N ← nondeterministic

A PDA (pushdown automaton)

$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$

where Q is a finite set of states

Σ is a finite input alphabet

Γ is a finite stack alphabet

$q_0 \in Q$ is the start state

$F \subseteq Q$ is the set of final states

δ: transition function

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \times (\Gamma \cup \{\lambda\}) \longrightarrow$$

Q: current state

$\Sigma \cup \{\lambda\}$: what to consume (current char. or empty string)

$\Gamma \cup \{\lambda\}$: what to pop (single symbol or empty string)

$$\longrightarrow \mathcal{P}(Q \times (\Gamma \cup \{\lambda\}))$$

$\mathcal{P}$: subsets

Q: next state

$\Gamma \cup \{\lambda\}$: what to push (single symbol or nothing)

A config. of a PDA is an element of

$$Q \times \Sigma^* \times \Gamma^*$$

- Q: current state
- Σ^*: what's left of input
- Γ^*: what's on stack

An initial config. of a PDA is an element of

$$\{q_0\} \times \Sigma^* \times \{\lambda\}$$

- q_0: start state
- Σ^*: input string
- $\{\lambda\}$: empty stack

JFLAP: $\{Z\}$, $Z \in \Gamma$, Z = end of stack marker

$$L(M) = \{ w \in \Sigma^* \mid (q_0, w, \lambda) \overset{*}{\vdash}_M (q_f, \lambda, \lambda) \text{ where } q_f \in F \}$$

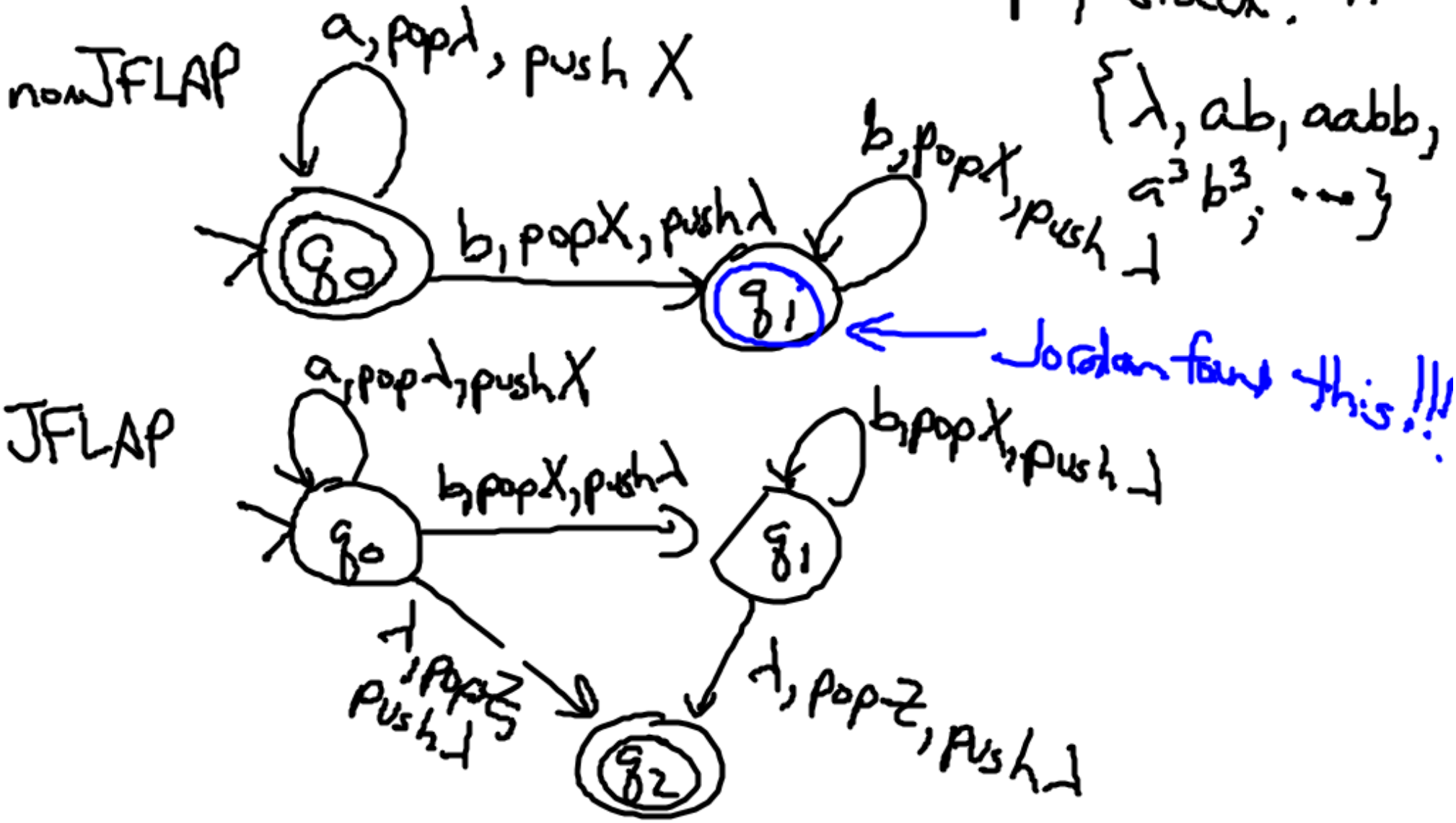
(λ = empty stack; Z in JFLAP)

Acceptance by <u>both</u> final state and empty stack!

An <u>accepting config.</u> is an element of $F \times \{\lambda\} \times \{\lambda\}$

Design a PDA $M \Rightarrow L(M) = \{a^n b^n \mid n \geq 0\}$

acceptance by final state and empty stack. !!



N ← nondeterministic

A PDA (pushdown automaton)

extended PDA

$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$

where Q is a finite set of states

Σ is a finite input alphabet

Γ is a finite stack alphabet

$q_0 \in Q$ is the start state

$F \subseteq Q$ is the set of final states

δ: transition function

Q (current state) $\times$ $(\Sigma \cup \{\lambda\})$ (what to consume (current char. or empty string)) $\times$ Γ^* (what to pop (single symbol or empty string)) $\rightarrow$ string

$P(Q \times \Gamma^*)$ string

P: subsets

Q: next state

Γ^*: what to push (single symbol or nothing)

$\{a^n b^n \mid n \geq 0\}$ ✓✓✓ $\{a^n b^{2n} \mid n \geq 0\}$ ✓✓✓

$\{a^{2n} b^n \mid n \geq 0\}$ ✓✓✓ $\{a^{2n} b^n \mid n \geq 1\}$ ✓✓✓

$\{a^n b^m \mid m,n \geq 0,\ n \leq m \leq 2n\}$

$S \to \lambda$

$S \to aSb$

$S \to aSbb$