Updating QR Factorization Under Sparse Updates.

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Overview

- In a least squares problem, the data might change. If the change occurs in a sparse way the QR decomposition can be updated faster than recomputation.
- First goal: Present a method that leverages the specialized structure of the update to accelerate QR computation
- Second goal: Experimentally determine the crossover point for different m, n, k values at which recomputation is faster than specialized updating.

Updating Algorithms

The total number of fill-in entries eliminated is

$$Lazy = \sum_{j=1}^{k} \sum_{t=1}^{j} (i_{j+1} - i_j - t) = \sum_{j=1}^{k} j(i_{j+1} - i_j - \frac{j+1}{2})$$

This is called the lazy elimination strategy because we defer elimination of fill-in entries until the last possible moment. The eager strategy eliminates fill-in as soon as it is created. The total number of fill-in entries removed is

$$Eager = \sum_{j=1}^{k} j(n - i_j) = n\sum_{j=1}^{k} j - \sum_{j=1}^{k} j i_j = \frac{nk(k+1)}{2} - \sum_{j=1}^{k} i_j$$

Both methods must eliminate the original nonzeros which are counted by $\sum_{j=1}^{k} n - i_j$.

Crossover Theory

For a fixed m large enough, the Householder based methods are cubic in n and the sparse updating methods are quadratic in n because we need to accumulate Q. Thus making this simplification we can model the performance ratio as

$$r_e(m, n, k) = \frac{C_m n^3}{n^2 k} = \frac{Cn}{k} \tag{1}$$

The relationship between m and n is accounted for in the dependence of C_m on m. When $r_e = 1$ the two methods will take the same amount of time, and the k for which this occurs is called the break even point.

Results

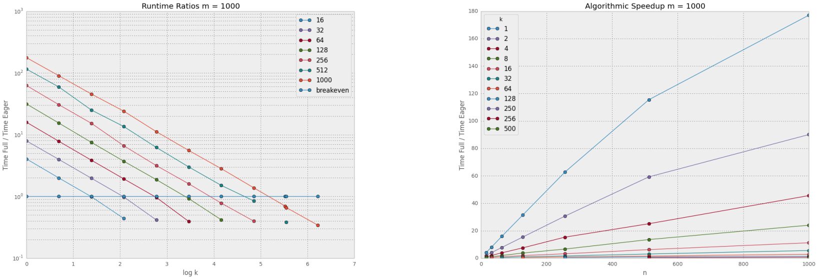

Figure: r_e as a function of k (left) and as a function of n (right)