- A Bayesian Model of the DNA Barcode Gap
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8 Abstract

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₂ 1 Introduction

Since its inception over 20 years ago, DNA barcoding (Hebert et al., 2003a,b) has emerged 13 as a robust method of specimen identification and species delimitation across myriad 14 taxonomic groups which have been sequenced at short, standardized gene regions like 5'-COI 15 for animals. However, the success of the approach depends crucially on two important factors: 16 (1) the availability of high-quality specimen records found in public reference sequence 17 databases such as the Barcode of Life Data Systems (BOLD) (Ratnasingham and Hebert, 18 2007), and (2) the establishment of a DNA barcode gap — the idea that the maximum 19 genetic distance observed within species is much smaller than the minimum degree of marker 20 variation found among species (Meyer and Paulay, 2005; Meier et al., 2008). Early work has demonstrated that the presence of a DNA barcode gap hinges strongly on extant levels of species haplotype diversity gauged from comprehensive specimen sampling at wide geographic and ecological scales. Despite this, many taxa lack adequate separation in their pairwise intraspecific and interspecific genetic distances, thereby compromising rapid matching of unknown samples to expertly-validated references.

Recent work has argued that DNA barcoding, in its current form, is lacking in statistical rigor, calling into question the existence of a true species' DNA barcode gap (Phillips et al., 2022). To support this notion, novel nonparametric locus-specific metrics based on the multispecies coalescent (Rannala and Yang, 2003; Yang and Rannala, 2017) were recently outlined and shown to hold strong promise when applied to predatory *Agabus* (Coleoptera: Dytiscidae) diving beetles (Phillips et al., 2024). The coalescent (Kingman, 1982)

encompasses a backwards continuous-time stochastic Markov process of allelic sampling within natural, neutrally-evolving, species populations towards the Most Recent Common Ancestor (MRCA). The metrics quantify the extent of asymmetric directionality of proportional genetic distance distribution overlap/separation for species within well-sampled 36 taxonomic genera based on a straightforward distance count. The metrics can be employed in a variety of ways, including to assess performance of marker genes for species identication, as well as to assess whether computed values are consistent with population genetic-level parameters like effective population size (N_e) , mutation rates (μ) and divergence times (τ) for species under study (Mather et al., 2019). However, what appears to be missing is an unbiased way to compute the statistical accuracy of the recommended estimators arising through problems inherent in frequentist maximum likelihood estimation for discrete probability distributions having bounded positive support on [0, 1]. To this end, here, a Bayesian model of the DNA barcode gap coalescent is introduced to rectify such issues. The 45 model allows accurate estimation of posterior means, posterior standard deviations, posterior quantiles, and credible intervals for the metrics given datasets of intraspecific and interspecific genetic distances for species of interest.

$_{49}$ 2 Methods

50 2.1 DNA Barcode Gap Metrics

Recently, Phillips et al. (2024) proposed novel nonparametric maximum likelihood estimators (MLEs) of proportional overlap/separation between intraspecific and interspecific pairwise genetic distance distributions for a given species (x) to aid assessment of the DNA barcode gap as follows:

$$p_x = \frac{\#\{d_{ij} \ge a\}}{\#\{d_{ij}\}} \tag{1}$$

$$q_x = \frac{\#\{d_{XY} \le b\}}{\#\{d_{XY}\}} \tag{2}$$

where d_{ij} and d_{XY} are distances within and among species, respectively, and the notation # reflects a count (**Figure 1**). Quantities a and b correspond to $\min(d_{XY})$ and $\max(d_{ij})$, 56 the minimum interspecific distance and the maximum intraspecific distance, respectively. 57 Notice that a and b are also the first and nth order statistics, respectively. Distances 58 are easily computed from a model of DNA sequence evolution, such as uncorrected or 59 corrected p-distances (Jukes and Cantor, 1969; Kimura, 1980). Similar expressions (denoted $p_x^{'}$ and $q_x^{'}$) for nearest neighbour species were also given (see Phillips et al. (2024)), in which d_{XY} included only interspecific distances between the species of interest and its closest neighbouring species. If a focal species is found to have multiple nearest neighbours, then 63 the species possessing the smallest average pairwise interspecfic distance is used. While these schemes differ considerably from the usual definition of the DNA barcode gap laid out by 65 Meyer and Paulay (2005) and Meier et al. (2008), they more accurately account for species' coalescence histories inferred from contemporaneous samples of DNA sequences. such as 67 interspecific hybridization/introgression events (Phillips et al., 2024). Note, distances (and hence the metrics) are constrained to the closed interval [0, 1]. Values of the estimators obtained from equations (1) and (2) close to or equal to zero give evidence for separation 70 between intraspecific and interspecific genetic distance distributions; that is, values suggest the presence of a DNA barcode gap for a target species. Conversely, values near or equal 72 to one give evidence for distribution overlap; that is, values likely indicate the absence of a 73 gap. Equations (1) and (2) can be expressed in terms of empirical cumulative distribution 74 functions (ECDFs)

$$p_x = \mathbb{P}(d_{ij} \ge a) = 1 - \hat{F}_{d_{ij}}(a) = \hat{F}_{d_{ij}}(b) - \hat{F}_{d_{ij}}(a)$$
(3)

$$q_x = \mathbb{P}(d_{XY} \le b) = \hat{F}_{d_{XY}}(b), \tag{4}$$

noting that $\hat{F}_{d_{XY}}(a) = 0$ (**Figure 1**). Given n increasing-ordered data points, the ECDF $\hat{F}_n(t) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}_{[x_i \leq t]}$ comprises a step function having jump discontinuities of $\frac{1}{n}$ at each sample observation (x_i) , excluding ties, where $\mathbb{1}(x)$ is the indicator function. From here, the asymmetric directionality of the metrics is obvious. As mentioned previously, similar equations for p'_x and q'_x can be easily derived.

81 2.2 A Bayesian Implementation

A major criticism of large sample (frequentist) theory is that it relies on asymptotic properties of the MLE (which is assumed to be a fixed but unknown quantity), such as estimator normality and consistency. This problem is especially pronounced in the case of binomial proportions. The estimated Wald SE of the sample proportion, is given by

$$\widehat{SE[\hat{p}]} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\tag{5}$$

where $\hat{p} = \frac{Y}{n}$ is the MLE, Y is the number of successes $(Y = \sum_{i=1}^{n} y_i)$ and n is the number of trials (i.e., sample size). However, the above formula is problematic for several reasons. First, Equation (5) makes use of the Central Limit Theorem (CLT); thus, large sample sizes are required for reliable estimation. When few observations are available, SEs will be large and inaccurate, leading to low statistical power. Further, resulting interval estimates could span values less than zero or greater than one, or have zero width, which is practically meaningless. Second, when proprtions are exactly equal to zero or one, resulting SEs will be exactly zero, rendering Equation (5) completely useless. In the context of the proposed DNA barcode gap metrics, values obtained at the boundaries of their support are often

encountered. Therefore, reliable calculation of SEs is not feasible. Given the importance of sufficient sampling of species genetic diversity for DNA barcoding initiatives, a different statistical estimation approach is necessary. Bayesian inference offers a natural path forward 97 in this regard since it allows for straightforward specification of prior beliefs concerning 98 unknown model parameters and permits the seamless propagation of uncertainty, when data is lacking, through integration with the likelihood function associated with true generating processes. As a consequence, Bayesian models are much more flexible and generally more 101 easily interpretable compared to frequentist approaches since entire posterior distributions, 102 along with their summaries, are outputted, rather than just sampling distributions, p-values, 103 and confidence intervals, allowing direct probability statements to be made. 104

$_{\scriptscriptstyle{5}}$ 2.3 The Model

Essentially, from a statistical perspective, the goal herein is to nonparametrically estimate 106 probabilities corresponding to extreme tail quantiles for positive highly skewed distributions 107 on the unit interval. Here, it is sought to numerically approximate the extent of 108 overlap/separation of intraspecific and interspecific pairwise genetic distance distributions 109 within [a, b]. This is a challenging computational problem within the current study as detailed 110 in subsequent sections. Counts, y, of overlapping genetic distances (as expressed in the 111 numerator of Equations (1) and (2)) are treated as binomially distributed with expectation 112 $\mathbb{E}[Y] = k\theta$, where $k = \{N, C\}$ are total count vectors of intraspecific and combined genetic 113 distances, respectively, for a target species along with its nearest neighbour species, and 114 k=M is a total count vector for all pairwise species comparisons. The quantity 115 $\theta = \{p_x, q_x, p_x^{'}, q_x^{'}\}.$ The metrics encompassing θ are presumed to follow a beta (α, β) distribution, with real shape parameters α and β , which is a natural choice of prior on 117 probabilities. Such a scheme is quite convenient since the beta distribution is conjugate to the 118 binomial distribution. Thus, the posterior distribution is also beta distributed. Parameters 119 were given an uninformative Beta(1, 1) prior, which is equivalent to a standard uniform 120

(Uniform(0, 1)) prior since it places equal probability on all parameter values within its support. As a result, the posterior is Beta(Y+1, n-Y+1), from which various moments and other quantities, such as the expected value $\mathbb{E}[Y] = \frac{Y+1}{n+2}$ and variance $\mathbb{V}[Y] = \frac{(Y+1)(n-Y+1)}{(n+2)^2(n+3)}$, can be easily calculated. In general however, when possible, it is always advisable to incorporate prior information, even if only weak, rather than simply imposing complete ignorance in the form of a flat prior distribution. With sufficient data, the choice of prior distribution becomes less important since the posterior will be dominated by the likelihood. The full univariate Bayesian model for species x is thus given by

$$\begin{aligned} y_{\text{lwr}} &\sim \text{Binomial}(N, p_{\text{lwr}}) \\ y_{\text{upr}} &\sim \text{Binomial}(M, p_{\text{upr}}) \\ y_{\text{lwr}}' &\sim \text{Binomial}(N, p_{\text{lwr}}') \\ y_{\text{upr}}' &\sim \text{Binomial}(C, p_{\text{upr}}') \\ p_{\text{lwr}}, p_{\text{upr}}, p_{\text{lwr}}', p_{\text{upr}}' &\sim \text{Beta}(1, 1). \end{aligned} \tag{6}$$

The model, which is inherently vectorized to allow processing of multiple species datasets 129 simultaneously, was fitted using the Stan probabilistic programming language (Carpenter 130 et al., 2017) framework for Hamiltonian Monte Carlo (HMC) via the No-U-Turn Sampler 131 (NUTS) sampling algorithm (Hoffman and Gelman, 2014) through the rstan R package (Stan 132 Development Team, 2023). Four chains were run for 2000 iterations each in parallel across 133 four cores with random parameter initializations. Within each chain, a total of 1000 samples 134 was discarded as warmup (i.e., burnin) to reduce dependence on starting conditions. Further, 135 1000 post-warmup draws were utilized per chain. Because HMC/NUTS results in dependent 136 samples that are minimally autocorrelated, Markov chain thinning is not required. Each of 137 these reflect default MCMC settings in Stan. Since the DNA barcode gap metrics often attain 138 values very close to zero and/or very near one, in addition to more intermediate values, a 139

noninformative Beta $(\frac{1}{2}, \frac{1}{2})$ prior, which is U-shaped symmetric and places greater probability density at the extremes of the distribution due to its heavier tails, while still allowing for variability in parameter estimates within intermediate values along its domain, was also attempted. However, this resulted in several divergent transitions, among other pathologies, imposed by complex geometry (*i.e.*, curvature) in the posterior space, despite remedies to resolve them, such as lowering the step size of the HMC/NUTS sampler. Note that this prior is Jeffreys' prior, which is proportional to the square root of the Fisher information and has several desirable statistical properties, most notably invariance to reparameterization.

To validate the overall correctness of the proposed statistical model given by Equation (4), in addition to generating MLEs as a means of comparison, posterior predictive checks were also employed to generate binomial random variates in the form of counts from the posterior predictive distribution; that is $\gamma = \{Np_x, Mq_x, Np'_x, Cq'_x\}$ to verify that the model adequately captures relevant features of the observed data.

153 2.4 Case Study

3 Results

155 4 Discussion

56 5 Conclusion

5 Supplementary Information

Information accompanying this article can be found in Supplemental Information.pdf.

Data Availability Statement

- Raw data, R, and Stan code can be found on GitHub at:
- https://github.com/jphill01/Bayesian-DNA-Barcode-Gap-Coalescent.

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Conflict of Interest

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174 Author Contributions

JDP wrote the manuscript, wrote R and Stan code, approved all developed code as well as analysed and interpreted all experimental results.

177 References

- ¹⁷⁸ Carpenter, B., A. Gelman, M. Hoffman, D. Lee, B. Goodrich, M. Betancourt, M. Brubaker,
- J. Guo, P. Li, and A. Riddell
- 2017. Stan: A probabilistic programming language. Journal of Statistical Software, 76:1.
- Hebert, P., A. Cywinska, S. Ball, and J. deWaard
- 2003a. Biological identifications through DNA barcodes. Proceedings of the Royal Society
- of London B: Biological Sciences, 270(1512):313-321.
- ¹⁸⁴ Hebert, P., S. Ratnasingham, and J. de Waard
- 2003b. Barcoding animal life: cytochrome c oxidase subunit 1 divergences among closely
- related species. Proceedings of the Royal Society of London B: Biological Sciences,
- 270(Suppl 1):S96–S99.
- 188 Hoffman, M. and A. Gelman
- 2014. The No-U-Turn Sampler: Adaptively setting path lengths in Hamiltonian Monte
- 190 Carlo. Journal of Machine Learning Research, 15:1593–1623.
- Jukes, T. H. and C. R. Cantor
- 1969. Evolution of protein molecules. In Mammalian Protein Metabolism, H. N. Munro,
- ed., Pp. 21–132. New York: Academic Press.
- 194 Kimura, M.
- 1980. A simple method for estimating evolutionary rates of base substitutions

- through comparative studies of nucleotide sequences. Journal of Molecular Evolution,
- 16(1):111-120.
- 198 Kingman, J.
- 1982. The coalescent. Stochastic Processes and Their Applications, 13:235–248.
- 200 Mather, N., S. M. Traves, and S. Y. W. Ho
- 2019. A practical introduction to sequentially Markovian coalescent methods for estimating
- demographic history from genomic data. Ecology and Evolution, 10(1):579–589.
- ²⁰³ Meier, R., G. Zhang, and F. Ali
- 2008. The use of mean instead of smallest interspecific distances exaggerates the size of
- the "barcoding gap" and leads to misidentification. Systematic Biology, 57(5):809–813.
- Meyer, C. and G. Paulay
- 2005. DNA barcoding: error rates based on comprehensive sampling. PLOS Biology,
- 3(12):e422.
- ²⁰⁹ Phillips, J., D. Gillis, and R. Hanner
- 2022. Lack of statistical rigor in DNA barcoding likely invalidates the presence of a true
- species' barcode gap. Frontiers in Ecology and Evolution, 10:859099.
- Phillips, J., C. Griswold, R. Young, N. Hubert, and H. Hanner
- 2024. A Measure of the DNA Barcode Gap for Applied and Basic Research, Pp. 375–390.
- New York, NY: Springer US.
- 215 Rannala, B. and Z. Yang
- 2003. Bayes estimation of species divergence times and ancestral population sizes using
- dna sequences from multiple loci. Genetics, 164:1645–1656.
- 218 Ratnasingham, S. and P. Hebert
- 2007. BOLD: The Barcode of Life Data System (http://www.barcodinglife.org). Molecular
- $Ecology\ Notes,\ 7(3):355-364.$

- 221 Stan Development Team
- 2023. RStan: The R interface to Stan. R package version 2.21.8.
- Yang, Z. and B. Rannala
- 2017. Bayesian species identification under the multispecies coalescent provides significant
- improvements to DNA barcoding analyses. *Molecular Ecology*, 26:3028–3036.

$\mathbf{Figures}$

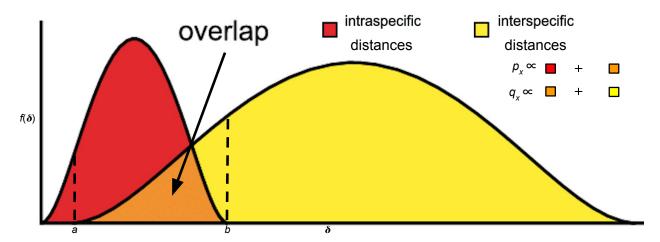


Figure 1: Modified depiction from Meyer and Paulay (2005) and Phillips et al. (2024) of the overlap/separation of pairwise intraspecific and interspecific genetic distances (δ) for calculation of the DNA barcode gap metrics $(p_x \text{ and } q_x)$ for species x. The minimum interspecific distance is denoted by a and the maximum intraspecific distance is indicated by b. The quantity $f(\delta)$ is akin to a kernel density estimate of the probability density function of pairwise genetic distances. A similar visualization can be displayed for p'_x and q'_x .