- A Bayesian Model of the DNA Barcode Gap
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- 7 Running Title: Bayesian inference for DNA barcode gap estimation

8 Abstract

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A simple statistical model of the DNA barcode gap is outlined. Specifically, accuracy of recently introduced nonparametric metrics, inspired by coalescent theory, to characterize the extent of proportional overlap/separation in maximum and minimum pairwise genetic distances within and among species, respectively, is explored in both frequentist and Bayesian contexts. The empirical cumulative distribution function (ECDF) is utilized to estimate probabilities associated with positively skewed extreme tail distribution quantiles bounded on the closed unit interval [0, 1] based on a straightforward binomial distance overlap count. Using R and Stan, the proposed maximum likelihood estimators and Bayesian model are demonstrated on cytochrome b (CYTB) gene sequences from two Aqabus diving beetle species exhibiting limits in the extent of representative taxonomic sampling. Large-sample theory and MCMC simulations show much uncertainty in parameter estimates, particularly when specimen sample sizes for target species are small. Findings highlight the promise of the Bayesian approach using a conjugate beta prior for reliable posterior uncertainty estimation when available data are sparse. Obtained results can shed light on foundational and applied research questions concerning DNA-based specimen identification and species delineation for studies in evolutionary biology and ecology, as well as biodiversity conservation and management, of wide-ranging taxa.

Keywords: Bayesian/frequentist inference, DNA barcoding, intraspecific genetic distance, interspecific genetic distance, specimen identification, species discovery

29 1 Introduction

The routine use of DNA sequences to support broad evolutionary hypotheses and questions concerning demographic processes like gene flow and speciation in diverse and spatially-distributed taxonomic lineages such as birds, fishes, insects, and arachnids took flight in the late 1980s (Avise et al., 1987). Despite this, the application of genomic data

to applied fields like biodiversity forensics, conservation, and management for the molecular identification of unknown specimen samples came later (e.q., Forensically Informative Nucleotide Sequencing (FINS); Bartlett and Davidson (1992)). Since its inception over 20 years ago, DNA barcoding (Hebert et al., 2003a,b) built on earlier work and has emerged as a 37 robust method of specimen identification and species delimitation across myriad Eukaryotic groups which have been sequenced at short, standardized gene regions like the cytochrome c oxidase subunit I (5'-COI) mitochondrial locus for animals. However, the success of the single-locus approach, particularly for regulatory and forensic applications, depends crucially on two important factors: (1) the availability of high-quality specimen records found in public reference sequence databases such as the Barcode of Life Data Systems (BOLD; http://www.barcodinglife.org) (Ratnasingham and Hebert, 2007) and GenBank (https://www.ncbi.nlm.nih.gov/genbank/), and (2) the establishment of a DNA barcode gap — the notion that the maximum genetic distance observed within species is much smaller 46 than the minimum degree of marker variation found among species (Meyer and Paulay, 2005; 47 Meier et al., 2008). Early work has demonstrated that the presence of a DNA barcode gap hinges strongly on extant levels of species haplotype diversity gauged from comprehensive specimen sampling at wide geographic and ecological scales (Bergsten et al., 2012; Čandek and Kuntner, 2015). Despite this, many taxa lack adequate separation in their pairwise 51 intraspecific and interspecific genetic distances due to varying rates of evolution in both genes 52 and taxa (Pentinsaari et al., 2016). Furthermore, it has been well-demonstrated that the 53 presence of a DNA barcode gap becomes less certain with increasing spatial scale of sampling since interspecific distances increase, while intraspecfic distances shrink (Phillips et al., 2022). This can pose problems in cases of rare species or monotypic taxa (Ahrens et al., 2016) and compromise rapid matching of unknown samples to expertly-validated references, leading to cases of false positives (taxon oversplitting) and false negatives (excessive lumping of taxa) as a result of incomplete lineage sorting, hybridization/introgression, species synonymy, cryptic species diversity, and misidentifications (Hubert and Hanner, 2015; Phillips et al., 2022).

Recent work has argued that DNA barcoding, in its current form, is lacking in statistical 61 rigor, as most studies rely strongly on heuristic distance-based measures to infer taxonomic 62 identity. Of these studies, few report measures of uncertainty, such as standard errors (SEs) and confidence intervals (CIs), around estimates of intraspecific and interspecific variation, calling into question the existence of a true species' DNA barcode gap (Čandek and Kuntner, 2015; Phillips et al., 2022). To support this notion, novel nonparametric locus- and species-specific metrics based on the multispecies coalescent (MSC) (Rannala and Yang, 2003; Yang and Rannala, 2010, 2017) were recently outlined. Unlike previously proposed MSC approaches introduced previously, Phillips et al.'s (2024) approach is tree-free and does not require judicious parameter setting. The statistics have been shown to hold strong promise 70 for reliable DNA barcode gap assessment when applied to predatory Agabus (Coleoptera: Dytiscidae) diving beetles (Phillips et al., 2024). Despite their ease of sampling and well-established taxonomy, this group possesses few morphologically-distinct taxonomic 73 characters that readily facilitate their assignment to the species level (Bergsten et al., 2012). Further, the proposed metrics indicate that sister species pairs from this taxon are often difficult to distinguish on the basis of their DNA barcode sequences (Phillips et al., 2024). Using sequence data from three mitochondrial cytochrome markers (5'-COI, 3'-COI, and cytochrome b (CYTB)) obtained from BOLD and GenBank, results highlight that DNA barcoding has been a one-sided argument. Phillips et al.'s (2024) findings point to the need 79 to balance both the sufficient collection of specimens, as well as the extensive sampling of 80 species: DNA barcode libraries are biased toward the latter (Phillips et al., 2024). The 81 coalescent (Kingman, 1982a,b) encompasses a backwards continuous-time stochastic Markov process of allelic sampling within natural, neutrally-evolving, species populations towards the 83 most recent common ancestor (MRCA). The estimators from Phillips et al. (2024) represent a clear improvement over simple, yet arbitrary, distance heuristics such as the 2\% rule noted by Hebert et al. (2003a) and the 10× rule (Hebert et al., 2004). The former asserts that DNA sequences differing by at least 2% at sequenced genomic regions should be expected

to originate from different biological species, whereas the latter suggests that sequences displaying 10 times more genetic variation among species than within taxa is evidence for a distinct evolutionary origin. However, the lack of adoption of an explicit, universally 90 agreed upon, species concept that governs lineage formation and evolution necessary to 91 establish rigorous taxon definitions for successful delimitation using these well-known criteria, is missing (Rannala, 2015). In addition, the reliance on visualization approaches, such as frequency histograms, dotplots, and quadrant plots to expose DNA barcoding's limitations, has also been criticized (Collins and Cruickshank, 2013; Phillips et al., 2022). Up until the work of Phillips et al. (2024), the majority of studies (e.g., Young et al. (2021)) have treated the DNA barcode gap as a binary response. However, given poor sampling depth for most 97 taxa, a Yes/No dichotomy is inherently flawed because it can falsely imply a DNA barcode gap is present for a taxon of interest when in fact no such separation in distances exists. The 99 proposed statistics quantify the extent of asymmetric directionality of proportional distance 100 distribution overlap/separation for species within well-sampled taxonomic genera based on a 101 straightforward distance count, in a similar vein to established measures of statistical 102 similarity such as the Kullback-Leibler (KL) divergence (Kullback and Leibler, 1951) and 103 other related statistics of f-divergence. The metrics can be employed in a variety of ways, 104 including to validate performance of marker genes for specimen identification to the species 105 level (as in Phillips et al. (2024)), as well as to assess whether computed values are consistent 106 with population genetic-level parameters like effective population size (N_e) , mutation rates 107 (μ) and divergence times (τ) for species under study in a statistical phylogeographic setting 108 (Knowles and Maddison, 2002; Mather et al., 2019). The most promising way forwards in 109 this regards seems to be through the use of the well-known BPP (Bayesian Phylogenetics 110 and Phylogeography) software program which permtt efficient simulations under various 111 MSC models (e.g., MSC-I (MSC with introgression) or MSC-M (MSC with migration) using MCMC (Flouri et al., 2018). 113

While introduction of the metrics is a step in the right direction, what appears to be

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missing is a rigorous statistical treatment of the DNA barcode gap. This includes an unbiased
way to compute the statistical accuracy of Phillips et al.'s (2024) estimators arising through
problems inherent in frequentist maximum likelihood estimation for probability distributions
having bounded positive support on the closed unit interval [0, 1]. To this end, here, a
Bayesian model of the DNA barcode gap coalescent is introduced to rectify such issues. The
model allows accurate estimation of posterior means, posterior standard deviations (SDs),
posterior quantiles, and credible intervals (CrIs) for the metrics given datasets of intraspecific
and interspecific distances for species of interest.

¹²³ 2 Methods

2.1 DNA Barcode Gap Metrics

The novel nonparametric maximum likelihood estimators (MLEs) of proportional overlap/separation between intraspecific and interspecific distance distributions for a given species (x) to aid assessment of the DNA barcode gap are as follows:

$$p_x = \frac{\#\{d_{ij} \ge a\}}{\#\{d_{ij}\}} \tag{1}$$

$$q_x = \frac{\#\{d_{XY} \le b\}}{\#\{d_{XY}\}} \tag{2}$$

$$p_x' = \frac{\#\{d_{ij} \ge a'\}}{\#\{d_{ij}\}} \tag{3}$$

$$q_x' = \frac{\#\{d_{XY}' \le b\}}{\#\{d_{XY}'\}} \tag{4}$$

where d_{ij} are distances within species, d_{XY} are distances among species for an entire genus of concern, and d'_{XY} are combined interspecific distances for a target species and its closest neighbouring species. The notation # reflects a count. Quantities a, a', and b correspond to $\min(d_{XY})$, $\min(d'_{XY})$, and $\max(d_{ij})$, the minimum interspecific distance, the minimum combined interspecific distance, and the maximum intraspecific distance, respectively
(Figure 1).

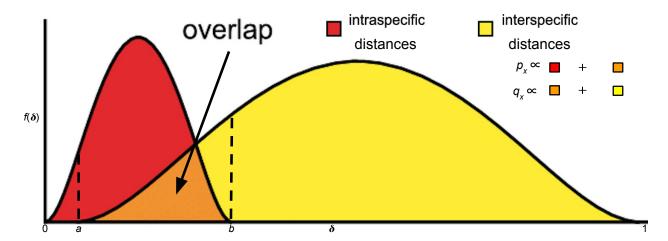


Figure 1: Modified depiction from Meyer and Paulay (2005) and Phillips et al. (2024) of the overlap/separation of intraspecific and interspecific distances (δ) for calculation of the DNA barcode gap metrics (p_x and q_x) for a hypothetical species x. The minimum interspecific distance is denoted by a and the maximum intraspecific distance is indicated by b. The quantity $f(\delta)$ is akin to a kernel density estimate of the probability density function of distances. A similar visualization can be displayed for p'_x and q'_x within the interval [a', b].

Hence, Equations (1)-(4) are simply empirical partial means of distances falling at and below, 134 or at and exceeding, given distribution thresholds. Notice further that a/a', and b are also 135 the first and nth order statistics, $X_{(1)}$ and $X_{(n)}$, respectively. Equations (1)-(4) can also 136 be expressed in terms of empirical cumulative distribution functions (ECDFs) (see next 137 section). Distances form a continuous distribution and are easily computed from a model of 138 DNA sequence evolution, such as uncorrected or corrected p-distances (Jukes and Cantor, 139 1969; Kimura, 1980); however, values are not independent and identically distributed (IID). 140 The approach of Phillips et al. (2024) differs markedly from the traditional definition of 141 the DNA barcoding gap laid out by Meyer and Paulay (2005) and Meier et al. (2008) in 142 that the proposed metrics incorporate interspecific distances which include the target species 143 of interest. Furthermore, if a focal species is found to have multiple nearest neighbours, 144 then the species possessing the smallest average distance is used. These schemes more 145 accurately account for species' coalescence processes inferred from contemporaneous samples 146

of DNA sequences leading to instances of barcode sequence sharing, such as interspecific hybridization/introgression events (Phillips et al., 2024). Within equations (3) and (4), the 148 degree of distance distribution overlap between a target taxon and its nearest neighbouring 149 species, gauged from magnitudes of p'_x and q'_x , is directly proportional to the amount of 150 time in which the two lineages diverged from the MRCA (Phillips et al., 2024). Thus, the 151 quantities can be used as a criterion to assess the failure of DNA barcoding in recently radiated taxonomic groups, among other plausible biological explanations. Note, distances 153 are constrained to the unit interval [0, 1], whereas the metrics are defined only on the interval 154 [a/a', b]. Values of the estimators obtained from equations (1)-(4) close to or equal to zero 155 give evidence for separation between intraspecific and interspecific distance distributions; 156 that is, values suggest the presence of a DNA barcode gap for a target species. Conversely, 157 values near or equal to one give evidence for distribution overlap; that is, values likely indicate 158 the absence of a DNA barcode gap. 159

160 2.2 The Model

Before delving into the derivation of the proposed DNA barcode gap metrics, review of some fundamental statistical theory is necessary.

For a given random variable X, its cumulative distribution function (CDF) is defined by

$$F_X(t) = \mathbb{P}(X \le t) = 1 - \mathbb{P}(X > t). \tag{5}$$

Rearranging Equation (5) gives

$$\mathbb{P}(X > t) = 1 - F_X(t),\tag{6}$$

165 from which it follows that

$$\mathbb{P}(X \ge t) = 1 - F_X(t) + \mathbb{P}(X = t). \tag{7}$$

Equations (1)-(4) can thus be expressed in terms of ECDFs as follows, since the true underlying CDFs, $F(\cdot)$, are unknown *a priori*, and therefore must be estimated using available data:

$$p_x = \mathbb{P}(d_{ij} \ge a)$$

$$= 1 - \hat{F}_{d_{ij}}(a) + \mathbb{P}(d_{ij} = a)$$

$$= \hat{F}_{d_{ij}}(b) - \hat{F}_{d_{ij}}(a) + \mathbb{P}(d_{ij} = a)$$
(8)

$$q_x = \mathbb{P}(d_{XY} \le b)$$

$$= \hat{F}_{d_{XY}}(b) \tag{9}$$

$$p'_{x} = \mathbb{P}(d_{ij} \ge a')$$

$$= 1 - \hat{F}_{d_{ij}}(a') + \mathbb{P}(d_{ij} = a')$$

$$= \hat{F}_{d_{ij}}(b) - \hat{F}_{d_{ij}}(a') + \mathbb{P}(d_{ij} = a')$$
(10)

$$q'_{x} = \mathbb{P}(d'_{XY} \le b)$$

$$= \hat{F}_{d'_{XY}}(b) \tag{11}$$

From this, it can be seen that $\hat{F}_{d_{ij}}(b) = 1$ in Equations (8) and (10). Given n increasing-ordered data points, the (discrete) ECDF, $\hat{F}_n(t) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}_{[x_i \leq t]}$, comprises a step function having jump discontinuities of size $\frac{1}{n}$ at each sample observation (x_i) , excluding ties (or steps of weight $\frac{i}{n}$ with duplicate observations), where $\mathbb{1}(x)$ is the indicator function. Note, $\mathbb{P}(X=t) \neq 0$. Equations (8)-(11) clearly demonstrate the asymmetric directionality of the proposed metrics. Furthermore, calculation of the DNA barcode gap estimators is convenient

as they implicitly account for total distribution area (including overlap).

A major criticism of large sample (frequentist) theory is that it relies on asymptotic 176 properties of the MLE (whose population parameter is assumed to be a fixed but unknown 177 quantity), such as estimator normality and consistency as the sample size approaches infinity. 178 This problem is especially pronounced in the case of binomial proportions (Newcombe, 1998). 179 The estimated Wald standard error (SE) of the sample proportion, is given by $SE[\hat{p}] =$ $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where $\hat{p} = \frac{Y}{n}$ is the MLE, Y is the total number of successes $(Y = \sum_{i=1}^{n} y_i)$ and n 181 is the total number of trials (i.e., sample size). However, the above formula for the standard 182 error is problematic for several reasons. First, it is a Normal approximation which makes 183 use of the central limit theorem (CLT); thus, large sample sizes are required for reliable 184 estimation. When few observations are available, SEs will be large and inaccurate, leading 185 to low statistical power to detect a true DNA barcode gap when one actually exists. Further, 186 resulting interval estimates could span values less than zero or greater than one, or have zero 187 width, which is practically meaningless. Second, when proportions are exactly equal to zero 188 or one, resulting SEs will be exactly zero, rendering $\widehat{SE[\hat{p}]}$ given above completely useless. 189 In the context of the proposed DNA barcode gap metrics, values obtained at the boundaries 190 of their support are often encountered. Therefore, reliable calculation of SEs is not feasible. 191 Given the importance of sufficient sampling of species genetic diversity for DNA barcoding 192 initiatives, a different statistical estimation approach is necessary. 193

Bayesian inference offers a natural path forward in this regard since it allows for straightforward specification of prior beliefs concerning unknown model parameters and permits the seamless propagation of uncertainty, when data are lacking and sample sizes are small, through integration with the likelihood function associated with true generating processes. The posterior distribution $(\pi(\theta|Y))$ is given by Bayes' theorem up to a proportionality $\pi(\theta|Y) \propto \pi(Y|\theta)\pi(\theta)$, where θ are unobserved parameters, Y are known data, $\pi(Y|\theta)$ is the likelihood, and $\pi(\theta)$ is the prior. As a consequence, because parameters are treated as random variables, Bayesian models are much more flexible and generally more easily interpretable compared to frequentist approaches. Under the Bayesian paradigm, entire
posterior distributions, along with their summaries (e.g., CrIs) are outputted, rather than just
long run behaviour reflected in sampling distributions, p-values, and CIs as in the frequentist
case, thus allowing direct probability statements to be made.

Essentially, from a statistical perspective, the goal herein is to nonparametrically estimate 206 probabilities corresponding to extreme tail quantiles for positive highly skewed distributions 207 on the unit interval (or any closed subinterval thereof). Here, it is sought to numerically 208 approximate the extent of proportional overlap/separation of intraspecific and interspecific 209 distance distributions within the subinterval [a/a', b]. This is a challenging computational 210 problem within the current study as detailed in subsequent sections. The usual approach 211 employs kernel density estimation (KDE), along with numerical or Monte Carlo integration 212 and invocation of extreme value theory (EVT); however, this requires careful selection of 213 the bandwidth parameter, among other considerations. This becomes problematic when 214 fitting finite mixture models where nonidentifiability is rampant. For DNA barcode gap 215 estimation, this would correspond to a two-component mixture (one for intraspecific distance 216 comparisons, and the other for interspecific comparisons), with one or more curve intersection 217 points between components, and the presence of zero distance inflation. This makes 218 parameter estimation difficult using methods like the Expectation-Maximization (EM) 219 algorithm (Dempster et al., 1977). Here, for simplicity, a different route is taken to avoid these 220 obstacles. Counts, y, of overlapping distances (as expressed in the numerator of Equations 221 (1)-(4)) are treated as binomially distributed with expectation $\mathbb{E}[Y] = k\theta$, where $k = \{N, C\}$ 222 are total count vectors of intraspecific and combined interspecific distances, respectively, for a 223 target species along with its nearest neighbour species, and k=M is a total count vector for 224 all interspecific species comparisons. This follows from the fact that the ECDF is binomially 225 distributed. The quantity $\theta = \{p_x, q_x, p_x', q_x'\}$. 226

The metrics encompassing θ are presumed to follow a Beta(α , β) distribution, with real shape parameters α and β , which is a natural choice of prior on probabilities. The beta

distribution has a prior mean of $\mathbb{E}[\theta] = \frac{\alpha}{\alpha + \beta}$ and a prior variance equal to $\mathbb{V}[\theta] = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$. In the case where $\alpha = \beta$, all generated Beta(α, β) distributions will possess the same prior 230 expectation, whereas the prior variance will shrink as both α and β increase. Such a scheme is 231 quite convenient since the beta distribution is conjugate to the binomial distribution. Thus, 232 the posterior distribution is also beta distributed, specifically, Beta($\alpha+Y$, $\beta+n-Y$), having 233 expectation $\mathbb{E}[\theta|Y] = \frac{\alpha+Y}{\alpha+\beta+n}$ and variance $\mathbb{V}[\theta|Y] = \frac{(\alpha+Y)(\beta+n-Y)}{(\alpha+\beta+n)^2(\alpha+\beta+n+1)}$. In the context of DNA barcoding, it is important that the DNA barcode gap metrics effectively differentiate 235 between extremes of no overlap/complete separation and complete overlap/no separation, 236 corresponding to values of the metrics equal to 0 and 1 (equivalent to total distance counts of 237 0 and n), respectively. These extremes yield a posterior expectation of $\mathbb{E}[\theta|Y=0] = \frac{\alpha}{\alpha+\beta+n}$ 238 and a posterior variance of $\mathbb{V}[\theta|Y] = 0$ = $\frac{\alpha(\beta+n)}{(\alpha+\beta+n)^2(\alpha+\beta+n+1)}$ and $\mathbb{E}[\theta|Y = n] = \frac{\alpha+n}{\alpha+\beta+n}$ 239 and $\mathbb{V}[\theta|Y=n]=\frac{(\alpha+n)\beta}{(\alpha+\beta+n)^2(\alpha+\beta+n+1)}$. Note, the posterior variances are equivalent at these 240 thresholds for all $\alpha = \beta$. 241 Parameters were given an uninformative Beta(1, 1) prior, which is equivalent to a standard 242 uniform (Uniform(0, 1)) prior since it places equal probability on all parameter values within 243 its support. This distribution has an expected value of $\mu = \frac{1}{2}$ and a variance of $\sigma^2 = \frac{1}{12}$. 244 Further, the posterior is Beta(Y + 1, n - Y + 1), from which various moments such as the 245 expected value $\mathbb{E}[Y] = \frac{Y+1}{n+2}$ and variance $\mathbb{V}[Y] = \frac{(Y+1)(n-Y+1)}{(n+2)^2(n+3)}$, and other quantities, can be 246 easily calculated. Clearly, $\mathbb{E}[\theta|Y=0] = \frac{1}{n+2}$ and $\mathbb{V}[\theta|Y=0] = \frac{n+1}{(n+2)^2(n+3)}$, and 247 $\mathbb{E}[\theta|Y=n]=\frac{n+1}{n+2}$ and $\mathbb{V}[\theta|Y=n]=\frac{n+1}{(n+2)^2(n+3)}$. In general however, when possible, 248 it is always advisable to incorporate prior information, even if only weak, rather than 249 simply imposing complete ignorance in the form of a flat prior distribution. In the case 250 of unimodal distributions, the (estimated) posterior mean often possesses the property that 251 it readily decomposes into a convex linear combination, in the form of a weighted sum, of the 252 (estimated) prior mean and the MLE. That is $\hat{\mu}_{posterior} = w\hat{\mu}_{prior} + (1-w)\hat{\mu}_{MLE}$, where for the 253 beta distribution, $w = \frac{\alpha + \beta}{\alpha + \beta + n}$. Therefore, with sufficient data, $w \to 0$ as $n \to \infty$, regardless 254 of the values of α and β , and the choice of prior distribution becomes less important since

the posterior will be dominated by the likelihood. For the Beta(1, 1), $w = \frac{2}{2+n}$, with n = 2 giving $w = \frac{1}{2}$; that is, the posterior is the arithmetic average of the prior and the likelihood.

The full Bayesian model for species x is thus given by

$$y_{\mathrm{lwr}} \sim \mathrm{Binomial}(N, p_{\mathrm{lwr}})$$

$$y_{\mathrm{upr}} \sim \mathrm{Binomial}(M, p_{\mathrm{upr}})$$

$$y'_{\mathrm{lwr}} \sim \mathrm{Binomial}(N, p'_{\mathrm{lwr}})$$

$$y'_{\mathrm{upr}} \sim \mathrm{Binomial}(C, p'_{\mathrm{upr}})$$

$$p_{\mathrm{lwr}}, p_{\mathrm{upr}}, p'_{\mathrm{upr}} \sim \mathrm{Beta}(1, 1).$$

$$(12)$$

Equation (12) for distinction between MLEs and Bayesian posterior estimates. The above 260 statistical theory and derivations lay a good foundation for the remainder of this paper. 261 The proposed model is inherently vectorized to allow processing of multiple species 262 datasets simultaneously. Model fitting was achieved using the Stan probabilistic 263 programming language (Carpenter et al., 2017) framework for Hamiltonian Monte Carlo 264 (HMC) via the No-U-Turn Sampler (NUTS) sampling algorithm (Hoffman and Gelman, 265 2014) through the rstan R package (version 2.32.6) (Stan Development Team, 2023) in R (version 4.4.1) (R Core Team, 2024). Four Markov chains were run for 2000 iterations each in 267 parallel across four cores with random parameter initializations. Within each chain, a total 268 of 1000 samples was discarded as warmup (i.e., burnin) to reduce dependence on starting 269 conditions and to ensure posterior samples are reflective of the equilibrium distribution. 270 Further, 1000 post-warmup draws were utilized per chain. Because HMC/NUTS results in 271 dependent samples that are minimally autocorrelated, chain thinning is not required. Each 272 of these reflect default Markov Chain Monte Carlo (MCMC) settings in Stan to control both 273 bias and variance in the resulting draws. All analyses in the present work were carried out 274

Note that p_x , q_x , $p_x^{'}$, and $q_x^{'}$ in Equations (1)-(4) are denoted p_{lwr} , p_{upr} , $p_{\text{lwr}}^{'}$, $q_{\text{upr}}^{'}$ within

on a 2023 Apple MacBook Pro with M2 chip and 16 GB RAM running macOS Ventura
13.2. A random seed was set to ensure reproducibility of model results. Outputted estimates
were rounded to three decimal places of precision. Posterior distributions were visualized as
KDE plots using the ggplot2 R package (version 3.5.1) (Wickham, 2016) with the default
Gaussian kernel and optimal smoothness selection. To successfully run the Stan program,
end users must have installed an appropriate compiler (such as GCC or Clang) which is
compatible with their operating system such as macOS.

Convergence was assessed both visually and quantitatively as follows: (1) through 282 examining parameter traceplots, which depict the trajectory of accepted MCMC draws 283 as a function of the number of iterations, (2) through monitoring the Gelman-Rubin R 284 statistic (Gelman and Rubin, 1992; Vehtari et al., 2021), which measures the concordance of 285 within-chain versus between-chain variance, and (3) through calculating the effective sample 286 size (ESS) for each parameter, which quantifies the number of independent samples generated 287 Markov chains are equivalent to. Mixing of chains was deemed sufficient when traceplots 288 looked like "fuzzy caterpillars", $\hat{R} < 1.01$, and effective sample sizes were reasonably large 289 (Gelman et al., 2020). After sampling, a number of summary quantities were reported, 290 including posterior means, posterior SDs, and posterior quantiles from which 95% CrIs 291 could be computed to make probabilistic inferences concerning true population parameters. To 292 validate the overall correctness of the proposed statistical model given by Equation (12), as 293 a means of comparison, posterior predictive checks (PPCs) were also employed to generate 294 binomial random variates in the form of counts from the posterior predictive distribution; that 295 is $\gamma = \{Np_x, Mq_x, Np_x^{'}, Cq_x^{'}\}$ to verify that the model adequately captures relevant features of the observed data. The proposed Bayesian model outlined here has a straightforward 297 interpretation (**Table 1**).

Table 1: Interpretation of the DNA barcode gap estimators within $[a/a',\,b]$

Parameter	Explanation		
$p_x/p_{ m lwr}$	When p_{lwr} is close to 0 (1), it suggests that the probability of intraspecific (interspecific) distances being larger (smaller) than interspecific (intraspecific) distances is low (high) on average, while the probability of interspecific (intraspecific) distances being larger (smaller) than intraspecific (interspecific) distances is high (low) on average; that is, there is (no) evidence for a DNA barcode gap.		
$q_x/p_{ m upr}$	When p_{upr} is close to 0 (1), it suggests that the probability of interspecific (interspecific) distances being larger (smaller) that intraspecific (interspecific) distances is high (low) on average, while the probability of intraspecific (interspecific) distances being large (smaller) than interspecific (intraspecific) distances is low (high) average; that is, there is (no) evidence for a DNA barcode gap.		
$p_x^{'}/p_{ m lwr}^{'}$	When $p'_{\rm lwr}$ is close to 0 (1), it suggests that the probability of intraspecific (combined interspecific distances for a target species and its nearest neighbour species) distances being larger than combined interspecific distances for a target species and its nearest neighbour species (intraspecific distances) is low (high) on average, while the probability of combined interspecific distances for a target species and its nearest neighbour species (intraspecific distances) being larger than intraspecific distances (combined interspecific distances for a target species and its nearest neighbour species) is high (low) on average; that is, there is (no) evidence for a DNA barcode gap.		
$q_x^{'}/p_{ m upr}^{'}$	When $p'_{\rm upr}$ is close to 0 (1), it suggests that the probability of combined interspecific distances for a target species and its nearest neighbour species (intraspecific distances) being larger than intraspecific distances (combined interspecific distances for a target species and its nearest neighbour species) is high (low) on average, while the probability of intraspecific distances (combined interspecific distances for a target species and its nearest neighbour species) being arger than combined interspecific distances for a target species and its nearest neighbour species (intraspecific distances) is low (high) on average; that is, there is (no) evidence for a DNA barcode gap.		

3 Results and Discussion

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The Agabus CYTB dataset analyzed by Phillips et al. (2024) is revisited herein. Briefly, using the R package MACER (Young et al., 2021), DNA sequences were downloaded 301 from GenBank and BOLD and processed to obtain a 343 bp FASTA alignment representing 302 46 unique haplotypes. Genetic distances were calculated using uncorrected p-distances. 303 Specifically, the proposed Bayesian model is demonstrated on the species A. bipustulatus 304 and A. nevadensis, since these taxa were the sole representatives for this locus, with the 305 most and the least specimen records, respectively (N = 701 and N = 2) across all three 306 assessed molecular markers. Further, A. bipustulatus comprised 46 total haplotypes, whereas 307 A. nevadensis possessed two haplotypes. Note, DNA barcode gap estimation is only possible 308 for species having at least two specimen records. This dataset is a prime illustrative example 309 highlighting the issue of inadequate taxon sampling, which arises frequently in large-scale 310 phylogenetic and phylogeographic studies, in several respects. First, from a statistical 311 viewpoint, sample sizes reflect extremes in reliable parameter estimation. Second, from a 312 DNA barcoding perspective, Agabus comprises about 200 extant species according to the 313 Global Biodiversity Information Facility (GBIF) (https://www.gbif.org); yet, due to the 314 level of convenience sampling inherent in taxonomic collection efforts for this genus, adequate representation of species and genetic diversity is far from complete. 316 MCMC parameter traceplots showed rapid mixing of chains to the stationary distribution 317 (Supplementary Figure 1). Further, all \hat{R} and ESS values (not shown) were close to their 318 recommended cutoffs of one and thousands of samples, respectively, indicating chains are 319 both well-mixed and have converged to the posterior distribution. 320 Bayesian posterior estimates were reported alongside frequentist MLEs, in addition to 321 SEs, posterior SDs, 95% CIs and 95% CrIs (**Table 2**). 322

Table 2: Nonparametric frequentist and Bayesian estimates of distance distribution overlap/separation for the DNA barcode gap coalescent model parameters applied to A. bipustulatus (N=701) and A. nevadensis (N=2) for CYTB, including 95% CIs and CrIs. CrIs are based on 4000 posterior draws. All parameter estimates are reported to three decimal places of precision.

Species	Parameter	MLE (SE, 95% CI)	Bayes Est. (SD; 95% CrI)
A. bipustulatus	$p_x/p_{ m lwr}$	1.000 (0.000; 1.000-1.000)	1.000 (0.000; 1.000-1.000)
$A.\ bipustulatus$	$q_x/p_{ m upr}$	1.000 (0.000; 1.000-1.000)	$1.000 \ (0.000; \ 0.999-1.000)$
$A.\ bipustulatus$	$p_x^{'}/p_{ m lwr}^{'}$	1.000 (0.000; 1.000-1.000)	1.000 (0.000; 1.000-1.000)
$A.\ bipustulatus$	$q_x^{'}/p_{ m upr}^{'}$	1.000 (0.000; 1.000-1.000)	$1.000 \ (0.000; \ 0.999-1.000)$
$A.\ nevadensis$	$p_x/p_{ m lwr}$	1.000 (0.000; 1.000-1.000)	$0.835\ (0.144;\ 0.470 - 0.996)$
$A.\ nevadensis$	$q_x/p_{ m upr}$	$0.010 \ (0.002; \ 0.006 - 0.014)$	$0.010 \ (0.002; \ 0.007 - 0.014)$
$A.\ nevadensis$	$p_x^{'}/p_{ m lwr}^{'}$	1.000 (0.000; 1.000-1.000)	$0.834\ (0.138;\ 0.481 - 0.994)$
A. nevadensis	$q_x^{'}/p_{ m upr}^{'}$	0.010 (0.070; -0.128-0.148)	0.010 (0.002; 0.007-0.014)

CIs were calculated using the usual large sample $(1 - \alpha)100\%$ -level interval estimate given by $\hat{p} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where $z_{1-\frac{\alpha}{2}} = 1.960$ for 95% confidence and α is the stated significance level (here, 5%). Given a $(1-\alpha)100\%$ CI, with repeated sampling, on average $(1-\alpha)100\%$ 325 of constructed intervals will contain the true parameter of interest; on the other hand, any 326 given CI will either capture or exclude the true parameter with 100% certainty. This in 327 stark contrast to a CrI, where the true parameter is contained within said interval with $(1-\alpha)100\%$ probability. Note, by default Stan computes equal-tailed (central) CrIs such 329 that there is equal area situated in the left and right tails of the posterior distribution. For 330 a 95% CrI, this corresponds to the 2.5th and 97.5th percent quantiles. However, constructed 331 intervals are usually only valid for symmetric or nearly symmetric distributions. Given the bounded nature of the DNA barcode gap metrics, whose posterior distributions, as expected, 333 show considerable skewness, a different approach to reporting CrIs, such as Highest Posterior 334 Density (HPD) intervals (Chen and Shao, 1999) or shortest probability intervals (SPIn) (Liu 335 et al., 2015) is warranted. As such asymmetric intervals generally attain greater statistical 336 efficiency (in the form of smaller Mean Squared Error (MSE) or variance) and higher coverage 337 probabilities than more standard interval estimates, careful in-depth comparison is left for 338 future work. 339

Findings based on nonparametric MLEs and Bayesian posterior means were quite

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comparable with one another and show evidence of complete overlap in intraspecific, interspecific, and combined interspecific distances for A. bipustulatus in both the p/q and p'/q' directions since the metrics attain magnitudes very close to one (**Table 2**). As a result, this likely indicates that no DNA barcode gap is present for this species. Such findings are strongly reinforced by the very tight clustering of posterior draws (**Figure 2**) and associated interval estimates owing to the large number of specimens sampled for this species.

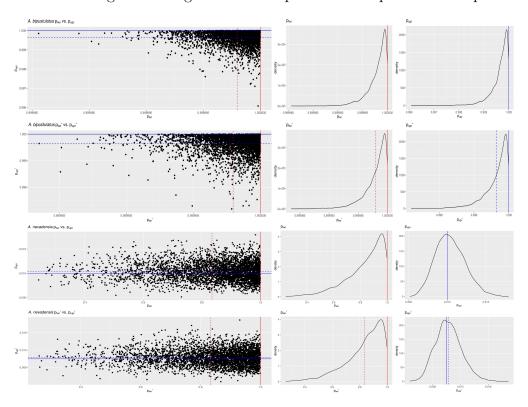


Figure 2: Scatterplots (black solid points) and distributions (black solid lines) depicting the DNA barcode gap metrics for A. bipustulatus (N = 701) and A. nevadensis (N = 2) across CYTB based on 4000 Bayesian posterior draws. MLEs and posterior means are displayed as coloured (red/blue) solid and dashed lines for the metrics, respectively.

On the other hand, the situation for *A. nevadensis* is more nuanced, as posterior values are further spread out (**Table 2** and **Figure 2**), suggesting less overall certainty in true parameter values given the low specimen sampling coverage for this taxon. Of note, the University CIs and 95% CrIs are quite wide for *A. nevadensis*, consistent with much uncertainty regarding the computed frequentist and Bayesian posterior means of the DNA barcode gap metrics. For instance, the Bayesian analysis for *A. nevadensis* suggests that the data are

consistent with both $p_{\rm lwr}$ and $p_{\rm lwr}^{'}$ ranging from approximately 0.250-1.000. Further, regarding the frequent ist analysis for the same species, the 95% CI for $q_x^{'}$ extends to negative values 354 at the left endpoint, due to the corresponding SE of 0.070 being too high as a result of 355 the extremely low sample size of n=2 individuals sampled (**Table 2**). Since the 95% CI 356 truncated at the lower endpoint includes the value of zero, the null hypothesis for the presence 357 of a DNA barcode gap cannot be rejected. Despite this, it is worth noting that truncation 358 is not standard statistical practice and will likely lead to an interval with less than 95% nominal coverage. In such cases, more appropriate confidence interval methods like the 360 Wilson score interval, the exact (Clopper-Pearson) interval, or the Agresti-Coull interval 361 should be employed (Newcombe, 1998; Agresti and Coull, 1998). KDEs for A. bipustulatus 362 are strongly left (negatively) skewed (**Figure 2**), whereas those for A. nevadensis exhibit 363 more symmetry, especially for $p_{\rm upr}$ and $p_{\rm upr}^{'}$ (**Figure 2**). These differences are likely due to the 364 stark contrast in sample sizes for the two examined species. Nevertheless, simulated counts 365 of overlapping specimen records from the posterior predictive distribution (Supplementary 366 **Table 1**) were found to be very close to observed counts for both species, indicating that the 367 proposed model adequately captures underlying variation. Obtained results suggest that use 368 of the Beta(1, 1) prior may not be appropriate given a low number of collected individuals 369 for most taxa in DNA barcoding efforts. This suggests that further consideration of more 370 informative beta priors is worthwhile. 371

4 Conclusion

Herein, the accuracy of the DNA barcode gap was analyzed from a rigorous statistical lens to expedite both the curation and growth of reference sequence libraries, ensuring they are populated with high quality, statistically defensible specimen records fit for purpose to address standing questions in ecology, evolutionary biology, management, and conservation. To accomplish this, recently proposed, easy to calculate nonparametric MLEs were formally

derived using ECDFs and applied to assess the extent of overlap/separation of distance distributions within and among two species of predatory water beetles in the genus Aqabus 379 sequenced at CYTB using a Bayesian binomial count model with conjugate beta priors. 380 Findings highlight a high level of parameter uncertainty for A. nevadensis, whereas posterior 381 estimates of the DNA barcode gap metrics for A. bipustulatus are much more certain. 382 Based on these results, it is imperative that specimen sampling be prioritized to better reflect actual species boundaries. More generally, apart from the metrics being employed to 384 better highlighting the importance of within-species genetic diversity versus between-species 385 divergence, it is expected that the approach developed herein will be of broad utility in 386 applied fields, such as DNA-based detection of seafood fraud within global supply chains. 387

Since the DNA barcode gap metrics often attain values very close to zero (suggesting no 388 overlap and complete separation of distance distributions) and/or very near one (indicating no 389 separation and complete overlap), in addition to more intermediate values, a noninformative 390 $Beta(\frac{1}{2},\frac{1}{2})$ prior may be more appropriate over complete ignorance imposed by a Beta(1,1)391 prior. The former distribution is U-shaped symmetric and places greater probability density 392 at the extremes of the distribution due to its heavier tails, while still allowing for variability 393 in parameter estimates within intermediate values along its domain. Note that this prior 394 is Jeffreys' prior density (Jeffreys, 1946), which is proportional to the square root of the 395 Fisher information $\mathcal{I}(\theta)$. That is $\pi(\theta) \propto \theta^{-\frac{1}{2}} (1-\theta)^{-\frac{1}{2}}$. Jeffreys' prior has several desirable 396 statistical properties as a prior: that it is inversely proportional to the standard deviation of 397 the binomial distribution, and most notably, that it is invariant to model reparameterization 398 (Gelman et al., 2014). However, this prior can lead to divergent transitions, among other 399 pathologies, imposed by complex geometry (i.e., curvature) in the posterior space since many 400 iterative stochastic MCMC sampling algorithms experience difficulties when exploring high 401 density distribution regions. Thus, remedies to resolve them, such as lowering the step size of 402 the HMC/NUTS sampler, should be attempted in future work, along with other approaches 403 such as empirical Bayes estimation to approximate beta prior hyperparameters from observed

data through the MLE or other methods of parameter estimation, such as the method of moments. Alternatively, hierarchical modelling could be employed to estimate separate 406 distribution model hyperparameters for each species and/or compute distinct estimates for 407 the directionality/comparison level of the DNA barcode gap metrics (i.e., lower vs. upper, 408 non-prime vs. prime) separately within the genus under study. This would permit greater 409 flexibility through incorporating more fine-grained structure seen in the data; however, low taxon sample sample sizes may preclude valid inferences to be reasonably ascertained due to 411 the large number additional parameters which would be introduced through the specification 412 of the hyperprior distributions. Methods outlined in Gelman et al. (2014), such as dealing 413 with non-exchangeability of observations and alternate model parameterizations like the logit, 414 may prove useful in this regard. Even though more work remains, it is clear that both 415 frequentist and Bayesian inference hold much promise for the future of molecular biodiversity 416 science. 417

⁴¹⁸ Supplementary Information

None declared.

420 Data Availability Statement

- Raw data, R, and Stan code can be accessed via Dryad at:
- http://datadryad.org/stash/share/
- RZIfMixcEODe0RWP7eyXWQewSVbqEIA9UTrH3ZVKyn4.
- A GitHub repository can be found at:
- https://github.com/jphill01/Bayesian-DNA-Barcode-Gap-Coalescent.

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Conflict of Interest

None declared.

438 Author Contributions

JDP wrote the manuscript, wrote R and Stan code, as well as analyzed and interpreted all model results.

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