MICROECONOMICS II.I – PROBLEM SET 5

NEW YORK UNIVERSITY, A.Y. 2012-2013

EXERCISE 1

Two quantity-setting firms with identical marginal cost $c > 0$ face an inverse demand curve $p = a - q_1 - q_2$. Firm 1 chooses q_1 knowing that Firm 2 will observe q_1 before choosing q_2.

(*a*). What is the unique subgame perfect equilibrium of this game? Either prove there are no other Nash equilibria of the game, or give an example of an NE of this game that is not an SPE.

(*b*). Suppose instead that it is commonly known that after Firm 1 chooses q_1, Firm 2 doesn't observe q_1 directly, but gets a signal x distributed as follows: with probability p the signal is correct, that is $x = q_1$; with probability $1 - p$ the signal is uninformative, that is x is uniformly distributed on $(0, a - c)$. Find a perfect Bayesian equilibrium **in pure strategies** for this model. How does it depend on p?

EXERCISE 2

Evaluating an infinite sequence of payoffs by its present discounted value captures the impatience of the decision maker. Koopmans (1960) showed axiomatically that there is no satisfactory way to model "perfect patience". A crude way to do this is by the limit of means criterion, which says a player cares only about the long-run average of the payoffs she receives. Suppose that each period $t = 1, 2, \ldots$ she gets some payoff x_t from the same bounded subset of $\mathbb{R}$. Define the associated sequence of averages by

$$a_t = \frac{1}{t} \sum_{j=1}^{t} x_j$$

(*a*). Give an example of a sequence that does not converge, although its sequence of averages does. Now give an example of a bounded sequence whose sequence of averages doesn't converge.

(*b*). To get around the problem that the "long-run average" may not be well-defined, the limit of means of the sequence (x_t) is defined to be the limit inferior

$$\lim_{T \to \infty} \inf_{t \geq T} x_t$$

Explain why this is always well-defined (assuming as above that the payoffs are bounded).

Due on March 11th by 7pm in Jacopo's mailbox (19W4 6th floor).

(*c*). Recall the definition of unimprovable strategy: A strategy σ_i for a player i is called unimprovable if there is no history of play after which player i can do strictly better by deviating from σ_i in the current period only (and conforming to σ_i thereafter) than by conforming with σ_i in the current period and thereafter. Now consider the following problem.

Samantha loves cookies. She gets 0 utils from eating no cookies at lunch, 1 from eating 1, and 2 from eating 2. Her friend Enabla is willing to give her up to 2 cookies each day to eat at lunch. Cookies cannot be stored for future consumption. Samantha's mother approves pleasure, but only in moderation. She gives Samantha the following instructions: "On any day t, eat one cookie at lunch, unless you ate two cookies the day before, in which case you eat no cookie on day t". View this as a strategy for Samantha in a one-person infinitely repeated game. Suppose that Samantha evaluates her payoff stream using the limit of means criterion. Show that although the strategy is not optimal, it is unimprovable.