

AN ERDOS SIMILARITY PROBLEM IN A TOPOLOGICAL SETTING

A thesis presented to the faculty of  
San Francisco State University  
In partial fulfilment of  
The Requirements for  
The Degree

Master of Arts  
In  
Mathematics

by

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## CERTIFICATION OF APPROVAL

I certify that I have read *AN ERDOS SIMILARITY PROBLEM IN A TOPOLOGICAL SETTING* by John P Gallagher and that in my opinion this work meets the criteria for approving a thesis submitted in partial fulfillment of the requirements for the degree: Master of Arts in Mathematics at San Francisco State University.

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2022

Your abstract comes here.

I certify that the Abstract is a correct representation of the content of this thesis.

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Chair, Thesis Committee

Date

## ACKNOWLEDGMENTS

I want to take a moment to list a few people who have shaped my pursuit of math. Firstly, thank you Mom and Dad for supporting. Mom, you sat with me at the dinner table teaching me

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# Chapter 1

## Introduction

Here's where you text starts. You might (but don't have to) have sections like...

### 1.1 Introduction to the Introduction

...or subsections...

#### 1.1.1 Introduction to the Introduction to the Introduction

To include text in your document, you just type. You don't have to worry about spacing between sentences;  $\text{\LaTeX}$  does that automatically. To start a new paragraph, just insert an empty line.

Here comes the new paragraph. You can include tables, such as Table 1.1, which can also be turned if necessary, as seen, e.g. in Table 1.2. (I think the latter is automatically on its own page; the former will just be put where  $\text{\LaTeX}$  thinks it fits

best.)

$d$	2	3	4	5	6	7	8	9
bound	3.6	8.5	15.8	25.7	38.3	53.5	71.4	92.0

Table 1.1: Bounds in various dimensions  $d$ .

To start a new chapter, you don't need to add a pagebreak...

$d$	2	3	4	5	6	7	8	9
bound	3.6	8.5	15.8	25.7	38.3	53.5	71.4	92.0

Table 1.2: More bounds in various dimensions  $d$ .

## Chapter 2

### The Real Stuff

...  $\text{\LaTeX}$  does that automatically.

#### 2.1 Various Tricks

Here are a few tidbits that came to my mind, in no particular order...

I'm sure most people know about `\label` and `\ref`; this feature is enough reason for me to use  $\text{\LaTeX}$ . For example, you might have something like...

**Theorem 2.1** (Euler). *Leonhard says  $e^{2\pi i} = 1$ .*

... which you can then later (or earlier) reference as Theorem 2.1 and you can point to page 4 on which it appears. If you label equations like

$$e^{2\pi i} = 1, \tag{2.1}$$

you can reference them most lazily using (2.1).

There are two ways to define internal macros: `\def` and `\newcommand`. The difference is that the former overwrites any possibly existing command, where as the latter induces a L<sup>A</sup>T<sub>E</sub>X complaint if you’re redefining an existing command (in which case you should use `\renewcommand` instead). Since I’m lazy, I tend to use `\def`, with one important exception: `\newcommand` allows you to use arguments. For example, the definition

```
\newcommand\floor[1]{\left\lfloor {#1} \right\rfloor}
```

produces a flexible floor function  $\lfloor \frac{3\pi}{2} \rfloor$ . And yes, the `[1]` can be replaced by `[n]` if you have use for `n` arguments (which get the placeholders `#1`, `#2`, `...`, `#n`).

The previous example reminds me to strongly recommend the use of `\left` and `\right` whenever you use parentheses:

$$\left(\frac{a}{b} - 2\right) \text{ just doesn't look good compared with } \left(\frac{a}{b} - 2\right).$$

There are three dashes in L<sup>A</sup>T<sub>E</sub>X—one like the one you just saw, one that’s used in “Berndt–Zaharescu’s Theorem” or “Chapter 7–9,” and one that’s used in “well-known identity.”

One of my favorite packages is `enumerate`:

1. first item

2. second item

3. third item

4. ...

TO DO: I stole the idea of a to-do box from a friend. This is useful when editing a paper and you want to remind your co-author or yourself about something...

## 2.2 Math

You can put a  $\square$  at the end of a math line if a proof happens to end with a math line. The version...

*Proof.* This follows from

$$e^{2\pi i} = 1.$$

$\square$

...wastes space and doesn't look nearly as cool as...

*Proof.* This follows from

$$e^{2\pi i} = 1.$$

$\square$

Speaking about proofs, sometimes you need a paragraph or two between a theorem and its proof, in which case you can use...

*Proof of Theorem 2.1.* This follows from

$$e^{2\pi i} = 1.$$

$\square$

You can (and should) define your own math operators with `\operatorname`.  
For example,

$$\mathrm{lcm}(2,3) = 6 \quad \text{looks better than} \quad \mathrm{lcm}(2,3) = 6.$$

Speaking about lcm's, I find it amusing that `\gcd` is a pre-defined operator, whereas `\lcm` is not.

The `\dots` command is relatively smart in  $\mathrm{\TeX}$ ; e.g., it knows automatically where to put the dots in  $\{1,2,\dots,n\}$  as compared to  $1+2+\dots+n$ . If you ever need to force a certain alignment of the dots, use `\ldots`, `\cdots`, `\vdots`, or `\ddots`.

For math stuff that takes several lines I prefer the environment `align` over `eqnarray`. The difference is minor but I still prefer

$$\begin{aligned} \sum_{j=1}^{k-1} \chi(j) \sin^2 \left( \frac{\pi j}{k} \right) \tan \left( \frac{2\pi j}{k} \right) &= \sqrt{k} \left( -\frac{1}{2} + (\chi(2) - 2) h(-k) \right) \\ &= \begin{cases} \sqrt{k} \left( -\frac{1}{2} - 3 h(-k) \right) & \text{if } k \equiv 3 \pmod{8}, \\ \sqrt{k} \left( -\frac{1}{2} - h(-k) \right) & \text{if } k \equiv 7 \pmod{8} \end{cases} \end{aligned}$$

over

$$\begin{aligned} \sum_{j=1}^{k-1} \chi(j) \sin^2 \left( \frac{\pi j}{k} \right) \tan \left( \frac{2\pi j}{k} \right) &= \sqrt{k} \left( -\frac{1}{2} + (\chi(2) - 2) h(-k) \right) \\ &= \begin{cases} \sqrt{k} \left( -\frac{1}{2} - 3 h(-k) \right) & \text{if } k \equiv 3 \pmod{8}, \\ \sqrt{k} \left( -\frac{1}{2} - h(-k) \right) & \text{if } k \equiv 7 \pmod{8}. \end{cases} \end{aligned}$$

## 2.3 Graphics

My favorite way to produce graphics is through a (public-domain) program called jPicEdt; it produces  $\text{\LaTeX}$  code that can be read right into the file (using the `\input` command). My second favorite way to go about graphics is through the package `graphicx`. I produce my graphics with a separate program, export them into pdf, and then overlay them with  $\text{\TeX}$  symbols if needed; see Figure 2.1 for an example.

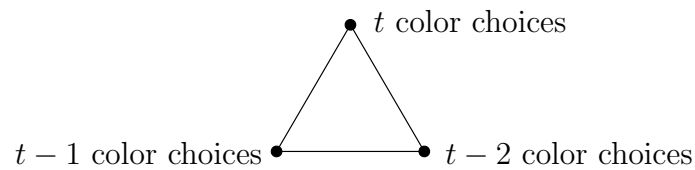


Figure 2.1: Proper  $t$ -colorings of  $K_3$ .

The overlaying of figures reminds me of two of my other favorite  $\text{\LaTeX}$  commands: `\vspace` and `\hspace`. For example, they allow you to place just



about

anywhere.

anything

This can be very useful, e.g., for presentations in which you might move pictures around.

## 2.4 Bibliographic Stuff

For references like [athanasiadismagic] I recommend using bibtex; it means that you have to do only minimal work, especially when you get the entries from *MathSciNet*. I keep all the references I've ever used in the same file and use this in all my documents...

If you could use an index at the end of your document, use makeindex---one of best reasons to use ~~TEX~~  $\LaTeX$  if you're writing a book.

## Appendix A: Triangulations of Polytopes

This creates an appendix, which is not numbered (and therefore has to be added to the table of contents semi-manually).

For code, the verbatim environment (maybe in conjunction with `\include`) is helpful:

It reproduces text

exactly

as

it

is

typed.

Good luck and enjoy writing!