What is a neural network?

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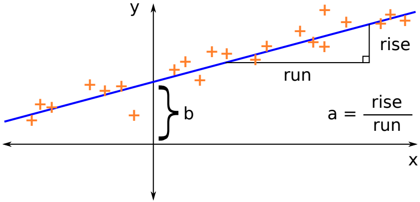
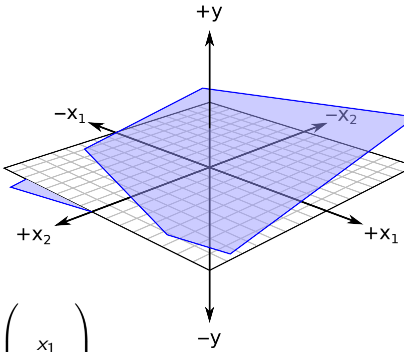
Equation of a line (fitting *a* and *b* to measurements *y* versus *x*)



$$\underbrace{a}_{\text{free parameters in the fit}} \cdot \underbrace{x}_{\text{input values}} + \underbrace{b}_{\text{free parameters}} = \underbrace{y}_{\text{output values}} = ax + b$$

Equation of a plane (height y versus 2D coordinates x_1 and x_2)



$$\underbrace{\begin{pmatrix} a_1 & a_2 & & \end{pmatrix}}_{\text{free parameters in the fit}} \cdot \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \\ \end{pmatrix}}_{\text{input values}} + \underbrace{b}_{\text{free parameters}} = \underbrace{y}_{\text{output values}} = a_1 x_1 + a_2 x_2 + b$$

Equation of a hyperplane (N-dimensional)

$$\underbrace{\begin{pmatrix} a_1 & a_2 & \dots & a_{10} \end{pmatrix}}_{\text{free parameters in the fit}} \cdot \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{10} \end{pmatrix}}_{\text{input values}} + \underbrace{b}_{\text{free parameters}} = \underbrace{y}_{\text{output values}} = a_1 x_1 + a_2 x_2 + \dots a_{10} x_{10} + b$$

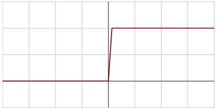
General linear transformation: many inputs, many outputs

$$\underbrace{\begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,10} \\ a_{2,1} & a_{2,2} & \dots & a_{2,10} \\ a_{3,1} & a_{3,2} & \dots & a_{3,10} \\ a_{4,1} & a_{4,2} & \dots & a_{4,10} \\ a_{5,1} & a_{5,2} & \dots & a_{5,10} \end{pmatrix}}_{\text{free parameters in the fit}} \cdot \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{10} \end{pmatrix}}_{\text{input values}} + \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix}}_{\text{free parameters}} = \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix}}_{\text{output values}} = \begin{matrix} a_{1,1}x_1 + a_{1,2}x_2 + \dots a_{1,10}x_{10} + b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 + \dots a_{2,10}x_{10} + b_2 \\ a_{3,1}x_1 + a_{3,2}x_2 + \dots a_{3,10}x_{10} + b_3 \\ a_{4,1}x_1 + a_{4,2}x_2 + \dots a_{4,10}x_{10} + b_4 \\ a_{5,1}x_1 + a_{5,2}x_2 + \dots a_{5,10}x_{10} + b_5 \end{matrix}$$

Pass through function f to make it non-linear

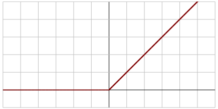
$$
f\left[\underbrace{\begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,10} \\ a_{2,1} & a_{2,2} & \dots & a_{2,10} \\ a_{3,1} & a_{3,2} & \dots & a_{3,10} \\ a_{4,1} & a_{4,2} & \dots & a_{4,10} \\ a_{5,1} & a_{5,2} & \dots & a_{5,10} \end{pmatrix}}_{\text{free parameters in the fit}} \cdot \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{10} \end{pmatrix}}_{\text{input values}} + \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix}}_{\text{free parameters}}\right] = \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix}}_{\text{output values}} = \begin{matrix} f[a_{1,1}x_1 + a_{1,2}x_2 + \dots a_{1,10}x_{10} + b_1] \\ f[a_{2,1}x_1 + a_{2,2}x_2 + \dots a_{2,10}x_{10} + b_2] \\ f[a_{3,1}x_1 + a_{3,2}x_2 + \dots a_{3,10}x_{10} + b_3] \\ f[a_{4,1}x_1 + a_{4,2}x_2 + \dots a_{4,10}x_{10} + b_4] \\ f[a_{5,1}x_1 + a_{5,2}x_2 + \dots a_{5,10}x_{10} + b_5] \end{matrix}
$$

The non-linear function f is called an "activation function"



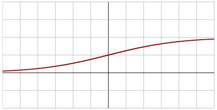
binary step

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$



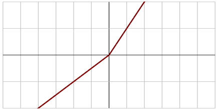
rectified linear unit (ReLU)

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$



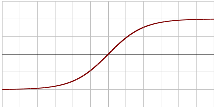
logistic (soft step)

$$f(x) = \frac{1}{1 + e^{-x}}$$



"leaky" ReLU

$$f(x) = \begin{cases} \alpha x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$



hyperbolic tangent

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

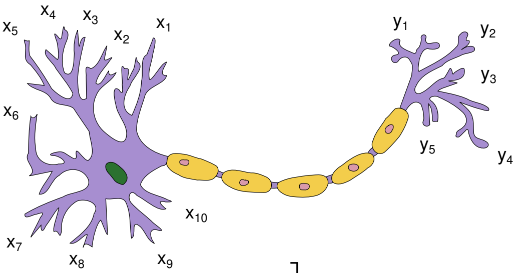
sigmoid linear unit ("swish")

$$f(x) = \frac{x}{1 + e^{-x}}$$

There are many choices, but ReLU is the simplest and most common.

Neural networks take inspiration from neurons in the brain

input signals
$x_1, x_2, \ldots, x_{10}$



output signals
y_1, y_2, y_3, y_4, y_5

$$f\left[\underbrace{\begin{pmatrix} a_{1,1} & a_{1,2} & \ldots & a_{1,10} \\ a_{2,1} & a_{2,2} & \ldots & a_{2,10} \\ a_{3,1} & a_{3,2} & \ldots & a_{3,10} \\ a_{4,1} & a_{4,2} & \ldots & a_{4,10} \\ a_{5,1} & a_{5,2} & \ldots & a_{5,10} \end{pmatrix}}_{\text{free parameters in the fit}} \cdot \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{10} \end{pmatrix}}_{\text{input values}} + \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix}}_{\text{free parameters}}\right] = \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix}}_{\text{output values}} = \begin{matrix} f[a_{1,1}x_1 + a_{1,2}x_2 + \ldots a_{1,10}x_{10} + b_1] \\ f[a_{2,1}x_1 + a_{2,2}x_2 + \ldots a_{2,10}x_{10} + b_2] \\ f[a_{3,1}x_1 + a_{3,2}x_2 + \ldots a_{3,10}x_{10} + b_3] \\ f[a_{4,1}x_1 + a_{4,2}x_2 + \ldots a_{4,10}x_{10} + b_4] \\ f[a_{5,1}x_1 + a_{5,2}x_2 + \ldots a_{5,10}x_{10} + b_5] \end{matrix}$$

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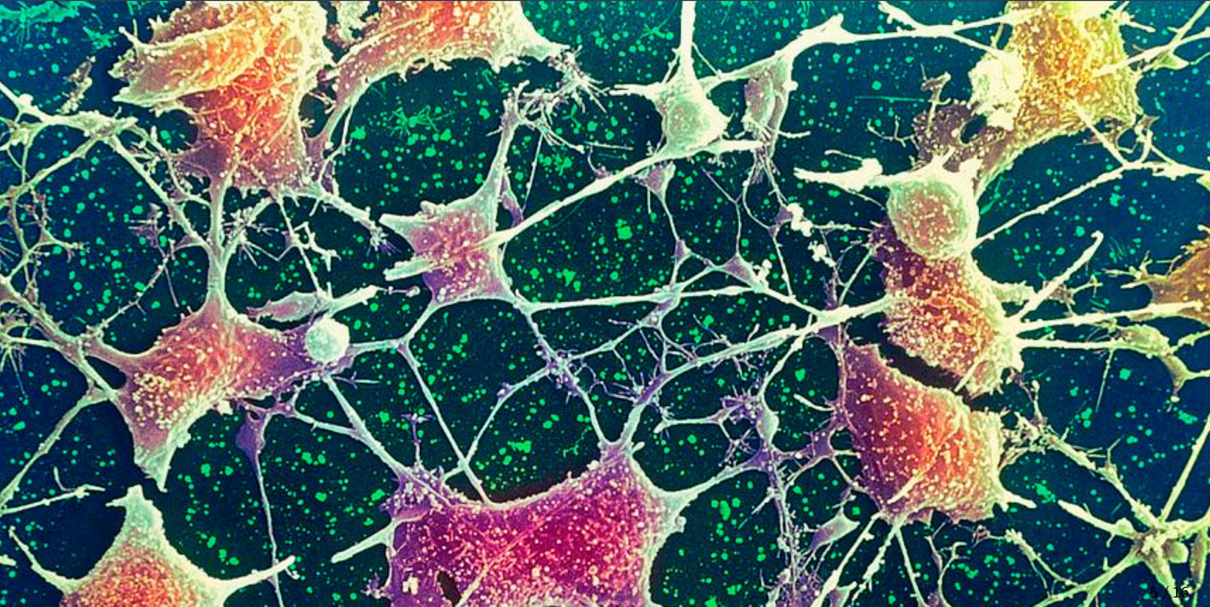

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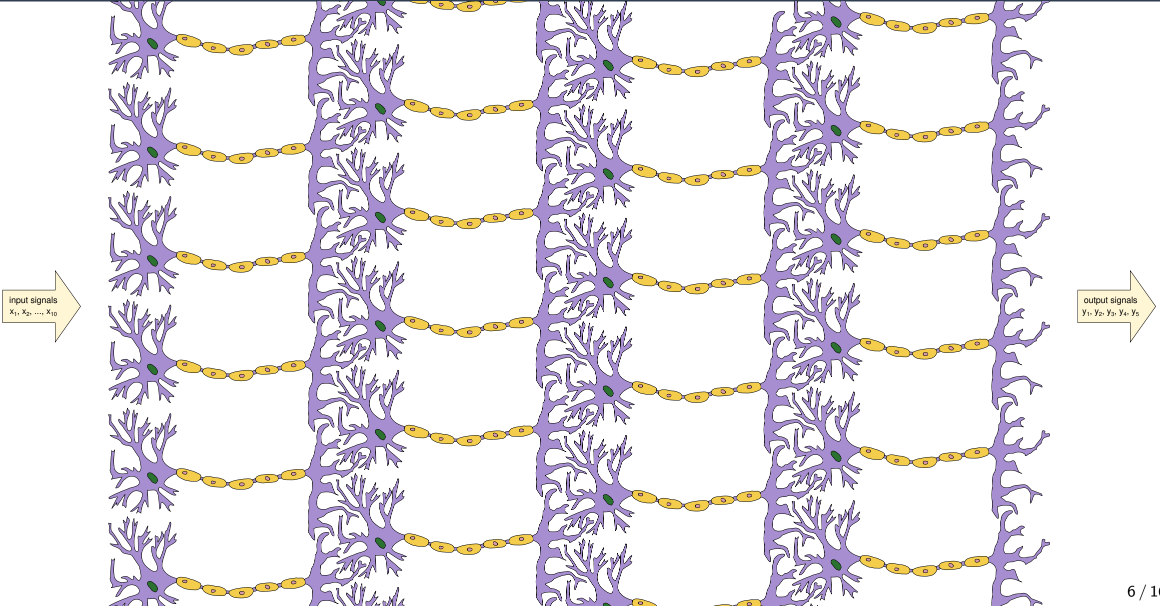

To do the same thing with our model, take the output of one "activation + linear transform" and use it as the input to the next:

$$f\left(a_{i,j}^{\text{layer 1}} \cdot x_j + b_i^{\text{layer 1}}\right)$$

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To do the same thing with our model, take the output of one "activation + linear transform" and use it as the input to the next:

$$f\left(a_{i,j}^{\text{layer 2}} \cdot \boxed{f\left(a_{i,j}^{\text{layer 1}} \cdot x_j + b_i^{\text{layer 1}}\right)} + b_i^{\text{layer 2}}\right)$$

Neural networks take inspiration from neurons in the brain

To do the same thing with our model, take the output of one "activation + linear transform" and use it as the input to the next:

$$f\left(a_{i,j}^{\text{layer 3}} \cdot \boxed{f\left(a_{i,j}^{\text{layer 2}} \cdot \boxed{f\left(a_{i,j}^{\text{layer 1}} \cdot x_j + b_i^{\text{layer 1}}\right)} + b_i^{\text{layer 2}}\right)} + b_i^{\text{layer 3}}\right)$$

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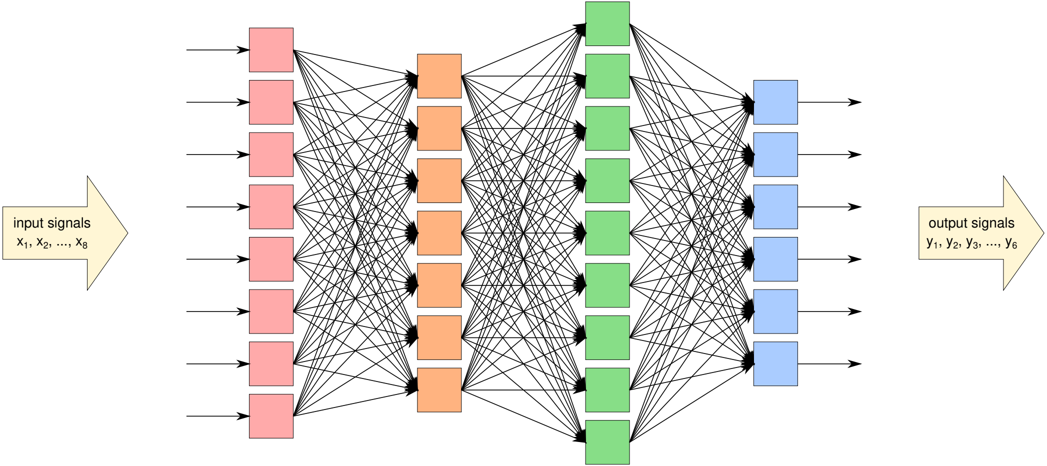
To do the same thing with our model, take the output of one "activation + linear transform" and use it as the input to the next:

$$f\left(a_{i,j}^{\text{layer 4}} \cdot \boxed{f\left(a_{i,j}^{\text{layer 3}} \cdot \boxed{f\left(a_{i,j}^{\text{layer 2}} \cdot \boxed{f\left(a_{i,j}^{\text{layer 1}} \cdot x_j + b_i^{\text{layer 1}}\right)} + b_i^{\text{layer 2}}\right)} + b_i^{\text{layer 3}}\right)} + b_i^{\text{layer 4}}\right)$$

It's usually drawn like this



The lines indicate that every output from one layer is included in the linear transformation of the next layer. ("There's an $a_{i,j}$ for every x_j and y_i.")

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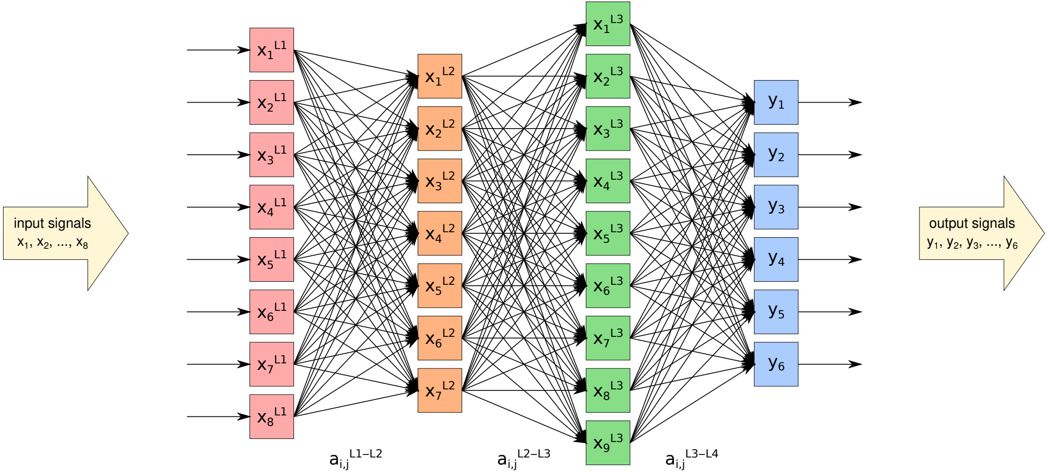

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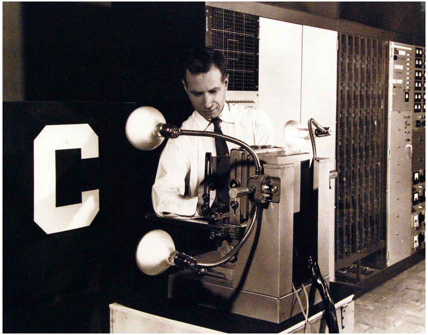
Connectivism: automatic learning by fitting the $a_{i,j}$, b_i parameters



Frank Rosenblatt's perceptron machine (1958) attempted to recognize images of letters.

The free parameters were adjusted with motors. This system eventually learned left-versus-right (not much more).

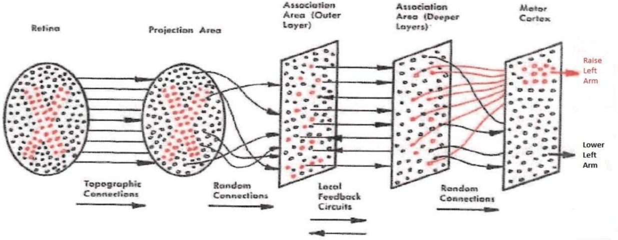


FIG. 1 — Organization of a biological brain. (Red areas indicate active cells, responding to the letter X.)

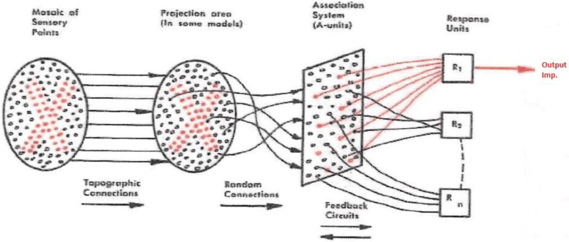
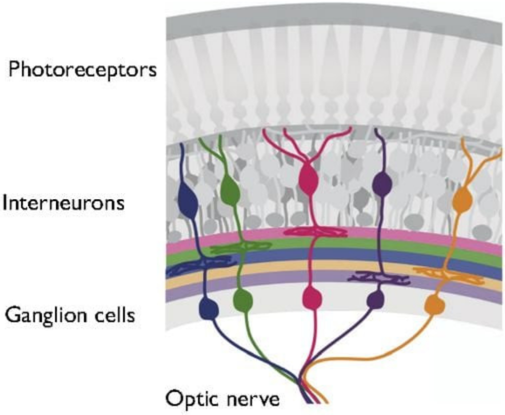


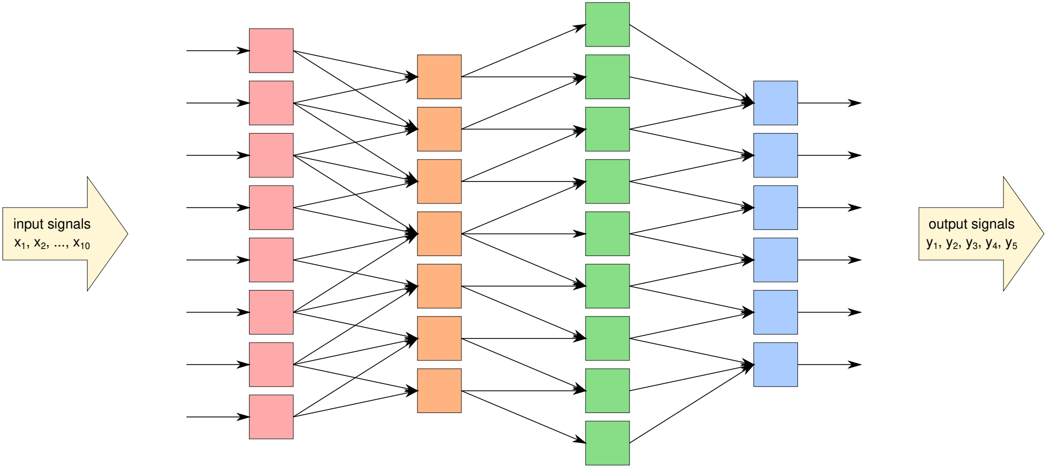
FIG. 2 — Organization of a perceptron.

More inspiration from nature: eye-neurons are not fully connected



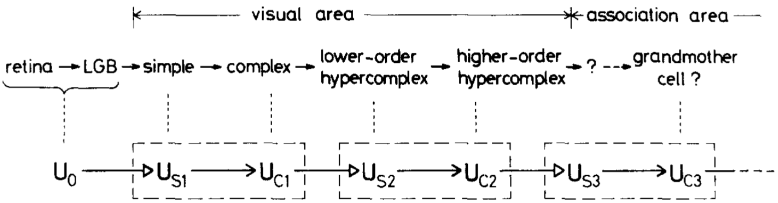
Neurons in one layer are connected to only a few of the neurons in the next layer.

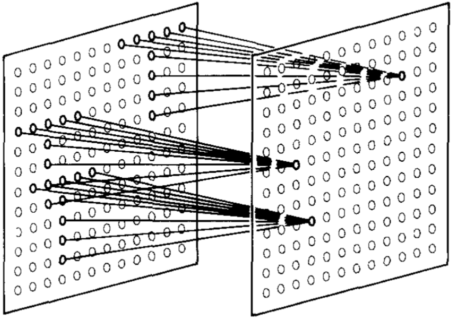
“Convolutional” (restricted) neural network

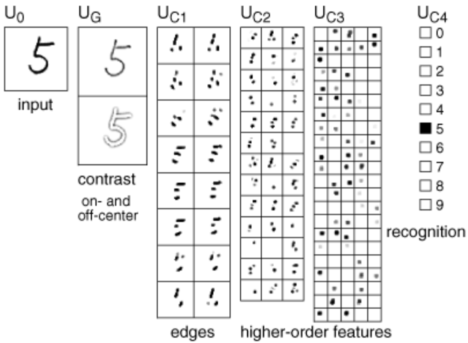


This sets a lot of $a_{i,j}$ parameters to zero and doesn't let them be tuned in the fit.

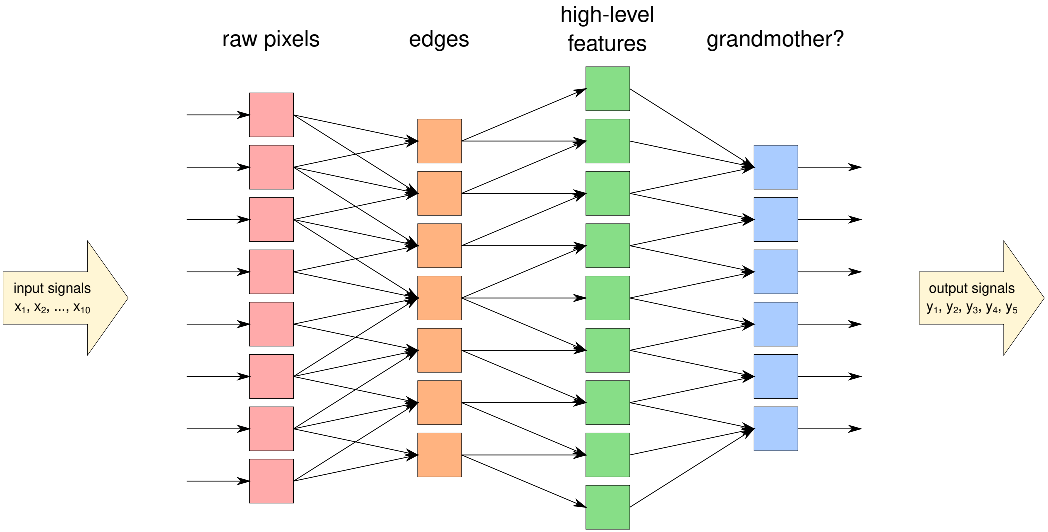
Kunihiko Fukushima's neocognitron (1980)



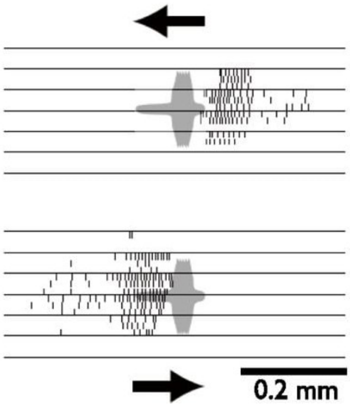




Each layer of a neural network is more abstract than the last

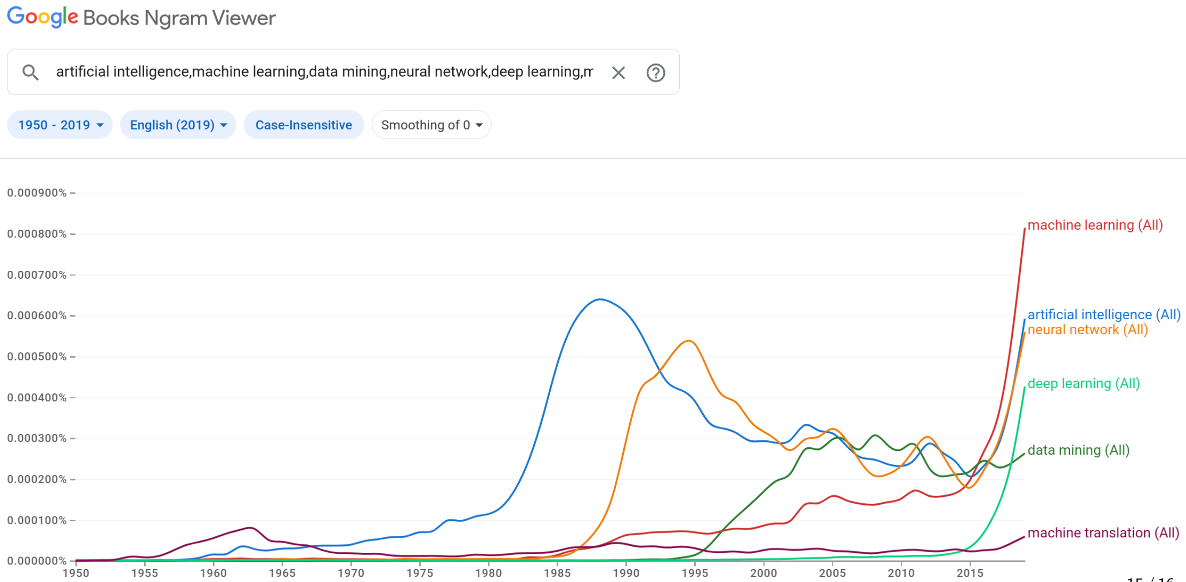


Fun fact: mice have hawk-shaped features in their visual networks



Y. Zhang, I. Kim, J. Sanes, *The most numerous ganglion cell type of the mouse retina is a selective feature detector* (2012), https://doi.org/10.1073/pnas.1211547109

These ideas have been studied for a long time



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4. Large enough compute farms and GPUs to analyze the above.