

a)

$$\langle \rho \rangle = -i\hbar \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x} dx \quad 1.33$$

$$\frac{d\langle \rho \rangle}{dt} = -i\hbar \frac{d}{dt} \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x} dx = -i\hbar \int_{-\infty}^{\infty} \frac{\partial}{\partial t} \left(\psi^* \frac{\partial \psi}{\partial x} \right) dx$$

$$\frac{d\langle \rho \rangle}{dt} = -i\hbar \int_{-\infty}^{\infty} \left(\frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial x} + \psi^* \left[\frac{\partial}{\partial t} \frac{\partial \psi}{\partial x} \right] \right) dx$$

$$\frac{d\langle \rho \rangle}{dt} = -i\hbar \int_{-\infty}^{\infty} \frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial x} dx - i\hbar \int_{-\infty}^{\infty} \psi^* \left[\frac{\partial}{\partial t} \frac{\partial \psi}{\partial x} \right] dx$$

The Schroedinger equation (slightly adjusted) tells us:

$$\frac{\partial \psi}{\partial t} = \frac{i\hbar}{2m} \frac{\partial^2 \psi}{\partial x^2} - \frac{iV}{\hbar} \psi$$

taking the complex conjugate,

$$\frac{\partial \psi^*}{\partial t} = -\frac{i\hbar}{2m} \frac{\partial^2 \psi^*}{\partial x^2} + \frac{iV}{\hbar} \psi^*$$

$$-i\hbar \int_{-\infty}^{\infty} \frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial x} dx = -i\hbar \int_{-\infty}^{\infty} \left(-\frac{i\hbar}{2m} \frac{\partial^2 \psi^*}{\partial x^2} + \frac{iV}{\hbar} \psi^* \right) \frac{\partial \psi}{\partial x} dx = -i\hbar \int_{-\infty}^{\infty} \left(-\frac{i\hbar}{2m} \frac{\partial^2 \psi^*}{\partial x^2} \frac{\partial \psi}{\partial x} + \frac{iV}{\hbar} \psi^* \frac{\partial \psi}{\partial x} \right) dx$$

$$-i\hbar \int_{-\infty}^{\infty} \frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial x} dx = -i\hbar \int_{-\infty}^{\infty} \left(-\frac{i\hbar}{2m} \frac{\partial^2 \psi^*}{\partial x^2} \frac{\partial \psi}{\partial x} \right) dx - i\hbar \int_{-\infty}^{\infty} \left(\frac{iV}{\hbar} \psi^* \frac{\partial \psi}{\partial x} \right) dx$$

$$-i\hbar \int_{-\infty}^{\infty} \frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial x} dx = -\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \left(\frac{\partial^2 \psi^*}{\partial x^2} \frac{\partial \psi}{\partial x} \right) dx + V \int_{-\infty}^{\infty} \left(\psi^* \frac{\partial \psi}{\partial x} \right) dx$$

we can integrate the first part of this using integration by parts. The second part looks like $iV \frac{\langle \rho \rangle}{\hbar}$

$$u = \frac{\partial \psi}{\partial x} \quad dv = \frac{\partial^2 \psi^*}{\partial x^2}$$

AAAAHHHHHHHHHHHH