1

WHY QUANTUM GRAVITY?

1.1 Quantum theory and the gravitational field

1.1.1 *Introduction*

Quantum theory seems to be a universal theory of Nature. More precisely, it provides a general framework for all theories describing particular interactions. Quantum theory has passed a plethora of experimental tests and is considered a well-established theory, except for the ongoing discussion about its interpretational foundations.

The only interaction that has not been fully accomodated within quantum theory is the gravitational field, the oldest known interaction. It is described very successfully by a classical (i.e. non-quantum) theory, Einstein's *general theory of relativity* (GR), also called *geometrodynamics*. From a theoretical, or even aesthetical, point of view, it is highly appealing, since the fundamental equations can be formulated in simple geometrical terms. Moreover, there exist by now plenty of experimental tests that have been passed by this theory without problems. One particular impressive example is the case of the binary pulsar PSR 1913+16: the decrease of its orbital period can be fully explained due to the emission of gravitational waves as predicted by GR. The accuracy of this test is only limited by the accuracy of clocks on Earth, which according to recent proposals for rubidium fountain clocks (Fertig and Gibble 2000) should approach an accuracy of about 10^{-16} (such a clock would go wrong by less than 1 s during a time as long as the age of the Universe). The precision is so high that one even needs to model the gravitational influence of the Milky Way on the binary pulsar in order to find agreement with the theoretical prediction (Damour and Taylor 1991).

The formalism of general relativity is discussed in many textbooks, see for example, Hawking and Ellis (1973), Misner *et al.* (1973), Straumann (1984), or Wald (1984). It can be defined by the Einstein–Hilbert action,

$$S_{\mathrm{EH}} = \frac{c^4}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^4x\ \sqrt{-g}\,(R-2\Lambda) \pm \frac{c^4}{8\pi G}\int_{\partial\mathcal{M}} \mathrm{d}^3x\ \sqrt{h}K\ . \tag{1.1}$$

Note that $c^4/16\pi G \approx 2.41\times10^{47}\ \mathrm{g\,cm\,s^{-2}} \approx 2.29\times10^{74}(\mathrm{cm\,s})^{-1}\ \hbar$. The integration in the first integral of (1.1) goes over a region $\mathcal{M}$ of the space–time manifold, and the second integral over its boundary $\partial\mathcal{M}$ (with the positive sign referring to a space-like, the negative sign referring to a time-like boundary). The integrand of the latter contains the determinant, h, of the three-dimensional metric on the boundary, and K is the trace of the second fundamental form (see Section 4.2.1).

That a surface term is needed in order to obtain a consistent variational principle had been already noted by Einstein (1916*a*).

In addition to the action (1.1) one considers actions for non-gravitational fields, in the following called $S_{\rm m}$ ('matter action'). They give rise to the energy–momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\rm m}}{\delta g^{\mu\nu}} \ , \tag{1.2}$$

which acts as a 'source' for the gravitational field. In general, it does not coincide with the canonical energy–momentum tensor. From the variation of $S_{\rm EH} + S_{\rm m}$, the Einstein field equations are obtained,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \ . \tag{1.3}$$

(Our convention is $R_{\mu\nu} = R^{\rho}_{\mu\rho\nu}$.)

A natural generalization of general relativity is the Einstein–Cartan theory, see for example, Hehl (1985) and Hehl *et al.* (1976) for details. It is found by gauging translations and Lorentz transformations (i.e. the Poincaré group), leading to the tetrad e^{μ}_{α} and the connection $\omega^{\alpha\beta}_{\mu}$ as gauge potentials, respectively. The corresponding gravitational field strengths are torsion and curvature. Torsion vanishes outside matter and does not propagate, but it is straightforward to formulate extensions with a propagating torsion. The occurrence of torsion is a natural consequence of the presence of spin currents. Its effects are tiny on macroscopic scales (which is why it has not been seen experimentally), but it should be of high relevance in the microscopic realm, for example, on the scale of the electronic Compton wavelength and in the very early universe. In fact, the Einstein–Cartan theory is naturally embedded in theories of supergravity (see Section 2.3), where a spin-3/2 particle (the 'gravitino') plays a central role.

In Chapter 2, we shall discuss some 'uniqueness theorems', which state that every theory of the gravitational field must contain GR (or the Einstein–Cartan theory) in an appropriate limit. Generalizations of GR such as the Jordan–Brans–Dicke theory, which contains an additional scalar field in the gravitational sector, are therefore mainly of interest as effective theories arising from fundamental theories such as string theory (see Chapter 9). They are usually not meant as classical alternatives to GR, except for the parametrization of experimental tests. That GR cannot be true at the most fundamental level is clear from the *singularity theorems* (cf. Hawking and Penrose 1996): under very general conditions, singularities in space–time are unavoidable, signalling the breakdown of GR.

The theme of this book is to investigate the possibilities of unifying the gravitational field with the quantum framework in a consistent way. This may lead to a general avoidance of space–time singularities.

1.1.2 *Main motivations for quantizing gravity*

The first motivation is **unification**. The history of science shows that a reductionist viewpoint has been very fruitful in physics (Weinberg 1993). The standard model of particle physics is a *quantum* field theory that has united in a certain sense all non-gravitational interactions. It has been very successful experimentally, but one should be aware that its concepts are poorly understood beyond the perturbative level; in this sense, the classical theory of GR is in a much better condition.

The universal coupling of gravity to all forms of energy would make it plausible that gravity has to be implemented in a quantum framework too. Moreover, attempts to construct an exact semiclassical theory, where gravity stays classical but all other fields are quantum, have failed up to now, see Section 1.2. This demonstrates, in particular, that classical and quantum *concepts* (phase space versus Hilbert space, etc.) are most likely incompatible.

Physicists have also entertained the hope that unification entails a solution to the notorious divergence problem of quantum field theory; as is shown in Chapter 2, perturbative quantum GR leads to even worse divergences, due to its non-renormalizability, but a full non-perturbative framework without any divergences may exist.

The second motivation comes from **cosmology** and **black holes**. As the singularity theorems and the ensuing breakdown of GR demonstrate, a fundamental understanding of the early universe—in particular, its initial conditions near the 'big bang'—and of the final stages of black-hole evolution requires an encompassing theory. From the historical analogy of quantum mechanics (which due to its stationary states rescued the atoms from collapse), the general expectation is that this encompassing theory is a *quantum* theory. Classically, the generic behaviour of a solution to Einstein's equations near a big-bang singularity is assumed to consist of 'BKL oscillations', cf. Belinskii *at al.* (1982) and the references therein. A key feature of this scenario is the decoupling of different spatial points. A central demand on a quantum theory of gravity is to provide a consistent quantum description of BKL oscillations.

The concept of an 'inflationary universe'[1] is often invoked to claim that the present universe can have emerged from generic initial conditions. This is only partly true, since one can of course trace back *any* present conditions to the past to find the 'correct' initial conditions. In fact, the crucial point lies in the assumptions that enter the *no-hair conjecture*, see for example, Frieman *et al.* (1997). This conjecture states that space–time approaches locally a de Sitter form for large times if a (probably effective) cosmological constant is present. It can be proved, provided some assumptions are made. In particular, it must be assumed that modes on very small scales (smaller than the Planck length, see below) are not amplified to cosmological scales. This assumption thus refers to the unknown regime of quantum gravity.

It must be emphasized that *if* gravity is quantized, the kinematical non-separability of quantum theory demands that the whole universe must be de-

[1] Following Harrison (2000), we shall write 'universe' instead of 'Universe' to emphasize that we talk about a *model* of the Universe, in contrast to 'Universe' which refers to 'everything'.

scribed in quantum terms. This leads to the concepts of quantum cosmology and the wave function of the universe, see Chapters 8 and 10.

A third motivation is the **problem of time**. Quantum theory and GR (in fact, every general covariant theory) contain a drastically different concept of time (and space–time). Strictly speaking, they are incompatible. In quantum theory, time is an external (absolute) element, *not* described by an operator (in special relativistic quantum field theory, the role of time is played by the external Minkowski space–time). In contrast, in GR, space–time is a dynamical object. It is clear that a unification of quantum theory with GR must lead to modifications of the concept of time. One might expect that the metric has to be turned into an operator. In fact, as a detailed analysis will show (Chapters 5 and 9), this will lead to novel features. Related problems concern the role of background structures in quantum gravity, the role of the diffeomorphism group (Poincaré invariance, as used in ordinary quantum field theory, is no longer a symmetry group), and the notion of 'observables'. That a crucial point lies in the presence of a more general invariance group was already noted by Pauli (1955)[2]:

> It seems to me ... that it is not so much the linearity or non-linearity which forms the heart of the matter, but the very fact that here a more general group than the Lorentz group is present ... .

1.1.3 *Relevant scales*

In a universally valid quantum theory, genuine quantum effects can occur on any scale, while classical properties are an emergent phenomenon only (see Chapter 10). This is a consequence of the superposition principle. Independent of this, there exist scales where quantum effects of a particular interaction should definitely be non-negligible.

It was already noted by Planck (1899) that the fundamental constants, speed of light (c), gravitational constant (G), and quantum of action ($\hbar$), can be combined in a unique way to yield units of length, time, and mass. In Planck's honour they are called Planck length, $l_{\rm P}$, Planck time, $t_{\rm P}$, and Planck mass, $m_{\rm P}$, respectively. They are given by the expressions

$$l_{\rm P} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\ \text{cm} , \tag{1.4}$$

$$t_{\rm P} = \frac{l_{\rm P}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \text{s} , \tag{1.5}$$

$$m_{\rm P} = \frac{\hbar}{l_{\rm P} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ \text{g} \approx 1.22 \times 10^{19}\ \text{GeV} . \tag{1.6}$$

The Planck mass seems to be a rather large quantity on microscopic standards. One has to keep in mind, however, that this mass (energy) must be concentrated

[2] 'Es scheint mir ..., daß nicht so sehr die Linearität oder Nichtlinearität Kern der Sache ist, sondern eben der Umstand, daß hier eine allgemeinere Gruppe als die Lorentzgruppe vorhanden ist ... .'

in a region of linear dimension l_P in order to see direct quantum-gravity effects. In fact, the Planck scales are attained for an elementary particle whose Compton wavelength is (apart from a factor of 2) equal to its Schwarzschild radius,

$$\frac{\hbar}{m_\mathrm{P}c} \approx R_\mathrm{S} \equiv \frac{2Gm_\mathrm{P}}{c^2} \ ,$$

which means that the space–time curvature of an elementary particle is not negligible. Sometimes (e.g. in cosmology), one also uses the Planck temperature,

$$T_\mathrm{P} = \frac{m_\mathrm{P}c^2}{k_\mathrm{B}} \approx 1.41 \times 10^{32} \ \mathrm{K} \ . \tag{1.7}$$

It is interesting to observe that Planck had introduced his units one year before he wrote the famous paper containing the quantum of action, see Planck (1899). How had this been possible? The constant $\hbar$ appears in Wien's law, $\hbar\omega_\mathrm{max} \approx 2.82 k_\mathrm{B}T$, which was phenomenologically known at that time. Planck learnt from this that a new constant of nature is contained in this law, and he called it b. Planck concludes his article by writing[3]:

These quantities retain their natural meaning as long as the laws of gravitation, of light propagation in vacuum, and the two laws of the theory of heat remain valid; they must therefore, if measured by all kinds of intelligent beings, always turn out to be the same.

It is also interesting that similar units had already been introduced by the Irish physicist Johnstone Stoney (1881). Of course, $\hbar$ was not known at that time, but one could (in principle) get the elementary electric charge e from Avogadro's number L and Faraday's number $F = eL$. With e, G, and c, one can construct the same fundamental units as with $\hbar, G$, and c (since the fine structure constant is $\alpha = e^2/\hbar c \approx 1/137$); therefore, Stoney's units differ from Planck's units by factors of $\sqrt{\alpha}$. Quite generally one can argue that there are three fundamental dimensional quantities (cf. Okun 1992).

The Planck length is indeed very small. If one imagines an atom to be of the size of the Moon's orbit, l_P would only be as small as about a tenth of the size of a nucleus. Still, physicists have already for a while entertained the idea that something dramatically happens at the Planck length, from the breakdown of the continuum to the emergence of non-trivial topology ('space–time foam'), see for example, Misner *et al.* (1973). We shall see in the course of this book how such ideas can be made more precise in quantum gravity. Unified theories may contain an intrinsic length scale from which l_P may be deduced. In string theory, for example, this is the string length l_s. A generalized uncertainty relation shows that scales smaller than l_s have no operational significance, see Chapter 9.

[3]'Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.'

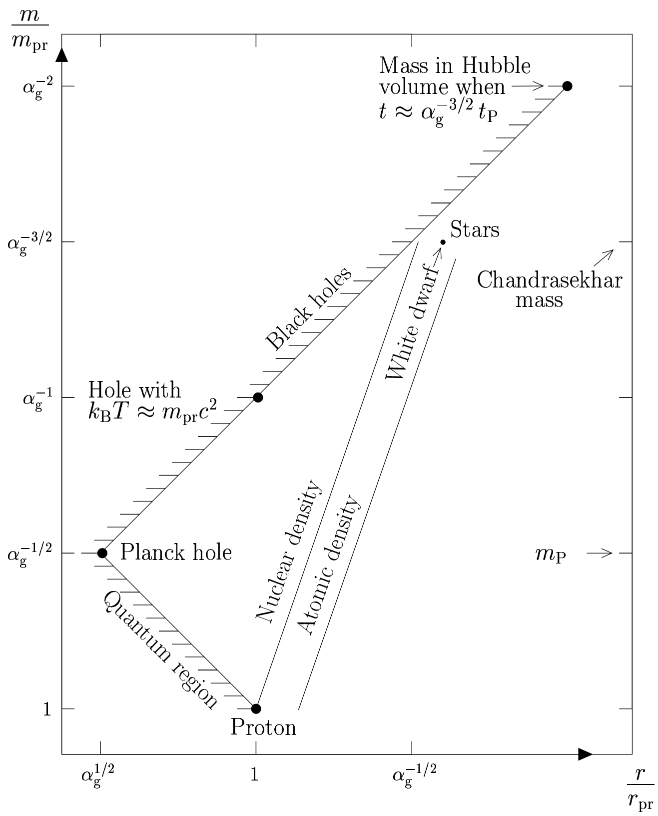

FIG. 1.1. Structures in the Universe (adapted from Rees (1995)).

Figure 1.1 presents some of the important structures in our universe in a mass-versus-length diagram. A central role is played by the 'fine structure constant of gravity',

$$\alpha_{\rm g} = \frac{Gm_{\rm pr}^2}{\hbar c} = \left(\frac{m_{\rm pr}}{m_{\rm P}}\right)^2 \approx 5.91 \times 10^{-39} \ , \tag{1.8}$$

where $m_{\rm pr}$ denotes the proton mass. Its smallness is responsible for the unimportance of quantum-gravitational effects on laboratory and astrophysical scales, and for the separation between micro- and macrophysics. As can be seen from the diagram, important features occur for masses that contain simple powers of $\alpha_{\rm g}$ (in terms of $m_{\rm pr}$), cf. Rees (1995). For example, the Chandrasekhar mass $M_{\rm C}$ is given by

$$M_{\rm C} \approx \alpha_{\rm g}^{-3/2} m_{\rm pr} \approx 1.4 M_{\odot} \ . \tag{1.9}$$

It gives the upper limit for the mass of a white dwarf and sets the scale for stellar masses. The minimum stellar life-times contain $\alpha_{\rm g}^{-3/2} t_{\rm P}$ as the important factor. It is also interesting to note that the size of human beings is roughly the

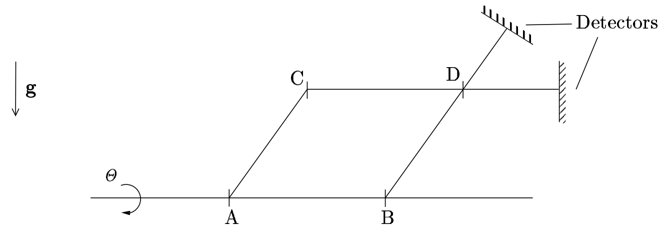

FIG. 1.2. Schematic description of the 'COW'-experiment for neutron interferometry in the gravitational field of the Earth.

geometric mean of Planck length and size of the observable part of the universe. It is an open question whether fundamental theories such as quantum gravity can provide an explanation for such values, for example, for the ratio $m_{\rm pr}/m_{\rm P}$, or not. We shall come back to this in Chapter 10.

As far as the relationship between quantum theory and the gravitational field is concerned, one can distinguish between different levels. The first, lowest level deals with quantum *mechanics* in *external* gravitational fields (either described by GR or its Newtonian limit). No back reaction on the gravitational field is taken into account. This is the only level where experiments exist so far. The next level concerns quantum *field* theory in *external* gravitational fields described by GR. Back reaction can be taken into account in a perturbative sense. These two levels will be dealt with in the next two subsections. The highest level, *full* quantum gravity, will be discussed in the rest of this book.

1.1.4 *Quantum mechanics and Newtonian gravity*

Consider first the level of Newtonian gravity. There exist experiments that test the classical trajectories of elementary particles, such as thermal neutrons that fall like mass points, see for example, Hehl *et al.* (1991). This is not so much of interest here. We are more interested in quantum-mechanical *interference* experiments concerning the motion of neutrons and atoms in external gravitational fields.

Historically, two experiments have been of significance. The experiment by Colella, Overhauser, and Werner ('COW') in 1975 was concerned with neutron interferometry in the gravitational field of the Earth. According to the equivalence principle, an analogous experiment should be possible with neutrons in accelerated frames. Such an experiment was performed by Bonse and Wroblewski in 1983. Details and references can be found in the reviews by Hehl *et al.* (1991) and Werner and Kaiser (1990).

In the following, we shall briefly describe the 'COW' experiment, see Fig. 1.2. A beam of neutrons is split into two parts, such that they can travel on different heights in the terrestrial gravitational field. They are then recombined and sent to detectors. The whole apparatus can be rotated with a varying angle θ around the horizontal axis. The interferences are then measured in dependence on θ.

The theoretical description makes use of the Schrödinger equation for neutrons (Hehl *et al.* 1991). The Hamiltonian in the system of the rotating Earth is given by

$$H = \frac{\mathbf{p}^2}{2m_{\rm i}} + m_{\rm g}\mathbf{gr} - \omega\mathbf{L} \ . \tag{1.10}$$

We have distinguished here between the inertial mass, $m_{\rm i}$, of the neutron and its (passive) gravitational mass, $m_{\rm g}$, because 'COW' have also used this experiment as a test of the equivalence principle. In the last term, ω and $\mathbf{L}$ denote the angular velocity of the Earth and the angular momentum of the neutron with respect to the centre of the Earth (given by $\mathbf{r} = 0$), respectively. This term describes centrifugal and Coriolis forces. Note that the canonical momentum is given by

$$\mathbf{p} = m_{\rm i}\dot{\mathbf{r}} + m_{\rm i}\omega \times \mathbf{r} \ . \tag{1.11}$$

The phase shift in the interferometer experiment is given by

$$\Delta\beta = \frac{1}{\hbar}\oint \ \mathbf{p}\mathrm{d}\mathbf{r} \ , \tag{1.12}$$

where the integration runs over the parallelogram ABDC of Fig. 1.2. According to (1.11), there are two contributions to the phase shift. The term containing ω describes the influence of the terrestrial rotation on the interference pattern ('neutron Sagnac effect'). It yields

$$\Delta\beta_{\rm Sagnac} = \frac{m_{\rm i}}{\hbar}\oint \ (\omega \times \mathbf{r})\mathrm{d}\mathbf{r} = \frac{2m_{\rm i}}{\hbar}\omega\mathbf{A} \ , \tag{1.13}$$

where $\mathbf{A}$ denotes the normal area vector of the loop ABDC.

Of main interest here is the gravitational part of the phase shift. Since the contributions of the sides $\overline{\rm AC}$ and $\overline{\rm DB}$ cancel, one has

$$\Delta\beta_{\rm g} = \frac{m_{\rm i}}{\hbar}\oint \ \mathbf{v}\mathrm{d}\mathbf{r} \approx \frac{m_{\rm i}(v_0 - v_1)}{\hbar}\overline{\rm AB} \ , \tag{1.14}$$

where v_0 and v_1 denote the absolute values of the velocities along $\overline{\rm AB}$ and $\overline{\rm CD}$, respectively. From energy conservation one gets

$$v_1 = v_0\sqrt{1 - \frac{2\Delta V}{m_{\rm i}v_0^2}} \approx v_0 - \frac{m_{\rm g}gh_0 \sin\,\theta}{m_{\rm i}v_0} \ ,$$

where $\Delta V = m_{\rm g}gh_0 \sin\theta$ is the potential difference, h_0 denotes the perpendicular distance between $\overline{\rm AB}$ and $\overline{\rm CD}$, and the limit $2\Delta V/m_{\rm i}v_0^2 \ll 1$ (about 10^{-8} in the experiment) has been used. The neutrons are prepared with a de Broglie wavelength $\lambda = 2\pi\hbar/p \approx 2\pi\hbar/m_{\rm i}v_0$ (neglecting the ω part, since the Sagnac effect contributes only 2 per cent of the effect), attaining a value of about 1.4

Å in the experiment. One then gets for the gravitational phase shift, the final result

$$\Delta\beta_{\mathrm{g}} \approx \frac{m_{\mathrm{i}} m_{\mathrm{g}} g \lambda A \sin\theta}{2\pi\hbar^2} , \tag{1.15}$$

where A denotes the area of the parallelogram ABDC. This result has been confirmed by 'COW' with 1 per cent accuracy. The phase shift (1.15) can be rewritten in an alternative form such that only those quantities appear that are directly observable in the experiment (Lämmerzahl 1996). It then reads

$$\Delta\beta_{\mathrm{g}} \approx \frac{m_{\mathrm{g}}}{m_{\mathrm{i}}} \mathbf{g}\mathbf{G} T T' , \tag{1.16}$$

where T (T') denotes the flight time of the neutron from A to B (from A to C), and $\mathbf{G}$ is the reciprocal lattice vector of the crystal layers (from which the neutrons are scattered in the beam splitter). Now m_{g} and m_{i} appear like in the classical theory as a ratio, not as a product. The COW experiment has also confirmed the validity of the (weak) equivalence principle in the quantum domain. Modern tests prefer to use atom interferometry because atoms are easier to handle and the experiments allow tests of higher precision (Lämmerzahl 1996, 1998). There the flight time is just the time between laser pulses, that is, the interaction time with the gravitational field; T is chosen by the experimentalist. Still, neutrons are useful to study quantum systems in the gravitational field. An experiment with ultracold neutrons has shown that their vertical motion in the gravitational field has discrete energy states, as predicted by the Schrödinger equation (Nesvizhevsky *et al.* 2002). The minimum energy is 1.4×10^{-12} eV, which is much smaller than the ground-state energy of the hydrogen atom.

It is also of interest to discuss the Dirac equation instead of the Schrödinger equation because this may give rise to additional effects. In Minkowski space (and cartesian coordinates), it reads

$$\left(\mathrm{i}\hbar\gamma^\mu \partial_\mu + \frac{mc}{\hbar}\right) \psi(x) = 0 , \tag{1.17}$$

where $\psi(x)$ is a Dirac spinor, and

$$[\gamma^\mu, \gamma^\nu]_+ \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} . \tag{1.18}$$

The transformation into an accelerated frame is achieved by replacing partial derivatives with covariant derivatives, see for example, Hehl *et al.* (1991),

$$\partial_n \longrightarrow D_n \equiv \partial_n + \frac{\mathrm{i}}{2}\sigma^{mk}\omega_{nmk} , \tag{1.19}$$

where $\sigma^{mk} = \mathrm{i}[\gamma^m, \gamma^k]$ is the generator of the Lorentz group, and ω_{nmk} denotes the anholonomic components of the connection. From the equivalence principle, one would expect that this gives also the appropriate form in curved space–time, where

$$[\gamma^\mu, \gamma^\nu]_+ = 2g^{\mu\nu} \ . \tag{1.20}$$

For the formulation of the Dirac equation in curved space–time, one has to use the tetrad ('vierbein') formalism, in which a basis $e_n = \{e_0, e_1, e_2, e_3\}$ is chosen at each space–time point. One can expand the tetrads with respect to the tangent vectors along coordinate lines ('holonomic basis') according to

$$e_n = e_n^\mu \partial_\mu \ . \tag{1.21}$$

Usually one chooses the tetrad to be orthonormal,

$$e_n \cdot e_m \equiv g_{\mu\nu} e_n^\mu e_m^\nu = \eta_{nm} \equiv \mathrm{diag}(-1,1,1,1) \ . \tag{1.22}$$

The reason why one has to go beyond the pure metric formalism is the fact that spinors (describing fermions) are objects whose wave components transform with respect to a two-valued representation of the Lorentz group. One therefore needs a local Lorentz group and local orthonormal frames.

One can define anholonomic Dirac matrices according to

$$\gamma^n \equiv e_\mu^n \gamma^\mu \ , \tag{1.23}$$

where $e_n^\mu e_\mu^m = \delta_n^m$. This leads to

$$[\gamma^n, \gamma^m]_+ = 2\eta^{nm} \ . \tag{1.24}$$

The Dirac equation in curved space–time or accelerated frames then reads

$$\left(\mathrm{i}\hbar\gamma^n D_n + \frac{mc}{\hbar}\right)\psi(x) = 0 \ . \tag{1.25}$$

In order to study quantum effects of fermions in the gravitational field of the Earth, one specializes this equation to the non-inertial frame of an accelerated and rotating observer, with acceleration $\mathbf{a}$ and angular velocity ω, respectively (see e.g. Hehl *et al.* 1991). A non-relativistic approximation with relativistic corrections is then obtained by the standard Foldy–Wouthuysen transformation. This leads to (writing $\beta \equiv \gamma^0$)

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = H_{\mathrm{FW}}\psi \ , \tag{1.26}$$

with

$$\begin{aligned} H_{\mathrm{FW}} &= \beta mc^2 + \frac{\beta}{2m}\mathbf{p}^2 - \frac{\beta}{8m^3c^2}\mathbf{p}^4 + \beta m(\mathbf{a}\,\mathbf{x}) \\ &\quad -\omega(\mathbf{L}+\mathbf{S}) + \frac{\beta}{2m}\mathbf{p}\frac{\mathbf{a}\,\mathbf{x}}{c^2}\mathbf{p} + \frac{\beta\hbar}{4mc^2}\vec{\sigma}(\mathbf{a}\times\mathbf{p}) + \mathcal{O}\left(\frac{1}{c^3}\right) \end{aligned} \tag{1.27}$$

($\vec{\sigma}$ denotes the Pauli matrices). The interpretation of the various terms in (1.27) is straightforward. The first four terms correspond to the rest mass, the usual

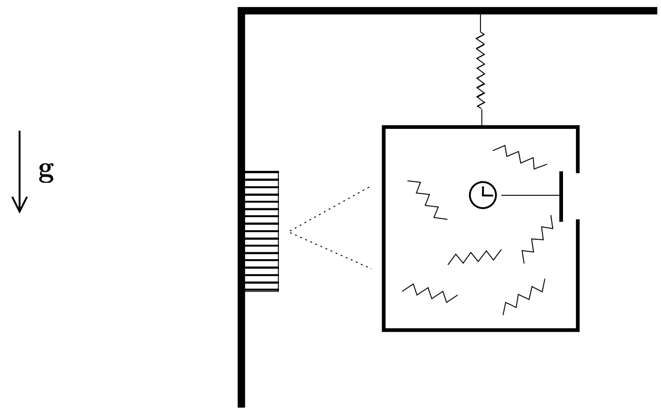

FIG. 1.3. Setting in the gedanken experiment of the Einstein–Bohr debate on the time–energy uncertainty relation.

non-relativistic kinetic term, the first relativistic correction to the kinetic term, and the 'COW' effect (or its analogue for pure acceleration), respectively. The term $\omega\mathbf{L}$ describes the Sagnac effect, while $\omega\mathbf{S}$ corresponds to a new spin-rotation effect ('Mashhoon effect') that cannot be found from the Schrödinger equation. One can estimate that for typical values of a neutron interferometer experiment, the Mashhoon effect contributes only 10^{-9} of the Sagnac effect. This is very small, but it has been indirectly observed (Mashhoon 1995). We mention that this framework is also of use in the study of a 'generalized' Dirac equation to parametrize quantum tests of general relativity (Lämmerzahl 1998) and to construct an axiomatic approach to space–time geometry, yielding a Riemann–Cartan geometry (see Audretsch *et al.* 1992).

In concluding this subsection, we want to discuss briefly one important occasion where GR seems to play a role in the foundations of quantum mechanics. This is the discussion of the time–energy uncertainty relations by Bohr and Einstein at the sixth Solvay conference which took place in Brussels in 1930 (cf. Bohr 1949).

Einstein came up with the following counter-argument against the validity of this uncertainty relation. Consider a box filled with radiation. A clock controls the opening of a shutter for a short time interval such that a single photon can escape at a fixed time t. The energy E of the photon is, however, also fixed because it can be determined by weighing the box before and after the escape of the photon. It thus seems as if the time–energy uncertainty relation were violated.

In his response to Einstein's attack, Bohr came up with the following arguments. Consider the details of the weighing process in which a spring is attached to the box, see Fig. 1.3. The null position of the balance is known with an accuracy Δq. This leads to an uncertainty in the momentum of the box, $\Delta p \sim \hbar/\Delta q$.

Bohr then makes the assumption that Δp must be smaller than the total momentum imposed by the gravitational field during the time T of the weighing process on the mass uncertainty Δm of the box. This leads to

$$\Delta p < v\Delta m = gT\Delta m \ , \tag{1.28}$$

where g is the gravitational acceleration. Now GR enters the game: the tick rate of clocks depends on the gravitational potential according to the 'redshift formula'

$$\frac{\Delta T}{T} = \frac{g\Delta q}{c^2} \ , \tag{1.29}$$

so that, using (1.28), the uncertainty in ΔT after the weighing process is

$$\Delta T = \frac{g\Delta q}{c^2} T > \frac{\hbar}{\Delta m c^2} = \frac{\hbar}{\Delta E} \ , \tag{1.30}$$

in accordance with the time–energy uncertainty relation. (After this, Einstein gave up to find an inconsistency in quantum mechanics, but focused instead on its possible incompleteness.) But are Bohr's arguments really consistent? There are, in fact, some possible loopholes (cf. Shi 2000). First, it is unclear whether (1.28) must really hold, since Δp is an intrinsic property of the apparatus. Second, the relation (1.29) cannot hold in this form, because T is *not* an operator, and therefore ΔT cannot have the same interpretation as Δq. In fact, if T is considered as a classical quantity, it would be more consistent to relate Δq to an uncertainty in g, which in fact would suggest to consider the quantization of the gravitational field. One can also change the gedanken experiment by using an electrostatic field instead of the gravitational field, where a relation of the form (1.29) no longer holds (see von Borzeszkowski and Treder 1988). It should also be emphasized that not much of GR is needed, in fact, the relation (1.29) follows from energy conservation and $E = h\nu$ alone. A general criticism of all these early gedanken experiments deals with their inconsistent interpretation of measurements as being related to uncontrollable interactions; see Shi (2000) and Chapter 10. The important feature is, however, entanglement between quantum systems.

It thus seems as if Bohr's analysis was mainly based on dimensional arguments. In fact, the usual application of the time–energy uncertainty relation relates linewidths of spectra for unstable systems with the corresponding half-life time. In quantum gravity, no time parameter appears on the fundamental level (see Chapter 5). A time–energy uncertainty relation can only be derived in the semiclassical limit.

1.1.5 *Quantum field theory in curved space–time*

Some interesting new aspects appear when quantum *fields* play a role. They mainly concern the notions of *vacuum* and *particles*. A vacuum is only invariant with respect to Poincaré transformations, so that observers that are not related by inertial motion refer in general to a different type of vacuum (Fulling 1973). 'Particle creation' can occur in the presence of external fields or for the state of non-inertial motion. An external electric field, for example, can lead to the creation of electron–positron pairs ('Schwinger effect'), see for example, Grib *et al.* (1994). We shall be mainly concerned with particle creation in the presence of external gravitational fields (Birrell and Davies 1982). This was first discussed by Schrödinger (1939).

One example of particular interest is particle creation from black holes (Hawking 1975), see for example, Frolov and Novikov (1998), Fré *et al.* (1999), and Hehl *et al.* (1998) for a detailed review. This is not only of fundamental theoretical interest, but could also lead to observational consequences. A black hole radiates with a universal temperature ('Hawking temperature') according to

$$T_{\mathrm{BH}} = \frac{\hbar\kappa}{2\pi k_{\mathrm{B}} c} \,, \tag{1.31}$$

where κ is the surface gravity of a stationary black hole which by the no-hair theorem is uniquely characterized by its mass M, its angular momentum J, and (if present) its electric charge Q. In the particular case of the spherically symmetric Schwarzschild black hole, one has $\kappa = c^4/4GM = GM/R_{\mathrm{S}}^2$ and therefore

$$T_{\mathrm{BH}} = \frac{\hbar c^3}{8\pi k_{\mathrm{B}} GM} \approx 6.17 \times 10^{-8} \left(\frac{M_\odot}{M}\right) \text{ K} \,. \tag{1.32}$$

This temperature is unobservationally small for solar-mass (and bigger) black holes, but may be observable for primordial black holes. It must be emphasized that the expression for T_{BH} contains all fundamental constants of Nature. One may speculate that this expression—relating the macroscopic parameters of a black hole with thermodynamic quantities—plays a similar role for quantum gravity as de Broglie's relations $E = \hbar\omega$ and $p = \hbar k$ once played for the development of quantum theory (Zeh 2001).

Hawking radiation was derived in the semiclassical limit in which the gravitational field can be treated classically. According to (1.32), the black hole loses mass through its radiation and becomes hotter. After it has reached a mass of the size of the Planck mass (1.6), the semiclassical approximation breaks down and the full theory of quantum gravity should be needed. Black-hole evaporation thus plays a crucial role in any approach to quantum gravity; cf. Chapter 7.

There exists a related effect to (1.31) in flat Minkowski space. An observer in uniform acceleration experiences the standard Minkowski vacuum not as empty, but as filled with *thermal* radiation with temperature

$$T_{\mathrm{DU}} = \frac{\hbar a}{2\pi k_{\mathrm{B}} c} \approx 4.05 \times 10^{-23} \; a \left[\frac{\mathrm{cm}}{\mathrm{s}^2}\right] \text{ K} \,. \tag{1.33}$$

This temperature is often called the 'Davies–Unruh temperature' after the work by Davies (1975) and Unruh (1976), with important contributions also by Fulling

(1973). Formally, it arises from (1.31) through the substitution of κ by a. This can be understood from the fact that *horizons* are present in both the black-hole case and the acceleration case, see for example, Kiefer (1999) for a detailed review. Although (1.33) seems to be a small effect, people have suggested to look for it in accelerators (Leinaas 2002) or in experiments with ultraintense lasers (Chen and Tajima 1999), without definite success up to now.

A central role in the theory of quantum fields on an external space–time is played by the semiclassical Einstein equations. These equations are obtained by replacing the energy–momentum tensor in (1.1) by the expectation value of the energy–momentum operator with respect to some quantum state Ψ,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle \ . \tag{1.34}$$

A particular issue is the regularization and renormalization of the object on the right-hand side (Birrell and Davies 1982). This leads, for example, to a flux of negative energy into the black hole, which can be interpreted as the origin of Hawking radiation. As we shall discuss in the next section, (1.34) is of limited value if seen from the viewpoint of the full quantum theory. We shall see in Section 5.4 that (1.34) can be derived approximately from canonical quantum gravity as a kind of mean-field equation.

1.2 Problems of a fundamental semiclassical theory

When dealing with approaches to quantum gravity, the question is sometimes asked whether it is really necessary to quantize the gravitational field. And even if it is, doubts have occasionally been put forward whether such a theory can operationally be distinguished from an 'exact' semiclassical theory.[4] As a candidate for the latter, the semiclassical Einstein equations (1.34) are often presented, cf. Møller (1962). The more general question behind this issue concerns the possibility of a consistent *hybrid dynamics* through which a quantum and a classical system is being coupled.

Eppley and Hannah (1977) argued that the coupling between a classical gravitational wave with a quantum system leads to inconsistencies. In fact, the gravitational nature of this wave is not crucial—it can be any classical wave. Without going into details, their arguments demonstrate that the quantum nature of the measuring apparatus has to be taken into account, in order to avoid inconsistencies. This resembles the old debate between Bohr and Einstein, in which Bohr had to impose the uncertainty relations also for a macroscopic object (the screen in the double-slit experiment), in order to save them from Einstein's attacks. In this sense the arguments by Eppley and Hannah give a general hint for the quantum nature of the gravitational field.

[4]'Semiclassical' here means a theory that couples exactly quantum degrees of freedom to classical degrees of freedom. It therefore has nothing to do with the WKB approximation which usually is referred to as the semiclassical approximation. The latter is discussed in Section 5.4.

It is often argued that the famous gedanken experiments by Bohr and Rosenfeld (1933) imply that a coupling between a classical and a quantum system is inconsistent. For this reason, here we shall briefly review their arguments (see also Heitler (1984) for a lucid discussion). Historically, Landau and Peierls (1931) had claimed that the quantum nature of the *electromagnetic field* cannot be tested, since there exists a fundamental minimal uncertainty for single field amplitudes, not only for conjugate pairs. Bohr and Rosenfeld have then shown that this is not true. Their line of thought runs as follows. Consider a charged body with mass M and charge Q, acting as a measuring device for the electric field $\mathcal{E}$ being present in a volume $V \equiv l^3$. Momentum measurements of the body are being made at the beginning and the end of the measurement time interval. In order to qualify the body as a measurement device, the following assumptions are made (in order to avoid back reaction, etc.); cf. also von Borzeszkowski and Treder (1988),

$$Q^2 \gtrsim \hbar c \approx 137 e^2 \ , \tag{1.35}$$

$$l > \frac{Q^2}{Mc^2} \ . \tag{1.36}$$

The latter condition expresses the fact that the electrostatic energy should be smaller than the rest mass. Bohr and Rosenfeld then found from their detailed analysis, the following conditions with $\mathcal{E}$ denoting the field average over the volume V,

$$\Delta\mathcal{E}\ l^2 \gtrsim \frac{\hbar c}{Q} \ , \tag{1.37}$$

$$\Delta\mathcal{E}\ l^3 \gtrsim \frac{\hbar Q}{Mc} \ . \tag{1.38}$$

One can now always choose a measurement device such that the ratio Q/M in the last expression can be made arbitrarily small. Therefore, $\mathcal{E}$ can be measured with arbitrary precision, contrary to the arguments of Landau and Peierls (1931). Bohr and Rosenfeld then show that Eqns (1.37) and (1.38) are in agreement with the uncertainty relations as being derived from the quantum commutators of quantum electrodynamics (QED). Their discussion, therefore, shows the *consistency* of the formalism with the measurement analysis. It does *not* provide a logical proof that the electromagnetic field must be quantized; cf. Rosenfeld (1963).[5]

Although the final formalism for quantum gravity is not yet at hand, the Bohr–Rosenfeld analysis can at least formally be extended to the gravitational field, cf. Bronstein (1936), DeWitt (1962) and von Borzeszkowski and Treder (1988). One can substitute the electric field $\mathcal{E}$ by the Christoffel symbols Γ

[5] From *empirical* arguments, we know, of course, that the electromagnetic field is of quantum nature.

('gravitational force'). Since one can then perform in the Newtonian approximation $\Gamma \sim GM_{\rm g}/r^2c^2$ ($M_{\rm g}$ denoting the gravitational mass) the substitutions

$$\Delta\mathcal{E} \to \frac{\Delta\Gamma c^2}{G}, \quad Q \to M_{\rm g} \ , \tag{1.39}$$

one gets from (1.38), the relation (writing $M_{\rm i}$ instead of M to emphasize that it is the inertial mass)

$$\Delta\Gamma\ l^3 \gtrsim \frac{\hbar G}{c^3}\frac{M_{\rm g}}{M_{\rm i}} \ . \tag{1.40}$$

Using the (weak) equivalence principle, $M_{\rm g} = M_{\rm i}$, and recalling the definition (1.4) of the Planck length, one can write

$$\Delta\Gamma \gtrsim \frac{l_{\rm P}^2}{l^3} \ . \tag{1.41}$$

The analogous relation for the metric g would then read

$$\Delta g \gtrsim \left(\frac{l_{\rm P}}{l}\right)^2 \ . \tag{1.42}$$

Thus, the measurement of a single quantity (the metric) is operationally restricted.[6] This is of course possible because, unlike QED, the fundamental length scale $l_{\rm P}$ is available. On the other hand, a gedanken experiment by Smith and Bergmann (1979) shows that the magnetic-type components of the Weyl tensor in linearized quantum gravity can be measured, provided a suitable average over space–time domains is performed.

Does (1.42) imply that the quantum nature of the gravitational field cannot be tested? We want to answer this question in the negative, for the following reasons. First, there might be other measurement devices which do not necessarily obey the above relations. Second, this analysis does not say anything about global situations (black holes, cosmology) and about non-trivial applications of the superposition principle. And third, in fact, quantum gravity seems to predict the existence of a smallest scale with operational meaning, see Chapters 6 and 9. Then, (1.42) could be interpreted as a confirmation of quantum gravity. It might of course be possible, as argued in von Borzeszkowski and Treder (1988), that quantum-gravitational analogues of effects such as Compton scattering or Lamb shift are unobservable in the laboratory. This has only little bearing on the above discussion, since one would expect that such quantum-gravitational effects are anyway not seen directly in laboratory experiments.

Returning to the specific equations (1.34) for a semiclassical theory, there are a number of problems attached with them. First, the expectation value of

[6]Equation (1.42) is similar to the heuristic relation $\Delta g \gtrsim l_{\rm P}/l$ of Misner *et al.* (1973), although the exponent is different.

the energy–momentum tensor that occurs on the right-hand side is usually divergent and needs some regularization and renormalization.[7] In this process, however, counter-terms arise that invoke higher powers of the curvature such as R^2, which may alter the semiclassical equations at a fundamental level. Second, (1.34) introduces the following element of non-linearity. The space–time metric g depends on the quantum state in a complicated way, since in (1.34) $|\Psi\rangle$ depends on g also through the (functional) Schrödinger equation (an equivalent statement holds in the Heisenberg picture). Consequently, if g_1 and g_2 correspond to states $|\Psi_1\rangle$ and $|\Psi_2\rangle$, respectively, there is no obvious relation between a superposition $A|\Psi_1\rangle + B|\Psi_2\rangle$ (which still satisfies the Schrödinger equation) and the metrics g_1 and g_2. This was already remarked by Anderson in Møller (1962) and by Belinfante in a discussion with Rosenfeld (see Infeld 1964). It was also the reason why Dirac strongly objected to (1.34); cf. von Borzeszkowski and Treder (1988).

Rosenfeld insisted on (1.34) because he strongly followed Bohr's interpretation of the measurement process for which classical concepts should be indispensable. This holds in particular for the structure of space–time, so he wished to have a *c*-number representation for the metric. He rejects a quantum description for the total system and answers to Belinfante in Infeld (1964) that Einstein's equations may merely be thermodynamical equations of state that break down for large fluctuations, that is, the gravitational field may only be an effective, not a fundamental, field, cf. also Jacobson (1995).

The problem with the superposition principle can be demonstrated by the following argument that has even been put to an experimental test (Page and Geilker 1981). One assumes that there is no explicit collapse of $|\Psi\rangle$, because otherwise one would expect the covariant conservation law $\langle\hat{T}_{\mu\nu}\rangle;^{\nu} = 0$ to be violated, in contradiction to (1.34). If the gravitational field were quantized, one would expect that each component of the superposition in $|\Psi\rangle$ would act as a source for the gravitational field. This is of course the Everett interpretation for quantum theory; cf. Chapter 10. On the other hand, eqn (1.34) depends on *all* components of $|\Psi\rangle$ simultaneously. Page and Geilker (1981) envisaged the following gedanken experiment, reminiscent of Schrödinger's cat, to distinguish between these options.

In a box, there is a radioactive source together with two masses that are connected by a spring. Initially, the masses are rigidly connected, so that they cannot move. If a radioactive decay happens, the rigid connection will be broken and the masses can swing towards each other. Outside the box, there is a Cavendish balance that is sensitive to the location of the masses and therefore acts as a device to 'measure' their position. Following Unruh (1984), the situation can be described by the following simple model. We denote with $|0\rangle$, the quantum state of the masses with rigid connection, and with $|1\rangle$, the corresponding state in which they can move towards each other. For the purpose of this

[7] This procedure leads to an essentially unique result for $\langle\hat{T}_{\mu\nu}\rangle$ if certain physical requirements are imposed, cf. Birrell and Davies (1982). The ambiguities can then be absorbed by a redefinition of constants appearing in the action.

experiment, it is sufficient to go to the Newtonian approximation of GR and to use the Hamilton operator $\hat{H}$ instead of the full energy–momentum tensor $\hat{T}_{\mu\nu}$. For initial time $t = 0$, it is assumed that the state is given by $|0\rangle$. For $t > 0$, the state then evolves into a superposition of $|0\rangle$ and $|1\rangle$,

$$|\Psi\rangle(t) = \alpha(t)|0\rangle + \beta(t)|1\rangle \ ,$$

with the coefficients $|\alpha(t)|^2 \approx \mathrm{e}^{-\lambda t}$, $|\beta|^2 \approx 1 - \mathrm{e}^{-\lambda t}$, according to the law of radioactive decay, with a decay constant λ. From this, one finds for the evolution of the expectation value,

$$\langle\Psi|\hat{H}|\Psi\rangle(t) = |\alpha(t)|^2\langle 0|\hat{H}|0\rangle + |\beta(t)|^2\langle 1|\hat{H}|1\rangle + 2\mathrm{Re}\left[\alpha^*\beta\langle 0|\hat{H}|1\rangle\right] \ .$$

If one makes the realistic assumption that the states are approximate eigenstates of the Hamiltonian, the last term, which describes interferences, vanishes. Anyway, this is not devised as an interference experiment (in contrast to Schrödinger's cat), and interferences would become small due to decoherence (Chapter 10). One is thus left with

$$\langle\Psi|\hat{H}|\Psi\rangle(t) \approx \mathrm{e}^{-\lambda t}\langle 0|\hat{H}|0\rangle + \left(1 - \mathrm{e}^{-\lambda t}\right)\langle 1|\hat{H}|1\rangle \ . \tag{1.43}$$

According to semiclassical gravity as described by (1.34), therefore, the Cavendish balance would follow the dynamics of the expectation value and slightly swing in the course of time. This is in sharp contrast to the prediction of linear quantum gravity, where in each component the balance reacts to the mass configuration and would thus be observed to swing instantaneously at a certain time. This is, in fact, what has been observed in the actual experiment (Page and Geilker 1981). This experiment, albeit simple, demonstrates convincingly that (1.34) cannot fundamentally be true.

In the above experiment, the reason for the deviation between the predictions of the semiclassical theory and the 'full' theory lies in the large fluctuation for the Hamiltonian. In fact, the experiment was devised to generate such a case. Large fluctuations also occur in another interesting situation—the gravitational radiation emitted by quantum systems (Ford 1982). The calculations are performed for linearized gravity, that is, for a small metric pertubation around flat space–time with metric $\eta_{\mu\nu}$, see for example, Misner *et al.* (1973) and Chapter 2. Denoting by $G_r(x, x')$ the retarded Green function, one finds for the *integrated* energy–momentum tensor $S_{\mu\nu}$ in the semiclassical theory described by (1.34), the expression,[8]

$$S^{\mu\nu}_{\mathrm{sc}} = -8\pi G\int \mathrm{d}^3x\mathrm{d}^4x'\mathrm{d}^4x'' \ \partial^\mu G_r(x, x')\partial^\nu G_r(x, x'')$$

[8]Hats on operators are avoided for simplicity; from now on we set $c = 1$ in most cases.

$$\times \left[\langle T_{\alpha\beta}(x')\rangle \langle T^{\alpha\beta}(x'')\rangle - 1/2\ \langle T(x')\rangle \langle T(x'')\rangle \right] \ , \qquad (1.44)$$

where $T \equiv T^{\mu\nu}\eta_{\mu\nu}$ denotes the trace of the energy–momentum tensor. On the other hand, quantization of the linear theory (see Chapter 2) yields

$$S_{\mathrm{q}}^{\mu\nu} = -8\pi G \int \mathrm{d}^3x \mathrm{d}^4x' \mathrm{d}^4x''\ \partial^\mu G_r(x,x')\partial^\nu G_r(x,x'') \\ \times \left\langle T_{\alpha\beta}(x')T^{\alpha\beta}(x'') - 1/2\ T(x')T(x'') \right\rangle \ . \qquad (1.45)$$

The difference in these results can be easily interpreted: in the semiclassical theory, $\langle T_{\mu\nu}\rangle$ acts as a source, and so no two-point functions $\langle T \dots T\rangle$ can appear, in contrast to linear quantum theory.

It is obvious that the above two expressions strongly differ, once the fluctuation of the energy–momentum tensor is large. As a concrete example, Ford (1982) takes a massless real scalar field as matter source. For coherent states there is no difference between (1.44) and (1.45). This is not unexpected, since coherent states are as 'classical' as possible, and so the semiclassical and the full theory give identical results. For a superposition of coherent states, however, this is no longer true, and the energies emitted by the quantum system via gravitational waves can differ by macroscopic amounts. For a number eigenstate of the scalar field, the semiclassical theory does not predict any radiation at all ($\langle T_{\mu\nu}\rangle$ is time-independent), whereas there is radiation in quantum gravity ($\langle T_{\mu\nu}T_{\rho\lambda}\rangle$ is time-dependent).[9] Therefore, one can in principle have macroscopic quantum-gravity effects even far away from the Planck scale!

Kuo and Ford (1993) have extended this analysis to situations where the expectation value of the energy density can be negative. They show that in such cases the fluctuations in the energy–momentum tensor are large and that the semiclassical theory gives different predictions than the quantum theory. This is true, in particular, for a squeezed vacuum state describing particle creation—a case that is relevant, for example, for structure formation in the universe, see the remarks in Section 10.1.3. Another example is the Casimir effect. Kuo and Ford (1993) show that the gravitational field produced by the Casimir energy is *not* described by a fixed classical metric.

It will be discussed in Section 5.4 to what extent the semiclassical equations (1.34) can be derived as approximations from full quantum gravity. Modern developments in quantum mechanics discuss the possibility of a consistent formulation of 'hybrid dynamics', coupling a quantum to a classical system, see for example, Diósi *et al.* (2000). This leads to equations that generalize mean-field equations such as (1.34), although no one has applied this formalism to the gravitational case. It seems that such a coupling can be formulated consistently if the 'classical' system is, in fact, a decohered quantum system (Halliwell 1998). However, this already refers to an effective and not to a fundamental level of description. It seems that DeWitt is right, who wrote (DeWitt 1962):

[9] Analogous results hold for electrodynamics, with the current j_μ instead of $T_{\mu\nu}$.

It is shown in a quite general manner that the quantization of a given system implies also the quantization of any other system to which it can be coupled.

1.3 Approaches to quantum gravity

As we have seen in the last sections, there exist strong arguments supporting the idea that the gravitational field is of *quantum* nature at the fundamental level. The major task, then, is the construction of a consistent quantum theory of gravity that can be subject to experimental tests.

Can one get hints how to construct such a theory from observation? A direct probe of the Planck scale (1.6) in high-energy experiments would be illusory. In fact, an accelerator of current technology would have to be of the size of several thousand lightyears in order to probe the Planck energy $m_{\rm P}c^2 \approx 10^{19}$ GeV. However, we have seen in Section 1.2 that macroscopic effects of quantum gravity could in principle occur at lower energy scales, and we will encounter some other examples in the course of this book. Among these are effects of the full theory such as non-trivial applications of the superposition principle for the quantized gravitational field or the existence of discrete quantum states in black-hole physics or the early universe. But one might also be able to observe quantum-gravitational correction terms to established theories, such as correction terms to the functional Schrödinger equation in an external space–time, or effective terms violating the weak equivalence principle. Such effects could potentially be measured in the anisotropy spectrum of the cosmic microwave background radiation or in the forthcoming satellite tests of the equivalence principle such as the mission STEP.

One should also keep in mind that the final theory (which is not yet available) will make its own predictions, some perhaps in a totally unexpected direction. As Heisenberg recalls from a conversation with Einstein[10]:

> From a fundamental point of view it is totally wrong to aim at basing a theory only on observable quantities. For in reality it is just the other way around. Only the theory decides about what can be observed.

A really fundamental theory should have such a rigid structure that all phenomena in the low-energy regime, such as particle masses or coupling constants, can be predicted in an unique way. As there is no direct experimental hint yet, most work in quantum gravity focuses on the attempt to construct a mathematically and conceptually consistent (and appealing) framework.

There is, of course, no a priori given starting point in the methodological sense. In this context, Isham (1987) makes a distinction between a 'primary theory of quantum gravity' and a 'secondary theory'. In the primary approach, one starts with a given classical theory and applies heuristic quantization rules. This

[10]'Aber vom prinzipiellen Standpunkt aus ist es ganz falsch, eine Theorie nur auf beobachtbare Größen gründen zu wollen. Denn es ist ja in Wirklichkeit genau umgekehrt. Erst die Theorie entscheidet darüber, was man beobachten kann.' (Einstein according to Heisenberg (1979))

is the approach usually adopted, which was successful, for example, in QED. Often the starting point is general relativity, leading to 'quantum general relativity' or 'quantum geometrodynamics', but one could also start from another classical theory such as the Brans–Dicke theory. One usually distinguishes between canonical and covariant approaches. The former employ at the classical level a split of space–time into space and time, whereas the latter aim at preserving four-dimensional covariance at each step. They will be discussed in Chapters 5, 6, and Chapter 2, respectively. The main advantage of these approaches is that the starting point is given. The main disadvantage is that one does not arrive immediately at a unified theory of all interactions.

The opposite holds for a 'secondary theory'. One starts with a fundamental quantum framework of all interactions and tries to derive (quantum) general relativity in certain limiting situations, for example, through an energy expansion. The most important example here is string theory (see Chapter 9). The main advantage is that the fundamental quantum theory automatically yields a unification, a 'theory of everything'; cf. Weinberg (1993). The main disadvantage is that the starting point is entirely speculative. A short review of the main approaches to quantum gravity is given by Carlip (2001).

In this book, we shall mainly focus on quantum GR because it is closer to established theories and because it exhibits many general aspects clearer. In any case, even if quantum GR is superseded by a more fundamental theory such as string theory, it should be valid as an *effective theory* in some appropriate limit. The reason is that far away from the Planck scale, classical general relativity is the appropriate theory, which in turn must be the classical limit of an underlying quantum theory. Except perhaps close to the Planck scale itself, quantum GR should be a viable framework (such as QED, which is also supposed to be only an effective theory). It should also be emphasized that string theory automatically implements many of the methods used in the primary approach, such as quantization of constrained systems and covariant perturbation theory.

An important question in the heuristic quantization of a given classical theory is which of the classical structures should be subjected to the superposition principle and which should remain classical (or absolute, non-dynamical) structures. Isham (1994) distinguishes the following hierarchy of structures; see also Butterfield and Isham (1999),

Point set of events $\longrightarrow$ topological structure $\longrightarrow$ differentiable manifold $\longrightarrow$ causal structure $\longrightarrow$ Lorentzian structure.

Most approaches subject the Lorentzian and the causal structure to quantization, but keep the manifold structure fixed. This is, however, not clear. More general approaches include attempts to quantize topological structure, see for example, Isham (1989), or to quantize causal sets, see for example, Sorkin (2003) and the references therein. We shall not discuss such approaches in this volume. According to the Copenhagen interpretation of quantum theory, all structures related to space–time would probably have to stay classical because they are thought to be necessary ingredients for the measurement process, cf. Chapter 10. For the purpose of quantum gravity, such a viewpoint is, however, insufficient and probably inconsistent.

Historically, the first remark on the necessity to deal with quantum gravity was made by Einstein (1916*b*). This was, of course, in the framework of the 'old' quantum theory and does not yet reflect his critical attitude against quantum theory, which he adopted later. He writes[11]:

> In the same way the atoms would have to emit, because of the inneratomic electronic motion, not only electromagnetic, but also gravitational energy, although in tiny amounts. Since this does hardly hold true in nature, it seems that quantum theory will have to modify not only Maxwell's electrodynamics, but also the new theory of gravitation.

[11]'Gleichwohl müßten die Atome zufolge der inneratomischen Elektronenbewegung nicht nur elektromagnetische, sondern auch Gravitationsenergie ausstrahlen, wenn auch in winzigem Betrage. Da dies in Wahrheit in der Natur nicht zutreffen dürfte, so scheint es, daß die Quantentheorie nicht nur die Maxwellsche Elektrodynamik, sondern auch die neue Gravitationstheorie wird modifizieren müssen.'