Practice Problems for Day 4

Problem 1: Integrals as area under the curve.

1. Graph the function defined by:

$$f(x) = \begin{cases} \frac{1}{10} & \text{if } x \in [0, 10] \\ 0 & \text{otherwise} \end{cases}$$

   This is an example of the uniform probability distribution.

2. By studying the graph and without using calculus, compute the area under the curve on the interval [2,7].

3. Now compute the same area using integral calculus.

Problem 2: Computing integrals.
Integrate the following functions, and check by differentiating. Hint: u-substitution may be useful for (5)-(7).

1. $\int x^7 dx$
2. $\int x^2 + 6x^5 dx$
3. $\int \frac{1}{x^2} dx$
4. $\int \frac{1}{x} dx$
5. $\int (3 - x)^{10} dx$
6. $\int \sqrt{7x + 9} dx$
7. $\int e^{5x+2} dx$

Problem 3: Integrals of probability distributions. Later in the course, we will learn that integrals can be used to answer questions about probability. For now, we will practice finding definite integrals for special functions called *probability distributions*.

1. Compute the area under the curve:

$$\int_{0.5}^{1} x(1-x)^2 dx$$

   This is an example of the beta distribution, a probability distribution which we'll see later this quarte.

2. Compute the area under the curve:

$$\int_{2}^{\infty} 4e^{-4x} dx$$

   This is an example of the exponential probability distribution, which we'll study later this quarter.