Part I:

#1. 2×3 #3. 3×2 #5. 1×4

#2. 2×2 #4. 2×3 #6. 4×1

Part II:

#1. $\bar{u} + \bar{v} = [9 \quad 7 \quad -4 \quad 9]$

#2. $\bar{u} - \bar{v} = [3 \quad -3 \quad -2 \quad 1]$

#3. $\alpha = 6$, hence:

$$\alpha \cdot \bar{u} = [36 \quad 12 \quad -18 \quad 30]$$

#4. $\bar{u} \cdot \bar{v} = \langle \bar{u}, \bar{v} \rangle = 18 + 10 + 3 + 20 = 51$

#5. $\|u\| = \sqrt{36 + 4 + 9 + 25} = \sqrt{74} =$

Part III:

#1. $A + C = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{pmatrix} + \begin{pmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{pmatrix}$ is undefined, matrices' dimensions don't match

#2. $A - C^t = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{pmatrix} - \begin{pmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -4 & -7 & -3 \\ 3 & 6 & 4 \end{pmatrix}$

#3. $C^t + 3D = \begin{pmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{pmatrix} + 3 \cdot \begin{pmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 14 & 3 & 3 \\ 2 & 7 & 9 \end{pmatrix}$

#4. $B \cdot A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{pmatrix} = \begin{pmatrix} -1 & -5 & -1 \\ 2 & 7 & 4 \end{pmatrix}$

#5. $B \cdot A^t = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 2 & 7 \\ 3 & 4 \end{pmatrix}$ is undefined. Inner dimensions of matrices in a product should match.

$\underline{2 \times 2} \cdot \underline{3 \times 2}$

#1. $B \cdot C = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{pmatrix}$ is undefined

$\underset{2 \times \underline{2}}{\quad} \cdot \underset{\underline{3} \times 2}{\quad}$

#2. $C \cdot B = \begin{pmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -6 \\ 9 & -8 \\ 6 & -6 \end{pmatrix}$

#3. $B^4 = B^2 \cdot B^2 = \left[\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \right]^2 =$

$= \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^2 = \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$

#4. $A \cdot A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 2 & 7 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 14 & 28 \\ 28 & 69 \end{pmatrix}$

#5. $D^T D = \begin{pmatrix} 3 & 1 \\ -2 & 2 \\ -1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 10 & -4 & 0 \\ -4 & 8 & 8 \\ 0 & 8 & 10 \end{pmatrix}$