

Image Gradients and Gradient Filtering

Computer Vision

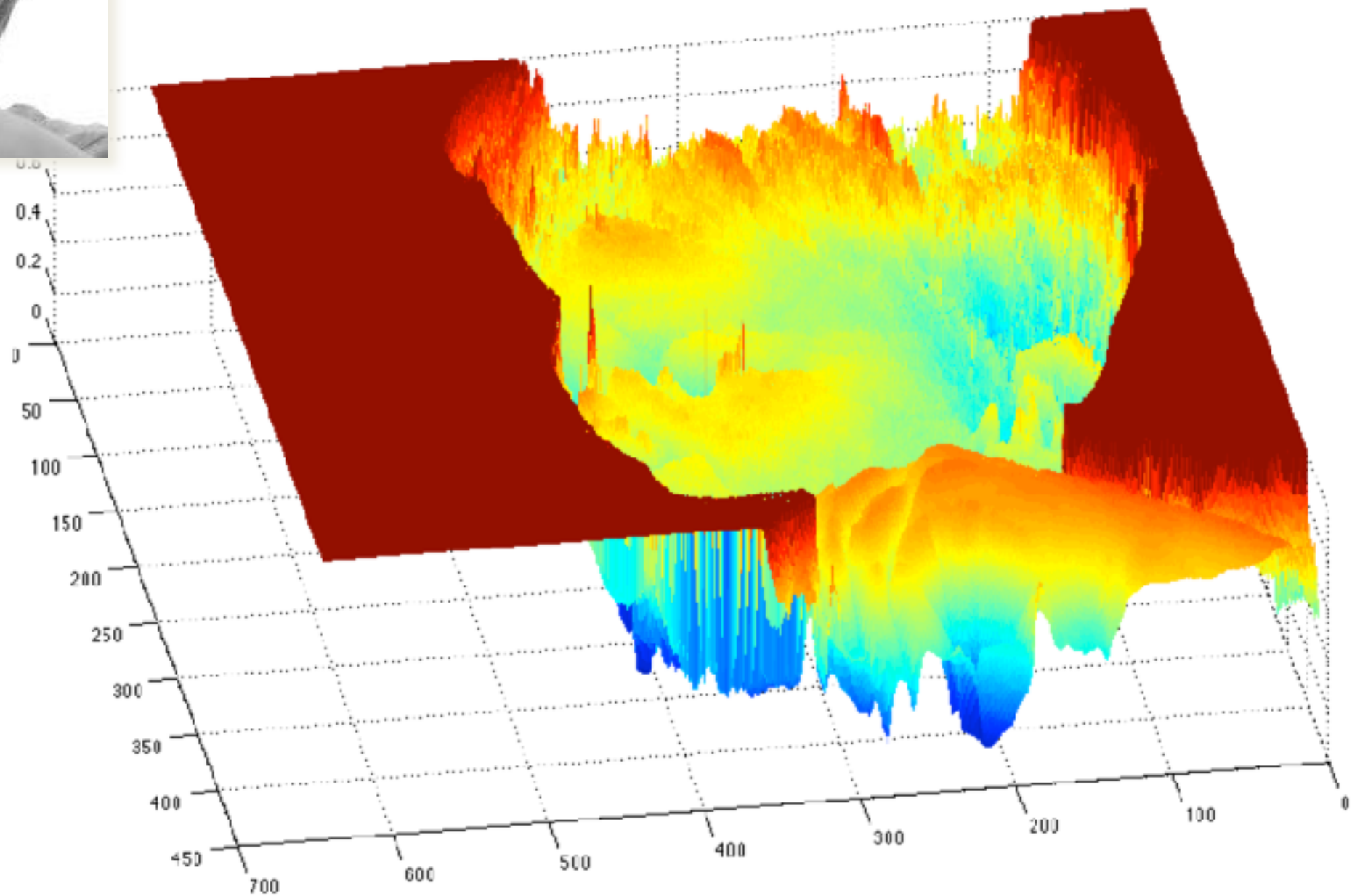
Carnegie Mellon University (Kris Kitani)

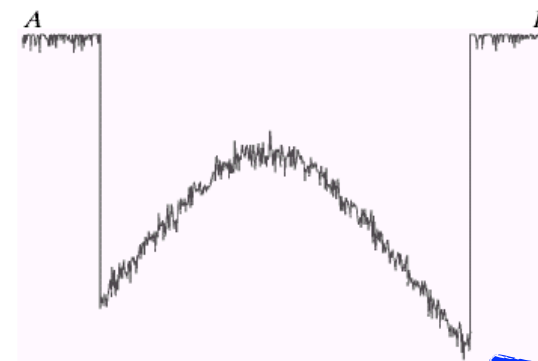
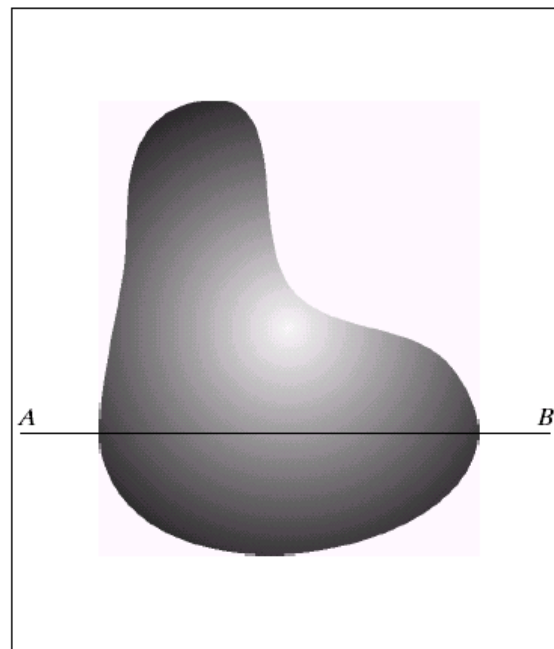
What is an image edge?

Recall that an image is a 2D function



$$f(x)$$





edge



edge

How would you detect an edge?
What kinds of filter would you use?

Do you remember this from high school?

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Do you remember this from high school?

The derivative of a function f at a point x is defined by the limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Approximation of the derivative when h is small


This definition is based on the ‘forward difference’ but ...

... it turns out that using the ‘central difference’
is more accurate

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

How do we compute the derivative of a discrete signal?

10	20	10	200	210	250	250
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... it turns out that using the 'central difference'
is more accurate

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

How do we compute the derivative of a discrete signal?

10	20	10	200	210	250	250
----	----	----	-----	-----	-----	-----



$$f'(x) = \frac{f(x+1) - f(x-1)}{2} = \frac{210 - 10}{2} = 100$$

-1	0	1
----	---	---

1D derivative filter

The 'Sobel' filter

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array}$$

a derivative filter
(with some smoothing)

Filter returns large response on vertical or horizontal edges?

The 'Sobel' filter

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

a derivative filter
(with some smoothing)

Filter returns large response on vertical or horizontal edges?

Is the output always positive?

The 'Sobel' filter

$$\frac{1}{8}$$

1	2	1
0	0	0
-1	-2	-1

a derivative filter
(with some smoothing)

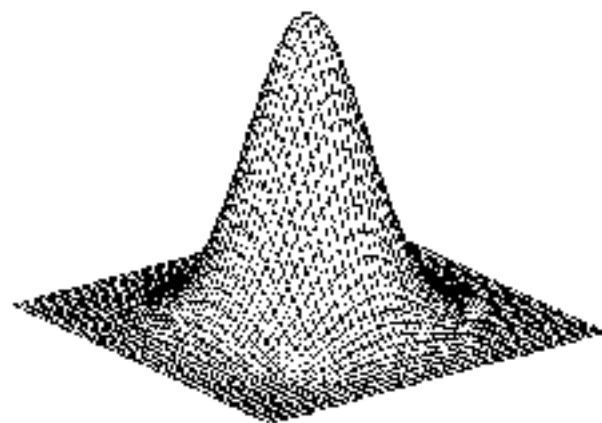
Responds to horizontal lines

Output can be positive or negative

The 'Sobel' filter

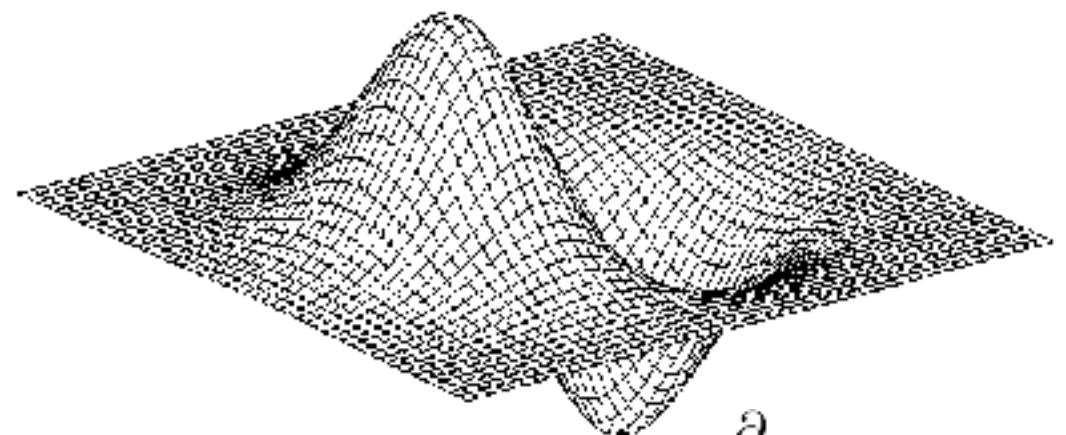
$$\frac{1}{8} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Approximation of the derivative of a Gaussian



Gaussian

$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



Derivative of Gaussian

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

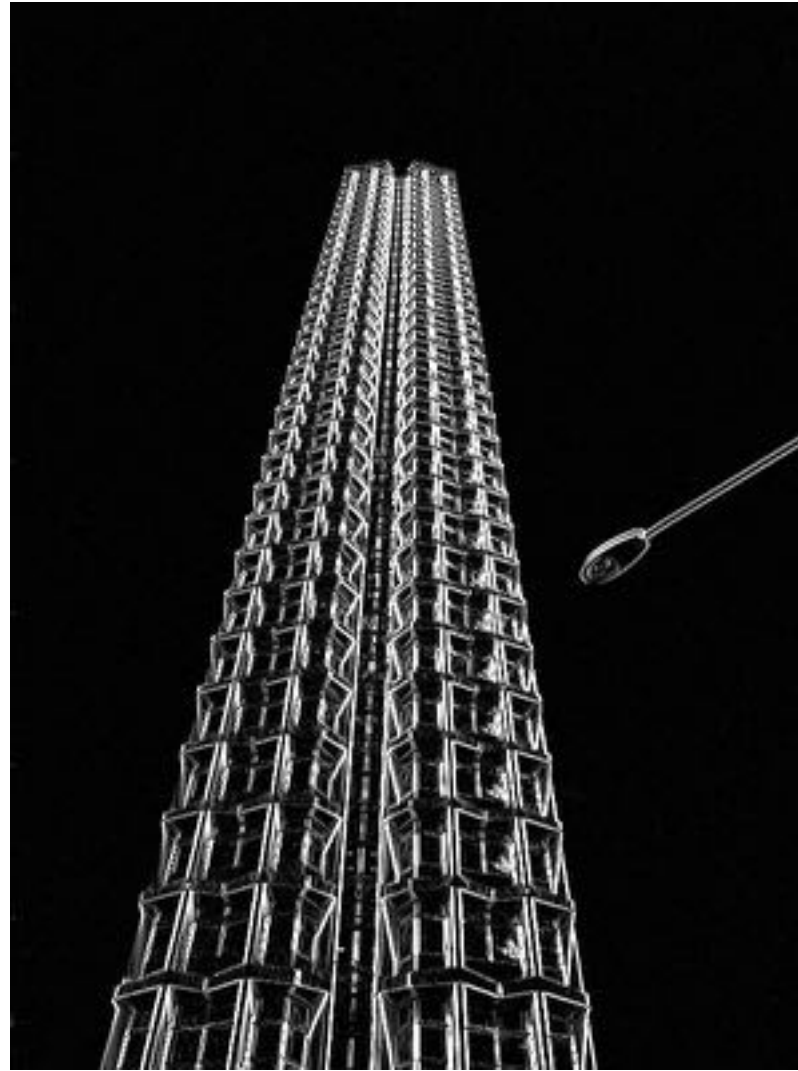


Output of which Sobel filter?



Output of which Sobel filter?

How do you visualize negative derivatives/gradients?



Derivative in X direction



Derivative in Y direction

Visualize with scaled absolute value

The 'Sobel' filter

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array}$$

Where does this filter come?

Decomposing the Sobel filter

$$\frac{1}{8} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Sobel

What this?

Decomposing the Sobel filter

$$\frac{1}{8} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Sobel

weighted average
and scaling

Decomposing the Sobel filter

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Sobel

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Decomposing the Sobel filter

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Sobel

weighted average
and scaling

What this?

x-derivative

The Sobel filter only returns the x and y edge responses.

*How can you compute the **image gradient**?*

How do you compute the image gradient?

Choose a derivative filter

$$S_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

What is this filter called?

Run filter over image

$$\frac{\partial f}{\partial x} = S_x \otimes f$$

$$\frac{\partial f}{\partial y} = S_y \otimes f$$

What are the dimensions?

Image gradient

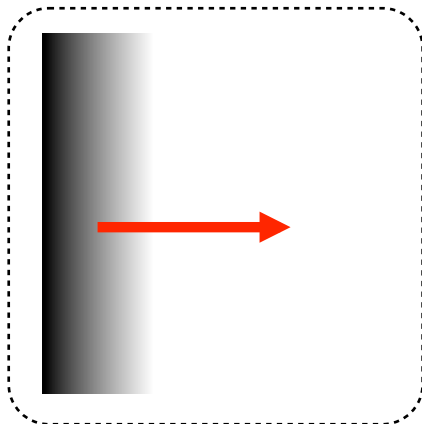
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

What are the dimensions?

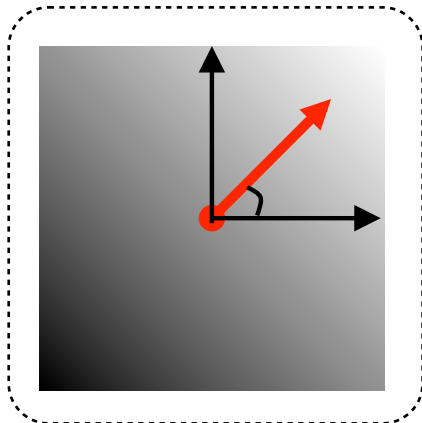
MATCH THAT GRADIENT !

MATCH THAT GRADIENT !

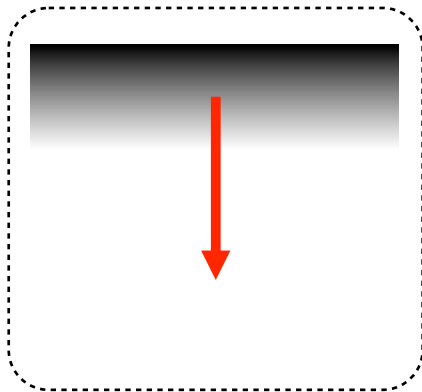
(a)



(b)



(c)



(1)

$$\nabla f = \left[0, \frac{\partial f}{\partial y} \right]$$

(2)

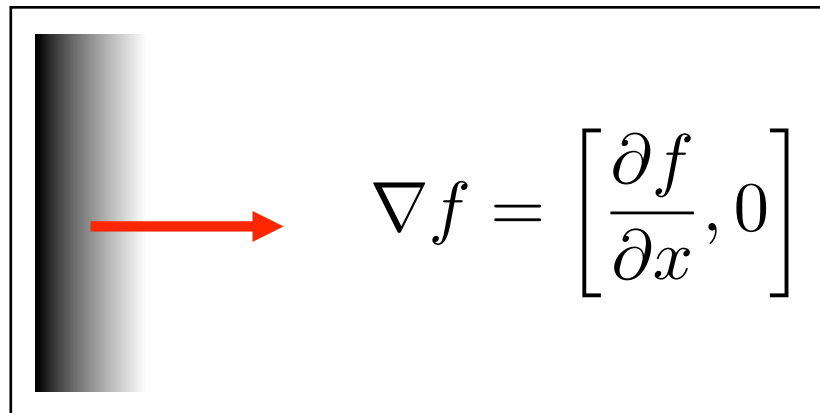
$$\nabla f = \left[\frac{\partial f}{\partial x}, 0 \right]$$

(3)

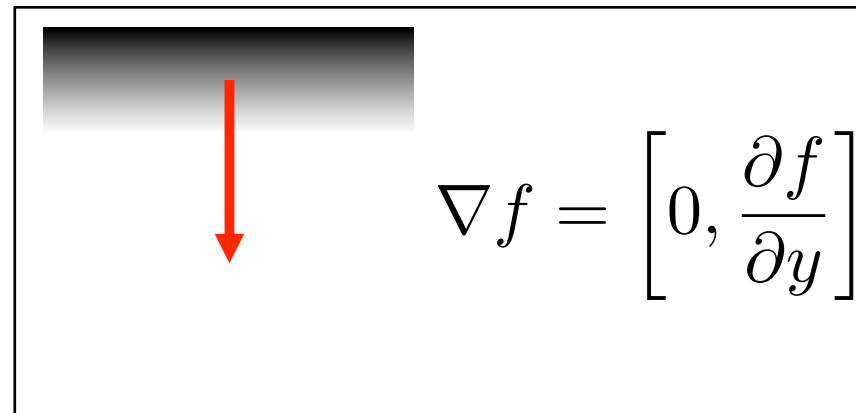
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

Image Gradient

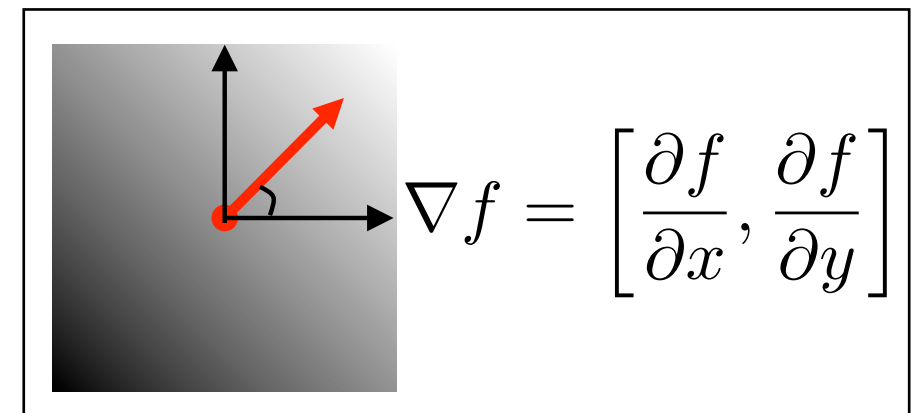
Gradient in x only



Gradient in y only



Gradient in both x and y



Gradient direction

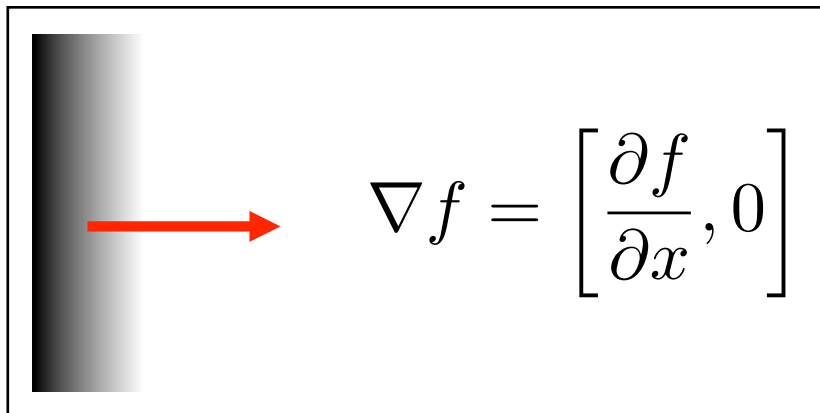
?

Gradient magnitude

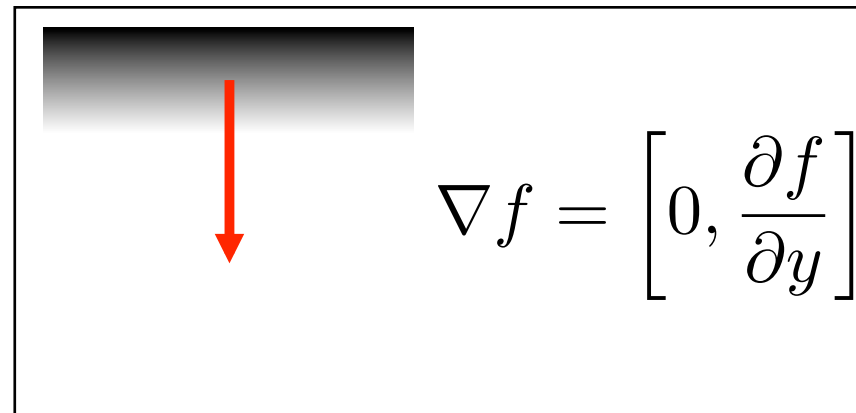
?

Image Gradient

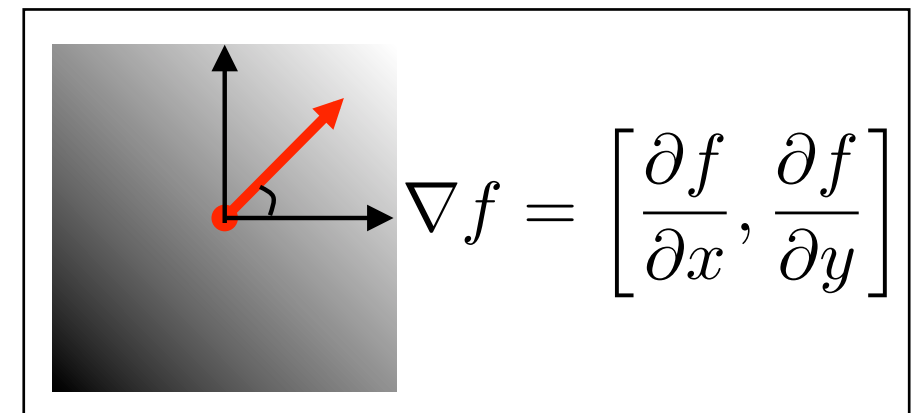
Gradient in x only



Gradient in y only



Gradient in both x and y



Gradient direction

$$\theta = \tan^{-1} \left(\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} \right)$$

Gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$$

How does the gradient direction relate to the edge?

What does a large magnitude look like in the image?

Common 'derivative' filters

Sobel

1	0	-1
2	0	-2
1	0	-1

1	2	1
0	0	0
-1	-2	-1

Scharr

3	0	-3
10	0	-10
3	0	-3

3	10	3
0	0	0
-3	-10	-3

Prewitt

1	0	-1
1	0	-1
1	0	-1

1	1	1
0	0	0
-1	-1	-1

Roberts

0	1
-1	0

1	0
0	-1

MACHINE PERCEPTION OF THREE-DIMENSIONAL SOLIDS

by

LAWRENCE GILMAN ROBERTS

S.B., Massachusetts Institute of Technology
(1961)

M.S., Massachusetts Institute of Technology
(1961)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1963

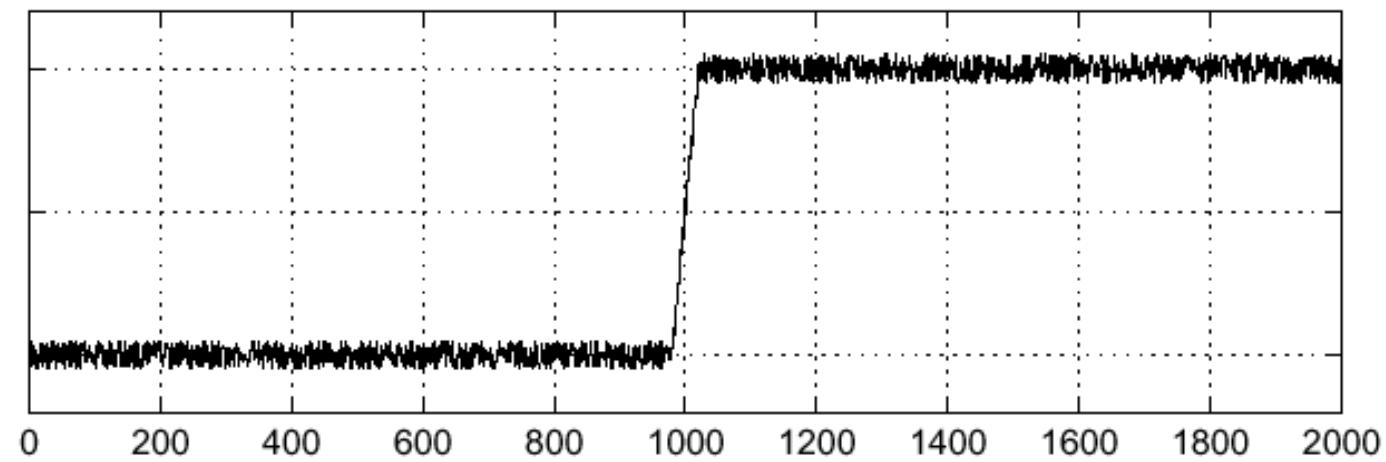
Signature of Author _____

Certified by _____ Thesis Supervisor

Accepted by _____
Chairman, Departmental Committee on Graduate Students

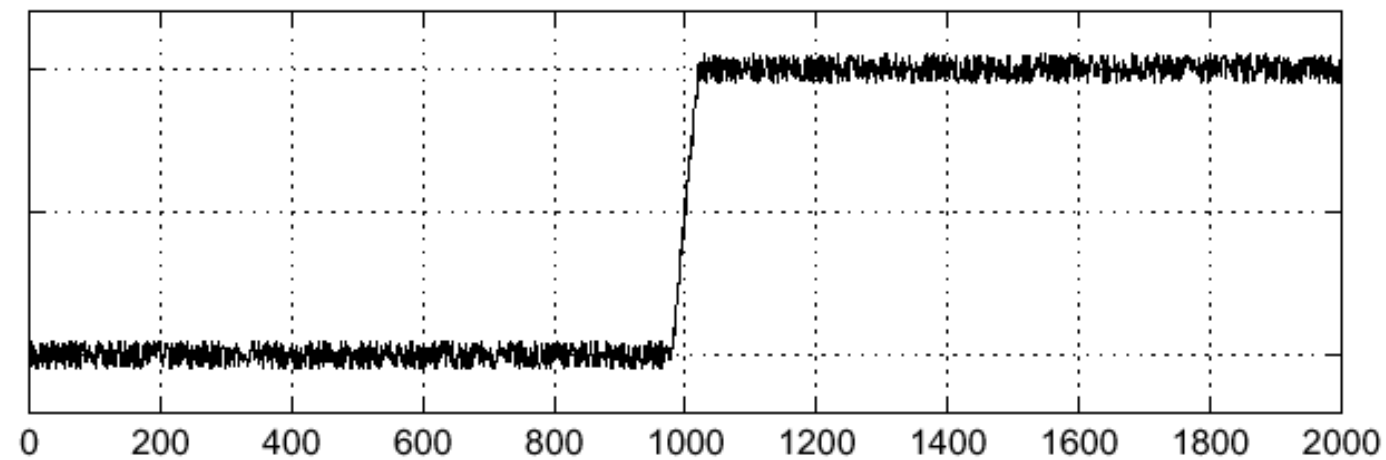
How do you find the edge from this signal?

Intensity plot



How do you find the edge from this signal?

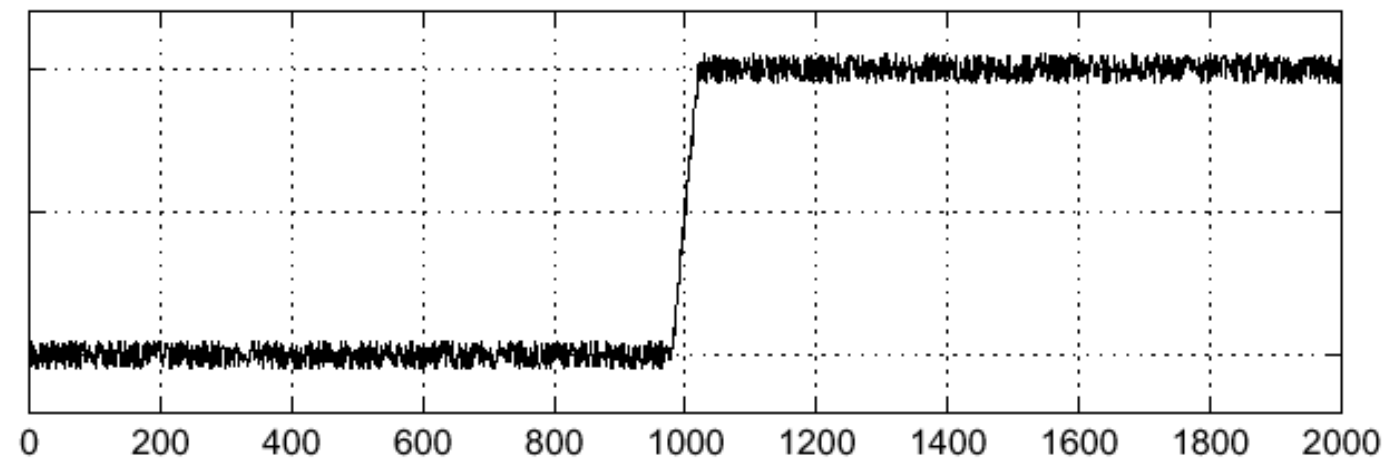
Intensity plot



Use a derivative filter!

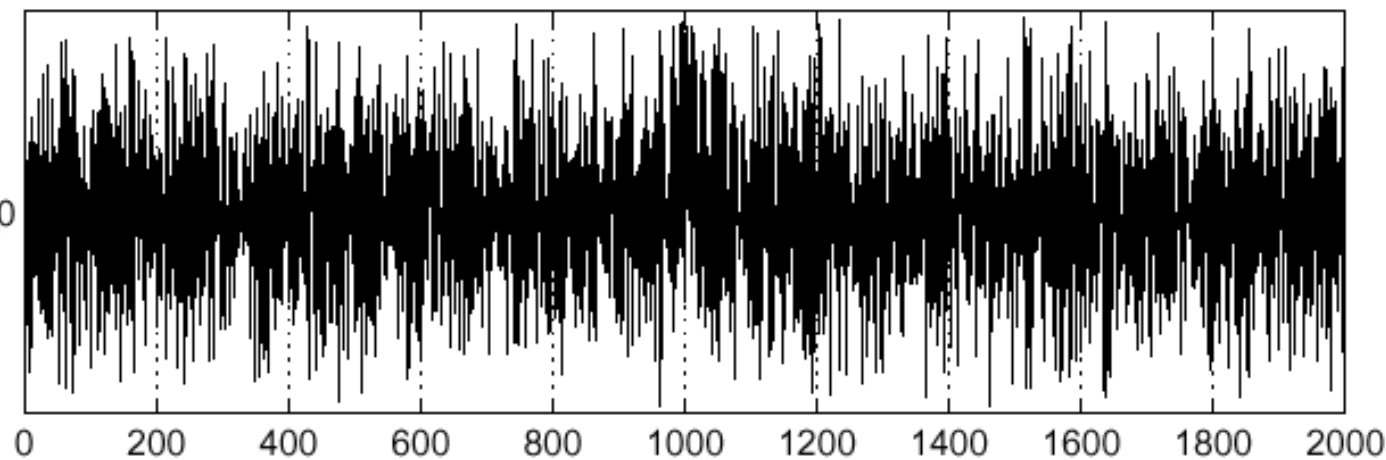
How do you find the edge from this signal?

Intensity plot



Use a derivative filter!

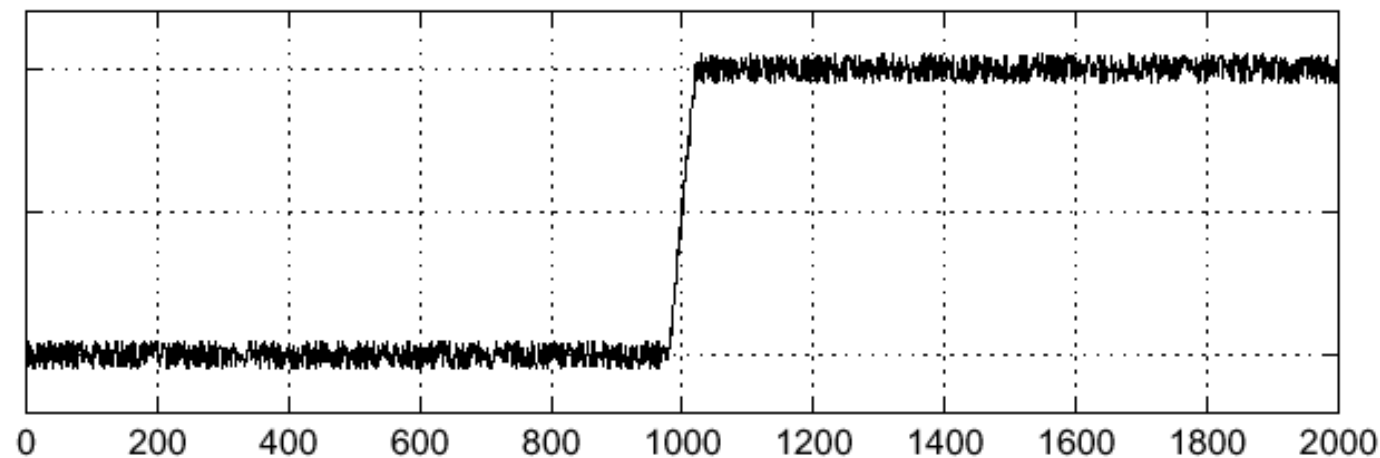
Derivative plot



What happened?

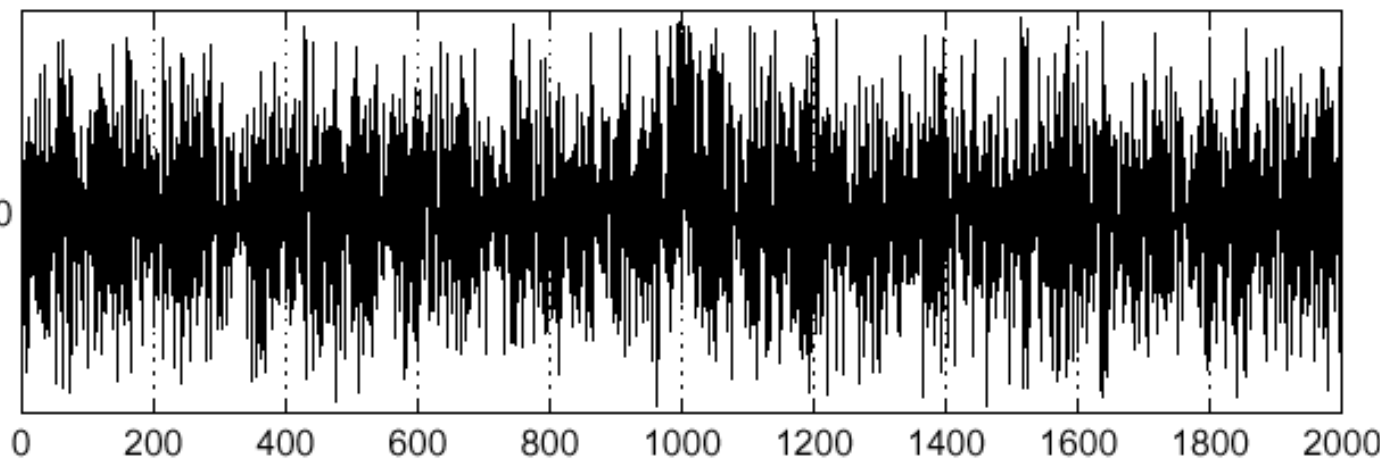
How do you find the edge from this signal?

Intensity plot



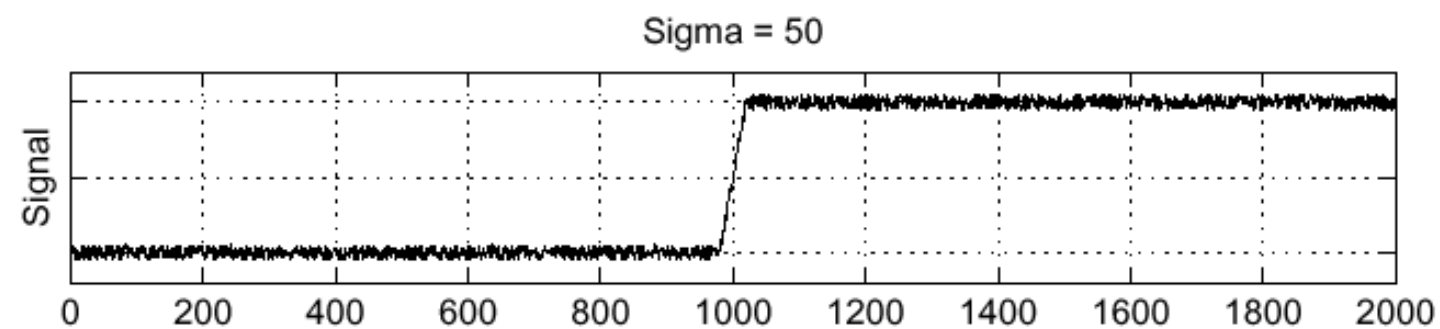
Use a derivative filter!

Derivative plot

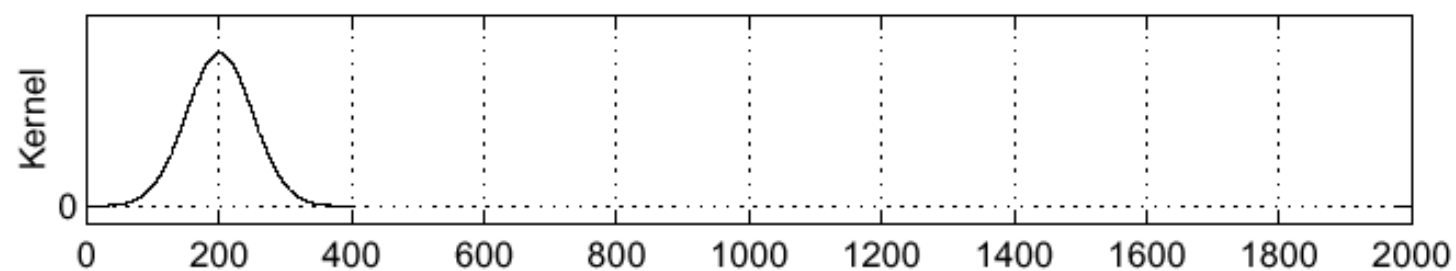


Derivative filters are sensitive to noise

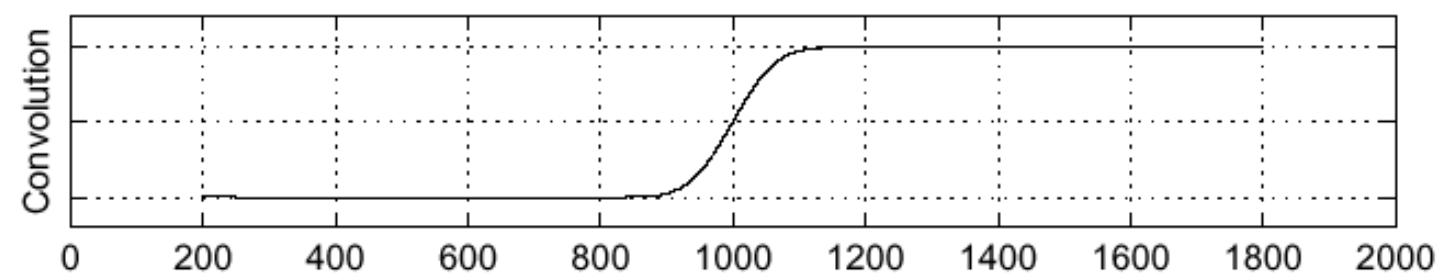
Input



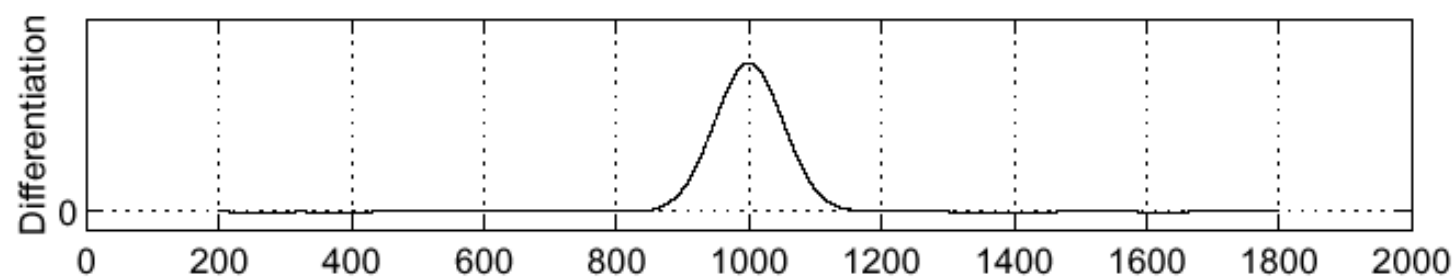
Gaussian



Smoothed input



Derivative

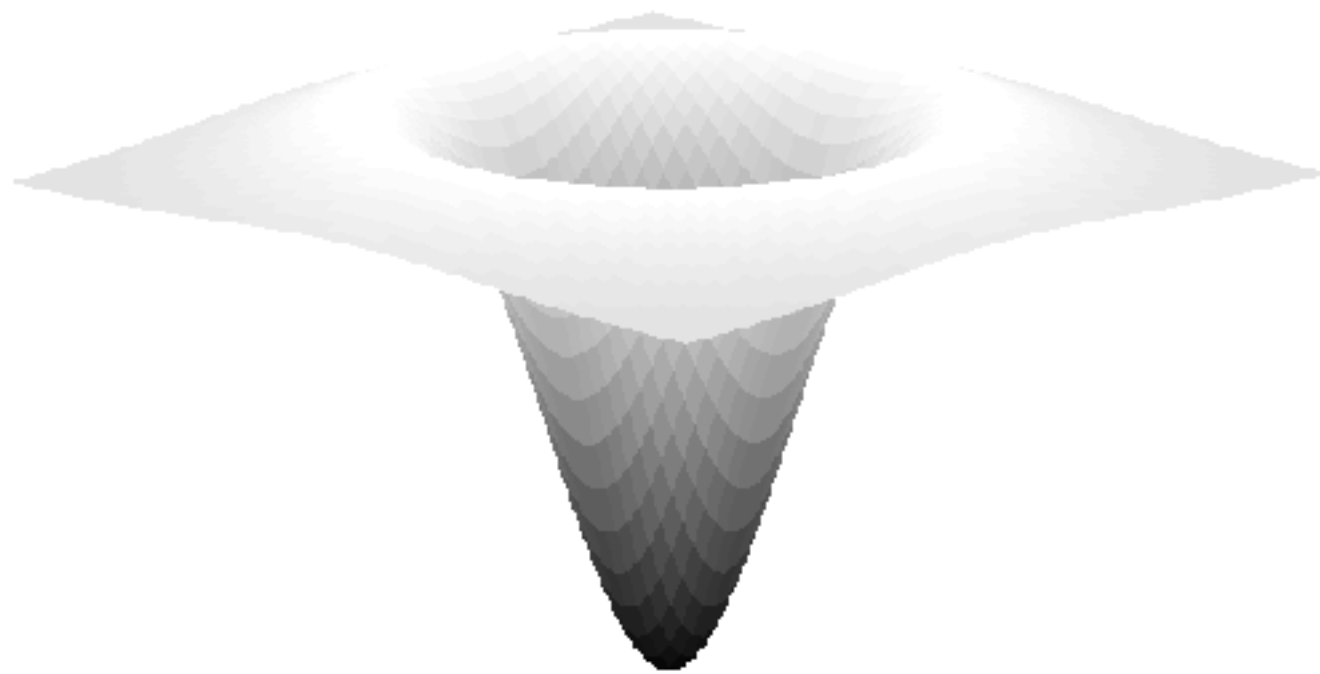


Output

Don't forget to smooth before running derivative filters!

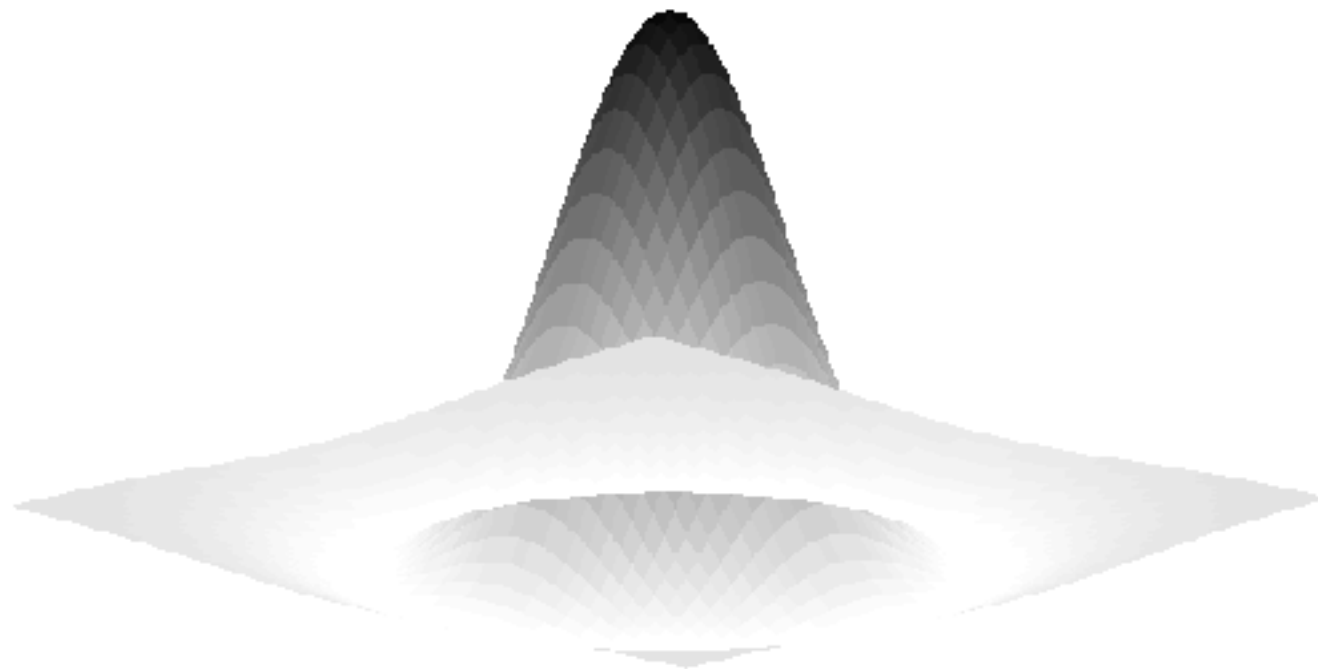
Laplace filter

A.K.A. Laplacian, Laplacian of Gaussian (LoG), Marr filter, Mexican Hat Function



Laplace filter

A.K.A. Laplacian, Laplacian of Gaussian (LoG), Marr filter, Mexican Hat Function



Laplace filter

A.K.A. Laplacian, Laplacian of Gaussian (LoG), Marr filter, Mexican Hat Function



first-order
finite difference

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

derivative filter

1	0	-1
---	---	----

second-order
finite difference

$$f''(x) \approx \frac{\delta_h^2[f](x)}{h^2} = \frac{f(x + h) - 2f(x) + f(x - h)}{h^2}.$$

Laplace filter

?	?	?
---	---	---

first-order
finite difference

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

derivative filter

1	0	-1
---	---	----

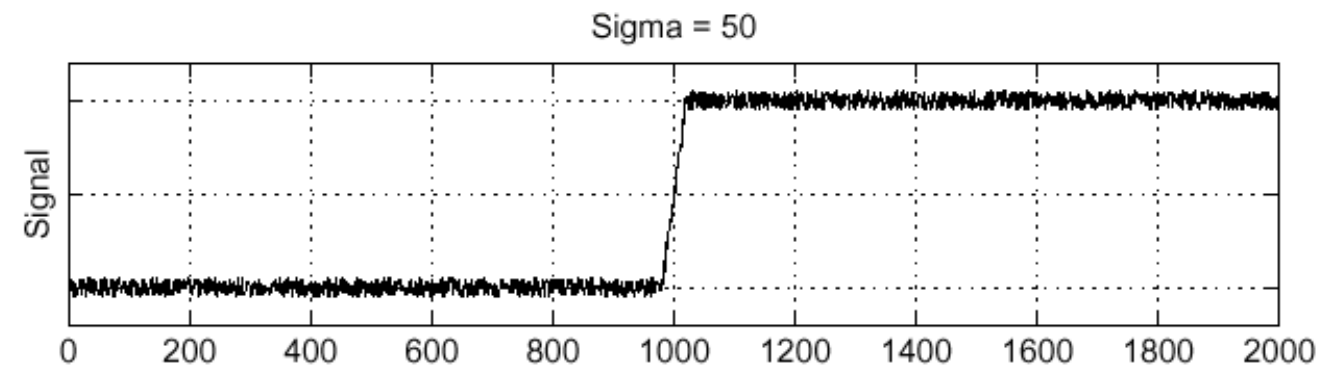
second-order
finite difference

$$f''(x) \approx \frac{\delta_h^2[f](x)}{h^2} = \frac{f(x + h) - 2f(x) + f(x - h)}{h^2}.$$

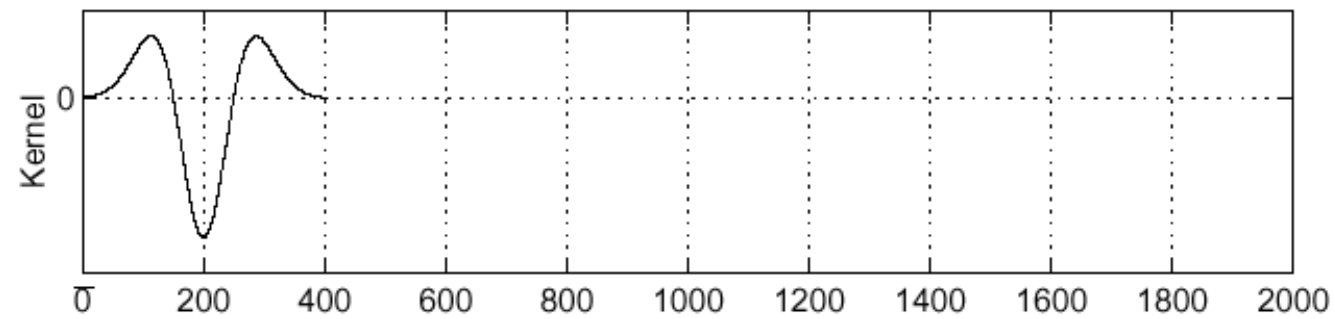
Laplace filter

1	-2	1
---	----	---

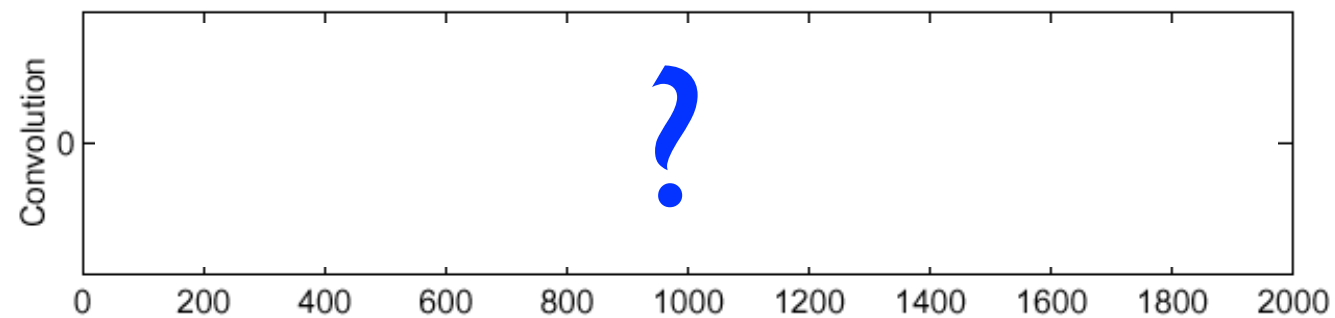
Input



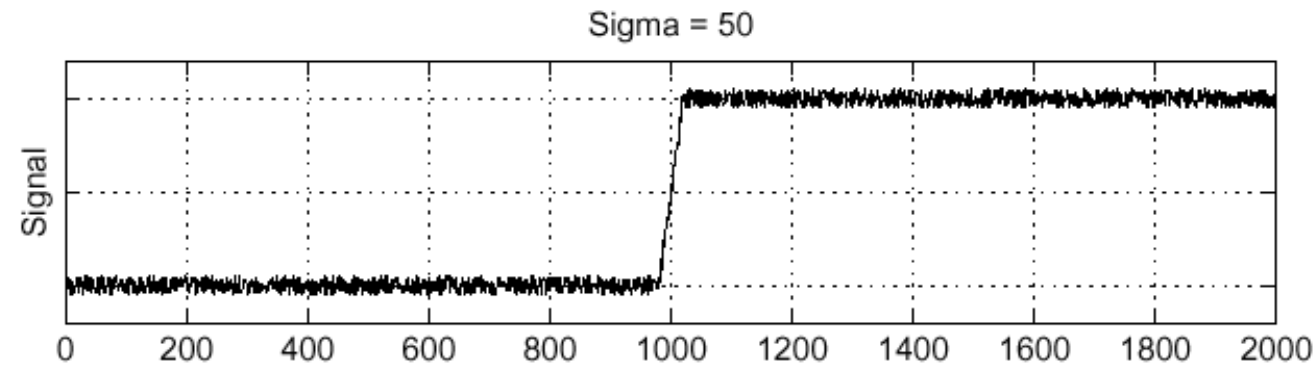
Laplacian



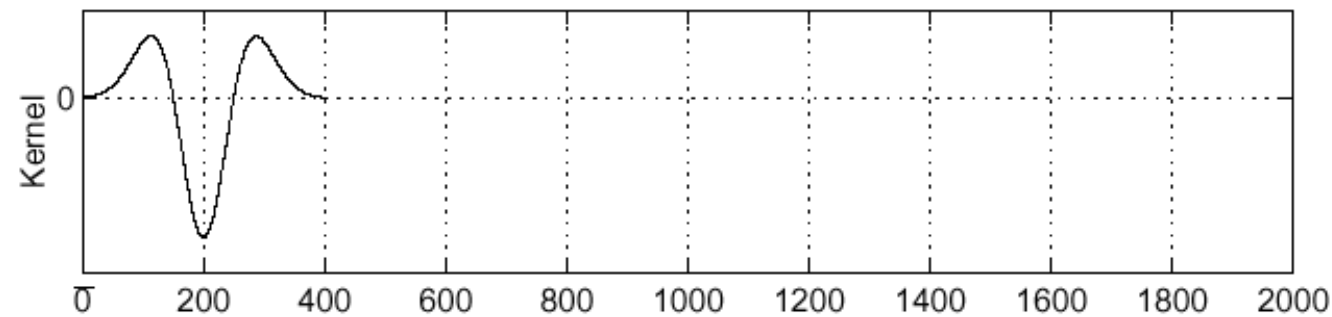
Output



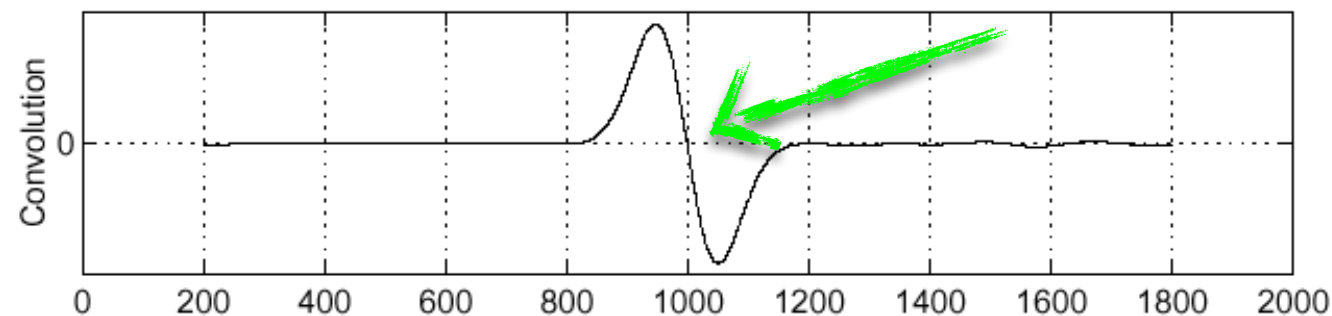
Input



Laplacian



Output



Zero crossings are more accurate at localizing edges
Second derivative is noisy

2D Laplace filter

1	-2	1
---	----	---

1D Laplace filter

?	?	?
?	?	?
?	?	?

2D Laplace filter

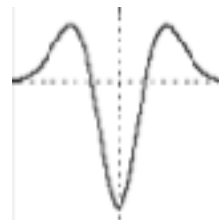
2D Laplace filter

1	-2	1
---	----	---

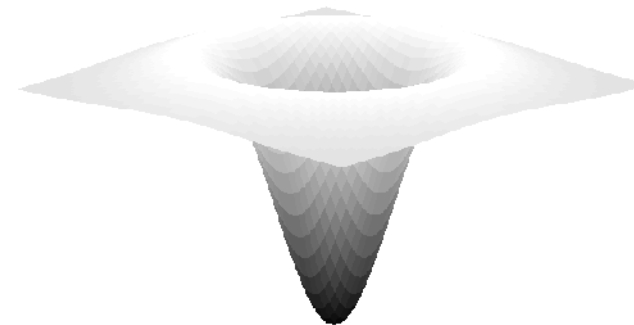
1D Laplace filter

?	?	?
?	?	?
?	?	?

2D Laplace filter



hint



2D Laplace filter

1	-2	1
---	----	---

1D Laplace filter

0	1	0
1	-4	1
0	1	0

2D Laplace filter

If the Sobel filter approximates the first derivative, the Laplace filter approximates?

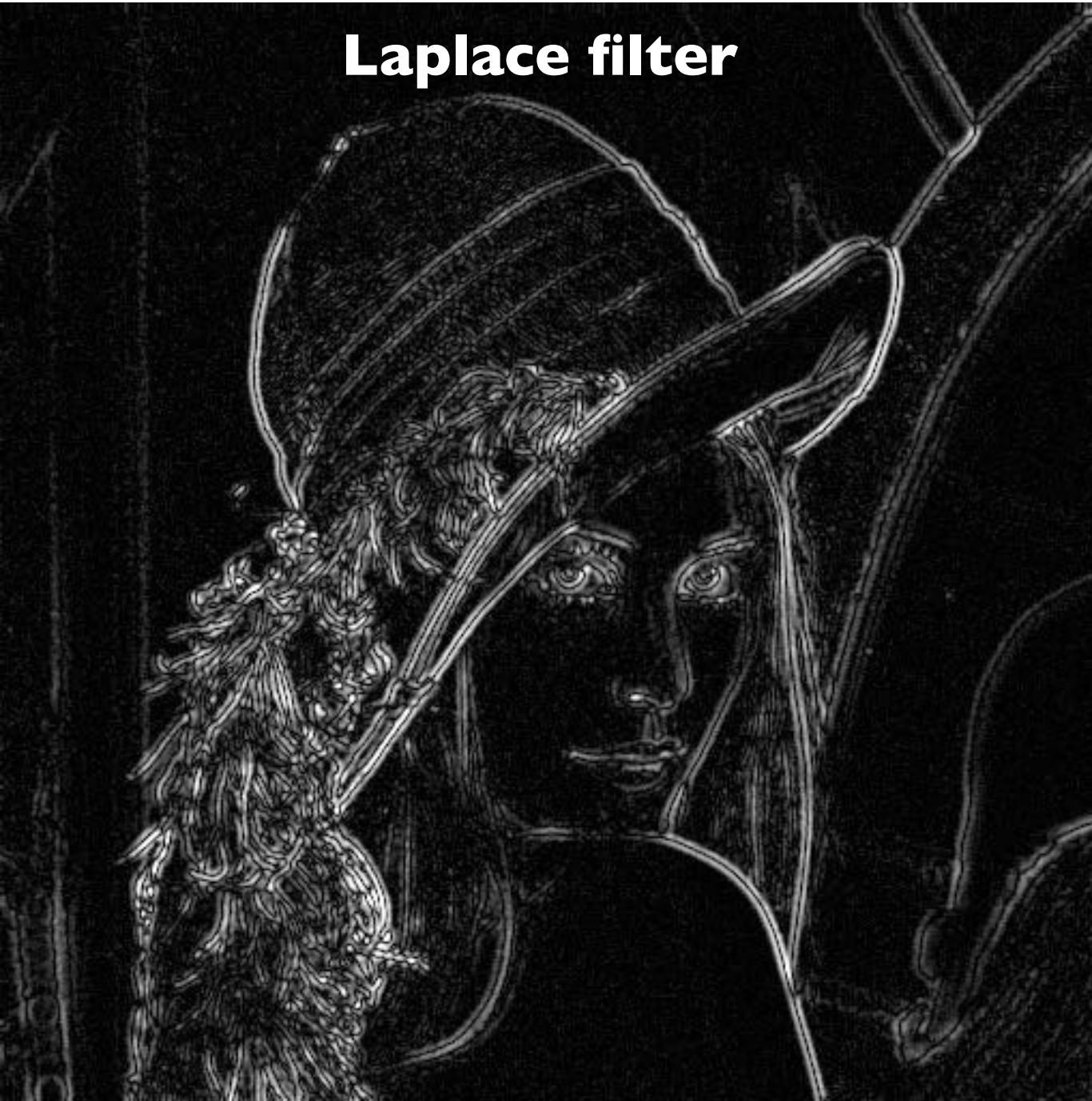
Laplace filter

with smoothing

Laplace filter

without smoothing

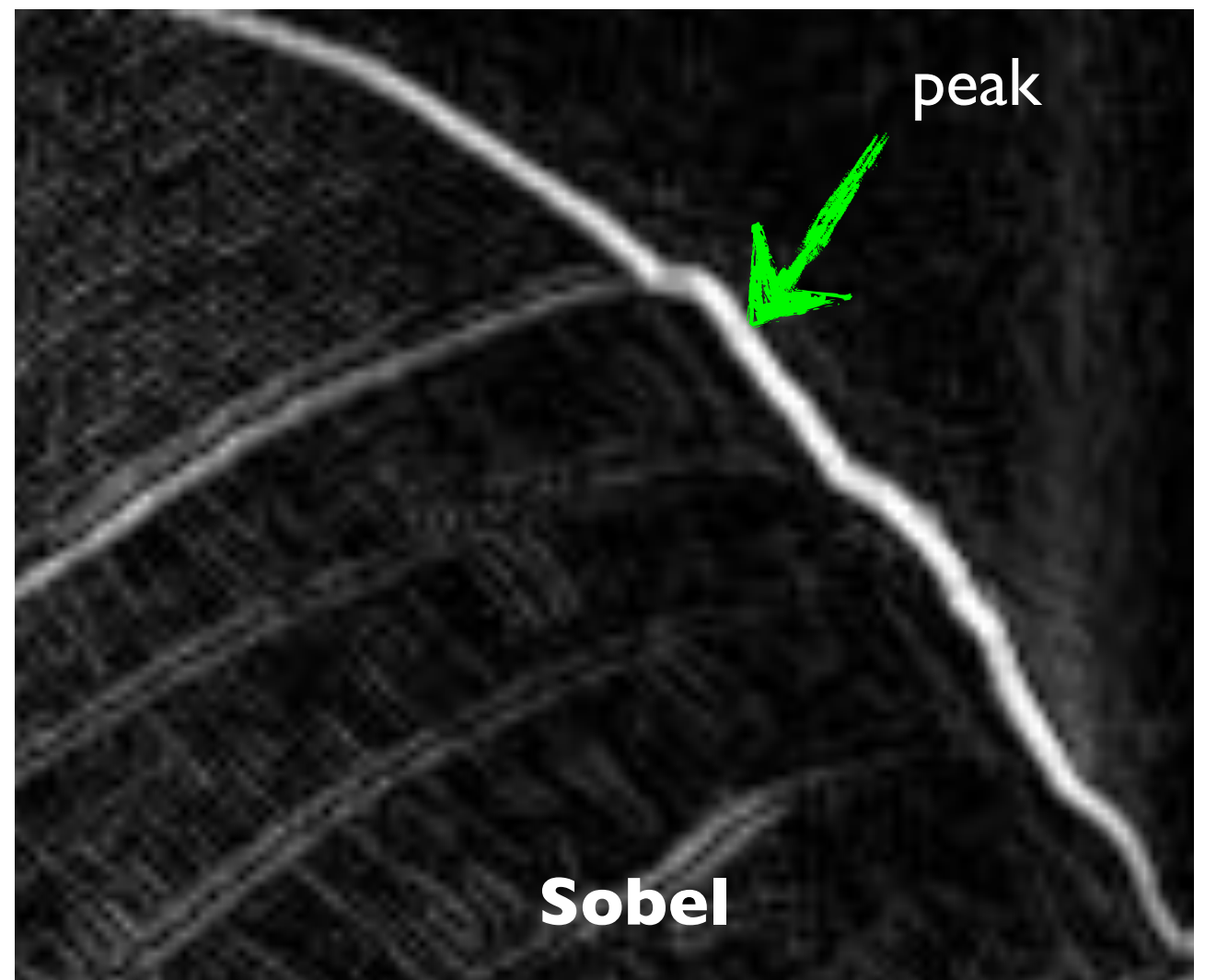
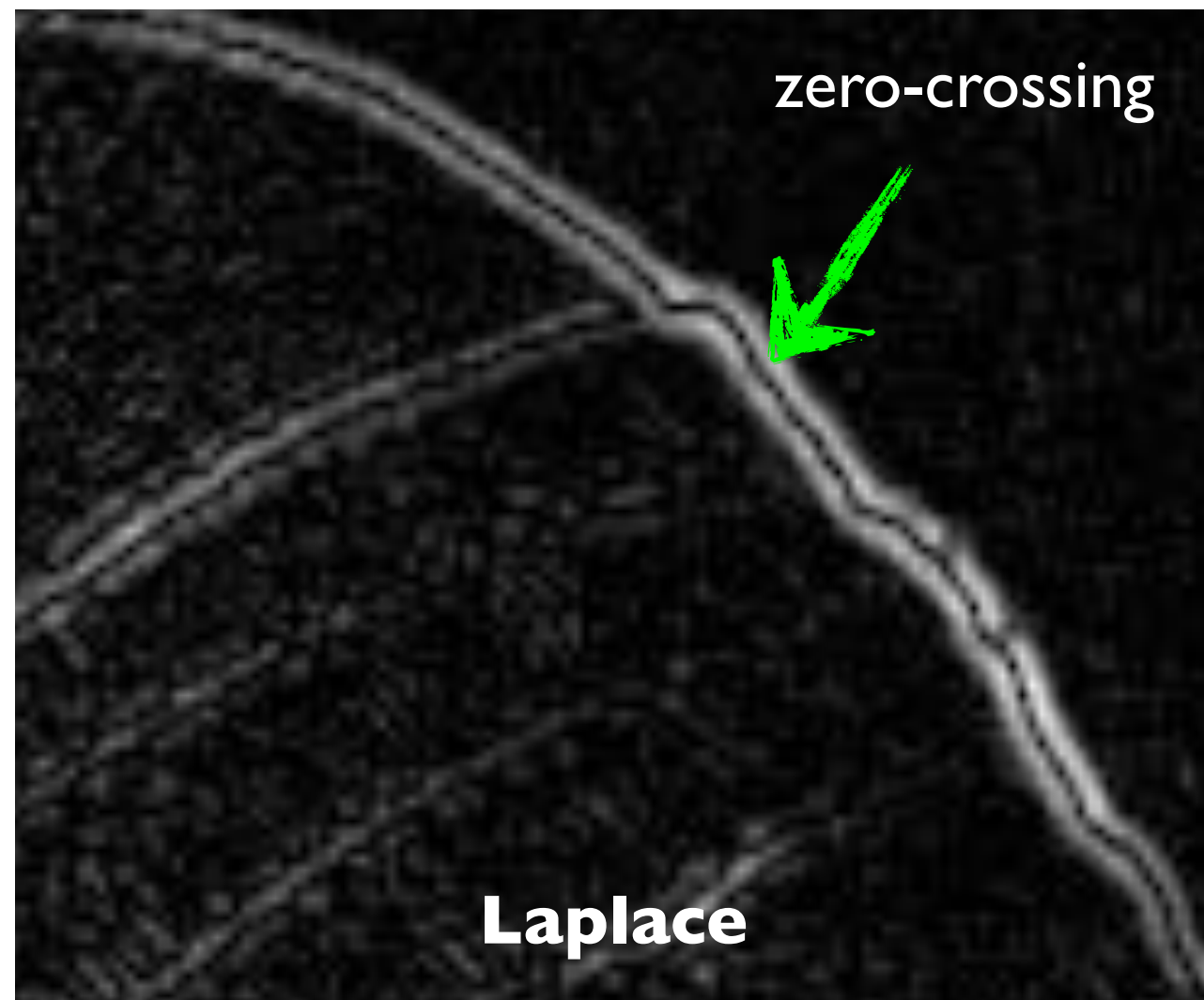
Laplace filter



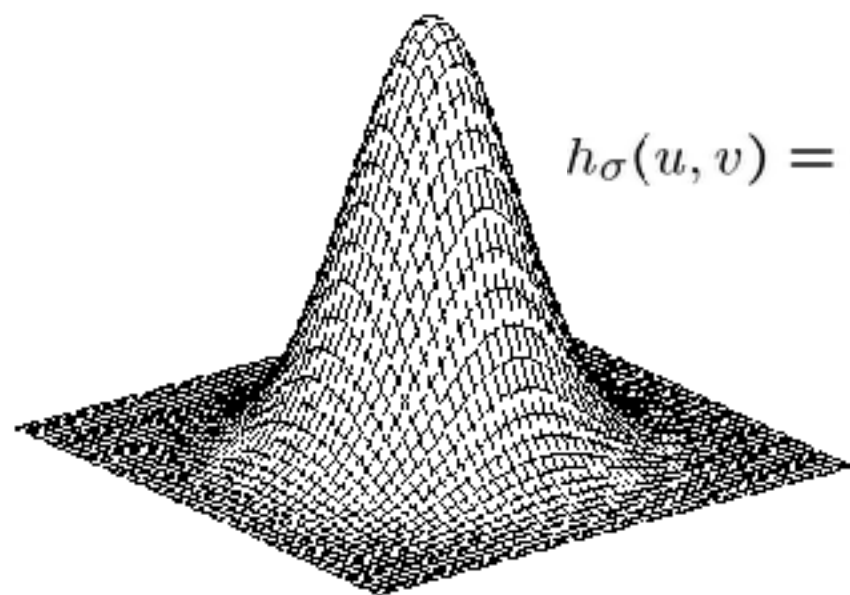
Sobel filter



What's different between the two results?



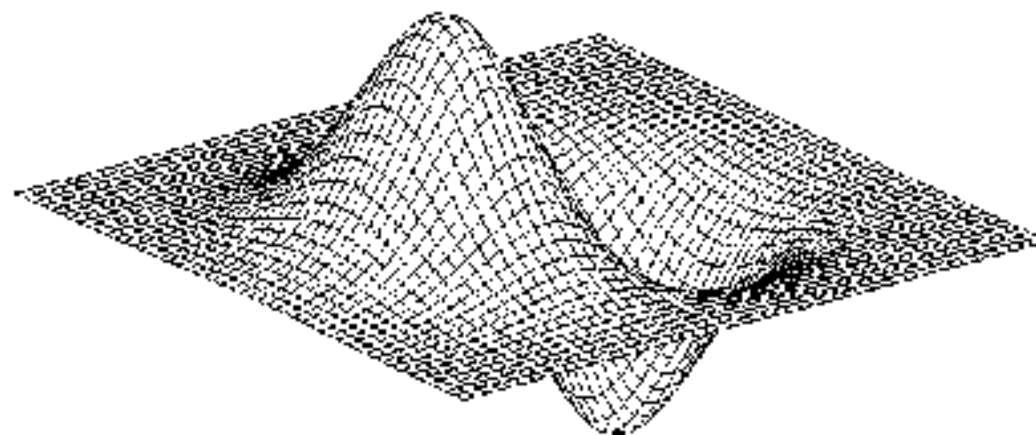
Zero crossings are more accurate at localizing edges
(but not very convenient)



$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

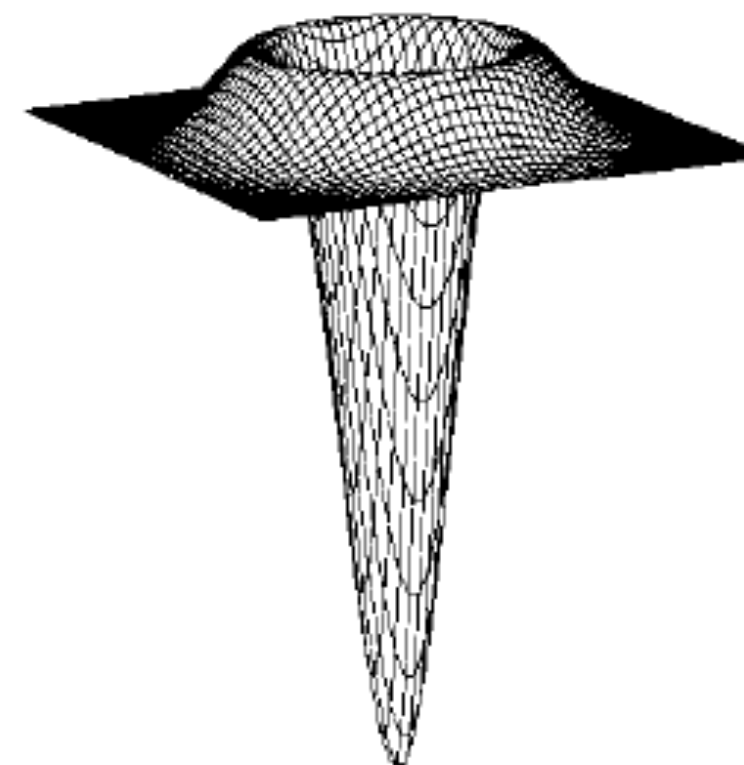
Gaussian

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$



Derivative of Gaussian

$$\nabla^2 h_{\sigma}(u, v)$$



Laplacian of Gaussian

2D Gaussian Filters