Updating QR Factorization Under Sparse Updates.

James Fairbanks[1] Domenic Carr[2]

[1]School of CSE

[2]School of ECE

March 25, 2014

Motivation

- In a least squares problem, the data might change. If the change occurs in a sparse way the QR decomposition can be updated faster than recomputation.
- The goal of our project is to present a method that is efficient and determine experimentally the crossover point at which recomputation is faster than updating.
- It is clear that a crossover point exists because when the update $\Delta A = B$ contains a nonzero entry in each column, the computation becomes a full factorization.

Problem Statement

Let $m \geq n$ $A \in R^{m \times n}$ and $A = QR$

$$\begin{aligned} Q^T(A+B) &= Q^T QR + Q^T B & (1) \\ &= R + Q^T B & (2) \end{aligned}$$

but the sparsity of B gives a structure to $Q^T B$

Lemma 1

Left multiplication by Q^T turns nonzero entries into nonzero columns.

$$Q^T \begin{bmatrix} & x & \\ x & & \\ & & \\ & & \end{bmatrix} = \begin{bmatrix} x & x & \\ x & x & \\ x & x & \\ x & x & \end{bmatrix}$$

Application of Givens rotations to $R + Q^T B$ gives an update method with out fill in.

Experimental Plan

Experiments

- Gather sample matrices
- Perturb them with a random perturbation of density $p \in (0, 1]$

Implementations

- Static QRD with Householder reflections.
- Sparse matrix dense matrix multiply for $Q^T B$.
- Givens rotations with knowledge of nonzero columns of $Q^T B$.

Contributions

- Analytic conditions of efficiency
- Experimental verification of crossover point.