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Heavy quark physics on the lattice

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I describe methods for dealing with b and c quarks within the lattice QCD approach and summarise recent results for phenomenologically important quantities.

1. INTRODUCTION

The previous speakers [1] have outlined the methodology of lattice QCD and some of the results. One of the main aims of this field is to calculate quantities for comparison to, or as predictions for, experiment. There is a long history of such calculations and the methods we use mean that there are systematic and statistical uncertainties associated with the results. These uncertainties have improved steadily over the years, helped particularly by recent approaches to the improvement of systematic errors from discretisation.

In many cases the largest remaining source of systematic error is that from the failure so far to include dynamical quarks in the vacuum with realistically low masses. We are now on the threshold of being able to overcome this source of uncertainty with new Teraflops computing power coming onstream in the next three years. This will mean an improvement of lattice systematic errors down to the level of a few percent (depending on the quantity calculated) and then lattice results can have a big impact on experiment. In particular I am concerned with calculations in heavy quark physics and there the determination of the sides of the CKM triangle will be dominated by theoretical uncertainties unless these are improved to a few percent.

Below I elaborate on heavy quark results from current lattice calculations. Most of these are still from the quenched approximation, but the methods being developed are directly applicable to configurations which include dynamical quarks once we have them.

2. HEAVY QUARKS ON THE LATTICE

Heavy (b or c) quarks present special challenges to lattice QCD because the quark mass in units of the lattice spacing, $m_Qa \geq 1$. (on typical lattices, $m_ba \approx 2-3, m_ca \approx 0.5-1$. This means that relativistic momenta $\vec{p} \approx m_Q$ are very distorted by the lattice discretisation and the use of a naive relativistic lattice action (Wilson or clover quarks) will give large errors that depend on m_Qa or $(m_Qa)^2$. However, one glance at the spectrum of bound states of b or c quarks shows that they are non-relativistic (the radial and orbital excitation energies are much smaller than the meson masses) and therefore it is possible to control these errors if we take a non-relativistic approach to the physics. Then b and c quarks can be treated accurately on the lattice and indeed heavy quarks physics is one of the most successful areas of lattice phenomenology.

There are several ways to proceed, each with its different proponents:

Static quarks. This is the $m_Q = \infty$ limit. Then the heavy quark is spinless and flavourless and the quark propagator becomes a string of gluon fields in the time direction [2]. This is a useful limit for comparison to continuum Heavy Quark Effective Theory.

NRQCD. This is a non-relativistic effective the-

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ory with a 2-component heavy quark field [3]

$$\mathcal{L}_Q = \overline{\psi}(D_t - \frac{\vec{D}^2}{2m_Q a} - c_4 \frac{\vec{\sigma}\cdot\vec{B}}{2m_Q a} + \ldots)\psi .$$

The c_i are fixed by matching to full continuum QCD. They represent the effect of states with momenta above the lattice cut-off, π/a, (missing from the lattice theory) and so this matching can be done in perturbation theory for large enough values of the lattice cut-off. It may also be done non-perturbatively. The value of the bare mass, $m_Q a$, is tuned non-perturbatively by requiring one hadron mass to be correct (the standard method for tuning lattice masses). The lattice spacing, a, cannot be taken to zero in this approach to get to the continuum limit; instead we must systematically improve the matching to full continuum QCD by adding higher order terms to remove discretisation errors. We can also improve by adding higher order terms in the non-relativistic expansion and, in general, these two improvements go together. As $m_Q a \to \infty$ we get the static limit.

Heavy Wilson Quarks (FNAL method). This approach uses the standard relativistic lattice quark action, but interprets the quark propagator in a non-relativistic way when fixing $m_Q a$, to remove a large part of the errors that this would otherwise give [4]

$$\mathcal{L}_Q = \overline{\psi}(\not{D} + m_Q a + \sigma_{\mu\nu}F^{\mu\nu})\psi ,$$

where the first two terms represent the lattice (Wilson) discretisation of the Dirac Lagrangian and the last term is the clover term. A systematic matching to full QCD is possible in this approach with $m_Q a$-dependent coefficients. In the small $m_Q a$ limit this approach yields standard light quark results. In the large $m_Q a$ limit it goes over essentially to NRQCD.

Wilson/clover quarks. This uses the standard clover light quark action (as above), improved to remove the leading discretisation errors. The residual errors are $\mathcal{O}(m_Q a)^2$ and may not be large for c quarks. HQET-inspired extrapolations to the b quark can be done provided that care is taken to untangle physical and unphysical (discretisation) dependence on the quark mass [5]. These extrapolations tend to give large errors. Improvements to this method may be possible using anisotropic lattice techniques. Then the lattice spacing in the temporal direction is much smaller than that in the spatial direction and $m_Q a_t$ can be small on manageable lattice volumes even for heavy quarks [7].

3. THE SPECTRUM

Extensive calculations of the heavyonium spectrum have been done with the NRQCD and heavy Wilson approaches and, more recently, with anisotropic clover. The static method gives the heavy quark potential which can then be solved within a potential model framework.

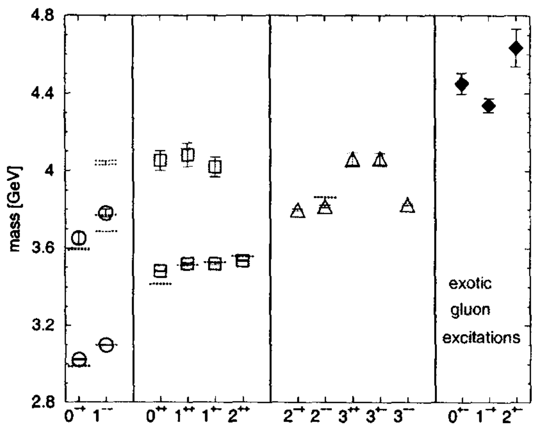

Figure 1. The charmonium spectrum from lattice QCD [7]. Experimental results are given by dotted lines.

Several of the radial and orbital excitation energies can be extracted if care is taken to use several different 'smearings' to excite different states from the vacuum. Figure 1 shows the charmonium spectrum obtained by the Columbia group using the anisotropic clover action in the quenched approximation. In addition to the standard S, P and D states they find a signal for 'hybrid' gluonic excitations, some with exotic quantum numbers. It is important to fix the mass of these more precisely in the presence of dynamical quarks so that they can be found in the next generation of experiments.

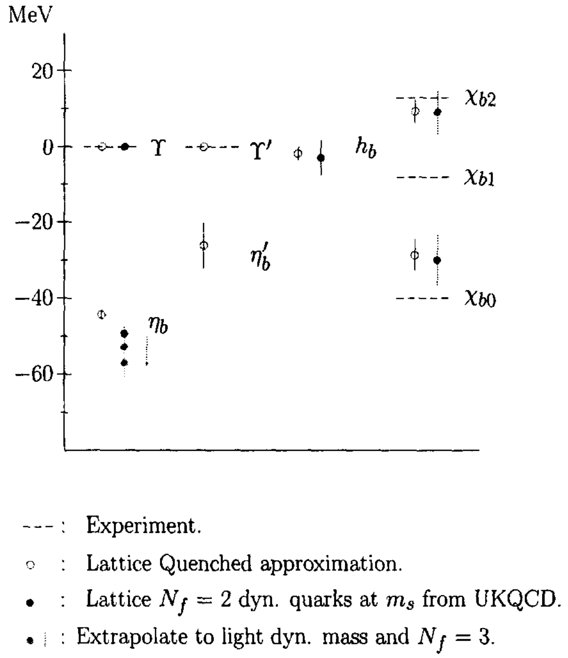

Figure 2. Fine structure in the bottomonium spectrum from lattice QCD [8].

Figure 2 focusses on the fine structure in the bottomonium spectrum using NRQCD for the b quarks and comparing results in the quenched approximation to those including two flavours of clover dynamical quarks with several masses down to a mass around the strange quark mass [8]. The hyperfine splitting, between the Υ and the η_b, is obtained quite precisely on the lattice and there is a clear increase on including dynamical quarks and as the dynamical quark mass decreases. If the results are extrapolated linearly in the dynamical quark mass to $m_s/3$ and the number of dynamical flavours is extrapolated linearly to 3, a hyperfine splitting of 60 ± 15 MeV is obtained. A much more precise result should be possible on configurations with 2+1 flavours of lighter dynamical quarks with improved determination of the c_i coefficients in the NRQCD Lagrangian. This will then provide a prediction for the mass of the η_b, yet to be seen experimentally.

The most complete determination of the spectrum of heavy-light bound states is based on using NRQCD for the b quark and the clover action for the light quark. Propagators for light quarks and heavy quarks are combined to make heavy-light mesons. Since it is not possible to generate light quark propagators with masses close to $m_{u,d}$ even in the quenched approximation, the results must be extrapolated to the chiral (light quark) limit to reach the B. Figure 3 shows the spectrum from ref. [9] in the quenched approximation. In common with results for the charmonium spectrum the hyperfine splitting, here between the B and the B^* mesons, is underestimated. This is believed to be, at least partially, a result of the quenched approximation. Another source of underestimation is the coefficient, c_4, which multiplies the coupling of the quark spin to the chromomagnetic field in a non-relativistic treatment of heavy quarks. More precise determinations of this coefficient are underway [10].

3.1. The b mass

Lattice QCD provides a non-perturbative method for fixing the bare quark mass in the lattice QCD Lagrangian. It can be adjusted until a particular hadron mass is correct. This bare mass can then be converted to any other mass desired, e.g. the quark mass in the $\overline{MS}$ scheme. In fact the best current determinations of the b quark mass use, rather than the bare mass, the binding energy in the static limit. Then

$$\overline{m}_b(\overline{m}_b) = Z_{cont}(m_B - E_B(\vec{p}=0) + E_0)\,.$$

E_0 is the lattice energy 'offset' and is known in the quenched approximation, in the static limit, to $\mathcal{O}(\alpha_s^3)$ [11]. Z_{cont} is the continuum renormalisation from M_{pole} to $\overline{m}_b$ in the $\overline{MS}$ scheme and is also known to $\mathcal{O}(\alpha_s^3)$ [12]. New non-perturbative methods are also being developed to fix E_0 [13]. The 'world average' for $\overline{m}_b(\overline{m}_b)$ is currently 4.30(10) GeV in the quenched approxi-

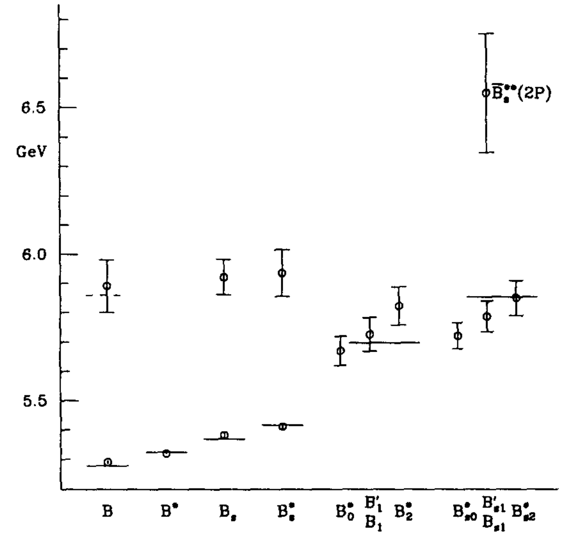

Figure 3. The spectrum of b-light mesons from lattice QCD [9].

mation [14].

4. MATRIX ELEMENTS

4.1. f_B

The simplest matrix element to calculate is that for the leptonic decay of the B, mediated by the heavy-light axial vector current. The decay rate can be determined experimentally in principle but is difficult in practice so a precise lattice determination is useful.

f_B is determined on the lattice from the matrix element of A_μ between the vacuum and a B meson

$$< 0|A_\mu|B >= p_\mu f_B \,.$$

We must match A_μ on the lattice to full continuum QCD. This has been done to $\mathcal{O}(\alpha_s, 1/m_Q, a)$ for NRQCD and heavy Wilson quarks, and these give currently the most precise results, in agreement with each other [15]. For light clover quarks the matching has been done non-perturbatively in the $ma \to 0$ limit and this can be used to obtain a result for f_D and extrapolate to f_B [16].

World averages quoted at the recent Lattice Conferences [14,17] are:

$$f_B^{(QA)} = 173 \pm 23 \text{ MeV}$$
$$f_{D_s}^{(QA)} = 203 \pm 14 \text{ MeV}$$
$$f_{B_s}/f_B = 1.15(5);\; f_{D_s}/f_D = 1.16(4)\,.$$

There is evidence that including dynamical quarks gives an increased value (by 20%) for f_B depending on how the overall scale is determined [17].

4.2. B_B

The matrix elements for the 4-quark operators of the effective weak Hamiltonian appropriate to $B^0 - \overline{B}^0$ mixing can be evaluated on the lattice. It is conventional to take the ratio of this to f_B^2 and call the answer B_B.

This is a somewhat harder calculation than f_B but the matching to the continuum has now been improved to the same level as that for f_B and the results from different methods are converging.

World averages from recent Lattice Conferences [14,17] are:

$$\hat{B}_{B_d} = 1.30(12)(13);\; f_{B_d}\sqrt{\hat{B}_{B_d}} = 230(40) \text{ MeV}.$$
$$\hat{B}_{B_s}/\hat{B}_{B_d} = 1.01(3);$$
$$\xi \equiv \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.16(5)\,.$$

4.3. B Semi-leptonic decay

B mesons decay semi-leptonically through the weak decay of the b quark to either a c quark or a u quark, and the decay of the virtual W particle to a lepton pair. The associated elements of the CKM matrix V_{cb} and V_{ub} can be determined by a comparison of the theoretical rate (proportional to the square of the unknown CKM element) with the experimental result. They are important inputs to constraints on the self-consistency of the Standard Model through the CKM triangle.

B decay to D or D^* mesons can usefully be discussed within a framework in which we consider both the b and the c quarks as heavy and determine corrections to the infinite mass limit in terms of inverse powers of m_b and m_c.

In the infinite mass limit, heavy quarks are spinless and flavorless, as discussed above. Then the form factors for decay B to D and B to D^* and elastic B to B scattering all become the same, when plotted against the variable $v \cdot v'$, where v is the initial meson and v' the final meson 4-velocity. This form factor is then known as the Isgur-Wise function and calculations on the lattice have been done using a variety of methods [18].

Of more direct use is a calculation of the form factor at the physical b and c masses in the zero-recoil limit (when $v_B \cdot v_D = 1$). There are a number of theoretical simplifications in this limit and the experimental results can be extrapolated to this point for the case of $B \to D^*$. A direct comparison of the two yields V_{cb}.

At this year's lattice conference there were new results from the FNAL group on the $B \to D^*$ form factor at zero recoil using the Heavy Wilson method [19]. After perturbative matching to the continuum they obtain a value for the form factor at zero recoil of $0.929(11)$ in the quenched approximation, with an error significantly smaller than the experimental errors on $V_{cb}\times$ this form factor. Unquenching affects the discrepancy of the theoretical result from unity and so is unlikely to change the result by more than one percent.

B semi-leptonic decay to light hadrons is harder to tackle on the lattice because most of the experimental rate occurs where the π or ρ meson has large momentum. When $\vec{p}a$ is large, there is the possibility of large discretisation errors. A number of groups have calculated the form factors for $B \to \pi$ decay but the results are still somewhat uncertain, depending markedly on how the extrapolation to physical light quark masses is done [14]. There is also some dispute about how well the soft pion theorem $f_0(q^2_{max}) = f_B/f_\pi$ is satisfied [17].

5. Conclusions

New calculations with much lighter dynamical quarks than before are on the horizon and this will result in significantly improved lattice results.

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