AN ERDOS SIMILARITY PROBLEM IN A TOPOLOGICAL SETTING

A thesis presented to the faculty of
San Francisco State University
In partial fulfilment of
The Requirements for
The Degree

Master of Arts
In
Mathematics

by

John P Gallagher

San Francisco, California

May2022

CERTIFICATION OF APPROVAL

I certify that I have read *AN ERDOS SIMILARITY PROBLEM IN A TOPOLOGICAL SETTING* by John P Gallagherand that in my opinion this work meets the criteria for approving a thesis submitted in partial fulfillment of the requirements for the degree: Master of Artsin Mathematicsat San Francisco State University.

_____
Dr. Chun-Kit Lai
Associate Professorof Mathematics

_____
Dr. Emily Clader
Assistant Professorof Mathematics

_____
Dr. Arek Goetz
Professorof Mathematics

AN ERDOS SIMILARITY PROBLEM IN A TOPOLOGICAL SETTING

John P Gallagher
San Francisco State University
2022

Your abstract comes here.

I certify that the Abstract is a correct representation of the content of this thesis.

_____

Chair, Thesis Committee   Date

ACKNOWLEDGMENTS

I want to take a moment to list a few people who have shaped my pursuit of math. Firstly, thank you Mom and Dad for supporting. Mom, you sat with me at the dinner table teaching me

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

Introduction

Here's where you text starts. You might (but don't have to) have sections like...

1.1 Introduction to the Introduction

...or subsections...

1.1.1 Introduction to the Introduction to the Introduction

To include text in your document, you just type. You don't have to worry about spacing between sentences; LaTeX does that automatically. To start a new paragraph, just insert an empty line.

Here comes the new paragraph. You can include tables, such as Table 1.1, which can also be turned if necessary, as seen, e.g. in Table 1.2. (I think the latter is automatically on its own page; the former will just be put where LaTeX thinks it fits

best.)

d	2	3	4	5	6	7	8	9
bound	3.6	8.5	15.8	25.7	38.3	53.5	71.4	92.0

Table 1.1: Bounds in various dimensions d.

To start a new chapter, you don't need to add a pagebreak...

d	2	3	4	5	6	7	8	9
bound	3.6	8.5	15.8	25.7	38.3	53.5	71.4	92.0

Table 1.2: More bounds in various dimensions d.

Chapter 2

The Real Stuff

...LaTeX does that automatically.

2.1 Various Tricks

Here are a few tidbits that came to my mind, in no particular order...

I'm sure most people know about `\label` and `\ref`; this feature is enough reason for me to use LaTeX. For example, you might have something like...

Theorem 2.1 (Euler)**.** *Leonhard says* $e^{2\pi i} = 1$.

...which you can then later (or earlier) reference as Theorem 2.1 and you can point to page 4 on which it appears. If you label equations like

$$e^{2\pi i} = 1\,, \tag{2.1}$$

you can reference them most lazily using (2.1).

There are two ways to define internal macros: `\def` and `\newcommand`. The difference is that the former overwrites any possibly existing command, where as the latter induces a LaTeX complaint if you're redefining an existing command (in which case you should use `\renewcommand` instead). Since I'm lazy, I tend to use `\def`, with one important exception: `\newcommand` allows you to use arguments. For example, the definition

`\newcommand\floor[1]{\left\lfloor {#1} \right\rfloor}`

produces a flexible floor function $\left\lfloor \frac{3\pi}{2} \right\rfloor$. And yes, the `[1]` can be replaced by `[n]` if you have use for `n` arguments (which get the placeholders `#1`, `#2`, ..., `#n`).

The previous example reminds me to strongly recommend the use of `\left` and `\right` whenever you use parentheses:

$$(\frac{a}{b} - 2) \quad \text{just doesn't look good compared with} \quad \left(\frac{a}{b} - 2\right).$$

There are three dashes in LaTeX—one like the one you just saw, one that's used in "Berndt–Zaharescu's Theorem" or "Chapter 7–9," and one that's used in "well-known identity."

One of my favorite packages is `enumerate`:

1. `first item`

2. `second item`

3. third item

4. ...

TO DO: I stole the idea of a to-do box from a friend. This is useful when editing a paper and you want to remind your co-author or yourself about something...

When writing text that require many sittings, I enjoy changing the font color, which is an easy way to show progress in a long project.

For writing big files, you may separate your files into subfiles. For example, this paragraph is in subfile.tex. I inputted this file into content.tex by using the "standalone" package and the following command where the sentence should be with the following command:

\input{subfile.tex}

2.2   Math

You can put a □ at the end of a math line if a proof happens to end with a math line. The version...

Proof. This follows from

$$e^{2\pi i} = 1\,.$$

□

...wastes space and doesn't look nearly as cool as...

Proof. This follows from

$$e^{2\pi i} = 1\,. \qquad\qquad\qquad\qquad\qquad \square$$

Speaking about proofs, sometimes you need a paragraph or two between a theorem and its proof, in which case you can use...

Proof of Theorem 2.1. This follows from

$$e^{2\pi i} = 1\,. \qquad\qquad\qquad\qquad\qquad \square$$

You can (and should) define your own math operators with \operatorname. For example,

$$\operatorname{lcm}(2,3) = 6 \quad \text{looks better than} \quad lcm(2,3) = 6\,.$$

Speaking about lcm's, I find it amusing that \gcd is a pre-defined operator, whereas \lcm is not.

The \dots command is relatively smart in LaTeX; e.g., it knows automatically where to put the dots in $\{1, 2, \ldots, n\}$ as compared to $1 + 2 + \cdots + n$. If you ever need to force a certain alignment of the dots, use $\ldots$, $\cdots$, $\vdots$, or $\ddots$.

For math stuff that takes several lines I prefer the environment align

over eqnarray. The difference is minor but I still prefer

$$\sum_{j=1}^{k-1} \chi(j) \sin^2\left(\frac{\pi j}{k}\right) \tan\left(\frac{2\pi j}{k}\right) = \sqrt{k}\left(-\frac{1}{2} + (\chi(2) - 2)\, h(-k)\right)$$

$$= \begin{cases} \sqrt{k}\left(-\frac{1}{2} - 3\, h(-k)\right) & \text{if } k \equiv 3 \bmod 8, \\ \sqrt{k}\left(-\frac{1}{2} - h(-k)\right) & \text{if } k \equiv 7 \bmod 8 \end{cases}$$

over

$$\sum_{j=1}^{k-1} \chi(j) \sin^2\left(\frac{\pi j}{k}\right) \tan\left(\frac{2\pi j}{k}\right) = \sqrt{k}\left(-\frac{1}{2} + (\chi(2) - 2)\, h(-k)\right)$$

$$= \begin{cases} \sqrt{k}\left(-\frac{1}{2} - 3\, h(-k)\right) & \text{if } k \equiv 3 \bmod 8, \\ \sqrt{k}\left(-\frac{1}{2} - h(-k)\right) & \text{if } k \equiv 7 \bmod 8. \end{cases}$$

2.3   Graphics

My favorite way to produce graphics is through a (public-domain) program
called jPicEdt; it produces LaTeX code that can be read right into the file
(using the \input command). My second favorite way to go about graphics
is through the package graphicx. I produce my graphics with a separate
program, export them into pdf, and then overlay them with TeX symbols if
needed; see Figure 2.1 for an example.

The overlaying of figures reminds me of two of my other favorite LaTeX

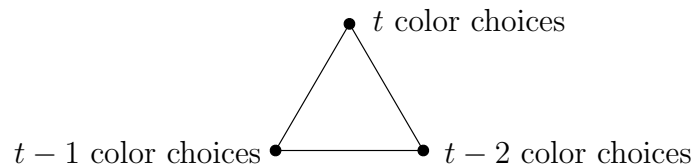Figure 2.1: Proper t-colorings of K_3.

```
commands:  \vspace and \hspace.  For example, they allow you to place just
about
```

anywhere.
anything

```
This can be very useful, e.g., for presentations in which you might move
pictures around.
```

2.4   Bibliographic Stuff

```
For references like [1, Section 2] I recommend using bibtex; it means that
you have to do only minimal work, especially when you get the entries from
MathSciNet.  I keep all the references I've ever used in the same file
and use this in all my documents...
```

```
    If you could use an index at the end of your document, use makeindex---one
of best reasons to use LaTeX if you're writing a book.
```

Appendix A: Triangulations of Polytopes

This creates an appendix, which is not numbered (and therefore has to be added to the table of contents semi-manually).

For code, the verbatim environment (maybe in conjunction with \include) is helpful:

```
It reproduces text
  exactly
    as
      it
        is
typed.

  Good luck and enjoy writing!
```