

# Practice Problems for Day 2

## CSSS Math Camp 2023

### Problem 1: Matrix operations

1. Give an example of two matrices  $A$  and  $B$  such that both  $AB$  and  $BA$  exist. What are the dimensions of  $AB$  and  $BA$ ?
2. Give an example of two matrices  $C$  and  $D$  such that  $CD$  exists but  $DC$  does *not* exist. What are the dimensions of  $CD$ ?
3. Give an example of a  $2 \times 2$  matrix with no zero entries whose determinant is 0.

### Problem 2: Solving systems of equations.

Consider the following system of equations:

$$2x + y = 10 \text{ and } x + 4y = 27.$$

1. Represent this system of equations in the form  $Az = w$ , where  $A$  is a  $2 \times 2$  matrix of numbers,  $w$  is a  $2 \times 1$  matrix of numbers, and  $z$  is a  $2 \times 1$  matrix of variables.
2. Find  $\det(A)$ . What does this tell you about whether or not  $A$  has an inverse?
3. Find  $A^{-1}$  and show that  $AA^{-1}$  is equal to the identity matrix.
4. Multiply both sides of the equation  $Az = w$  by  $A^{-1}$  to find  $z$ .
5. Check your answer by plugging into the system of equations above.

**Problem 3: Linear regression.** Suppose that you wish to study the association between students' GPA and the number of hours they sleep per night. To study this question, you ask three students to report their GPA and their average hours of sleep. Below is the data:

Student ID	Sleep (hrs)	GPA
1	8	3.8
2	5	3.3
3	10	3.9

1. Plot GPA versus sleep and draw a line that seems to fit the data. Is the slope of this line positive or negative? What does this tell us about the relationship between sleep and GPA?
2. Write down the  $X$  and  $y$  matrices for a linear regression of GPA on sleep. Don't forget to include a column of 1s in  $X$ .
3. Find  $X^T X$ .
4. Find  $(X^T X)^{-1}$ .
5. Write down how you would estimate  $\beta$  (you do not need to simplify, simply write down the matrices that you would multiply to find  $\beta$ ).