```
(* Copyright (C) 2015 Francois P. LANDES  *)


MathcalM := 2  (*THIS CANNOT BE CHANGED !*)
```

Explanation for the use of the Gamma[x1,x2] function:

$$\text{Sum}\left[\frac{1}{x!}\,\text{lambda}^{x},\ \{x,\ 0,\ \text{Xmax}\}\right]$$

$$\frac{e^{\text{lambda}}\,\text{Gamma}[1+\text{Xmax},\ \text{lambda}]}{\text{Xmax}!}$$

```
Clear[NormalizationOld, NormalizationNew,
 ps1Tot, ps2Tot, M1, M2, NN, Step1, Step2, Step, cci, NNi]
P1[z1_, z2_, i_] := (1 / NormalizationOld) 1 / (Factorial[z1] * Factorial[z2]) *
  (M1 / (NN * (ps1Tot)))^z1 (M2 / (NN * (ps2Tot)))^z2

EE := 1.2
eeStep := 0.100001
errorBar := 0.0001
MathcalNStep = 5

Array[psResults, 100, 0]
Array[psMathcalN, 100, 0]
For[ MathcalN = 0, MathcalN < 100, MathcalN++, {psResults[MathcalN] = 0}]
psResultsCounter = 0

Timing[For[ beta = 1.8, beta < 4.1,
  (*MathcalN=4,MathcalN<100,*)
  (*ee=EE,ee<EE+eeStep,*)
  Timing[{
    Clear[NormalizationOld, NormalizationNew,
     ps1Tot, ps2Tot, M1, M2, NN, Step1, Step2, Step, cci, NNi],
    (*P1[z1_,z2_,i_]:=(1/NormalizationOld)  1/(Factorial[z1]*Factorial[z2])  *
      (M1/(NN*(ps1Tot)))^z1  (M2/(NN*(ps2Tot)))^z2 ,*)
    (*Initial Guess:  *)


    beta = beta + 0.2,
    ee = EE,

    MathcalN = 32,

    (*MathcalN =MathcalN + MathcalNStep,
    (* Number of different kinds of agents *)*)
    (*beta=1.5, *)
    Array[partialSums, MathcalN + 1, 0],
```

```
base = 1.1,

rr = 30.0,
gg = 1.0 / rr,

pi1 = 0.01,
MagicN = 10 000,

(*Initial Guess:  *)
NormalizationOld := 1.0,
ps1Tot := 0.5,
ps2Tot := 0.5,

cci[i_] = base^(i + 0.5),
NNi[i_] = MagicN * (1 - base^(-beta)) (base^i )^(-beta) ,

Print["Maximum Capital= ", cci[MathcalN - 1] , "        average capital=",
 Sum[NNi[iii] cci[iii] / NN, {iii, 0, MathcalN - 1}]],

(*DiscretePlot[NNi[iii]cci[iii] ,{iii,0,MathcalN-1}], *)
pi2 = rr pi1,
M1 =
 Floor[Sum[cci[iii] NNi[iii], {iii, 0, MathcalN - 1}] / (pi1 ee (1 + gg rr))],
M2 = Floor[gg M1],
NN = Sum[NNi[iii], {iii, 0, MathcalN - 1}],




(*cci[iii_]= base^(iii+0.5)+pi1/10.,*)
(*cci[iii_]=base^(iii+0.5)+pi1/2.,(* CHEAT, to handle the heaviside and
 other sums correctly (boundary conditions are troublemakers)  *)*)


(*(*Initial Guess:  *)
NormalizationOld=1.0,
ps1Tot=0.5,*)
(*NormalizationOld=7.136775864327738`*^7,
ps1Tot=0.9986766815185548`,*)

Step1 := 0.1,
Step2 := 0.05,
Step := Step1 + Step2,
iterationNumber := 0,
While [Abs[Step] > errorBar,
 {iterationNumber += 1,
  lambda1 = (M1 / (NN * (ps1Tot))),
  lambda2 = (M2 / (NN * (ps2Tot))),
  For[ iii = 0, iii < MathcalN,
   {partialSums[iii] =
     e^(lambda2) / NormalizationOld Sum[  1 / (Factorial[z1]) * lambda1^z1
```

```
        (Gamma[1 + Floor[(cci[iii] - pi1 * z1) / pi2], lambda2]) /
         Factorial[Floor[(cci[iii] - pi1 * z1) / pi2]] ,
       {z1, 0, Floor[cci[iii] / pi1]}],
    iii += 1 }],
  NormalizationNew = Sum[
    partialSums[iii],
    (* Sum[P1[z1,iii] ,{z1,0,Floor[cci[iii] /pi1]+1}],*)
    {iii, 0, MathcalN - 1}],
  NormalizationOld *= NormalizationNew,


  (*ps1Goal=Sum[ NNi[iii]/NN - NNi[iii]/NN P1[Floor[(cci[iii]/pi1)],iii]
      NormalizationNew/ (partialSums[iii])   ,{iii,0,MathcalN-1}],*)
  ps1Goal = Sum[ NNi[iii]/NN - NNi[iii]/NN Sum[ P1 [Floor[(cci[iii] - pi2 * z2) / pi1],
         z2, iii] NormalizationNew / (partialSums[iii]),
       {z2, 0, Floor[cci[iii] / pi2]}], {iii, 0, MathcalN - 1}],
  ps2Goal = Sum[ NNi[iii]/NN - NNi[iii]/NN Sum[ P1[z1, Floor[(cci[iii] - pi1 * z1) /
           pi2], iii] NormalizationNew / (partialSums[iii]),
       {z1, 0, Floor[cci[iii] / pi1]}], {iii, 0, MathcalN - 1}],

  SignOfDifference1 = ps1Goal - ps1Tot,
  SignOfDifference2 = ps2Goal - ps2Tot,
  (*Step1 =
   If[Abs[Step1]> Abs[SignOfDifference1], SignOfDifference1, Step1],*)
  Step1 = If[SignOfDifference1 * Step1 > 0, Step1, -Step1 / 1.2],
  Step2 = If[SignOfDifference2 * Step2 > 0, Step2, -Step2 / 1.2],

  ps1Tot = ps1Tot + Step1,
  ps2Tot = ps2Tot + Step2,
  ps1Tot = If[ps1Tot ≥ 1, 1, ps1Tot],
  ps1Tot = If[ps1Tot ≤ 0, errorBar / 10 000., ps1Tot],
  ps2Tot = If[ps2Tot ≥ 1, 1, ps2Tot],
  ps2Tot = If[ps2Tot ≤ 0, errorBar / 10 000., ps2Tot],

  Step1 = If [ps1Tot == errorBar / 10 000., Step1 / 1.2, Step1],
  Step2 = If [ps2Tot == errorBar / 10 000., Step2 / 1.2, Step2],
  (*VERBOSE MODE: UNCOMMENT THE FOLLOWING LINE:*)
  (*Print[ "ps1Tot= ",ps1Tot, "  ps2Tot= ",ps2Tot,"    ee=", ee,
   "      Step1=", Step1, "      Step2=", Step2, "      Step=", Step],*)
  (*Print["partialSums[1]= ",partialSums[1]],*)


  (*Step = Abs[Step1],             (* BOTH SHOULD WORK*)*)
  Step = Abs[Step2] + Abs[Step1], (*+Abs[1-NormalizationNew],*)
  (*Step = Min[Abs[Step1],Abs[Step2]],*)
  Step = If[iterationNumber ≥ 1000, 0, Step]
 }],
Print["    ee=", ee, "     beta=", beta, "      MathcalN=",
```

```
      MathcalN, "   ps1Tot= ", ps1Tot, "  ps2Tot= ", ps2Tot],
     Print[ "ps1Tot= ", ps1Tot, "  ps2Tot= ", ps2Tot, "    ee=", ee,
      "      Step1=", Step1, "      Step2=", Step2, "      Step=", Step],
     psMathcalN[psResultsCounter] = MathcalN,
     psResults[MathcalN] = ps1Tot,
     psResultsCounter = psResultsCounter + 1,
     ee = ee + eeStep
    }]]]
cci[MathcalN - 1]
```

```
5
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{psResults[0], psResults[1], psResults[2], psResults[3], psResults[4],
 psResults[5], psResults[6], psResults[7], psResults[8], psResults[9],
 psResults[10], psResults[11], psResults[12], psResults[13], psResults[14],
 psResults[15], psResults[16], psResults[17], psResults[18], psResults[19],
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 psResults[95], psResults[96], psResults[97], psResults[98], psResults[99]}
```

```
{psMathcalN[0], psMathcalN[1], psMathcalN[2], psMathcalN[3], psMathcalN[4],
 psMathcalN[5], psMathcalN[6], psMathcalN[7], psMathcalN[8], psMathcalN[9],
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 psMathcalN[88], psMathcalN[89], psMathcalN[90], psMathcalN[91],
 psMathcalN[92], psMathcalN[93], psMathcalN[94], psMathcalN[95],
 psMathcalN[96], psMathcalN[97], psMathcalN[98], psMathcalN[99]}
```

0

Maximum Capital= 20.1312 average capital= $\frac{19\,074.4}{NN}$

ee=1.2 beta=2. MathcalN=32 ps1Tot= 0.942214 ps2Tot= 0.212088

ps1Tot= 0.942214 ps2Tot= 0.212088 ee=1.2 Step1=
0.0000472485 Step2=-0.0000408227 Step=0.0000880711

Maximum Capital= 20.1312 average capital= $\frac{17\,884.5}{NN}$

ee=1.2 beta=2.2 MathcalN=32 ps1Tot= 0.947173 ps2Tot= 0.21977

ps1Tot= 0.947173 ps2Tot= 0.21977 ee=1.2 Step1=
0.0000472485 Step2=-0.0000408227 Step=0.0000880711

Maximum Capital= 20.1312 average capital= $\frac{16\,927.5}{NN}$

ee=1.2 beta=2.4 MathcalN=32 ps1Tot= 0.951883 ps2Tot= 0.226085

ps1Tot= 0.951883 ps2Tot= 0.226085 ee=1.2 Step1=
0.0000472485 Step2=-0.0000408227 Step=0.0000880711

Maximum Capital= 20.1312 average capital= $\frac{16\,152.2}{NN}$

ee=1.2 beta=2.6 MathcalN=32 ps1Tot= 0.955601 ps2Tot= 0.230947

ps1Tot= 0.955601 ps2Tot= 0.230947 ee=1.2 Step1=
0.0000472485 Step2=-0.0000408227 Step=0.0000880711

Maximum Capital= 20.1312 average capital= $\frac{15\,518.3}{NN}$

ee=1.2 beta=2.8 MathcalN=32 ps1Tot= 0.958093 ps2Tot= 0.234143

ps1Tot= 0.958093 ps2Tot= 0.234143 ee=1.2 Step1=
-0.0000566982 Step2=0.0000340189 Step=0.000090717

Maximum Capital= 20.1312 average capital= $\frac{14\,994.7}{NN}$

ee=1.2 beta=3. MathcalN=32 ps1Tot= 0.959586 ps2Tot= 0.235916

ps1Tot= 0.959586 ps2Tot= 0.235916 ee=1.2 Step1=
0.0000472485 Step2=0.0000489872 Step=0.0000962357

Maximum Capital= 20.1312 average capital= $\frac{14\,557.4}{NN}$

$Aborted

20.1312

0

Maximum Capital= 20.1312 average capital= $\frac{19\,074.4}{NN}$

ee=1.2 beta=2. MathcalN=32 ps1Tot= 0.942214 ps2Tot= 0.212088

ps1Tot= 0.942214 ps2Tot= 0.212088 ee=1.2 Step1=
0.0000472485 Step2=-0.0000408227 Step=0.0000880711

Maximum Capital= 20.1312 average capital= $\frac{17\,884.5}{NN}$

```
    ee=1.2     beta=2.2      MathcalN=32   ps1Tot= 0.947173  ps2Tot= 0.21977

ps1Tot= 0.947173  ps2Tot= 0.21977    ee=1.2      Step1=
 0.0000472485      Step2=-0.0000408227      Step=0.0000880711

Maximum Capital= 20.1312        average capital=16 927.5
                                                  NN

    ee=1.2     beta=2.4      MathcalN=32   ps1Tot= 0.951883  ps2Tot= 0.226085

ps1Tot= 0.951883  ps2Tot= 0.226085    ee=1.2      Step1=
 0.0000472485      Step2=-0.0000408227      Step=0.0000880711

Maximum Capital= 20.1312        average capital=16 152.2
                                                  NN

    ee=1.2     beta=2.6      MathcalN=32   ps1Tot= 0.955601  ps2Tot= 0.230947

ps1Tot= 0.955601  ps2Tot= 0.230947    ee=1.2      Step1=
 0.0000472485      Step2=-0.0000408227      Step=0.0000880711

Maximum Capital= 20.1312        average capital=15 518.3
                                                  NN

    ee=1.2     beta=2.8      MathcalN=32   ps1Tot= 0.958093  ps2Tot= 0.234143

ps1Tot= 0.958093  ps2Tot= 0.234143    ee=1.2      Step1=
 -0.0000566982      Step2=0.0000340189      Step=0.000090717

Maximum Capital= 20.1312        average capital=14 994.7
                                                  NN

    ee=1.2     beta=3.      MathcalN=32   ps1Tot= 0.959586  ps2Tot= 0.235916

ps1Tot= 0.959586  ps2Tot= 0.235916    ee=1.2      Step1=
 0.0000472485      Step2=0.0000489872      Step=0.0000962357

Maximum Capital= 20.1312        average capital=14 557.4
                                                  NN

    ee=1.2     beta=3.2      MathcalN=32   ps1Tot= 0.960186  ps2Tot= 0.236351

ps1Tot= 0.960186  ps2Tot= 0.236351    ee=1.2      Step1=
 -0.0000393737      Step2=0.0000489872      Step=0.0000883609

Maximum Capital= 20.1312        average capital=14 188.4
                                                  NN

    ee=1.2     beta=3.4      MathcalN=32   ps1Tot= 0.960305  ps2Tot= 0.235694

ps1Tot= 0.960305  ps2Tot= 0.235694    ee=1.2      Step1=
 -0.0000393737      Step2=0.0000489872      Step=0.0000883609

Maximum Capital= 20.1312        average capital=13 873.6
                                                  NN

    ee=1.2     beta=3.6      MathcalN=32   ps1Tot= 0.960007  ps2Tot= 0.234516

ps1Tot= 0.960007  ps2Tot= 0.234516    ee=1.2      Step1=
 0.0000472485      Step2=0.0000489872      Step=0.0000962357

Maximum Capital= 20.1312        average capital=13 602.7
                                                  NN

    ee=1.2     beta=3.8      MathcalN=32   ps1Tot= 0.959266  ps2Tot= 0.232778

ps1Tot= 0.959266  ps2Tot= 0.232778    ee=1.2      Step1=
 -0.0000566982      Step2=-0.0000408227      Step=0.0000975208

Maximum Capital= 20.1312        average capital=13 367.2
                                                  NN
```

```
    ee=1.2     beta=4.      MathcalN=32   ps1Tot= 0.958418  ps2Tot= 0.230904

ps1Tot= 0.958418  ps2Tot= 0.230904    ee=1.2      Step1=
 -0.0000393737      Step2=-0.0000587846      Step=0.0000981584

Maximum Capital= 20.1312        average capital= 13161./NN

    ee=1.2     beta=4.2      MathcalN=32   ps1Tot= 0.957459  ps2Tot= 0.228997

ps1Tot= 0.957459  ps2Tot= 0.228997    ee=1.2      Step1=
 -0.0000393737      Step2=-0.0000587846      Step=0.0000981584

{692.050346, Null}

20.1312
```

```
For[ MathcalN = 0, MathcalN < 100, MathcalN++, {Print[psResults[MathcalN], "  "]}]
For[ MathcalN = 0, MathcalN < 100,
 MathcalN++, {Print[psMathcalN[MathcalN], "  "]}]
psResults[32]
psMathcalN[]


Plot[NNi[i], {i, 0, MathcalN}]
Plot[cci[i], {i, 1, MathcalN}]
Plot[NNi[i] cci[i], {i, 1, MathcalN}]
```