A Stitch in Time

I never really understood General Relativity until I learned to sew, just as I never really understood thermodynamics until I learned to cook. As with cooking, I was driven to sewing by necessity: my jeans didn't fit. My first modifications were small ones: first, darts here and there to pull in material, then adding elastic waistbands, then suspenders. Eventually I got so fed up with the bad fit that I decided to make my own jeans from scratch. I bought a few yards of fabric, wrapped it around myself, and started cutting. Mistakes didn't worry me: I knew how to patch and I figured I would just keep cutting and patching until the thing fit around my body. It took about a week for the carpet to slowly evolve into pantaloons. I finished them just in time to be wearing them when my wife came back from a conference, whereupon she exclaimed, "They're purple!" Pants 1.0 have been described as part-Joker, part-Catwoman, and lasted almost a year.

Spending all that time with fabric forced me to think a lot about topology and curvature. I had to somehow replicate my own curvature in cloth: too much curvature and it would be bulgy, too little and it would be tight. The most difficult part is the crotch, which bends upward in the front and back, but downward toward the legs. "Oh," I realized, "That's what it means to be negatively curved!" Zero and positive curvature are easy to understand from textbooks, but negative curvature is always drawn as a twisted bit of graph paper, mysteriously ~~labeled~~ called a saddle-point. "Of course it's shaped like a saddle," I said to myself. "What else ~~is~~ are saddles meant to fit?"

In popular explanations of General Relativity, people often say that space-time is like a rubber sheet, curving under the weight of the planets. The planets are marbles that someone has set on the rubber sheet, to show that the sheet does bend under them. This explanation has always bothered me because the marbles are outside of the sheet, but if the sheet is supposed to represent space, then the planets ought to be inside the sheet. Moreover, the demonstration only works because the real Earth is under the rubber sheet, drawing marbles toward itself.

¶ And while I'm comfortable with the idea that sheets are curved, what could it possibly mean to say that space is curved? How would I even visualize that?

¶ In school, I learned a lot about the mathematics of curvature and its application to space-time — the theory of gravitation known as General Relativity. But I never felt that I had an intuitive understanding of it until I learned the ancient art of shaping cloth with thread. Video games helped, too.

¶ How is cloth like space? For this metaphor to work, we have to imagine being a drawing on the cloth — in other words, two-dimensional! I've only ever been two-dimensional while playing video games, such as the kind where you're in a dungeon with many rooms. Each room has north, south, east, and west, but up and down are meaningless. There are doors at the edges of the room; these lead to other rooms, some with hidden treasure or monsters. It looked a bit like this:

The video game was not projected on a piece of cloth, but there's no conceptual reason against that — only technological. Cloth computer screens would be pretty cool. The logic of the game would not be altered by folding the cloth: adjacent stitches (pixels) are still adjacent when cloth is folded, so our hero would crawl around the crease without ever knowing that his world isn't spread flat.

¶ His world is changed, however, by how we sew it. Suppose that we sew the east door of the room to the west door: when our hero walks out through one door, he finds himself walking in through the other. In fact, his whole dungeon appears to be a long east-west corridor of identical rooms.

(picture)

In a video game these kinds of connections can be made arbitrarily: the programmer just needs to type "if (exit through west) then { enter through east;}." It's like a hyperlink on a web page. We could imagine something similar in our three-dimensional world without having to visualize a four-dimensional suture. While there isn't any known way to make wormholes or looped spaces in reality, this idea is all over science fiction, from Madeline L'Engle's *A Wrinkle in Time* to Neil Gaiman's *Coraline*.

¶ In this article, I'd like to talk about a different kind of connection. The looped space is made from one wierd connection — I'd like to talk about curvature, which is made from wierd connections all over the place.

¶ Suppose that we build a dungeon from a network of linked rooms with two north doors for every one south door. Every step in latitude has twice as many rooms as the previous. I could draw it like this:

with blue lines representing the links between doors. Since the two north doors lead in different directions, I'm calling them "ronth" and "ornth." The rooms get crowded at the top, so I've shrunk them in my map, but to the video game hero, they're all the same size.

¶ Here's a better way to represent them:

¶ When these rooms are sewed together, they form a lumpy surface like a lettuce leaf. Our hero doesn't know that: to him, it's just a two dimensional world. But there are many more places to explore than a grid with only north, south, east, and west doors.

¶ This is what it means for a space to be curved. The cloth is said to be curved because it can't be laid flat due to the way that it is stitched — this is different from merely folding a normal sheet of cloth.

¶ The two-dimensional space corresponding to that cloth is also curved. To make the correspondence explicit, imagine a computer screen pixel for every stitch in the cloth. The essential thing is how pixels — points in space — are connected, who their neighbors are.

¶ Despite his two-dimensionalness, the video game character knows that his world is curved. One experiment he can do to find out is to measure the lengths of paths between two points:

In an uncurved space, the shortest path between two rooms at the same lattitude is a straight east·west line. To get from room A to room B, we might imagine that the shortest way would be to go east·east·east·east·east·east·east: 7 rooms. But in this space, south·south·east·orth·orth is shorter: 5 rooms. Even to our video game hero, this does not seem to be a straight path: first he's going south, then he's going orth! But it is undeniably shorter: breadcrumbs prove it.

¶ Experiments like this can measure the degree of curvature. The more a shortest path appears to be curved, the greater the magnitude of space-curvature. An uncurved space has curvature = 0, and spherical curvature, like the surface of a globe, has positive curvature. The video game space we've been considering here, like the crotch of my pants, is negatively curved. If we had three north doors for every one south door, then it would be more negatively curved. Negatively curved spaces are significant because the three-dimensional space we live in, you and I, is negatively curved. The curvature of our world is only slightly negative, but it is <u>not</u> zero.

¶ To see the effect of this curvature in our everyday lives, we need to consider time as a dimension, like length, breadth, and height. Time intervals are distances, ~~just~~ much like distances in space ~~(apart from a minus sign that I'm not going to tell you about)~~. Our perception of time is extremely foreshortened, compared to our perception of space: one nanosecond of time is about as long as one foot. It is mostly this separation of scales that makes time intervals seem different from distances.

¶ When we move, we trace out paths in space-time. Seen from the side, we are long, skinny creatures that grow to be about six feet tall but thirty trillion nanoseconds long. And we lace around each other, threading through our lives like a mass of spaghetti.

¶ We have seen a negatively curved space; here is what a negatively curved space-time looks like:

In this piece of cloth, the horizontal direction represents time and the vertical direction represents height. I drew a video game character incorrectly — in space-time, he would look like a snake, rather than having a back and a front that faces forward in time. I plead artistic license. In the time interval represented by this cloth, he leaps directly upward and falls back down. Apart from the initial leap, the course of his path through space-time is dictated solely by gravity.

¶ That path is an arc. He rises, slows to a stop, and then falls back down at an ever-increasing rate. Though it is curved, this path is shorter than it would be if he stayed on the ground (the lower arrow). In fact, all of the shortest paths through space-time are falls.

This is how gravity works. Skydivers feel weightless as they fall because they are: they are following the same kind of path through space-time as they would in deep space, far from the Earth. When we stand on the ground, we feel pushed into our feet. When we drive a car around a tight corner, we feel a sideways push for exactly the same reason: both are unnaturally long paths through space-time. Something must deflect our path from the natural one. When we stand on the ground, it is the solidity of the earth, preventing our feet from following the shortest path through space-time, ~~that~~ which leads straight down. When we turn a corner in a car, it is the seat belt that prevents us from following the shortest path: straight ahead.

¶ The ~~space-time~~ curvature that causes a flying arc through the air to be a shorter space-time path than just staying on the ground is an extremely small curvature. On the second floor of a building, time intervals are one part in a quadrillion longer than they are on the first floor ~~(longer, not shorter, because of the minus sign I didn't tell you about)~~. Satellites in orbit lose a microsecond every ten minutes and need to be specially programmed to stay in sync with computers on Earth. This tiny curvature is only noticible to us because we have such a foreshortened perception of time. It takes billions of nanoseconds — that is, a few seconds — to see the path of a tossed object deflect a few feet (1 foot = 1 nanosecond).

¶ The gravitational curvature of the Earth is tiny, but we are sensitive to the passage of time at exactly the right rate to be aware of it. It makes me wonder if this is a biological adaptation to living on Earth, much like the fact that we ~~can~~ only see the colors of light that can penetrate Earth's atmosphere. Would creatures that evolve in low-gravity environments — asteroids — sense time much more slowly than us? Would creatures in high gravity — the cooled husk of an ancient white dwarf star — seem to live at a maniacal pace?