- A Bayesian Model of the DNA Barcode Gap
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8 Abstract

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₂ 1 Introduction

Since its inception over 20 years ago, DNA barcoding (Hebert et al., 2003a,b) has emerged 13 as a robust method of specimen identification and species delimitation across myriad 14 taxonomic groups which have been sequenced at short, standardized gene regions like 5'-COI 15 for animals. However, the success of the approach depends crucially on two important factors: 16 (1) the availability of high-quality specimen records found in public reference sequence 17 databases such as the Barcode of Life Data Systems (BOLD) (Ratnasingham and Hebert, 18 2007), and (2) the establishment of a DNA barcode gap — the idea that the maximum 19 genetic distance observed within species is much smaller than the minimum degree of marker 20 variation found among species (Meyer and Paulay, 2005; Meier et al., 2008). Early work has demonstrated that the presence of a DNA barcode gap hinges strongly on extant levels of species haplotype diversity gauged from comprehensive specimen sampling at wide geographic and ecological scales (Bergsten et al., 2012). Despite this, many taxa lack adequate separation in their pairwise intraspecific and interspecific genetic distances, thereby compromising rapid matching of unknown samples to expertly-validated references.

Recent work has argued that DNA barcoding, in its current form, is lacking in statistical rigor, calling into question the existence of a true species' DNA barcode gap (Phillips et al., 2022). To support this notion, novel nonparametric locus-specific metrics based on the multispecies coalescent (Rannala and Yang, 2003; Yang and Rannala, 2017) were recently outlined and shown to hold strong promise when applied to predatory *Agabus* (Coleoptera: Dytiscidae) diving beetles (Phillips et al., 2024). The coalescent (Kingman, 1982)

encompasses a backwards continuous-time stochastic Markov process of allelic sampling within natural, neutrally-evolving, species populations towards the Most Recent Common Ancestor (MRCA). The metrics quantify the extent of asymmetric directionality of proportional genetic distance distribution overlap/separation for species within well-sampled 36 taxonomic genera based on a straightforward distance count. The metrics can be employed in a variety of ways, including to assess performance of marker genes for species identication, as well as to assess whether computed values are consistent with population genetic-level parameters like effective population size (N_e) , mutation rates (μ) and divergence times (τ) for species under study (Mather et al., 2019). However, what appears to be missing is an unbiased way to compute the statistical accuracy of the recommended estimators arising through problems inherent in frequentist maximum likelihood estimation for discrete probability distributions having bounded positive support on [0, 1]. To this end, here, a Bayesian model of the DNA barcode gap coalescent is introduced to rectify such issues. The 45 model allows accurate estimation of posterior means, posterior standard deviations, posterior quantiles, and credible intervals for the metrics given datasets of intraspecific and interspecific genetic distances for species of interest.

$_{49}$ 2 Methods

50 2.1 DNA Barcode Gap Metrics

Recently, Phillips et al. (2024) proposed novel nonparametric maximum likelihood estimators (MLEs) of proportional overlap/separation between intraspecific and interspecific pairwise genetic distance distributions for a given species (x) to aid assessment of the DNA barcode gap as follows:

$$p_x = \frac{\#\{d_{ij} \ge a\}}{\#\{d_{ij}\}} \tag{1}$$

$$q_x = \frac{\#\{d_{XY} \le b\}}{\#\{d_{XY}\}} \tag{2}$$

where d_{ij} are pairwise genetic distances within species and d_{XY} are genetic distances among species for an entire genus of concern. The notation # reflects a count (**Figure 1**). Quantities 56 a and b correspond to $\min(d_{XY})$ and $\max(d_{ij})$, the minimum interspecific distance and the 57 maximum intraspecific distance, respectively. Notice that a and b are also the first and 58 nth order statistics, respectively. Distances are easily computed from a model of DNA 59 sequence evolution, such as uncorrected or corrected p-distances (Jukes and Cantor, 1969; 60 Kimura, 1980). Similar expressions (denoted $p_{x}^{'}$ and $q_{x}^{'}$) for nearest neighbour species were 61 also given (see Phillips et al. (2024)), in which d_{XY} included only interspecific distances 62 between the species of interest and its closest neighbouring species. If a focal species is found 63 to have multiple nearest neighbours, then the species possessing the smallest average pairwise 64 interspecific distance is used. While these schemes differ considerably from the usual definition of the DNA barcode gap laid out by Meyer and Paulay (2005) and Meier et al. (2008), they more accurately account for species' coalescence histories inferred from contemporaneous 67 samples of DNA sequences, such as interspecific hybridization/introgression events (Phillips et al., 2024). Note, distances (and hence the metrics) are constrained to the closed interval [0, 1]. Values of the estimators obtained from equations (1) and (2) close to or equal to zero give evidence for separation between intraspecific and interspecific genetic distance distributions; that is, values suggest the presence of a DNA barcode gap for a target species. Conversely, 72 values near or equal to one give evidence for distribution overlap; that is, values likely indicate 73 the absence of a gap. Equations (1) and (2) can be expressed in terms of empirical the 74 cumulative distribution functions (ECDF) as follows

$$p_x = \mathbb{P}(d_{ij} \ge a) = 1 - \hat{F}_{d_{ij}}(a) = \hat{F}_{d_{ij}}(b) - \hat{F}_{d_{ij}}(a)$$
(3)

$$q_x = \mathbb{P}(d_{XY} \le b) = \hat{F}_{d_{XY}}(b),\tag{4}$$

noting that $\hat{F}_{d_{ij}}(b) = 1$ and $\hat{F}_{d_{XY}}(a) = 0$ (**Figure 1**). Given n increasing-ordered data points, the ECDF $\hat{F}_n(t) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}_{[x_i \leq t]}$ comprises a step function having jump discontinuities of $\frac{1}{n}$ at each sample observation (x_i) , excluding ties, where $\mathbb{1}(x)$ is the indicator function. From here, the asymmetric directionality of the metrics is obvious. As mentioned previously, similar equations for p'_x and q'_x can be easily derived.

81 2.2 A Bayesian Implementation

A major criticism of large sample (frequentist) theory is that it relies on asymptotic properties of the MLE (which is assumed to be a fixed but unknown quantity), such as estimator normality and consistency. This problem is especially pronounced in the case of binomial proportions. The estimated Wald SE of the sample proportion, is given by

$$\widehat{SE[\hat{p}]} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\tag{5}$$

where $\hat{p} = \frac{Y}{n}$ is the MLE, Y is the number of successes $(Y = \sum_{i=1}^{n} y_i)$ and n is the number of trials (i.e., sample size). However, the above formula is problematic for several reasons. First, Equation (5) makes use of the Central Limit Theorem (CLT); thus, large sample sizes are required for reliable estimation. When few observations are available, SEs will be large and inaccurate, leading to low statistical power. Further, resulting interval estimates could span values less than zero or greater than one, or have zero width, which is practically meaningless. Second, when proprtions are exactly equal to zero or one, resulting SEs will be exactly zero, rendering Equation (5) completely useless. In the context of the proposed DNA barcode gap metrics, values obtained at the boundaries of their support are often

encountered. Therefore, reliable calculation of SEs is not feasible. Given the importance of sufficient sampling of species genetic diversity for DNA barcoding initiatives, a different statistical estimation approach is necessary. Bayesian inference offers a natural path forward 97 in this regard since it allows for straightforward specification of prior beliefs concerning 98 unknown model parameters and permits the seamless propagation of uncertainty, when data is lacking, through integration with the likelihood function associated with true generating 100 processes. As a consequence, Bayesian models are much more flexible and generally more 101 easily interpretable compared to frequentist approaches since, under the latter paradigm, 102 entire posterior distributions, along with their summaries, are outputted, rather than just 103 sampling distributions, p-values, and confidence intervals as in the former case, thus allowing 104 direct probability statements to be made. 105

$_{06}$ 2.3 The Model

Essentially, from a statistical perspective, the goal herein is to nonparametrically estimate 107 probabilities corresponding to extreme tail quantiles for positive highly skewed distributions 108 on the unit interval. Here, it is sought to numerically approximate the extent of 109 overlap/separation of intraspecific and interspecific pairwise genetic distance distributions 110 within [a, b]. This is a challenging computational problem within the current study as detailed 111 in subsequent sections. Counts, y, of overlapping genetic distances (as expressed in the 112 numerator of Equations (1) and (2)) are treated as binomially distributed with expectation 113 $\mathbb{E}[Y] = k\theta$, where $k = \{N, C\}$ are total count vectors of intraspecific and combined genetic 114 distances, respectively, for a target species along with its nearest neighbour species, and 115 k=M is a total count vector for all pairwise species comparisons. The quantity 116 $\theta = \{p_x, q_x, p_x^{'}, q_x^{'}\}.$ The metrics encompassing θ are presumed to follow a beta (α, β) 117 distribution, with real shape parameters α and β , which is a natural choice of prior on 118 probabilities. Such a scheme is quite convenient since the beta distribution is conjugate to the 119 binomial distribution. Thus, the posterior distribution is also beta distributed. Parameters 120

were given an uninformative Beta(1, 1) prior, which is equivalent to a standard uniform 121 (Uniform(0, 1)) prior since it places equal probability on all parameter values within its 122 support. As a result, the posterior is Beta(Y+1, n-Y+1), from which various moments and 123 other quantities, such as the expected value $\mathbb{E}[Y] = \frac{Y+1}{n+2}$ and variance $\mathbb{V}[Y] = \frac{(Y+1)(n-Y+1)}{(n+2)^2(n+3)}$, 124 can be easily calculated. In general however, when possible, it is always advisable to 125 incorporate prior information, even if only weak, rather than simply imposing complete ignorance in the form of a flat prior distribution. With sufficient data, the choice of prior 127 distribution becomes less important since the posterior will be dominated by the likelihood. 128 The full univariate Bayesian model for species x is thus given by 129

$$y_{\mathrm{lwr}} \sim \mathrm{Binomial}(N, p_{\mathrm{lwr}})$$

$$y_{\mathrm{upr}} \sim \mathrm{Binomial}(M, p_{\mathrm{upr}})$$

$$y'_{\mathrm{lwr}} \sim \mathrm{Binomial}(N, p'_{\mathrm{lwr}})$$

$$y'_{\mathrm{upr}} \sim \mathrm{Binomial}(C, p'_{\mathrm{upr}})$$

$$p_{\mathrm{lwr}}, p_{\mathrm{upr}}, p'_{\mathrm{lwr}}, p'_{\mathrm{upr}} \sim \mathrm{Beta}(1, 1).$$
(6)

The model, which is inherently vectorized to allow processing of multiple species datasets 130 simultaneously, was fitted using the Stan probabilistic programming language (Carpenter et al., 2017) framework for Hamiltonian Monte Carlo (HMC) via the No-U-Turn Sampler 132 (NUTS) sampling algorithm (Hoffman and Gelman, 2014) through the rstan R package (Stan 133 Development Team, 2023). Four chains were run for 2000 iterations each in parallel across 134 four cores with random paramester initializations. Within each chain, a total of 1000 samples 135 was discarded as warmup (i.e., burnin) to reduce dependence on starting conditions. Further, 136 1000 post-warmup draws were utilized per chain. Because HMC/NUTS results in dependent 137 samples that are minimally autocorrelated, Markov chain thinning is not required. Each of 138 these reflect default MCMC settings in Stan. Since the DNA barcode gap metrics often attain 139

values very close to zero and/or very near one, in addition to more intermediate values, a noninformative $\text{Beta}(\frac{1}{2}, \frac{1}{2})$ prior, which is U-shaped symmetric and places greater probability density at the extremes of the distribution due to its heavier tails, while still allowing for variability in parameter estimates within intermediate values along its domain, was also attempted. However, this resulted in several divergent transitions, among other pathologies, imposed by complex geometry (*i.e.*, curvature) in the posterior space, despite remedies to resolve them, such as lowering the step size of the HMC/NUTS sampler. Note that this prior is Jeffreys' prior, which is proportional to the square root of the Fisher information and has several desirable statistical properties, most notably invariance to reparameterization.

To validate the overall correctness of the proposed statistical model given by
Equation (6), in addition to generating MLEs as a means of comparison, posterior predictive
checks were also employed to generate binomial random variates in the form of counts from
the posterior predictive distribution; that is $\gamma = \{Np_x, Mq_x, Np'_x, Cq'_x\}$ to verify that the
model adequately captures relevant features of the observed data.

4 2.4 Model Interpretation

The proposed Bayesian model outlined herein has a straightforward interpretation:

- When p_{lwr} is close to 0, it suggests that the probability of intraspecific distances being larger than interspecific distances is low on average, while the probability of interspecific distances being larger than intraspecific distances is high on average; that is, there is evidence for a DNA barcode gap;
- When p_{lwr} is close to 1, it suggests that the probability of intraspecific distances being larger than interspecific distances is high on average, while the probability of interspecific distances being larger than intraspecific distances is low on average; that is, there is no evidence for a DNA barcode gap;
 - When p_{upr} is close to 0, it suggests that the probability of interspecific distances

being larger than intraspecific distances is high on average, while the probability of intraspecific distances being larger than interspecific distances is low on average; that is, there is evidence for a DNA barcode gap;

- When p_{upr} is close to 1, it suggests that the probability of interspecific distances being larger than intraspecific distances is low on average, while the probability of intraspecific distances being larger than interspecific distances is high on average; that is, there is no evidence for a DNA barcode gap;
 - When p'_{lwr} is close to 0, it suggests that the probability of intraspecific distances being larger than combined interspecific distances for a target species and its nearest neighbour species is low on average, while the probability of combined interspecific distances for a target species and its nearest neighbour species being larger than intraspecific distances is high on average; that is, there is evidence for a DNA barcode gap;
 - sWhen p'_{lwr} is close to 1, it suggests that the probability of intraspecific distances being larger than combined interspecific distances for a target species and its nearest neighbour species is high on average, while the probability of combined interspecific distances for a target species and its nearest neighbour species being larger than intraspecific distances is low on average; that is, there is no evidence for a DNA barcode gap;
 - When p'_{upsr} is close to 0, it suggests that the probability of combined interspecific distances for a target species and its nearest neighbour species being larger than intraspecific distances is high on average, while the probability of intraspecific distances being larger than combined interspecific distances for a target species and its nearest neighbour species is low on average; that is, there is evidence for a DNA barcode gap; and,

• When $p'_{\rm upr}$ is close to 1, it indicates that the probability of combined interspecific distances for a target species and its nearest neighbour species being larger than intraspecific distances is low on average, while the probability of intraspecific distances being larger than combined interspecific distances for a target species and its nearest neighbour species is high on average; that is, there is no evidence for a DNA barcode gap.

Framed more succinctly,

- If b is relatively large and a is relatively small, p_{lwr} represents the extent to which intraspecific distances tend to be larger than interspecific distances at and beyond a and at and below b;
- If a is relatively large and b is relatively small, p_{upr} represents the extent to which interspecific distances tend to be larger than intraspecific distances at and below b and at and beyond a;
- If b is relatively small and min_comb is relatively large, p'_{lwr} represents the extent to which intraspecific distances tend to be larger than combined interspecific distances for a target species and its nearest neighbour species at and beyond min_comb and at and below b; and,
 - If min_comb is relatively small and b is relatively large, p'_{upr} represents the extent to which combined interspecific distances for a target species and its nearest neighbour species tend to be larger than intraspecific distances at and below b and at and beyond min_comb .

- 2.1 2.5 Case Study
- 212 **3** Results
- 213 4 Discussion
- 5 Conclusion

215 Supplementary Information

Information accompanying this article can be found in Supplemental Information.pdf.

217 Data Availability Statement

- Raw data, R, and Stan code can be found on GitHub at:
- https://github.com/jphill01/Bayesian-DNA-Barcode-Gap-Coalescent.

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230 Conflict of Interest

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232 Author Contributions

JDP wrote the manuscript, wrote R and Stan code, approved all developed code as well as analysed and interpreted all experimental results.

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Figures Figures

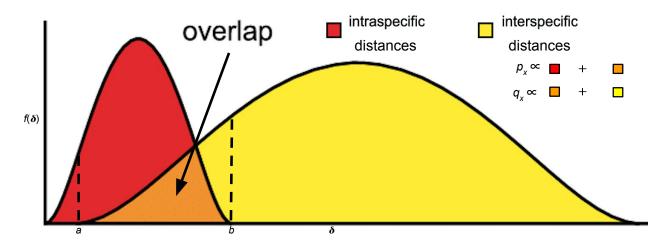


Figure 1: Modified depiction from Meyer and Paulay (2005) and Phillips et al. (2024) of the overlap/separation of pairwise intraspecific and interspecific genetic distances (δ) for calculation of the DNA barcode gap metrics (p_x and q_x) for a hypothetical species x. The minimum interspecific distance is denoted by a and the maximum intraspecific distance is indicated by b. The quantity $f(\delta)$ is akin to a kernel density estimate of the probability density function of pairwise genetic distances. A similar visualization can be displayed for p_x' and q_x' .