Problem Set 3: Differential Calculus

Problem 1: Approximating the derivative. In this problem, we will approximate the derivative of a function at a point by computing the slope of lines between two points. Consider the function $f(x) = x^2$.

1. Use the derivative rules to find $f'(x)$. What is $f'(1)$?
2. We can approximate the derivative at $x = 1$ as the slope of the line between the points $(1, f(1))$ and $(1 + \delta, f(1 + \delta))$. For $\delta = 1, \frac{1}{2}, \frac{1}{4}.\frac{1}{8}, \frac{1}{16}$, approximate $f'(1)$ as

$$f'(1) \approx \frac{f(1+\delta) - f(1)}{\delta}.$$

3. What happens to the estimate of $f'(1)$ as δ gets closer to 0?

Problem 2: Calculating derivatives. Compute the derivatives of the following functions:

1. $f(x) = x^5 + 2$
2. $f(\mu) = 2\mu^4 + \mu^2$
3. $f(y) = ye^y$
4. $f(x) = \frac{e^x}{e^x+1}$
5. $f(y) = e^{3x^2}$

Problem 3: Finding critical points. Find all critical points of the following functions by setting the derivative equal to 0. For each critical point, find the second derivative and use this to identify the point as a maximum, minimum, or saddle.

1. $f(x) = x^3 + 12$
2. $f(x) = x^3 - 3x$
3. $f(x) = x^3 + x$