume for contradiction that the agent is not consuming the closest news source. We show that the agent will be better off using the same ω but replacing the farthest news source with the closest one. Call the old news source o, and the new one n. Let $\delta = (\delta_x, \delta_y) = (\cos(t_n) - \cos(t_o), \sin(t_n) - \sin(t_o))$. Without loss of generality, we can assume that $t = 0$. Let $v_x = \sum \omega_i \cos(t_i)$ and $v_y = \sum \omega_i \sin(t_i)$. Call the new vector after the switch $\tilde{\omega}$ with associated $\tilde{v}_x = \sum \tilde{\omega}_i \cos(t_i)$ and $\tilde{v}_y = \sum \tilde{\omega}_i \sin(t_i)$. Since only change from ω to $\tilde{\omega}$ was the switch of one source, the following holds $\tilde{v} = v + \omega_0 \delta$ where ω_0 is the weight put on this source. Note that conditional on the news sources chosen, an agent is choosing ω to minimize $\mathbb{E}(\mathbb{E}(f(\theta_{id}, t)|s) - f(\theta_{id}, t))^2$. Note that given our assumption on t, this is always equal to $(1 - v_x)^2 + v_y^2 + \sum_i \omega_i^2 \frac{1}{\tau_i}$. Going from ω to $\tilde{\omega}$ the last term doesn't change, only the first two terms change. It is sufficient for the result to show that $(1 - v_x)^2 + v_y^2 - (1 - \tilde{v}_x)^2 + \tilde{v}_y^2 > 0$.

$$(1 - v_x)^2 + v_y^2 - (1 - \tilde{v}_x)^2 + \tilde{v}_y^2 = (1 - \tilde{v}_x + \omega_0 \delta_x)^2 + (\tilde{v}_y - \omega_0 \delta_y)^2 - (1 - \tilde{v}_x)^2 + \tilde{v}_y^2$$
$$= \omega_0^2 \delta_x^2 + \omega_0^2 \delta_y^2 + 2\omega_0 \delta_x (1 - \tilde{v}_x) - 2\omega_0 \delta_y \tilde{v}_y \tag{9}$$

It is sufficient to focus on the case where $(\delta_x, \delta_y) = (1 - \cos(2\beta), \sin(2\beta))$ with $2\beta = \pi/n$, as in the other cases the desired condition will be easier to satisfy.

$$= \omega_0 [\omega_0 (1 - \cos(2\beta))^2 + \omega_0 \sin^2(2\beta) + 2(1 - \cos(2\beta))(1 - \tilde{v}_x) - 2\sin(2\beta)\tilde{v}_y]$$
$$= 4\omega_0 \sin^2(\beta)[\omega_0 \sin^2(\beta) + \omega_0 \cos^2(\beta) + (1 - \tilde{v}_x) - \frac{\cos(\beta)}{\sin(\beta)}\tilde{v}_y] \tag{10}$$

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Now we use the fact that $\tilde{v}_y \leq \frac{\sin(\beta)}{\cos(\beta)}\tilde{v}_x$, otherwise, we can always rotate the new sources that are chosen without ω.

$$
\begin{aligned}
&> 4\omega_0 \sin^2(\beta)[\omega_0 + (1 - \tilde{v}_x) - \tilde{v}_x] \\
&> 4\omega_0 \sin^2(\beta)[\omega_0 + 1 - 2\tilde{v}_x]
\end{aligned}
\tag{11}
$$

It is sufficient for the result to show that $\tilde{v}_x \leq 1$.

Claim 6. $v_x < 1$

Proof. Look at the best case where there are κ news sources that are all perfectly targeting $t = 0$ (which cannot happen for finite n). It is easy to see that the optimal strategy would be to choose $\omega = (\omega, \omega, ..\omega)$ to minimize $(1 - \kappa\omega)^2 - \kappa\omega^2\sigma^2$. Solving this problem gives us $\omega = \frac{1}{\kappa+\sigma^2}$ which implies that $v_x = \kappa\omega = \frac{\kappa}{\kappa+\sigma^2}$. By our normalization assumption, $\sigma^2 = \kappa$ which implies $v_x \leq 0.5$. $\qquad\square$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Lemma A5. *In the optimal learning strategy, an agent consumes the closest news sources.*

Proof. In Lemma A4, we already showed that there cannot be a gap in the news sources consumed to the right and to the left. Now assume for contradiction that the set of new sources chosen is not actually the set closest to the agent. Let $t_m = \frac{1}{\kappa}\sum = t_i$ be the mean type of the chosen set of new sources. Without loss of generality, assume that t_m is to the right of t. By assumption $t - t_m = \frac{\pi}{2n} + \theta$ for some $\theta > 0$. This also suggests that t is closer to the mid point of an alternative set of new sources that have been shifted to the left. First we show that the original set of news sources provides a more effective learning strategy for all types between t and t_m. We can always take the most right and left news sources, and we can replace $\tilde{v}_1 = (1 - \rho)v_1 + \rho\frac{v_1+v_\kappa}{2}$ and $\tilde{v}_\kappa = (1 - \rho)v_\kappa + \rho\frac{v_1+v_\kappa}{2}$. We can do this iteratively for all other news sources as well. As $\rho \to 1$, the estimated type shifts towards t_m and the variance goes down. Using symmetry, we've demonstrated that this set of news sources to be better for $t_m - \frac{\pi}{2n} + \theta$. But this implies that the news sources can be shifted to the left to the get a better learning strategy. $\qquad\square$

Lemma A5 implies that when firms locate equidistantly, and choose the same τ, with n news sources, each firm serves $\frac{\kappa}{n}$ of the market. Each firm is competing over threshold types with neighbors that are κ to the right an left. Hence, existence of symmetric equilibria and the comparative results can be shown following the same strategy for the $\kappa = 1$ case taking the adjustment with respect to the threshold types into account. $\square$