CS 161 Homework 5

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Question 1

Part (A);

NOTE: I've shortened the propositional symbols from "Smoke" to S and from "Fire" to F

S	F	$\neg S$	$\neg F$	$S \Rightarrow F$	$\neg S \Rightarrow \neg F$	$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
F	F	T	T	T	T	T
F	T	T	F	T	F	F
T	F	F	T	F	T	T
T	T	F	F	T	T	T

The sentence is NEITHER valid nor unsatisfiable because it is neither true for all models nor false for all models (as seen by the presence of both T and F in the rightmost column).

Part (B):

NOTE: proposition symbol abbreviation is the same as Part (A), with the addition of shortening "Heat" to H.

S	F	H	$S \Rightarrow F$	$S \vee H$	$(S \vee H) \Rightarrow F$	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
F	F	F	T	F	T	T
F	F	T	T	T	F	F
F	T	F	T	F	T	T
F	T	T	T	T	T	T
T	F	F	F	T	F	T
T	F	T	F	T	F	T
T	T	F	T	T	T	T
T	T	T	T	T	T	T

The sentence is NEITHER valid nor unsatisfiable because it is neither true for all models nor false for all models (as seen by the presence of both T and F in the rightmost column).

Part (C):

NOTE: The symbol abbreviations are the same as those from Part (B).

S	F	H	$S \wedge H$	$(S \wedge H) \Rightarrow F$	$S \Rightarrow F$	$H \Rightarrow F$	$(S \wedge H) \Rightarrow F$	$(S \Rightarrow F) \vee (H \Rightarrow F)$	$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$
F	F	F	F	T	T	T	T	T	T
F	F	T	F	T	T	F	T	T	T
F	T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	T	T	T
T	F	T	T	F	F	F	F	F	T
T	T	F	F	T	T	T	T	T	T
T	T	T	T	T	T	T	T	T	T

The sentence is VALID because it is true for all models (as seen by the presence of strictly T in the rightmost column).

Question 2

Part (A):

Propositional symbols:

- P = the unicorn is mythical
- Q = the unicorn is mortal
- R = the unicorn is a mammal
- S = the unicorn is horned
- T = the unicorn is magical

Propositional logic knowledge base:

- $P \Rightarrow \neg Q$
- $\neg P \Rightarrow (Q \wedge R)$
- $(\neg Q \vee R) \Rightarrow S$
- $S \Rightarrow T$

Part (B):

$$
\begin{aligned}
&\big[P \Rightarrow \neg Q\big] \wedge \big[\neg P \Rightarrow (Q \wedge R)\big] \wedge \big[(\neg Q \vee R) \Rightarrow S\big] \wedge \big[S \Rightarrow T\big] \\
&\Leftrightarrow \big[\neg P \vee \neg Q\big] \wedge \big[P \vee (Q \wedge R)\big] \wedge \big[\neg(\neg Q \vee R) \vee S\big] \wedge \big[\neg S \vee T\big] \\
&\Leftrightarrow \big[\neg P \vee \neg Q\big] \wedge \big[P \vee Q\big] \wedge [P \vee R] \wedge \big[(Q \wedge \neg R) \vee S\big] \wedge \big[\neg S \vee T\big] \\
&\Leftrightarrow \big[\neg P \vee \neg Q\big] \wedge \big[P \vee Q\big] \wedge [P \vee R] \wedge \big[Q \vee S\big] \wedge \big[\neg R \vee S\big] \wedge \big[\neg S \vee T\big]
\end{aligned}
$$

Part (C):

Part (i):

By contradiction, we assume that the knowledge base does not entail that the unicorn is mythical. i.e. We assume that $KB \wedge \neg P$.

Making this assumption, we generate the following clauses using resolution:

1.	$\neg P$	Premise
2.	$\neg P \vee \neg Q$	Premise
3.	$P \vee Q$	Premise
4.	$P \vee R$	premise
5.	$Q \vee S$	Premise
6.	$\neg R \vee S$	Premise
7.	$\neg S \vee T$	Premise
8.	Q	Resolution on 1, 3
9.	R	Resolution on 1, 4
10.	$\neg Q \vee Q$	Resolution on 2, 3
11.	$\neg P \vee P$	Resolution on 2, 3
12.	$\neg Q \vee R$	Resolution on 2, 4
13.	$\neg P \vee S$	Resolution on 2, 5
14.	$P \vee S$	Resolution on 4, 6
15.	$Q \vee T$	Resolution on 5, 7
16.	$\neg R \vee T$	Resolution on 6, 7
18.	$\neg Q \vee S$	Resolution on 2, 14

19.	$\neg P \vee T$	Resolution on 2, 15
20.	$R \vee S$	Resolution on 4, 13
21.	$P \vee T$	Resolution on 4, 16
22.	S	Resolution on 5, 17
23.	$\neg Q \vee T$	Resolution on 7, 17
24.	$R \vee T$	Resolution on 7, 19
25.	T	Resolution on 7, 21
26.	$S \vee T$	Resolution on 13, 20

We cannot generate any more unique clauses using resolution, and we did not generate the empty set.
$\therefore$ it is not possible to derive from the knowledge base that the unicorn is mythical.

Part (ii):

By contradiction, we assume that the knowledge base does not entail that the unicorn is magical. i.e. We assume that $KB \wedge \neg T$.

Making this assumption, we generate the following clauses using resolution:

1.	$\neg T$	Premise
2.	$\neg P \vee \neg Q$	Premise
3.	$P \vee Q$	Premise
4.	$P \vee R$	premise
5.	$Q \vee S$	Premise
6.	$\neg R \vee S$	Premise
7.	$\neg S \vee T$	Premise
8.	$P \vee S$	Resolution on 4, 6
9.	$\neg Q \vee S$	Resolution on 2, 8
10.	S	Resolution on 5, 9
11.	T	Resolution on 7, 10
12.	Empty set	resolution on 1, 11

We generated the empty set, so $KB \wedge \neg T$ is unsatisfiable.
$\therefore$ It is possible to derive from the knowledge base that the unicorn is magical.

Part (iii):

By contradiction, we assume that the knowledge base does not entail that the unicorn is horned. i.e. We assume that $KB \wedge \neg S$.

Making this assumption, we generate the following clauses using resolution:

1.	$\neg S$	Premise
2.	$\neg P \vee \neg Q$	Premise
3.	$P \vee Q$	Premise
4.	$P \vee R$	premise
5.	$Q \vee S$	Premise
6.	$\neg R \vee S$	Premise
7.	$\neg S \vee T$	Premise
8.	$P \vee S$	Resolution on 4, 6
9.	$\neg Q \vee S$	Resolution on 2, 8
10.	S	Resolution on 5, 9
11.	Empty set	resolution on 1, 10

We generated the empty set, so $KB \wedge \neg S$ is unsatisfiable.
$\therefore$ It is possible to derive from the knowledge base that the unicorn is horned.

Question 3

Propositional Symbols:
- O = oil is present
- G = natural gas is present
- T = result of geological test (True for positive, False for negative)

Given Information:
- $P(O \wedge \neg G) = 0.5$
- $P(\neg O \wedge G) = 0.2$
- $P(\neg O \wedge \neg G) = 0.3$
- $P(O \wedge G) = 0$

- $P(T \mid O) = 0.9$
- $P(T \mid G) = 0.3$
- $P(T \mid (\neg O \wedge \neg G)) = 0.1$

"Suppose the test comes back positive. What's the probability that oil is present?"

$$P(O) = P(O \wedge \neg G) + P(O \wedge G) = 0.5 + 0 = 0.5$$
$$P(G) = P(\neg O \wedge G) + P(O \wedge G) = 0.2 + 0 = 0.2$$

$$\begin{aligned}
P(O \mid T) &= \frac{P(O \wedge T)}{P(T)} \\
&= \frac{P(T \mid O)P(O)}{P(O \wedge G \wedge T) + P(O \wedge T) + P(G \wedge T) + P(\neg O \wedge \neg G \wedge T)} \\
&= \frac{P(T \mid O)P(O)}{0 + P(T \mid O)P(O) + P(T \mid G)P(G) + P(T \mid (\neg O \wedge \neg G))P(\neg O \wedge \neg G)} \\
&= \frac{(0.9)(0.5)}{(0.9)(0.5) + (0.3)(0.2) + (0.1)(0.3)} \\
&= 0.8\bar{3}
\end{aligned}$$

Given that the test comes back positive, the probability that oil is present is $0.8\bar{3}$.