Measuring *Differences* in Momentum Scale as a Function of Track Parameters

Jim Pivarski

Texas A&M University

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Motivation

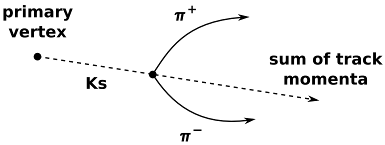
- Alignment optimizes tracks very well locally, but less so globally
  - any two regions that are rarely crossed by the same track can develop different momentum scales through weak modes
- Masses of J/ψ, Υ, and Z set the momentum scale, but with some complications
  - each daughter samples a different region of the tracker: needs to be untangled
  - shape is not symmetric due to final state radiation
  - backgrounds must be part of the fit
- In this talk, I'll present a procedure that can equalize momentum scales in all regions of the tracker, but not set the absolute scale
- How it fits into a complete track-correction program:

  alignment $\rightarrow$ this correction $\rightarrow$ absolute scale correction

- Also provides uncertainties and correlations in the remaining bias, which are important for setting systematic errors on physics quantities

The idea

- Alignment makes module positions and track parameters mutually consistent (optimizes track shapes)
- Overall momentum scale correction adjusts observed resonance peaks to externally known values
- Another physics constraint: momentum direction of daughters of a decay-in-flight must be consistent with the flight direction



  - sensitive to different momentum scales in different regions of the detector, but not absolute scale
  - symmetric in the case of no misalignment (unlike mass peak)
- $K_S \to \pi^+\pi^-$ for low momenta, perhaps $B \to$ "all charged" for higher?
  - boosted K_S are not sufficient because we need the daughters to sample different parts of the detector: need high mass

Untangling daughters (1/2)

- Same formalism as alignment (HIP for a single alignable):

$$(N \text{ track residuals}) = (6\times N \text{ matrix}) \cdot (\text{6-DOF parameters})$$

- N decays with angle difference $\Delta\vec{\phi}$, track-parameter corrections described by M parameters $\vec{p}$ ($N \gg M$):

$$\begin{pmatrix} \Delta\phi_1 \\ \vdots \\ \Delta\phi_N \end{pmatrix} = A \cdot \begin{pmatrix} p_1 \\ \vdots \\ p_M \end{pmatrix}$$

- A is the transformation from parameters $\vec{p}$ to observables $\Delta\vec{\phi}$ and is therefore the derivative $\frac{\partial(\Delta\phi_i)}{\partial p_j}$, which can be computed numerically:
  1. for each decay i, compute nominal $\Delta\phi_i$
  2. for each parameter j, apply ϵp_j to all tracks for $\Delta\phi_i(\epsilon p_j)$
  3. $A_{ij} = \dfrac{\Delta\phi_i - \Delta\phi_i(\epsilon p_j)}{\epsilon}$

- The values of $\vec{p}$ which minimize quadratic χ^2 (assumes Gaussian $\Delta\vec{\phi}$)

$$\chi^2 = \left(\Delta\vec{\phi} - A \cdot \vec{p}\right)^T \left({\sigma_{\Delta\phi}}^2\right)^{-1} \left(\Delta\vec{\phi} - A \cdot \vec{p}\right)$$

are $\vec{p} = \left(A^T \left({\sigma_{\Delta\phi}}^2\right)^{-1} A\right)^{-1} \left(A^T \left({\sigma_{\Delta\phi}}^2\right)^{-1} \Delta\vec{\phi}\right) \quad \sim \frac{\sum w_i x_i}{\sum w_i}$

with uncertainty/covariance $\left(A^T \left({\sigma_{\Delta\phi}}^2\right)^{-1} A\right)^{-1} \quad \sim \frac{1}{\sum w_i}$

  - "convert $\Delta\vec{\phi}$ into $\vec{p}$ space and compute weighted mean"
  - the ${\sigma_{\Delta\phi}}^2$ weights are propagated uncertainties in $\Delta\vec{\phi}$

- The final $\vec{p}$ correct biases in tracking, but perhaps more importantly, the covariance provides a rigorous systematic error in tracking

- Potential early application: $\Xi^\pm$ mass measurement; tracking bias is the only systematic error

Parameterization of $\vec{p}$

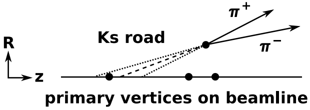
- Biases are small and slowly varying (largest alignment weak modes are global distortions)
- Expand general $\vec{p}(\phi, \kappa, \cot\theta, d_{xy}, d_z)$ function and keep low-order
  - Fourier-expand in ϕ: $\sin\phi$, $\cos\phi$, $\sin 2\phi$, $\cos 2\phi$
  - Taylor-expand in other parameters: κ, $\kappa\cot\theta$, $(\cot\theta)^2$
  - κ and d_{xy} are not sampled near zero due to physics signature: exclude κ^2 and ${d_{xy}}^2$ terms
- 33 terms for $\delta\kappa$

$$\begin{aligned}\delta\kappa = {} & p_1 + p_2\sin(\phi) + p_3\cos(\phi) + p_4\sin(2\phi) + p_5\cos(2\phi) + \\ & p_6\kappa + p_7\cot\theta + p_8 d_{xy} + p_9 d_z + p_{10}\sin(\phi)\kappa + \\ & p_{11}\sin(\phi)\cot\theta + p_{12}\sin(\phi)d_{xy} + p_{13}\sin(\phi)d_z + p_{14}\cos(\phi)\kappa + p_{15}\cos(\phi)\cot\theta + \\ & p_{16}\cos(\phi)d_{xy} + p_{17}\cos(\phi)d_z + p_{18}\sin(2\phi)\kappa + p_{19}\sin(2\phi)\cot\theta + p_{20}\sin(2\phi)d_{xy} + \\ & p_{21}\sin(2\phi)d_z + p_{22}\cos(2\phi)\kappa + p_{23}\cos(2\phi)\cot\theta + p_{24}\cos(2\phi)d_{xy} + p_{25}\cos(2\phi)d_z + \\ & p_{26}\kappa\cot\theta + p_{27}\kappa d_{xy} + p_{28}\kappa d_z + p_{29}\cot\theta\cot\theta + \\ & p_{30}\cot\theta d_{xy} + p_{31}\cot\theta d_z + p_{32}d_{xy}d_z + p_{33}d_z d_z\end{aligned}$$

- Same for $\delta\phi$ and δd_{xy}: 99 parameters, 99×99 matrix inversion
- $\delta\cot\theta$ and δd_z cannot be optimized; used to select primary vertex

$K_S \to \pi^+\pi^-$ selection

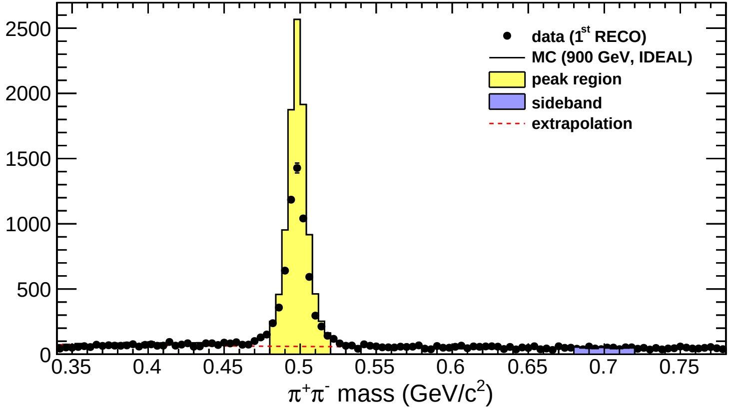
- Identify $K_S \to \pi^+\pi^-$ by $\pi^+\pi^-$ invariant mass (it therefore cannot be used in the optimization)
- Use sideband for a background control sample; sideband events enter the minimization with negative weight, cancelling the effect of backgrounds under mass peak
- Identify the correct primary vertex by propagating K_S to beamline in $r - z$ plane ($\delta \cot\theta$ and δd_z cannot be optimized)



- Require exactly one primary vertex in the road from K_S
- Require K_S vertex to be within $1.5 < |v_{xy}| < 3$ cm; away from beamspot and within first pixel layer
- From $K_S \to \pi^+\pi^-$, typical curvatures are $0.5 \lesssim |\kappa| \lesssim 5$ GeV^{-1}

Feasibility study (1/6)

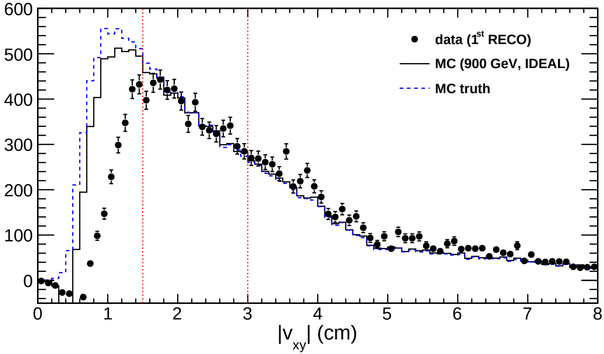
- Quick check to make sure that there aren't any show-stoppers
- MC and 1st RECO data (non-optimal tracking: large APEs)



- Upper sideband only: there can be signal events below peak due to final state radiation

Feasibility study (2/6)

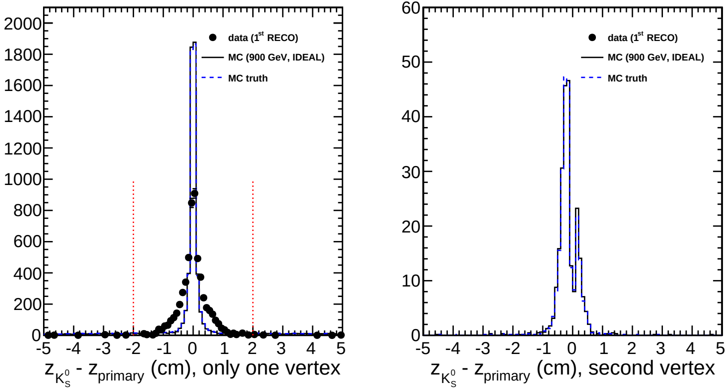
- All plots from this point onward are background-subtracted and normalized within cut windows
- "MC truth" is matched to true K_S in MC (no sideband subtraction)



- Could loosen track flight significance and lower $|v_{xy}|$ boundary
- Upper $|v_{xy}|$ boundary must be within first pixel layer so that resolution is smooth as a function of track d_{xy}

Feasibility study (3/6)

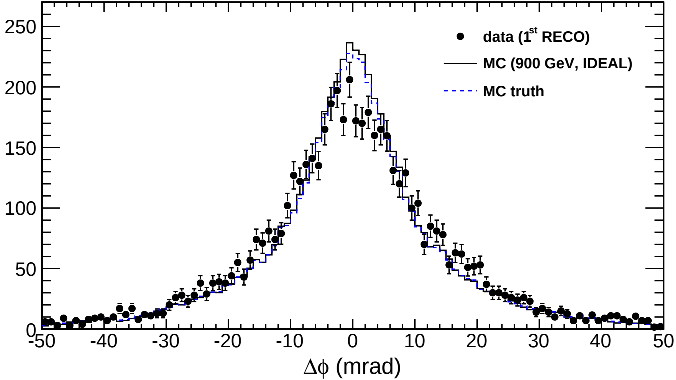
- Propagation of K_S to beamline in two cases: when there is only one primary vertex (left), and when there is also a second (right)



- In MC, 3.4% of events have a second primary vertex, in data, several do, but well beyond 5 cm (negligible pile-up plus beam-gas?)
- Efficiency of "only one vertex" requirement is dependent on luminosity

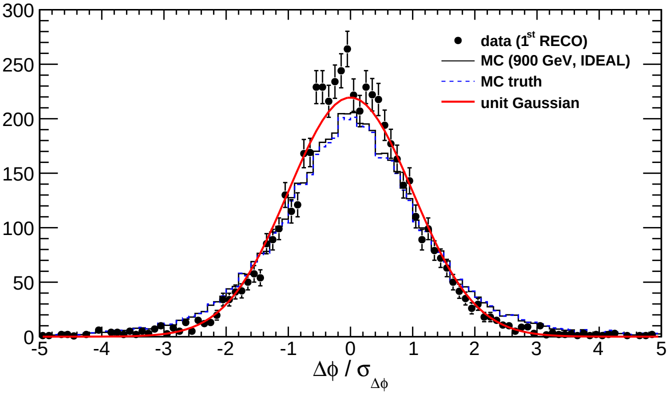
Feasibility study (4/6)

- Distribution of all $\Delta\phi$ is not Gaussian: is quadratic χ^2 the right estimator? Does the minimization need to be made non-linear (replace 99×99 matrix inversion with Minuit)?



- Some indication of $\Delta\phi$ variation in data? Remember, these are unrealistic APEs

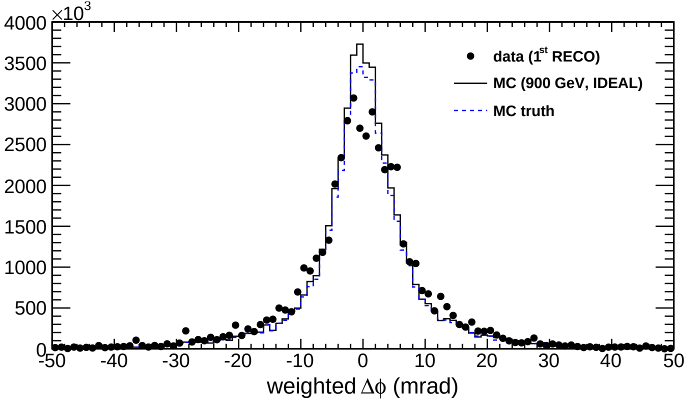
- Calculation of $\sigma_{\Delta\phi}{}^2$ weights must include all correlations between two-track intersection and momentum sum, starting from 5×5 track parameter covariances



- Uncertainty is slightly underestimated in MC, larger in data

Feasibility study (6/6)

- $\Delta\phi$ histogram weighted by $1/{\sigma_{\Delta\phi}}^2$



- Much narrower, indicating that the largest outliers have the largest uncertainties
- Data are still broader than MC (and distribution still not Gaussian)

Conclusions

- Proposal: multi-step track corrections
  1. alignment corrects track shapes, but can allow relative biases in regions of the tracker that are not connected by cosmic rays or collisions (assuming no resonance constraints)
  2. parameterize track parameter space and apply regional corrections to make momentum scale uniform, and also acquire matrix of uncertainties
  3. apply overall correction to set momentum scale with resonance masses, also with uncertainties
- Even if no track parameter biases are observed, uncertainty in bias is an important systematic for physics analyses
  - early analysis: CMS can improve $\Xi^{\pm}$ mass measurement in an analysis with only one systematic uncertainty— tracking
- Demonstrated selection and $\Delta\phi$, ${\sigma_{\Delta\phi}}^2$ calculation with $K_S \to \pi^+\pi^-$
  - next steps would be to calculate $\vec{p}$ and its correlations; is it consistent with zero in MC? How much uncertainty per $\sqrt{N}$?
- For most analyses, a higher-mass metastable particle would be more relevant: $B \to$ all charged hadrons?