Antenna Location Problem

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1 Introduction

Forth (4G) and now fifth (5G) generation cellular networks are being implemented worldwide, the never-ending increasing demand of these wireless services is traduced in the fast-paced deployment of infrastructure that can satisfy everyone's needs. The smart distribution of the infrastructure is a must, since every user needs to be served with minimum quality of service (QoS) specifications but achieving cost-efficient resource provisioning.

The key objective of a cellular system is to provide sizable capacity and ubiquitous coverage over the service area Wang and Ran (2016). Using distributed antenna systems (DAS) improves the capacity and coverage of a cell in a cellular network Clark et al. (2001). The question then arises, "What is the optimal selection of locations of an antenna to satisfy performance indicators (e.g. coverage, capacity among others) while minimizing the infrastructure cost?".

To deploy or keep expanding a cellular network, one part of the networking planning process is to address the Antenna Placement Problem (APP), which consists in determining the number, locations, and type of base stations considering installation costs, coverage constraints, inter-cell interference, and other network planning parameters.

This document aims to present the formulation and further solution of a simple ***Antenna Location Problem***. We will restrict our problem to compute the optimal cost-efficient number of installed antennas in a regular square (cell) grid, where for each square a traffic demand should be satisfied. Our optimization framework imposes no constraints on the location of the antennas and no inter-cell interference will be considered.

2 Problem Description

Let us consider a set of squared cells, each one characterized by a different demand $\mathbf{R_{mn}}$. The antennas can be installed in the vertexes of each cell as shown in Figure 1. Each antenna absorbs demand by all of the surrounding cells that share the vertex where the antenna is located.

If more than one antenna covers a cell, the demand $\mathbf{R_{mn}}$ is equally distributed among all the antennas as presented in Figure 2. Moreover, each antenna has a maximum capacity $\mathbf{Q_{ij}}$, which must not be exceeded, and installation costs $\mathbf{c_{ij}}$. The objective is to minimize the total cost while covering all the demands.

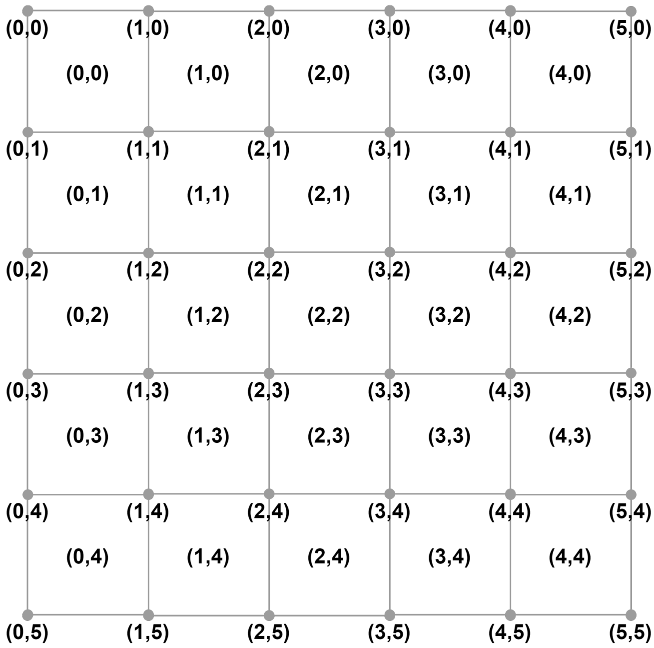


Figure 1: Possible locations of antennas in a 5×5 example.

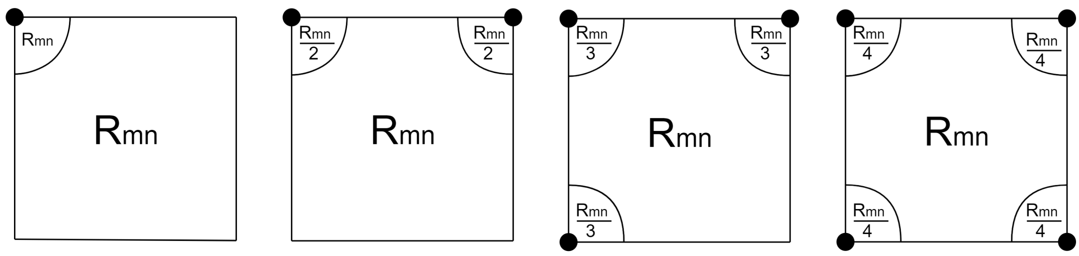


Figure 2: Demand distribution among the number of antennas.

3 Mathematical Model

This section presents the mathematical model proposed to solve the Antenna Location Problem stated in Section 2.

Parameters:

$\mathbf{R_{mn}}$:= total demand of cell $\mathbf{(m,n)}$.
$\mathbf{c_{ij}}$:= cost of installation of antenna $(\mathbf{i},\mathbf{j})$.
$\mathbf{Q_{ij}}$:= maximum capacity of antenna $(\mathbf{i},\mathbf{j})$.

Decision Variables:

$\mathbf{x_{ij}}$:= takes value 1 if antenna $(\mathbf{i},\mathbf{j})$ is built, 0 otherwise.
$\mathbf{z^{k}_{mn}}$:= takes value 1 if cell $\mathbf{(m,n)}$ is covered by k antennas, 0 otherwise.
$\mathbf{q_{ij}}$:= covered demand by antenna $(\mathbf{i},\mathbf{j})$.
$\mathbf{q^{NE}_{ij}}$:= covered portion of demand by antenna $(\mathbf{i},\mathbf{j})$ from its North-East cell.
$\mathbf{q^{NW}_{ij}}$:= covered portion of demand by antenna $(\mathbf{i},\mathbf{j})$ from its North-West cell.
$\mathbf{q^{SE}_{ij}}$:= covered portion of demand by antenna $(\mathbf{i},\mathbf{j})$ from its South-East cell.
$\mathbf{q^{SW}_{ij}}$:= covered portion of demand by antenna $(\mathbf{i},\mathbf{j})$ from its South-West cell.

Model:

$$\min \quad \sum_i \sum_j \mathbf{c_{ij}x_{ij}} \tag{1}$$

$$\text{s.t.} \quad \sum_{k=0}^{4} \mathbf{z^k_{mn}} = 1 \qquad \forall m,n \tag{2}$$

$$4 - \sum_{i=m}^{m+1}\sum_{j=n}^{n+1} \mathbf{x_{ij}} = \sum_{k=0}^{4}(4-k)\mathbf{z^k_{mn}} \qquad \forall m,n \tag{3}$$

$$\mathbf{q^{SE}_{ij}k - R_{mn}x_{ij}} \leq (1-\mathbf{z^k_{mn}})\mathbf{Q_{ij}M} \qquad \forall m,n,i=m,j=n, M \gg max(Q_{i,j}) \tag{4}$$

$$\mathbf{q^{SW}_{ij}k - R_{mn}x_{ij}} \leq (1-\mathbf{z^k_{mn}})\mathbf{Q_{ij}M} \qquad \forall m,n,i=m,j=n+1, M \gg max(Q_{i,j}) \tag{5}$$

$$\mathbf{q^{NE}_{ij}k - R_{mn}x_{ij}} \leq (1-\mathbf{z^k_{mn}})\mathbf{Q_{ij}M} \qquad \forall m,n,i=m+1,j=n, M \gg max(Q_{i,j}) \tag{6}$$

$$\mathbf{q^{NW}_{ij}k - R_{mn}x_{ij}} \leq (1-\mathbf{z^k_{mn}})\mathbf{Q_{ij}M} \qquad \forall m,n,i=m+1,j=n+1, M \gg max(Q_{i,j}) \tag{7}$$

$$\mathbf{R_{mn}} = \mathbf{q^{SE}_{ij}} + \mathbf{q^{SW}_{ij+1}} + \mathbf{q^{NE}_{i+1j}} + \mathbf{q^{NW}_{i+1j+1}} \qquad \forall m,n,i=m,j=n \tag{8}$$

$$\mathbf{q_{ij}} = \mathbf{q^{NW}_{ij}} + \mathbf{q^{NE}_{ij}} + \mathbf{q^{SW}_{ij}} + \mathbf{q^{SE}_{ij}} \qquad \forall i,j \tag{9}$$

$$\mathbf{q_{ij}} \leq \mathbf{Q_{ij}x_{ij}} \qquad \forall i,j \tag{10}$$

$$\begin{aligned}
&\mathbf{x_{ij}} \in 0,1 && \forall i,j\\
&\mathbf{z^k_{mn}} \in 0,1 && \forall m,n\\
&\mathbf{q_{ij}} \in \mathbb{R}^+ && \forall i,j\\
&\mathbf{q^{NE}_{ij}} \in \mathbb{R}^+ && \forall i,j\\
&\mathbf{q^{NW}_{ij}} \in \mathbb{R}^+ && \forall i,j\\
&\mathbf{q^{SE}_{ij}} \in \mathbb{R}^+ && \forall i,j\\
&\mathbf{q^{SW}_{ij}} \in \mathbb{R}^+ && \forall i,j
\end{aligned}$$

- The objective function is presented in Equation (1). In concordance with the problem, the total cost is minimized. This cost is equal to the sum of installation costs of all the installed antennas.
- Equation (2) ensures that each cell is covered by a determined number of k antennas. Since the sum of $\mathbf{z^k_{mn}}$ should be equal to one, this variable can have a value equal to 1 only for one value of k and it would be 0 for all the other values. For instance, if a cell has three installed antennas of the four possible, $\mathbf{z^3_{mn}} = 1$ and $\mathbf{z^k_{mn}} = 0 \quad \forall k \neq 3$.
- Equation (3) counts how many antennas have been installed to define the value of k for which $\mathbf{z^k_{mn}}$ is equal to 1. To illustrate the performance of this constraint, suppose 3 antennas have been installed in a cell. On the left side of the equation, the result would be 1, being that the sum of the installed antennas ($\mathbf{x_{ij}}$) that surround the cell is 3. On the other side, the only way to satisfy the equality is that $\mathbf{z^3_{mn}} = 1$. In this side, the value would be $(4-3)\times 1 = 1$.
- With Equations (4) to (7) the demand of each cell is equitably distributed among the antennas that cover the cell. For example, Equation (4) provides a value for $\mathbf{q^{SE}_{ij}}$ where $i=m$ and $j=n$, this means that for each cell this antenna will be the one located at the upper left vertex. If the antenna is built, $\mathbf{x_{ij}} = 1$ and if the current value of k is equal to the number of installed antennas $\mathbf{z^k_{mn}} = 1$. Subsequently, $\mathbf{q^{SE}_{ij}} \leq \frac{\mathbf{R_{mn}}}{k}$.

- Equation (8) declares that the sum of the absorbed demand for each antenna in a cell is equal to the demand of the cell. With this constraint, it is guaranteed that the entire demand is absorbed.
- Constraint (9) establishes that the sum of the portions of absorbed demand for each antenna (from each surrounding cell) is equal to the total demand absorbed by the antenna.
- Equation (10) ensures that the antenna does not exceed its maximum capacity.

Figure 3 presents a diagram that lets us understand better the constraints and variables. Each antenna can absorb the demand of its four surrounding cells. NW, NE, SW, and SE refer to the direction of the cells. For example, antenna $(1,1)$ covers in total $\mathbf{q_{11}}$ demand, where $\mathbf{q_{11}^{NW}}$ is the portion of demand absorbed from cell $(0,0)$, $\mathbf{q_{11}^{NE}}$ is the portion of demand absorbed from cell $(1,0)$, $\mathbf{q_{11}^{SW}}$ is the portion of demand absorbed from cell $(0,1)$, and $\mathbf{q_{11}^{SE}}$ is the portion of demand absorbed from cell $(1,1)$.

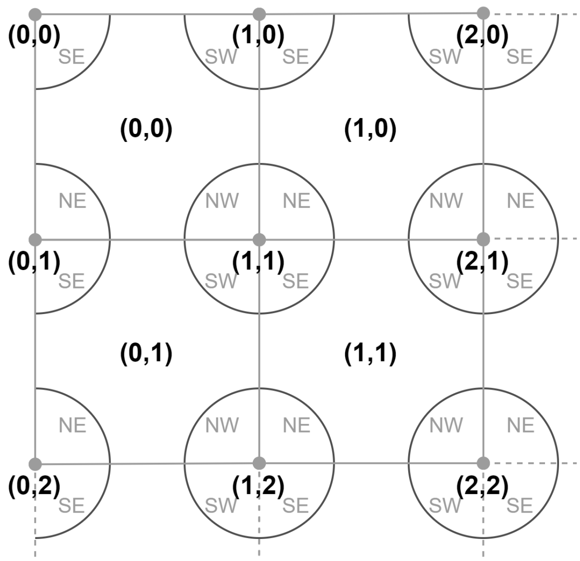


Figure 3: Reference diagram of the model.

Notice also that the four vertexes of a cell $(\mathbf{m},\mathbf{n})$ have always coordinates equal to $(\mathbf{i},\mathbf{j})$, $(\mathbf{i+1},\mathbf{j})$, $(\mathbf{i},\mathbf{j+1})$, and $(\mathbf{i+1},\mathbf{j+1})$ with $i = m$ and $j = n$. This indexing helps to not lose track of the constraints of a cell for its surrounding antennas. For instance, the cell (0,0) has vertexes (0,0), (1,0), (0,1), and (1,1).

4 Instance Generation

Every instance generation needs configuration settings. The configured parameters will define the nature of the input instance. Table 1 summarizes the definition of every setting.

Variable	Description
`antenna_row`	Number of rows of the grid
`antenna_column`	Number of columns of the grid
`max_capacity`	Maximum capacity for each antenna
`min_capacity`	Minimum capacity for each antenna
`max_demand`	Maximum traffic demand for each cell
`min_demand`	Minimum traffic demand for each cell
`max_cost`	Maximum installation cost for each antenna
`min_cost`	Minimum installation cost for each antenna

Table 1: Configuration settings.

There are three input instances: $\mathbf{R_{mn}}$, $\mathbf{c_{ij}}$, and $\mathbf{Q_{ij}}$. The generation of these inputs is based on the specific test that will be performed to the exact and heuristic solutions. Each specific test is explained in Section 6. Moreover, each entry sample can be generated using different distributions.

4.1 Uniform distribution

We assumed that installation costs $\mathbf{c_{ij}}$, traffic demand $\mathbf{R_{mn}}$, and antenna capacity $\mathbf{Q_{ij}}$ are uniformly distributed. The reasoning of assigning a uniform distribution for every instance is connected to simulating a first rough approximation of the reality.
The upper and lower bounds for the uniform distribution for $\mathbf{c_{ij}}$ is given by the configuration parameters `min_cost` and `max_cost`. Equally, for $\mathbf{Q_{ij}}$ and $\mathbf{R_{mn}}$ the upper and lower bound is indicated by `min_capacity`, `max_capacity` and `min_demand`, `max_demand` respectively.

4.2 Normal distribution

Assuming that the cost $\mathbf{c_{ij}}$, demand $\mathbf{R_{mn}}$, and capacity $\mathbf{Q_{ij}}$ of the antennas in the grid have values around a mean that are determined by external factors such as location, the terrain conditions or legal permits of building, a normal distribution was used for the three input instances.
The mean was set equal to the average of the configuration parameters `max_capacity` and `min_capacity` for $\mathbf{Q_{ij}}$, `min_cost` and `max_cost` for $\mathbf{c_{ij}}$, and `max_demand` and `min_demand` for $\mathbf{R_{mn}}$, and the standard deviation equal to the difference of these parameters.

4.3 Realistic distribution

The instance generation described in Sections 4.1 and 4.2 did not consider any relation between cost $\mathbf{c_{ij}}$, nor capacity $\mathbf{Q_{ij}}$, nor demand $\mathbf{R_{mn}}$. The purpose of this new instance generation is to associate the installation cost with the antenna capacity.
First, the antenna capacity $\mathbf{Q_{ij}}$ is generated with a normal distribution with mean equal to the average between the configuration parameters `max_capacity` and `min_capacity` and standard deviation equal to the difference of `max_capacity` and `min_capacity`. Once generated $\mathbf{Q_{ij}}$, the relation with antenna costs $\mathbf{c_{ij}}$ is a linear interpolation. In which the configuration parameters `min_cost` and `max_cost` set the boundaries of the interpolation to generate the $\mathbf{c_{ij}}$ instance.
The traffic demand instance $\mathbf{R_{mn}}$ was assumed to have an exponential distribution behavior with exponential parameter $\frac{2}{3} \times$ `max_demand`, simulating a scenario where the majority of cells have low demand but a few of them have higher demand because of their location (e.g. cells with stadiums, coliseums, museums...).

5 Heuristic Methods

Five different heuristic methods were implemented. For each method, the stop condition is given by a maximum number of iterations. The initial solution corresponds to the installation of all the antennas in the grid, then the cost will be the maximum. On each iteration a new possible solution is generated according to the logic of each method, the feasibility of the possible solution is validated and the total cost is computed. If this possible solution is feasible and the total cost is less than in the previous solution, the output solution is replaced for the possible solution.

5.1 Random

In the search of a simple and fast method implementation, the **Random** heuristic was formulated. It consists in generating a random x_{ij} instance with probability P_{ij} of installing an antenna and $1 - P_{ij}$ to not install an antenna, in this case, $P_{ij} = 2/3$. The method is repeated until the configured number of iterations is completed.

5.2 Probability Density Function Type 1 (PDFT1)

In this method, a probability matrix, where each element p_{ij} corresponds to the probability of installing antenna x_{ij}, is created. Then, each new possible solution is generated according to the probability matrix. To compute the probability matrix three different probabilities were considered.

- **Cost probability**: According to the installation cost of each antenna, a probability of being installed is assigned. For instance, the least expensive antennas have a greater probability of being installed. Concordantly, a low probability is given to the most expensive antennas. Equation (11) defines the probability for each antenna according to the cost. In this method, to avoid the most expensive antenna to be designated with probability 0, Equation (12) is used to shift the probabilities so that the maximum is one.

$$p_{1,ij} = 1 - \frac{c_{ij}}{max(c_{ij})} \tag{11}$$

$$p_{1,ij} = 1 - max(p_{1,ij}) + p_{1,ij} \tag{12}$$

- **Demand probability**: Equation (13) describes the second probability to assign a second "weight" to place each antenna, i.e for each antenna the four nearest cell demands are added and divided into the maximum value of all antennas demands. In this way, an antenna absorbing higher demand will have a higher probability of being installed. Figure 4 illustrates with a dotted blue line, the sum of the four demands for antenna x_{ij}.

$$p_{2,ij} = \frac{\sum_{m=i-1}^{i} \sum_{n=j-1}^{j} R_{mn}}{max(\sum_{m=i-1}^{i} \sum_{n=j-1}^{j} R_{mn})} \tag{13}$$

- **Nearest cost probability**: For each antenna, the costs of the eight nearest antennas are added, excluding the own cost of the antenna, as described in Equation (14). Figure 4 depicts, with a dotted red line, the sum of the nearest antennas cost.

$$p_{3,ij} = 1 - \frac{(\sum_{m=i-1}^{i+1} \sum_{n=j-1}^{j+1} c_{mn}) - c_{mn}}{max((\sum_{m=i-1}^{i+1} \sum_{n=j-1}^{j+1} c_{mm}) - c_{mn})} \tag{14}$$

$$p_{3,ij} = 1 - max(p_{3,ij}) + p_{3,ij} \tag{15}$$

Using the three probabilities, each element of the probability is computed using $p_{ij} = p_{1,ij} * p_{2,ij} * p_{3,ij}$. To avoid probabilities equal to 0 or 1, meaning building an antenna or not always, for all probabilities a one-dimensional linear interpolation is implemented, i.e all probabilities will be mapped between 0.2 and 0.8.

5.3 Probability Density Function Type 2 (PDFT2)

Looking for a lighter heuristic, removing the **Nearest Cost probability**, and not considering Equation (12), which essentially is avoiding the probabilities to be equal to zero for the Cost probability, there is no need to interpolate as in the **PDFT1**. In this method, only **Cost probability** and **Demand probability** are used to create the probability matrix. Therefore, each element of the probability is calculated as $p_{ij} = p_{1,ij} * p_{2,ij}$.

5.4 N to 1 (N21)

As it is desired to minimize the cost while covering all demand, a method in which initially all antennas are installed is implemented. Right after, one antenna is removed from x_{ij}. The removal of antennas is repeated until no feasible solution can be achieved. This heuristic is denominated N to 1 (N21) since N antennas are removed until 1 is left. N is the maximum possible number of antennas that can be built.

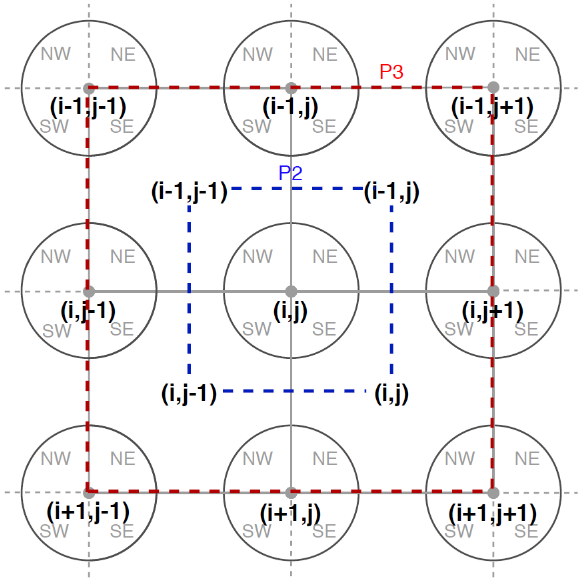


Figure 4: Probabilities description.

For every antenna removal j, a maximum of J iterations are performed. x_{ij} is randomly shuffled for each iteration (e.g. when the first antenna is removed, this removal is shuffled for J iterations). The drawback of this approach is that for small grids and a large number of iterations, the antenna elimination instance may be repeated.

5.5 1 to N (12N)

This method is similar to the N to 1 explained in the previous section. Nevertheless, the number of built antennas is increased from 1 to N. In essence, the algorithm starts by placing randomly one antenna. Then, the location of the antenna is shuffled a determined number of iterations. After each iteration, the solution is validated and the best solution in terms of the objective function is saved. Later, the number of antennas is increased by one, and once again the algorithm iterates changing the position of the two antennas. The entire process is repeated until the number of antennas reaches N. At the end, the best solution is returned.
Notice that some instances never have a feasible solution for a small number of antennas. For example, if the instance contains 5×5 cells, it is not possible to cover the demands with just one antenna. In this case, every solution generated over the iterations is going to be unfeasible. This is a disadvantage of the algorithm since it makes the iterations even when the solution is not possible. However, by using this method it is possible to know the minimum required number of antennas to obtain a feasible solution.

5.6 Destroy and Rebuild

When a new possible solution is classified as unfeasible, the Destroy and Rebuild algorithm is used. This can be considered as a heuristic algorithm that repairs other heuristics. In particular, we use this repair method in the heuristics defined in Sections 5.1, 5.2 and 5.3. The function used to classify each new possible solution return the constraint that is not being satisfied. Thereby, the constraint identifier that is not met, and the location of the cell or antenna are known.
This implementation focuses on modifying the number of built antennas close to the cell or antenna involved to try to satisfy the missing constraint. This procedure is performed a maximum of 5 times for the same constraint, if after the fifth attempt a feasible solution is not found, the new possible solution is definitely classified as unfeasible.
For the validation of the feasibility of a new possible solution, the matrix x_{ij} is taken as input. From the problem definition, the constraint (3) determines the value k of z^k_{mn}, and constraints (4), (5), (6) and (7) defines $q^{SE}_{ij}, q^{SW}_{ij}, q^{NE}_{ij}$, and q^{NW}_{ij} respectively. Similarly, the constraint (9) establishes the q_{ij}

value. Therefore, the constraints that really evaluates the feasibility of the instance are (8) and (10) which do not define any value for the *decision variables* but determine a viable solution. For these two constraints, the following solutions are proposed to try to fulfill them:

- **Constraint (8):** When this constraint is not met it means that the R_{mn} demand is not covered by the nearby antennas at a cell $(\mathbf{m}, \mathbf{n})$. Therefore, from the antennas that have not been installed yet, the least expensive antenna is built. If all four antennas are already installed but the constraint is not met, the instance does not have a feasible solution and antennas capacity must be increased.

- **Constraint (10):** This constraint does not satisfy the requirements when the capacity of a built antenna is exceeded by the demand of the four adjacent cells. To try to comply this constraint, the cell with the highest demand is found and the cheapest antenna in this cell is installed (if not installed yet). If the cell with the highest demand has already the four antennas built, the procedure continues with the second highest demand cell, the process is repeated until a feasible solution is achieved or the eight nearby antennas are installed. If all eight antennas are built and the constraint is still not met, the instance does not have a feasible solution, which means that the antenna capacity must be increased.

6 Computational Results

In this section, the test scenarios are described and the results for each case are presented and analyzed. With the purpose of visualizing the solution given for every instance and test, it was generated a grid as shown in Figure 5. The green dots symbolizes that the antenna was installed, while the red represents the non-installation of them.
In every vertex, it is indicated the installation cost (c), the portion of capacity given (q), and the total capacity (Q) of each antenna. The demand required by each cell is depicted in the center of every four adjacent antennas.

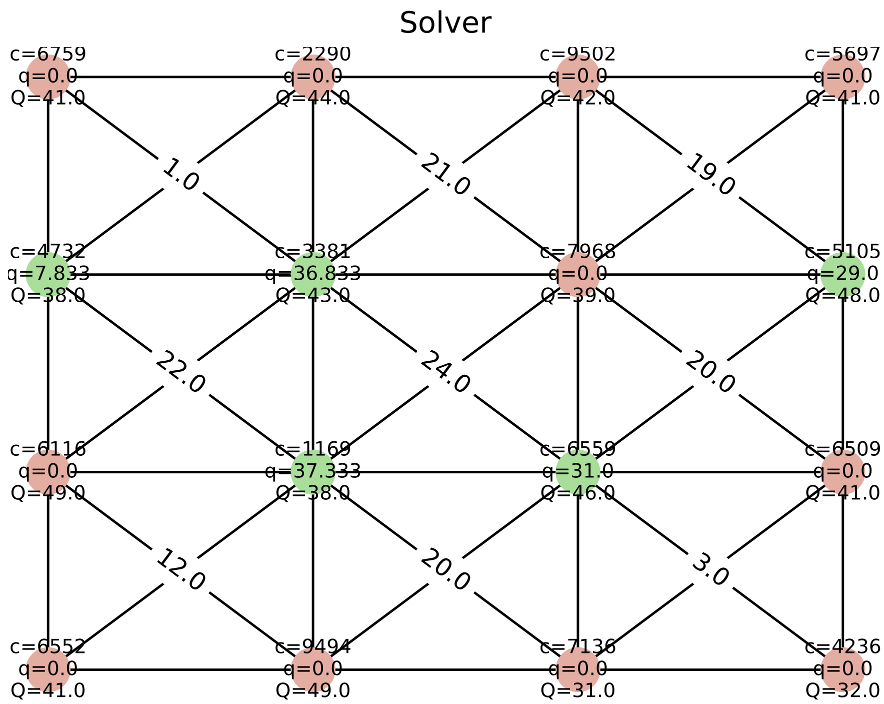


Figure 5: 4x4 Solver instance.

6.1 Test 1

The first test consists in keeping the total antenna capacity $\mathbf{Q_{ij}}$ fixed and the installation cost $\mathbf{c_{ij}}$ and traffic demand $\mathbf{R_{ij}}$ uniformly distributed. This could be a first approximation to reality since in some cases, for a specific region the required antennas will have the same specifications, thus, having the same capacity. However, installation costs may vary for a particular portion of land due to regulations. Similarly, traffic demand can change according to the needs of a given cell.

Figures 6 to 10 show the ratio between the average execution time (mean of time with different seeds) of each heuristic and the average execution time of the solver for different dimensions of the grid between 3×3 to 10×10. The higher the ratio, the slower the heuristic with respect the solver time. In general, it is observed that the difference between the execution time of the heuristic methods and the solver decreases as the dimensions of the grid become higher. That happens because all the heuristics are limited by a determined number of iterations, whereas the solver runs until it finds the optimal solution. Therefore, even when the execution time is lower in the heuristics for high dimensions, their solution could be each time farthest from optimal.

Another observation is that the ratio for the random heuristic and the PDFT1 ranges from around 0 to 6 and with PDFT2 the maximum ratio is 8. However, it is observed that with N21 and 12N the execution time becomes even 30 times higher than the execution time of the solver. This suggests that these methods do not have a suitable performance in terms of time.

To complement the previous analysis, the average execution time and its variance for all the methods for instances with an equal number of columns and rows were plotted. The resulting graph is depicted in Figure 11, it is observed that the variance of the solver is very large for some dimensions. In those cases, there were a few seeds for which the solver found the solution faster. To understand this behavior, a new test was created. This test is presented in Section 6.5. It is also seen that overall the execution time increases as the dimensions become higher. Comparing with the solver, it takes less time in execution than the other methods for small instances, but there is a point where the methods become faster.

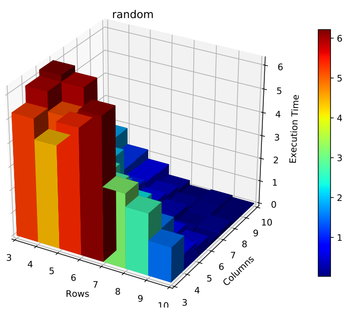


Figure 6: Random heuristic time with respect to solver time.

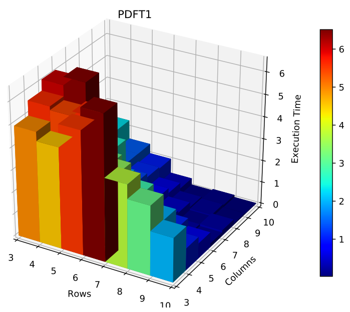


Figure 7: PDFT1 heuristic time with respect to solver time.

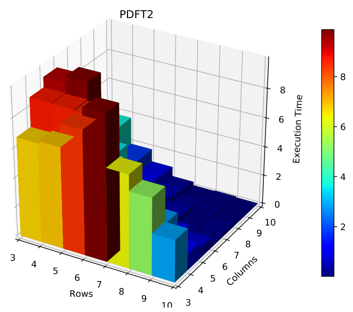


Figure 8: PDFT2 heuristic time with respect to solver time.

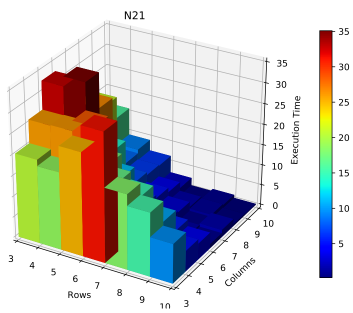


Figure 9: N21 heuristic time with respect to solver time.

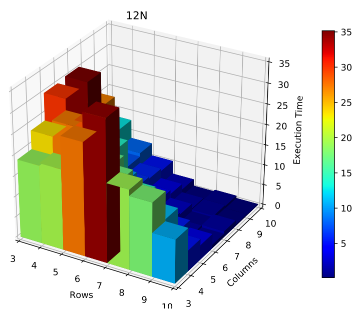
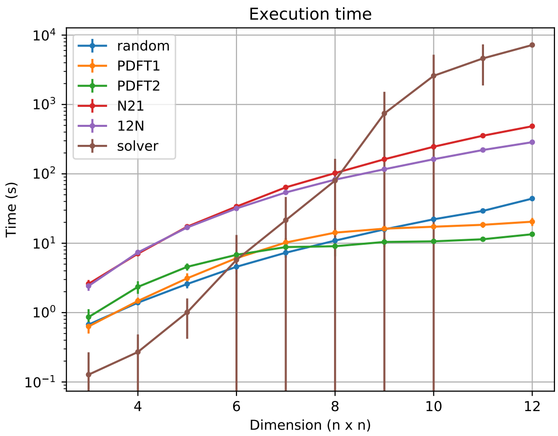


Figure 10: 12N heuristic time with respect to solver time.



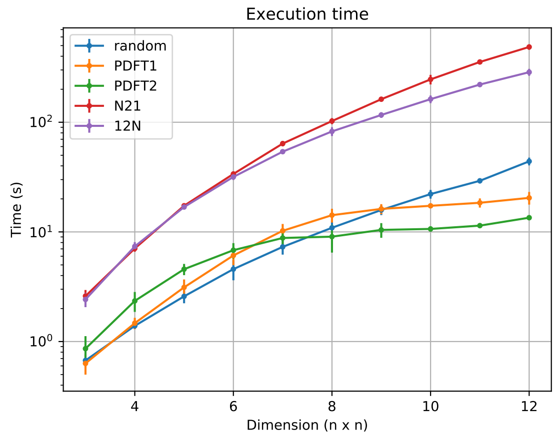
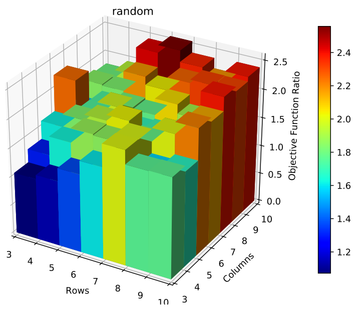
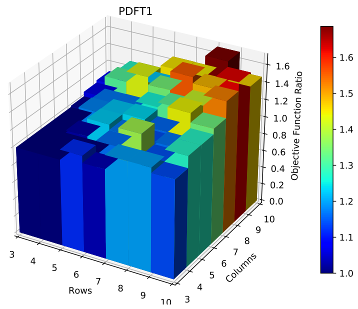


Figure 11: Execution time and error for all methods.

Figure 12: Execution time and error for heuristic methods.





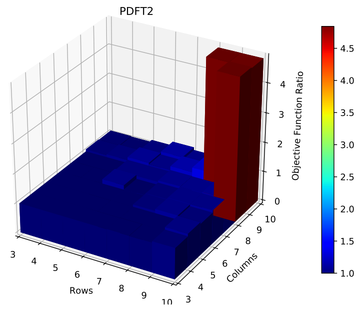
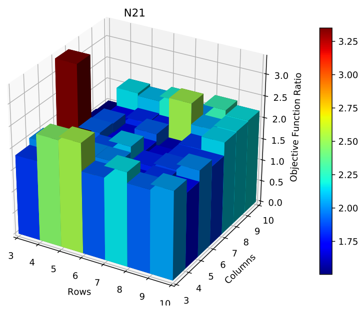


Figure 13: Random heuristic cost with respect to solver cost.

Figure 14: PDFT1 heuristic cost with respect to solver cost.

Figure 15: PDFT2 heuristic cost with respect to solver cost.



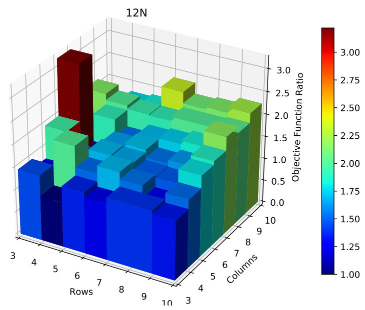


Figure 16: N21 heuristic cost with respect to solver cost.

Figure 17: 12N heuristic cost with respect to solver cost.

To appreciate with more detail the heuristic method curves, Figure 12 was plotted. In this graph, the solver was removed to see with an improved resolution the other curves. It is observed that, as it was seen in the 3-dimensional curves, the methods N21 and 12N are the most time-consuming heuristics. On the other hand, for dimensions lower than 8, the lowest execution time was obtained with the random method, followed by PDFT1, and PDFT2. For higher dimensions, the lowest execution time is provided by the PDFT1 and the PDFT2.
The ratio between the total cost of each heuristic method and the total cost of the solver for different dimensions and seed 0 is presented in Figures 13 to 17. In this case, the lower the ratio the better the solution given by each method. It is seen that overall the total cost with respect to the solver solution becomes higher as the dimension of the grid increases. Nevertheless, it is possible to identify the difference between different heuristic methods. In particular, the PDFT1 and PDFT2 methods had the best performance in terms of the objective function. It is observed that the ratio in these cases

is equal to 1 for smaller dimensions. For higher dimensions, the PDFT1 method reaches maximum a solution which is around 1.6 times the cost provided by the solver, whereas the PDF has a drastic increase with respect to the solver for dimensions with more than 8 rows and 8 columns.
Regarding the random methods (Random, N21, and 12N), it is evident that when the grid is 3×6, the values of the objective function are particularly higher. An explanation for this behavior can be based on the seed. In this case, the generated instance could be more difficult to solve. Additionally, it is observed that these methods in general provide higher total costs than the PDF methods.

6.2 Test 2

The second test focuses on testing the performance of heuristic methods, by modifying the number of iterations, with respect to the solver solution. For this, one instance is generated with a uniform distribution and constant capacity, to generate instances with similar complexities, the solver computes the optimal solution for the instance and then each heuristic method will give a solution using a different maximum number of iterations. In this way, the performance of each heuristic method can be compared with respect to the solver, using the execution and the objective function as metrics. All heuristic methods are evaluated using two instances, with dimensions 4x4 and 9x9.
For the 4x4 instance, Figure 18 depicts the average execution time for heuristic methods, the dotted line represents the execution time of the solver to find the optimal solution and Figure 19 plots the cost comparison, in percentage, the optimal solution given by the solver (dotted line) and the heuristic methods. It can be seen that only PDFT1 and PDFT2 methods will reach the optimal solution using approximately 5000 and 500 iterations, respectively. However, PDFT1 will take more time than the solver to find the optimal solution but for a short number of iterations, it will give a good approximation. In the case of PDFT2, it will find the optimal solution faster than the solver and for a reduced number of iterations, it will find a fairly close value. The random method is approaching the optimal value slowly, so it would require more iterations to give a better approximation. The 12N and N21 methods take the longest time to give a solution and the solution is not good regarding the optimal cost of the solver.

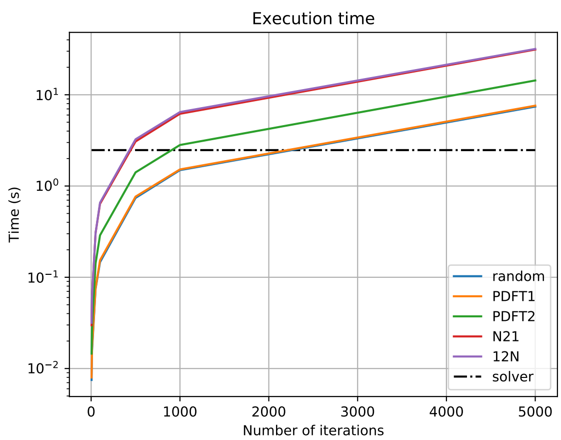


Figure 18: Execution time using different iterations for heuristic methods. Dimensions 4x4.

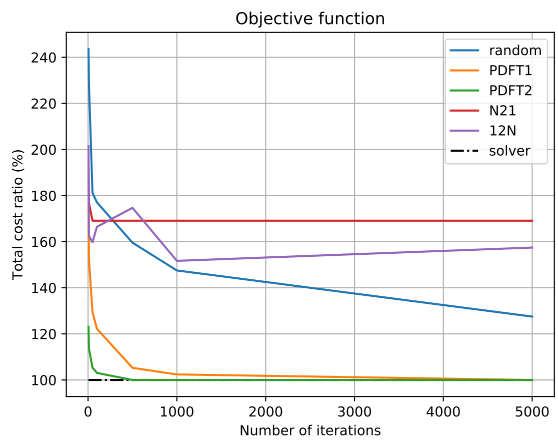


Figure 19: Objective function using different iterations for heuristic methods. Dimensions 4x4.

Figures 20 and 21 plot the performance for a 9x9 instance. The 12N and N21 were stopped in 10000 iterations due to the excessive time spent to give a solution and the high objective function values that were almost doubling the optimal solution. It is seen that the random method obtains a solution in a short time but the estimation is not good. On the other hand, the PDFT1 and PDFT2 methods give a good approximation in a reasonable time. It can be seen that the more iterations, the cost is closer to the optimal value but these methods would take more time than the solver for a very large number of iterations.

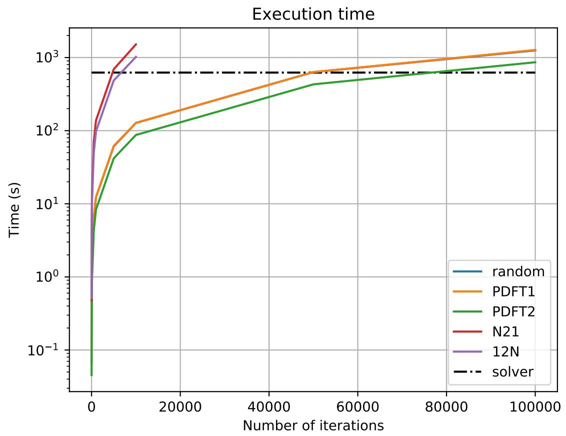


Figure 20: Execution time using different iterations for heuristic methods. Dimensions 9x9.

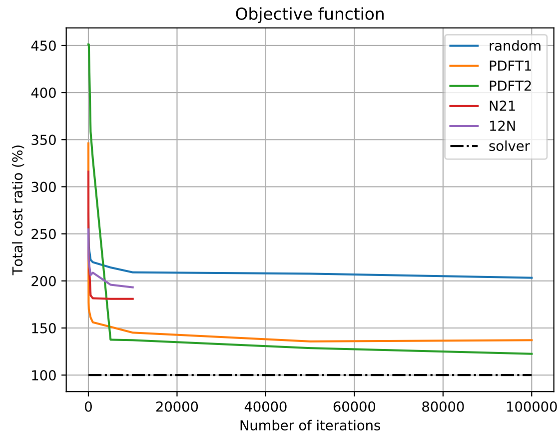


Figure 21: Objective function using different iterations for heuristic methods. Dimensions 9x9.

6.3 Test 3

One of the formulated hypothesis was that the ratio between the traffic demand $\mathbf{R_{mn}}$ and the total capacity of the antenna $\mathbf{Q_{ij}}$ affected the solver's time execution to compute the optimal problem. Figure 22 illustrates the situation, while every element in $\mathbf{Q_{ij}}$ was kept constant at maximum capacity 40, $\mathbf{R_{mn}}$ was increasing the traffic demand from 1 to 40.

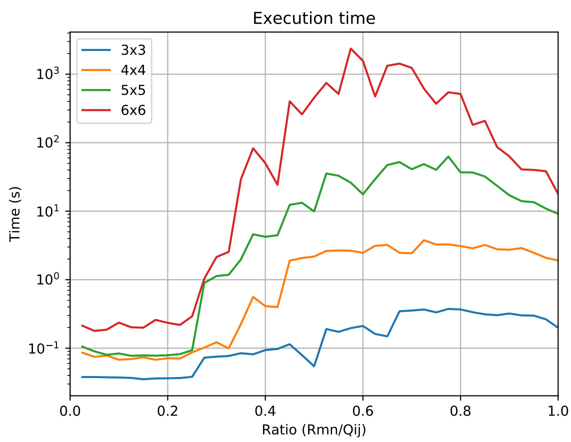


Figure 22: Execution time for different ratio between $\mathbf{R_{mn}}$ and $\mathbf{Q_{ij}}$.

Figure 22 compares the execution time for instances 3×3 up to 6×6. As the required demand of each cell is small compared to the maximum capacity of the antenna; the solver takes a short time to find the optimal solution. As demand rises with respect to the maximum capacity the solver reaches an execution time peak. After this highest point, the solver decrements its run time.

During the phase in which the demand is little compared to the capacity (i.e. ratio range 0.025 to 0.2 approximately) the solver easily finds an optimal solution. The optimal combination of antennas can be placed anywhere due to whichever antenna satisfies the demand. On the other hand, the stage when the execution time starts increasing until the peak, it becomes harder for the solver to find an optimal solution. In contrast to the previous stage, the solver now cannot place anywhere the antennas but needs to be more selective. The solver needs to evaluate more combinations to identify the optimal arrangement of antennas.

Following the time execution peak, the solver comes to a solution faster. The reasoning behind this is that the solver is finding only optimal solutions placing all the surrounding antennas to meet the cell's demand, therefore, the solution becomes trivial when the relation between $\mathbf{R_{mn}}$ and $\mathbf{Q_{ij}}$ approaches one because all the antennas are required. It is clear that as the dimension of the instance increases the execution time increases accordingly (see Section 6.1). However, the execution time peak is more evident in bigger dimension grids, for smaller grids is harder to discriminate where this peak is, the curve tends to be planar.

6.4 Test 4

One of the tests for the optimization framework was to evaluate the system's behavior feeding it with various instances $\mathbf{c_{ij}}$, $\mathbf{Q_{ij}}$, and $\mathbf{R_{mn}}$, simulating in this way several real-world scenarios. For this test, the instance generation mentioned in Sections 4.1, 4.2, and 4.3 was employed. The **Uniform distribution**, **Normal distribution**, and **Realistic distribution** time execution behavior is depicted in Figure 23.

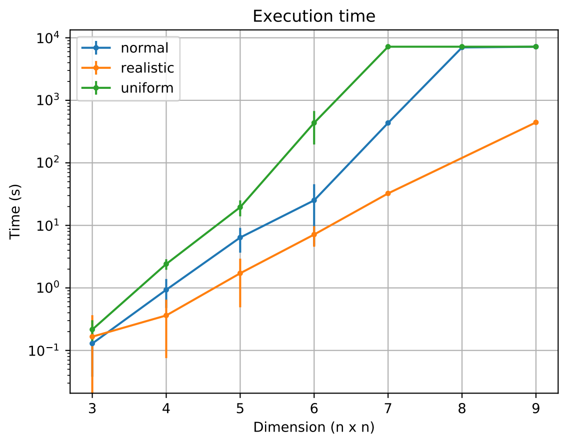


Figure 23: Solver's execution time for different instance generation.

The test run was executed using 10 different seeds for grid sizes 3×3 up to 9×9 setting a time limit of 2 hours. The **Realistic distribution** was in average the less time consuming among all. While the **Uniform distribution** took the longest to achieve an optimal solution. In this case, the demand might overwhelm the total capacity (matter discussed in test of Section 6.3).
Simulating the traffic demand $\mathbf{R_{mn}}$ with an exponential distribution advantages the **Realistic distribution** because the antenna capacity $\mathbf{Q_{ij}}$, as it is simulated with a normal distribution with mean equal to the average of `max_capacity` and `min_capacity`, overtakes in average the cell demand.
The **Uniform distribution** on the other hand, as the demand and capacity are generated under the same conditions, the solver takes more time. The medium point is the **Normal distribution** that neither takes a lot of time to solve the problem nor solves it in a short period.

6.5 Test 5

The objective of this test is to observe the behavior of the solver in terms of execution time, using different seeds. For this purpose, a 6×6 grid was used, the total antenna capacity $\mathbf{Q_{ij}}$ was kept fixed, and the installation cost $\mathbf{c_{ij}}$ and traffic demand $\mathbf{R_{ij}}$ had a uniform distribution. Additionally, the seeds were varied from 0 to 29, and each one was iterated 100 times. In Figure 24, the results of this test are depicted. It is observed that almost all seeds have the same distribution with some outliers in the upper part. Moreover, the majority of boxes are below 10 seconds, a few of them between 10 and 20 seconds and only 4 above 20 seconds.
It is possible to say that the instances generated by seeds below 10 seconds are less complex than the generated above it. Therefore, the solver spend less time in finding the optimal solution. Considering that, if all the boxes are averaged, the resulting box would have a big standard deviation of around 35 seconds. This explain the behavior that was observed in Figure 11 in Section 6.1.

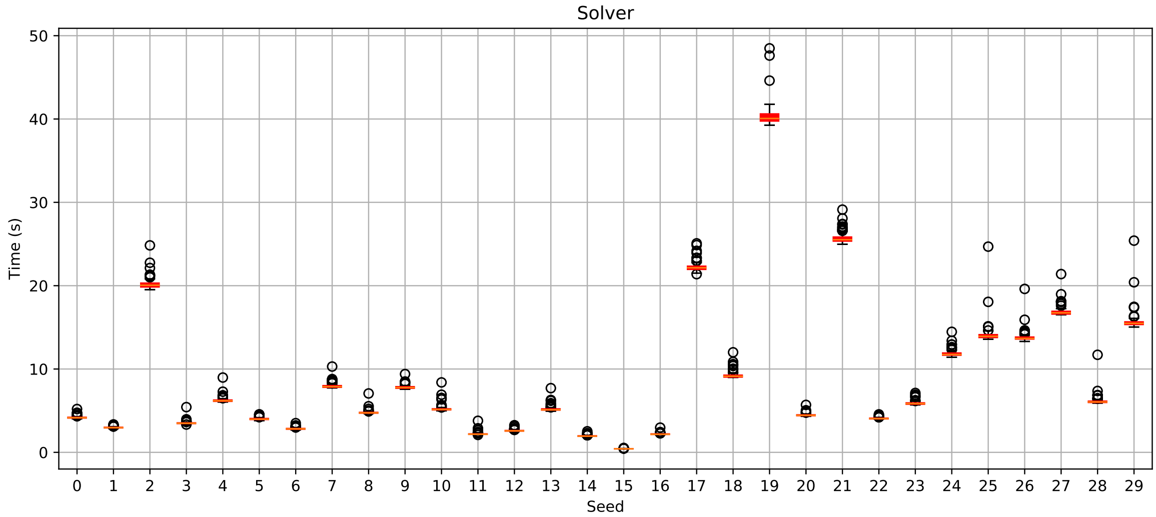


Figure 24: Execution time for different seeds with a 6 × 6 instance.

7 Conclusions

We have presented a general optimization framework to achieve an optimal antenna location solution for a square grid. Moreover, for the proposed problem we have discussed several heuristic methods that could outperform the exact solution in terms of time while approaching to an optimal solution. Overall, the best-performing heuristic was the PDFT2, since it reaches the optimal solution for small instances first than the other heuristics in polynomial time. For bigger instances, when an optimal solution is not found, the objective function will be up to 50% greater than the optimal solution which is the lowest among all the methods.

In general, we are considering two main networking planning variables, the traffic demand for every cell (region) and the capacity given by the antennas in a flat territory. This could be a good **first** approximation for the solution of a real problem. However, we are still far from obtaining a real-world Antenna Placement Solution. There are still many factors to be considered such as the geographical deformations of the land, the coverage of an antenna and the obstructions affecting it, inter-cell interference, among many other parameters. And we might still generalize the problem not just to restricting placing the antennas in a square grid but in any irregular polygon form that describes any possible location where an antenna could be.

References

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