CSCI 3022-002 Intro to Data Science
Discrete pdfs

$$f(x) = P(X = x)$$

$$F(x) = P(X \leq x)$$

Example: Suppose we are given the following pmf:

$$P(X = x) = f(x) = \begin{cases} .5 & x = 0 \\ .167 & x = 1 \\ .333 & x = 2 \\ 0 & else \end{cases}$$

1. Calculate: $F(0), F(1), F(2)$.
2. What is $F(1.5)$? $F(20.5)$?
3. Is $P(X < 1) = P(X \leq 1)$?

Opening Example; Soln

Example: Suppose we are given the following pmf:

$$P(X = x) = f(x) = \begin{cases} .5 & x = 0 \\ .167 & x = 1 \\ .333 & x = 2 \\ 0 & else \end{cases}$$

$F(2) = 1$

1. Calculate: $F(0), F(1), F(2)$.

$F(0) = P(X \le 0) = .5$

$F(1) = P(X \le 1) = P(X=0) + P(X=1) = .5 + 1/6 = 2/3$

2. What is $F(1.5)$? $F(20.5)$?

$P(X \le 1.5) = P(X \le 1) + \cancel{P(1 < X \le 1.5)} = .667$

$P(X \le 20.5) = 1$

3. Is $P(X < 1) = P(X \le 1)$?

Opening Example; Soln

Example: Suppose we are given the following pmf:

$$P(X = x) = f(x) = \begin{cases} .5 & x = 0 \\ .167 & x = 1 \\ .333 & x = 2 \\ 0 & else \end{cases}$$

1. Calculate: $F(0), F(1), F(2)$. non-decreasing
   $F(0) = P(X \leq 0) = .5$; $F(0) = P(X \leq 1) = .667$; $F(0) = P(X \leq 2) = 1$
2. What is $F(1.5)$? $F(20.5)$?
   $F(1.5) = P(X \leq 1.5) = P(X \leq 1) = .667$; $F(0) = P(X \leq 2) = 1$
3. Is $P(X < 1) = P(X \leq 1)$?
   Most certainly not!

$$P(X \leq 1) = P(X < 1) + \overset{.167}{P(X = 1)}$$

Announcements and To-Dos

Announcements:

1. HW 3, posted.
2. Another nb day this Friday.

Last time we learned:

1. Bayes Theorem, and introduced pdfs and cdfs.

To do:

1. Start that HW!

Last Time...

We got a cool formula that was secretly just the definition of conditional probability rewritten!

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

We got two functions to play with:

- A *Probability density function* (pdf) is a function $f(x)$ that describes the probability distribution of a random variable X. Discrete case: $f(x) = P(X = x)$.
- The *cumulative density function* (cdf), denoted $F(x)$, is defined for every real number x to be the probability that the observed value of X will be at most x, or $\underline{F(x)} = P(\underline{X} \leq \underline{x})$. Discrete case: a sum of values of $f(x)$!

$F(a) = P(X \leq a)$

pdf to cdf

R.V. X w/ pdf f ; cdf F

The relationship between pdf and cdf is very important!

$$f(x) = \begin{cases} 1/36 & 2 \\ 2/36 & 3 \\ \vdots & \vdots \\ 0 & 1 \end{cases}$$

$P(X = .) + P(X = \cdots) + \cdot\cdot P(X = a)$

$$F(a) = P(X \underline{\le a}) = \sum_{\underline{x \le a}} P(X = x)$$

Example: What is the probability that if I roll two dice, they add up to at least 9. Write in terms of $F(x)$, then compute.

X

9

$X = \{\cancel{1}, 2, \cdots, 12\}$

X: sum of 2 dice

$$P(X \ge 9) = 1 - P(\underbrace{(X \ge 9)^c}_{\text{complement}}) = 1 - P(X < 9)$$

$$= 1 - P(X \le 8)$$

$$= 1 - F(8)$$

pdf to cdf

The relationship between pdf and cdf is very important!

$$F(a) = P(X \leq a) = \sum_{x \leq a} P(X = x)$$

Example: What is the probability that if I roll two dice, they add up to at least 9. Write in terms of $F(x)$, then compute. Fair!

$X :=$ the sum of the two dice, we want

$$\underline{P(X \geq 9)} = 1 - P(X < 9) = \boxed{1 - P(X \leq 8)} = 1 - F(8).$$

The easier probability is probably the

$$P(X \geq 9) = \underset{4/36}{P(X = 9)} + \underset{3/36}{P(X = 10)} + \underset{2/36}{P(X = 11)} + \underset{1/36}{P(X = 12)} = 10/36.$$

2d6; Ω and X

Suppose we roll two fair, 6-sided dice. Let $X :=$ the value representing the maximum of the two dice.

1. What are the possible values of X? $= X \in \{1, 2, 3, 4, 5, 6\}$
2. Which elements of the sample space map to which values of X? $P(1/36) \to (2, 3) \Rightarrow X = 3$
3. What is the pmf of X?

Blue

Red	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

↙ 36 possibilities, each tile equally likely

2d6; Ω and X

Suppose we roll two fair, 6-sided dice. Let $X :=$ the value representing the maximum of the two dice.

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2. Which elements of the sample space map to which values of X?
3. What is the pmf of X?

1. $X \in \{1, 2, 3, 4, 5, 6\}$

2.

Die 2

Die 1

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

3. The pmf is: $P(X = x)$; or

$$f(x) = \begin{cases} 1/36 & X = 1 \\ 3/36 & X = 2 \\ 5/36 & X = 3 \\ 7/36 & X = 4 \\ 9/36 & X = 5 \\ 11/36 & X = 6 \end{cases}$$

$f(x) = 0$ ⇗ for all other X.

2d6; The Max

$\boxed{X}$ the max of 2 die rolls,

$F(\boxed{2}) = P$ that the max of 2 die rolls is exactly $\boxed{2}$

Now we have

$$f(x) = \begin{cases} 1/36 & X = 1 \\ 3/36 & \boxed{X = 2} \\ 5/36 & X = 3 \\ 7/36 & \boxed{X = 4} \\ 9/36 & X = 5 \\ 11/36 & \boxed{X = 6} \end{cases}$$

lower case!

What are:

1. $P(X \text{ is even})$?

$P(X \text{ is } 2 \text{ or } 4 \text{ or } 6)$

$= P(X=2) + P(X=4) + P(X=6) = \frac{3+7+11}{36} = 21/36.$

2. $P(X \text{ is 3 or less})$?

$P(X \le 3) = F(3)$

3. What is the cdf for X?

$$F(x) = \begin{cases} 0 & x < 1 \\ P(X=1) = 1/36 & 1 \le x < 2 \\ P(X \le 2) = 1/36 + 3/36 & 2 \le x < 3 \end{cases}$$

$F_X(a)$

$= P(X \le a)$

2d6; The Max

Now we have

lower-case!

↓

$$f(x) = \begin{cases} 1/36 & X = 1 \\ 3/36 & X = 2 \\ 5/36 & X = 3 \\ 7/36 & X = 4 \\ 9/36 & X = 5 \\ 11/36 & X = 6 \end{cases}$$

What are:

1. $P(X$ is even$)$?

$$\frac{3+5+7}{36}$$

2. $P(X$ is 3 or less$)$?

$$\frac{1+3+5}{36}$$

3. What is the cdf for X?

F(a)

$$F(x) = \begin{cases} 0 & \cancel{X}^{a} < 1 \\ 1/36 & 1 \leq \cancel{X}^{x} < 2 \\ 4/36 & 2 \leq \cancel{X}^{x} < 3 \\ 9/36 & 3 \leq \cancel{X}^{a} < 4 \\ 16/36 & 4 \leq \cancel{X}^{x} < 5 \\ 25/36 & 5 \leq \cancel{X}^{x} < 6 \\ 36/36 & X > 6 \end{cases}$$

PDF/CDF recap

pmf ∈ pdf

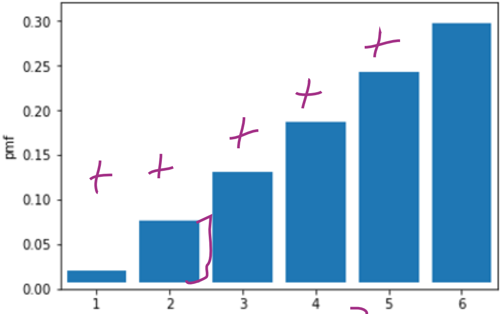
(jumps)

A picture denoting the pdf and cdf of our X:

pmf ~~PDF~~: pmf: histogram

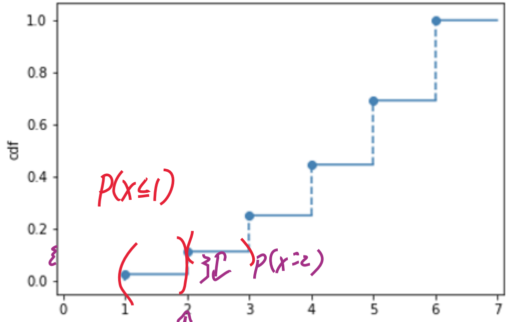
always a line

CDF:



↑ P(X=2) ≈ .08?

pdf cont's: line



P(X≤1)

P(X=2)

↑ P(X<3)

= P(X≤2)

= P(X<2)+P(X=2)

Discrete Random Variables

Discrete random variables can be categorized into different types or classes.
Each type/class models many different real-world situations. They can loosely be broken down into a few groups:

1. The *Discrete Uniform* for modeling n equally likely (*fair*) outcomes np.random.choice
2. Distributions built on counting trials-until-event (how rolls until I get a 6, etc.) when the trials are identical and repeated

   Examples: Binomial, Geometric, etc. binary outcomes
3. Counting occurrences of an event over fixed areas of time/space.

   Example: Poisson

The Bernoulli : X

A random variable whose only possible values are 0 or 1.

This is a discrete random variable – why?

→ 2 outcomes

This distribution is specified by a single parameter:

$P(\{H\})$

The probability of a heads/"success" p! This gives the pdf:

$$f(a) = \begin{cases} p & a=1 \\ 1-p & a=0 \end{cases}$$

"is distributed as"

We denote the Bernoulli random variable X by $X \sim \text{bern}(p)$

The Bernoulli

A random variable whose only possible values are 0 or 1.

This is a discrete random variable – why?

Countable outcomes

This distribution is specified by a single parameter:

The probability of a heads/“success” p! This gives the pdf:

$$P(X=x)=f(x)=\begin{cases} p & x=1 \\ (1-p) & x=0 \\ 0 & else \end{cases}$$

We denote the Bernoulli random variable X by $\underline{X \sim Bern(p)}$

The Bernoulli

A random variable whose only possible values are 0 or 1.

This is a discrete random variable – why?

This distribution is specified by a single parameter:
The probability of a heads/“success” p! This gives the pdf:

awkward!

$$P(X = x) = f(x) = \begin{cases} p & x = 1 \\ (1-p) & x = 0 \\ 0 & else \end{cases}$$

It turns out, it’s nice to write the pdf as a single line whenever possible. The nicest way to do so for the Bernoulli:

$$f(x) = p^x (1-p)^{1-x}$$

$x=0$

$$\cancel{p^0}(1-p)^{1-0} = 1-p$$

$x=1$

$$p^1 (\cancel{1-p})^{\cancel{1-1}} = p$$

which works as long as we remember x can only be 0 or 1.
We denote the Bernoulli random variable X by $X \sim Bern(p)$

Overview

The Bernoulli random variable is the building block for numerous important probability distributions that reflect **repeated** measurements.

Statistics and data science on repeated measurements requires us understand principles of **counting**!

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1. Some counting is easy: how many integers are there in $[0, 9]$?

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2. Zach, Felix, Rachel, and Ioana line up at a coffee stand. How many different orders could they stand in?

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2. Zach, Felix, Rachel, and Ioana line up at a coffee stand. How many different orders could they stand in?
   This is a *permutation:* it counts distinct orderings

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3. There are 10 problems on an exam, and you need 7 correct to pass. How many different ways are there to pass?

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Statistics and data science on repeated measurements requires us understand principles of **counting**!

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   This is a *combination*: it counts ways a set can be split into subsets

Permutations

How many ways can you order a set of one object; e.g. $\{A\}$?

1: {A}

How many ways can you order a set of two objects; e.g. $\{A, B\}$?

A,B or BA

How many ways can you order a set of three objects; e.g. $\{ABC\}$?

CBA, BCA, BAC

CA,B or ACB or ABC

What's the pattern? How many ways could you order n objects?

Permutations

How many ways can you order a set of one object; e.g. $\{A\}$?
A: 1 way. $\{A\}$.

How many ways can you order a set of two objects; e.g. $\{A, B\}$?
A: 2 ways. $\{AB, BA\}$.

How many ways can you order a set of three objects; e.g. $\{ABC\}$?
A: 6 ways. $\{ABC, ACB, BAC, BCA, CBA, CAB\}$. $= 3!$

What's the pattern? How many ways could you order n objects?
A: $n!$

Permutations; General

What if you have n objects, but only want to permute r of them?

How many 3-character strings can we make if each character is a distinct letter from the English alphabet? $\{S, A, D\} \neq \{A, P, S\}$

order matters!

What is the general form for an r-permutation of n objects?

Permutations; General

What if you have n objects, but only want to permute r of them?

How many 3-character strings can we make if each character is a distinct letter from the English alphabet?
A: There are 24 that start with $\boxed{\{AB\}}$. There are 25 letters (including B) that could have followed an A. There are 26 options to start with. That multiplies to $26 \cdot 25 \cdot 24$.

What is the general form for an r-permutation of n objects?
A: $P(n, r) = \frac{n!}{\underline{(n-r)!}}$

This should feel a lot like **sampling without replacement**.. because it is, only without probabilities.

Combinations

Counting *combinations* means counting the number of ways an object can be sliced into subsets. The big difference: **order doesn't matter**.

> How many 3-character *combinations* can we make if each character is a distinct letter from the English alphabet?

$$\{S, A, D\} \overset{\text{set}}{=} \{D, S, A\}$$

Combinations

Counting *combinations* means counting the number of ways an object can be sliced into subsets. The big difference: **order doesn't matter**.

> How many 3-character *combinations* can we make if each character is a distinct letter from the English alphabet?
> Start with the number of permutations: $P(n,r) = 26 \cdot 25 \cdot 24$, then ask how many times we "overcounted," because now we don't want subsets with the same elements.
> **Ex:** How many times did we include a subset with $\{A, B, C\}$?
> $6 = 3!$

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> Start with the number of permutations: $P(n, r) = 26 \cdot 25 \cdot 24$, then ask how many times we "overcounted," because now we don't want subsets with the same elements.
> **Ex:** How many times did we include a subset with $\{A, B, C\}$?
>
> Our permutation set had $\{ABC\}, \{ACB\}, \{BAC\}, \{BCA\}$,$\{CBA\}$, and $\{CAB\}$ as distinct... or all 6 orderings of those 3 elements! So:

$$C(n, r) = \frac{n!}{(n-r)!\underset{\nearrow \text{extra overcounting}}{(r!)}}$$

Combinations; Example

Combinations often use a variety of notations, including

$$C(n,r) = \binom{n}{k} = \frac{n!}{(n-r)!r!} := \text{“n choose k”}$$

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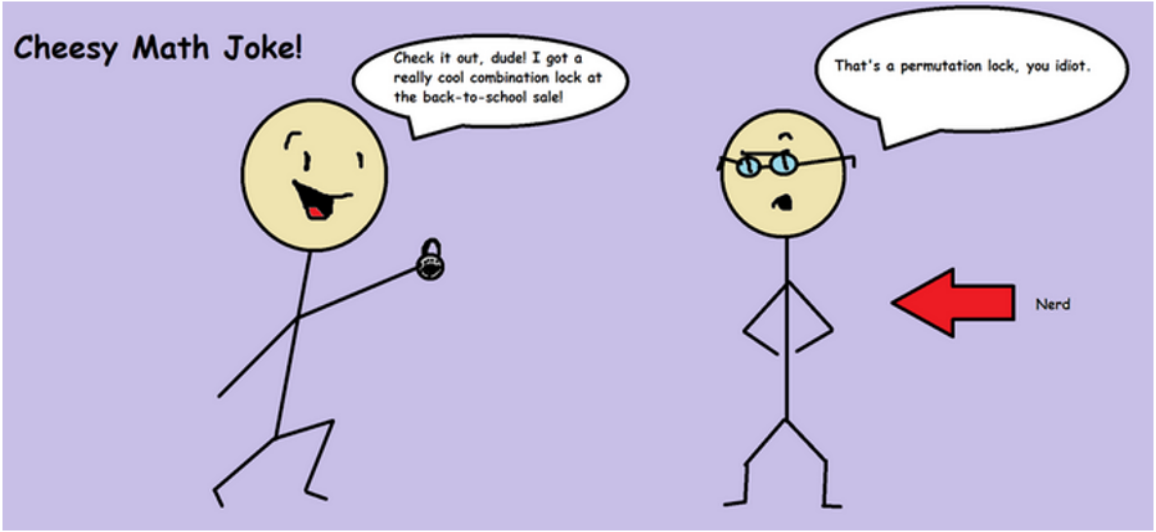
Example: If there are 10 problems on an exam, and you need 7 correct to pass, how many different ways are there to pass?

Answer: $C(10,7) + C(10,8) + C(10,9) + C(10,10)$

$$= \frac{10!}{(10-10)!(10!)} = \frac{10!}{1 \cdot 10!} = 1$$

# of ways to choose 7 correct out of 10 + 8 correct + ...

Perms and Combs; Summary



Binomials

Exponents are useful, and pretty common: a lot of both data science and computational problems often involve solutions that look like polynomials. For example, let's consider:

1. Expand $(x+y)^1$
2. Expand $(x+y)^2$
3. Expand $(x+y)^3$
4. Expand $(x+y)^4$

Binomials

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↓ binomial: 2 things to a power

1. Expand $(x+y)^1$
   Solution: $(x+y)^1 = x + y$
2. Expand $(x+y)^2$
   Solution: $(x+y)^2 = x^2 + 2xy + y^2$
3. Expand $(x+y)^3$
   Solution: $(x+y)^1 = (x+y)(x^2+2xy+y^2) = x^3+3x^2y+3xy^2+1$
4. Expand $(x+y)^4$
   Solution: $(x+y)^1 = (x+y)(x^3+3x^2y+3xy^2+1) = x^4+4x^3y+6x^2y^2+4xy^3+1$

Binomials

Exponents are useful, and pretty common: a lot of both data science and computational problems often involve solutions that look like polynomials. For example, let's consider:

1. Expand $(x+y)^1$
   Solution: $(x+y)^1 = x + y$
2. Expand $(x+y)^2$
   Solution: $(x+y)^2 = x^2 + 2xy + y^2$
3. Expand $(x+y)^3$
   Solution: $(x+y)^1 = (x+y)(x^2+2xy+y^2) = x^3 + 3x^2y + 3xy^2 + 1$
4. Expand $(x+y)^4$
   Solution: $(x+y)^1 = (x+y)(x^3+3x^2y+3xy^2+1) = \underset{1}{x^4} + \underset{4}{4x^3y} + \underset{6}{6x^2y^2} + \underset{4}{4xy^3} + \underset{1}{1}$

What are some patterns? It's definitely symmetric - the coefficient are palindromic - and it seems to always start with 1 and then n (the power)

Binomials

One way to think about a binomial (two term) expansion is using "choose." Think about foiling:

$$(x_0 + x_1)(a + b) = \underbrace{ax_0}_{\text{first}} + \underbrace{bx_0}_{\text{outer}} + \underbrace{ax_1}_{\text{inner}} + \underbrace{bx_1}_{\text{last}}$$

There are 4 terms, but these are the same 4 terms as we would get from a multiplication rule: "choose" one of the first 2 terms and "choose" one of the second 2 terms for $2 \cdot 2$ total.

For our problem, we have to worry about repeating terms, though! If we think about:

$$(x + y)^4 = \boxed{(\overline{\underline{x}} + y)}\,\boxed{(\underline{x} + \overline{y})}\,\boxed{(\overline{x} + \underline{y})}\,\boxed{(x + \overline{\underline{y}})}$$

it's making 4 choices: "choose x or y," then "choose x or y," then "choose x or y," then "choose x or y." The coefficient of the x^2y^2 term is the number of ways we could "choose x or y" 4 times and end up with 2 x's and 2 y's.

Binomials, Cont'd

So we're expanding

$$\begin{aligned}(x+y)^4 &= (x+y)\,(x+y)\,(x+y)\,(x+y)\\ &= (x+y)(x^3+3x^2y+3xy^2+1)\\ &= x^4+4x^3y+6x^2y^2+4xy^3+1\end{aligned}$$

and the coefficient of the x^2y^2 term is the number of ways we could "choose x or y" 4 times and end up with 2 x's and 2 y's.

Let's check. We're looking for all of the ways you could get e.g. $xxyy$, $yyxx$, $xyyx$, etc. This is the same as asking for the number of ways to choose 2 of the 4 "slots" to be x or choosing 2 of the 4 slots to be y, or $\underbrace{C(4,2) = \frac{4!}{2!}.}$

Binomial Theorem

Theorem: Let x and y be variables and n be a non-negative integer. Then

$$(x+y)^n = \sum_{k=0}^{n} C(n,k)x^{n-k}y^k = C(n,0)x^n y^0 + C(n,1)x^{n-1}y^1 + \cdots + C(n,n)x^0 y^n$$

In other words, $C(n,k)$ is the coefficient of $x^k y^{n-k}$ *and* $x^{n-k}y^k$. We usually write the C numbers in choose notation:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k}y^k = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \cdots + \binom{n}{n}x^0 y^n$$

Pascal's Triangle

For small expansions, an easy trick to find the binomial coefficients is Pascal's triangle. Each entry of the triangle is the sum of the two entries above it:

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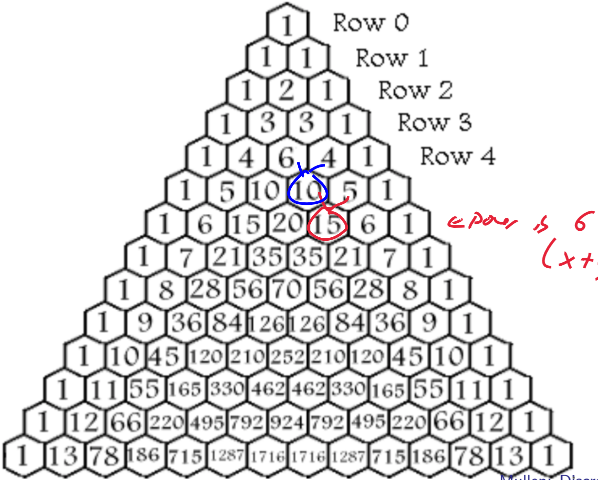

epower is 6
$(x+y)^6$

Finding/using:

$$C(n,k) = \frac{n!}{(n-k)!(k!)}$$

The Binomial

$P(\{H\}) = p$

$\rightsquigarrow P(X=1) = p$

Example: A fair Bernoulli coin is tossed eight times. A "successful toss" is defined to be the coin landing on heads.
Let $X = \#$ of successes or heads in 8 tosses.

equally likely

1. How many *ways* in Ω can $X = 3$?

counting : i.e. $\left\{\begin{matrix} 1\,1\,1\,0\,0\,0\,0\,0 \\ 1\,0\,1\,1\,0\,0\,0\,0 \end{matrix}\right\} = C(8,3)$

2. What is $P\underline{(X = 3)}$ for each *one* of those ways?

$P(\{H \underset{\text{and}}{} H \underset{\text{and}}{} H\,T\,T\,T\,T\,T\}) = P(\{T\,T\,T\,T\,T \underset{\text{and}}{} H \underset{\text{and}}{} H \underset{\text{and}}{} H\})$

$= P(\{H\})^3 \cdot P(\{T\})^5$

3. What is $P(X = 3)$?

The Binomial

Example: A fair Bernoulli coin is tossed eight times. A "successful toss" is defined to be the coin landing on heads.
Let $X = \#$ of successes or heads in 8 tosses.

1. How many *ways* in Ω can $\underline{X = 3}$?

   $C(8, 3)$ OR $C(8, 5)$

2. What is $P(X = 3)$ for each *one* of those ways?

   One such way is $\{HHHTTTTT\}$ which has probability $\underline{P(\{H\})^3 \cdot P(\{T\})^5}$.

3. What is $P(X = 3)$? The product of these two things!

The Binomial

Lets generalize those ideas to derive the Binomial pdf for n trials of an underlying Bern(p).

Let $X :=$ the number of successes of n trials of a Bern(p). Then:

P of X successes in n tries.

NOTATION: We write $\text{bin}(\overset{\text{\# of tries}}{\overset{\downarrow}{n}}, p)$ to indicate that X is a Binomial rv with success probability p and n trials.

The Binomial

Lets generalize those ideas to derive the Binomial pdf for n trials of an underlying Bern(p).
Let $X :=$ the number of successes of n trials of a Bern(p). Then:

$$P(X = i) = (\# \text{ of ways that } X = i) \cdot P(\text{of one such outcome})$$

NOTATION: We write $\underline{X \sim bin(n, p)}$ to indicate that X is a Binomial rv with success probability p and n trials.

The Binomial

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Let $X :=$ the number of successes of n trials of a Bern(p). Then:

$$P(X = i) = (\#\text{ of ways that } X = i) \cdot P(\text{of one such outcome})$$

$$P(X = i) = \binom{n}{i} \cdot P(n \text{ successes}) \cdot P(n - i \text{ failures}).$$

$$P(X = i) = \binom{n}{i} p^i (1-p)^{(n-i)}$$

$$f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{(n-x)}; \quad x \in \{0, 1, 2, \ldots, n\}$$

NOTATION: We write $\underline{X \sim bin(n,p)}$ to indicate that X is a Binomial rv with success probability p and n trials.

The Binomial

The Binomial r.v. counts the total number of successes out of n trials, where X is the number of successes.

Important Assumptions:

1. Each trial must be *independent* of the previous experiment.
2. The probability of success must be *identical* for each trial.

The binomial is often defined and derived as the sum of n *independent, identically distributed* Bernoulli random variables.

In practice, any time we try to study a proportion on an underlying population, we gather a smaller sample where the observed proportion can often be thought of as a binomial random variable.

Daily Recap

Today we learned

1. Bernoullis, Binomials!

Moving forward:

- nb day Friday!

Next time in lecture:

- More special and common pdfs!