bel := belief

or

and

Not

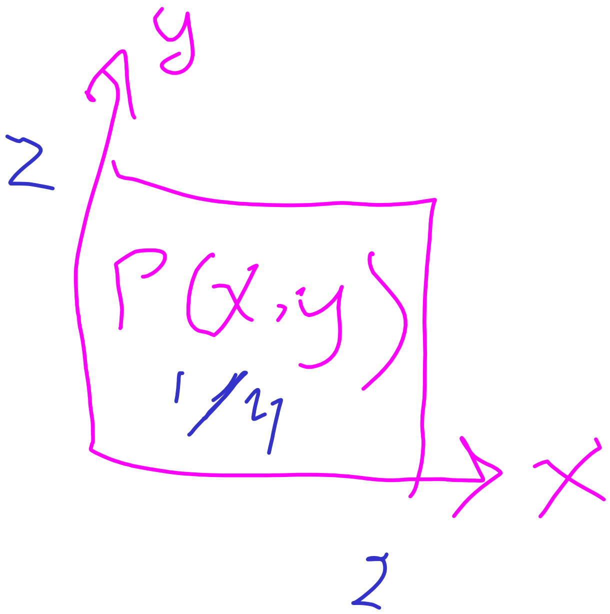
using case to denote discrete vs. continuous

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$P(X \leq x) = \int_{-\infty}^{x} p(x')\,dx'$$

$$p(x) = \frac{d}{dx} P(X \leq x)$$

Joint and conditional probability
P(x,y) uniform over square




$$P(1,1) = \frac{1}{4},$$

P(x,y) = 1/4 if 0<x<2 ; 1/4 if 0 < y <2 ; 0 otherwise

$$P(x=1, y=1) = 0$$

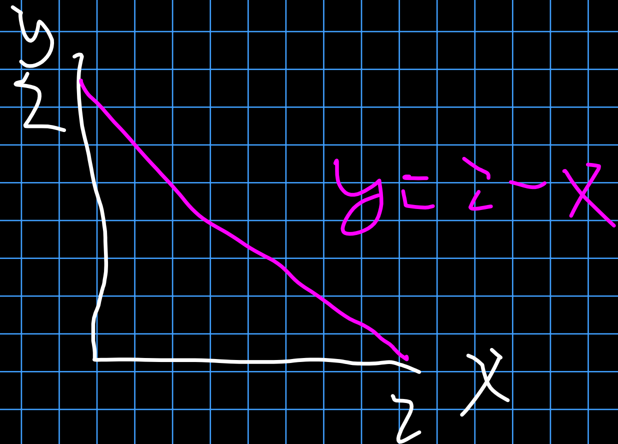
$$p(x) = \int_y P(x,y)\,dy$$

$$= \int_0^2 \frac{1}{4}\, dy = \begin{cases} \frac{1}{2} & 0 < x < 2 \\ & 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$P(x \mid y) = \frac{P(x, y)}{P(y)} \qquad \begin{matrix} \longrightarrow 1/4 \\ \\ \longrightarrow 1/2 \end{matrix}$$

$$= \begin{cases} 1/2 & \text{if } 0 < x < 2 \\ \text{"} & \phantom{\text{if }} 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

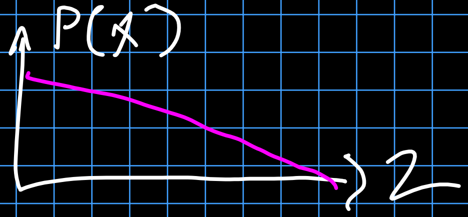
$P(x \mid y) = P(x)$ if $x, y$ independent

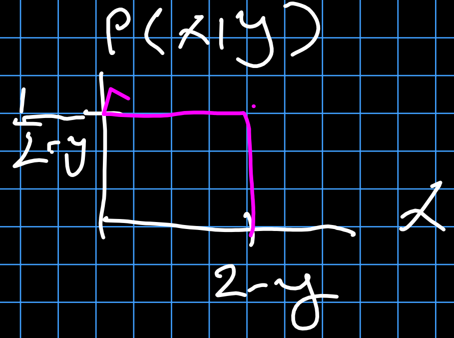



$$P(x,y) = \begin{cases} 1/2 & \text{if } 0<x<2 \\ & \quad 0<y<2 \\ 0 & \text{otherwise} \end{cases}$$

$$P(x) = \int_0^{2-x} \frac{1}{2}\,dy = (2-x)\frac{1}{2} \quad \text{if } 0<x<2$$

$$P(x|y) = \frac{\frac{1}{2}}{(2-x)\frac{1}{2}} = \begin{cases} \frac{1}{2-x} & \text{if } 0<y<2 \\ & \quad 0<x<2-y \\ 0 & \text{otherwise} \end{cases}$$




TL;DR

TLDR: x y not independent, p(x) depends on y vice versa