Measuring Track Resolution from $K_S \to \pi^+\pi^-$

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1 Motivation

Optimizing the resolution of tracking detectors is only the first step in searches for new phenomena; once the resolution has been improved as much as possible, it must then be precisely known. The significance of new multi-track resonances is inversely proportional to the resolution of such peaks: if observed, the resolution of the peak can be derived from the observation, but if not, it will need to be estimated to set the correct upper limit. The easiest way to estimate the resolution is from a Monte Carlo simulation, but in the early stages of an experiment, the simulation might be missing important effects. One would be much more confident in the simulation if it is confirmed by an independent measurement.

In simple cases, such as searches for new dimuon resonances, one could imagine extrapolating from the widths of known peaks, J/ψ, Υ, and Z, without needeeding to know the resolution of the individual tracks. However, new particles by definition have unfamiliar kinematics: either the mass is much higher than any known resonance or the momentum is, as new low-mass dimuons would have to be produced from heavy top-down decays, necessarily giving them a much higher boost than the J/ψ or Υ mesons produced by strong interactions. Dimuon extrapolations would not be valid, because boosted low-mass resolution depends on the underlying tracks' curvature and initial directions in a different way from unboosted low-mass resolution.

To derive the resolution of any prompt multi-track decay, it is sufficient to know the track 3-momentum resolution as a probability distribution, that is

$$P(\Delta q/p_T, \Delta\phi, \Delta\cot\theta; p_T, \phi, \eta, d_{xy}, d_z) \tag{1}$$

where $\Delta q/p_T$, $\Delta\phi$, and $\Delta\cot\theta$ are the differences between measured curvature, azimuthal angle, and cotangent of the polar angle ($\cot\theta = p_z/p_T$) and their true values, such that

$$\int d(\Delta q/p_T)\, d(\Delta\phi)\, d(\Delta\cot\theta)\, P(\Delta q/p_T, \Delta\phi, \Delta\cot\theta; qp_T, \phi, \eta, d_{xy}, d_z) = 1. \tag{2}$$

The arguments after the semicolon, qp_T, ϕ, η, d_{xy} and d_z, characterize the tracks in terms of their signed transverse momentum, azimuthal angle, pseudorapidity, and transverse and longitudinal impact parameters. The above distribution can be parameterized or expressed in bins of the most important arguments, usually qp_T, η, and d_{xy}.

We can obtain the resolution distribution from Monte Carlo by subtracting generator-level values from the reconstructed values of each momentum component. However, if it can also be derived directly from the data, then we can perform an important check on the Monte Carlo and also replace it in kinematic simulations, if the data-driven distribution is known with sufficient precision.

1.1 Split cosmic rays

One way to derive the distribution in Equation 1 is to acquire a sample of cosmic rays traversing the detector close to the interaction point. By reconstructing the two halves of the cosmic ray's trajectory as separate tracks, we obtain independent measurements of not only the three momentum components, but also the two impact parameters; in other words,

$$P_{\text{cosmic}}(\Delta q/p_T, \Delta\phi, \Delta\cot\theta, \Delta d_{xy}, \Delta d_z; qp_T, \phi, \eta, d_{xy}, d_z). \tag{3}$$

If one of the two tracks could be perfectly measured, then the resolution of the other one would be derived by simply subtracting each of the five parameters, track by track. This would yield the full 5-D distribution P_{cosmic}, with all the correlations between the parameters intact. Neither track can be perfectly measured, but we can assume that they have the same resolution and derive the final distribution by deconvolution. For Gaussian distributions centered on zero, convolution simply adds widths in quadrature, so the single track resolution is approximately

$$\begin{pmatrix} \Delta q/p_T \\ \Delta\phi \\ \Delta\cot\theta \\ \Delta d_{xy} \\ \Delta d_z \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} (q/p_T)_1 - (q/p_T)_2 \\ \phi_1 - \phi_2 \\ \cot\theta_1 - \cot\theta_2 \\ (d_{xy})_1 - (d_{xy})_2 \\ (d_z)_1 - (d_z)_2 \end{pmatrix}. \tag{4}$$

Though it is useful to have such a simple method to measure all five track parameters, cosmic rays do have some disadvantages:

- Their distribution is primarily vertical, illuminating the top and bottom of the detector well, but not the sides or endcaps. To extend the measurement to the sides of the detector, we can assume symmetry in ϕ. But it would be meaningless to extrapolate a barrel measurement (low η) to the endcaps (high η), where a different geometry and a different detector technology is used.

- Only a small fraction pass close enough to the interaction point to resemble the tracks from collision products, whose resolution we want to determine. One must either apply a tight cut on d_{xy} and d_z or compute a $d_{xy} \to 0$, $d_z \to 0$ limit, which is essentially the same thing.

- The pair of tracks under comparison always includes one top track and one bottom track, so it cannot resolve symmetric errors.

- The top track traverses the detector in the "wrong" direction, opposite to tracks from collisions. While this has several consequences, the most important is that cones from any associated showers, such as from TeV muons, point toward the interaction point instead of away from it. This could make the resolution of the top track different from that of the bottom track.

- The number of collected cosmic rays does not scale with luminosity, so the statistical gain diminishes with time.

The two advantages which are unique to this cosmic ray technique are: (1) access to all five track parameters, and (2) the cosmic ray spectrum is power-law, rather than exponential, so it can supply higher-energy events than any other technique.

1.2 Dimuon resonance masses

The J/ψ and Υ families and the Z boson decay into muon pairs with known masses, providing an a priori constraint for each event. Unfortunately, one constraint is not enough to fully analyze the tracks: the other parameters would need to be presumed to be correctly measured or deconvolved from an assumed resolution.

For example, in $Z \to \mu^+\mu^-$ decays, the Z boson has an unknown 3-momentum, though its vertex can be assumed to coincide with the nearest primary vertex (constructed using all tracks except the two muons). To compute $\Delta q/pT$ of the first track, we must use all the measured track parameters of the second track, as well as ϕ and $\cot\theta$ of the first track. To account for the uncertainty in these five reference parameters, as well as the intrinsic width of the Z, one can deconvolve the final distribution using a technique described later in this note. Applying this method would improve our knowledge of curvature q/pT, but not $\Delta\phi$ or $\Delta\cot\theta$.

For tracks with high enough momenta to be regarded as being essentially straight, such as high-energy Z daughters, the momentum resolution is dominated by the second-order curvature measurement, rather than first-order $\Delta\phi$ or $\Delta\cot\theta$. However, for lower momenta, $\Delta q/pT$ and $\Delta\phi$ are more tightly coupled, so a different technique would be needed.

2 Kinematic constraints from $K_S \to \pi^+\pi^-$

Two-body decays of K_S differ from the dimuon resonances considered above in that K_S has a macroscopic flight path of several centimeters— long enough to accurately measure its direction but short enough to resemble the prompt tracks of interest. A diagram of the event topology is given in Figure 1.

The direction of the K_S momentum (ϕ_K, θ_K) can be determined from the displacement vector between the primary vertex and the $\pi^+\pi^-$ vertex (defined by their point of closest approach). Though the magnitude of that displacement is random, its direction must coincide with the K_S momentum. These two constraints combine with the known K_S mass to completely determine its kinematics.

To demonstrate the usefulness of this topology, we can show that the 3-momentum $\vec{p}_+$ of one daughter π^+ can be completely determined from the 3-momentum $\vec{p}_-$ of the other and

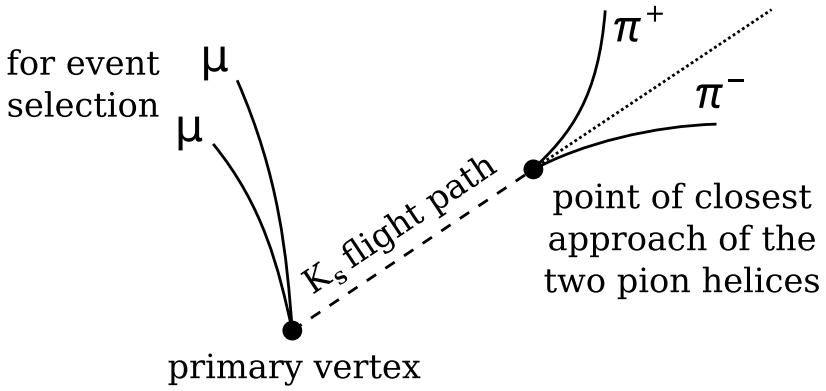

Figure 1: Schematic diagram of $K_S \to \pi^+\pi^-$ from a hadron collision.

the displacement vector $\vec{s}$ between the two vertices. The K_S 3-momentum is the sum of $\vec{p}_+$ and $\vec{p}_-$, and also $\vec{s}$ scaled by a factor α:

$$\vec{p}_+ + \vec{p}_- = \alpha\vec{s}. \tag{5}$$

The mass constraint determines α:

$$\left(\sqrt{|\vec{p}_+|^2 + {m_\pi}^2} + \sqrt{|\vec{p}_-|^2 + {m_\pi}^2}\right)^2 - \left(\vec{p}_+ + \vec{p}_-\right)^2 = {m_{K_S}}^2 \tag{6}$$

$$\left(\sqrt{|\alpha\vec{s} - \vec{p}_-|^2 + {m_\pi}^2} + \sqrt{|\vec{p}_-|^2 + {m_\pi}^2}\right)^2 - \alpha^2|\vec{s}|^2 = {m_{K_S}}^2. \tag{7}$$

Expand and collect terms, with many cancellations:

$$\alpha^2\left(|\vec{s}|^2\left(|\vec{p}_-|^2 + {m_\pi}^2\right) - \left(\vec{s}\cdot\vec{p}_-\right)^2\right) + \alpha\left(-{m_{K_S}}^2\left(\vec{s}\cdot\vec{p}_-\right)\right) + \left({m_{K_S}}^2\left(|\vec{p}_-|^2 + {m_\pi}^2\right) - \frac{{m_{K_S}}^4}{4}\right) = 0 \tag{8}$$

$$\alpha = \frac{{m_{K_S}}^2\left(\vec{s}\cdot\vec{p}_-\right) \pm 2m_{K_S}\sqrt{\left(|\vec{p}_-|^2 + {m_\pi}^2\right)\left[\left(\vec{s}\cdot\vec{p}_-\right)^2 - |\vec{s}|^2\left(|\vec{p}_-|^2 + {m_\pi}^2 - \frac{{m_{K_S}}^2}{4}\right)\right]}}{2\left(|\vec{s}|^2\left(|\vec{p}_-|^2 + {m_\pi}^2\right) - \left(\vec{s}\cdot\vec{p}_-\right)^2\right)}. \tag{9}$$

And thus we can cross-check each component of $\vec{p}_+$ with data derived from $\vec{p}_-$ and the $\pi^+\pi^-$ vertex, with only a twofold degeneracy in solutions ($\pm$ in the solution for α).

$$\begin{pmatrix} \Delta p_{x+} \\ \Delta p_{y+} \\ \Delta p_{z+} \end{pmatrix} \text{ should be equal to } \begin{pmatrix} \alpha s_x - p_{x-} \\ \alpha s_y - p_{y-} \\ \alpha s_z - p_{z-} \end{pmatrix} \tag{10}$$

The right-hand side of Equation 10 contains a hidden dependency on the π^+ track, in that the $\pi^+\pi^-$ vertex, and therefore $\vec{s}$, cannot be determined without knowing all of the π^+ track parameters. However, that calculation depends crucially on the π^+ impact parameters, so the test is not trivially satisfied.

3 Deriving track resolution from constraints

More generically, we can say that we observe four constraints in the $K_S \to \pi^+\pi^-$ decay which are a function f of the measured track parameters of the pions (randomly labeled track 1 and track 2)

$$\begin{pmatrix} m_{K_S}|_{\text{true}} - m_{K_S}|_{\text{tracks}} \\ \phi_{K_S}|_{\text{vertex}} - \phi_{K_S}|_{\text{tracks}} \\ \cot\theta_{K_S}|_{\text{vertex}} - \cot\theta_{K_S}|_{\text{tracks}} \\ z|_{\pi^+} - z|_{\pi^-} \end{pmatrix} = f \begin{pmatrix} q/p_T|_1 \\ \phi|_1 \\ \cot\theta|_1 \\ d_z|_1 \\ d_{xy}|_1 \\ q/p_T|_2 \\ \phi|_2 \\ \cot\theta|_2 \\ d_z|_2 \\ d_{xy}|_2 \end{pmatrix} \tag{11}$$

where the first three terms on the left-hand-side are a rearrangement of the momentum constraint we discussed in the previous section, and $z|_{\pi^+} - z|_{\pi^-}$ is an additional constraint on the impact parameters: the distance of closest approach of the two helices gives us information about the tracks' d_{xy} and d_z because that distance ought to be zero. We can only measure one, and we choose d_z because it will be more significant after deconvolving the effect of d_{xy} than the reverse.

We are interested in the small region where the left-hand-side is nearly zero, where we can expand the right-hand-side linearly around the true values of the parameters. (The left-hand-side is exactly zero when the measured values are equal to the true values.)

$$\begin{pmatrix} m_{K_S}|_{\text{true}} - m_{K_S}|_{\text{tracks}} \\ \phi_{K_S}|_{\text{vertex}} - \phi_{K_S}|_{\text{tracks}} \\ \cot\theta_{K_S}|_{\text{vertex}} - \cot\theta_{K_S}|_{\text{tracks}} \\ z|_{\pi^+} - z|_{\pi^-} \end{pmatrix} = \left(\frac{\partial f_i}{\partial q_j}\right) \begin{pmatrix} \Delta q/p_T|_1 \\ \Delta\phi|_1 \\ \Delta\cot\theta|_1 \\ \Delta d_z|_1 \\ \Delta d_{xy}|_1 \\ \Delta q/p_T|_2 \\ \Delta\phi|_2 \\ \Delta\cot\theta|_2 \\ \Delta d_z|_2 \\ \Delta d_{xy}|_2 \end{pmatrix} \tag{12}$$

The 4×10 Jacobian matrix of the four f_i components is represented above as $\left(\frac{\partial f_i}{\partial q_j}\right)$. We would like to derive the above errors in track parameters from the constraints on the left, but the Jacobian cannot be inverted— we can only determine 4 of the 10 track parameters.

To select the first four, we split the matrix into a 4×4 part that we can invert and a 4×6 part corresponding to parameters whose uncertainty will need to be deconvolved from the final result.

To split the matrix, we make two copies of it and mask alternate blocks with zeros:

$$\left(\frac{\partial f_i}{\partial q_j}\right) = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} & 0 & 0 & 0 & 0 & 0 & 0 \\ f_{21} & f_{22} & f_{23} & f_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\ f_{31} & f_{32} & f_{33} & f_{34} & 0 & 0 & 0 & 0 & 0 & 0 \\ f_{41} & f_{42} & f_{43} & f_{44} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & f_{15} & f_{16} & f_{17} & f_{18} & f_{19} & f_{10} \\ 0 & 0 & 0 & 0 & f_{25} & f_{26} & f_{27} & f_{28} & f_{29} & f_{20} \\ 0 & 0 & 0 & 0 & f_{35} & f_{36} & f_{37} & f_{38} & f_{39} & f_{30} \\ 0 & 0 & 0 & 0 & f_{45} & f_{46} & f_{47} & f_{48} & f_{49} & f_{40} \end{pmatrix} \quad (13)$$

$$(\partial f_{4\times 4}) = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{pmatrix} \text{ and } (\partial f_{4\times 4}) = \begin{pmatrix} f_{15} & f_{16} & f_{17} & f_{18} & f_{19} & f_{10} \\ f_{25} & f_{26} & f_{27} & f_{28} & f_{29} & f_{20} \\ f_{35} & f_{36} & f_{37} & f_{38} & f_{39} & f_{30} \\ f_{45} & f_{46} & f_{47} & f_{48} & f_{49} & f_{40} \end{pmatrix} \quad (14)$$

which splits Equation 12 into

$$\begin{pmatrix} m_{K_S}|_{\text{true}} - m_{K_S}|_{\text{tracks}} \\ \phi_{K_S}|_{\text{vertex}} - \phi_{K_S}|_{\text{tracks}} \\ \cot\theta_{K_S}|_{\text{vertex}} - \cot\theta_{K_S}|_{\text{tracks}} \\ z|_{\pi^+} - z|_{\pi^-} \end{pmatrix} = (\partial f_{4\times 4}) \begin{pmatrix} \Delta q/p_T|_1 \\ \Delta\phi|_1 \\ \Delta\cot\theta|_1 \\ \Delta d_z|_1 \end{pmatrix} + (\partial f_{4\times 6}) \begin{pmatrix} \Delta d_{xy}|_1 \\ \Delta q/p_T|_2 \\ \Delta\phi|_2 \\ \Delta\cot\theta|_2 \\ \Delta d_z|_2 \\ \Delta d_{xy}|_2 \end{pmatrix}. \quad (15)$$

With a little rearrangement,

$$(\partial f_{4\times 4})^{-1} \begin{pmatrix} m_{K_S}|_{\text{true}} - m_{K_S}|_{\text{tracks}} \\ \phi_{K_S}|_{\text{vertex}} - \phi_{K_S}|_{\text{tracks}} \\ \cot\theta_{K_S}|_{\text{vertex}} - \cot\theta_{K_S}|_{\text{tracks}} \\ z|_{\pi^+} - z|_{\pi^-} \end{pmatrix} = \begin{pmatrix} \Delta q/p_T|_1 \\ \Delta\phi|_1 \\ \Delta\cot\theta|_1 \\ \Delta d_z|_1 \end{pmatrix} + (\partial f_{4\times 4})^{-1} (\partial f_{4\times 6}) \begin{pmatrix} \Delta d_{xy}|_1 \\ \Delta q/p_T|_2 \\ \Delta\phi|_2 \\ \Delta\cot\theta|_2 \\ \Delta d_z|_2 \\ \Delta d_{xy}|_2 \end{pmatrix}. \quad (16)$$

The right-hand-side is the sum of deviates from two distributions, the first is the distribution that we want, and the second is a distribution that we can assume from a reasonable prior (the output of a Monte Carlo simulation), transformed by matrices derived purely from

geometry. The distribution of a sum of deviates from two distributions is their convolution, so we will compute the desired result by deconvolution. The left-hand-side is the error distribution measured in data, also transformed by a purely geometric matrix. This matrix would be difficult but possible to compute analytically, so a calculation from numerical derivatives would be well-behaved. To derive the desired distribution, we will apply the transformations to each measurement separately on the left-hand-side and each randomly-generated deviate in the second term of the right-hand-side, and then deconvolve the distribution of the latter from the distribution of the former.

Although it was necessary to assume a distribution for the track 2 parameters and one of the impact parameters from track 1, the labels 1 and 2 were randomly assigned: the track 2 parameters must be distributed in the same way as the track 1 parameters (in the same p_T, η, and d_{xy} bins). This procedure improves our knowledge of track 1 parameters with data: it can be repeated with the assumed input being the result from iteration 1. The only missing piece is one of the two impact parameters, Δd_{xy} in the derivation above. Applying this procedure will not improve our knowledge of Δd_{xy} (perhaps a 3-body decay would be necessary).

4 Deconvolving distributions

Deconvolution of of parameterized distributions is much less susceptible to numerical error than raw distributions in histograms, so we begin by fitting distributions of the two terms in Equation 16 with the following ansatz:

$$g(\vec{x}) = \int d\vec{y} \left(\prod_i \frac{1}{\pi} \frac{\Gamma_i/2}{(x_i - y_i)^2 + (\Gamma_i/2)^2} \right) \left(\prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \right) \exp\left(\sum_i \frac{(x_i - a_i - \sum_{j>i} b_{ij} x_j)^2}{2\,{\sigma_i}^2} \right) \tag{17}$$

The function is a convolution of a Cauchy-Lorentz distribution and a Gaussian distribution for each parameter, with correlations b_{ij} between the Gaussians (to allow the error ellipse to be angled in the i, j plane). This is a highly descriptive function, with the Lorentz Γ_i absorbing any non-linear tails. If there is non-negligible background after applying the cuts described in the next section, we should also include a constant or linear term to parameterize the background, but not include it in the deconvolution.

When all functions are Fourier-transformed, convolution is simply pointwise multiplication, so the easiest way to proceed is to Fourier-transform each of the factors in Equation 17. For the sake of this short note, we will operate on a simplified ansatz:

$$g'(\vec{x}) = \int d\vec{y} \left(\prod_i \frac{1}{\pi} \frac{\Gamma_i/2}{(x_i - y_i)^2 + (\Gamma_i/2)^2} \right) \left(\prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \right) \exp\left(\sum_i \frac{(x_i - a_i)^2}{2\,{\sigma_i}^2} \right) \tag{18}$$

which reduces the complexity of the algebra (which would otherwise involve a lot of polynomial fractions). The Fourier transform of the Cauchy-Lorentz distribution is

$$\frac{1}{\pi} \frac{\Gamma_i/2}{{x_i}^2 + (\Gamma_i/2)^2} \to \frac{1}{\sqrt{2\pi}} \exp\left(\frac{\Gamma_i}{2} |k_i| \right), \tag{19}$$

and the transform of the Gaussian is

$$\frac{1}{\sqrt{2\pi}\sigma_i}\exp\left(\frac{-(x_i-a_i)^2}{2{\sigma_i}^2}\right) \rightarrow \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-{k_i}^2{\sigma_i}^2}{2}\right)\exp(ia_ik_i) \tag{20}$$

The Fourier transformation of our ansatz $g'(\vec{x})$ is simply the product of these:

$$\hat{g}'(\vec{k}) = \prod_i \frac{1}{\sqrt{2\pi}}\exp\left(\frac{\Gamma_i}{2}|k_i|\right)\ \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-{k_i}^2{\sigma_i}^2}{2}\right)\exp(ia_ik_i) \tag{21}$$

and the convolution of two such distributions with different parameters, $g^A(\vec{x})$ with a_i^A, σ_i^A, Γ_i^A and $g^B(\vec{x})$ with a_i^B, σ_i^B, Γ_i^B, is

$$\hat{g}^{A\otimes B}(\vec{k}) = \prod_i \frac{1}{\sqrt{2\pi}}\exp\left(\frac{\Gamma_i^A+\Gamma_i^B}{2}|k_i|\right)\ \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-{k_i}^2({\sigma_i^A}^2+{\sigma_i^B}^2)}{2}\right)\exp\left(i(a_i^A+a_i^B)k_i\right). \tag{22}$$

The convolution $g^{A\otimes B}(\vec{x})$ therefore has the same form as $g^A(\vec{x})$ and $g^B(\vec{x})$, but with

$$a_i^{A\otimes B} = a_i^A + a_i^B \tag{23}$$

$$\sigma_i^{A\otimes B} = \sqrt{(\sigma_i^A)^2+(\sigma_i^B)^2} \tag{24}$$

$$\Gamma_i^{A\otimes B} = \Gamma_i^A + \Gamma_i^B. \tag{25}$$

(As previously mentioned, this result is complicated by including b_{ij} correlation terms, but it is not unsolvable.)

To deconvolve the two distributions described by their deviates in Equation 16, we fit each to an ansatz and subtract the fitted parameter values. If the assumption that one is a convolution of the other is valid, then σ_i and Γ_i will always be positive, though the inequality may be violated due to numerical error. If a σ_i or Γ_i is found to be imaginary or negative, it will be replaced by zero. (That corresponds to the case in which the width of distribution A is much larger than distribution B, hiding the precise value of B's width.)

5 Selecting $K_S \rightarrow \pi^+\pi^-$

This method requires a large sample of relatively background-free $K_S \rightarrow \pi^+\pi^-$ events, which could be hard to find in hadron collisions. Following branching fractions in the PDG, at least 0.5% of (charged or uncharged) B meson decays have the following form:

$$B \rightarrow \mu DX \text{ with } D \rightarrow \mu KY \text{ with } K_S \rightarrow \pi^+\pi^-. \tag{26}$$

This follows the Cabbibo-favored pathway of a b quark into u and d, with all virtual W bosons decaying leptonically. The two muons are distinctive and are usually included in triggers and skims for low-mass dilepton events.

Background can be further reduced by requiring a large distance between the primary vertex and the $\pi^+\pi^-$ candidate vertex. The length of the K_S flight path was not used to determine resolution, only its direction, so such a cut would not bias the results. Other quality cuts, such as cuts on the $\pi^+\pi^-$ invariant mass or consistency in direction between the $\pi^+\pi^-$ and the K_S flight path must be loose, because they do bias the resolution measurement.

6 Conclusions

In this note, we described several methods to determine resolution of tracks from data, focusing on the one which is both the most complicated and the most relevant for low-mass searches. The formalism of section 3 and the deconvolution method of section 4 can be applied to the split cosmic ray technique (section 1.1) for all track parameters and high-energy tracks, and to the dimuon resonance masses (section 1.2) for curvature resolution at medium energies, throughout the detector. A combination of all three would map the resolution at all orders of magnitude in p_T, with different assumptions in each region.