Formalism for another source of interference with an arbitrary phase relative to $q\bar{q}$

The total amplitude is the sum of four amplitudes $A = A_c + A_{q\bar{q}} + A_{ggg} + A_{ggg\text{-noint}}$:

1. A_c: continuum $q\bar{q}$,
2. $A_{q\bar{q}}$: $\Upsilon \to q\bar{q}$,
3. A_{ggg}: $\Upsilon \to ggg$ decays which interfere with $q\bar{q}$ at the hadron level, and
4. $A_{ggg\text{-noint}}$: $\Upsilon \to ggg$ decays which do not interfere with $q\bar{q}$ at the hadron level.

The squared amplitude is

$$|A|^2 = |A_c|^2 + |A_{q\bar{q}}|^2 + |A_{ggg}|^2 + |A_{ggg\text{-noint}}|^2 + 2\mathcal{R}e({A_c}^* A_{q\bar{q}}) + 2\mathcal{R}e({A_c}^* A_{ggg}) + \\ + 2\mathcal{R}e({A_{q\bar{q}}}^* A_{ggg}) + {A_{ggg\text{-noint}}}^*(A_c + A_{q\bar{q}} + A_{ggg}) \quad (1)$$

The only two relevant terms are $2\mathcal{R}e({A_c}^* A_{q\bar{q}})$ and $2\mathcal{R}e({A_c}^* A_{ggg})$. The rest are non-interfering or are interference between two Υ final states, which is just part of the Υ cross-section (a part of the Γ_{ee} we wish to measure).

To use Karl's existing code, I need to make these two interference terms look like one. Karl's lineshape takes two inputs, y_{int} and ϕ to define interference with the background:

$$2\mathcal{R}e({A_c}^* A_{BW}) = y_{int}\sigma_{BW}\left(2\frac{W_{res} - M_\Upsilon}{\Gamma}\cos\phi + \sin\phi\right) \quad (2)$$

With only $q\bar{q}$ interference, y_{int} is proportional to $\sqrt{\mathcal{B}_{q\bar{q}}}$ and the amplitude of $\Upsilon \to q\bar{q}$, and ϕ is the phase difference between continuum $q\bar{q}$ and $\Upsilon \to q\bar{q}$ when $\sqrt{s} \ll M_\Upsilon$. My A_{BW} has two terms, $A_{q\bar{q}} + A_{ggg}$. I will parameterize them like this:

$$A_{q\bar{q}} = a_{q\bar{q}} e^{i\phi_r} \text{ and } A_{ggg} = a_{ggg} e^{i\phi_r + i\phi_{ggg}} \quad (3)$$

where ϕ_r is the resonance phase (which varies from 0 to π as a function of beam energy) and ϕ_{ggg} is a constant offset due to $\Upsilon \to q\bar{q}$ and $\Upsilon \to ggg$ not being coherent (ϕ_{ggg} might be π). Now

$$A_{q\bar{q}} + A_{ggg} = (a_{q\bar{q}} + a_{ggg} e^{i\phi_{ggg}}) e^{i\phi_r} = (a_{q\bar{q}} + a_{ggg}\cos\phi_{ggg} + i a_{ggg}\sin\phi_{ggg}) e^{i\phi_r} \quad (4)$$

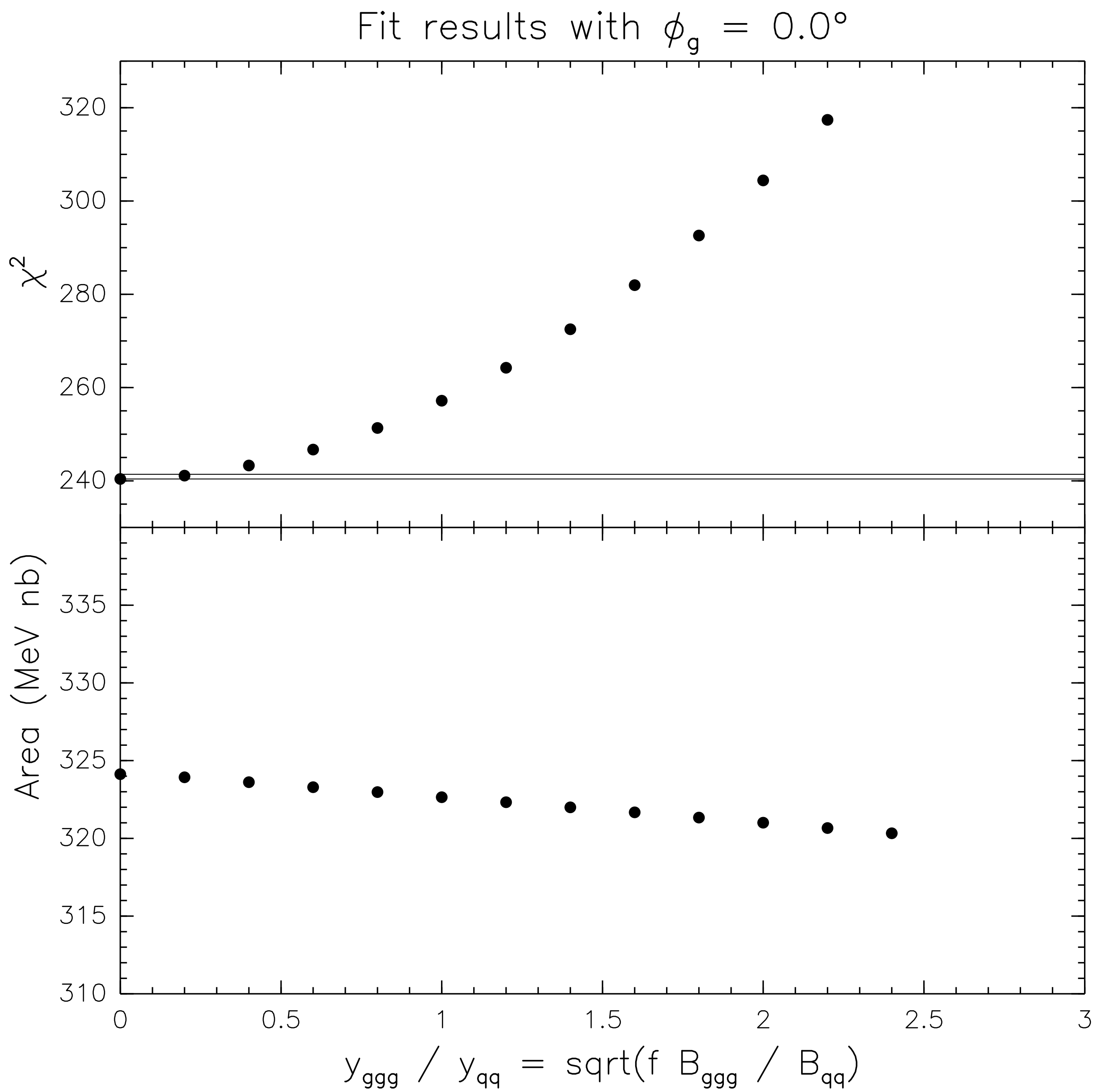
$$= \sqrt{{a_{q\bar{q}}}^2 + {a_{ggg}}^2 + 2a_{q\bar{q}}a_{ggg}\cos\phi_{ggg}}\,\exp\left(i\phi_r + i\tan^{-1}\left(\frac{a_{ggg}\sin\phi_{ggg}}{a_{q\bar{q}} + a_{ggg}\cos\phi_{ggg}}\right)\right) \quad (5)$$

To use Karl's function, I need to replace

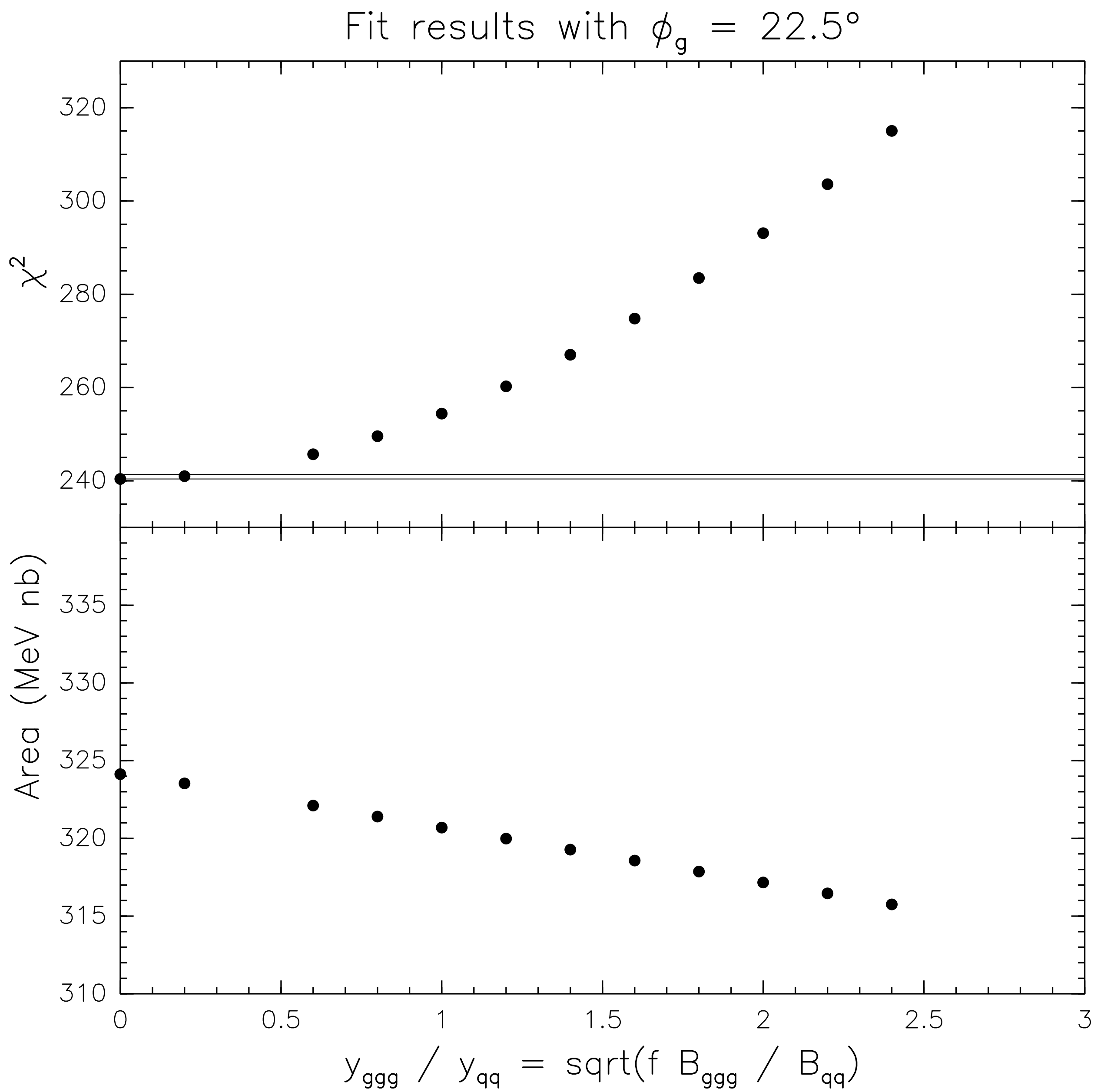
$$y_{int} \;\to\; \sqrt{{y_{q\bar{q}}}^2 + {y_{ggg}}^2 + 2y_{q\bar{q}}y_{ggg}\cos\phi_{ggg}} \text{ and} \quad (6)$$

$$\phi \;\to\; \phi_{q\bar{q}} + tan^{-1}\left(\frac{y_{ggg}\sin\phi_{ggg}}{y_{q\bar{q}} + y_{ggg}\cos\phi_{ggg}}\right) \quad (7)$$

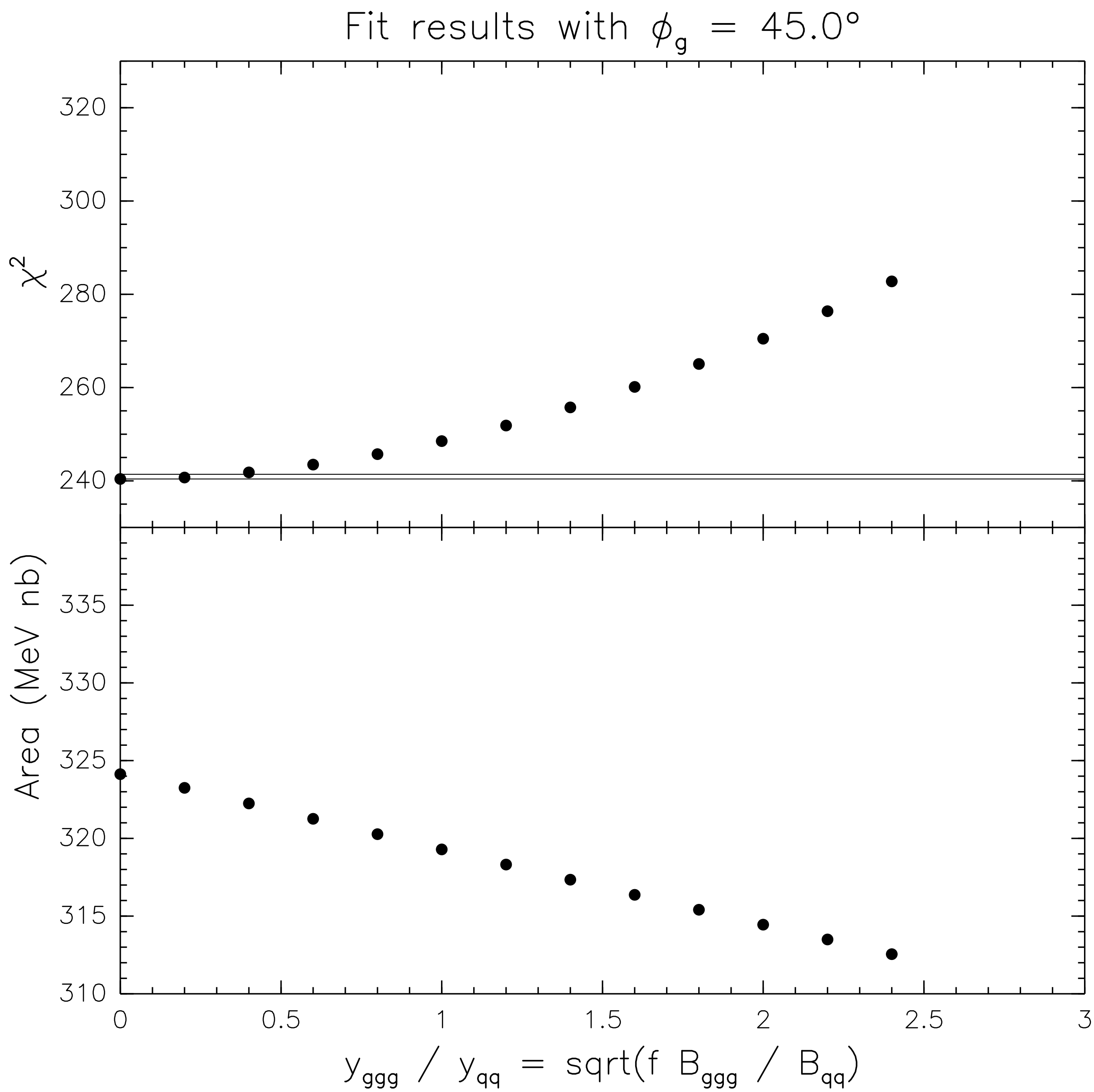
where $y_{q\bar{q}}$ is the usual interference parameter, y_{ggg} is the equivalent for the fraction of $\Upsilon \to ggg$ that interferes, and $\phi_{q\bar{q}}$ is the usual starting interference (assumed to be zero, as usual).

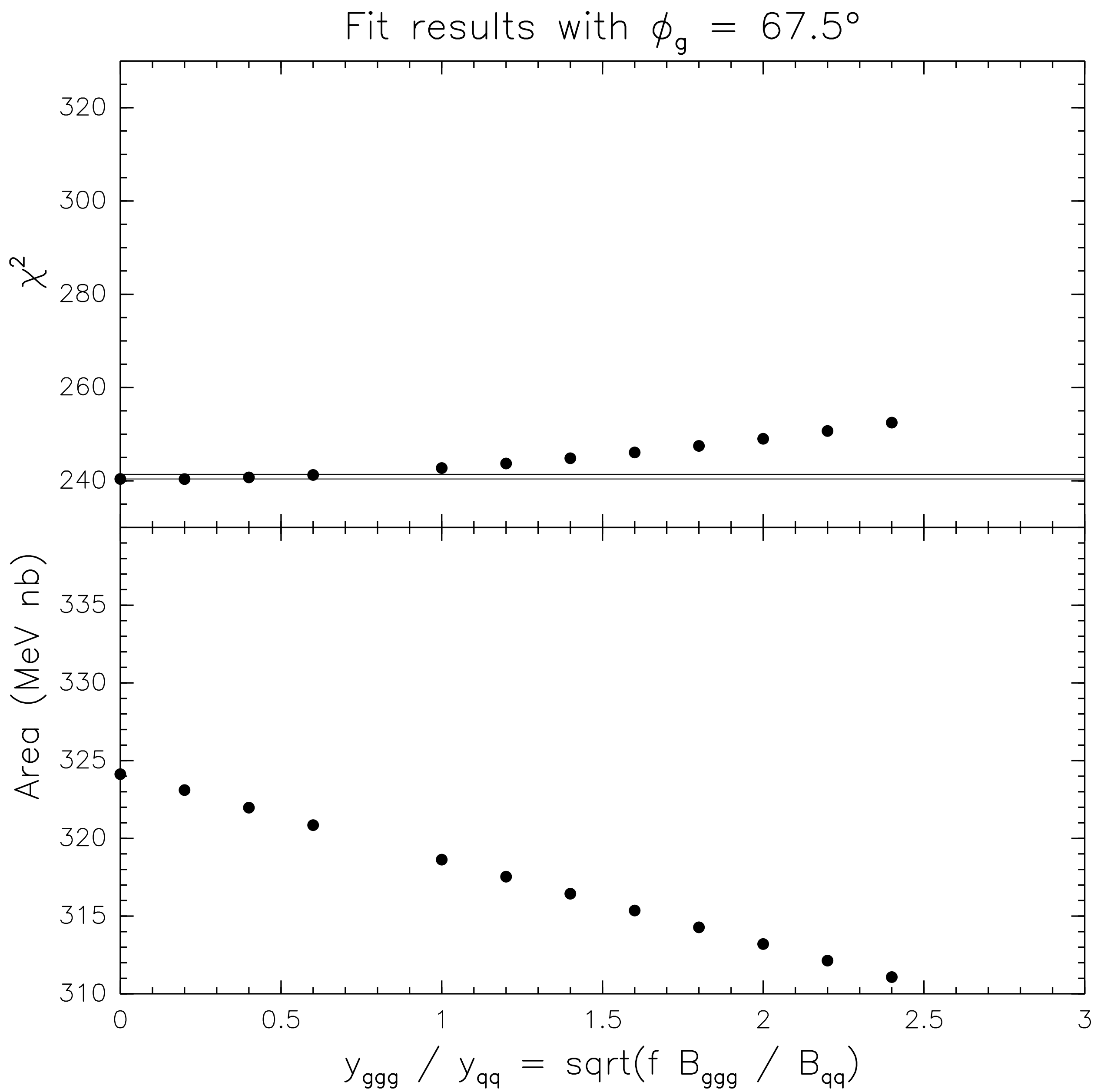


(The horizontal lines represent one unit in χ^2)

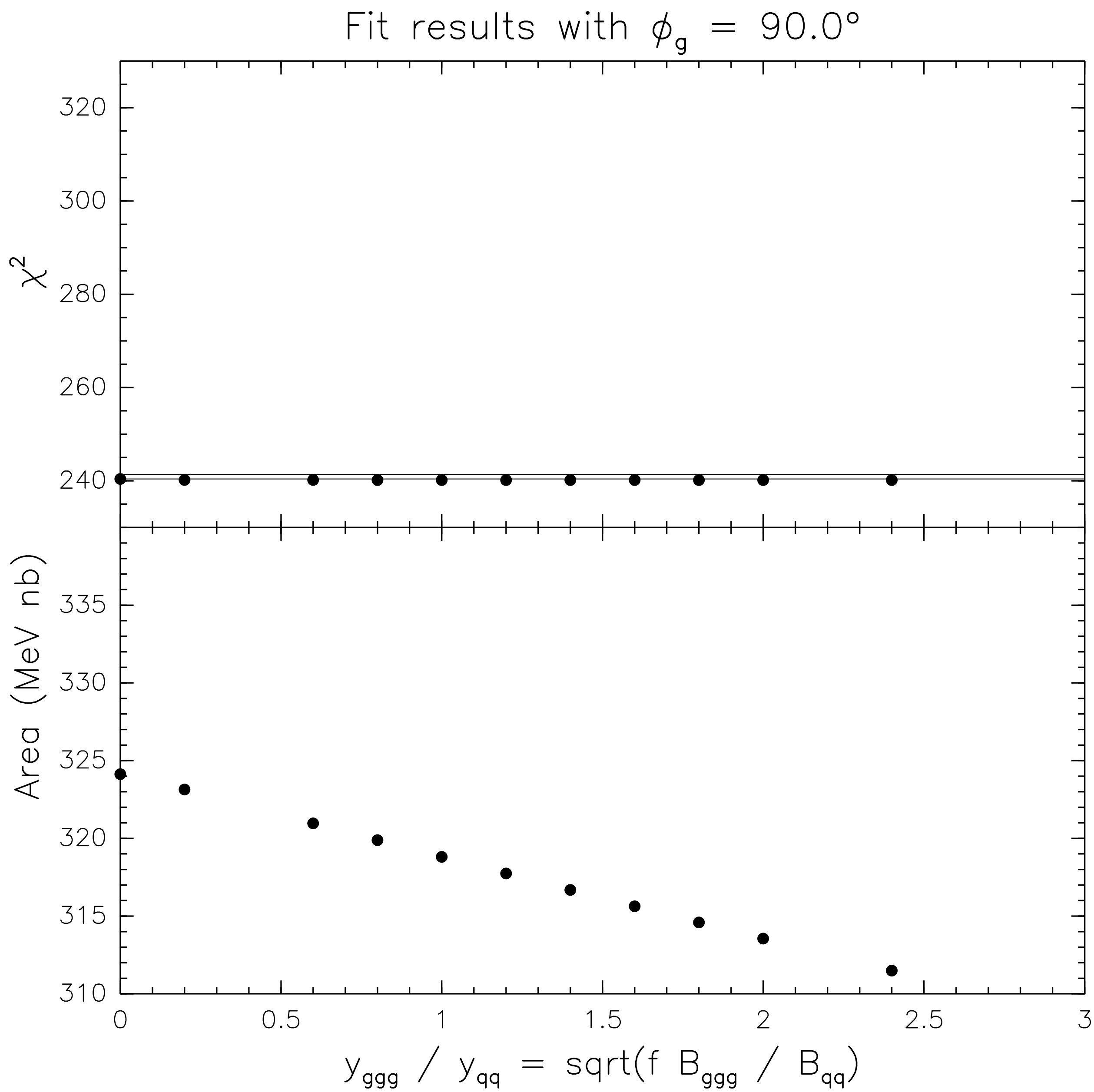


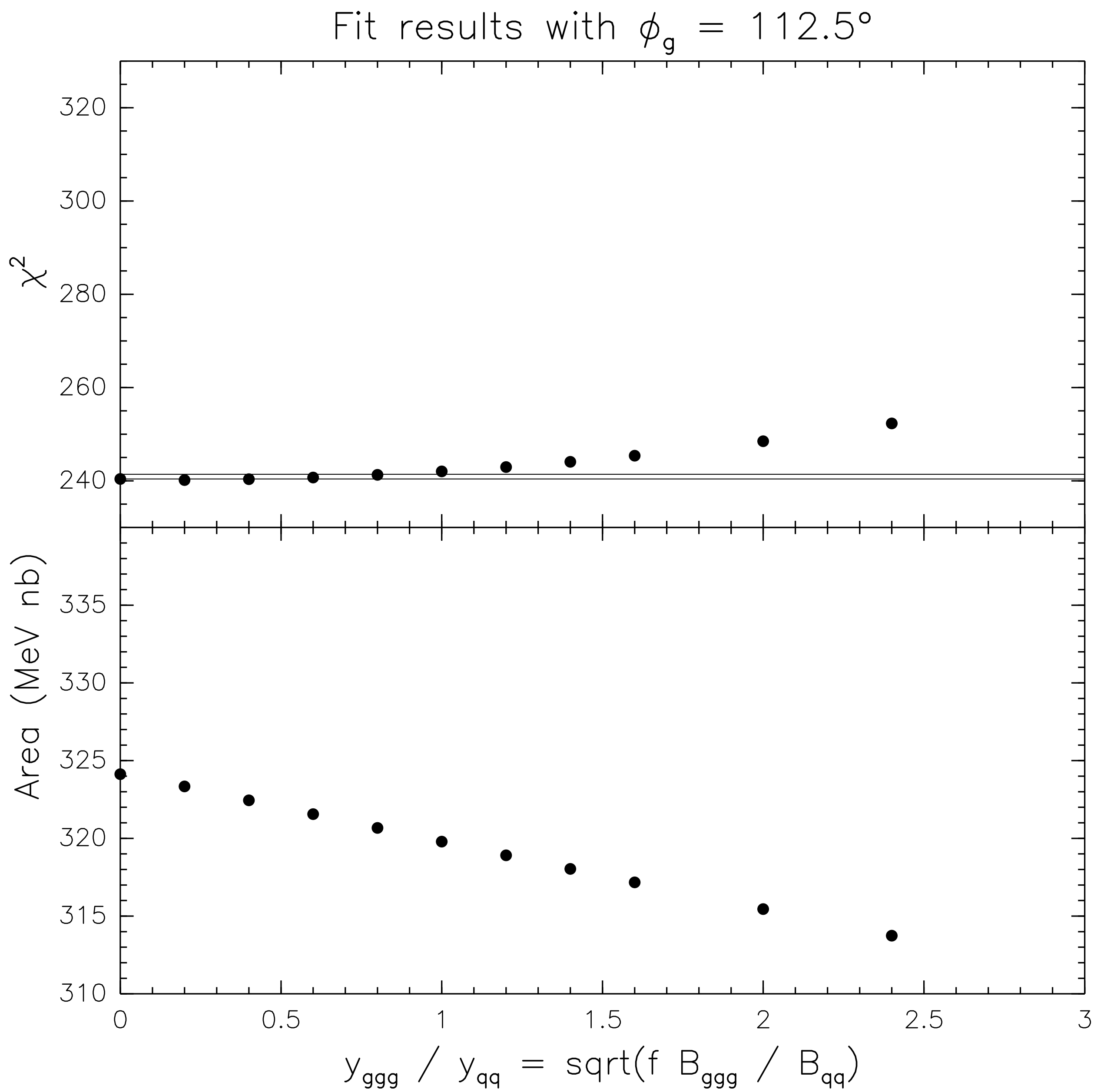
Fit results with ϕ_g = 22.5°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

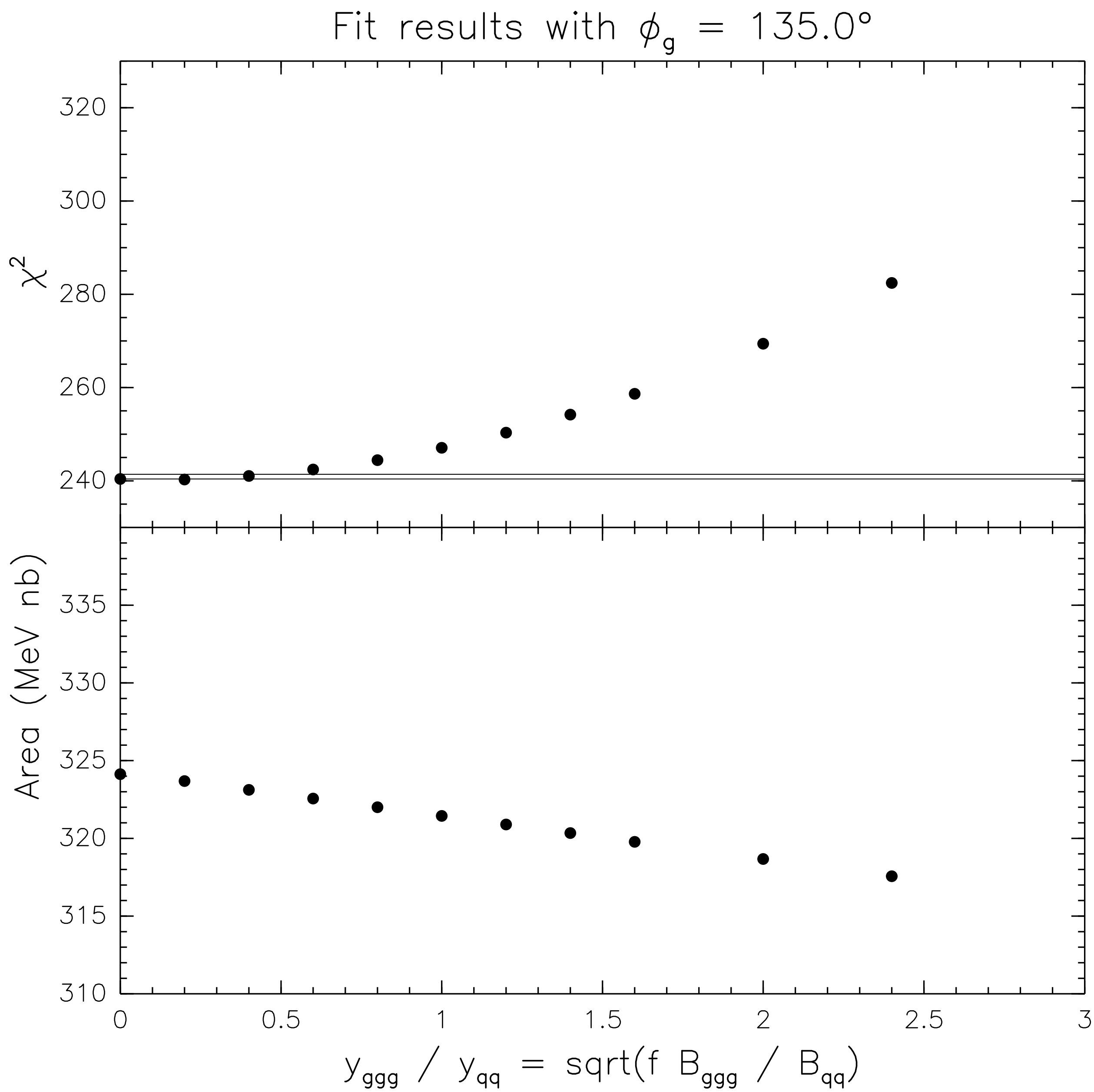

Fit results with ϕ_g = 45.0°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

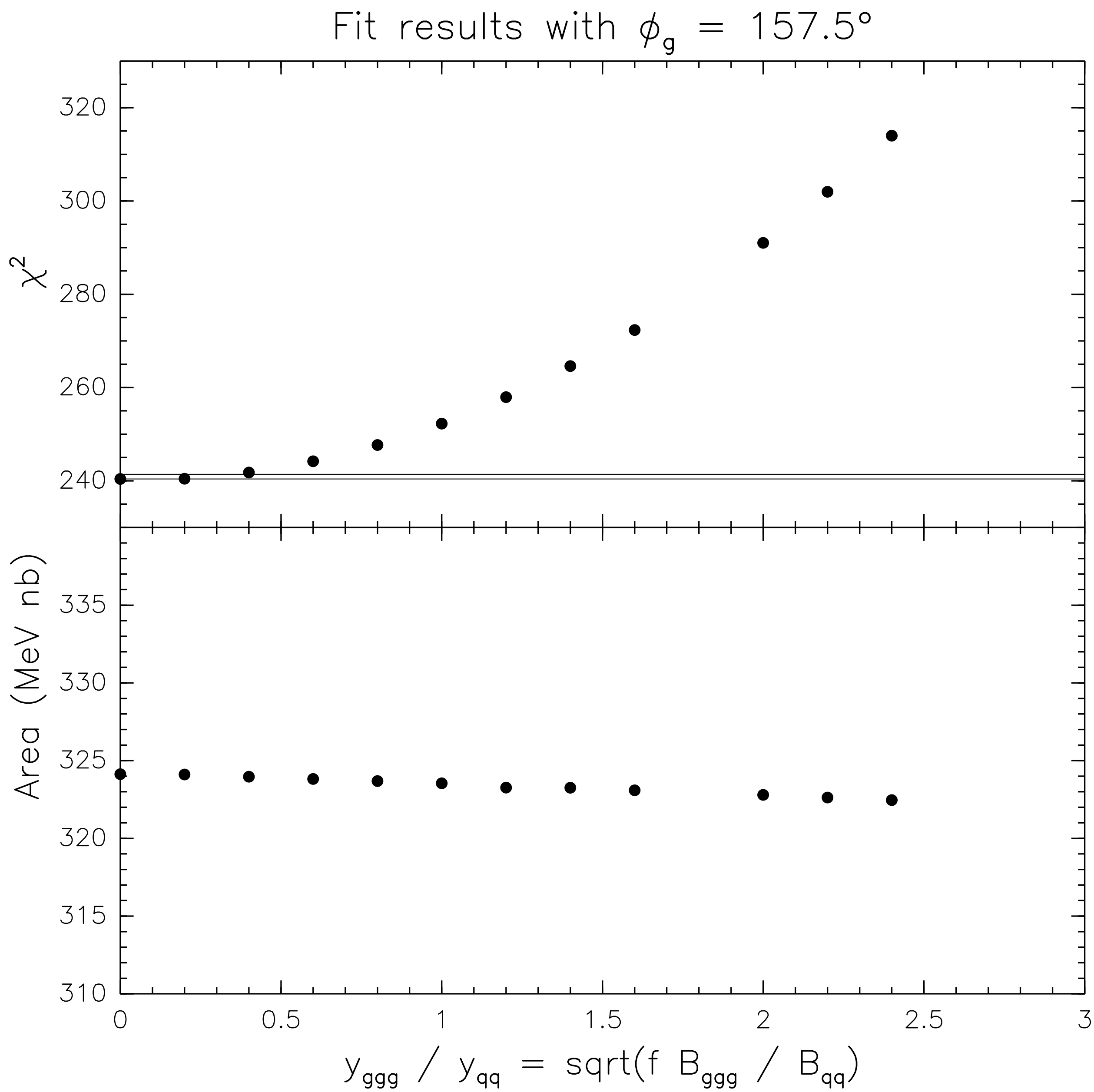


Fit results with $\phi_g = 67.5°$
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
$y_{ggg} / y_{qq} = \mathrm{sqrt}(f\ B_{ggg} / B_{qq})$

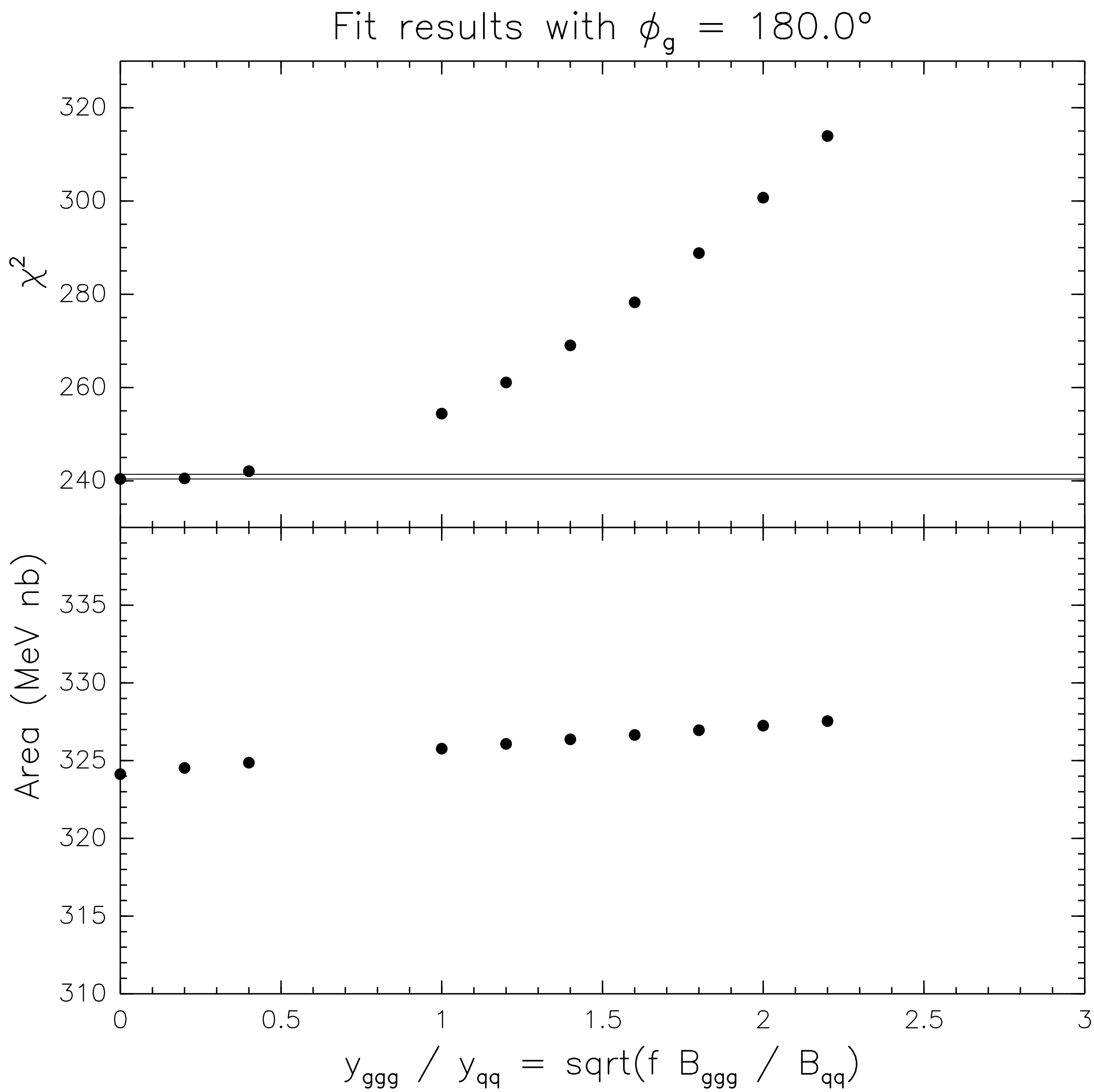

Fit results with ϕ_g = 90.0°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})


Fit results with ϕ_g = 112.5°
320
300
280
260
240
χ^2
335
330
325
320
315
310
Area (MeV nb)
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

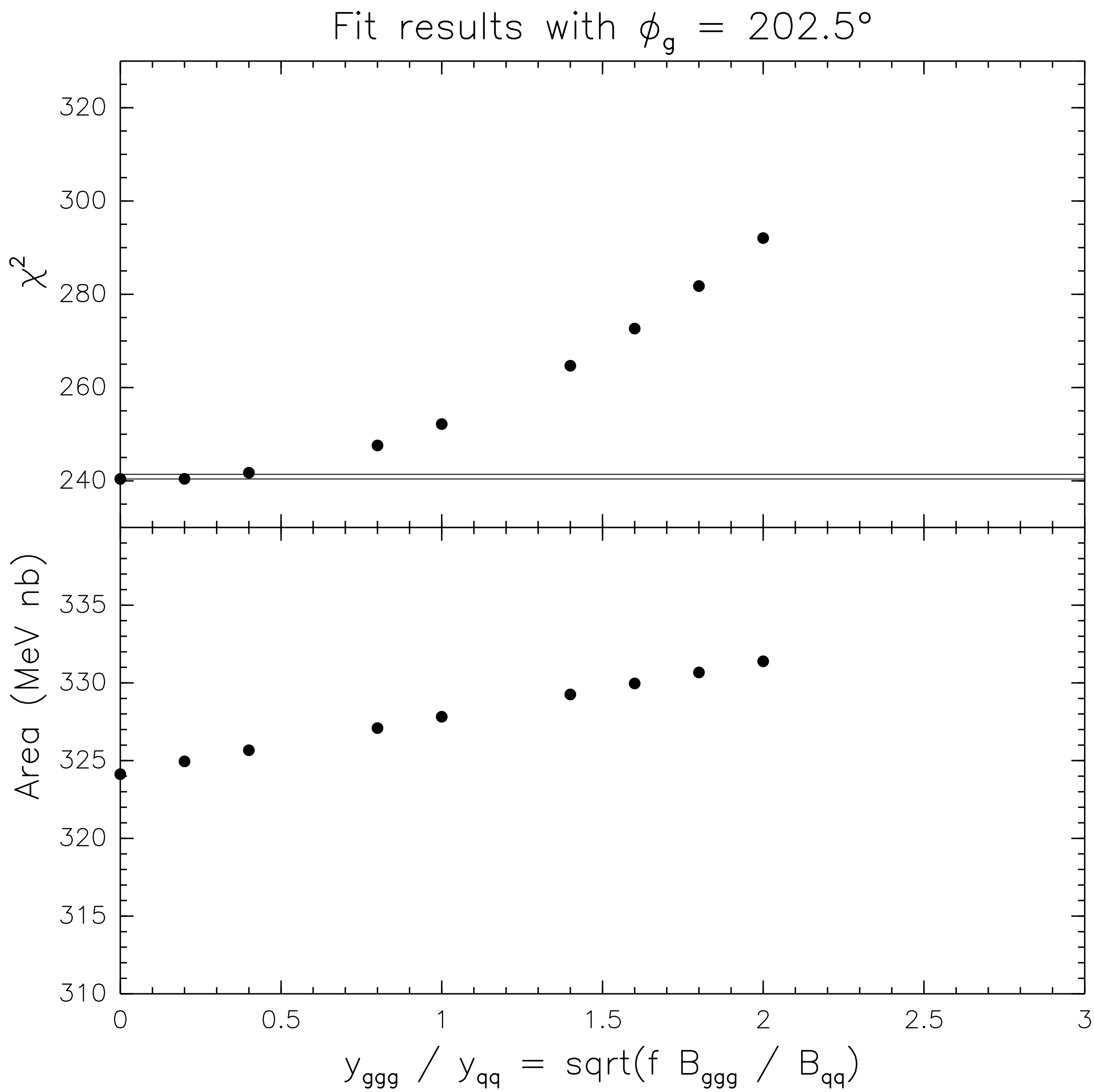

Fit results with ϕ_g = 135.0°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

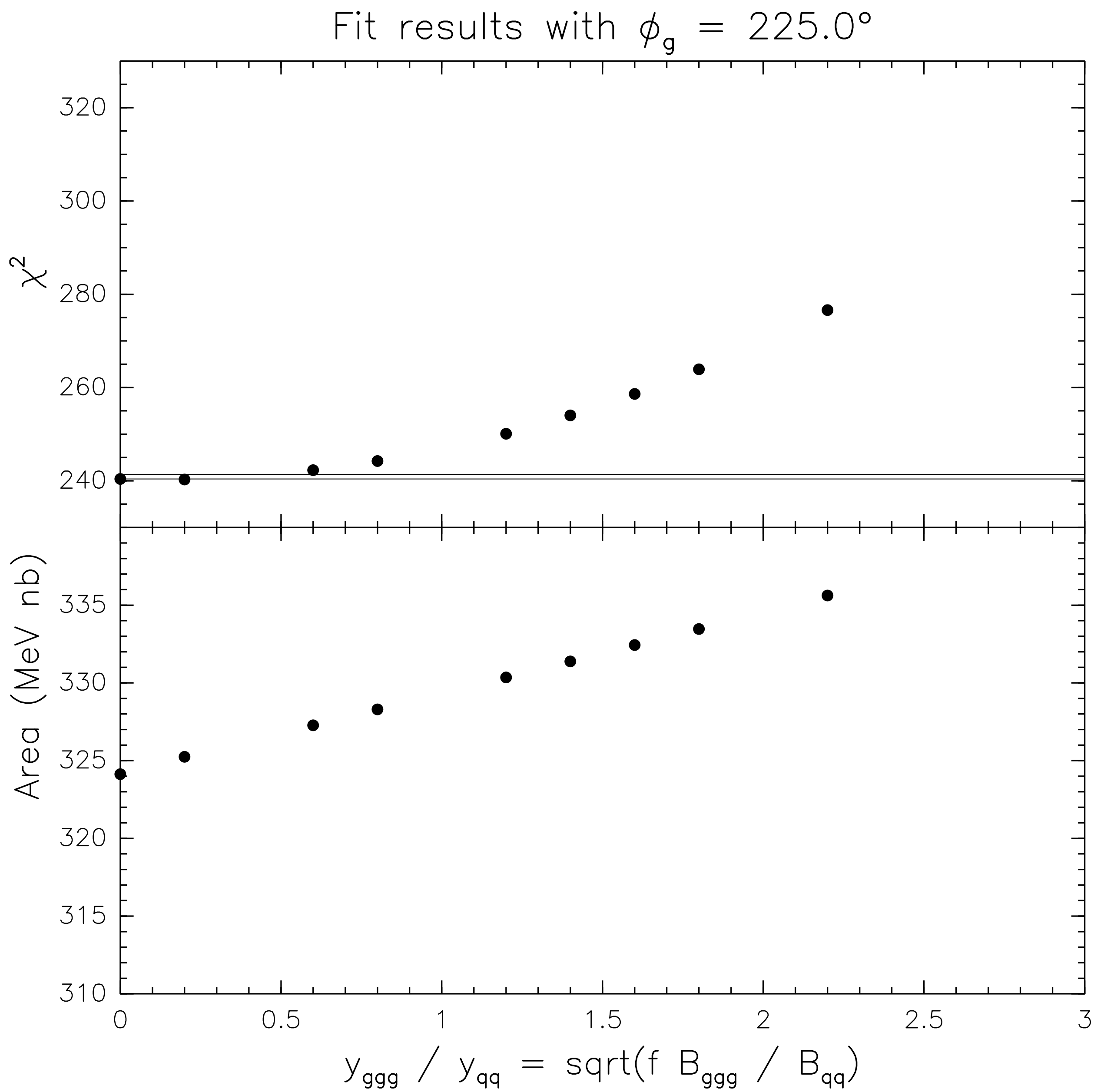


Fit results with ϕ_g = 157.5°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})



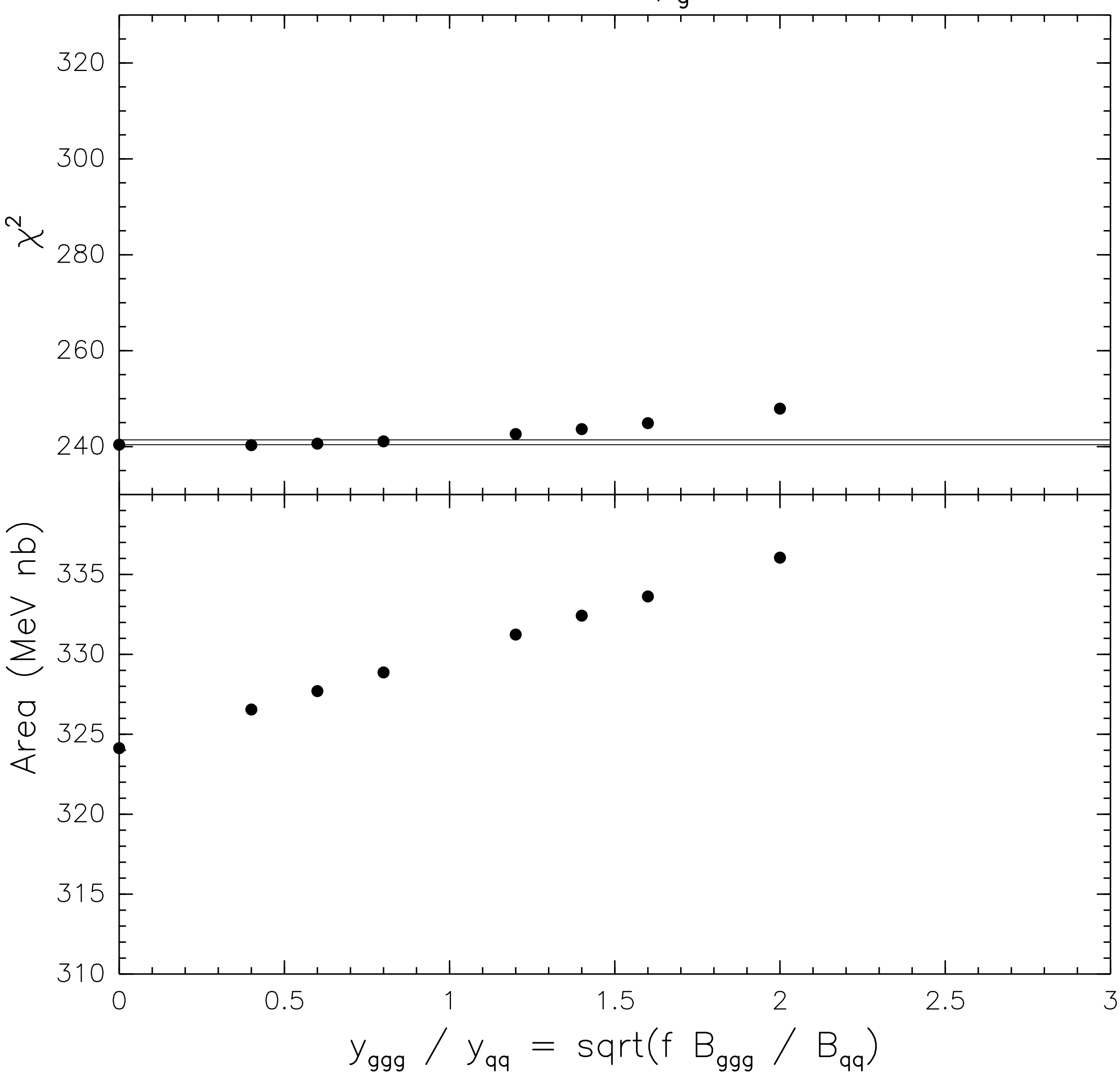
Fit results with ϕ_g = 180.0°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

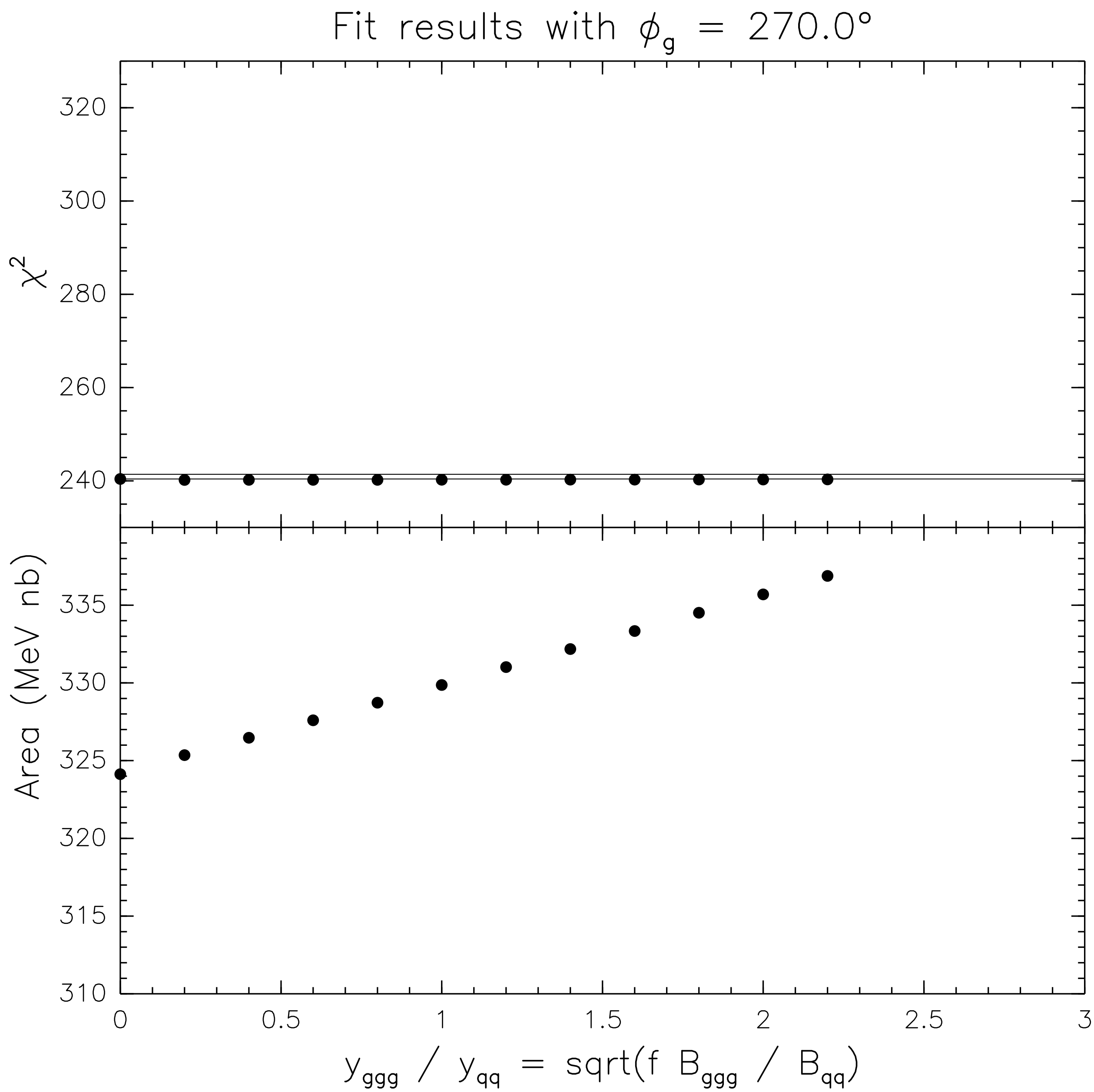

Fit results with ϕ_g = 202.5°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

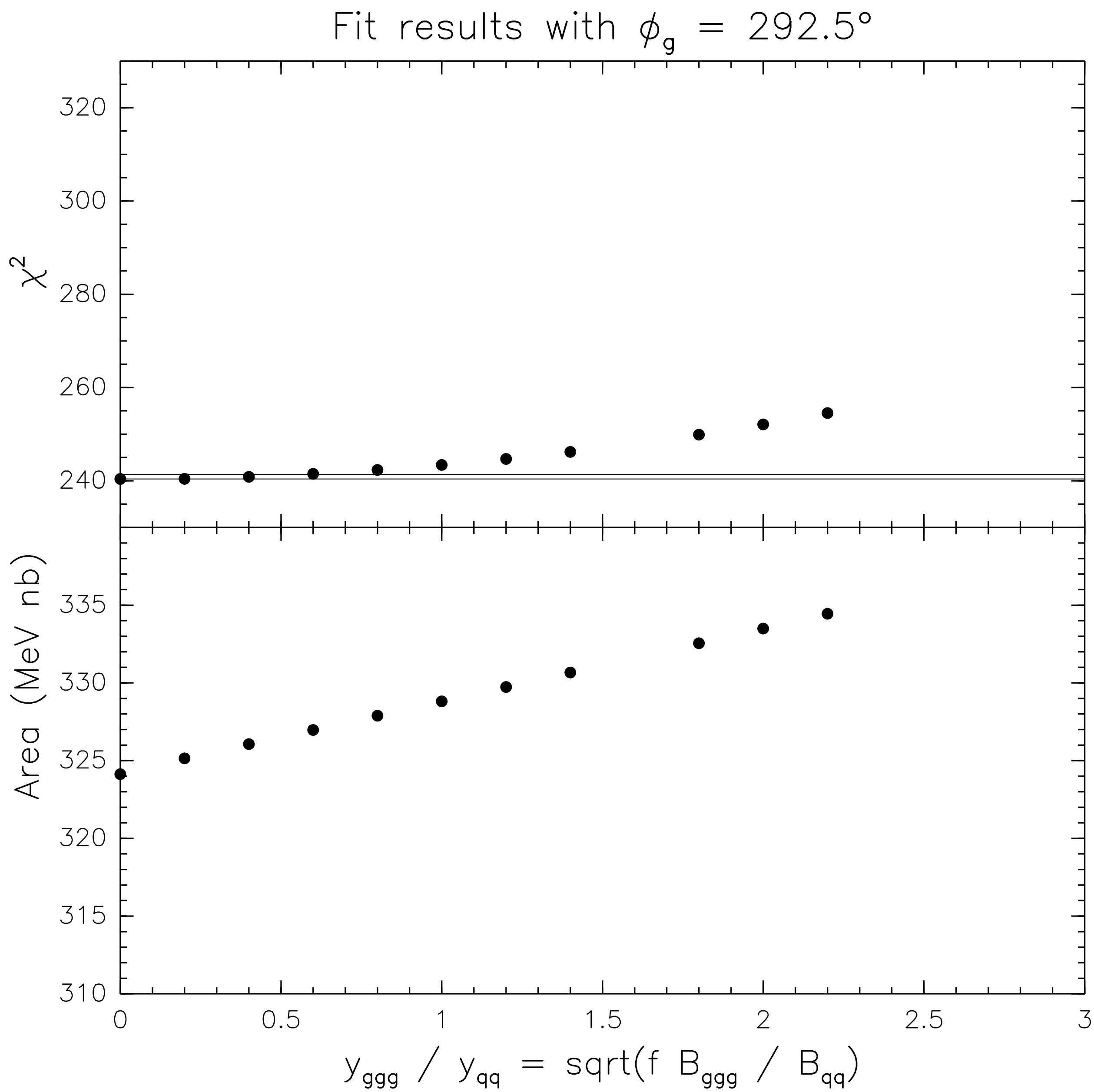


Fit results with ϕ_g = 225.0°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

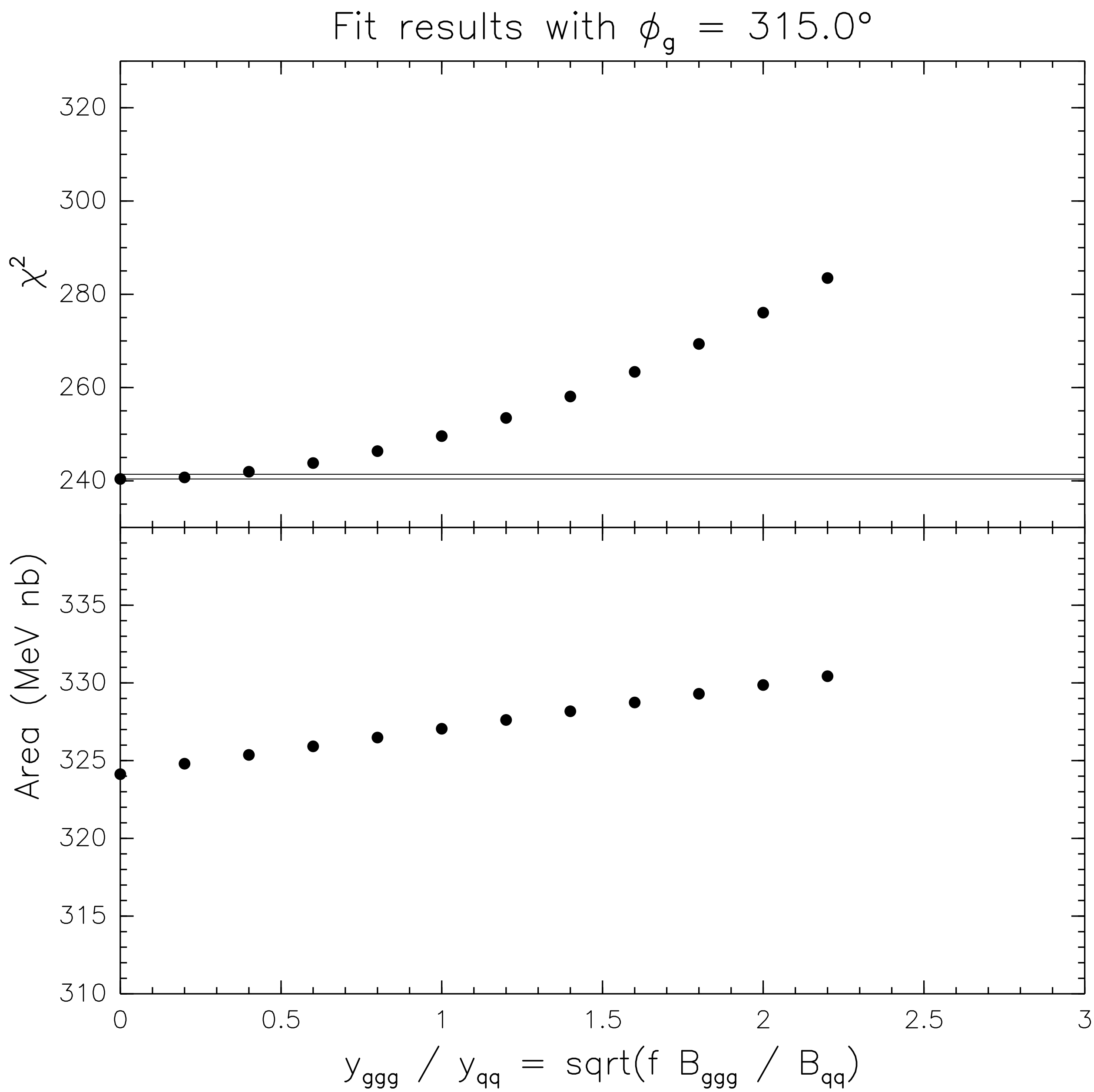
Fit results with $\phi_g = 247.5°$

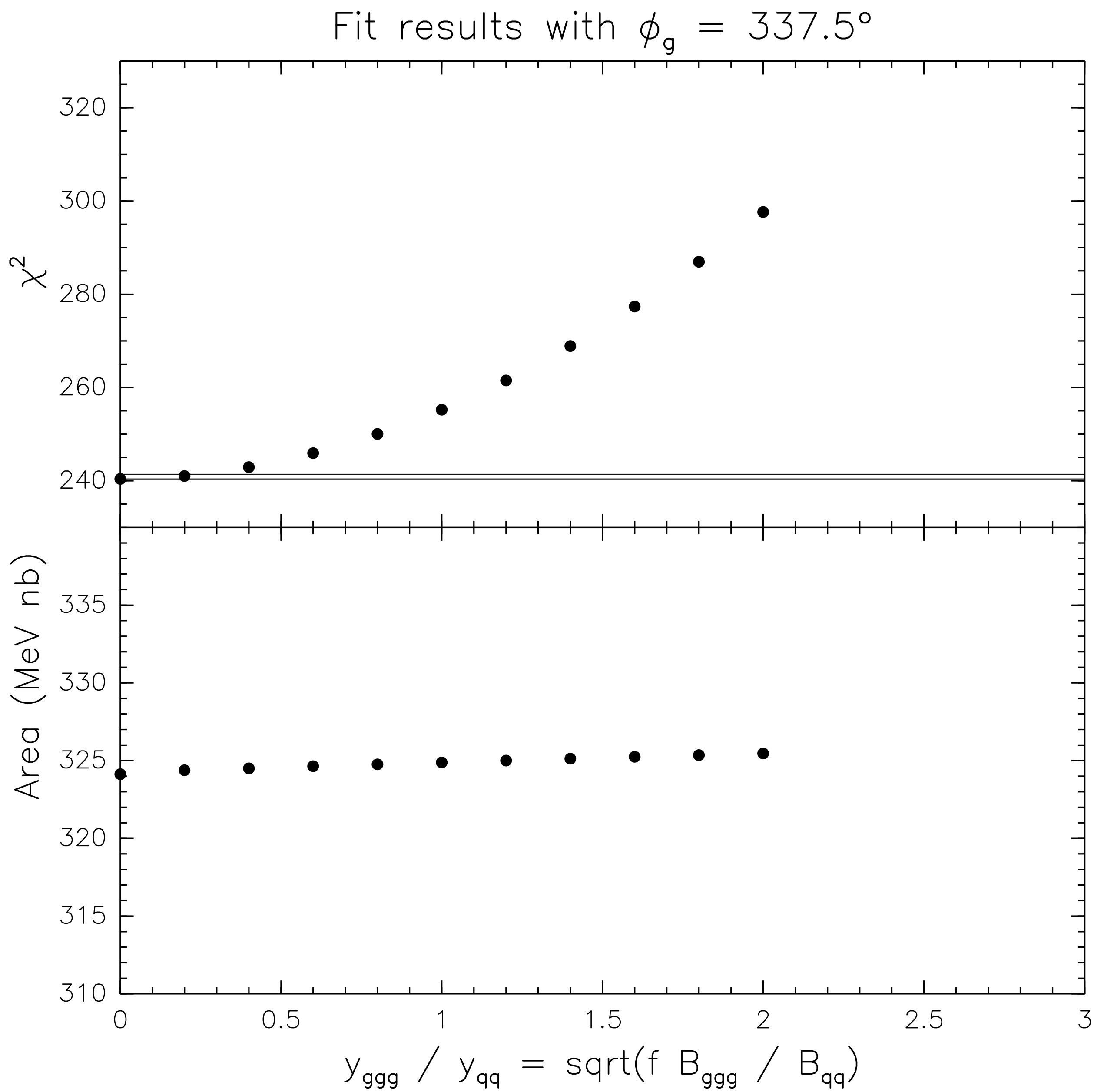



Fit results with ϕ_g = 270.0°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

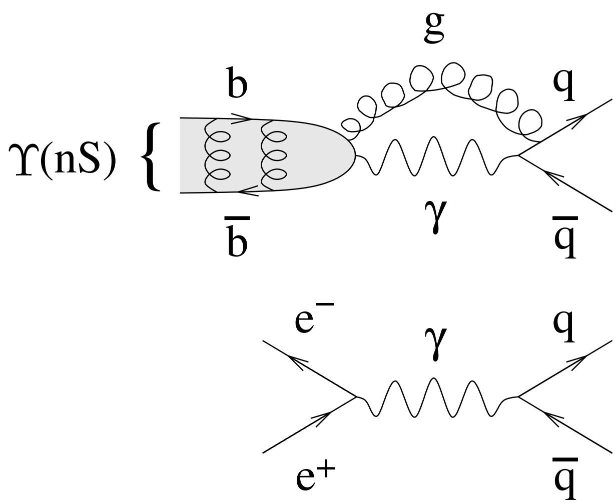


Fit results with ϕ_g = 292.5°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

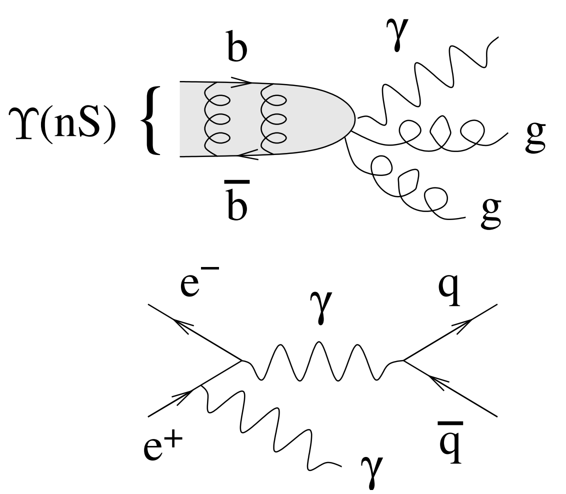

Fit results with ϕ_g = 315.0°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})


Fit results with ϕ_g = 337.5°
χ^2
320
300
280
260
240
Area (MeV nb)
335
330
325
320
315
310
0
0.5
1
1.5
2
2.5
3
y_{ggg} / y_{qq} = sqrt(f B_{ggg} / B_{qq})

Other Forms of Interference



We have assumed that the $\sqrt{s} \ll M_\Upsilon$ phase difference between continuum $q\bar{q}$ and $\Upsilon \to q\bar{q}$ is zero, as it is for $\mu^+\mu^-$. A gluon connecting $b\bar{b}$ with $q\bar{q}$ could spoil that. However, the final state quarks are light (how often are they $c\bar{c}$?), with $\sim$5 GeV of momentum, so α_s is 0.2. The top diagram is therefore of order 4% on a 10% decay mode.



This is a very small effect because the $gg\gamma$ mode has a branching fraction of some 3%.

Did I leave anything out? Are my arguments wrong?