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Robot Localization and Mapping
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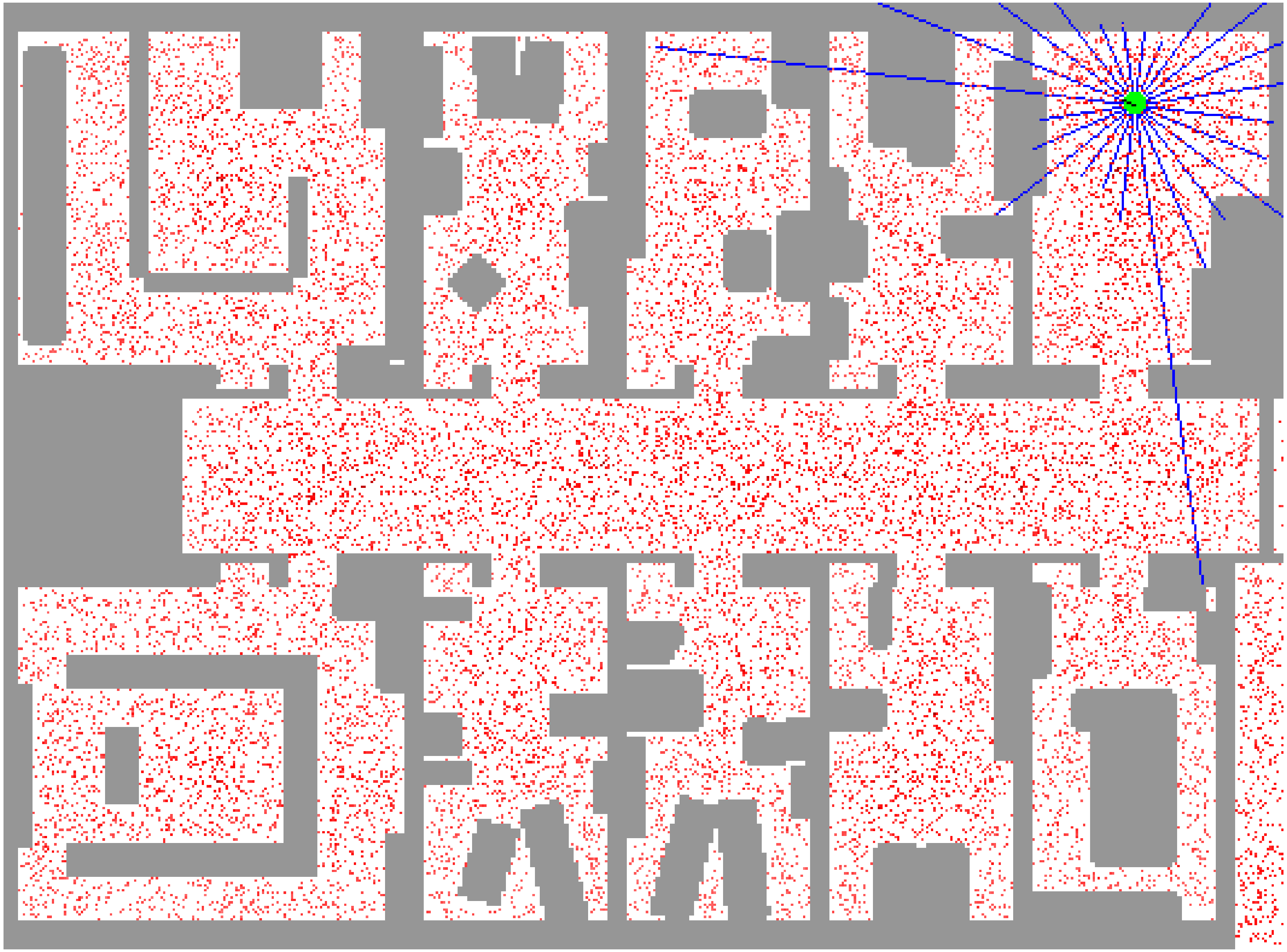
Monte Carlo Localization



Monte Carlo Localization

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Monte Carlo Localization: Efficient Position Estimation for Mobile Robots

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Abstract

This paper presents a new algorithm for mobile robot localization, called Monte Carlo Localization (MCL). MCL is a version of Markov localization, a family of probabilistic approaches that have recently been applied with great practical success. However, previous approaches were either computationally cumbersome (such as grid-based approaches that represent the state space by high-resolution 3D grids), or had to resort to extremely coarse-grained resolutions. Our approach is computationally efficient while retaining the ability to represent (almost) arbitrary distributions. MCL applies sampling-based methods for approximating probability distributions, in a way that places computation "where needed." The number of samples is adapted on-line, thereby invoking large sample sets only when necessary. Empirical results illustrate that MCL yields improved accuracy while requiring an order of magnitude less computation when compared to previous approaches. It is also much easier to implement.

Introduction

Throughout the last decade, sensor-based localization has been recognized as a key problem in mobile robotics (Cox 1991; Borenstein, Everett, & Feng 1996). Localization is a

While the majority of early work focused on the tracking problem, recently several researchers have developed what is now a highly successful family of approaches capable of solving both localization problems: *Markov localization* (Nourbakhsh, Powers, & Birchfield 1995; Simmons & Koenig 1995; Kaelbling, Cassandra, & Kurien 1996; Burgard *et al.* 1996). The central idea of Markov localization is to represent the robot's belief by a probability distribution over possible positions, and use Bayes rule and convolution to update the belief whenever the robot senses or moves. The idea of probabilistic state estimation goes back to Kalman filters (Gelb 1974; Smith, Self, & Cheeseman 1990), which use multivariate Gaussians to represent the robot's belief. Because of the restrictive nature of Gaussians (they can basically represent one hypothesis only annotated by its uncertainty) Kalman-filters usually are only applied to position tracking. Markov localization employs discrete, but *multi-modal* representations for representing the robot's belief, hence can solve the global localization problem. Because of the real-valued and multi-dimensional nature of kinematic state spaces these approaches can only *approximate* the belief, and accurate approximation usually requires prohibitive amounts of computation and memory.

In particular, *grid-based* methods have been devel-

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Probabilistic State Estimation

- Uncertain observations
  - Sensor noise & non-idealities
- Uncertain beliefs
  - Derived from sensor observations
  - Approximate algorithms
- *Probabilistic State Estimation*
  - Identify the quantities (state variables) we care about.
    - e.g., “study time” versus “exam grade”
  - Determine probability for every possible simultaneous assignment

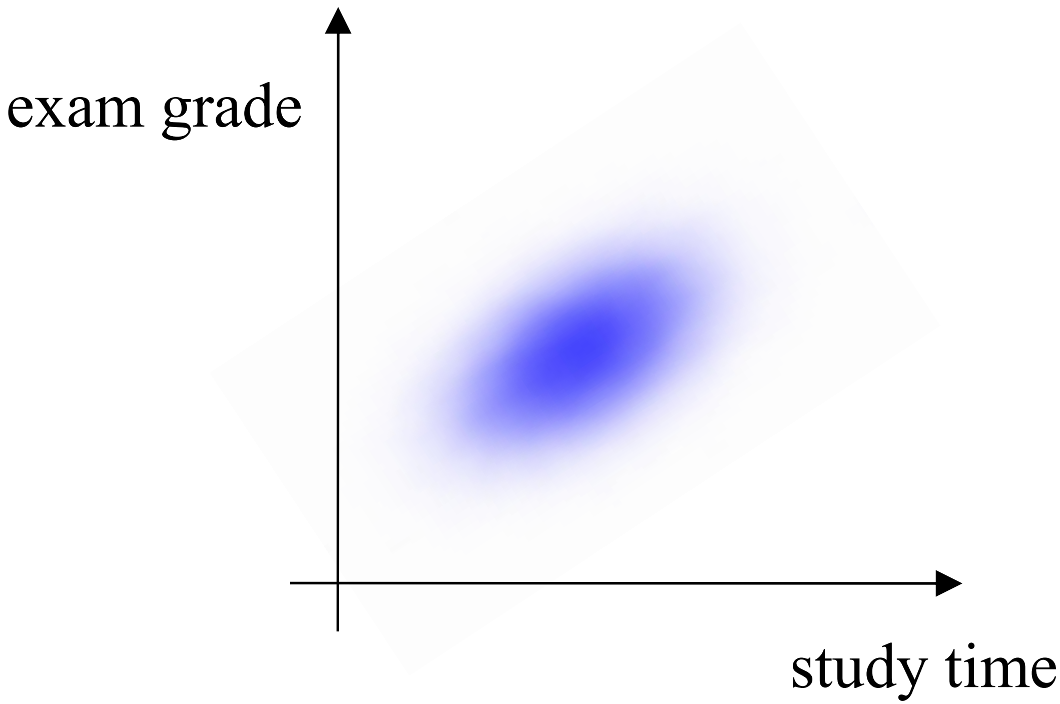
Representing State

- Represent everything we need to know in terms of a vector of quantities
  - "State vector"
  - Usually continuous-valued in this course
- The "meaning" of the variables is up to us
  - e.g., index 7 is the temperature in Seattle.
  - Bookkeeping work for us.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$$

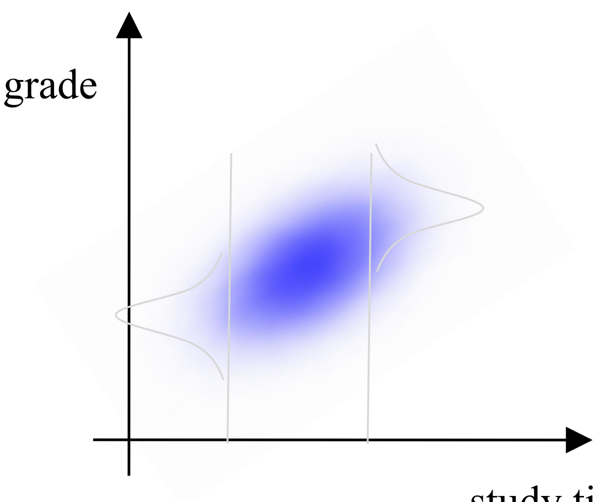
Representing Uncertainty

- In principle, distribution of unknown quantities can be arbitrary



Correlations

- Estimates of variables tend to become correlated over time
  - Observation: Study time is 4 hours
  - Belief about study time *and* exam grade are affected
- Distribution of exam grade depends on study time: two are correlated
  - We'll look at correlations closer later today
  - The data does not necessarily imply any *causal* relationship.



Probability Basics

Discrete Probability	Continuous Probability
P(x) = Probability of event occurring	p(x) = Probability *density* at x
$Prob(x) = P(x)$	$Prob(x) = 0$
$0 \leq P(x) \leq 1$	$0 \leq p(x) < \infty$
$\sum_{-\infty}^{\infty} P(x) = 1$	$\int_{-\infty}^{\infty} p(x)dx = 1$

Probability Basics: Expectation

- Weighted average according to probability

$$\mu_x = E[x] = \int_{-\infty}^{\infty} xp(x)dx$$

- Basic properties of expectation

$$E[\alpha] = \alpha$$

$$E[\alpha x] = \alpha E[x]$$

$$E[\alpha + x] = \alpha + E[x]$$

$$E[x + y] = E[x] + E[y]$$

Joint Expectation

$$E[xy] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, p(x,y)dxdy$$

- Uncorrelated: $E[xy] = E[x]E[y]$
  - Independence → Uncorrelated
  - Uncorrelated $\not\to$ Independence
    - e.g.

$$p(x,y) = \tfrac{1}{4}\delta(x,y-1) + \tfrac{1}{4}\delta(x,y+1) + \tfrac{1}{4}\delta(x-1,y) + \tfrac{1}{4}\delta(x+1,y)$$

- Conditional Expectation: $E[x|y] = \int_{-\infty}^{\infty} x\, p(x|y)dx$
  - $E[x|y] = E[x]$ implies neither independence nor uncorrelatedness
    - e.g. $p(x,y) = \frac{1}{3}\delta(x,y+1) + \frac{1}{3}\delta(x+1,y) + \frac{1}{3}\delta(x-1,y)$

Variance & Covariance

- Average squared deviation from the mean.

- (Auto) covariance
  - Scalar: $$\sigma_x^2 = E[(x - E[x])^2]$$
  - Vector: $$\Sigma_{\mathbf{xx}} = E[(\mathbf{x} - E[\mathbf{x}])(\mathbf{x} - E[\mathbf{x}])^\top]$$

- (Cross) covariance
  - Scalar: $$\sigma_{xy}^2 = E[(x - E[x])(y - E[y])]$$
  - Vector: $$\Sigma_{\mathbf{xy}} = E[(\mathbf{x} - E[\mathbf{x}])(\mathbf{y} - E[\mathbf{y}])^\top]$$

Expectation Exercise

- We know that:

$$\Sigma_{\mathbf{xx}} = E[(\mathbf{x} - E[\mathbf{x}])(\mathbf{x} - E[\mathbf{x}])^\top]$$

- Suppose we measure a bunch of samples of $\mathbf{x}$. We compute the first and second moments of $\mathbf{x}$, i.e.,

$$M_{\mathbf{x}} = \sum \mathbf{x}$$

$$M_{\mathbf{xx}} = \sum \mathbf{x}\mathbf{x}^\top$$

- How do we compute Σ using only these moments and the number of samples?

Projecting Covariances

- Suppose I know $\mathbf{x} \sim \mu_{\mathbf{x}}, \Sigma_{\mathbf{x}}$

- How do we handle $\mathbf{y} = \mathrm{A}\mathbf{x} + \mathbf{b}$???

$$\Sigma_{\mathbf{yy}} = E[(\mathbf{y} - E[\mathbf{y}])(\mathbf{y} - E[\mathbf{y}])^\top]$$

- (Algebra) ➔ $\Sigma_{\mathbf{yy}} = \mathrm{A}\Sigma_{\mathbf{xx}}\mathrm{A}^\top$

Properties of the Covariance Matrix

$$\Sigma = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

- Symmetric

  $B = C^\top$ why?

- Positive (semi) definite

  $\mathbf{a}^\top \Sigma \mathbf{a} \geq 0$ why?

- Inverse is also positive definite
  - Proof: see next slide

- Determinant → Volume of uncertainty (Product of the Eigenvalues)

Positive (Semi) Definite Properties

1. If $A \geq 0$ and $B \geq 0$ then $A+B \geq 0$

$$\mathbf{x}^\top (A + B)\mathbf{x} = \mathbf{x}^\top A\mathbf{x} + \mathbf{x}^\top B\mathbf{x}$$

2. If either A <u>or</u> B is positive definite, then so is $A+B$; this follows from 1.

3. If $A>0$, then $A^{-1}>0$

$$\mathbf{x}^\top A\mathbf{x} = \mathbf{x}^\top A A^{-1} A\mathbf{x} = (A\mathbf{x})^\top A^{-1}(A\mathbf{x}) > 0 \quad \text{if} \quad \mathbf{x} \neq 0$$

4. If $A \geq 0$, then $F^T A F \geq 0$ for <u>any</u> (not necessarily square) matrix F for which $F^T A F$ is defined.

$$\mathbf{x}^\top (F^\top A F)\mathbf{x} = (F\mathbf{x})^\top A(F\mathbf{x}) \geq 0$$

5. If $A>0$ and F invertible, then $F^T A F>0$. This follows from 3 and 4.

Correlation Coefficient

- The correlation coefficient is defined as:

$$\rho_{xy} = \frac{\mathrm{cov}(x, y)}{\sigma_x \sigma_y} \qquad |\rho_{xy}| \leq 1$$

- Covariance matrix in terms of correlation coefficients

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \rho_{n2}\sigma_n\sigma_2 & \cdots & \sigma_n^2 \end{bmatrix}$$