Media Competition and the Source of Disagreement

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Abstract

We study the competitive provision and endogenous acquisition of political information. Our main result identifies a natural equilibrium channel through which a more competitive market for information increases social disagreement. A critical insight we put forward is that competition among information providers leads to a particular kind of informational specialization: firms provide relatively less information on issues that are of common interest, and relatively more information on issues along which agents' preferences are more heterogeneous. This enables agents to find information providers that are better aligned with their preferences. While agents become individually better informed, the social value of the information provided in equilibirum decreases, thereby decreasing the probability that the society implements socially optimal policies.

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1. Introduction

We present a simple model to study the competitive provision and endogenous acquisition of political information. Our primary goal is to illustrate the effects of increased competition in the market for news at three distinct levels: differentiation in the types of information supplied by news sources, consequences of this differentiation for the distribution of opinions in the society, and its eventual impact on the efficiency of voting outcomes. Our interest is motivated by a growing public debate on the consequences of a fast-changing media landscape and information consumption habits on our democracies.[1] So far, the literature on the political economy of mass media has not evaluated competition in the context of a full-fledged equilibrium model in which an arbitrary number of non-partisan media firms compete for the attention of rational voters. This paper fills this gap, and presents a particularly simple model whose analysis leads to two main conclusions. First, we show that competition leads to *content* specialization, even when consumers are rational and news sources are unbiased. The novel insight we put forward is that competition forces information providers to become relatively less informative on issues that are particularly important from a social point of view. Second, we analyze the downstream effects of such specialization and show that, while agents becomes individually better informed, competition amplifies social disagreement. Next, we highlight the welfare implications of increased disagreement. In contrast with most of the previous results which emphasize inefficiencies associated with supply or demand side frictions in the media market, we illustrate a natural channel through which increased competition systematically decreases the probability that the society is able to successfully implement socially optimal policies.

In our model, a finite number of firms compete to provide information to a continuum of Bayesian agents about a newly proposed policy with uncertain prospects. Whether the the new policy is implemented to replace the known status quo depends on its approval rate. The policy contains a vertical component, *valence*, along which preferences are identical, and two horizontal components, *ideology*, along which preferences are heterogeneous. Firms generate signals about these components, but in doing so face a budget constraint on how informative these signals can be. Specifically, being more precise about one of these components requires the firm to trade off precision on the other two. To fix ideas, imagine a new health care bill is being discussed whose details are yet not fully known by the public. The bill potentially affects many dimensions of social life, and voters might evaluate these dimensions differently. For example, it could promote an increase in the overall quality of health care (vertical),

[1] See Sunstein (2017), Nichols (2017), and Pew Research Center (2016) for recent descriptions of the media landscape and the modern news consumer.

expand the budget deficit (horizontal), and induce more redistribution via increasing the share of the population covered (horizontal). Voters gather information from the media and voice their opinions. A larger public consensus increases the probability that the bill successfully goes through Congress. Media compete for the attention of the public and allocate their limited resources (journalists, airtime etc.) on a possibly different mix of these policy dimensions to maximize their readership.

The equilibrium of our model demonstrates how competition among information providers affects content specialization. While each agent wants to learn about the underlying state of nature (the details of the policy), different types would like to learn about different aspects of it. But firms compete for readership, and thus, have an incentive to generate information that is simultaneously valuable for agents of different types. They can do so by being informative about dimensions of common interest, namely valence. However, as the market becomes more competitive, the effectiveness of such a generalist approach declines - agents of different types get targeted by different firms which provide signals tailored towards their informational needs. We believe this aspect of the interaction between producers and consumers of information is not special to our set up, but rather it is a generic feature of the competitive provision of information by profit maximizing firms to a heterogeneous Bayesian audience. This leads to novel insights. First, competition creates social value in the sense that it makes each agent better informed about his own evaluation of the policy. However, agents become more informed on increasingly different aspects of the state space: the ones that they specifically care about. Second, the market never overspecializes. As the number of firms in the market grows to infinity, the equilibrium converges to a *daily-me* paradigm, a situation in which each agent finds a news source perfectly designed to meet her unique informational needs. Third, because agents become better informed about different sub-dimensions of the state space, they disagree more and therefore the probability they implement policies that are socially optimal decreases.

The equilibrium mechanism behind our main results can be decomposed into two distinct parts. The first part exploits a simple idea. Suppose a group of agents individually choose among two options: a safe option, which ensures a payoff of zero, and a risky option, which yields an uncertain payoff that depends on both a common component and a private one. In this simple setup, we can imagine more information about the common (private) component to increase (decrease) the correlation in agents choices, *ceteris paribus*. A similar force is behind our main results. Valence acts as a common component, while ideology *can* act like the private component. While simple, this mechanism is incomplete. In this paper, we illustrate how competition among information providers, and the resulting need to specialize when there is heterogeneity in the informational needs of the agents making up the society, can generate a shift in the types of information provided in the market.

This brings us to the second part of our equilibrium mechanism. In our model, specialization comes about because information on valence is, by definition, equally useful to all agents. Therefore, information on ideology must be targeted instead. We construct a model where ideological preferences are diverse, but correlated to different degrees among different subpopulations of the society. While this can be achieved in multiple ways, we posit that ideology is multi-dimensional. As agents are heterogeneously interested in these dimensions, this creates the scope for firms to specialize in different mixtures of these dimensions. The interactions between these two parts of the mechanisms that we just described generate an equilibrium channel through which an increase in competition leads to an increase in disagreement among the population, a result that is at the heart of our contribution.

We then use our model as a benchmark to study the effects of media competition on social welfare. From an ex-ante point of view, we find that competition – via specialization– creates a larger spectrum of informational options for agents, enabling them to select news sources that are better aligned with their needs. In this sense, competition increases ex-ante welfare as it makes agents individually better informed. This result conforms with the classic view that sees the market for news as a "marketplace of ideas," promoting knowledge and the discovery of truth,[2] and more generally with previous results in this literature that we review below. However, the welfare effects of media competition extend well beyond the individual information acquisition stage. The market for political news differs from other markets partly because it has an indirect effect on welfare through information externalities imposed on the policy process (Prat (Forthcoming)). Consistent with this view, our model predicts that, while agents become individually more informed, their opinions diverge as they become informed on increasingly diverging aspects of the unknown policy. This triggers the increase in social disagreement documented above and, with it, drives a number of socially undesirable outcomes.

It is useful to first highlight that any inefficiency in the market for political information originates from heterogeneity in the preferences of the agents. The political process, by definition, involves aggregation over the opinions of agents who are potentially in conflict with each other. We show that this creates a wedge between the *social* and the *individual* value of information about valence, irrespective of the distribution of ideological preferences. Competition necessarily results in a supply of information on valence that is inefficiently low. As a consequence, we show that a more competitive market for news decreases the probability that society is able to discern between socially "good" and "bad" policies. Note,

[2]See Posner (1986) for a wide-ranging and introduction to this classic view.

however, that this result depends critically on the shift in the *type* of information provided to the agents. While agents get individually better informed as the number of information providers in the market increase, it is not the case that that they are able to find information structures that are "more informative" in a Blackwell sense. The market becomes relatively more informative on dimensions that are socially inefficient, i.e. ideology, and less informative on dimensions that are socially efficient, i.e. valence. To make this point formal, we consider a special subset of policies: those that are Pareto dominant under complete information. In these policies, if the state of the world was known all agents would agree on the desirable political outcome. We show that, even when conditioning on such a policy, an increase in competition decreases the probability that the policy is implemented.

Literature. Our paper contributes to the burgeoning literature on the political economy of mass media, a field that has been particularly active in the last decade.[3] Specifically, we contribute to the branch of this literature that studies the effects of endogenous provision of political information and its externalities on the political process. One robust finding of this literature is that when information providers are partisans – namely, they are interested in persuading the public to take a certain action – competition generally brings about better social outcomes. Intuitively, competition forces firms to better align with what consumers demand, thus reducing their inherent biases. Results along this line echo in the works of Baron (2006), Chan and Suen (2009), Anderson and McLaren (2012).[4] Similarly to these papers, Duggan and Martinelli (2011) finds that slanting is an equilibrium outcome in a richer model that allows for electoral competition, but otherwise abstracts away the problem of competitive information provision. While not modeling competition, the works of Alonso and Câmara (2016) and Bandyopadhyay et al. (2017) also belong to this strand of the literature. Instead, a general treatment of competition among biased sender is introduced in Gentzkow and Kamenica (2017). Our work differs from these papers; we assume that information providers are non-partisans and they compete for consumers' attention to maximize advertisement revenues. Chan and Suen (2008) consider a model with features that can be mapped back to our setup. Their primary interest, however, is to study the effects of exogenously located firms on the electoral competition. They show that a new entrant increases the probability that parties choose the policy favored by the median voter, thereby increasing welfare. In an extension, they also endogenize competition, but the only industry structure they can feasibly analyze (a duopoly) typically leads to higher welfare. Closer to

[3]See Prat and Strömberg (2013) and Gentzkow et al. (2015) for recent and comprehensive reviews of the literature.

[4]The welfare-increasing effects of competition are also illustrated in Besley and Prat (2006), Corneo (2006) and Gehlbach and Sonin (2014), although for orthogonal reasons from those discussed here, namely the potential risks of media capture by the government.

our work, Chen and Suen (2018) study a competition model in which a number of biased media firms compete for the scarce attention of readers finding that an increase in competition leads to an increase in the overall informativeness of the industry. Similarly, results consistent with the idea that competition is welfare-increasing are discussed in Burke (2008), Gentzkow and Shapiro (2006), Gentzkow et al. (2014). Sobbrio (2014) does not analyze the welfare implications of media competition, but shows that competition can lead to specialization. Galperti and Trevino (2018) study a model of endogenous provision and acquisition of information and show how competition for attention can lead to a homogeneous supply of information, even when consumers would value accessing heterogeneous sources. Overall, when consumers are rational, evidence is stacked in favor of the welfare-increasing effects of media competition. Our paper contributes to this literature by solving for a full-fledged competition model illustrating a novel and natural channel through which competition generates welfare-decreasing effects.[5] Departing from the assumption of rationality, Mullainathan and Shleifer (2005) consider a simple model in which heterogeneous consumers derive psychological utility when their prior views are confirmed by new observations. Consistent with the findings discussed above, they also find that more competition leads to specialization and a decrease in prices. Bernhardt et al. (2008) studies the welfare implications of competition in a related model with behavioral agents showing that competition increase the probability the society makes mistakes.

Empirical implications. Finally, our paper also relates to the large empirical literature that specifically studies the effects of media competition on political participation and electoral outcomes (Stromberg (2004), Gentzkow (2006), Stone and Simas (2010), Gentzkow et al. (2011), Falck et al. (2014), Drago et al. (2014), Miner (2015), Cagè (2017), Gavazza et al. (Forthcoming), Campante et al. (Forthcoming)).[6] To this literature, our model contributes with three distinct testable predictions. First, it predicts that a stronger media competition leads to more *content* specialization. So far, the literature and the broader public debate on the changes in the media landscape have focused on differentiation of new sources in terms of ideological biases. In this paper, we highlight that even in the absence of distortionary biases, firms specialize their product by creating content that is increasingly targeted to different audiences. Novel empirical methods, that exploit machine learning techniques to analyze textual data, have provided preliminary evidence of the effect that we highlight in this paper. Angelucci et al. (2018) explore the content production of French

[5]In the context of a voting game with heterogeneous voters and information externalities, some of the results in Ali et al. (2017) echoes a similar intuition and can be though of as complementary to ours.

[6]While not explicitly focusing on media competition, DellaVigna and Kaplan (2007) and, more recently, Martin and Yurukoglu (2017) study the effect of biased news on voting behavior. Relatedly, Boxell et al. (2017) study the relationship between social media use and polarization.

newspapers as the market in which they operated became more competitive. They find evidence of content specialization, taking the form of an increased focus on local news, as opposed to national ones. Similarly, Martin and Yurukoglu (2017) find evidence of content specialization for major cable outlets and provide evidence of correlation with competition. Finally, Nimark and Pitschner (2018) provide a comprehensive account of the extent to which American newspapers specialize in the production of their content. While more research on this topic is needed, the use of these techniques is expected to grow in the field (Gentzkow et al. (Forthcoming)). Second, our model predicts that an increase in media competition is correlated with an increase in disagreement even among rational agents. There is a growing literature analyzing polarization in public opinion. A number of papers have investigated this channel (Prior (2013), Campante and Hojman (2013)), but evidence is mixed and more research is needed. Finally, our model predicts that content specialization will develop at the expense of information about valence, a dimension of the policy space along which agents have homogeneous preferences.

The rest of paper is organized as follows. Section 2 introduces our model and discusses its main assumptions. In Section 3, we solve for the equilibrium of the information provision game for an arbitrary number of competing firms. Moreover, we establish how competition affects the equilibrium supply of information. Our main results are presented in Section 4, where we discuss how competition affects the value of information, social disagreement and welfare. Sections 5 and 6 discuss important extensions of our model and conclude. All proofs are relegated in the Appendix.

2. Model

A society with a unit mass of Bayesian agents evaluates an unknown policy $\theta := (\theta_0, \theta_1, \theta_2) \in \mathbb{R}^3$, the components of which are believed to be mutually independent and identically distributed as standard normals. Agents have heterogeneous preferences about the policy θ. Specifically, an agent of type $t \in T := [-\pi, \pi]$, drawn from a uniform distribution F, evaluates policy θ according to the utility function

$$u(\theta, t) := \lambda\theta_0 + (1-\lambda)\theta_t^{id} \qquad \text{with} \qquad \theta_t^{id} := \cos(t)\theta_1 + \sin(t)\theta_2. \tag{1}$$

We refer to dimension θ_0 – along which agents' preferences are perfectly aligned – as the *valence* component of policy θ and to dimensions θ_1 and θ_2 – along which agents' preferences are type dependent– as the *ideological* components. $\lambda \in (0,1)$ denotes the importance of valence relative to ideology.[7]

[7]In the political science literature, this distinction goes back to Downs (1957) and Stokes (1963).

A set N with $n \in \mathbb{N}$ firms provide information on θ by committing to an information structure. To do so, each firm $i \in N$ chooses a vector of weights $b_i \in \mathbb{R}^3$ that satisfies the budget constraint $\|b_i\| \leq 1$, where $\| \cdot \|$ is the Eucledian norm. A choice of a b_i induces an information structure $s_i(\theta) := (s_i^v(\theta), s_i^{id}(\theta))$, composed by signals $s_i^v(\theta) := b_{i,0}\theta_0 + \varepsilon$, informative about valence, and $s_i^{id}(\theta) := b_{i,1}\theta_1 + b_{i,2}\theta_2 + \varepsilon$, informative about ideology. The error term ε is assumed to be standard normal and independent both across firms and agents. A firm's profits are increasing in its readership, namely the measure of agents $S \subset T$ who acquire information from it.

Each agent selects a single firm $i \in N$ and receives a costless signal realization $s := (s^v, s^{id})$.[8] She evaluates the unknown policy θ relative to a known status quo whose utility is normalized at zero. We assume that agents vote sincerely and receive utility $\mathbb{E}(u(\theta, t)|s)$ if they approve the policy and zero otherwise. $z(\theta, t)$ is a random variable describing type t's approval behavior of the policy given θ.[9] Finally, we assume that policy θ is implemented with a probability that is strictly increasing in $\Gamma(\theta) := \mathbb{E}(\int_T z(\theta, t)dF(t))$, the expected *approval rate* of policy θ. Figure 1 depicts the timeline of the game. The solution concept we use is Perfect Bayesian Equilibrium.

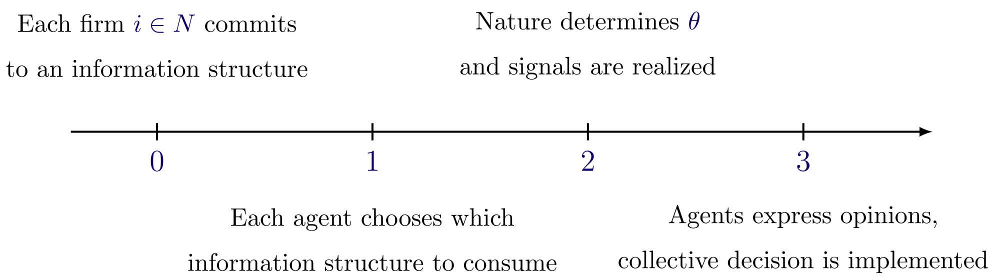

FIGURE 1: Timeline of the game.

We pause for a discussion of our model and the implications of our assumptions. As detailed in the following sections, a key aspect of the equilibrium mechanism is how competition affects firms' incentives for *specialization*. Non-price competition among firms leads to product specialization in markets with two features (Tirole (1988)): firms are constrained in their supply and there is heterogeneity in consumers' demand. There are several different ways in which such constraints and heterogeneity can be introduced and modeled. The choices we made serve two main purposes: they provide enough tractability to solve for the equilibrium with an arbitrary number of firms, and they allow for a particularly transparent and clean depiction of the main forces at play. We should, however, emphasize that, as discussed

[8] As we show in Section 5, our results extend to the case where each agent can choose finitely many firms.

[9] Our aim is to keep the notation simple here, but note that the approval decision depends on the information structure chosen, and the signal realization.

below, the key forces driving our result are more general than the model we present.

Signals. Firms solve a finite resource allocation problem in which they trade-off precision between two signals, s^v and s^{id}. Absent this substitutability imposed by the budget constraint, there would exists a pair of signals that Blackwell dominate all the others. Such pair would be the only one offered in equilibrium and the competition problem would become uninteresting. More importantly, this assumption captures aspects of the real world that are ubiquitous: firms face both supply-side constraints (in the number of journalists they can employ, the number of pages they can fill, their allotted airtime, etc.) and demand-side side ones (e.g. consumers' attention). Also, building on the work of Duggan and Martinelli (2011), in providing a information on ideology, our firms reduce a two-dimensional state (θ_1, θ_2) to a one-dimensional signal. While being immaterial for our main results, this assumption proves to be extremely convenient as it allows us to analyze the firm's problem as choosing a location on circle (Salop (1979)).[10]

Preferences. Despite being Bayesian, agents in our model may assign different valuations to the same information structure. This is a byproduct of the heterogeneity in their preferences. Heterogeneity in voter preferences is well-documented and has been studied in different contexts.[11] Of course, there are several different ways to model such heterogeneity, all leading to the similar conclusions that different types assign different values to the same experiment. Our preferences are designed to capture heterogeneity in a tractable way, i.e. via a one-dimensional type $t \in T$, while retaining the "circumference" representation as it is particularly convenient to solve the firms' problem. Incidentally, this assumption also captures an important aspect of the real-world: agents can disagree both on which issues are important to them (their "agenda", so to speak) and on how each issue in their agenda should be addressed (their "slant").[12] An agent's type t simultaneously captures both the relative weights she puts on different issues and her position on each of these issues. For example, given our preference specification in Equation 1, type $t = \pi/4$ prefers higher realizations of both θ_1 and θ_2 and attaches equal weights to both dimensions (Figure 2). In contrast, type $t = -\pi/4$ prefers higher realizations of θ_1 and lower realizations of θ_2, but

[10]Other kinds of constraints have been discussed in the literature. For example, Chan and Suen (2008) consider a one dimensional state that is mapped into a binary signal. This induces a similar constraint on the firm's choice set as signals cannot be equally informative to all agents. More generally, constraints can be introduced via a cost function that price precision.

[11]In this literature, the first paper noting this is probably Suen (2004).

[12]Pew Research Center (January, 2017) provides plenty of evidence of the differences in voters agendas. Consistently, multidimensional preferences are common in this literature. See for example Groseclose (2001), Eyster and Kittsteiner (2007), Carillo and Castanheira (2008), Ashworth and de Mesquita (2009) Dragu and Fan (2015), Aragones et al. (2015), Yuksel (2015) for applications in the context of party competition, and Alesina et al. (1999), Lizzeri and Persico (2005), Fernandez and Levy (2008) for applications to public goods.

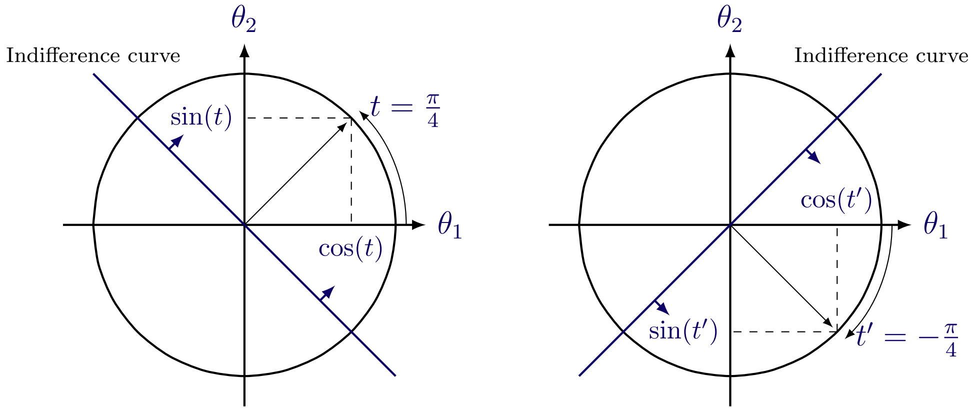

FIGURE 2: The ideological trade-offs between (θ_2, θ_3) for different types $t \in T$.

still attaches equal weights to both issues. This way of modeling heterogeneity has other desirable features. First, the distance $|t-t'|$ between two types on the circumference provides us with a simple measure of how correlated their preferences are. This allows us to map the competition game among firms into a spatial problem. Second, the preferences give us a natural normalization in which all agents, albeit heterogeneous, ex ante dislike uncertainty in the same way.[13] Finally, λ measures how important the valence dimension is relative to the ideology and, thus can be interpreted as an ex ante measure of polarization.

Distribution over types. We assume types are uniformly distributed over T. This assumption is most important for the tractability of the model. In fact, the existence of a *symmetric* equilibrium heavily hinges on such assumption. From a conceptual point of view, however, this assumption is unproblematic for two important reasons. First, content specialization is a robust consequence of competition that generalizes beyond the uniform distribution. In our model, independent of the assumptions on the distribution, each agent receives her own most-preferred information structure as the number of firms in the market go to infinity. (see Proposition 4 and Remark 3.) Second, a key feature of our welfare result is that the *social* value of information on valence exceeds its *individual* value. As we show in Remark 4, this continues to be true under any non-degenerate distribution. Finally, conditional on θ, our model can always be mapped into a more standard, one-dimensional model in which there is heterogeneity in the evaluation of the policy. Uniformity assumption on F guarantees that the distribution is symmetric around the median evaluation which also corresponds to $\lambda\theta_0$. We return to these points in the final discussion of Section 6.

Readership. To keep the model tractable, we do not allow firms to compete on prices. This assumption is sensible, for at least, three reasons. First, the largest share of revenues in the

[13]Indeed, the variance $\mathbb{V}\big(u(\theta,t)\big)$ is independent of t. To see this, notice that $\theta_1 \cos(t) + \theta_2 \sin(t) \sim \mathcal{N}(0,1)$ for all t.

media industry are generated through advertisement, which mainly depends on readership. Whenever positive, the price for political news is nevertheless often negligible. Second, whenever present, price competition is the media industry is often highly regulated.[14] Third, price competition would provide an additional incentive for product differentiation, thereby exacerbating disagreement in the society (Section 4). In this sense, our result is surprising in that - even in the absence of price competition - incentives for differentiation are strong enough to produce negative welfare implications.

Sincere Voting. Agents express either a favorable or an unfavorable opinion about the policy and receive direct utility from such activity. Equivalently, one can think of this as voting for or against a policy in a referendum or a political challenger who takes positions on several issues. We put aside the question of why people express their preferences and vote. Indeed, in a model with a continuum of voters, no individual has an impact on the election outcome. A direct utility from honest voting (perhaps rising from a sense of civic responsibility) is the most straightforward and possibly most realistic assumption in this context. In Section 6, we discuss the robustness of our main results to strategic voting.

3. Equilibrium

This section is devoted to the analysis of the equilibrium of our game and it is divided into three parts. We begin by reducing the firm's problem to a location on a disc. Then, we solve the agents' information acquisition problem and characterize its properties. Finally, we solve for the equilibrium of the game.

3.1. Problem of the Firm: Consumer-Targeting

Firm $i \in N$ chooses a vector of weights $b_i \in \mathbb{R}^3$ to maximize readership, while respecting the budget constraint $\|b_i\| \leq 1$. In this subsection, we reduce this problem to a *consumer-targeting* problem on the unit disc, something that will prove to be extremely convenient in the equilibrium analysis. Denote the action set $A_i := T \times [0, 1] \in A_i$ with typical element $a_i = (x_i, \tau_i)$.

Remark 1. *Fix b_i s.t. $\|b_i\| = 1$. There exists a unique action $a_i = (x_i, \tau_i) \in A_i$ such that the signals induced by b_i are distributed as $s^v(\theta) \sim \mathcal{N}\left(\theta_0, \frac{1}{\tau_i}\right)$ and $s^{id}(\theta) \sim \mathcal{N}\left(\theta^{id}_{x_i}, \frac{1}{1-\tau_i}\right)$.*

[14] A major example of this is the Newspaper Preservation Act of 1970 in the United States exempting competing newspapers from certain provisions of antitrust laws.

Notice that it is without loss of generality to focus attention on b_i such that $\|b_i\| = 1$. When that is not the case, i.e. when $\|b_i\| < 1$, there exists a scalar $c > 1$ such that $\|cb_i\| \leq 1$ is still feasible and induces an information structure that Blackwell-dominates the one induced by b_i. By Blackwell theorem, every type $t \in T$ would assign a higher value to cb_i and therefore the readership of firm i would weakly increase and this cannot be consistent with equilibrium behavior. Therefore, the result in Remark 1 allows us to think of the problem of the firm "as if" firm i was choosing (1) a consumer $x_i \in T$ to target and (2) how much to invest in the precision of the signal about valence, $\tau_i \in [0, 1]$. This representation will allow us to think of the firm's problem as a location on a disc. Like in a standard location model, agents will acquire information from the firm located "closer" to them. However, unlike most location models, the notion of distance is not given exogenously, e.g. via a so called transportation cost. In our model, the notion of distance is a by-product of the fact that agents are Bayesian: it will be measured via the "value of information."

3.2. The Value of Information

In our model, agents behave in a straightforward way: an agent of type t will acquire information from the firm that produces the highest *value of information* conditional on her type t. In this section, our main task is to analytically compute the value of a generic information structure for a generic agent of type t. Consider the information structure induced by $a_i = (x_i, \tau_i)$. After observing the realization of signals $s = (s^v, s^{id})$, agent $t \in T$ can compute the expected utility of policy θ, denoted $\mathbb{E}\big(u(\theta, t)\big|s\big)$. Given the agent's incentives specified in Section 2, she either approves the policy or rejects it to receive utility $\max\big\{0, \mathbb{E}\big(u(\theta, t)\big|s\big)\big\}$. The value of the information structure induced by a_i is therefore the *ex ante* expectation of such future expected payoff, namely

$$V(a_i|t) := \mathbb{E}\Big(\max\big\{0, \mathbb{E}\big(u(\theta, t)\big|s\big)\big\}\Big) \geq 0$$

where the outermost expectation is taken with respect to the possible realizations of s defined by a_i and the prior beliefs on θ. Intuitively, the value of an information structure for a type $t \in T$, i.e. $V(a_i|t)$, measures how much better off she expects to be after receiving the signals, relative to receiving no signals whatsoever.[15] The function $V(a_i|t)$ is the information-theoretic counterpart of the "transportation cost" in a location model. This functions represents a key building block in our solution, and our model allows us to compute it in an explicit way, as the next proposition shows.

[15]The value of receiving no information is of course $\mathbb{E}\Big(\max\big\{0, \mathbb{E}\big(u(\theta, t)\big)\big\}\Big) = 0$.

Proposition 1. *The value of information $a_i = (\tau_i, x_i) \in A_i$ for an agent of type $t \in T$ is*

$$V(a_i|t) = \frac{\sigma(a_i|t)}{\sqrt{2\pi}} \tag{2}$$

with $\sigma^2(a_i|t) := \lambda^2 g(\tau_i) + (1-\lambda)^2 \cos^2(t - x_i) g(1 - \tau_i)$ *and* $g(\tau_i) := \dfrac{\tau_i}{1+\tau_i}$.

In equilibrium, given a profile of actions $a \in A := \prod_i A_i$, agent t chooses the firm offering the highest value of information $V(a_i|t)$. Formally, given the profile of information structures induced by a, we denote the the information acquisition behavior of type t by $r(a,t) \in \Delta(N)$.

Proposition 1 is an important building block in the solution of our model as it provides us with a full characterization of the value of information in our game. The first insight is that $\sigma^2(a_i|t)$, namely the variance of the random variable $\mathbb{E}\big(u(\theta,t)|s\big)$, is in a one-to-one relation with the value of information. The intuition is simple: when the variance of interim utility is high, the relative gain of choosing one option versus another conditional on the signal realizations are likely to be high as well. This implies that agent t's ex ante expectation of future payoffs are high too. The second insight is that the value of information can be reduced to two components, both with precise economic meaning. These two components will reveal the main trade-off that firms face when designing these information structures.

Remark 2. *The value of information $V(a_i|t)$ is determined by two components:*

1. *A* Generalist *component $\lambda^2 g(\tau_i)$, strictly increasing in τ_i and type-independent.*
2. *A* Specialist *component $(1-\lambda)^2 \cos^2(t - x_i) g(1-\tau_i)$, strictly decreasing in τ_i and increasing in $\cos^2(t-x)$.*

To understand this result, consider an increase in τ_i. This corresponds to an increase in the precision allocated to the valence signal s^v. Remark 2 shows that an increase in τ_i makes the information structure more *generalist*: since all agents (equally) care about valence, an information structure that is more informative on the valence dimension will benefit all agents, irrespective of their type t. In contrast, a decrease in τ_i makes the information more *specialist*. However, there are a multitude of different ways in which an information structure can become specialist, according to the *ideological mix* it provides information about. In Remark 2, this is captured formally by the term $\cos^2(t - x_i)$ which, again, has a precise economic interpretation. In fact, we have that the statistical correlation between the agent t's preferences and those of the targeted agent x_i is $\mathbb{C}\big(u(\theta,t), u(\theta,x_i)\big) = \lambda^2 + (1-\lambda^2)\cos(t-x)$, implying that the length of the arc between agents t and x_i measures their relative *ideological distance*.[16] The value of information is ultimately affected by $\cos^2(t - x_i)$, the square of the

[16] Under this measure, two agents can be ideologically similar even if when their respective "bliss-points"

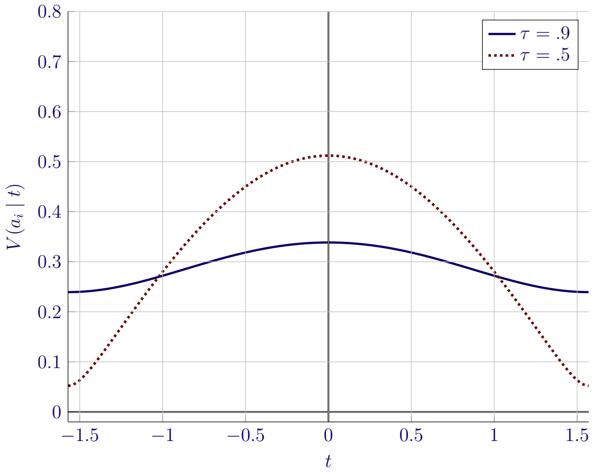

FIGURE 3: The value of information (τ, x) for type t. On the horizontal axis, $t-x$ represents the distance of given type from her information provider.

correlation. This implies that two types with perfectly negatively correlated preferences, i.e. t and $t+\pi$, value information equally as they agree on the weights they attach to θ_1 and θ_2. This is a by-product of two assumptions we made: agents are Bayesian and information structures are unbiased. This suggests that, from the point of view of the firms, targeting agent x_i is equivalent to targeting agent $x_i + \pi$: therefore, the problem of the firm can be further reduced with no loss of generality to choosing a target $x_i \in \tilde{T} := [-\pi/2, \pi/2]$.

The trade-off between generalism and specialism in the choice of the information structure is depicted in Figure 3. We fix $x_i = 0$ and plot the value of two information structures as a function of t. When τ_i is high, the information structure is more *generalist*. The value associated with this news source is not particularly high, even for the agents that are close to the target x_i, but remains steady even for agents that are far away from it. On the other hand, when τ_i is low, the information structure is more *specialist*. The associated value is high for types that are close to the target x_i, but drops rapidly for voters whose ideological preferences are farther away. The problem of the firm can be visualized as the choice of a location on a disk (Figure 4). Each firm chooses a precision on valence τ_i – implying a distance from the center – and a target $x_i \in \tilde{T}$ – implying a particular angle in the circle.[17]

are far apart. This happens when they trade-off different payoff-relevant dimensions in similar ways, hence their preferences are highly correlated. This way of measuring ideological distance adds to the existing literature on polarization (see Gordon and Landa (2017)), which mostly focuses on bliss-points distance, thereby offering a more nuanced way of measuring polarization empirically.

[17]For example, choosing $x_i = 0$ (resp. $x_i = \pm\frac{\pi}{2}$) implies that the firm only reports about dimensions (θ_0, θ_1) (resp. (θ_0, θ_2)).

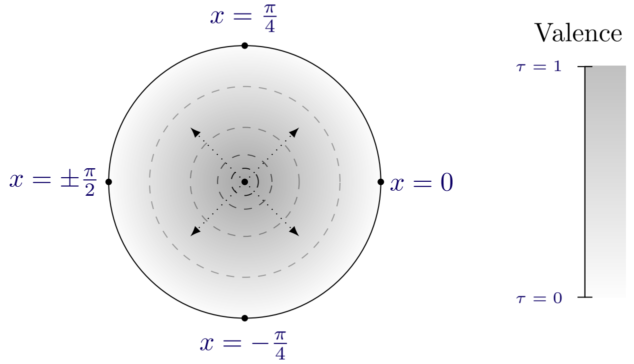

FIGURE 4: Mapping the firm's problem into a location choice.

3.3. The Equilibrium Design of Information Structures

After having characterized the information acquisition problem, we proceed by characterizing the problem of the firm, that consists in optimally designing an information structure, by choosing $a_i \in A$, to maximize its readership. The profits of firm $i \in N$ are given by:

$$\Pi_i(a_i, a_{-i}) := \int_T r(a,t)[i] \; dF(t).$$

where $r(a,t)[i]$ is the probability that type t consumes information structure i given a.

We focus on equilibria in pure strategies. Such equilibrium are defined as follows.

Definition 1. *A PBE is a triple* $(a^\star, r^\star, z^\star)$ *such that:*

(i) for all $i \in N$ *and* $a_i \in A_i$, $\Pi_i(a_i^\star, a_{-i}^\star) \geq \Pi_i(a_i, a_{-i}^\star)$;

(ii) for all $t \in T$ *and* $a \in A$, *supp* $r^\star(t,a) \subseteq \arg\max_{i \in N} V(a_i|t)$;

We say that an equilibrium is symmetric *if* $a^\star$ *is such that* $\tau_i = \tau_j$ *for every* $i, j \in N$.

In equilibrium, each firm maximizes its readership relative to the behavior of other firms, agents only consume information structures that yield the highest value and conditional on the signals received, sincerely approve or disapprove the policy θ. Figure 4 provides a graphical illustration of the concept of symmetric equilibrium. In a symmetric equilibrium, firms are located on the same inner circle ($\tau_i = \tau_j$ for all $i, j \in N$). The next result establishes existence and generic uniqueness of the equilibrium in our game.

Proposition 2. *A PBE always exists and any PBE of the game is necessarily symmetric. Moreover, there exists* $\bar{n} \in \mathbb{N}$ *such that, for all* $n \geq \bar{n}$, *firms target a set of agents equidistant from each other and the equilibrium is unique up to* modular rotations *of the firms'* *locations.*[18]

The existence of only symmetric equilibria, as defined in Definition 1, heavily relies on the uniformity of the type-distribution F.[19] Our focus on the uniformity of F, is due to two important reasons. The first one is tractability: by providing a full characterization of how firms locates on the circle, symmetry allows us to explicitly solve for the equilibrium for arbitrary $n \in \mathbb{N}$, hence study the effects of competition. The second implication is salience: uniformity creates an environment in which the equilibrium tensions that we highlight are extreme, thereby making the channel behind our results most transparent. While technically useful, from a qualitative point of view uniformity is inessential in shaping our results, as we shall argue in Section 6. Finally, it is worth noticing that, while a continuum of equilibria can be generated by simply rotating the location of firms, this multiplicity is immaterial for the problem we study. Due to uniformity of F, it is not the the firms' absolute location that matters, but rather their relative distance, which is not affected by modular rotations.

Our goal is to analyze the equilibrium as the number of firms $n \in \mathbb{N}$ in the market changes. In particular, we shall study the effects of an increase in competition on the type of information that firms produce and, ultimately, on the agents' behavior.

3.4. Increasing Competition in the Market for News

In the last part of this section, we provide the main characterization of the effects of competition on *firms*'s behavior. The next result will serve as the building block for our study of how competition affects *agents*' behavior and, more generally, welfare. We study increase in competition in a simple way, by comparing equilibria as the number of firms in the market increases. In the next result we show that, as the number of firms increases, a firm's optimal response to this increase in competition is to specialize, something that in our model is achieved by decreasing precision on valence.

Proposition 3. *As competition increases, firms become less informative about the valence dimension. More specifically, there exists a* $\bar{n} \in \mathbb{N}$ *such that the equilibrium* $\tau^\star(n)$ *is maximal for* $n < \bar{n}$ *and strictly decreasing otherwise.*

The intuition behind this result is closely linked to Remark 2. As n increases the market becomes more competitive and firms compete over an increasingly smaller market share.

[18] More formally, if $(x_i)_{i\in N}$ are the firms' equilibrium locations, any modular rotation $(x_1 \oplus k, \ldots, x_n \oplus k)$ for $k \in \mathbb{R}$ still constitutes an equilibrium.

[19] The mere existence of an equilibrium, in fact, is guaranteed by standard existence results, via Glicksberg's Theorem, for any atom-less distributions of types F.

The market share of each firm has a well-defined structure when mapped into the circle of circumference $\tilde{T}$, namely it is a connected set of types (Figure 5). Graphically, this means that the arc of agents serviced by the firms becomes smaller as n increases. This implies that the preferences of any agent in this arc become increasingly correlated with those of the targeted agent x_i. That is, the firms provide information to a set of agents whose ideological preferences are increasingly aligned. The best response to a market share that becomes increasingly homogeneous is to decrease τ_i, that is to disinvest in valence in favor of ideology. In fact, as n increases, the marginal benefit on profits of decreasing τ_i is higher since $\cos^2(t - x_i)$ is higher for all $t \in T$ such that $r(t,a)_i > 0$ (Proposition 1). Figure 5 illustrate graphically these effects of increased competition on the location of firms.

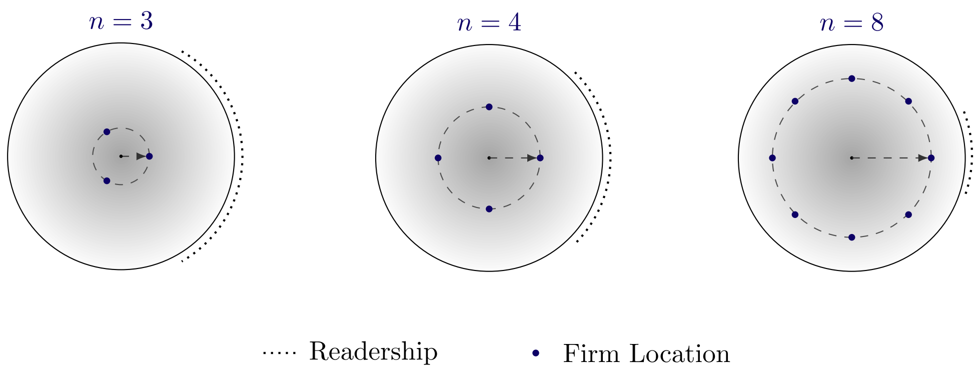

FIGURE 5: The representation of the symmetric equilibrium for several values of n.

It is important to notice that, while decreasing τ_i, the firm increases the value of information for all consumers who are ex-post still acquiring information from firm i. This is the reason for why such disinvestment in valence is a best response for the firm. In this sense, the market does not *over-differentiate*, it never provides "too much" information about ideology relative to the demand of the market share of a given firm. The equilibrium mechanism identified by Proposition 3 can be thought of as an information-theoretic counterpart of classic *product differentiation*. Differentiation is a ubiquitous feature of competition games with heterogeneous consumers. In Proposition 3, we extend this standard result to the realm of information structures. Firms differentiate their products by making the information structure they sell increasingly independent. In the parametrized world of our model, this happens by decreasing the investment on θ_0 and increasing the investment on different combinations of (θ_1, θ_2).

4. Competition, Disagreement and Welfare

In the previous section, we showed that, as competition increases, the equilibrium supply of information on the valence dimension decreases, allowing firms to diversify more aggressively. This result is at the core of the contribution of this paper and, in this section, we will investigate the consequences of such diversification at three different levels. First, we will study whether agents become more or less informed as competition increases. Second, we will study whether competition fosters agreement in the beliefs that agents hold regarding the policy. Third, we will study the overall welfare implications of an increase in competition.

4.1. Competition and the Value of Information

Our main result in this section highlights the *positive* effect of increasing competition in the market for news. We show that competition brings about more information for the agents. This force manifests in our model in three different ways. From an aggregate point of view, increased competition increases the *aggregate* value of information, i.e. the society as a whole becomes better informed. Moreover, from an individual point of view, each agent becomes progressively more informed. Finally, as the market for news becomes infinitely competitive, each agent receives her first-best information structure. We refer to this limit result as the *daily-me* effect, a situation in which every consumer can find an information structure on the market that is exactly tailored towards her needs (Sunstein (2001)).

Proposition 4. *For any sequence of symmetric equilibria* $(a^{\star}(n))_n \in \mathcal{A}^{\infty}$*:*

- (a) *The* total value of information $\mathcal{V}(a^{\star}(n)) := \int_T V(a^{\star}(n)\,|\,t)dF(t)$ *increases in* n*. That is, as competition increases, the society as a whole becomes overall more informed.*
- (b) *For any agent* $t \in T$*, there exists a subsequence* $a^{\star}(n_k)$ *such that* $V(a^{\star}(n_k)\,|\,t)$ *is increasing in* n_k*. That is, as competition increases, every single agent becomes progressively more informed.*
- (c) (Daily-me effect)*. For every agent* $t \in T$*,* $\lim_{n\to\infty} V(a^{\star}(n)\,|\,t) = v^{\star} := \max_{\tau_i} V(\tau_i, x_i = t\,|\,t)$*.*

The first result shows that the aggregate value of information increases in the number of firms. That is, competition brings about more information to the society and, overall, agents appreciate that. The positive effect of competition is, however, stronger than this. Indeed, competition not only increases the value of information in the aggregate, but it progressively increases the value of information agent-by-agent. To formalize this idea, we consider any arbitrary sequence of equilibria for increasing n and show that, for an arbitrary agent $t \in T$, there always is a subsequence along which the equilibrium value of her information increases.[20] Furthermore, as the number of firms in the market grows arbitrarily large, the value of information for each agent $t \in T$ converges to the *same* limit value $v^\star$. This value represents the first-best value of information, the value that would be achieved if an agent t could choose τ_i and x_i to maximize her value of information.

These results shed additional light on the equilibrium forces behind our Proposition 3. The force that push firms to decrease $\tau^\star$ is, in fact, demand-driven. As the number of firms increase, each firm serves an increasingly smaller set of agents and provides them them with a product, an information structure, that is increasingly better suited to their needs, increasing their value of information. This result is in contrast with Mullainathan and Shleifer (2005) which stems from our two main working assumptions: that firms are non-partisan and that consumers are Bayesian. Finally, Proposition 4(c) shows that the market does not *over-differentiate*. That is, even when n grows arbitrarily large, firms lower τ only in so far as this makes their target-consumers better off. Incidentally, this provides intuition for why, as n grows to infinity, no firm has an incentive to deviate away from the symmetric equilibrium and locate back in the center of the disk (Figure 5). As n grows large, in fact, each agents $t \in T$ can find a firm $i \in N$ whose location x_i is arbitrarily close to t, and whose precision on valence $\tau_i^\star(n)$ is arbitrarily close her own first-best. Proposition 4 points out that the inefficiency identified in this paper is not due to some form of market failure. On the contrary, competition enables voters to learn more effectively. However, they learn about different aspects of the state space, i.e. shifting attention to θ_1 and θ_2. This will lead their opinion on the policy to become increasingly independent as we show in the next Section.

Importantly, in Remark 3, we highlight that the limit result of Proposition 4(c), which states that as n grows large, firms maximally specialize and consumers receive their first-best information structure, does not depend on the assumption of uniformity of F.

Remark 3. *Let F be a continuous and strictly increasing distribution on T and $\big(a^\star(n)\big)_n$ be any sequence of equilibria:*

- (c') *For all types $t \in T$, $\lim_{n\to\infty} |x^\star_{r^\star(a^\star(n),t)} - t| = 0$. That is, the distance between type t and the closest firm vanishes in the limit.*
- (c'') (Daily-me effect) *For all types $t \in T$, $\lim_{n\to\infty} V(a^\star(n)\,|\,t) = v^\star := \max_{\tau_i} V(\tau_i, x_i = t\,|\,t)$.*

[20] The use of a subsequence is due to the fact that any rotation of the firms' equilibrium locations, that affects the value of information of a given agent t, is itself an equilibrium of our game. A conceptually equivalent way to capture the same idea is to say that, fixed an agent $t \in T$, the worst-case equilibrium value of information, from agent t's perspective, is increasing in n.

4.2. Competition as the Source of Disagreement

While competition increases the value of information for each agent, different agents become informed about different sub-dimensions of the state space, those that they specifically care about. This has an effect on their approval behavior and, ultimately, on the probability that the policy is implemented. In this section, we analyze these effects and show that increased competition unequivocally increases disagreement in the society. We define social disagreement as the probability $D_n(\theta_0)$ that, conditional on θ_0 and at an arbitrary equilibrium with n-firms, two randomly selected agents $t, t' \sim F$ disagree on whether the policy should be implemented. More formally, fix n and an equilibrium $a^\star(n)$. $z^\star(\theta, t)$ is the equilibrium random variable, describing the voting behavior of agent t as a function of the realized (θ_1, θ_2) and of the idiosyncratic shocks. The approval rate $\Gamma_n(\theta_0) = \mathbb{E}\big(\int_T z(\theta, t) dF(t) \big)$ can be interpreted as the probability that, conditional on θ_0, a randomly selected agent is in favor of implementing the policy. Therefore, we define disagreement as $D_n(\theta_0) := 2\Gamma_n(\theta_0)(1 - \Gamma_n(\theta_0))$. Intuitively, a society features low (high) disagreement, if it is relatively unlikely to find two agents who disagree (agree). Our next result formalizes the idea that more competition leads to higher disagreement.

Proposition 5. *Fix an arbitrary sequence of equilibria $(a^\star(n))_n$. As the number of competing firms n increases, social disagreement $D_n(\theta_0)$ increases. That is, the probability that two randomly selected agents disagree about implementing a policy with valence θ_0 increases.*

To understand this result it is instructive to look at the interim expected utility of an agent $t \in T$ after receiving the signal realization s. For concreteness, we focus attention on an equilibrium $(\tau^\star, x^\star)$ in which agent t is exactly targeted by firm i, i.e. $x_i^\star = t$. Her interim expected utility is then

$$\mathbb{E}\big(u(\theta, t)|s\big) = \lambda g(\tau^\star) s^v(\theta) + (1 - \lambda) g(1 - \tau^\star) s^{id}(\theta).$$

When $\tau^\star = 1$, agent t's approval decision is entirely based on s^v which is identically distributed for all agents $t' \in T$, irrespective of the firm they are gathering information from. Therefore, integrating out the random errors in s^v, the probability that agent t is in favor of θ is entirely determined by θ_0. As n increases, however, $\tau^\star$ decreases (Proposition 3). A rational agent responds by increasing the weight she puts on signal s^{id}, that is now more informative, at the expenses of the weight assigned to s^v, that is now less informative. Crucially, different types, when gathering information from different sources are subject to different distributions of s^{id}. Therefore, their voting decisions assign an increasing weight on signals that are decreasingly correlated with another. For example, when for some agent t signal s^{id} is negative, she demands an increasingly higher, i.e. less likely, realization of s^v in order to overturn her ideological distaste of θ. Vice versa, an agent t' for which s^{id} is positive demands an increasingly lower, i.e. less likely, realization of s^v in order to disapprove policy θ.

Proposition 5 represents a crucial insight of this paper. It is based on two distinct mechanisms that are working together. First, all disagreement is driven by the fact that preferences of the agents are heterogeneous on dimensions (θ_1, θ_2). This implies that, as agents become better informed about (θ_1, θ_2), they will disagree more. And yet, there is no a priori reason for why we should expect agents to become more informed precisely on those dimensions on which they disagree. This is where the second part of the mechanism kicks in. Competition among an increasing number of firms creates incentives to provide more information precisely on those dimensions (θ_1, θ_2) on which agents disagree. Hence, our result in the Proposition above.

4.3. Disagreement and its Welfare Consequences

In this section, we conclude our analysis of the effects of increased competition by studying how increased disagreement affects the welfare of a society. There are multiple reasons for why increased disagreement could create social inefficiencies: deliberation time could be longer, parliamentary representation could become increasingly fragmented, the necessity of compromise could reduce the effectiveness of proposed policies. While our model is silent about the details of the institutional arrangements, we have assumed monotonicity of the implementation rule: a larger public support for a policy translates into a higher probability of implementation.[21] Given this, we shall measure welfare according to two distinct welfare criteria.

Utilitarian Welfare. We begin by assessing the impact of increased competition by measuring the social ex-post welfare of implementing a given policy θ, denoted $W(\theta) := \int_T u(\theta, t)dF(t)$. Since the status quo yields utility that is normalized at zero for all agents, this means that, from a social point of view it is optimal to implement a policy if and only if $W(\theta) \geq 0$. From this perspective, which differs from our analysis in Section 4.1 focusing on individual values, information on different dimensions of the state space potentially generate social externalities. To understand this point in general terms, we temporarily drop our

[21]It is straightforward to provide a natural micro-foundation for this implementation rule. Under a classic majority rule, preferences may be subject to an aggregate interim shock (a political scandal, a terroristic attack, etc.) that sway preferences in favor or against the policy. In such case, $\Gamma(\theta)$ determines the probability that the will of the people is not overturn by a particular realization of the aggregate shock. See Baron (1994) and Grossman and Helpman (1996) for more details.

working assumption of F being uniform and allow for a more general type-distributions. We say that a distribution F on T is *symmetric* around a type $t^\star \in T$ if its density f satisfies $f(t^\star + \delta) = f(t^\star - \delta)$ for all $\delta \geq 0$.[22] In the next result, we analytically compute $W(\theta)$ and establish the existence of a fictitious "representative agent" of our society.

Remark 4. *Let F be symmetric around type $t^\star \in T$. Then,*

$$W(\theta) := \int_T u(\theta, t) dF(t) = \lambda\theta_0 + \beta_F (1 - \lambda)\big(\theta_1 \cos(t^\star) + \theta_2 \sin(t^\star)\big),$$

where $\beta_F := \displaystyle\int_T \cos(t) dF(t) \in [0, 1]$.

The result above illustrates an important feature of our model. For a general class of type-distributions, the social planner evaluates social welfare associated with implementing a given policy in way that is similar to the representative type $t^\star$, namely $u(\theta, t^\star)$, but with an important difference: the scalar β_F. Notice that $\beta_F = 1$ if and only if F is degenerate on a point $t^\star$. In this case, the society is perfectly homogeneous and there are no informational externalities. In all other cases, the planner attaches a strictly higher relative weight to valence compared to any other agent in the society.[23] In a nutshell, no agent in the economy cares about valence as much as the social planner. Valence has "superior" value in the eyes of the social planner because it avoids the kind of trade-offs among types with different preferences in the society that are inevitable with ideology. Any increase in the vector (θ_1, θ_2) will necessarily make some agents better off at the expense of some other agents, and the exact proportions depend on the distribution F. Instead, an increase in θ_0 unambiguously makes everyone better off. The discrepancy between the *social* and *private* value of valence becomes extreme when F is uniform, the case we have so far analyzed. In this case, we have that $\beta_F = \int_T \cos(t) dF(t) = 0$ and the planner cares infinitely more about valence than any other agent in the society. Intuitively, ideology is not valued by the planner because the benefits to any group of agents are exactly offset by losses to another group. This case is instructive because our result becomes extreme: *any* increase in competition leads to a welfare loss.

Proposition 6. *Fix θ_0 and an arbitrary sequence of equilibria $(a^\star(n))_n$. As the number of competing firms n increases, the probability that the collective decision conditional on θ matches the socially optimal one decreases.*

While valence has a superior social value relative to ideology, an increasingly competitive

[22] Note that given how preferences are represented in our model, type $t + 2\pi$ is identical to $t - 2\pi$ and t. Hence, we can always define a symmetric distribution around any $t^\star$.

[23] For example, when $f = U[0, \pi/2]$, we have that $t^\star = \pi/4$ and $\beta_F = \frac{\pi}{2}$.

market creates incentive to disinvest resources away from valence. As shown in Section 4.3, competition increases ideological voting and intensifies social disagreement. This implies that the approval decision of each agent, $z(\theta, t)$, is increasingly less dependent on θ_0. However, as we argued above, θ_0 is the only relevant dimension when the distribution of types is uniform. Therefore, the social planner approves a policy θ if and only if $\theta_0 \geq 0$. By Proposition 3, an increasingly competitive market supplies less information about valence. Hence, agents' approval decisions become increasingly uncorrelated with those of the social planner, generating the inefficiency illustrated above.

Complete Information Benchmark. In the last part of this section, we discuss another result which, while complementing Proposition 6, highlights the inefficiency that pervades our model in a more striking, perhaps even unexpected way. To do so, we focus on a special class of policies, those that Pareto-dominate the status-quo under complete information. More formally, a policy θ Pareto-dominates the status-quo if, $u(\theta, t) \geq 0$ for all $t \in T$. These policies are characterized by a particularly large realization of θ_0 relative to (θ_1, θ_2). A similar definition can be made for policies that are Pareto-dominated by the status-quo. By construction, the set of such policies is non empty and has strictly positive measure. These policies are such that full social agreement would be achieved (i.e. $\Gamma(\theta) = 1$), if the society could perfectly learn the state. We show next that the probability that the society is able to implement even these particularly straightforward class of policies also decrease with competition.

Proposition 7. *Fix a Pareto-dominant policy θ, and for each n assume all equilibria are equally likely. The expected approval rate $\Gamma_n(\theta)$ decreases with n.*

This result is important because it illustrates two distinct features of our model. First, it demonstrates that there is plenty of scope for information to play a positive role in decreasing social disagreement our model. As the class of Pareto-dominant policies shows, it is misleading to think that the complete information benchmark is the worst of all possible worlds. The second feature of our model that Proposition 7 illustrates is that it would be equally misleading to expect competition to bring about "more" information to the agents, thereby pushing the society closer to its complete information benchmark. The result above shows that exactly the opposite is true. In fact, along any sequence of equilibria, each agent $t \in T$ is offered a sequence of information structures that cannot be Blackwell ranked. The society does not move closer to its complete information benchmark. Instead, the inefficiency lies in the fact that the market supplies increasingly more imprecise information about valence. A Bayesian agent reacts by magnifying the weight she attaches to her ideology signal and therefore even ex-post Pareto dominant policies become harder to implement.

On a more technical note, the result in Proposition 7 is formulated *conditional* on the state θ and, a fortiori, conditional on some ideology pair (θ_1, θ_2). When doing so, it is no longer true that our equilibrium multiplicity is immaterial, as the exact sequence of equilibria $(a^\star(n))_n$ matters when computing $\Gamma_n(\theta)$. In Proposition 7 we take care of this issue by taking an expectation over the set of equilibria (which are assumed to be uniformly distributed). In Remark 5, we show that we can always construct an actual equilibrium sequence $(a^\star(n))_n$ along which the approval rate declines.

Remark 5. *Fix a Pareto-dominant policy θ, there always exists a sequence of equilibria $(a^\star(n))_n$ along which the approval rate $\Gamma_n(\theta)$ decreases with n.*

Polarization. We conclude this section with a final comparative static, this time with respect to λ. In our model, λ represents the weight agents put on valence (See Equation 1). For this reason, one can think of $1-\lambda$ as a simple measure of ex-ante "polarization" in the political preferences of the electorate. Our next result finds that in more polarized societies the inefficiency created by competition is even larger.

Remark 6. *Fix $n > 1$. As heterogeneity in agents' preferences increases (a decrease in λ), the expected approval rate decreases.*

Remark 6 shows that the inefficiency associated with competition is exacerbated by polarization in the distribution of political preferences in the society. As polarization increases, demand for information on ideology increases. In a competitive market, firms respond to this demand by shifting precision from valence to ideology.

5. Consuming Multiple News Sources

We have maintained so far the assumption that each agent chooses one and only one information provider. In this section, we relax this assumption and show how our results extend to the case in which agents consume multiple information structures. When agents consume only one product, we showed that each agent chooses to consume the information provider that is "located" closest to her, that is the information structure that is most correlated with her own preferences. This significantly reduced the complexity of the game played by the firms and allowed us to demonstrate the main forces driving our results in a simple and transparent way. When agents consume multiple news sources, a possibly counteracting force is introduced, namely the fact that as n increases agents consume more information. If agents can freely access the products of all n information providers that are competing on the market, then firms are not facing any congestion problem, hence no specialization is expected to occur. In fact, in such case, since *all* agents will consume *all* products, firms won't be actually competing with each other. Yet, assuming that agents can consume *every* signal produced by the market, irrespective of n, is possibly even more extreme than assuming they can only acquire one. More realistically, agents have limitations on how many signals they can process due to time or cognitive constraints, opportunity costs, etc. In this section, we show that our main results generalize to allowing agents to consume an arbitrarily large but finite number of different news sources. We assume there is a cap $\kappa \in \mathbb{N}$ on the number of news sources an agent can consume, and we study how competition affects social outcomes when this constraint is binding, namely when $n \geq \kappa$. In this extension, we assume as before that each agent faces a constraint on the total precision of information available to her which is normalized to 1. This implies that if they can consume κ products, each information provider faces a budget constraint of $1/\kappa$.

Proposition 8. *Let $n \geq \kappa$. A symmetric equilibrium in which agents consume the closest new sources always exists. Moreover, as competition increases, information providers specialize by increasing precision on ideology.*

Once again, firms compete for readership. When agents can consume multiple news sources, the technical challenge that we face is to carefully define the readership for each firm. The normalization on the total precision we have made above imposes constraints on how much information agents are able to extract from κ news sources. This provides a sufficient condition under which agents always pick the firms that are closest to them. That is, the optimal learning strategy entails choosing the κ firms that are individually ranked highest (for type t) in terms of the value associated with the information structure they provide.[24] Once this is established, the game played among the firms can be mapped back to the $\kappa = 1$ case we've solved before with adjustments on how market share is defined. For example, if $n = 8$ and $\kappa = 2$ as shown in Figure 6, each firm will cater to a quarter of the market with neighboring firms serving overlapping shares of the population. However, once these adjustments are made, forces underlying the structure of the symmetric equilibria are identical to the $\kappa = 1$ case. Each firm will choose their reporting strategy to maximize the value of the signals they provide for their most extreme readers - the threshold types that are indifferent between consuming this news source and another. The only difference will be that each news source will effectively be competing over these threshold types with news sources that are κ

[24]Without any constraints on how much information can be transmitted with κ news sources, we can encounter situations where agents optimally choose to consume a different set of news sources. In these examples, agents care about how symmetrically distributed the news sources are around t more than how informative the news sources are individually.

to the right and to the left.

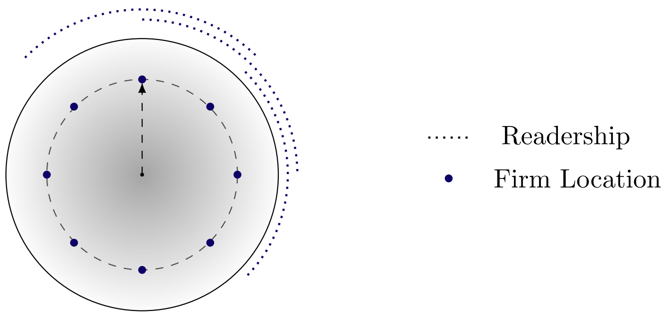

FIGURE 6: The representation of a symmetric equilibrium for $n = 8$ and $\kappa = 2$. Overlapping readership is marked for the three adjacent firms located in the first quadrant.

The competitive tensions that this situation generates are very similar to the $\kappa = 1$ case. In fact, as before, firms will choose precision of their signal on ideology relative to valence depending on how correlated the preferences of their readers are on ideology. For any κ, as n increases the market will be segmented into smaller and smaller groups with more correlated preferences. Consequently, news sources will shift focus to ideological issues, foregoing those consumers that are "far away" from their location, thus creating more value for those that are close by.

6. Discussion and Concluding Remarks

Distribution of preferences. Our model highlights how competition leads to informational specialization. A key insight that our model brings forward is that firms specialize by providing relatively less information on dimensions that are of common interest, namely less information on valence. Our primary goal in the paper has been to demonstrate how this specialization can lead to amplification of social disagreement and analyze its welfare consequences. Of course, the assumptions we made on the structure and the distribution of agents' preferences are only a coarse description of reality, as they intend to capture its most general features. For example, the stark distinction that we assumed between valence and ideology is just a useful theoretical construct, borrowed from the political science literature. In reality, people's preferences may well depend on complex and heterogeneous combinations of many different issues. From this perspective, we think of valence as capturing the principal component of such heterogeneity, a statistical dimension along which agents' preferences are maximally correlated, and with respect to which the residual heterogeneity, which we call ideology, is indeed orthogonal. Our simplified way of modeling the policy space aims at capturing this generic feature of the real world, while making the illustration of the mechanism behind our analysis tractable and our main results particularly transparent.

The most substantive assumptions we have made are those that lead to firms' specialization following an increase in competition. In our model, this result is driven by supply-side constraints on the information structures that can be provided the firms, and by demand-side heterogeneity in agents' preferences.[25] This allows us to capture a world in which, due to external constraints (budget, airtime, attention etc.) and heterogeneous tastes, information needs to be *targeted*. Voters who trade off different dimensions of the policy space in different ways, consequently demand different types of news sources that match their informational needs. Competition leads to specialization, which brings out components of voters' preferences that are heterogeneous. We find this natural and compelling. From this perspective, it is important to notice that, in our model, the news sources do not *create* social disagreement out of the blue. Rather, they uncover and amplify that primitive heterogeneity that's already present in agents' preferences.

Another important simplifying assumption of our model is the uniformity of the type-distribution F. Specifically, this assumption allows us to analytically solve for the equilibrium of our game, for *any* number of competing firms. This allows us to characterize the effects of increased competition for all industry models, spanning the whole space from monopoly to perfect competition. However, as we established in the previous Sections, the details of the type-distribution are not essential for comparing monopoly and perfect competition, two qualitatively important benchmarks. In Remark 3, we showed that, as the market becomes arbitrarily large, firms do specialize irrespective of the details of the types distribution, in fact converging to the agent-preferred information structure (*daily-me effect*). Similarly, in Remark 4 we illustrated that, for any non-degenerate type-distribution, agents care about valence necessarily less than the social planner. Such a wedge will drive the perfectly competitive market to over-provide information on ideology relative to what the social planner would have prescribed.

Competition and the provision of public goods. Our main result relates to the traditional literature on public goods provision. In the context of our model, however, the distinction between what is public or private is more abstract and depends on the *content* of the information provided by the news sources. The literature on public goods has emphasized how competitive markets favor the provision of private goods over public ones, leading to the under-provision of the latter. In our context, a similar force is at work: the heterogeneity

[25]Absent any constraints, all firms would want to fully reveal the state θ. Absent heterogeneity in agents' preferences, all firms want to provide exactly the same information structure, at least in the baseline version of our model. In either case, we don't have specialization.

in voters' preferences gives information providers, who face a shrinking market share, incentives to design informational products that are increasingly more *private*, as they cater more exclusively to a smaller portion of the electorate (Proposition 1). As discussed at the beginning of this Section, the subdivision of the preference space into a public and a private dimension is more general than the specific model introduced in this paper. Therefore, we would expect similar mechanisms, characterized by differentiation of information providers through the oversupply of private information, to have significant consequences in other contexts as well where information is disseminated to a population of agents with heterogeneous preferences. Yet, our model also highlights that the market for political news is unique in that information is consequential to how people vote. Since agents do not take this externality into account, changes in the types of information provided to the voters, purely due to competitive forces, can have significant negative effects on aggregate voting patterns.

Strategic Voting. While solving the agent's problem, we assumed sincere voting. This is a particularly common assumption in the political economy literature and possibly the most realistic description of voters' behavior. It allows us to work with a continuum of voters and to abstract away from the specifics of the electoral rule. Yet, strategic voting could potentially affect our results at multiple levels. First, it could affect the way voters vote given the information they acquired. As a result of this, it could affect how voters value information, and hence which news source they decide to acquire. Finally, given all the above, it could affect the information provision stage in which firms compete with each other. The literature on strategic voting assumes that voters are motivated by instrumental considerations of how their voting behavior can affect the electoral outcome.[26] Importantly, the key insights of the paper, in particular that competition leads to more disagreement, would also apply to a version of the model in which we consider the majoritarian electoral rule in combination with strategic voting. Under the majority rule, an agent's vote is pivotal when there is maximal disagreement within the society on whether the policy should be approved. Conditioning on such an event would be highly informative on the common component of agent's preferences. As a result, a strategic voter would put even more weight on signals that are informative on how her preferences differ from the rest of the society. This reinforces the demand for divisive information, and consequently provides further incentives for product differentiation as a result of competition.

Introducing a government-funded news source. Our main results highlight how competition in the information market can amplify social disagreement by shifting focus from valence

[26] Such effects crucially hinge on the mechanism that maps vote shares into electoral outcome. In this paper, we focused on settings where the *distribution* of votes - and not only who wins the majority - has an impact on agents' welfare. It is easy to see that, in this case, strategic voting moves closer to sincere voting.

issues to ideological issues. Our model demonstrates that this can be a natural consequence of the contrast in the *social* and *individual* value of information on these different dimentions. Profit maximizing firms (news sources in our model) shift their informational products to cater more to individual demands as the market gets more and more segmented. It is interesting to consider what role government-funded news sources that are not affected by competitive forces can play in such an environment. After all, despite the dramatic increase in the number of news sources available to voters, government-funded news sources remain in most countries.[27] For example, we can consider how our results would be affected by the presence of a non-strategic player (a government-funded news source) that uniquely provides information on valence to the agents. While the presence of such player could increase social welfare by moving agents' voting behavior closer to the socially optimal one, we would expect our main results on the effects of competition to still go through. In fact, an increase in competition would still induce firms to decrease their precision on valence, a dimension on which information creates less value for agents due to the presence of the government-funded news source, and provide more information on ideology, triggering the increase in social disagreement.

This simple example also provides some additional context for one of the assumptions we made on agents' preferences. We have assumed that the weight λ that agents puts on valence relative to ideology is type independent.[28] While relaxing this assumption would make the analytical solution to the firms' equilibrium strategies significantly more cumbersome, it would not alter the main tension in our model. Mirroring the government-funded new source example we just discussed, as the number of competing firms increase and the marginal gains from catering to the informational needs of the "moderates" become negligible, *some* firms will to find it profitable to specialize. This would trigger the increase in ideological voting and, therefore, as by Proposition 5, an increase in social disagreement.

While we do not solve these extensions formally, there is clear intuition on how a public news source, by providing information on valence, can play an important role counteracting market forces that emphasize ideological issues.[29] Furthermore, our results also suggest that the role played by public news sources can change with the level of competition in the market. Public news sources have historically been founded on principles that emphasize "universal geographic accessibility," "attention to minorities," "contribution to national identity and

[27] BBC, PBS/NPR, Deutsche Welle, CBC, VOA are some prominent examples.

[28] Relaxing this assumption in the framework of our model would imply the distribution of types to correspond to a distribution on a disk rather than on a circle.

[29] Government funded news sources can also be manipulated and censored more easily. In this discussion, we assumed the public source to be unbiased. We refer the reader to Besley and Prat (2006) for a study of competition and media capture.

sense of community," and "distance from vested interests."[30] As acquisition of political news shifts online and the number of news sources simultaneously available to voters dramatically increases, there is arguably less concern on some of the issues addressed above. Nonetheless, our model demonstrates that, as the level of competition in the market increases, public news sources can have a critical role to play in shaping public opinion by refocusing public discourse on issues which are of relevance to the population as a whole and on which there is general agreement.

[30]These highly referenced principles were first stated by the Broadcasting Research Unit in Britain in 1985.

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A. Proofs

Proof of Remark 1. Fix α_i with $\|\alpha_i\| = 1$ and define $\tau_i := \alpha_{0,i}^2$. The equation $\alpha_{1,i}^2 + \alpha_{2,i}^2 = 1 - \tau_i$ implicitly defines a circle in $(\alpha_{1,i}, \alpha_{2,i})$ that has amplitude $\sqrt{1-\tau_i}$. Therefore, for any given pair $(\alpha_{1,i}, \alpha_{2,i})$ there exists a unique $x_i \in T$ such that $\alpha_{1,i} := \sqrt{1-\tau_i}\cos(t)$ and $\alpha_{2,i} := \sqrt{1-\tau_i}\sin(t)$. To conclude the proof, let s_i and $\tilde{s}_i$ be the information structured induced by α_i and a_i, respectively. Notice that $\tilde{s}_i$ arises as a linear transformation of s_i: $\tilde{s}_i^v = \frac{1}{\sqrt{\tau}_i} s_i^v$ and $\tilde{s}_i^{id} = \frac{1}{\sqrt{1-\tau_i}} s_i^v$. Therefore, the two information structures are equivalent for a Bayesian agent. □

Lemma A1. *Fix an information structure s induced by some $a_i \in A_i$ and a type $t \in T$. The interim expected utility is given by:*

$$\mathbb{E}\big(u(\theta,t)|s\big) = \lambda g(\tau_i)s^v + (1-\lambda)\cos(t - x_i)g(1-\tau_i)s^{id}$$

with $g(\tau_i) = \frac{\tau_i}{1+\tau_i}$.

Proof of Lemma A1. Fix $t \in T$ and $a_i \in A_i$. Recall that $\theta \sim \mathcal{N}(0, I_3)$, where I_3 is the identity matrix. We consider signals defined as $s^v(\theta) = \theta_0 + \frac{1}{\sqrt{\tau_i}}\varepsilon_i^v$ and $s^{id}(\theta) = \theta_1\cos(x_i) + \theta_2\sin(t) + \frac{1}{\sqrt{1-\tau_i}}\varepsilon_i^{id}$. We can explicitely compute this interim expectation:

$$\begin{aligned}\mathbb{E}\big(u(\theta,t)|s\big) &= \mathbb{E}\Big(\lambda\theta_0 + (1-\lambda)\big(\theta_1\cos(t) + \theta_2\sin(t)\big)\big|s\Big)\\ &= \lambda\mathbb{E}\big(\theta_0|s^v\big) + (1-\lambda)\cos(t)\mathbb{E}\big(\theta_1|s^{id}\big) + (1-\lambda)\sin(t)\mathbb{E}\big(\theta_2|s^{id}\big).\end{aligned}$$

where we used independence of $(\theta_0, \theta_1, \theta_2)$. Computing these conditional expectations and letting $g(\tau_i) := \frac{\tau_i}{1+\tau_i}$ we get:

$$\mathbb{E}\big(\theta_0|s^v\big) = g(\tau_i)s_{\mathrm{v}} \qquad \mathbb{E}\big(\theta_1|s^{id}\big) = \cos(x_i)g(1-\tau_i)s_{\mathrm{id}} \qquad \mathbb{E}\big(\theta_2|s^{id}\big) = \sin(x_i)g(1-\tau_i)s_{\mathrm{id}}.$$

Using the trigonometric identity $\cos(t)\cos(x_i) + \sin(t)\sin(x_i) = \cos(t - x_i)$ and putting everything together concludes the proof. □

Proof of Proposition 1. Let X be a random variable distributed according to $X \sim \mathcal{N}(0, \sigma^2)$. We have that:

$$\mathbb{E}\big(\max\{0, X\}\big) = \frac{1}{2}\mathbb{E}\big(X\big|X \geq 0\big) = \frac{\sigma}{\sqrt{2\pi}}.$$

Now fix some $a_i \in A_i$ and let $s = (s^v, s^{id})$ be the information structure that is induced by it. Also, let $X := \mathbb{E}(u(\theta,t)|s)$. From Lemma A1, we know that $X = \lambda g(\tau_i)s^v + (1-\lambda)\cos(t - x_i)g(1-\tau_i)s^{id}$. That is, X is the sum of independent normally distributed random variables. By definition, we have $s^v \sim \mathcal{N}(0, 1 + \frac{1}{\tau_i})$ and $s^{id} \sim \mathcal{N}(0, 1 + \frac{1}{1-\tau_i})$, where we used the trigonometric identity $\cos(x_i)^2 + \sin(x_i)^2 = 1$. Therefore, $X \sim \mathcal{N}(0, \sigma^2)$ where

$$\sigma^2 := \lambda^2 g(\tau_i) + (1-\lambda)^2\cos^2(t - x_i)g(1-\tau_i).$$

Therefore, $V(a_i|t) = \frac{\sigma}{\sqrt{2\pi}}$. $\square$

Proof of Proposition 2. When $n = 1$, the firm can choose $\tau_1 = 1$ and select an arbitrary location x_1. In such way, the value produced by $a_1 = (\tau_1, x_1)$ is equal to $V(a_1|t) = \frac{\lambda}{2\sqrt{\pi}} > 0$ for all $t \in T$. Therefore, all agents acquire information from the firm. Profits are maximized, $\Pi_1(a_1) = 1$, and the firm has no incentive to deviate. This equilibrium is trivially symmetric since there is only one firm.

Now let $n \geq 2$. First we construct a candidate symmetric equilibrium and, then, we prove it is indeed a symmetric equilibrium. With no loss of generality, we focus on firm $i = 1$ and normalize its location in the candidate equilibrium strategy profile to $x_1 = 0$. Condition $|x_i - x_j| \geq \pi/n$ for all $i, j \in N$ pins down exactly the location of all other firms in this candidate symmetric equilibrium. Specifically, firm $i \in \{1, \ldots, n\}$ is located at $x_i^\star = \frac{(i-1)\pi}{n}$. In our candidate symmetric equilibrium, all firms but $i = 1$ play a $\tau \in [0, 1]$. Firm $i = 1$ instead plays some, possibly identical, τ_1, and maximize its profit conditional on all other firms playing τ. Denote $t_r \in T$ (resp. $t_l \in T$) to be the unique solution of $V(a_1|t_r) = V(a_2|t_r)$ (resp. $V(a_1|t_l) = V(a_n|t_l)$), namely the type indifferent between acquiring information from either firm 1 and firm 2 (resp. n). The problem of the firm is equivalent to maximizing $t_r - t_l \geq 0$. The first-order condition is $\frac{\partial t_r}{\partial \tau_1} = \frac{\partial t_l}{\partial \tau_1}$. By definition of t_r, we have that

$$\frac{\partial}{\partial \tau_1}\Big(V\big((\tau_1, x_1^\star)|t_r\big) - V\big((\tau^\star, x_2^\star)|t_r\big)\Big)\Big|_{\tau_1 = \tau} = 0$$

This equilibrium condition allows us to retrieve an expression for $\frac{\partial t_r}{\partial \tau_1}$. In fact

$$\frac{\partial t_r}{\partial \tau_1} = \frac{\lambda^2 g'(\tau_1) - (1-\lambda)^2 \cos^2(t_r - x_1) g'(1-\tau_1)}{(1-\lambda)^2\big(g(1-\tau_1)\sin 2(t_r - x_1) + g(1-\tau^\star)\sin 2(x_2^\star - t_r)\big)}$$

Similarly, from $V(\tau_1, x_1^\star|t_l) - V(\tau^\star, x_n^\star|t_l) = 0$, we can retrieve an expression for for $\frac{\partial t_l}{\partial \tau_1}$:

$$\frac{\partial t_l}{\partial \tau_1} = -\frac{\lambda^2 g'(\tau_1) - (1-\lambda)^2 \cos^2(x_1 - t_l) g'(1-\tau_1)}{(1-\lambda)^2\big(g(1-\tau_1)\sin 2(x_1 - t_l) + g(1-\tau^\star)\sin 2(t_l - x_n^\star)\big)}$$

By symmetry, $t_l - x_n^\star = x_2^\star - t_r$ and $t_r - x_1^\star = x_1^\star - t_l$. Therefore, $\lambda^2 g'(\tau_1) = (1-\lambda)^2 \cos^2(t_r - x_1) g'(1-\tau_1)$, or

$$\frac{\lambda^2}{(1-\lambda)^2}\frac{g'(\tau_1)}{g'(1-\tau_1)} = \cos^2(t_r - x_1)$$

Since our candidate equilibrium is symmetric, it must be that $\tau_1 = \tau^\star$ and $t_r - x_1 = \frac{\pi}{2n}$, that is

$$\frac{\lambda^2}{(1-\lambda)^2}\frac{g'(\tau^\star)}{g'(1-\tau^\star)} = \cos^2\Big(\frac{\pi}{2n}\Big). \tag{3}$$

The left-hand side is strictly decreasing in $\tau^\star$ and the right-hand side is constant. Therefore, there exists a unique solution $\tau^\star$ to the equation above that. Together with $x^\star$ defined above, the pair $a^\star = (\tau^\star, x^\star)$ constitute our candidate symmetric equilibrium. Importantly, not only $\tau^\star$ is unique, but it is derived independently of the specific locations $x^\star$ chosen for our candidate equilibrium. That is, once we verify that $a^\star$ is indeed an equilibrium, uniqueness up to modular rotations of the locations $x^\star$ will follow. Notice that, by construction, we have already shown that at such a candidate symmetric equilibrium firm 1, and a fortiori any firm i, has no incentive to unilaterally deviate to any other $\tau_1' \neq \tau^\star$. To prove, $(\tau^\star, x^\star)$ is an equilibrium it remains to be shown that firm 1 has no profitable deviation neither on x_1 nor on (x_1, τ_1).

In equilibrium, it must be that $\frac{\partial}{\partial x_1} V(\tau_1, x_1|t_r) = \frac{\partial}{\partial x_1} V(\tau^\star, x_2^\star|t_r)$. Therefore, we can derive expressions for $\frac{\partial t_r}{\partial x_1}$. Indeed, we get

$$(1-\lambda)^2 g(1-\tau_1)\sin 2(t_r - x_1)\big(1 - \frac{\partial t_r}{\partial x_1}) = (1-\lambda)^2 g(1-\tau_2)\sin 2(x_2 - t_r)\frac{\partial t_r}{\partial x_1}$$

giving us the following expression:

$$\frac{\partial t_r}{\partial x_1} = \frac{g(1-\tau_1)}{g(1-\tau_1) + \psi_r g(1-\tau_2)}, \quad \text{with } \psi_r := \frac{\sin 2(x_2 - t_r)}{\sin 2(t_r - x_1)}.$$

Similarly, we can get

$$\frac{\partial t_l}{\partial x_1} = \frac{g(1-\tau_1)}{g(1-\tau_1) + \psi_l g(1-\tau_n)}, \quad \text{with } \psi_l := \frac{\sin 2(t_l - x_n)}{\sin 2(x_1 - t_l)}.$$

In a symmetric equilibrium, thresholds t_r and t_l fall at the midpoints between locations, namely $t_r = (x_1 + x_2)/2$ and $t_r = (x_1 + x_n)/2$. This implies that $\psi_r = \psi_l = 1$ and therefore $\frac{\partial t_r}{\partial x_1} - \frac{\partial t_l}{\partial x_1} = 0$. Thus, firm 1, and a fortiori any firm i, does not strictly gain by locating itself away from $x_1^\star$.

It remains to be shown that there are no joint deviations in τ_i *and* x_i that could make firm i better off. We do this in the next two claims.

Claim 1. *For all $\tau_1 > \tau^\star$ and locations x_1, $\Pi(a_1', a_{-i}^\star) < \pi/n$.*

Fix $\tau_1 > \tau^\star$ and $x_1 > x_1^\star$ (the case in which $x_1 < x_1^\star$ is symmetric). Consider the type $\tilde{t} := (x_1 + x_2^\star)/2$ which is midway between x_1 and $x_2^\star$. We want to show that $\tilde{t}$ does prefer $i+1$ to i. Notice that since $x_1 > x_1^\star$ and, by Definition 4.1, $x_2^\star - x_1^\star = \pi/2n$, we have that $\tilde{t} - x_1 = x_2^\star - \tilde{t} < \pi/2n$. By construction, $\tau^\star$ is the optimal level of valence for a type t who is $\pi/2n$-away from the information provider. All types that are closer than $\pi/2n$ would prefer less valence. Thus, $\tilde{t}$ strictly prefers firm $i+1$ since, compared with firm i, it offers a lower level of valence, $\tau^\star < \tau_1$. We conclude that $x_2^\star - t_r > t_r - x_1 > 0$, hence $\psi_r > 1$. Now let's consider t_l. If it is such that $t_l - x_n > x_1 - t_l$ then firm i's profits are necessarily less than $\pi/2n$. Thus, the only case we need to consider is the one in which $t_l - x_n < x_1 - t_l$. In this case, $\psi_l < 1$. Summing up, we have that $\psi_r > 1$ and $\psi_l < 1$, implying that $\frac{\partial t_r}{\partial x_1} - \frac{\partial t_l}{\partial x_1} < 0$.

Claim 2. *For all $\tau_1 < \tau^\star$ and all locations x_1, firm 1's profit are smaller than π/n.*

Fix $\tau_1 < \tau^\star$ and $x_1 > x_1^\star$ (the case in which $x_1 < x_1^\star$ is symmetric). There are two subcases to consider here. Either the left threshold type t_l is indifferent between firm 1 and n (as it was in the previous Claim), or it is indifferent between firm 1 and 2. This second case is possible because firm 1 is now deviating to a lower τ_1 than its neighbors. On the other side, the right threshold t_r will always correspond to a type who is indifferent between firm 1 and 2.

Subcase 1: Let's assume t_l is indifferent between 1 and n. A similar argument to the one in the Claim above will show that $t_l - x_n^\star > x_1 - t_l > 0$. In fact the midpoint $\tilde{t} := (x_1 + x_n^\star)/2$ is now more than $\frac{\pi}{2n}$-away from both x_1 and $x_n^\star$. Thus she would prefer more valence than $\tau^\star$. Since $\tau_1 < \tau^\star$, type $\tilde{t}$ prefers x_n. This shows $t_l - x_n^\star > x_1 - t_l > 0$ and implies that $\psi_l > 1$. Now we look at t_r. Once again, either *(a)* firm 1 is conquering more than half of the market between 1 and 2, i.e. $x_2 - t_r < t_r - x_1$ or *(b)* firm 2 does, i.e. $x_2 - t_r > t_r - x_1$. If *(b)* is the case, then firm 1's profits are necessarily less than π/n and we are done. If *(a)* holds, instead, $\psi_r < 1$ and therefore $\frac{\partial t_r}{\partial x_1} - \frac{\partial t_l}{\partial x_1} > 0$. Since $x_1 \in [x_1^\star, x_2^\star]$ was arbitrary, we proved that the derivative of profits is strictly increasing in such region. Thus, firm 1 will keep increasing x_1, getting closer and closer to x_2. Eventually, firm x_1 will locate in the same spot of x_2, but with a lower τ_1. Thus the threshold type t_l will be no longer indifferent between firm 1 and n, but rather with firm 1 and 2. This is Subcase 2, which we analyze next.

Subcase 2: Let's assume t_l is indifferent between 1 and 2. It must be that t_l is closer to 2 than n. If not, t_l should prefer n to 2, a contradiction. Now consider $\tilde{t} = \frac{x_2^\star + x_{i+2}^\star}{2}$, which is the midpoint between firm 2 and $i+2$. Notice that since $x_1 \in [x_1^\star, x_2^\star]$, $\tilde{t} - x_2^\star \geq \tilde{t} - x_i$. Since firm 1, relative to firm 2, is offering lower valence τ_1 and it is weakly farther away to $\tilde{t}$, then such type will prefer firm 2 to 1. Since by construction $\tilde{t} - t_l \leq \pi/n$, firm 1 's profit are lower than π/n.

This shows that the candidate symmetric strategy profile $a^\star = (\tau^\star, x^\star)$ is indeed a symmetric equilibrium of our game.

Now we show non-existence of asymmetric equilibria. We'll make use of the following lemma.

Lemma A2. *Readership for all firms in equilibrium is a connected interval.*

Proof. Assume for contradiction that there exists a firm i such that there exists an interval $[\underline{x}, \bar{x}]$ and $\epsilon > 0$, such that i has no readers in the interval, but it's readership includes $[\underline{x} - \epsilon, \underline{x})$, and $(\bar{x}, \bar{x} + \epsilon]$. There has to be at least two firms targeting voters in $[\underline{x}, \bar{x}]$. (If there was only one firm, it would increase τ.) We can also assume that these firms have readership that is a connected interval. Otherwise we would focus on one of them and relabel i. Wlog assume that x_k is weakly to the left of $\frac{\underline{x}+\bar{x}}{2}$ Pick the firm in $[\underline{x}, \bar{x}]$ most to the right. Denote it's readership as $[x - \delta_l, x - \delta_r]$. Note that moving x the right without changing τ would increase readership for this firm which contradicts the equilibrium assumption. $\square$

Now assume for contradiction that an asymmetric equilibrium exists where firms choose different τ values. Then there has to be a firm 0 such that $\tau_0 \leq \tau_i$ for all i and $\tau_i > \tau_0$ for either it's neighbor to the right or to the left. Denote the strategy of this firm as (x_0, τ_0). Our proof strategy is to construct a profitable deviation for either firm 0 or one of it's neighbors.

Denote 0's readership as $[x_0 - \delta_l, x_0 + \delta_r]$. Since τ_0 is chosen in equilibrium, it must be that $\frac{\partial \delta_l}{\partial \tau_0} \leq 0$ or $\frac{\partial \delta_r}{\partial \tau_0} \leq 0$. Otherwise, τ_0 would have been higher. Note that both of these cannot be 0, because that would imply the neighbor with the higher τ to be closer to the threshold type, but then firm 0 would definitely deviate to locate where this firm is located and choose a τ slightly lower than this firm's and increase it's readership.

Hence, we can assume wlog that $\frac{\partial \delta_r}{\partial \tau_0} < 0$. Denote the strategy of the closest firm to the right as (x_1, τ_1). There are two cases to consider. (1) $\tau_0 = \tau_1$; (2) $\tau_0 < \tau_1$. Denote the strategy of the closest firm to the left as (x_{-1}, τ_{-1}). Let $d_{-1} = x_0 - x_{-1}$ and $d_1 = x_1 - x_0$.

Case 1: $(\tau_0 = \tau_1)$ Note that by assumption $\tau_{-1} > \tau_0$. Since threshold types $x_0 - \delta_l$ and $x_0 + \delta_r$ are indifferent between firms the following holds.

$$\lambda^2 g(\tau_0) + (1-\lambda)^2 cos^2(\delta_l) g(1-\tau_0) = \lambda^2 g(\tau_{-1}) + (1-\lambda)^2 cos^2(d_{-1} - \delta_l) g(1-\tau_{-1})$$

$$\lambda^2 g(\tau_0) + (1-\lambda)^2 cos^2(\delta_r) g(1-\tau_0) = \lambda^2 g(\tau_1) + (1-\lambda)^2 cos^2(d_1 - \delta_r) g(1-\tau_1)$$

Note that moving x_0 for example to the right implies a decrease in d_1 and an equivalent increase in d_{-1}. If we differentiate these equations with respect to d_1 ad d_{-1} accounting for how δ_r and δ_l change respectively, we get

$$\frac{\partial \delta_r}{\partial d_1} = \frac{1}{1 + \frac{\cos(\delta_r) sin(\delta_r) g(1-\tau_0)}{\cos(d_1-\delta_r) sin(d_1-\delta_r) g(1-\tau_1)}}$$

$$\frac{\partial \delta_l}{\partial d_{-1}} = \frac{1}{1 + \frac{\cos(\delta_l) sin(\delta_l) g(1-\tau_0)}{\cos(d_{-1}-\delta_l) sin(d_{-1}-\delta_l) g(1-\tau_{-1})}}$$

In equilibrium it must be that

$$\frac{\partial \delta_r}{\partial d_1} = \frac{\partial \delta_l}{\partial d_{-1}}$$

which implies

$$\frac{\cos(\delta_r) sin(\delta_r) g(1-\tau_0)}{\cos(d_1-\delta_r) sin(d_1-\delta_r) g(1-\tau_1)} = \frac{\cos(\delta_l) sin(\delta_l) g(1-\tau_0)}{\cos(d_{-1}-\delta_l) sin(d_{-1}-\delta_l) g(1-\tau_{-1})}$$

which can be rewritten as

$$\frac{\cos(\delta_r) sin(\delta_r)}{\cos(d_1-\delta_r) sin(d_1-\delta_r)} = \frac{\cos(\delta_l) sin(\delta_l) g(1-\tau_0) g(1-\tau_1)}{\cos(d_{-1}-\delta_l) sin(d_{-1}-\delta_l) g(1-\tau_{-1})}$$

Since $\tau_1 = \tau_0 < \tau_{-1}$, it must be that $\frac{g(1-\tau_1)}{g(1-\tau_{-1})} > 1$, but since $\tau_1 = \tau_0$, $\delta_r = d_1 - \delta_r$, which implies $\frac{\cos(\delta_r)sin(\delta_r)}{\cos(d_1-\delta_r)sin(d_1-\delta_r)} = 1$. But then it must be that $\frac{\cos(\delta_l)sin(\delta_l)g(1-\tau_0)}{\cos(d_{-1}-\delta_l)sin(d_{-1}-\delta_l)} < 1$. This implies $\delta_l < d_{-1} - \delta_l$. This means that firm 0 is getting less than half of $d_1 + d_{-1}$. Consider the following deviation for firm 0: $(x_{-1} + \frac{d_1+d_{-1}}{2}, \tau')$ where τ' is the optimal τ for a type who is $\frac{d_1+d_{-1}}{4}$ far from the firm. This will clearly guarantee readership at least equal to $\frac{d_1+d_{-1}}{2}$ which generates the contradiction.

Case 2: ($\tau_0 < \tau_1$) We'll replicate the same proof above relabeling firm 1 as firm 0, and 0 as -1. The following condition should still hold.

$$\frac{\cos(\delta_r)sin(\delta_r)}{\cos(d_1-\delta_r)sin(d_1-\delta_r)} = \frac{\cos(\delta_l)sin(\delta_l)g(1-\tau_0)g(1-\tau_1)}{\cos(d_{-1}-\delta_l)sin(d_{-1}-\delta_l)g(1-\tau_{-1})}$$

$\delta_l < d_{-1} - \delta_l$ and $\tau_1 \geq \tau_{-1}$ imply that the left hand side less than 1, which implies that the right side is also less than 1, and hence $\delta_r < d_1 - \delta_r$, so a similar profitable deviation can be constructed.

□

Lemma A3. *For each type $t \in T$ and sequence of equilibria $(a^\star(n))_{n\in\mathbb{N}} \in A^\infty$ there exist increasing sequences $\underline{V}_t(n)$ and $\bar{V}_t(n)$ such that, for all $n \in \mathbb{N}$, $\underline{V}_t(n) \leq V(a_{i_n(t)}|t) \leq \bar{V}_t(n)$ and $\lim_n \underline{V}_t(n) = \lim_n \bar{V}_t(n) = v^\star := \max_\tau V(\tau, t\,|\,t)$.*

Proof. Let $(a^\star(n))_{n\in\mathbb{N}} \in A^\infty$ be a sequence of symmetric equilibria of the competition game and fix an arbitrary type $t \in T$. In this proof, we shall denote $i_n(t) \in N$ the label of the firm chosen by type t at equilibrium $a^\star(n)$. Because any modular rotation of equilibrium locations $x^\star(n)$ still constitutes an equilibrium at n, there exists two sequences of equilibria $(\bar{a}^\star(n))_n$ and $(\underline{a}^\star(n))_n$, that depend on the type t and have the following two properties: (1) for each $n \in \mathbb{N}$, the uniqueness part in Proposition 2 implies that equilibria $a^\star(n)$, $\bar{a}^\star(n)$ and $\underline{a}^\star(n)$ have the same component $\tau^\star(n)$; (2) for each $n \in \mathbb{N}$, $\bar{a}^\star(n)$ is such that $x_i = t$, for some $i \in N$; (3) for each $n \in \mathbb{N}$, $\underline{a}^\star(n)$ is such that type t is indifferent between i and $i+1$, for some $i \in N$. For each $n \in \mathbb{N}$, denote $\underline{V}_t(n)$ and $\bar{V}_t(n)$ the value of information for type t at equilibrium $\underline{a}^\star(n)$ and $\bar{a}^\star(n)$, respectively. For all $n \in \mathbb{N}$, we have that $\underline{V}_t(n) \leq V(a_{i_n(t)}|t) \leq \bar{V}_t(n)$. To see this, fix $n \in \mathbb{N}$. By construction, in equilibrium $\bar{a}^\star(n)$, $x_i = t$, hence in the expression for $\bar{V}_t(n)$ we have a term, $\cos^2(t-t) = 1 \geq \cos^2(t - x_{i_n(t)})$. Since $\tau^\star(n)$ is the same under both $\bar{a}^\star(n)$ and $a^\star(n)$, we conclude that $V(a_{i_n(t)}|t) \leq \bar{V}_t(n)$. Similarly, by construction of $\underline{a}^\star(n)$, type t is indifferent between two firms and, therefore, $\cos^2(t - x_i) = \cos^2(\frac{\pi}{2n}) \leq \cos^2(t - x^\star_{i_n(t)})$. Since $\tau^\star(n)$ is the same under both $\underline{a}^\star(n)$ and $a^\star(n)$, we conclude that $\underline{V}_t(n) \leq V(a_{i_n(t)}|t)$. Next, we show that sequence $\underline{V}_t(n)$ is increasing in n. Showing that $\underline{V}_t(n)$ is increasing amounts to show that, the value of information for an indifferent type, type $t \in T$ in our construction, is increasing when going from n to $n+1$. Suppose not, that is suppose $\underline{V}_t(n) > \underline{V}_t(n+1)$. Fix a firm i and its location $x_i \in T$ in both n and $n+1$. Denote $\tilde{t}(n)$ and $\tilde{t}(n+1)$ to be the indifferent types to firm i at n and $n+1$. Without loss of generality assume that $\tilde{t}(n), \tilde{t}(n+1) \leq x_i$. This implies that $\tilde{t}(n) \leq \tilde{t}(n+1)$. Notice that the value of information is always decreasing in the distance between a type and its firm. This implies that at $\tau^\star(n)$, type $\tilde{t}(n+1)$ is better off than at $\tau^\star(n+1)$. Since, by definition, $\tilde{t}(n+1)$ is indifferent between i and $i+1$, firm i can deviate from $\tau^\star(n+1)$, increasing its precision on valence, and strictly gain from this deviation, a contradiction. Therefore, $\underline{V}_t(n) \leq \underline{V}_t(n+1)$. Next, we show that also the sequence $\bar{V}_t(n)$ is increasing in n. This amounts to show that, as $\tau^\star(n)$ decreases, the value of information of the targeted type, i.e. $x_i = t$ is increasing. Suppose this is not the case. That is suppose that $\bar{V}_t(n) > \bar{V}_t(n+1)$. This implies that $V(\tau^\star(n), t \,|\, t) > V(\tau^\star(n+1), t \,|\, t)$. This is can be the case if and only if $\lambda^2 g(\tau^\star(n)) + (1-\lambda)^2 g(1-\tau^\star(n)) > \lambda^2 g(\tau^\star(n+1)) + (1-\lambda)^2 g(1-\tau^\star(n+1))$, which can be the case if and only if, $\tau^\star(n) \leq 3\lambda + 1 =: \arg\max V(\tau, t \,|\, t)$. This contradicts the assumption that $a^\star(n)$ is an equilibrium, since if $\tau^\star(n) \leq 3\lambda+1$, firm i can deviate to a $\tau' > \tau^\star(n)$ and increase the value of information for the indifferent type. Therefore, we conclude that $\bar{V}_t(n)$ is increasing in n. Next, we show that $\lim_n \underline{V}_t(n) = \lim_n \bar{V}_t(n)$. First, notice that both sequence converge to some limit point, since they are increasing and bounded by 1. Second, suppose these two limits point are such that $\lim_n \bar{V}_t(n) - \lim_n \underline{V}_t(n) > \epsilon$. This implies that for arbitrarily large $n \in \mathbb{N}$, $\bar{V}_t(n) - \underline{V}_t(n) > \epsilon$, which, in turn, implies that there exists a $\delta > 0$, independent of n, such that $\cos^2(\pi/2n) < 1-\delta$. Since n was chosen arbitrarily, we have a contradiction. We conclude that $\epsilon = 0$, that is the sequences have the same limit point, denoted $v^\star$. Finally, by solving Equation 3 for τ, we know that as $n \to \infty$, $\tau^\star(n) \to 3\lambda + 1$. We conclude that the limit point $v^\star$ of the sequences $\underline{V}_t(n)$ and $\bar{V}_t(n)$ is the agent t's preferred τ when $x_i = t$, that is $v^\star = \max_\tau V(\tau, t \,|\, t)$. $\square$

Proof of Proposition 4. Consider a sequence of symmetric equilibria $(a^\star(n))_{n\in\mathbb{N}} \in A^\infty$ and an arbitrary type $t \in T$. For each $n \in \mathbb{N}$, denote $V_t(n)$ the value of information for agent t at equilibrium $a^\star(n)$.

$(a.)$ Fix $n \in \mathbb{N}$ and consider a firm $i \leq n$. It is without loss of generality to assume that for both n and $n+1$ firm i is located at zero, that is $x_i^\star(n) = x_i^\star(n+1) = 0$. This is true because modular rotations of equilibrium locations constitute equivalent equilibria (Proposition 2). Thus, firm i provides information to agents of type $t \in T_i(n+1) := [-\frac{\pi}{2n}, \frac{\pi}{2n}]$ at equilibrium $a^\star(n)$, and to agents of type $t \in T_i(n+1) := [-\frac{\pi}{2(n+1)}, \frac{\pi}{2(n+1)}]$ at equilibrium $a^\star(n)$. Clearly, $T_i(n+1) \subset T_i(n)$. For all types $t \in T_i(n+1)$, we have that $V(\tau^\star(n), x_i = 0 \,|\, t) \leq V(\tau^\star(n+1), x_i = 0 \,|\, t)$. To see that, we compute the solution to the maximization problem $\max_{\tau\in[0,1]} V(\tau, x_i = 0 \,|\, t = \frac{\pi}{2(n+1)})$. This pins down agent $t = \frac{\pi}{2(n+1)}$'s preferred τ, given that she receives information from firm i, with $x_i = 0$. The first-order condition of this maximization problem is:

$$\frac{\lambda^2}{(1-\lambda)^2} \frac{g'(\tau)}{g'(1-\tau)} = \cos^2\left(\frac{\pi}{2(n+1)}\right).$$

This equation is also the equilibrium condition we have derived in Equation 3 (Proposition 2), evaluated at $n+1$. This implies that agent $t = \frac{\pi}{2(n+1)}$'s preferred τ, given that she receives information from firm i, is precisely $\tau^\star(n+1)$, the equilibrium precision on valence when $n+1$. Therefore, agent $t = \frac{\pi}{2(n+1)}$'s value of information, conditional on receiving information from firm i, is maximized at $\tau^\star(n+1)$. A fortiori, all agents $t \in T_i(n+1)$, we have that $V(\tau^\star(n), x_i = 0 \,|\, t) \leq$

$V(\tau^{\star}(n+1), x_i = 0 \,|\, t)$. Now consider the agents that are served by firm i at $a^{\star}(n)$, but not at $a^{\star}(n+1)$. These are types in the set $T_i(n) \setminus T_i(n+1)$. Fix $\bar{t}$, one of such types. Notice that $V(\tau^{\star}(n), x_i = 0 \,|\, \bar{t}) \leq V(\tau^{\star}(n), x_i = 0 \,|\, t = \frac{\pi}{2(n+1)}) \leq V(\tau^{\star}(n+1), x_i = 0 \,|\, t')$ for all $t' \in T_i(n+1)$. At $a^{\star}(n+1)$, type $\bar{t}$ will receive information from a firm $j \neq i$ and, by definition of the symmetric equilibrium, $|\bar{t} - x_j| \leq \frac{\pi}{2n}$. Therefore, there exists some $t \in T_i(n+1)$, such that $V(\tau^{\star}(n+1), x_i = 0 \,|\, \bar{t}) = V(\tau^{\star}(n+1), x_i = 0 \,|\, t)$. Therefore, $V(\tau^{\star}(n+1), x_i = 0 \,|\, \bar{t}) \geq V(\tau^{\star}(n), x_i = 0 \,|\, \bar{t})$. We conclude that all types $t \in T_i(n)$ are better off at $a^{\star}(n+1)$ relative to $a^{\star}(n)$. Since the identity of firm i was chosen arbitrary, this also concludes the proof.

$(b.)$ Fix an arbitrary $n_k \in \mathbb{N}$. We want to show that there is a $n_{k+1} > n_k$ such that $V_t(n_{k+1}) \geq V_t(n_k)$. Suppose not. That is, for all $n > n_k$, $V_t(n) < V_t(n_k)$. By Lemma A3, we have that, for all $n \geq n_k$, $\underline{V}_t(n) \leq V_t(n) < V_t(n_k) \leq \bar{V}(n_k)$. This implies that the sequence $\underline{V}_t(n)$ is bounded away from $\lim_n \bar{V}_t(n)$, a contradiction. Therefore, there exists a $n_{k+1} > n_k$ such that $V_t(n_{k+1}) \geq V_t(n_k)$. By the induction principle, we can construct a subsequence $(V_t(n_k))$ that is increasing.

$(c.)$ By Lemma A3, we have that $\underline{V}_t(n) \leq V_t(n) \leq \bar{V}(n)$ for all $n \in \mathbb{N}$ and that $\lim_n \underline{V}_t(n) = \lim_n \bar{V}(n) = v^{\star}$. These two facts imply that $\lim_n V_t(n) = v^{\star}$. □

Proof of Remark 3: Define Δ_n in the following way. $\min_{t\in T} F(t+\Delta_n) - F(t-\Delta_n) = \frac{1}{n-1}$. Clearly assumption on F imply $\Delta_n > 0$ for all n and $\lim_{n\to\infty} \Delta_n = 0$.

Fix any t. Let δ_n be the closest firm when there are n firms. We show that $\delta_n < 2\Delta_n$. Assume not for contradiction. There has to be at least one firm in equilibrium whose readership is weakly less than $\frac{1}{n}$ in equilibrium. Consider this firm deviating to locate at t and choosing the optimal τ targeting types $t - \Delta_n$ and $t + \Delta_n$. By construction, all types between them would strictly prefer this firm, which means that the firm can guarantee readership that is weakly higher than $\frac{1}{n-1}$ which generates the contradiction. Hence we have shown that farthest firm must be closer than $2\Delta_n$ at any n which is sufficient for the result.

Fix any t. We show that $V(a^{\star}(n) \;\; t) > V_n$ for some sequence of $(V_n)_n$ such that $\lim_{n\to\infty} V_n = 0$. Let a^t denote the best information structure for type t such that $V(a^t \,|\, t) = v^{\star}$. Since a_n is an equilibrium, it must be that no firm wants to deviate to a^t. This means that deviating to a^t cannot guarantee readership for the firm from $[t - \Delta_n, t + \Delta_n]$ (otherwise there would have to be a firm that would deviate as it would guarantee $\frac{1}{n-1}$ readership which would clearly be profitable.) This means that types $t - \Delta_n$ and $t + \Delta_n$ have an information structure that creates value for them more than what a^t would, $V(a^t \,|\, t - \Delta_n)$. Define

$$V_n = \min_{\{a_i \in A_i | V(a_i \,|\, t-\Delta_n) > V(a^t \,|\, t-\Delta_n)\}} V(a_i \,|\, t)$$

Basically, we search over all information structures that create value of at least $V(a^t \,|\, t - \Delta_n)$ for type $t - \Delta_n$, then among those identify the lowest value one for type t. Clearly, $\Delta_n \to 0$, is sufficient to show that $V_n \to V^{\star}$. □

Proof of Proposition 3: In the Proof of Proposition 2, we have derived the symmetric solution for $\tau^\star$ as a function of the number of player in the game n:

$$\frac{\lambda^2}{(1-\lambda)^2}\frac{g'(\tau^\star)}{g'(1-\tau^\star)} = \cos^2\left(\frac{\pi}{2n}\right)$$

Notice that the right-hand side is increasing in n, while the left-hand side is strictly decreasing in $\tau^\star$. Therefore, an increase in n is compensated by a decrease in $\tau^\star$. □

Proof of Proposition 5. Fix an equilibrium where firms locate on $x = (x_1, ...x_n)$. Note that given x_1, $x_2 = x_1 + \frac{\pi}{n}$ and so on. Also, since types t and $t + \pi$ choose the same firm, without loss of generality, we'll focus on half the type distribution, $[0, \pi]$. Conditional on θ, aggregate approval rate conditional on θ is

$$\int_0^\pi \Phi\left(\frac{a(\tau)\theta_0 + b(\tau, x_t)(\cos(x_t)\theta_1 + \sin(x_t)\theta_2)}{c(\tau, x_t)}\right) + \Phi\left(\frac{a(\tau)\theta_0 - b(\tau, x_t)(\cos(x_t)\theta_1 + \sin(x_t)\theta_2)}{c(\tau, x_t)}\right)\frac{dt}{\pi}$$

where the two terms refer to expected approval rates for type t and $t + \pi$ with

$$a(\tau) = \lambda g(\tau) \qquad b(\tau, x_t) = (1-\lambda)g(\bar{\tau}-\tau)\cos(x_t-t) \qquad c(\tau, x_t) = \sqrt{\frac{\lambda^2 g^2(\tau)}{\tau} + \frac{(1-\lambda)^2 g^2(\bar{\tau}-\tau)cos^2(x_t-t)}{\bar{\tau}-\tau}}$$

Now we look at approval rate conditional only on θ_0 and apply the following identity twice $\int_{\mathbb{R}} \Phi(\alpha + \beta x)d\Phi(x) = \Phi\left(\frac{\alpha}{\sqrt{1+\beta^2}}\right)$.

which simplifies to

$$\int_0^\pi 2\Phi\left(\frac{a(\tau)\theta_0}{\sqrt{c^2(\tau, x_t) + b^2(\tau, x_t)(sin^2(x_t) + cos^2(x_t))}}\right)\frac{dt}{\pi} = \int_0^\pi 2\Phi\left(\frac{a(\tau)\theta_0}{\sqrt{c^2(\tau, x_t) + b^2(\tau, x_t)}}\right)\frac{dt}{\pi}$$

It will be useful to change the notation focusing on $\delta = |x_t - t|$. We use $b(\tau, \delta) = (1-\lambda)g(\bar{\tau}-\tau)\cos(\delta)$ and $c(\tau, \delta) = \sqrt{\frac{\lambda^2 g^2(\tau)}{\tau} + \frac{(1-\lambda)^2 g^2(\bar{\tau}-\tau)cos^2(\delta)}{\bar{\tau}-\tau}}$.

Note that in any equilibrium δ is uniformly distributed between $[0, \frac{\pi}{2n}]$. Thus,

$$\int_0^\pi 2\Phi\left(\frac{a(\tau)\theta_0}{\sqrt{c^2(\tau, x_t) + b^2(\tau, x_t)}}\right)\frac{dt}{\pi} = \int_0^{\frac{\pi}{2n}} 2\Phi\left(\frac{a(\tau)\theta_0}{\sqrt{c^2(\tau, \delta) + b^2(\tau, \delta)}}\right)\frac{2n}{\pi}d\delta$$

$$c^2(\tau, \delta) + b^2(\tau, \delta) = \lambda^2\frac{\tau}{(1+\tau)^2} + (1-\lambda)^2 cos^2(\delta)\frac{(\bar{\tau}-\tau) + (\bar{\tau}-\tau)^2}{(1+\bar{\tau}-\tau)^2}$$

taking the derivate give us

$$\lambda^2\left(\frac{1-\tau}{(1+\tau)^3}\right) - (1-\lambda)^2 cos^2(\delta)\left(\frac{1}{(1+\bar{\tau}-\tau)^2}\right)$$

I use $\frac{\lambda^2}{(1-\lambda)^2}\frac{g'(\tau)}{g'(\bar{\tau}-\tau)} = cos^2\left(\frac{\pi}{2n}\right)$ which can be written as $\frac{\lambda^2}{(1-\lambda)^2}\frac{(1+\bar{\tau}-\tau)^2}{(1+\tau)^2} = cos^2\left(\frac{\pi}{2n}\right)$ which comes from the optimality condition of the firms.

$$< \lambda^2\left(\frac{1-\tau}{(1+\tau)^3}\right) - \lambda^2\left(\frac{1}{(1+\tau)^2}\right) = \frac{\lambda^2}{(1+\tau)^2}\left(\frac{1-\tau}{1+\tau} - 1\right) < 0$$

So we have shown that $c^2(\tau,\delta) + b^2(\tau,\delta)$ increases when τ decreases. We also know that $a(\tau)$ also decreases when τ decreases. We also know that $c^2(\tau,\delta) + b^2(\tau,\delta)$ is higher for smaller values for δ. As n increases, the limits of the integral shrink. Moreoever, τ decreases. Both of these force the approval rate to move closer to one half. □

Proof of Remark 4. Let F be symmetric around $t^\star \in T$. We need to show that

$$\int_T \big(\theta_1 \cos(t) + \theta_2 \sin(t)\big) dF(t) = \big(\theta_1 \cos(t^\star) + \theta_2 \sin(t^\star)\big) \int_T \cos(t) dF(t)$$

Consider

$$\theta_1 \int_T \cos(t) dF(t) = \theta_1 \int_{t^\star - \bar{\delta}}^{t^\star + \bar{\delta}} \cos(t) f(t) dt$$

With a change of variable and using symmetry of F, we have that

$$\int_{t^\star - \bar{\delta}}^{t^\star + \bar{\delta}} \cos(t) f(t) dt = \int_{-\bar{\delta}}^{\bar{\delta}} \cos(t^\star + \delta) f(t^\star + \delta) d\delta = \int_0^{\bar{\delta}} \Big(\cos(t^\star + \delta) + \cos(t^\star - \delta)\Big) f(t^\star + \delta) d\delta$$

Using the trigonometric identity $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$, we can write

$$\begin{aligned}
\cos(t^\star + \delta) + \cos(t^\star - \delta) &= \cos(t^\star)\Big(\cos(\delta) + \cos(-\delta)\Big) - \sin(t^\star)\Big(\sin(\delta) + \sin(-\delta)\Big) \\
&= 2\cos(\delta)\cos(t^\star).
\end{aligned}$$

We can rewrite the integral above as $\theta_1 \cos(t^\star) \int_0^{\bar{\delta}} (2\cos(\delta)) f(t^\star + \delta) d\delta$. Finally, using symmetry again and another change of variable we can write $\int_0^{\bar{\delta}} (2\cos(\delta)) f(t^\star + \delta) d\delta = \int_T \cos(t) dF(t)$. This gives us the final expression

$$\theta_1 \int_T \cos(t) dF(t) = \theta_1 \cos(t^\star) \int_T \cos(t) dF(t). \tag{4}$$

Now we consider the second term $\theta_2 \int_T \sin(t) dF(t)$. By performing the same manipulations and using the trigonometric identity $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$, we can write that

$$\theta_2 \int_T \sin(t) dF(t) = \theta_2 \sin(t^\star) \int_T \cos(t) dF(t). \tag{5}$$

Summing equations 4 and 5, we get

$$\int_T \big(\theta_1 \cos(t) + \theta_2 \sin(t)\big) dF(t) = \big(\theta_1 \cos(t^\star) + \theta_2 \sin(t^\star)\big)\beta_F$$

where $\beta_F := \int_T \cos(t)dF(t)$. $\square$

Proof of Proposition 6. This follow from Proposition 5. We have shown that the approval rate moves closer to $1/2$ conditional on θ_0, we also know whether or not it is larger than $1/2$ is determined by the sign of θ_0. $\square$

Proof of Proposition 7. Let $\rho(t|\theta, \delta, \tau_n)$ be the probability that type t approves the policy conditional on θ which will depend on the location of the closest new source $\delta = |x_t - t|$ and τ_n. As we have shown in the proof of Proposition 5, we can write

$$\rho(t|\theta, \delta, \tau_n) = \Phi\left(\frac{a(\tau)\theta_0 + b(\tau,\delta)(\cos(t+\delta)\theta_1 + \sin(t+\delta)\theta_2)}{c(\tau,\delta)}\right)$$

with

$$b(\tau,\delta) = (1-\lambda)g(\bar{\tau}-\tau)\cos(\delta) \qquad c(\tau,\delta) = \sqrt{\frac{\lambda^2 g^2(\tau)}{\tau} + \frac{(1-\lambda)^2 g^2(\bar{\tau}-\tau)cos^2(\delta)}{\bar{\tau}-\tau}}$$

We use this to specify the expected approval rate

$$\Gamma(\theta) = \int_0^{\pi}\int_{\frac{-\pi}{2n}}^{\frac{\pi}{2n}} \rho(t|\theta,\delta,\tau_n) + \rho(t+\pi|\theta,\delta,\tau_n)\frac{nd\delta}{\pi}\frac{dt}{\pi} \tag{6}$$

Call $f(t) = \cos(t)\theta_1 + \sin(t)\theta_2$.

Using this we can write:

$$\begin{aligned}\Gamma(\theta) &= \int_0^{\pi}\int_{\frac{-\pi}{2n}}^{\frac{\pi}{2n}} \left(\Phi\left(\tfrac{a(\tau)\theta_0 + b(\tau,\delta)f(t+\delta)}{c(\tau,\delta)}\right) + \Phi\left(\tfrac{a(\tau)\theta_0 - b(\tau,\delta)f(t+\delta)}{c(\tau,\delta)}\right)\right)\frac{nd\delta}{\pi}\frac{dt}{\pi} \\ &= \int_0^{\pi}\int_{\frac{-\pi}{2n}}^{\frac{\pi}{2n}} \left(\Phi\left(\tfrac{a(\tau)\theta_0 + b(\tau,\delta)f(t)}{c(\tau,\delta)}\right) + \Phi\left(\tfrac{a(\tau)\theta_0 - b(\tau,\delta)f(t)}{c(\tau,\delta)}\right)\right)\frac{nd\delta}{\pi}\frac{dt}{\pi}\end{aligned} \tag{7}$$

The second equality is due to the symmetry. Note that what I've done is basically to collect all terms such that $t' + \delta = t$. This must be uniformly distributed with $t' \in [t-\delta, t+\delta]$.

Claim 3. $\frac{a(\tau)}{c(\tau,\delta)}$ *increases with* τ *and increases with* $|\delta|$.

Proof. Let's look at $\left(\frac{a(\tau)}{c(\tau,\delta)}\right)^2$ which gives us

$$\frac{1}{\frac{1}{\tau} + \frac{(1-\lambda)^2 g^2(\bar{\tau}-\tau)cos^2(\delta)}{\lambda^2 g^2(\tau)(\bar{\tau}-\tau)}}$$

Using $\frac{\lambda^2}{(1-\lambda)^2}\frac{(1+\bar{\tau}-\tau)^2}{(1+\tau)^2} = cos^2\left(\frac{\pi}{2n}\right)$ the expression above becomes (setting $\bar{\delta} = \frac{\pi}{2n}$)

$$\frac{1}{\frac{1}{\tau}+\frac{cos^2(\delta)(\bar{\tau}-\tau)^2}{cos^2(\bar{\delta})\tau^2(\bar{\tau}-\tau)}}$$

we simplify

$$\frac{1}{\frac{1}{\tau}+\frac{cos^2(\delta)(\bar{\tau}-\tau)}{cos^2(\bar{\delta})\tau^2}}$$

This is clearly increasing in τ and increases with $|\delta|$ and increases in $\bar{\delta}$. $\square$

Claim 4. $\frac{b(\tau,\delta)}{c(\tau,\delta)}$ *decreases with* τ *and decreases with* $|\delta|$*.*

Proof. Let's look at $\left(\frac{b(\tau,\delta)}{c(\tau,\delta)}\right)^2$ which gives us

$$\frac{1}{\frac{\lambda^2 g^2(\tau)}{(1-\lambda)^2 g^2(\bar{\tau}-\tau)cos^2(\delta)\tau}+\frac{1}{\bar{\tau}-\tau}}$$

Using $\frac{\lambda^2}{(1-\lambda)^2}\frac{(1+\bar{\tau}-\tau)^2}{(1+\tau)^2}=cos^2\left(\frac{\pi}{2n}\right)$ the expression above becomes

$$\frac{1}{\frac{cos^2(\bar{\delta})\tau}{(\bar{\tau}-\tau)^2 cos^2(\delta)}+\frac{1}{(\bar{\tau}-\tau)}}$$

we see how everything is opposite of the previous case giving us the desired result. $\square$

Claim 5. *Call* $\Gamma(\theta)=\Phi(\alpha\theta_0+\beta f(t))+\Phi(\alpha\theta_0-\beta f(t))$*.* $\Gamma(\theta)$ *moves away from one half as* α *increases, and moves towards one half as* β *increases.*

Proof. First note that $\Gamma(\theta)>0$ whenever $\theta_0>0$.

$$\begin{aligned}\frac{\partial\Gamma(\theta)}{\partial\alpha} &= \theta_0\left(\phi(\alpha\theta_0+\beta f(t))+\phi(\alpha\theta_0-\beta f(t))\right)\\ \frac{\partial\Gamma(\theta)}{\partial\beta} &= f(t)\left(\phi(\alpha\theta_0+\beta f(t))-\phi(\alpha\theta_0-\beta f(t))\right)\end{aligned} \tag{8}$$

Note that in the first equation $\phi(\alpha\theta_0+\beta f(t))+\phi(\alpha\theta_0-\beta f(t))$ is always positive, so the sign depends on the sign of θ_0 giving us the desired result with respect to α.

In the second equation, assume that $f(t)>0$ and $\theta_0>0$. Then the expression in the parentheses must be negative. Otherwise if $f(t)<0$ and $\theta_0>0$, then the expression in the parentheses must be positive, but it's multiplied by a negative term. The cases where $\theta_0<0$ work the same way. $\square$

Combining the three claims gives us the desired result for any t conditional on θ, which implies that $A(\theta)$ moves towards one half. $\square$

Proof of Remark 5. Let $\Gamma(\theta|\vec{x_n})$ be the approval rate when the firm locations are fixed by $\vec{x_n}$. In the proof above we have shown that $\Gamma_n(\theta) = \int\limits_{\vec{x_n}} \Gamma(\theta|\vec{x_n})\frac{ndx_n}{\pi}$ is decreasing with n. Note that $\Gamma(\theta|\vec{x}_n)$ changes continuously when we vary firm 1's location in $[0, \frac{\pi}{n}]$. This means that we can always find a $\vec{x_n^\star}$ such that $\Gamma(\theta|\vec{x_n^\star}) = \Gamma_n(\theta)$. Construct a sequence of equilibria $(\vec{x_n})_{n\in\mathbb{N}}$ by setting each $\vec{x_n}$ as described above. By construction approval rate is decreasing along the sequence. □

Proof of Remark 6. Proof of Proposition 3 reveals that $\tau^\star$ is lower when λ is lower. Using this we can show that in the proof of Proposition 7, $\frac{a(\tau,\delta)}{c(\tau,\delta)}$ is lower and $\frac{b(\tau,\delta)}{c(\tau,\delta)}$ is higher which would give us the desired result. □

Proof of Proposition 8. We'll make use of the following lemmas.

Lemma A4. *For any two news sources on the same side of t, if the agent is consuming the farthest one, then she must be consuming the one closer as well.*

Proof. Fix the learning strategy used by an agent of type t. Since we are focusing on symmetric equilibria where all news sources provide the same precision of signals on valence vs. ideology, we can focus on learning from the signals associated with ideology. Any learning strategy consists of two parts. Set of κ chosen news sources and a vector $\omega = (\omega_1, \omega_2, ...\omega_\kappa)$ which specifies how the signals from news sources are used in calculating the expected $f(\theta_{id}, t)$. Namely, $\mathbb{E}(f(\theta_{id}, t)|s) = \sum \omega_i s$ where linearity follows from the fact that the signals are normally distributed. Assume for contradiction that the agent is not consuming the closest news source. We show that the agent will be better off using the same ω but replacing the farthest news source with the closest one. Call the old news source o, and the new one n. Let $\delta = (\delta_x, \delta_y) = (\cos(t_n) - \cos(t_o), \sin(t_n) - \sin(t_o))$. Without loss of generality, we can assume that $t = 0$. Let $v_x = \sum \omega_i \cos(t_i)$ and $v_y = \sum \omega_i \sin(t_i)$. Call the new vector after the switch $\tilde{\omega}$ with associated $\tilde{v}_x = \sum \tilde{\omega}_i \cos(t_i)$ and $\tilde{v}_y = \sum \tilde{\omega}_i \sin(t_i)$. Since only change from ω to $\tilde{\omega}$ was the switch of one source, the following holds $\tilde{v} = v + \omega_0\delta$ where ω_0 is the weight put on this source. Note that conditional on the news sources chosen, an agent is choosing ω to minimize $\mathbb{E}(\mathbb{E}(f(\theta_{id}, t)|s) - f(\theta_{id}, t))^2$. Note that given our assumption on t, this is always equal to $(1 - v_x)^2 + v_y^2 + \sum_i \omega_i^2 \frac{1}{\tau_i}$. Going from ω to $\tilde{\omega}$ the last term doesn't change, only the first two terms change. It is sufficient for the result to show that $(1 - v_x)^2 + v_y^2 - (1 - \tilde{v}_x)^2 + \tilde{v}_y^2 > 0$.

$$\begin{aligned}(1 - v_x)^2 + v_y^2 - (1 - \tilde{v}_x)^2 + \tilde{v}_y^2 &= (1 - \tilde{v}_x + \omega_0\delta_x)^2 + (\tilde{v}_y - \omega_0\delta_y)^2 - (1 - \tilde{v}_x)^2 + \tilde{v}_y^2 \\ &= \omega_0^2\delta_x^2 + \omega_0^2\delta_y^2 + 2\omega_0\delta_x(1 - \tilde{v}_x) - 2\omega_0\delta_y\tilde{v}_y\end{aligned} \tag{9}$$

It is sufficient to focus on the case where $(\delta_x, \delta_y) = (1 - \cos(2\beta), \sin(2\beta))$ with $2\beta = \pi/n$, as in the other cases the desired condition will be easier to satisfy.

$$\begin{aligned}&= \omega_0[\omega_0(1-\cos(2\beta))^2 + \omega_0\sin^2(2\beta) + 2(1-\cos(2\beta))(1-\tilde{v}_x) - 2\sin(2\beta)\tilde{v}_y] \\ &= 4\omega_0\sin^2(\beta)[\omega_0\sin^2(\beta) + \omega_0\cos^2(\beta) + (1-\tilde{v}_x) - \tfrac{\cos(\beta)}{\sin(\beta)}\tilde{v}_y]\end{aligned} \tag{10}$$

Now we use the fact that $\tilde{v}_y \leq \frac{\sin(\beta)}{\cos(\beta)}\tilde{v}_x$, otherwise, we can always rotate the new sources that are chosen without ω.

$$\begin{aligned}&> 4\omega_0\sin^2(\beta)[\omega_0 + (1-\tilde{v}_x) - \tilde{v}_x] \\ &> 4\omega_0\sin^2(\beta)[\omega_0 + 1 - 2\tilde{v}_x]\end{aligned} \tag{11}$$

It is sufficient for the result to show that $\tilde{v}_x \leq 1$.

Claim 6. $v_x < 1$

Proof. Look at the best case where there are κ news sources that are all perfectly targeting $t = 0$ (which cannot happen for finite n). It is easy to see that the optimal strategy would be to choose $\omega = (\omega, \omega, ..\omega)$ to minimize $(1-\kappa\omega)^2 - \kappa\omega^2\sigma^2$. Solving this problem gives us $\omega = \frac{1}{\kappa+\sigma^2}$ which implies that $v_x = \kappa\omega = \frac{\kappa}{\kappa+\sigma^2}$. By our normalization assumption, $\sigma^2 = \kappa$ which implies $v_x \leq 0.5$. □

□

Lemma A5. *In the optimal learning strategy, an agent consumes the closest news sources.*

Proof. In Lemma A4, we already showed that there cannot be a gap in the news sources consumed to the right and to the left. Now assume for contradiction that the set of new sources chosen is not actually the set closest to the agent. Let $t_m = \frac{1}{\kappa}\sum = t_i$ be the mean type of the chosen set of new sources. Without loss of generality, assume that t_m is to the right of t. By assumption $t - t_m = \frac{\pi}{2n} + \theta$ for some $\theta > 0$. This also suggests that t is closer to the mid point of an alternative set of new sources that have been shifted to the left. First we show that the original set of news sources provides a more effective learning strategy for all types between t and t_m. We can always take the most right and left news sources, and we can replace $\tilde{v_1} = (1-\rho)v_1 + \rho\frac{v_1+v_\kappa}{2}$ and $\tilde{v_\kappa} = (1-\rho)v_\kappa + \rho\frac{v_1+v_\kappa}{2}$. We can do this iteratively for all other news sources as well. As $\rho \to 1$, the estimated type shifts towards t_m and the variance goes down. Using symmetry, we've demonstrated that this set of news sources to be better for $t_m - \frac{\pi}{2n} + \theta$. But this implies that the news sources can be shifted to the left to the get a better learning strategy. □

Lemma A5 implies that when firms locate equidistantly, and choose the same τ, with n news sources, each firm serves $\frac{\kappa}{n}$ of the market. Each firm is competing over threshold types with neighbors that are κ to the right an left. Hence, existence of symmetric equilibria and the comparative results can be shown following the same strategy for the $\kappa = 1$ case taking the adjustment with respect to the threshold types into account. □