Statistical Mechanics of Economic Systems

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ABSTRACT

Short summary of the contents of your thesis.

To someone special

ACKNOWLEDGEMENTS

Put your acknowledgements here.

DECLARATION

Put your declaration here.

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ACRONYMS

DRY Don't Repeat Yourself

API Application Programming Interface

UML Unified Modeling Language

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ECONOMICS AND THE GENERAL EQUILIBRIUM THEORY

In this chapter we will discuss some of the economic theories behind the work of the thesis. Mainly, we will introduce General Equilibrium Theory and some of its main theorems.

General Equilibrium Theory is a mature and consolidated field of Economics [1, 5, 6] which aims to characterize the existence and properties of equilibria in certain market settings. Economic systems are assumed to frequently have actors with opposing goals: the owner of a good wants to sell it for the highest possible price, while its potential buyers would like to purchase it for as low as possible. Fishermen would like to catch as many fishes as possible as long as their peers care to not also overdo it otherwise they may extinguish the oceans. In this way, one expects economies and markets to converge to a certain steady state and among other things, General Equilibrium Theory characterizes these steady states in a rigorous manner. In this sense, it's also a theory in Microeconomic, because it explains macrobehavior from the incentives of microscopic agents.

In General Equilibrium Theory, an economy is defined through the following components: we assume I consumers, J firms and L goods. Each consumer has a consumption set X_i which contains all possible consumption bundles $x_i = (x_i^1, \dots x_i^L)$ that the consumer has access to, ie, each bundle x_i is a $L-$dimensional vector with nonnegative entries (we are assuming he cannot consume a negative amount of a good). X_i is limited by "physical" contraints, such as no access to water or bread, but not monetary constraints, which will arise later.

The consumer also has an utility function $U_i(x)$ that takes every element $x_i \in X_i$ to a real number, representing how much the consumer values each bundle of his consumption set. This allows us to define define a preference relationship over the elements in X_i (ie, if the consumer prefers bundle x to x'), which is *complete*[1] and *transitive*[2], two standard requirements in Economics for rational behavior.

Finally, the consumer is also endowed with an initial bundle of goods $\omega_i = (\omega_i^1, \dots, \omega_i^L)$, $\omega_i^l \geq 0$ which will define his budget given a set of prices for the goods and will constraint his choices on X_i.

Each firm j has a production set Y_j of technologies $y_j = (y_j^1, \dots, y_j^L)$ which it is able to operate. Unlike consumption bundles, which are final allocations and therefore must be nonnegative, technologies can be any real number: the negative entries are inputs and the positive entries are outputs that the firm can operate. Y_j is also limited only by "physical" constraints, not by monetary constraints.

[10] [9] [8] [5] [7, 3] [11, 4, 12]

1 For every $x, x' \in X_i$, either $U_i(x) \geq U_i(x')$ or $U_i(x) \leq U_i(x')$.
2 For every $x, y, z \in X_i$, if $U_i(x) \geq U_i(y)$ and $U_i(y) \geq U_i(z)$, then $U_i(x) \geq U_i(z)$.

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THE RANDOM LINEAR ECONOMY MODEL

In this chapter we will discuss the Random Linear Economy model [2] presented by Andrea De Martino, Matteo Marsili and Isaac Pérez Castillo which will be the basis of some of the applications discussed in the rest of this thesis.

An economy in this model is defined by M goods, N firms, with a technological density parameter of $n = N/M$, and one representative consumer. The consumer has an initial random wealth $x_0 = (x_0^1, \ldots, x_0^M)$, $x_0^\mu \geq 0$ drawn independently from an exponential distribution, and wishes to improve its welfare in the market according to a separable utility function $U(x) = \sum_{\mu=1}^{M} u(x^\mu)$.

Each firm in the market has a random technology $\xi_i = (\xi_i^1, \ldots, \xi_i^\mu)$, where $\xi_i^\mu < 0$ represents an input and $\xi_i^\mu > 0$ represents an output. Each firm i only has one technology, and its only decision is the scale s_i at which it operates this technology, ie, each company represents a transformation in the space of goods given by the M dimensional vector $s_i\xi_i$. The elements ξ_i^μ are drawn from a $\mathcal{N}(0, M^{-1})$ normal distribution, normalized so that $\sum_{\mu=1}^{M} \xi_i^\mu = -\epsilon$, that is, all technologies are a little inefficient. This is too guarantee that there's no way to combine two (or more) technologies and produce infinite goods.

This economy is closed and therefore we must have a market clearing condition: the N dimensional production scale vector s has to be such that

$$x = x_0 + \sum_{i=1}^{N} s_i \xi_i \tag{1}$$

ie, all the inputs the firms use have to come from the consumer's initial endowment. As with traditional General Equilibrium scenarios, the market clearing condition makes it so that solving the consumer maximum utility problem, $x^* = \arg\max_x U(x)$, simultaneously solves the firms maximum profits problem, $s_i^* = \arg\max_{s_i} s_i p_i \cdot \xi_i$, with the prices being set also by the first order condition of the consumer's maximization problem, $p_\mu = \frac{\partial U}{\partial x_\mu}$. In brief, market clearing makes so that the firms production and the market prices are set to satisfy consumer's desired demand, and no actor in the market has an incentive to deviate from this equilibrium.

Market clearing also carries a strong restriction. If we multiply both sides of the equation (1) by p, we get

$$p \cdot (x - x_0) = \sum_i s_i p \cdot \xi_i, \tag{2}$$

The left side of the above equation has to always be smaller or equal to zero, because of the budget condition. But the right hand side has to be always greater or equal to zero, because this term represents the sum of the individual firms' profits and they can always choose $s_i = 0$ if a firm is losing money. Therefore, we must have that both sides are equal to zero, and the consequence is that the agent completely spends all his available budget (ie, $p \cdot x = p \cdot x_0$, he has not "leftover" cash after choosing x) and that the firms either have zero profit ($p \cdot \xi_i = 0$) or leave the market ($s_i = 0$).

One of the important implications of equation (2) is that we may not have more than M firms active at any given equilibrium realization. This is because $p \cdot \xi_i = 0$ is an equation on p_μ with M variables, and if we have more than M equations, this system has no solution.

The model has some very interesting properties which are described at length in [2]. In particular, it's possible to analytically calulate the distribution probabilities of x and s (and therefore of p) and see that all macroscopic quantities derived from these two quantities depend on the number of firms per good $n = N/M$. The model displays a regime change at $n = 2$, ie, two random technologies per good. When $n < 2$, the market is competitive and the fraction of active firms $\phi = \sum_i \mathbb{I}(s_i > 0)/N$ is around $\phi = 0.5$. Because each firm has on average half the goods as inputs and half as outputs, when $n < 2$ you don't have enough firms to span the whole M dimensional space in order to be able to fine tune the quantities desired for all the goods.

When $n > 2$, however, there are many firms to choose from and statistically it's possible to choose M linear independent firms that span the whole good space. In this regime, the market becomes monopolistic and ϕ assymptotically goes to zero with n, due to a saturation of active firms on M.

The change in the model economy's GDP also reflects the qualitative change in allocation. The authors define the gross product for the model as the total value of goods produced, that is, the sum of $(x_\mu - x_0^\mu)p_\mu$ for all goods μ that are produced, ie, $x_\mu > x_0^\mu$. However, the market clearing condition (2) makes the value of goods produced equal to the value of goods used as input, so we calculate the GDP Y by averaging over the absolute value of all trades:

$$Y = \frac{\sum_{\mu=1}^{M} |x_\mu - x_0^\mu| p_\mu}{2\sum_{\mu=0}^{M} p_\mu}, \tag{3}$$

where the denominator also includes a normalization for the prices.

What is shown in [2] is that in the competitive regime when $n < 2$, a new firm will have a significant positive effect on Y, while in the monopolistic regime $n > 2$ a new firm will have negligible impact on the gross product. We will revisit this result later in this paper.

The Random Linear Economies model is particularly suitable for further analysis because it's a General Equilibrium setting with few ingredients, but the introduction of stochastic elements offers a non-trivial phase transition which is not observed in similar "simple" economic models in the literature.

Part II

APPLICATIONS

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