

Geog 509: Bayesian Data Analysis

Chapter 2 Problem Set

Jesse Piburn

2020-01-27

2E1

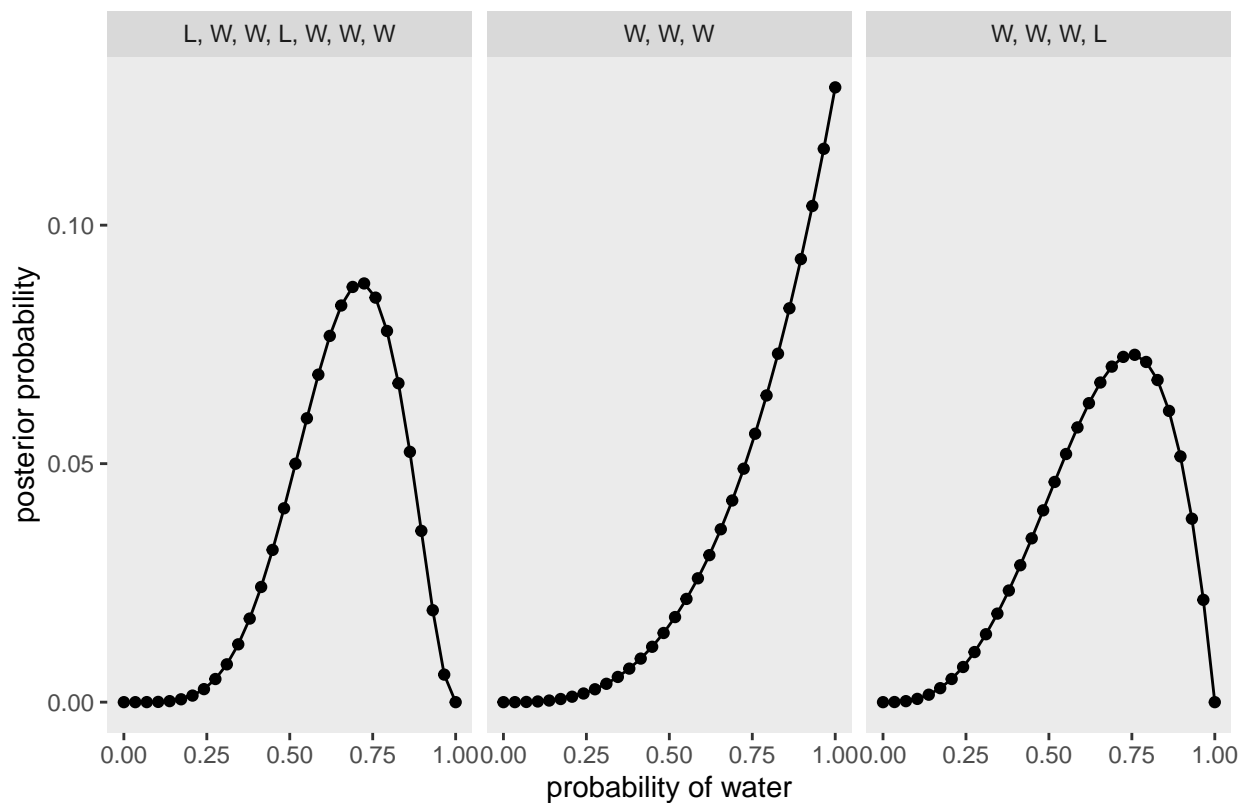
2 and 4 are both correct $\Pr(\text{rain}|\text{monday})$ and $\frac{\Pr(\text{rain}|\text{monday})}{\Pr(\text{monday})}$

2M1

```
library(tidyverse)

tibble(
  label = c("W, W, W", "W, W, W, L", "L, W, W, L, W, W, W"),
  w = c(3, 3, 5),
  n = c(3, 4, 7),
  prior = 1
) %>%
  expand(
    nesting(w, n, prior, label),
    p_grid = seq(from = 0, to = 1, length.out = 30)
  ) %>%
  group_by(w, n) %>%
  mutate(
    likelihood = dbinom(w, size = n, prob = p_grid),
    unstd_posterior = likelihood * prior,
    posterior = unstd_posterior / sum(unstd_posterior)
  ) %>%
  ggplot(aes(x = p_grid, y = posterior)) +
  geom_point() +
  geom_line() +
  labs(
    subtitle = "Grid Approximated Posterior Distrubtions",
    x = "probability of water",
    y = "posterior probability"
  ) +
  theme(panel.grid = element_blank()) +
  facet_wrap(~label)
```

Grid Approximated Posterior Distributions



2H1

$$\Pr(\text{twins}|A) = 0.1$$

$$\Pr(\text{twins}|B) = 0.2$$

$$\Pr(A) = 0.5$$

$$\Pr(B) = 0.5$$

$$\Pr(\text{twins}) = \Pr(\text{twins}|A) \Pr(A) + \Pr(\text{twins}|B) \Pr(B) = 0.1(0.5) + 0.2(0.5) = 0.15$$

$$\Pr(A|\text{twins}) = \frac{\Pr(\text{twins}|A) \Pr(A)}{\Pr(\text{twins})} = \frac{0.1(0.5)}{0.15} = \frac{1}{3}$$

$$\Pr(B|\text{twins}) = \frac{\Pr(\text{twins}|B) \Pr(B)}{\Pr(\text{twins})} = \frac{0.2(0.5)}{0.15} = \frac{2}{3}$$

$$\Pr(\text{twins}) = \Pr(\text{twins}|A) \Pr(A) + \Pr(\text{twins}|B) \Pr(B) = 0.1\left(\frac{1}{3}\right) + 0.2\left(\frac{2}{3}\right) = \frac{1}{6}$$

2H2

$$\Pr(A|twins) = \frac{\Pr(twins|A) \Pr(A)}{\Pr(twins)} = \frac{0.1(0.5)}{0.15} = \frac{1}{3}$$

2H3

$$\Pr(single|A) = 1 \smallsetminus \Pr(twins|A) = 1 \smallsetminus 0.1 = 0.9$$

$$\Pr(single|B) = 1 \smallsetminus \Pr(twins|B) = 1 \smallsetminus 0.2 = 0.8$$

$$\Pr(A) = \frac{1}{3}$$

$$\Pr(B) = \frac{2}{3}$$

$$\Pr(single) = \Pr(single|A) \Pr(A) + \Pr(single|B) \Pr(B) = 0.9(\frac{1}{3}) + 0.8(\frac{2}{3}) = \frac{5}{6}$$

$$\Pr(A|single) = \frac{\Pr(single|A) \Pr(A)}{\Pr(single)} = \frac{0.9(1/3)}{5/6} = 0.36$$