Practice Problems for Day 1
CSSS Math Camp 2023

Problem 1: Sum and product notation. Wrtie out the following sums and products and simplify when possible.

1. $\sum_{k=1}^{4}(k-1)^2$
2. $\prod_{i=2}^{10}\frac{(i+1)}{i}$
3. $\prod_{j=1}^{3}x^j$

Problem 2: Exponents and logarithms.

1. Using the definition of raising a number to a power, explain why $a^3 \cdot a^4 = a^{3+4}$.
2. Simplify the following:
   (a) $\log(e^2)$
   (b) $\log_2(4 \cdot 16^2)$
   (c) y^3y^{-2}
   (d) $z^{\frac{1}{2}}$

Problem 3: Linear equations. Suppose the supply curve for oil is expressed with the following linear equation:

$$-x + 4y = 8$$

And the demand curve is expressed with this equation:

$$2x + 5y = 15$$

1. Find the slope of the supply curve using two points.
2. Find the slope and intercept of the supply curve and the demand curve by rearranging the equations to be in the form $y = mx + b$.
3. Plot the two lines (a simple sketch is fine, be sure to put numbers on the x and y axes). Visually estimate the x and y value of the point where the two lines intersect.
4. Solve for the x and y value where the two lines intersect.

Problem 4: Quadratic equations. Consider the equation $x^2 - 8x + 15 = 0$

1. Compute $b^2 - 4ac$. What does this tell you about how many roots the equation has?
2. Find the root(s) of the equation using the quadratic formula.
3. Plot the function $f(x) = x^2 - 8x + 15$ by plugging in the x values $1, 2, 3, 4, 5, 6, 7$.
4. What are the domain and range of $f(x)$. Hint: $f(x)$ achieves its minimum y-value at $x = 4$.

Problem 5: Limits Limits can be useful for getting a general sense of what a function looks like. Consider the function $g(x) = \frac{1}{x-1}$.

1. Compute the following limits:
   (a) $\lim_{x\to\infty} g(x)$
   (b) $\lim_{x\to-\infty} g(x)$
   (c) $\lim_{x\downarrow 1} g(x)$
   (d) $\lim_{x\uparrow 1} g(x)$
2. Is there any value of x such that $g(x) = 0$ (i.e. does $g(x)$ cross over the y-axis)?
3. Based only on your answers to the above, sketch what the graph of $g(x)$ might look like.