TFES Lab (ME EN 4650)
Flat Plate Convection

Textbook Reference: Section 7.1–7.2 (pp. 402–414) from Incropera, DeWitt, Bergman, Lavine, 6$^{\text{th}}$ ed., John Wiley & Sons (2007).

Objectives

(i) measure the temperature as a distance of length along a heated flat plate using embedded thermocouples,

(ii) calculate the local and average heat transfer coefficients and Nusselt numbers, and compare with the theoretical predictions, and

(iii) calculate the net heat flux from the surface.

Background

Determining the convective heat transfer from a surface in a forced or free flow is an important practical problem encountered in a wide variety of engineering applications. Figure 1 illustrates the mechanism of convective heat transfer from a surface. The *local* surface heat flux is expressed as q_s'' and occurs over a differential area segment on the surface dA_s. The *total* heat transfer rate, q_s, between the surface and surrounding fluid flow is obtained by integrating the local heat flux over the entire surface

$$q_s = \int_{A_s} q_s''\, dA_s \,. \tag{1}$$

From Newton's Law of Cooling, we know that the local surface heat flux is given by

$$q_s'' = h\,(T_s - T_\infty), \tag{2}$$

where h denotes the *local* heat transfer coefficient, T_s is the local surface temperature, and T_∞ is the fluid temperature far from the surface.

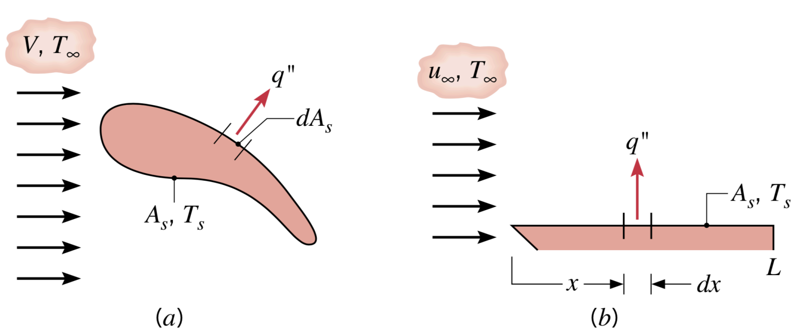


Figure 1. Local convection heat transfer from (a) surface of arbitrary shape and (b) flat plate.

In this laboratory experiment, we will heat a surface using resistive heaters that supply a known amount of heat q_s. We will use a wind tunnel to generate forced flow over the surface; and, then measure the temperature of the surface T_s as a function of location, as well as the freestream temperature T_∞. We will subsequently calculate the local heat transfer coefficient from (2) based on measurements of q_s, T_s, and T_∞. We can define the *average* heat transfer coefficient for the surface as

$$\overline{h} = \frac{1}{A_s} \int_{A_s} h \, dA_s. \tag{3}$$

Note, in many practical applications, we want to determine the total heat transfer rate to/from a surface instead, given a series of temperature measurements along the surface. In this scenario, we would have to select an appropriate h value for the surface shape and flow configuration of interest. To do so, we would utilize empirical Nusselt number relations. Such relations for very basic shapes and configurations may be found in an undergraduate textbook. Those for more complicated shapes might be found in the primary literature. However, for truly unique shapes, the engineer may be compelled to perform custom experiments, using procedures similar to the ones described in this laboratory exercise.

Parallel flow over a flat plate, as shown in Figure 2, provides a good approximation to the flow over many types of surfaces, especially ones that are only slightly contoured, such as airfoils or turbine blades, for example. In this type of external flow, the boundary layers develop freely, without any constraints imposed by adjacent surfaces. Consequently, there will always be a region of the flow outside the boundary layer, in which the velocity and temperature gradients are negligible. The is referred to as the freestream flow and is assumed to have a velocity U_∞ and temperature T_∞.

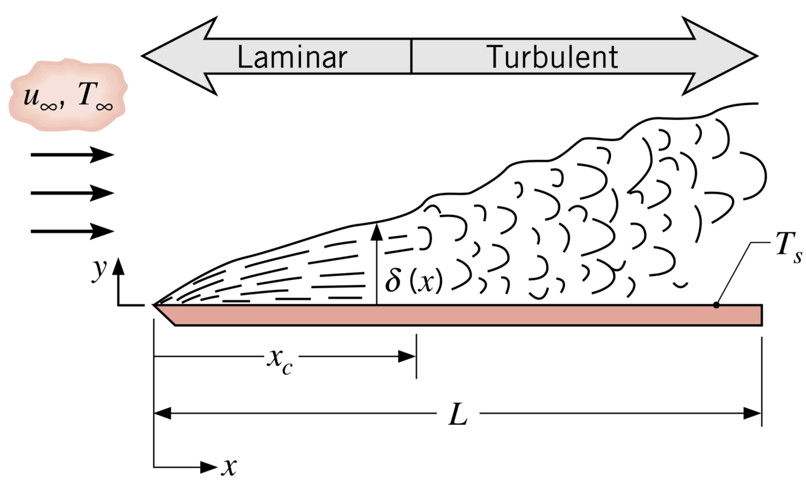


Figure 2. Schematic of parallel flow over a heated flat plate illustrating the transition from laminar to turbulence. The boundary layer thickness is denoted as $\delta(x)$. The streamwise coordinate x is defined from the leading edge. The total length of the plate is L.

The Nusselt number is a nondimensional parameter of the flow that provides a ratio of convection to conduction. The *local* Nusselt number for flow over a flat plate is defined as

$$Nu_x = \frac{h_x\, x}{k}, \tag{4}$$

where h_x denotes the *local* heat transfer coefficient at a distance x from the leading edge. By nondimensionalizing the boundary layer equations, it can be shown that the *local* Nusselt number satisfies the following similarity relationship

$$Nu_x = f(x^*,\, Re_x,\, Pr), \tag{5}$$

where x^* is the nondimesional length along the plate, Re_x is the Reynolds number, and Pr is the Prandtl number, defined respectively as

$$x^* = \frac{x}{L}, \qquad Re_x = \frac{U_\infty\, x}{\nu}, \qquad Pr = \frac{\nu}{\alpha}. \tag{6}$$

Here, x is defined from the leading edge of the plate, L is the total length of the plate, ν $(= \frac{\mu}{\rho})$ is the kinematic viscosity of the air, and α $(= \frac{k}{c_p\,\rho})$ is the thermal diffusivity of the air, where c_p, μ, ρ, and k denote the material properties of the air, namely the specific heat at constant pressure, the dynamic viscosity, the density, and thermal conductivity, respectively. In the section on "Data Analysis", the functional relationships for $f(x^*,\, Re_x,\, Pr)$ for laminar and turbulent boundary layer flow are presented. In many engineering applications, we are not necessarily interested in the detailed behavior of the local Nusselt number, and are only interested in understanding the bulk effect of convection averaged over the entire surface of the object.

In the current experiment, we will examine the case where the heated section is located at some downstream distance from the leading edge of the plate, as shown in Figure 3. The length of the unheated starting segment is denoted as $x = \xi$. Here, x is still defined as

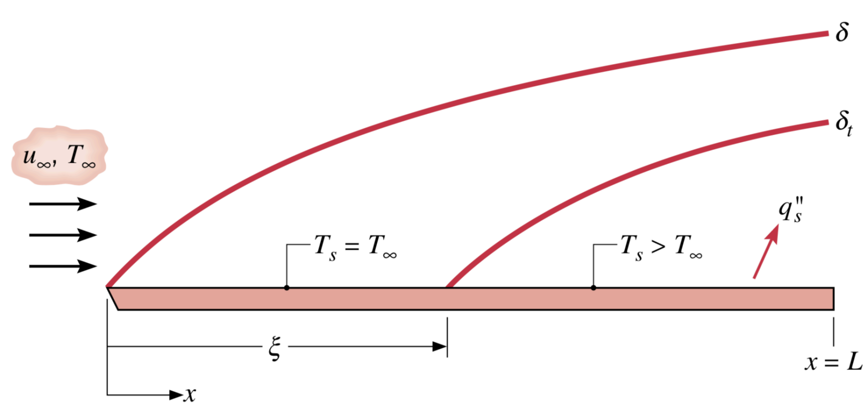


Figure 3. Schematic of parallel flow over a flat plate with an unheated starting length of $x = \xi$. The growth of the momentum and thermal boundary layers with heights δ and δ_t, respectively, are highlighted.

the distance from the leading edge of the plate. The total length of the plate from the leading edge to the end of the heated section is denoted as L. In this configuration, the hydrodynamic boundary layer starts at $x=0$, and leads the thermal boundary layer, which does not start growing until $x=\xi$. This type of scenario provides a better model of many electronic chip cooling applications, for example, in which the chip is typically mounted on a much larger cooling plate so that the heat source remains offset from the leading edge of the plate.

The *average* heat transfer coefficient $\overline{h}_L$ is usually calculated over the heated portion only, and given by

$$\overline{h}_L = \frac{1}{(L-\xi)} \int_{x=\xi}^{L} h_x \, dx \tag{7}$$

The corresponding average Nusselt number is defined as

$$\overline{Nu}_L = \frac{\overline{h}_L \, L}{k}. \tag{8}$$

Note, by convention, the characteristic length scale used in the definition of $\overline{Nu}_L$ in (8) is the total plate length L, even though heating only occurs over a portion of the plate. This is done in practice, in order to obtain consistent comparisons of the effect of ξ on $\overline{Nu}_L$, so that all cases utilize the same characteristic lengthscale L regardless of the length of the unheated section. The theoretical local and average Nusselt number relations can be obtained by applying a correction to the relations for a flat plate with uniform heating over the entire plate. We will present the mathematical form of these relations in the "Data Analysis" section.

Experimental Setup

In this laboratory, convection heat transfer from the surface of a heated plate will be calculated at a single Reynolds number. Table 1 lists the measurements that will be acquired, along with their native units. Thermocouples will be used to measure the surface temperature of the plate as well as the air temperature in the freestream. A Pitot-static probe connected to a differential pressure transducer will be used to measure the dynamic pressure (P_{dyn}) in the freestream. From this, the freestream velocity can be calculated using

Table 1. List of measurements acquired in the experiment with their native units.

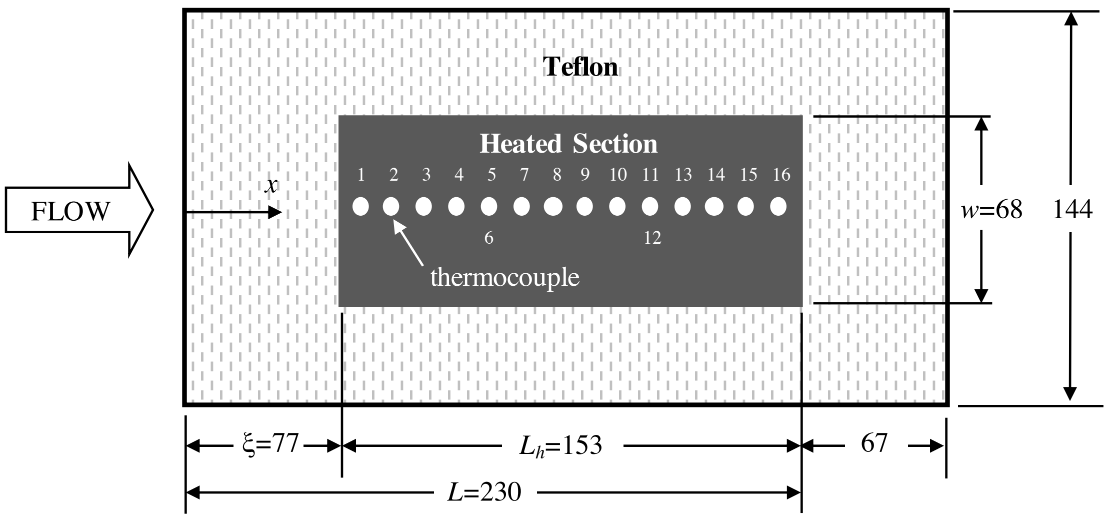
Quantity	Symbol	Units	Instrument
Freestream dynamic pressure	P_{dyn}	mmHg	Pitot-static probe
Plate surface temperature	$T_s(x)$	°C	thermocouple
Freestream temperature	T_∞	°C	thermocouple
Heater voltage	V	VAC	multimeter
Heater resistance	Ω	Ohm	multimeter

Bernoulli's equation,

$$U_\infty = \sqrt{2\,P_{\rm dyn}/\rho}, \tag{9}$$

where ρ is the air density based on the freestream temperature and barometric pressure in the laboratory. Note, the values for the quantities used in (9) must have consistent units. It is recommended that the measured value of $P_{\rm dyn}$ be converted from its native units of mmHg to SI units (i.e., Pa) and ρ be calculated in SI units of kg/m^3, which will yield a value for U_∞ in SI units of m/s.

A small heated flat plate has been designed and constructed to provide experimental conditions that closely mimic those assumed in the theoretical solutions for forced convection over a flat plate with an unheated starting length. The plate is installed in the center of the wind tunnel test section. Air flow is controlled using the wind tunnel fan. In order to eliminate difficulties associated with insulating one of the surfaces, the flat plate has been designed to produce symmetric thermal boundary conditions (i.e., heat transfer occurs from both the top and bottom surfaces).



Thermocouple	Location (mm)	Thermocouple	Location (mm)
1	85	9	153
2	92	10	162
3	102	11	173
4	112	12	173
5	123	13	186
6	123	14	196
7	134	15	209
8	143	16	219

Figure 4. Schematic of the experimental setup showing the locations of the thermocouples. Note, thermocouples #6 and #12 are located on the underside of the plate. The vertical striped section represents the teflon support material. Only the inner grey area is heated. All dimensions are in mm.

The flat plate used in the present study consists of a smooth rectangular surface, 297 mm long by 144 mm wide, as shown in Figure 4. The thickness of the plate is 13.9 mm. A heated section is located a distance $\xi = 77$ mm from the leading edge. The heated section measures $L_h = 153$ mm long by $w = 68$ mm wide, and extends the entire thickness of the plate. The edges of the heated section are surrounded by Teflon that acts like an insulating material. Heat is generated by six flat electric-resistance strip heaters that are embedded within the heated section. The strip heaters each have dimensions of 76.2 mm by 25.4 mm, and are mounted side-by-side so that the air flows across the heater width. The heater arrangement provides a nearly iso-flux boundary condition on the exposed top and bottom surfaces of the plate. The strip heaters are wired in parallel and powered using AC power through a variable transformer that allows the voltage to be adjusted. The total power to the heaters, q_{p}, is determined by measuring the voltage, V, supplied to the heaters and the total resistance of the heater strips R,

$$q_{\mathrm{p}} = V^2/R\,. \tag{10}$$

The net heat flux through either the top or bottom surface is obtained by assuming that the heat is dissipated equally from the two heated plate surfaces,

$$q''_s = \frac{q_{\mathrm{p}}}{2\,(L_h\,w)}\,. \tag{11}$$

Figure 5 shows a top-down photograph of the plate next to an infrared image of the plate during heating. Striations in the temperature contour plot are noticeable due to the individual heater strips. Nonuniform heating can be observed, as well some flux (temperature gradient) into the surrounding Teflon material, implying that horizontal conduction is not completely negligible.

The heated section of the plate is made of a composite material with the layers oriented in a vertical direction such that the thermal conductivity in the vertical direction is approximately 10 times that in the horizontal direction. This design feature reduces axial conduction along

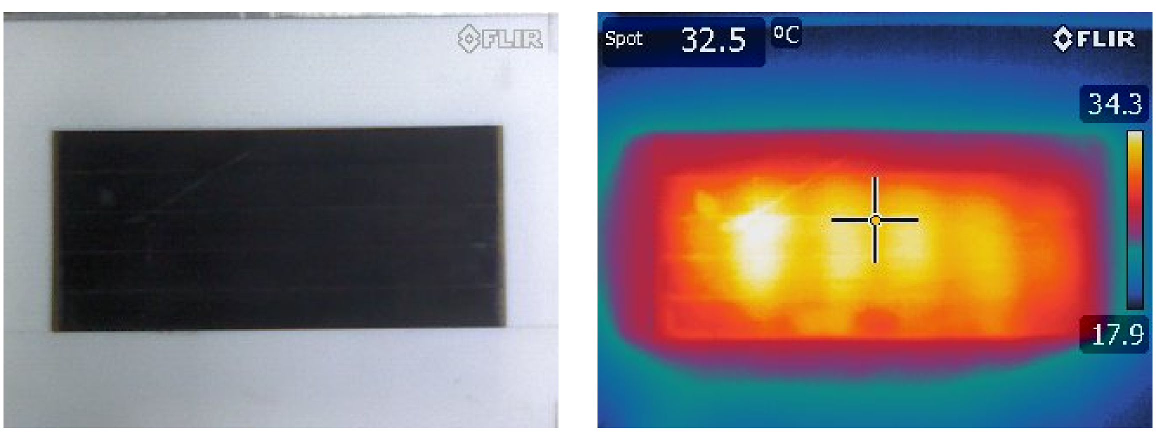


Figure 5. (a) Photograph of the flat plate looking down from above as viewed using the IR camera. The black area represents the heated section; and the white area represents the teflon support material. The flow would be left to right over the surface. (b) Infrared image of the flat plate during heating without air flow.

the heated surface. The adjacent Teflon material minimizes heat loss in the horizontal direction to the surroundings. An aerodynamic leading edge is attached to the front of the heated plate and is designed to prevent boundary layer separation, such that a classical hydrodynamic boundary layer will be established along the top surface of the plate.

Thermocouples are embedded down the centerline of the heated section of the plate according to the x-spacing shown in Figure 4. The locations are referenced from the leading edge. All thermocouples are located on the top surface of the plate, except #6 and #12 that are located on the bottom surface (at the same x-distance as thermocouples #5 and #11, respectively). The thermocouple junction beads have been inserted into small holes drilled from the plate backside to a depth nearly equal to the plate thickness. A high thermal conductivity epoxy secures the thermocouples within the holes. The thermocouple wires routed outside the wind tunnel and connected to a computer data acquisition system for readout. Measurements are recorded once the system has reached steady-state.

Laboratory Procedure

1. Measure and record the local atmospheric pressure P_{atm} using the barometer in the lab. Note, the ambient temperature should be recorded by the thermocouples on the flat plate prior to heating the plate or turning on the wind tunnel, as described below.
2. Collect ambient temperature measurements.
   (a) On the data acquisition computer, start the LabView software program, and then open the file named `WindTunnelConvection2020.vi`.
   (b) Select the RUN button (white arrow in the upper left corner of the workspace window).
   (c) Observe the variations in temperature measurements between the 16 thermocouples. Wait a few minutes to ensure the temperatures have all equilibrated.
   (d) Save the data. Take the average of these data and use as your value for the freestream temperature T_∞.
3. Measure the resistance of the heating strips.
   (a) Ensure the variable transformer power supply is switched OFF.
   (b) Unplug the white power cord connecting the metal junction box to the variable transformer.
   (c) Connect a BNC cable from the multimeter to the top of the metal junction box.
   (d) Turn the dial on the multimeter to the Resistance setting: **Ω**, as shown in Figure 6. Record the resistance reading in Ohm. Note, the resistance should be around 160 Ω. If the resistance is greater than approximately 200 Ω, one or more heaters is disconnected.
4. Set and measure the voltage applied to the heating strips.
   (a) Ensure the variable transformer power supply is switched to the OFF position.

Figure 6. Photographs showing the multimeter settings to measure the (left) heater strip resistance and (right) heater strip voltage. The green circles highlight the correct settings to use on the dial switch.

(b) Reconnect the metal junction box to the variable transformer using the white power cord.

(c) Turn the dial on the variable transformer to a voltage level between 30% – 60%, as suggested by the TA.

(d) Ensure a BNC cable is still connecting the multimeter to the top of the metal junction box.

(e) Switch the variable transformer to the ON position.

(f) Turn the dial on the multimeter to the AC Voltage setting: $\tilde{\mathbf{V}}$, as shown in Figure 6. Record the voltage reading. Note, the setting on the variable transformer dial should roughly correspond to the voltage in Volts, i.e., a setting of 30% should yield a supply voltage of about 30 VAC.

5. In the data acquisition program, select the RUN button (white arrow in the upper left corner of the workspace window). Observe the temperature measurements. The temperature of all thermocouples should be increasing with time.

6. Set the flow speed.

   Depending on the voltage level selected for powering the heating strip in the prior step, the freestream flow speed and wind tunnel fan frequency should be set according to Table 2, in order to ensure sufficient convective cooling on the low end, but not too much as to diminish the temperature gradients in the x-direction. Assuming that the boundary layer will transition to turbulence at $Re_L \approx 5 \times 10^5$, the settings in Table 2 will guarantee that the flow remains laminar over the entire surface of the heated segment of the flat plate.

Table 2. Recommended flow settings.

Freestream Velocity (m/s)	Fan Frequency (Hz)	Heating Voltage (V)
5 – 10	9 – 16	30 – 40
10 – 15	16 – 23	40 – 50
15 – 30	23 – 50	50 – 60

The following describes how to turn on the wind tunnel and set the flow speed. Note, the wind tunnel fan is controlled by adjusted the frequency of electrical power supplied between 0–60 Hz, where 0 Hz represents no power and 60 Hz represents full power. The wind tunnel fan requires a minimum amount of power to start, and will likely not start below a power frequency of 8 Hz.

(a) On the wind tunnel control panel, as shown in Figure 7, press the CTRL key if the PANEL CONTROL light is NOT illuminated green.

(b) Type in the desired fan frequency using the numeric keypad.

(c) To start the fan, press the RUN key.

(d) While running, you can dynamically change the speed using the △ and ▽ keys.

(e) To stop the fan, press the STOP key.

Figure 7. Control panel to adjust wind tunnel speed. The PANEL CONTROL light is underneath the LCD display and highlighted by the white arrow.

7. Record the plate temperatures.

   (a) In the Labview data acquisition program, select the RUN button (white arrow in the upper left corner of the workspace window), if it has not already been selected.

   (b) Monitor the thermocouple measurements. Importantly, the plate should be allowed to reach a maximum local temperature of more than 70°C, as this will damage the components.

   (c) Wait until the plate temperatures have reached steady-state. This will take approximately 10–15 minutes. Note, temperatures should not change by more than ±0.2°C over any given time interval once the flow has reached equilibrium.

   (d) Save the temperatures to a data file.

   (e) Record the dynamic pressure reading from the pressure transducer. You can use the voltmeter to obtain an average. To do this, connect the BNC cable from the back of the pressure transducer box to the voltmeter. Turn the dial switch on the voltmeter to DC Voltage: $\overline{\overline{\text{V}}}$. Then, follow the instructions in Figure 8 to perform an average.

8. Once the experiment is complete, lower the voltage level on the variable transducer and turn the switch to the OFF position.

9. Run the wind tunnel for 5–10 additional minutes to allow the the plate to cool. To stop the fan, press the STOP key on the wind tunnel control panel.

To use MIN MAX AVG recording:

1. Set the desired measurement function and range. (Autoranging is disabled in the MIN MAX AVG mode.)
2. Press MIN MAX to activate MIN MAX AVG mode.

   MIN MAX and MAX come on, and the highest reading detected since entering MIN MAX AVG shows on the display.
3. To step through the low (MIN), average (AVG), and present readings, press MIN MAX.
4. To pause MIN MAX AVG recording without erasing stored values, press HOLD.

   HOLD comes on.
5. To continue MIN MAX AVG recording, press HOLD again. HOLD turns off.
6. To erase stored readings and exit, press MIN MAX for 1 second or turn the rotary switch.

Figure 8. Instructions for using the real-time averaging feature on the Fluke 179 multimeter.

Data Analysis

The primary quantities of interest in the analysis of the experimental data are the (i) local and average heat transfer coefficient, (ii) the local and average Nusselt number and (iii) net heat transfer rate between the plate and the air flow. We will compare the values obtained from the measurements to those predicted by the theory, following the steps described below.

1. Calculations from Measurement Data

A. Heat Flux from Top Surface (q_s'')

The *net* heat flux from the top surface, q_s'', can be calculated from (10) and (11) as

$$q_s'' = \frac{V^2}{2\,R\,L_h\,w}\,, \tag{12}$$

where V and R are the measured heater supply voltage and electrical resistance, respectively. The expression in (12) assumes 100% efficiency in converting electrical power to heat, which is a good assumption for the case of resistive heating elements.

B. Heat Transfer Coefficient (h_x and $\overline{h}_L$)

The local heat transfer coefficient is calculated from Newton's Law of Cooling given in (2), using the measured surface and freestream temperatures as well as the heat flux from (12),

$$h_x(x_i) = \frac{q_s''}{T_s(x_i) - T_\infty} \qquad \text{for} \qquad i = 1, \ldots, N\,, \tag{13}$$

where x_i denotes the x-location of the i^{th} data point as illustrated in Figure 9 and N denotes the total number of data points, which is equal to 14 for the top surface.

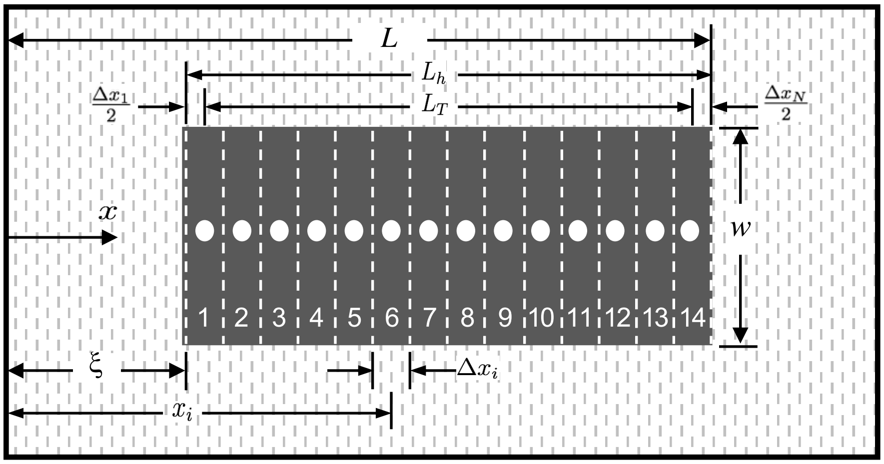


Figure 9. Drawing of the flat plate showing the temperature data discretized into individual measurement cells (delineated by the white dashed lines), each with an area $w\,\Delta x_i$ for $i=1,\ldots,N$ where i is the index number of the cell and $N=14$. The numbering scheme denotes the index of each measurement cell, and may not be the same as the thermocouple number. Note Δx_i is not necessarily the same for each cell either (see the dimensions listed in Figure 4). The white dots represent the thermocouple positions.

The average heat transfer coefficient $\overline{h}_L$ for the entire heated section of the plate is the integral of h_x as given by (7). We will use the trapezoidal rule to perform the numerical integration. Since data are not available exactly at the end points of the heated section at $x=\xi$ and $x=L$ (see Figure 9), we will adjust the limits of the integration accordingly so that the average is performed only over the length of the heated plate section L_T spanning the actual data points,

$$\overline{h}_L = \frac{1}{L_T}\int_{x=x_1}^{x=x_N} h_x(x)\,dx\,, \tag{14}$$

where x_1 and x_N are the locations of the first and last thermocouples, respectively, and $L_T=x_N-x_1$.

C. Nusselt Number (Nu_x and $\overline{Nu}_L$)

The local Nusselt number is calculated based on (4) using the local heat transfer coefficient values from (13) in the previous section,

$$Nu_x(x_i) = \frac{h_x(x_i)\,x_i}{k_f} \qquad \text{for} \qquad i=1,\ldots,N\,, \tag{15}$$

where k_f is the thermal conductivity of the air evaluated at the local film temperature $T_f(x_i)$, defined as

$$T_f(x_i) = \frac{T_s(x_i)+T_\infty}{2}\,. \tag{16}$$

A Matlab function will be provided by the instructor to aid in calculating air properties (see the APPENDIX). The average Nusselt number for the heated plate section is calculated based on the definition of the Nusselt number as given in (8), and using the value for the average heat transfer coefficient given by (14) in the previous section,

$$\overline{Nu}_L = \frac{\overline{h_L}\, L}{\overline{k}_f}, \tag{17}$$

where $\overline{k}_f$ denotes the thermal conductivity of the air based on the average film temperature over the entire heated portion of the plate. Note, by convention, the characteristic lengthscale used in (17) should be the total plate length L (unheated length plus heated length), for reasons explained in the “Background” section.

2. Theoretical Predictions

A. Local Nusselt Number ($Nu_{x,\mathrm{th}}$) and Heat Transfer Coefficient ($h_{x,\mathrm{th}}$)

For flow over a flat plate, the Nusselt number can be related to the Reynolds number and Prandtl number via a universal similarity function. For flow over a flat plate with an unheated starting length of ξ, where the heated portion is subjected to a uniform surface heat flux (iso-flux) boundary condition, the theoretical relationships for the *local* Nusselt number are given by[†]

$$\text{Laminar:} \quad Nu_{x,\mathrm{th}}(x) = \frac{0.453\, Re_x^{1/2}\, Pr^{1/3}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}}, \tag{18}$$

$$\text{Turbulent:} \quad Nu_{x,\mathrm{th}}(x) = \frac{0.031\, Re_x^{4/5}\, Pr^{1/3}}{\left[1-(\xi/x)^{9/10}\right]^{1/9}}. \tag{19}$$

The above relationships are only valid for $x > \xi$, where x is measured from the leading edge, and for $0.6 \leq Pr \leq 60$. Note, Re_x and Pr are defined as in (6). All air properties appearing in Re_x and Pr should be assumed constant and evaluated at the **average film temperature** (averaged over the length of the heated portion of the plate). Recall that the critical Reynolds number for flow over a flat plate is $Re_c \approx 5 \times 10^5$. In situations where $Re_x < Re_c$, the laminar relationship should be used; while for $Re_x > Re_c$, the turbulent relationship should be used.

The *local* theoretical heat transfer coefficient, $h_{x,\mathrm{th}}$, is then determined from the definition of the Nusselt number,

$$h_{x,\mathrm{th}}(x) = \left(\frac{\overline{k}_f}{x}\right) Nu_{x,\mathrm{th}}(x)\,, \tag{20}$$

where $\overline{k}_f$ is the thermal conductivity of the fluid evaluated based on the *average* film temperature.

[†] W. Kays and M. Crawford, *Convective Heat and Mass Transfer*, 3rd ed., McGraw Hill (1993)

B. Average Nusselt Number ($\overline{Nu}_{L,\text{th}}$) and Heat Transfer Coefficient ($\overline{h}_{L,\text{th}}$)

The average theoretical heat transfer coefficient is obtained by substituting the theoretical relations for Nu_x from (18) or (19) into (20), integrating over the length of the heated portion of the plate from $x = \xi$ to $x = L$, and then dividing by $L - \xi$ as written in (7). The integration can be done analytically[‡] to yield the following relations

$$\text{Laminar:} \quad \overline{h}_{L,\text{th}} = 2 \left(\frac{\overline{k}_f}{L - \xi} \right) \left(0.453\, Re_L^{1/2}\, Pr^{1/3} \right) \left[1 - \left(\frac{\xi}{L} \right)^{3/4} \right]^{2/3} , \tag{21}$$

$$\text{Turbulent:} \quad \overline{h}_{L,\text{th}} = \frac{5}{4} \left(\frac{\overline{k}_f}{L - \xi} \right) \left(0.031\, Re_L^{4/5}\, Pr^{3/5} \right) \left[1 - \left(\frac{\xi}{L} \right)^{9/10} \right]^{8/9} . \tag{22}$$

where $\overline{k}_f$ and all other fluid properties should be evaluated at the average film temperature calculated over the heated portion of the plate. In the above relations, $Re_L = U_\infty L/\nu$.

The average theoretical Nusselt number is then given by substituting (21) or (22) into (8). The subsequent equation is repeated here for convenience,

$$\overline{Nu}_{L,\text{th}} = \frac{\overline{h}_{L,\text{th}}\, L}{\overline{k}_f} . \tag{23}$$

C. Predicted Surface Temperature Distribution ($T_{s,\text{th}}$)

If the heat flux from the surface is known, then the surface temperature distribution may be predicted. This may be useful, for example, if you are trying to design a component such that the temperature remains within a specified range.

Based on the **measured heat flux** from the heated surface as given in (12), we can calculate the predicted surface temperature distribution, $T_{s,\text{th}}(x)$, along the top surface using Newton's Law of Cooling (2), coupled with the theoretical value for the heat transfer coefficient $h_{x,\text{th}}$ from (20). This yields the following equation

$$T_{s,\text{th}}(x) = T_\infty + \frac{q_s''}{h_{x,\text{th}}(x)} , \tag{24}$$

where T_∞ is the measured freestream temperature, and q_s'' is the measured net heat flux to the top surface as given by (12).

D. Predicted Heat Transfer due to Convection ($q_{s,\text{th}}''$ and $q_{s,\text{th}}$)

Alternatively, if the surface temperature is known, then the heat flux and net heat transfer rate may be predicted. This may be useful, for example, if you are trying to design a cooling device to reject a specified amount of heat.

Based on the **measured temperatures**, we can calculate the predicted heat flux $q_{s,\text{th}}''(x)$ along the top surface using Newton's Law of Cooling (2), coupled with the

[‡]T. Ameel, "Average effects of forced convection over a flat plate with an unheated starting length", *Int. Comm. Heat Mass Transfer*, V. 24, No. 8, pp. 1113-1120 (1997)

theoretical value for the heat transfer coefficient, $h_{x,\mathrm{th}}$, from (20). This yields the following equation

$$q''_{s,\mathrm{th}}(x_i) = h_{x,\mathrm{th}}(x_i)\,[T_s(x_i) - T_\infty] \ , \tag{25}$$

where x_i represents the locations of the thermocouples and T_∞ is the measured freestream temperature. To obtain a prediction for the net heat transfer rate from the plate, we will use the trapezoidal rule to average $q''_{s,\mathrm{th}}$ over the temperature data from the top surface, similar to what was done for h_L in (14), and then multiply by the area of the heated portion of the plate,

$$q_{s,\mathrm{th}} = \frac{w\,L_h}{L_T}\int_{x=x_1}^{x=x_N} q''_{s,\mathrm{th}}(x)\,dx \, , \tag{26}$$

where x_1 and x_N are the locations of the first and last thermocouples and $L_T = x_N - x_1$.

3. Estimated Heat Transfer due to Radiation

The accuracy of the theoretical predictions may be affected by radiation heat transfer. The local heat flux due to radiation from the heated surface to the surroundings can be estimated from the following equation, using the **measured temperatures**

$$q''_{\mathrm{rad}}(x_i) = \epsilon\,\sigma\left(T_s^4(x_i) - T_\infty^4\right) , \tag{27}$$

where x_i represents the locations of the measurements, ϵ denotes the emissivity of the plate, and $\sigma = 5.6703 \times 10^{-8}$ W/m^2·K^4 is the Stefan-Botzmann constant. Note, the values for T_s and T_∞ in (27) must be specified as absolute temperatures (i.e., in terms of degree Kelvin, if using SI units). The heated section of the flat plate is comprised of a composite material. The exact emissivity of this material is not known, but most carbon fiber composite materials have a relatively high emissivity. Therefore, we will take an approximate value of $\epsilon = 0.7$ for the emissivity here.

Of interest is the percent contribution of heat flux lost to the surroundings via radiation compared to the total heat flux into the system from the resistive heaters. To determine this, q''_{rad} needs to be averaged over the heated surface. We will use the trapezoidal rule, similar to what was done for h_L in (14),

$$q''_{\mathrm{rad},L} = \frac{1}{L_T}\int_{x=x_1}^{x=x_N} q''_{\mathrm{rad}}(x)\,dx \, , \tag{28}$$

where x_1 and x_N are the locations of the first and last thermocouples and $L_T = x_N - x_1$. Finally, the heat transfer rate due to radiation can be obtained by multiplying the predicted net heat flux from (28) by the area of the heated portion of the plate

$$q_{\mathrm{rad},L} = L_h\,w\,q''_{\mathrm{rad},L} \, . \tag{29}$$

Required Plots and Tables

1a. Plot the surface temperature T_s in units of K (on the vertical axis) versus the nondimensional length x' (on the horizontal axis), comparing the measured data with the theoretical prediction, where $x' = (x - \xi)/L_h$. The limits on the horizontal axis should go from 0 to 1. Use the following linestyles: ○ 14 measurements obtained on the *top* surface, □ 2 measurements obtained on the *bottom* surface, — theoretical prediction assuming convection only, and − − theoretical prediction that corrects for the effect of radiation. Do not connect the data points with a line. Include a legend.

1b. Plot the local heat transfer coefficient h_x in units of W/m^2·K (on the vertical axis) versus the nondimensional length x' (on the horizontal axis) comparing the experimentally determined values to the theoretical prediction, where $x' = (x - \xi)/L_h$. The limits on the horizontal axis should go from 0 to 1. Use open circles for the experimental values (do not connect with a line) and a solid line for the theoretical prediction. Include a legend.

1c. Plot the local Nusselt number Nu_x (on the vertical axis) versus the nondimensional length x' (on the horizontal axis) comparing the experimentally determined values to the theoretical prediction, where $x' = (x - \xi)/L_h$. The limits on the horizontal axis should go from 0 to 1. Use open circles for the experimental values (do not connect with a line) and a solid line for the theoretical prediction. Include a legend.

1d. Create a table as shown below with the values from your calculations for the average Nusselt number $\overline{Nu}_L$, average heat transfer coefficient $\overline{h}_L$, and net heat transfer rate q_s from the *top surface*. Note, the average heat transfer coefficient should be calculated over the heated portion only from $x = \xi$ to $x = L$.

	$\overline{Nu}_L$	$\overline{h}_L$ (W/m^2·K)	q_s (W)
Measured			
Theoretical			

Short-Answer Questions

2a. Calculate the percent difference between the theoretical and experimental values of Nu_x, h_x, and T_s as a function of x',

$$\epsilon_Y = \frac{Y_{\text{exp}} - Y_{\text{th}}}{Y_{\text{th}}} \cdot 100, \tag{30}$$

where Y denotes the quantity of interest. State the ranges of ϵ_Y (min & max percentage values) for each quantity (Nu_x, h_x, and T_s). Describe the trend in ϵ_Y with x', and comment whether there are regions along the plate (in terms of x') where the agreement between theory and experiment are more/less favorable for each quantity. [2–4 sentences]

2b. State the percent difference between the experimental and theoretical values for the average heat transfer coefficient and average Nusselt number based on the values given in your table from 1d. When calculating the percent difference, use the same form of the equation as given above for 2a. Offer a viable explanation as to why these differences are so high, and suggest one modification to the experiment or data analysis methods that might lead to better agreement. [3–4 sentences]

2c. State the percentage contribution of heat flux lost to the surroundings via radiation compared to the net heat flux to the top surface by the resistive heaters,

$$\frac{q''_{\mathrm{rad},L}}{q''_s} \cdot 100, \tag{31}$$

where q''_s is the net heat flux to the top surface based on the power supply measurements as given in (12), and $q''_{\mathrm{rad},L}$ is the average radiation flux as given in (28). Does radiation heat transfer help to explain any discrepancies that are observed between the experimental and theoretical data? Explain why or why not. [2–3 sentences]

2d. State the Reynolds number (based on L) for the flow over the heated surface, where L is measured from the leading edge to the end of the heated surface. Comment on whether the boundary layer is expected to be laminar over the entire heated surface. Comment on how you could verify (experimentally) that the boundary layer is indeed laminar or turbulent. [2–3 sentences]

APPENDIX: Calculating Air Fluid Properties

Students can utilize a custom Matlab function entitled `AirProperties`, provided by the instructor, in order to calculate the air fluid properties for a given absolute temperature (in degrees K) and absolute pressure (in Pa). The "help" information for the function is printed below.

```
%--------------------------------------------------------------------
% function [rho,mu,k,Cp] = AirProperties(T,P)
%
% Properties of DRY air based on the ideal gas law. Note, for ideal
% gases, Cp, k, and mu are independent of pressure.
%
% INPUTS:
%    T      Temperature in Kelvin
%    P      Atmospheric pressure in Pascal
%
% OUTPUTS:
%    rho    density in kg/m^3
%    mu     absolute viscosity in kg/(m*s)
%    k      thermal conductivity in W/(m*K)
%    Cp     specific heat in J/(kg*K)
%
% M Metzger
%--------------------------------------------------------------------
```

To call this function in Matlab, consider the following example where the temperature is 30°C and the atmospheric pressure is 655 mmHg:

```
T=30+273.15;                        %temperature in K
P=655*133.3224;                     %pressure in Pa

[rho,nu,k,Cp]=AirProperties(T,P);  %call function
```

The output arguments, in the order returned, are: density, dynamic viscosity, thermal conductivity, and specific heat. The values for all quantities are given in standard SI units.