TFES Lab (ME EN 4650)
Flow Around a Circular Cylinder

Textbook Reference: Section 6.3 (pp. 241–253), Section 9.4 (pp. 428–433), Section 9.7 (pp. 445–459) from Pritchard, 8$^{\text{th}}$ ed.

Objectives

(a) measure the pressure distribution around a circular cylinder, and integrate that to find the drag force,

(b) measure the wake profiles (mean horizontal velocity and turbulence intensity) downstream of a circular cylinder using a Pitot-static probe,

(c) calculate the drag force acting on a circular cylinder by using the wake profile measurement along with the conservation of momentum and mass equations, and

(d) compare the drag coefficient obtained from the experimental data to that published in the literature.

Background

Flow fields associated with bluff (non-streamlined) bodies are of great significance in many engineering applications. Typical applications include the flow around aircraft struts, flow past cabling systems, vehicle aerodynamics, building aerodynamics, and the flow around heat transfer surfaces, to name a few. In the present lab, we investigate the flow past a circular cylinder in a uniform crossflow, which represents a classical configuration that displays many of the features generic to bluff body flows. Sample flow visualization images of this flow field at a moderat Reynolds number are shown in Figure 1. Of particular interest in bluff body flows is understanding how the drag coefficient varies with Reynolds number, and how the pressure coefficient varies with angular position around the object. For the case of flow around a circular cylinder, the Reynolds number (Re_D), drag coefficient (C_D), and pressure

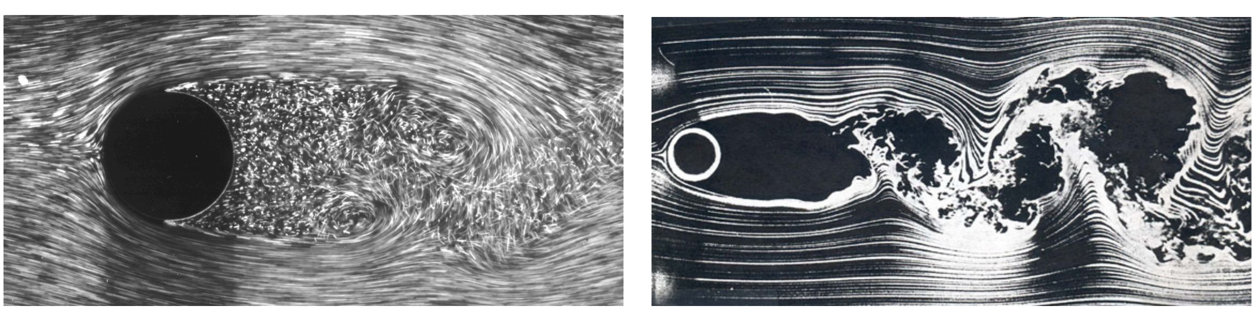

Figure 1. Images of the flow around a circular cylinder at moderate Reynolds number, clearly showing the turbulent wake region on the backside of the cylinder. Flow is from left to right in both images.

coefficient (C_p) are defined, respectively, as

$$Re_D = \frac{\rho\, U_\infty\, D}{\mu}\,, \qquad C_D = \frac{F_D}{\frac{1}{2}\,\rho\, U_\infty^2\, A_f}\,, \qquad \text{and} \qquad C_p = \frac{P_{\text{cyl}} - P_\infty}{\frac{1}{2}\,\rho\, U_\infty^2}, \tag{1}$$

where ρ denotes the fluid density, μ denotes the fluid dynamic viscosity, U_∞ is the freestream velocity (i.e., velocity of the flow far from the object), P_∞ is the freestream static pressure far from the object, D is the cylinder diameter, F_D is the drag force acting on the cylinder, A_f $(= D\,H)$ represents the frontal area (i.e., the area of the object projected onto a plane that is perpendicular to the freestream flow), and P_{cyl} is the static pressure acting on the cylinder. For the case of a circular cylinder where the mean flow is normal to the axis of the cylinder, the projected area looks like a rectangle with dimensions D (cylinder diameter) and H (cylinder span length). Note, the quantity $\frac{1}{2}\rho U_\infty^2$ represents the dynamic pressure of the freestream.

For many bluff bodies, the drag coefficient decreases with Reynolds number, until it reaches a plateau value at Reynolds numbers equal to or greater than about 1000. Figure 2 illustrates the behavior of C_D versus Re_D for a two-dimensional circular cylinder (i.e., the span of the cylinder is assumed to be long enough that end effects may be neglected). Data in the present lab will be acquired within the range $1\times10^3 \leq Re_D \leq 1\times10^4$, where according to Figure 2, $C_D \approx 1$.

Physically, the drag force represents the reaction force required to hold the cylinder stationary in the flow. In the experiment, the reaction force is being applied by the cylinder mounting points in the wind tunnel side wall. We will NOT measure this reaction force directly.

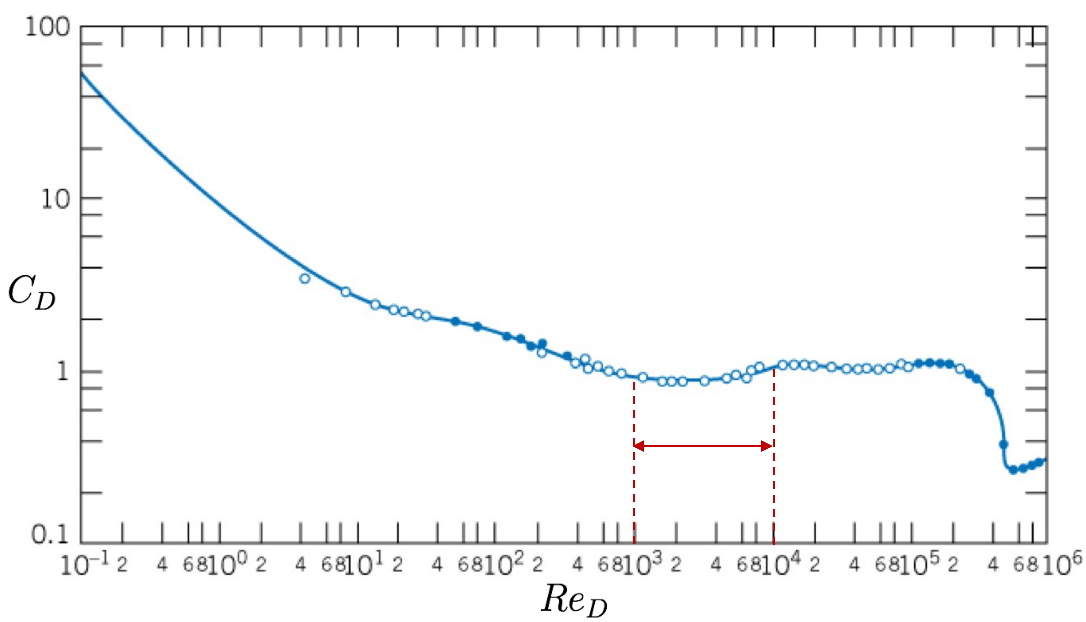

Figure 2. Drag coefficient versus Reynolds number for a two-dimensional circular cylinder, based on data from Schlichting, 1979. The region between the dashed lines represents the Reynolds number range of interest in the present experiment.

However, we can still measure the resultant drag force *indirectly* using one of two methods: (i) applying the conservation of x-momentum equation along with the conservation of mass equation to a control volume around the cylinder, and (ii) integrating the static pressure distribution around the surface of the cylinder. In both cases, the only data that needs to be measured is the pressure. Figure 3 illustrates an example of the expected C_p distribution around a circular cylinder.

Experimental Setup

Experiments will be conducted in the wind tunnel shown in Figure 4 that consists of a 12 inch × 12 inch × 24 inch plexiglass test section. A schematic of the experiment is given in Figure 5 showing the coordinate system that will be used with the origin at the cylinder center. The circular cylinder has a diameter of D = 0.75 inch. The cylinder is mounted horizontally across the test section of the wind tunnel with a span length of H = 12 inch. The approach flow velocity U_∞ (and hence Reynolds number) may be varied by controlling the power supplied to the wind tunnel fan via a variable frequency drive. The freestream static pressure of the flow is denoted by P_∞. While, the static pressure acting on the surface of the cylinder is denoted by P_{cyl}. Note, the fan driving the flow through the wind tunnel is located at the end of the facility. This means that the flow is drawn through the wind

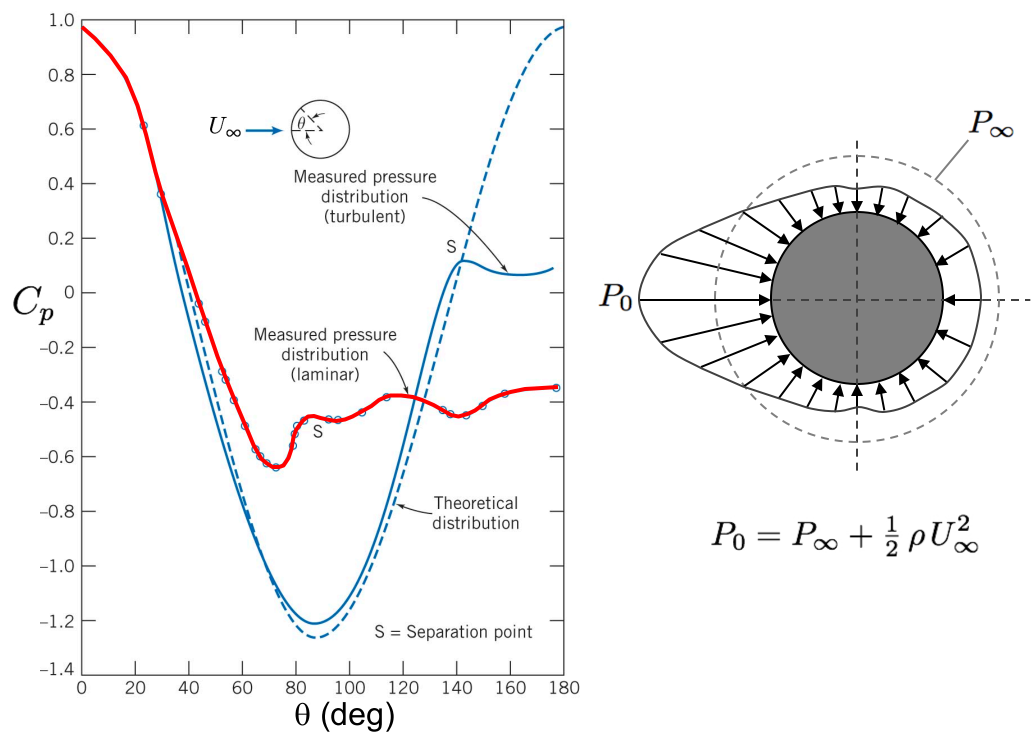

Figure 3. (left) Pressure coefficient and (right) pressure distribution versus polar angle θ for flow around a long circular cylinder. The red line indicates the expected behavior for cylinder Reynolds numbers in the range $1\times10^3 \leq Re_D \leq 1\times10^5$. S marks the location at which the flow separates, and P_0 denotes the stagnation pressure, which is equal to the sum of the freestream static and dynamic pressures, i.e., $P_0 = P_\infty + \frac{1}{2}\rho U_\infty^2$.

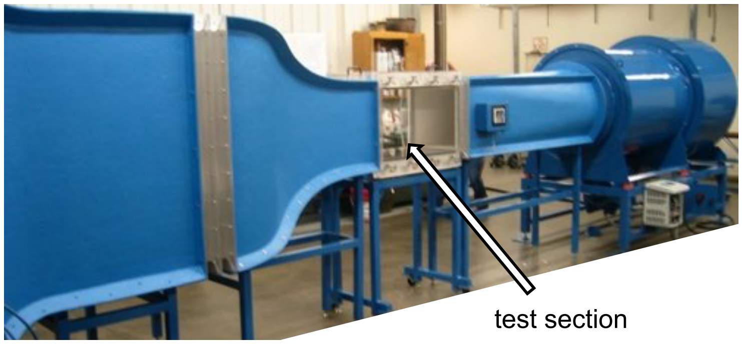

Figure 4. Photograph of the wind tunnel used in the experiment. The arrow indicates the test section where the cylinder and instrumentation will be placed.

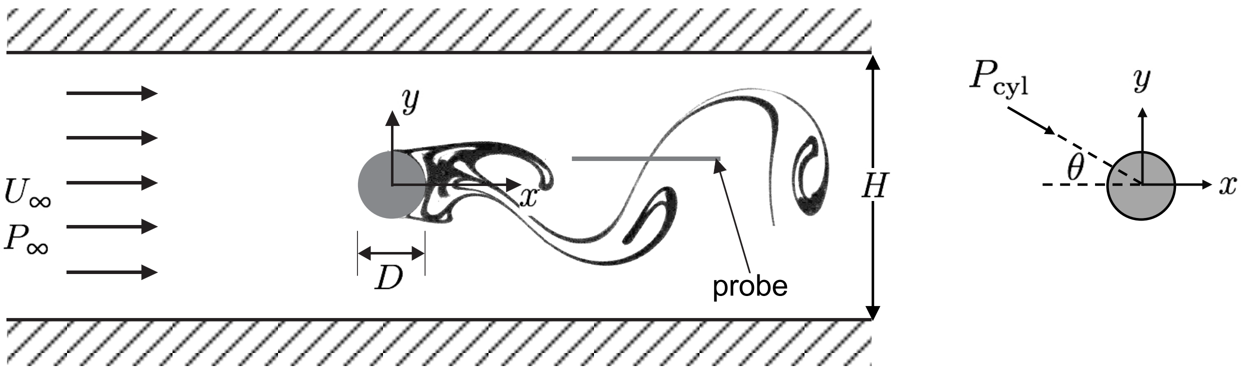

Figure 5. Schematic (side view) of the cylinder and measurement probe set up in the wind tunnel, showing the origin of the coordinate system, the direction of the freestream velocity U_∞, and the direction of the static pressure P_{cyl} acting on the cylinder surface.

tunnel via suction; or, in other words P_∞ (static pressure inside the wind tunnel) will be slightly less than the atmospheric pressure P_{atm} in the room. Table 1 lists the different sets of measurements that will be acquired in the experiment: (i) the static *vacuum* pressure acting on the cylinder, $P_{\text{atm}} - P_{\text{cyl}}$, (ii) the dynamic pressure ΔP of the flow in the freestream and in the wake of the cylinder, and (iii) the static *vacuum* pressure of the flow in the freestream, $P_{\text{atm}} - P_\infty$, and in the wake of the cylinder, $P_{\text{atm}} - P_{\text{wake}}$.

Table 1. List of measurements acquired in the experiment with their native units. All measurements utilize the same differential pressure transducer.

Quantity	Symbol	Units	Instrument
Static *vacuum* pressure around cylinder	$P_{\text{atm}} - P_{\text{cyl}}$	mm Hg	tap on cylinder surface
Dynamic pressure in wake & freestream	ΔP	mm Hg	Pitot-static probe
Static *vacuum* pressure in wake & freestream	$P_{\text{atm}} - P_\infty$	mm Hg	Pitot-static probe (static holes only)

Measuring Static Pressure using Surface Tap

The static pressure P_{cyl} along the surface of the cylinder will be measured using a surface tap (i.e., a small hole drilled into the cylinder) as shown in Figure 6. The hole is routed through the cylinder body to one of its ends that has a hose barb fitting. Flexible tubing is connected from the fitting on the end of the cylinder to a digital manometer or pressure transducer. The pressure transducer used in the present experiment is a MKS Baratron 398A unit, capable of measuring the **differential pressure** between the two input ports, labeled $\mathsf{P_A}$ and $\mathsf{P_B}$. The pressure transducer generates a voltage signal proportional to the difference in pressures ($\mathsf{P_A}$-$\mathsf{P_B}$) detected across the two input ports. This voltage signal is then connected via a BNC cable to a data acquisition system and sampled by a computer program. Note, if the $\mathsf{P_B}$ port on the pressure transducer is left open to atmosphere, then the unit will display/measure the *gauge pressure*. Similarly, if the $\mathsf{P_A}$ port on the pressure transducer is left open to atmosphere, then the unit will display/measure the *vacuum pressure*.The range on the pressure transducer is 10 mm Hg (or 1.333 kPa). The static pressure distribution around the cylinder as a function of polar angle θ is obtained by manually rotating the cylinder mounting plug to vary θ.

Measuring Velocity using a Pitot-Static Probe

Figure 7 shows a photograph of the tip of a Pitot-static probe and a schematic of how the probe can be connected to a manometer in order to measure the dynamic pressure (and hence velocity) of the flow. A Pitot-static probe consists of a tube inside of another tube, where the inner tube is open to the oncoming flow and the end of the outer tube is capped. The inner tube measures the *stagnation pressure*. While, small holes drilled around the periphery of the outer tube measure the *static pressure*. When using a Pitot-static probe, it is important that the tip be aligned with the mean flow direction. This guarantees that the stagnation streamline is parallel to the tip body such that the flow actually stagnates at point 1 (as

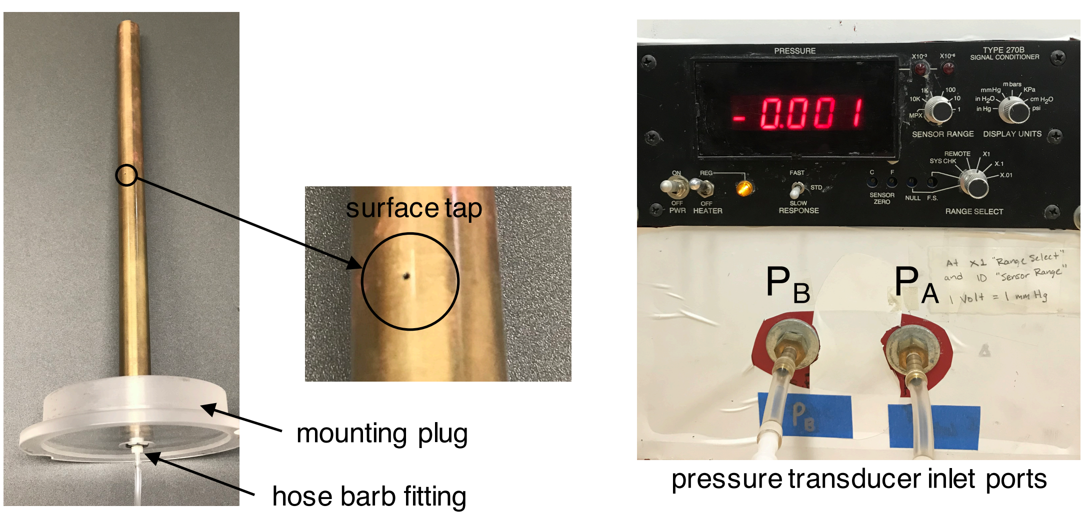

Figure 6. (left) Photograph of the cylinder showing the surface tap used to measure static pressure. (right) Photograph of the pressure transducer display unit and inlet ports labeled $\mathsf{P_B}$ and $\mathsf{P_A}$.

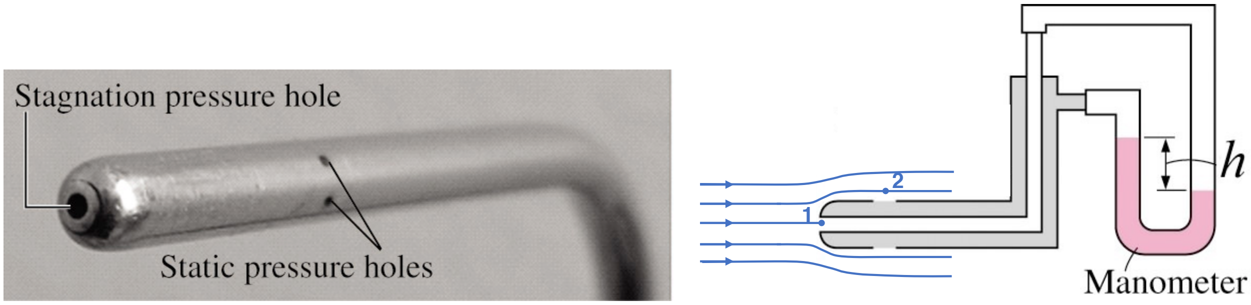

Figure 7. (left) Photograph of a Pitot-static probe. (right) Schematic of a Pitot-static probe connected to a manometer with five sets of streamlines drawn around the tip (right).

shown in Figure 7), and that the static pressure measured at point 2 is perpendicular to the streamline.

We write Bernoulli's equation between points 1 and 2, assuming that they are on the same streamline, which is valid if the probe tip is small enough,

$$P_1 = P_2 + \tfrac{1}{2}\,\rho\, u^2\,, \tag{2}$$

In (2), u denotes the velocity over the probe at point 2, ρ denotes the density of the fluid (in this case, air), P_1 denotes the stagnation pressure at point 1 coincident with the tip of the probe, and P_2 denotes the local static pressure of the flow at point 2. In (2), we have neglected the velocity at point 1 since it is zero at the tip of the probe where the flow stagnates. We define the pressure difference as $\Delta P = P_1 - P_2$. Substituting this into (2) gives

$$\Delta P = \tfrac{1}{2}\,\rho\, u^2\,. \tag{3}$$

Physically, ΔP represents the *dynamic pressure* of the flow, and can be measured by the manometer liquid level height h, as shown Figure 7. In the present experiment, h is measured using a differential pressure transducer. Solving (3) for u yields

$$u = \sqrt{\frac{2\,\Delta P}{\rho}}\,. \tag{4}$$

The relation in (4) gives us a way to determine the velocity at a "point" in the flow from a differential pressure measurement. Importantly, ΔP in (4) MUST be in proper SI units of Pa. Since the pressure transducer used in this experiment outputs in units of mm Hg, we must use the following conversion: 1 mm Hg = 133.32 Pa.

Laboratory Procedure

1. Determine the air density.
   a. Measure the room temperature using the thermometer and the atmospheric pressure using the barometer. Record values on your data sheet.
   b. Use the ideal gas law to calculate the local air density, ρ. Note, in Salt Lake City, $\rho \approx 1\ \mathrm{kg/m^3}$.

2. Inspect display units.

   IMPORTANT: students should not need to adjust any of the knobs on the display units, simply inspect that the gain and range settings are as listed below and indicated in Figure 8.

   On the MKS pressure transducer display:

   → set the SENSOR RANGE knob to "10",
   → set the DISPLAY UNITS knob to "mmHg", and
   → set the RANGE SELECT knob to "X1".

   On the ELD instrument display, verify that the X AXIS POSITION readout and Y AXIS POSITION readout respond to movement of the Pitot-static probe traverse. Verify that the X AXIS POSITION and Y AXIS POSITION readouts display the correct calibrated values when the Pitot-static probe is positioned at the cylinder center. The correct calibrated values will be taped to the wind tunnel test section. **Do not adjust the "span" knobs on the ELD instrument display**, as this will invalidate the traverse system calibration (see next step).

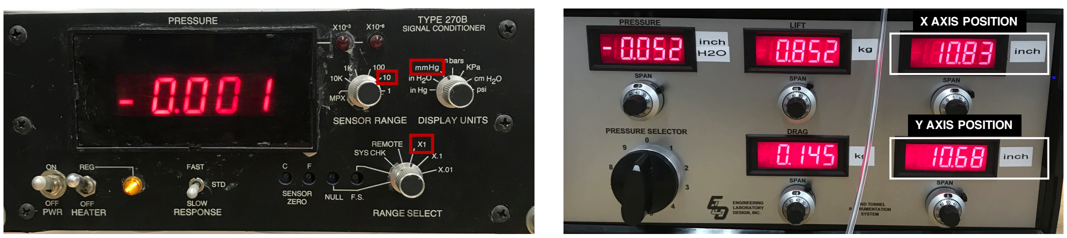

Figure 8. (left) MKS pressure transducer display. (right) ELD instrument display.

3. Calibrate the traverse system, if necessary (only needed if you have mistakenly turned the span knobs on the ELD instrument display).

   a. Place a ruler vertically in the wind tunnel with the bottom edge resting on top of the cylinder.
   b. Using the Y-traverse knob, position the center of the Pitot-static probe tip so that it is colinear with the bottom edge of the ruler.
   c. Record the readout from the Y-traverse display.
   d. Without moving the ruler, use the Y-traverse knob to move the Pitot-static probe upwards a distance equal to $1D$ (or 0.75 inch).
   e. Record the readout from the Y-traverse display.
   f. The difference between the two readouts, divided by 0.75 inch, provides the conversion between display units to actual inches.
   g. Repeat the procedure for the X-traverse by rotating the ruler 90°, so that it lies horizontally along the top of the cylinder.

4. Measure the freestream velocity $U_\infty(y)$ and static pressure $P_\infty(y)$ upstream of the cylinder.

   a. Set the wind tunnel speed so that your Reynolds number based on cylinder diameter is in the range $1\times10^3 \leq Re_D \leq 1\times10^4$. The approach flow speed should be between U_∞ =5–10 m/s.

      The following describes how to turn on the wind tunnel and set the flow speed. Note, the wind tunnel fan is controlled by adjusted the frequency of electrical power supplied between 0–60 Hz, where 0 Hz represents no power and 60 Hz represents full power. The wind tunnel fan requires a minimum amount of power to start, and will likely not start below a power frequency of 8 Hz. A fan frequency between 15–20 Hz should be suitable for this experiment.

      i. On the wind tunnel control panel, as shown in Figure 9, press the CTRL key if the PANEL CONTROL light is NOT illuminated green.
      ii. Type in the desired fan frequency using the numeric keypad.
      iii. To start the fan, press the RUN key.
      iv. While running, you can dynamically change the speed using the $\triangle$ and ∇ keys.
      v. To stop the fan, press the STOP key.

Figure 9. Control panel to adjust wind tunnel speed.

   b. The Pitot-static probe is oriented in the shape of a L where the lower leg contains the sensing tip. Verify that the sensing tip is pointing upstream and that the lower leg of the probe is parallel to the wind tunnel side walls. This will ensure that the probe measures the horizontal x-component of the velocity.

   c. Move the Pitot-static probe to an X-location that is 4.125 inches (or 5.5D) upstream of the cylinder center. Note, if the traverse system has been pre-calibrated by the instructor, then the ELD instrument display should readout X=1.00 inches at this location. Set the Y-location of the Pitot-static probe so that it is at the cylinder center. The ELD instrument display should read out Y=5.875 inches at this location.

   d. Connect the tubes from the Pitot-static probe to the pressure transducer. The TOTAL (stagnation) pressure side should be connected to the port labeled P_A and the STATIC pressure side should be connected to the port labeled P_B. Make sure the output cable from the pressure transducer is connected to channel ai0 of the BNC connector block on the computer cart.

e. Measure the freestream dynamic pressure ΔP (= $\mathsf{P_A} - \mathsf{P_B}$) from the pressure transducer using the provided LabView program on the computer. Collect 15 seconds of data and save it in the directory for your lab section as `Uinf_yD0.txt`, or something else descriptive.

f. Convert the measured dynamic pressure ΔP to a velocity using (4) and calculate the Reynolds number to verify that it is in the appropriate range. Note, the dynamic viscosity of air at room temperature is 1.83×10^{-5} N·s/m^2.

g. Disconnect the TOTAL pressure side of the Pitot-static probe from port $\mathsf{P_A}$ on the pressure transducer and leave that port open to atmospheric pressure. The display on the pressure transducer is now reading the static *vacuum* pressure of the freestream ($P_{\text{atm}} - P_\infty$). Collect 15 seconds of data and save it in the directory for your lab section as `Pinf_yD0.txt`, or something else descriptive.

h. Move the Pitot-static probe upward in the Y-direction by an amount $1D$ (or 0.75 inch) and repeat the measurements. Keep the X-location fixed. Be sure to increment the file name used when saving the data. Repeat until the Pitot-static probe has reached a Y-location of $4D$ above the cylinder center.

5. Measure the wake velocity $u(y)$ and static pressure $P_{\text{wake}}(y)$ downstream of the cylinder.

   (a) Move the Pitot-static probe to an X-location that is $7D$ downstream of the cylinder center, and a Y-location coincident with the cylinder center. The ELD instrument display should readout X=10.375 inch and Y=5.875 inch, if it has been properly calibrated.

   (b) Connect the tubes from the Pitot-static probe to the pressure transducer. The TOTAL (stagnation) pressure side should be connected to the port labeled $\mathsf{P_A}$ and the STATIC pressure side should be connected to the port labeled $\mathsf{P_B}$.

   (c) Collect 15 seconds of data and save it in the directory for your lab section as `Uwake_yD0.txt`, or something else descriptive.

   (d) Disconnect the TOTAL pressure side of the Pitot-static probe from port $\mathsf{P_A}$ on the pressure transducer and leave that port open to atmospheric pressure. The display on the pressure transducer is now reading the static *vacuum* pressure in the wake ($P_{\text{atm}} - P_{\text{wake}}$). Collect 15 seconds of data and save it in the directory for your lab section as `Pwake_yD0.txt`, or something else descriptive.

   (e) Determine the Y-increments that you will move the Pitot-static probe in order to get about 20 data points across the upper-half of the wake, extending from $y/D = 0$ (cylinder center) to $y/D = 4$. Record these on your data sheet. Note, you should concentrate your points in the wake region immediately behind the cylinder, where the velocity profile changes most rapidly. This region is only a few inches high in the Y-direction.

   (f) Turn the Y-traverse knob to move the Pitot-static probe to each of the Y locations determined in the previous step. Use the Y-traverse readout and the Y-traverse calibration determined earlier to set the y location accurately.

(g) Measure both the dynamic pressure ΔP and the static vacuum pressure $P_{\text{atm}} - P_{\text{wake}}$ by collecting 15 seconds of data for each, and saving the files in the directory for your lab section as `Uwake_yDXX.txt` and `Pwake_yDXX.txt` where `XX` denotes the corresponding y/D position of the probe, or some other descriptive naming convention of your choice.

(h) Repeat until all Y locations have been measured. Take care to ensure that the proper tubes are connected to the pressure transducer for each measurement.

6. Measure static pressure distribution around cylinder surface P_{cyl}.

   (a) Keep the wind tunnel at the same speed used in the previous steps.

   (b) Connect the pressure tap on the cylinder to the port on the pressure transducer labeled $\mathsf{P_B}$, and leave the port labeled $\mathsf{P_A}$ open to atmospheric pressure. The display on the pressure transducer is now showing the static *vacuum* pressure acting on the cylinder ($P_{\text{atm}} - P_{\text{cyl}}$).

   (c) Rotate the cylinder such that the cylinder pressure tap is located at an angle $\theta = 0^\circ$. Use the notch on the mounting plug and the protractor tape on the wind tunnel side wall to obtain an accurate angular position.

   (d) Collect 15 seconds of data and save it in the directory for your lab section as `Pcyl_deg0.txt`, or some other descriptive name.

   (e) Repeat by rotating the cylinder in 5° increments from $0^\circ < \theta \le 90^\circ$. Then, increase by 10° to $\theta = 100^\circ$. Then, increment by 20° in the range $100^\circ < \theta \le 180^\circ$. Collect 15 seconds of data at each angle. Be sure to change the file name accordingly to reflect the actual angular position.

Data Analysis

1. Turbulence in the Wake

   At the Reynolds number investigated in this experiment, the flow in the wake behind the cylinder is highly turbulent. This means the velocity field is unsteady (changing instantaneously in time at any given point). Figure 10 shows the instantaneous velocity signal as a function of time for one point in the wake. One way that engineers and scientists deal with turbulent flow is to decompose it into a mean (time-averaged) component plus a fluctuating component,

$$u(x, y, t) = \overline{u}(x, y) + u'(x, y, t), \tag{5}$$

   where, the overline denotes a time-averaged quantity. The data collected from the pressure transducer is digitized by the computer, and stored as a series of data points, one velocity for each instant in time. The sampling frequency of the data is 100 Hz (i.e., 100 samples are collected by the computer every second). In other words, the time between data points collected is 10 ms. Therefore, the time-averaged velocity at each point can be calculated as

$$\overline{u} = \frac{1}{N} \sum_{i=1}^{N} u_i \tag{6}$$

where N denotes the total number of data points collected. For example, if the sampling frequency is 100 Hz and the data were collected for 20 s, then $N = 100 \cdot 20 = 2000$. The standard deviation of u provides a measure of the amplitude of the fluctuating velocity u' part of the signal. The standard deviation is calculated as

$$\sigma_u = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(u_i - \overline{u})^2}. \tag{7}$$

The *turbulence intensity* is then defined as $\sigma_u/\overline{u}$ and represents the relative strength of the fluctuating part of the signal to the mean part. Note, the turbulence intensity at the point shown in Figure 10 is 0.032 (or, 3.2%).

2. Post-Processing the Raw Data

Before we can proceed to calculating the drag coefficient of the cylinder, we need to post-process the collected raw data. The steps below illustrate how to do this for a single data file. You can create a for-loop to read in all of your data files and expedite this process, as shown in the Appendix I.

i. Calculate the air density from the ideal gas law using the measured room temperature, T_{amb}, and atmospheric pressure, P_{atm}, from the barometer.

ii. Load the measurements from each data file into Matlab:

```
data=load('MyData.txt');
```

Note, the above operation creates a column-vector `data` with a size (N,1) where N is the total number of measurements acquired in 15 seconds.

→ For the wake profile measurements and the freestream velocity measurement, `data` contains the differential pressures ΔP from the Pitot-static probe.

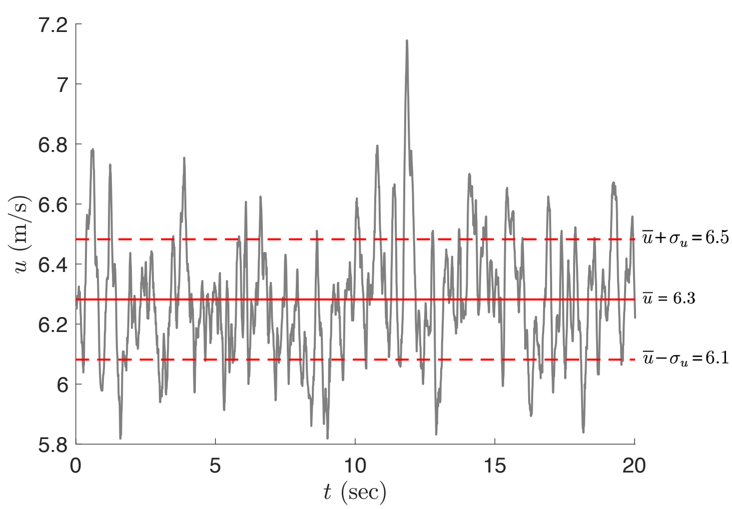

Figure 10. Instantaneous velocity signal recorded from the Pitot-static probe located in the wake at x/D=7 and y/D=0. The horizontal red line indicates the mean velocity: $\overline{u}$ = 6.3 m/s. While, the two horizontal red dashed lines represent one standard deviation: $\sigma_u = \pm 0.2$ m/s. Note, the turbulence intensity associated with this signal is (0.2/6.3)·100=3.2%.

- → For the static pressure of the flow upstream/downstream of the cylinder, `data` contains the static *vacuum* pressure: $P_{\text{atm}} - P_\infty$ (upstream) and $P_{\text{atm}} - P_{\text{wake}}$ (downstream). Therefore, to calculate P_∞ (the absolute freestream static pressure), you would perform this operation:

```
Pinf=Patm-data;
```

  Similarly, to calculate P_{wake} (the absolute static pressure in the wake), you would preform this operation:

```
Pwake=Patm-data;
```

  Note, both of the above operations assumes that `Patm` and `data` are in the same units of mmHg, since the pressure transducer outputs readings in mmHg.

- → For the static pressure measurements around the cylinder, `data` contains the static *vacuum* pressure $P_{\text{atm}} - P_{\text{cyl}}$. Therefore, to calculate P_{cyl} (the absolute static pressure on the cylinder), you would perform this operation:

```
Pcyl=Patm-data;
```

iii. For the wake profile measurements, convert the measured differential pressures ΔP from the Pitot-static probe to u velocities using (4). Be sure to convert pressure to units of Pa.

```
u=sqrt(2*data*Pconvert/rho);
```

In the above operation, `Pconvert` is the unit conversion for pressure (from mmHg to Pa) and `rho` is the air density in proper SI units. Note, the above operation creates an array `u` that is the same size as `data`.

iv. Compute the statistics of u (mean and standard deviation).

```
umean=mean(u); ustd=std(u);
```

The value in `umean` is the one you will use when calculating the drag force; while, the value in `ustd` will be used in plotting the turbulence intensity (and, the uncertainty as described below).

v. For the case of the static pressure measurements around the cylinder, you only need to convert to SI units and then calculate the statistics.

```
p=data*Pconvert; pmean=mean(p); pstd=std(p);
```

3. <u>Uncertainty Estimates</u>

Because of the turbulent nature of the flow in the wake, the mean velocity measurements will have some uncertainty. We would like to estimate how accurately we can measure $\overline{u}$ at each of the points in the wake profile. To do this, we need to calculate the *standard error of the mean*, denoted as $\sigma_{\overline{u}}$. Recall from the Central Limit Theorem of statistics that the distribution of the mean is closely approximated by the Gaussian or normal distribution as shown in Figure 11. Here, $\tilde{u}$ denotes the *true* mean value, which we are estimating by $\overline{u}$ based on a finite-time sample of the data using the calculation in (6). The standard error of $\overline{u}$ is given by

$$\sigma_{\overline{u}} = \frac{\sigma_u}{\sqrt{n}}, \tag{8}$$

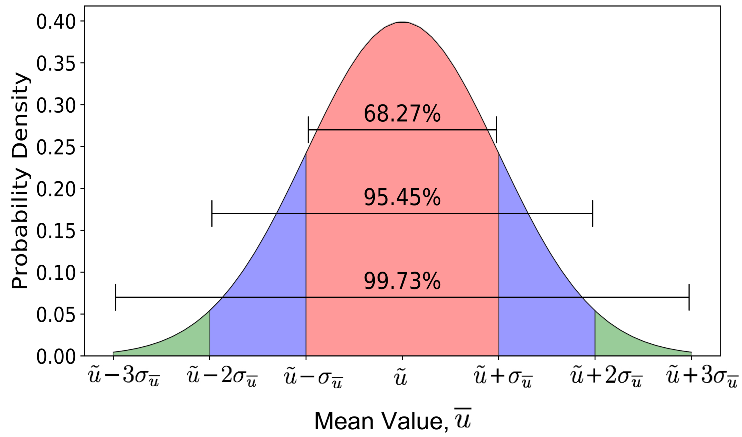

Figure 11. The standard Gaussian or normal probability density function where $\tilde{u}$ denotes the *true* mean value and $\sigma_{\overline{u}}$ denotes the standard error of $\overline{u}$, which is an estimate of $\tilde{u}$. This plot shows that 95.5% of all the possible mean values $\overline{u}$ calculated from the population will fall within a range of $\tilde{u}+2\sigma_{\overline{u}}$ and $\tilde{u}-2\sigma_{\overline{u}}$.

where n denotes the number of *independent* observations in the sample and σ_u is the same standard deviation of the velocity signal that was calculated in (7).

Here is where the analysis with regard to turbulence gets a little tricky. Turbulence is not a true random process, even though it behaves in a random-like fashion. The velocity at any given point in a turbulent flow does not jump randomly from one value to the next, but rather, changes continuously in time. In fact, the turbulence is correlated over some timescale, τ, as highlighted in Figure 12. When we expand the

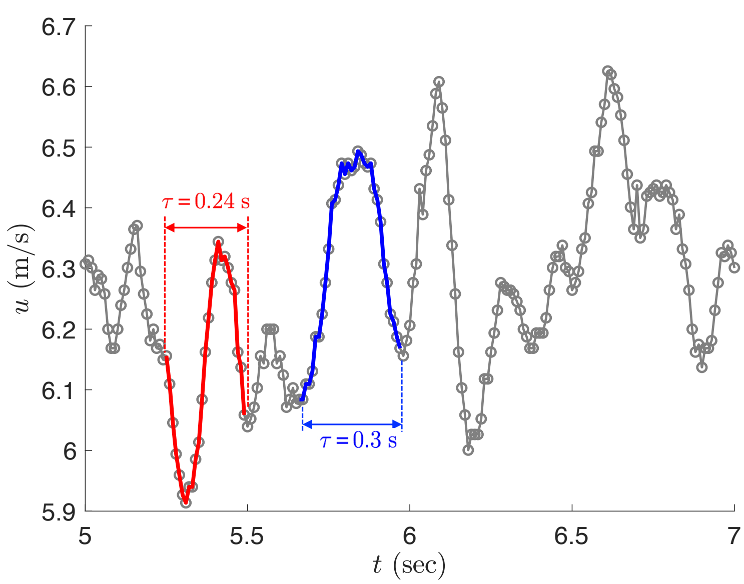

Figure 12. The same velocity signal from Figure 10 expanded to show a 2-second time segment. The blue and red highlighted regions indicate sections in the time series where the velocity appears correlated with itself over a time period of τ. This is referred to as the correlation timescale of the turbulence.

velocity signal, we see that it looks somewhat sinusoidal with a period (or correlation timescale) of $\tau \approx 0.3$ s in this flow. Therefore, we will only have a series of independent velocity measurements, if we take a data point every τ seconds. Since the total duration of the velocity signal is 20 s, $n = 20\,\mathrm{s}/0.3\,\mathrm{s} = 67$ in this case. This yields a value of $\sigma_{\overline{u}} = 0.2/\sqrt{67} = 0.024$ m/s, which represents the uncertainty in the mean value at this point in the wake profile. When analyzing the uncertainty in your data, assume $n = 50$ for all data points, since your data files will only be 15-s long.

Finally, we would like to specify a confidence interval to our uncertainty. This way, we can say that the mean value falls within a certain range with some percentage of confidence. As shown in Figure 11, 95.5% of all possible mean values in the population fall within a range of $\pm 2\sigma_{\overline{u}}$ about the true mean. Therefore, if we use $\pm 2\sigma_{\overline{u}}$ for our errorbars, we can claim with 95% confidence that the mean value of the velocity at this point in the wake is 6.3 ± 0.05 m/s. Note, we can follow a similar procedure to specify the uncertainty in the static pressure measurements around the cylinder.

4. Drag Force: From Conservation of Mass and Momentum

The net drag force on the cylinder is determined through an application of the integral form of the conservation of x-momentum equation. A sketch of the problem including the control volume is shown in Figure 13. Conservation of x-momentum for the control volume shown is

$$-F_D + \int_{\mathrm{CS}} P\, dA = \int_{\mathrm{CS}} u\, \rho\, \vec{V} \cdot d\vec{A}, \tag{9}$$

where u denotes the x-component of velocity, F_D denotes the force required to hold the cylinder in place (and is equivalent to the drag force acting on the cylinder), P is the static pressure acting on the control surface, and ρ is the air density. Note, in writing (9), we have assumed steady flow. This will be a good assumption if we use the time-averaged velocities and static pressures. We have further assumed that the control volume is large enough (i.e., the control surfaces are far enough away from the cylinder), such that the flow along the top and bottom control surfaces remains essentially parallel. This allows us to neglect the shear stress contribution in (9). In

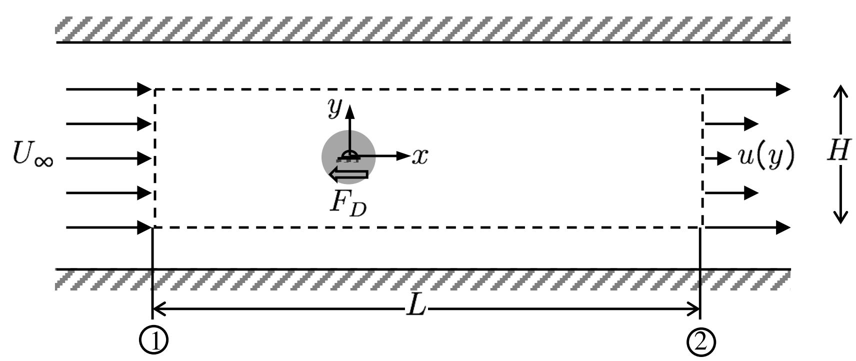

Figure 13. Control volume (dashed line) for the integral conservation of momentum analysis applied to the circular cylinder mounted in the center of the wind tunnel.

terms of the measurements acquired in the lab, the pressure along control surfaces ① and ② are $P_1 \equiv P_\infty$ and $P_2 \equiv P_{\text{wake}}$, respectively.

The integrals in (9) can be split into two parts, for the upstream ① and downstream ② control surfaces. Note, in reality neither the freestream velocity U_∞ nor the freestream static pressure P_∞ nor the static pressure in the wake P_{wake} are constant. Therefore, we need to measure those quantities as a function of y. Solving (9) for F_D, and expanding the integrals yields an equation for the drag force per unit length acting on the cylinder,

$$F_D = 2\int_{y=0}^{H/2} \left[P_\infty(y) - P_{\text{wake}}(y)\right] dy + 2\rho \int_{y=0}^{H/2} U_\infty^2(y)\, dy - 2\rho \int_{y=0}^{H/2} u(y)^2\, dy \tag{10}$$

where we have taken advantage of symmetry so that we only have to integrate over the top half of the domain, thus the factor of 2 appearing in front of the integral terms. The first term on the righthand side represents the NET pressure acting on the control volume. The second term represents the momentum flux through the upstream control surface. And, the third term represents the momentum flux in the wake.

To determine F_D in (10), the $P_\infty(y)$, $P_{\text{wake}}(y)$, $U_\infty(y)$ and $u(y)$ must be measured and integrated numerically. The numerical integration should be performed using a second-order accurate **trapezoidal rule** or a third-order accurate Simpson's rule. In Matlab, the trapezoidal rule method can be performed using the `trapz()` command. For example, for the momentum flux in the wake, you would write the following:

```
MomFlux = trapz(y,u.^2);
```

Note, the height values stored in `y` must be referenced to the origin at the center of the cylinder and NOT the wind tunnel floor. Recall that the trapezoidal rule is a second-order accurate numerical method, the accuracy of which depends strongly on the spacing between the data points. This is why we acquired so many data points in the wake, in order to obtain an accurate calculation of the integral momentum flux and the static pressure in the wake. One could possibly get away with fewer data points if a higher order integration method such as Simpson's rule is used for the integration. In addition, it turns out that the control-volume method is highly sensitive to the static pressure measurements. Assuming constant static pressure across the control surface leads to large errors in the drag force calculation.

Remember that <u>conservation of mass</u> also needs to be satisfied for this same control volume,

$$\rho \int_{y=0}^{H/2} U_\infty(y)\, dy - \rho \int_{y=0}^{H/2} u(y)\, dy = 0. \tag{11}$$

The diligent engineer will check to ensure that the data, when numerically integrated, yield a value close to zero (to within the uncertainty of the measurements at least).

One last comment should be made about the size of the control volume. The x-component of the flow velocity will be measured in the lab using a Pitot-static probe connected to a differential pressure transducer. The wake profile $u(y)$ and freestream velocity profile $U_\infty(y)$ are obtained by traversing the Pitot-static probe in the positive y direction starting from the cylinder center and moving toward the top wall of the test section. The wake profile should be measured at an x location between $7D$–$10D$ downstream of the cylinder. This downstream distance is chosen because Pitot-static probe data cannot be trusted in the wake region directly behind the cylinder, where the flow recirculates. Similarly, the freestream profile should be measured at an x location $> 5D$ upstream of the cylinder to reduce the chance of probe interference with the cylinder.

5. Drag Force: From the Static Pressure Distribution around Cylinder

At high Reynolds number ($Re_D > 1 \times 10^3$), the drag force acting on a circular cylinder is dominated by the **form drag** (or pressure drag), with the viscous drag (or skin friction) contributing much less. Therefore, we can estimate the drag force on the circular cylinder by integrating the static pressure distribution P_{cyl} around the surface of the cylinder. The highest pressure is expected at the stagnation point ($\theta = 0^\circ$) and the lowest pressure is expected where the flow separates near the top and bottom of the cylinder ($\theta \approx \pm 90^\circ$). This expectation is confirmed by the C_p versus θ plot shown previously in Figure 3.

The resultant drag force on the cylinder is simply the integral of the component of pressure projected into the mean flow direction. Since the mean flow is in the x-direction, we need to integrate the x-component of the pressure. Recall that the pressure acts **normal** to the surface. Therefore, the drag coefficient can be calculated as

$$C_D = \int_{\theta=0}^{\pi/2} C_p(\theta)\, d\theta. \tag{12}$$

Importantly, θ must be expressed in units of radians in the (12). Based on the coordinate system shown in Figure 3, $\theta = 0$ denotes the location when the cylinder tap is located at the stagnation point (i.e., at the front of the cylinder); while, $\theta = \pi$ denotes the location where the cylinder tap is located directly at the back of the cylinder. This integration should be performed numerically using either a second-order trapezoidal rule or a third-order Simpson's rule.

Figures

Captions

A meaningful and comprehensive figure caption must accompany all figures. The figure caption must include the following label: Figure 1X. where X denotes the letter a – d according to the plot order listed below. Use the following guidelines when writing figure captions:

→ Ideally, the figure caption should provide a "standalone" description of the plot, and contain enough information that the reader can understand what is being shown without having to refer back to the main text of the document (report, memo, etc.).

→ The caption should start with a statement of the quantities plotted on both axes. Typically the axis labels will utilize mathematical symbols (with appropriate units) for the quantities being plotted; however, in the caption, these quantities are described in words.

→ Include relevant contextual information about the plot. For example, for the wake profile, you should state the x/D location and Re_D value at which the measurements were taken. Additionally, you should state that the measurements are for the case of flow around a two-dimensional circular cylinder. Finally, adding some text about the measurement technique used to obtain the data would be appropriate.

→ When your plot includes errorbars, it is helpful to provide information in the caption about the errorbars. For example, you should include the fact that the errorbars represent a 95% confidence interval.

→ When comparing with published data, it is appropriate to provide a reference for the publication.

→ If in doubt about what information to include in the caption, one should err on the side of providing more information, rather than less.

Plots

1a. Plot the mean horizontal velocity profile in the wake $\overline{u}/U_\infty$ (on the vertical axis) versus y/D (on the horizontal axis). Since both $\overline{u}/U_\infty$ and y/D are nondimensional, your axis labels will not have units. Use markers, such as circles, for your data. Include vertical errorbars on the data for $\overline{u}/U_\infty$ that represent the standard error in the mean to within a 95% confidence interval. That is, your errorbars should have a total length of $\pm 2\,\sigma_{\overline{u}}/U_\infty$, where $\sigma_{\overline{u}}$ denotes the standard error of the mean velocity. Note, $\sigma_{\overline{u}}$ will vary with y. For reference, include a solid black (horizontal) line corresponding to $\overline{u}/U_\infty = 1$ on your plot as well. This line represents the case where the mean velocity in the wake is equal to the freestream velocity.

1b. Plot the horizontal turbulence intensity $\sigma_u/\overline{u}$ (on the vertical axis) versus y/D (on the horizontal axis). Since both $\sigma_u/\overline{u}$ and y/D are nondimensional, your axis labels will not have units. Use markers, such as circles, for your data.

1c. Plot the pressure coefficient C_p (on the vertical axis) versus angular position θ (in degrees, on the horizontal axis). Use markers, such as circles, for your data. Include vertical errorbars on the data for C_p that represent the standard error of C_p to within a 95% confidence interval. For reference, include a solid black (horizontal) line corresponding to $C_p = 0$ on your plot as well. This line represents the case where the static pressure on the cylinder is equal to the freestream static pressure.

1d. On a single figure, plot the following as listed below. Include an appropriate legend.

→ C_D (on the vertical axis) versus Re_D (on the horizontal axis) for the published results. Use a solid black line. Both C_D and Re_D should be plotted on logarithmic coordinates, similar to Figure 2. Include grid lines on your plot. Note, the published results can be downloaded as a text file from CANVAS.

→ C_D versus Re_D based on your analysis of the cylinder static pressure measurements. Plot your single data point using the ■ marker.

→ C_D versus Re_D based on your analysis of the conservation of mass and momentum equations. Plot your single data point using the ● marker.

Short-Answer Questions

In each of the short-answer questions below that require additional calculations, be sure to include the calculations in your Matlab code and have the results displayed to the screen.

2a. State the thickness of the wake at the x/D location of your measurements. The wake thickness should be stated in terms of the number of cylinder diameters D and not in dimensional units. Include the cylinder Reynolds number for your measurements as well. Finally, provide a technical description how you quantified the wake thickness. [2 sentences]

Your response should be of the following form, where XX denotes the values from your analysis:

> "The wake thickness behind the cylinder was found to be XXD, based on Pitot-static probe measurements taken at a downstream distance of x/D=XX and Reynolds number of Re_D=XX. The wake thickness was determined as the y-location in the wake profile where the wake velocity u reached a plateau, i.e., became independent of y."

2b. State the maximum uncertainty in $\overline{u}$ and C_p and their corresponding y/D locations. Calculate the maximum uncertainties in terms of a percentage using the following expressions:

$$\max\left(\frac{2|\sigma_{\overline{u}}|}{\overline{u}}\right) \cdot 100 \qquad \text{and} \qquad \max\left(\frac{2|\sigma_{P_r}|}{P_r}\right) \cdot 100 \,, \tag{13}$$

where P_r $(= \overline{P}_{\text{cyl}} - \overline{P}_\infty)$ is the average relative static pressure acting on the cylinder. Explain why the error bars on $\overline{u}/U_\infty$ are not uniform with respect to y/D, and appear larger in some regions of the wake. Similarly, explain why the error bars on C_p appear larger at some θ values but less at others. [3 sentences]

2c. State the y/D range at which $\overline{u}/U_\infty \geq 1$ for your data in the wake. In your statement, include the x/D location at which the wake profile was measured, along with the cylinder Reynolds number Re_D. Explain why there are regions of the flow in the wake where the $\overline{u}$ velocity is greater than U_∞. [2–3 sentences]

2d. State the percent difference in the drag coefficients C_D obtained from your data analysis using the two different methods, compared with the "expected value" at the same cylinder Reynolds number based on the published results,

$$\epsilon_{C_D}\,(\%) = \frac{|(C_D)_{\text{Pub}} - (C_D)_{\text{Data}}|}{(C_D)_{\text{Pub}}} \cdot 100$$

Note, you will need to interpolate the published C_D results to find the value of C_D corresponding to the cylinder Reynolds number used in your experiment. When writing your response, explicitly state both methods used to calculate C_D. Also, in your response, explicitly state the value of the Reynolds number for your experiment. Finally, state which method yields a more accurate measure of C_D. [3 sentences]

Your response should be of the following form, where XX denotes the values from your analysis:

> "At a cylinder Reynolds number of Re_D=XX, the percent difference in C_D between the published results and the present data based on conservation of mass/momentum from the wake profile measurements is XX%. At the same Reynolds number, the percent difference in C_D between the published results and the present data based on the static pressure measurements around the cylinder is XX%. From this, it is clear that the XX method yields a more accurate measure of the drag coefficient."

Appendix I: MATLAB code

When collecting data in the wake profile and static pressure experiments, it is helpful to follow a file-naming scheme that allows the data files to be easily read into Matlab using a `for-loop`. For example, consider the case where the following data files were created for the static pressure experiment: P0deg, P5deg, P10deg, ..., P180deg. We can use the following code snippet to read in these files and perform the post-processing calculations.

```
% define array of polar angles examined in the experiment
theta = [0:5:100,120:20:180];

% total number of data files collected
N = length(theta);

% create arrays to store the mean and standard deviation
pmean = zeros(size(theta));
pstd = zeros(size(theta));

% loop through all data files
for (i=1:N)
    % filename of ith file
    FileName=['P',num2str(theta(i)),'deg'];

    % read in data from the file
    data = load(FileName);

    % calculate mean and std
    pmean(i) = mean(data);
    pstd(i) = std(data);
end
```