CSSS Math Camp Lecture 1a

Review of Math Notation and Algebra

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Introductions

- Name and pronouns
- Program / school / department
- One goal you have for math camp
- One thing you are nervous about for math camp (optional)

Math camp overview

Monday:	Algebra, Linear equations and systems of equations, Functions and limits
Tuesday:	Matrices
Wednesday:	Derivatives
Thursday:	Introduction to probabability
Friday:	Integrals

Lecture 1a: Review of Math Notation and Algebra

- Math notation
- Fractions
- Rules of exponents, logarithms
- Order of operations

Notation

Real Numbers

- Any number that falls on the continuous line. Often represented by a, b, c, d
- Examples: 2, 3.234, $1/7$, $\sqrt{5}$, π
- The set of real numbers is denoted by $\mathbb{R}$. Then $a \in \mathbb{R}$ means a is in the set of real numbers.

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Variables

- Can take on different values
- Often represented by x, y, z

Notation

Functions

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- Often represented by $\sum$ and summed over some integer
- Example:
  $$\sum_{i=1}^{3}(i+1)^2 = (1+1)^2 + (2+1)^2 + (3+1)^2 = 2^2 + 3^2 + 4^2 = 29$$

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Products

- Often represented by $\prod$ and multiplied over some integer
- Example: $\prod_{k=1}^{3}(y_k + 1)^2 = (y_1 + 1)^2 \times (y_2 + 1)^2 \times (y_3 + 1)^2$

Fractions

Multiplying & Dividing

Fractions are used to describe parts of numbers. They are comprised of two parts:

$$\frac{\texttt{numerator}}{\texttt{denominator}}$$

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Simplify: $\frac{8}{14} =$

Fractions

Adding & Subtracting

Adding and subtracting requires that fractions must have the same denominator. If not, we need to find a common denominator (a larger number that has both denominators as factors) and convert the fractions. Then add (or subtract) the two numerators.

Examples:

$\frac{1}{7} + \frac{4}{7} =$

$\frac{1}{3} + \frac{1}{4} =$

$\frac{17}{20} - \frac{3}{4} =$

Exponents

a^n is 'a to the power of n'. a is multiplied by itself n times. Often a is called the base, n the exponent. Examples:

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

$$6^4 = 6 \cdot 6 \cdot 6 \cdot 6 = 1296$$

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Exponents do not have to be whole numbers. They can be fractions or negative.
Examples:

$$4^{1/2} = \sqrt{4} = 2$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

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- $a^{1/2} = \sqrt{a}$
- $a^{1/k} = \sqrt[k]{a}$
- $a^0 = 1$

Logarithms

A logarithm is the power (x) required to raise a base (c) to a given number (a).

$$\log_c(a) = x \Rightarrow c^x = a$$

Examples:

- $2^3 = 8 \Rightarrow \log_2(8) = 3$

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- $4^6 = 4096 \Rightarrow \log_4(4096) = 6$
- $9^{1/2} = 3 \Rightarrow \log_9(3) = \frac{1}{2}$

Logarithms

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Examples:

- $10^2 = 100 \Rightarrow \log_{10}(100) = 2$
- $2^3 = 8 \Rightarrow \log_2(8) = 3$
- $e^2 = 7.3891... \Rightarrow \log(7.3891) = 2$

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The natural logarithm ($\log_e$) is the most common; used to model exponential growth (populations, etc). If no base is specified, i.e. $\log(a)$, most often the base is e. Sometimes written as $\ln(a)$.

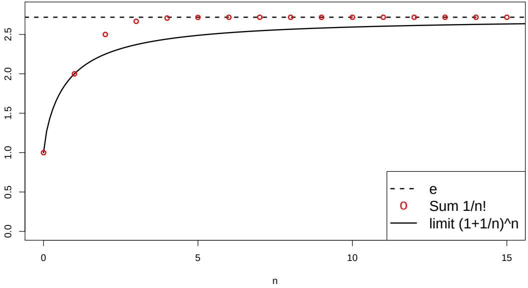
Logarithms

What is e?

The number e is a famous irrational number. The first few digits are $e = 2.718282...$

Two ways to express e:

- $lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$
- $\sum_{n=0}^{\infty} \frac{1}{n!}$



Logarithms
Rules

$$\log_c(a \cdot b) = \log_c(a) + \log_c(b)$$

Proof:

Logarithms

Rules

$$\log_c(a^n) = n \cdot \log_c(a)$$

Proof for $n = 2$:

$$\begin{aligned}
x &= \log_c(a^2) \iff c^x = a^2 \\
&\Rightarrow c^{x_1+x_2} = a \cdot a \text{ where } x_1 + x_2 = x \\
&\Rightarrow c^{x_1} \cdot c^{x_2} = a \cdot a \Rightarrow c^{x_1} = a; c^{x_2} = a \\
&\Rightarrow x_1 = \log_c(a); x_2 = \log_c(a) \\
&\Rightarrow x = x_1 + x_2 \Rightarrow \log_c(a^2) = \log_c(a) + \log_c(a) = 2 \cdot \log_c(a)
\end{aligned}$$

Logarithms

Rules

$$\log_c\left(\tfrac{a}{b}\right) = \log_c(a) - \log_c(b)$$

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&\Rightarrow c^{x_1} \cdot c^{x_2} = \frac{a}{b} \Rightarrow c^{x_1} = a; c^{x_2} = \frac{1}{b} = b^{-1} \\
&\Rightarrow x_1 = \log_c(a); x_2 = (-1) \cdot \log_c(b) \\
&\Rightarrow x = x_1 + x_2 \Rightarrow \log_c\left(\frac{a}{b}\right) = \log_c(a) - \log_c(b)
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Logarithms

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- $\log_2(8 \cdot 4) = \log_2(8) + \log_2(4) = 3 + 2 = 5$
- $\log_{10}\left(\frac{1000}{10}\right) = \log_{10}(1000) - \log_{10}(10) = 3 - 1 = 2$
- $\log_4(6^4) = 4 \cdot \log_4(6)$
- $\log(x^3) = 3 \cdot \log(x)$

Order of Operations

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

- **P**arentheses
- **E**xponents
- **M**ultiplication
- **D**ivision
- **A**ddition
- **S**ubtraction

Order of Operations

Examples

When looking at an expression, work from the left to right following **PEMDAS**. Note: multiplication and division are interchangeable; addition and subtraction are interchangeable.

- $\left((1+2)^3\right)^2 = (3^3)^2 = 27^2 = 729$

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- $\left((1+2)^3\right)^2 = (3^3)^2 = 27^2 = 729$
- $4^3 \cdot 3^2 - 10 + 27/3 = 64 \cdot 9 - 10 + 9 = 576 - 10 + 9 = 575$

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- $4^3 \cdot 3^2 - 10 + 27/3 = 64 \cdot 9 - 10 + 9 = 576 - 10 + 9 = 575$
- $(x+x)^2 - 2x + 3 = (2x)^2 - 2x + 3 = 4x^2 - 2x + 3$