# **An Accuracy Argument for Self-Trust**

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December 11, 2024

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**Seld-Doubt and Self-Trust** 

## **Self-Doubt**

Two kinds of self-doubt

- 1. **Alethic self-doubt**: doubting that my beliefs are accurate.
- 2. **Normative**: doubting that my beliefs are rational.

I will focus on alethic self-doubt here.

#### **Rational Self-Doubt**

It seems rational to doubt the accuracy of my own beliefs.

- Plenty of evidence that I have been wrong, and that my peers are wrong.
- Preface-like cases: I'm confident that some of my beliefs about biology are *false* (e.g. "Mammals don't lay eggs").
- Cartesian Circle: No non-circular way to rule out the possibility that our beliefs are thoroughly inaccurate.





### **Irrational Self-Doubt**

Some cases of extreme self-doubt seem irrational.

E.g. believing a Moorean sentence:

"It's raining, but it's not the case that I believe that it's raining".

 $\mathsf{Bel}(A \land \neg \mathsf{Bel}(A))$ 



## Questions

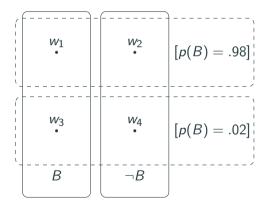
- Why are certain kinds of self-doubt irrational? Why is some amount of self-trust rationally required?
- How much may we rationally doubt ourselves? What is the minimum amount of self-trust that is rationally required?
- What about graded doxastic states? E.g. being very confident in "It's raining, and I'm very confident that it's not raining" seems nearly as bad as believing a Moorean sentence.

**Goal**: Use accuracy to answer these questions.

#### **Notation**

- $W = \{w_1, ..., w_n\}$  finite set of *possible worlds*.
- Greek letters  $\pi, \gamma$  denote *rigidly designated credence functions*, i.e. vectors in  $\mathbb{R}^n$ .
- Latin letters p, q denote definite descriptions of credence functions.
  - p is a function from possible worlds to credence functions. So  $p(w_i)$  is a credence function for every  $w_i \in \mathcal{W}$ .
  - Can think of them as vector-valued random variables.
  - Abuse notation:  $p_i$  instead of  $p(w_i)$ .
- If  $\phi$  is a property of credence functions,  $[\phi(p)]$  is the proposition  $\{w_i : \phi(p_i)\}$

- p = My radiologist's credence function.
- B = I have a broken bone.



$$p_1 = p_2 = (.97, .01, .01, .01)$$
  $p_3 = p_4 = (.01, .01, .01, .97)$ 

## A self-trust requirement

- Let *p* be a definite description of your credence function.
- Let  $\pi$  be your actual credence function (i.e.  $\pi = p_i$  where  $w_i$  is the actual world)

#### **Total Trust**

You Totally Trust yourself iff:

$$\mathbb{E}_{\pi}(X|[\mathbb{E}_{p}(X) \ge r]) \ge r \tag{1}$$

whenever  $X: \mathcal{W} \to \mathbb{R}$ ,  $r \in \mathbb{R}$ , and the above conditional expectation is defined.

I want to argue that rational agents Totally Trust themselves.

# **Consequences of Total Trust**

Coherence + Total Trust  $\implies$  you cannot be maximally confident in A as well as in  $[p(A) \le low]$ . Because:

$$\pi(A \land [p(A) \le \mathsf{low}]) = 1 \tag{2}$$

$$\iff \frac{\pi(A \land [p(A) \le \text{low}])}{\pi([p(A) \le \text{low}])} = 1$$
(3)

$$\iff \pi(A|[p(R) \le \text{low}]) = 1 > \text{low}$$
 (4)

which violates Total Trust.

More generally: Coherence + Total Trust  $\implies$  you cannot be very highly confident of both A and  $[p(A) \le low]$ .

Total Trust rules out high confidence in Moore-like sentences.

**An Accuracy Argument for Total** 

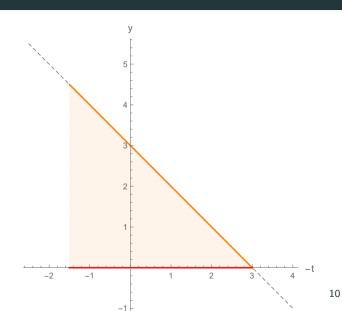
**Trust** 

## **Generalised Strictly Proper Scores**

How inaccurate is expectation  $\mathbb{E}_{\pi}(X)$  when X has value  $X(w_i) = x_i$ ?

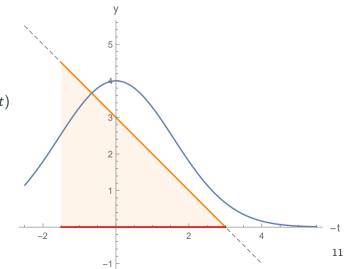
- Interpret  $\mathbb{E}_{\pi}(X)$  as a unique fair price for gamble X.
- (X-t) is desirable whenever  $t<\mathbb{E}_{\pi}(X)$
- "Add up" the losses resulting from these desirability judgements when  $w_1$  is the case.

- $W = \{w_1, w_2\}.$
- X = (-3, 6).
- $\pi = (1/2, 1/2)$ , so  $\mathbb{E}_{\pi}(X) = 1.5$ .
- Say  $w_1$  is the case, so X = -3



$$S(\mathbb{E}_{\pi}(X),x_i)=\int_{x_i}^{\mathbb{E}_{\pi}(X)}-(x_i-t)\lambda(dt)$$

Different  $\lambda$  yield different GSP measures of inaccuracy.



## An Accuracy Argument for Total Trust

## Theorem (Dorst et al. 2012, Th.3.2)

 $\pi$  Totally Trusts p iff for every GSP measure of inaccuracy,  $\pi$  expects p to be at least as accurate as  $\pi$ .

- Suppose I don't Totally Trust myself, i.e.  $\pi$  does not Totally Trust p.
- Then there is a rigidly designated credence function  $\pi$  (e.g. (1/3, 1/3, 1/3)) that I think is more accurate than p under some GSP measure S.
- I expect that I would be more accurate, as measured by S, by having credence function  $\pi$  at all possible worlds!

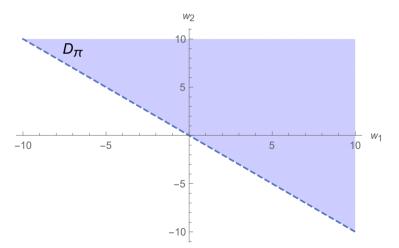
## **Problem**

Why should we care about that measure?

Improving the Argument

# From credences to desirability judgements

The desirability judgements induced by a coherent credence function  $\pi$  via its expectation  $\mathbb{E}_{\pi}$ , interpreted as unique fair price, are **extremely structured**.



## From credences to desirability judgements

Represent opinions via sets of desirable gambles for more expressive power.

**Subtle Point**: We want to show that **rational** agents Totally Trust themselves.

- Rational agents have coherent and (for this talk) precise doxastic states.
- So we need to show that agents with coherent, precise doxastic states should trust themselves.
- Your beliefs are still representable by a coherent credence function  $\pi$ .
- Added expressive power lets us consider ways your beliefs could be that don't correspond to any coherent credence function.

# From credences to desirability judgements

We can express Total Trust in desirability terms.

- Let p be a definite description of your credence function.  $D_p = \{X : p(X) > 0\}$  is a definite description of a set of gambles.
- Let  $\pi$  be your actual credence function, rigidly designated. So  $D_{\pi} = \{X : \pi(X) > 0\}$  is a rigidly designated set of gambles.

#### **Total Trust**

 $\pi$  Totally Trusts p iff:

$$X \in D_{\pi(\cdot|[X \in D_p])} \tag{5}$$

whenever  $X: \mathcal{W} \to \mathbb{R}$ ,  $r \in \mathbb{R}$ , and the above conditional expectation is defined.

# From GSP to INSERT NAME HERE measures of inaccuracy

We can use [INSERT NAME HERE] to measure the inaccuracy of an arbitrary set of desirable gambles at a world.

- For every *X* which you find desirable, you get a penalty if *X* is not actually desirable.
- For every X which you don't find desirable, you get a penalty if X is actually desirable.

$$S(\pi, w_i) = \int_{D_{w_i} \sim D_{\pi}} x_i d\mu - \int_{D_{\pi} \sim D_{w_i}} x_i d\mu$$
 (6)

#### **GSP vs INSERT NAME HERE**

#### GSP:

- The single value  $\mathbb{E}_{\pi}(X)$  determines the desirability of all gambles in form (X-t).
- These judgements jointly determine the inaccuracy of the expectation value  $\mathbb{E}_{\pi}(X)$ .
- Structural assumption: desirable gambles are a half-space through the origin.

#### **INSERT NAME HERE**

- Each desirability judgement contributes individually to your total score.
- No structural assumptions on desirability judgements.

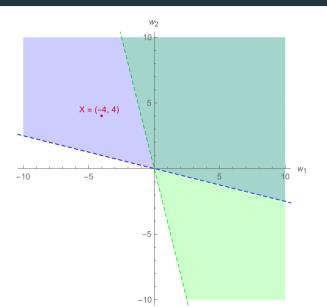
### A useful fact

## Fact 1

If Total Trust fails on some gamble, then it fails on some open set of gambles.

• 
$$W = \{w_1, w_2\}$$

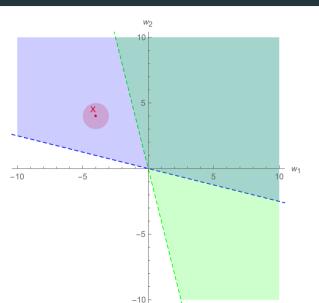
- w<sub>2</sub> is the actual world.
- $p_1 = (.2, .8)$
- $p_2 = \pi = (.8, .2)$
- X = (-4, 4)



• 
$$[X \in D_p] = \{w_1\}.$$

- $\pi(X|\{w_1\}) = -4$ .
- So  $X \notin D_{\pi(\cdot|\{w_1\})}$ .
- Similarly for X + Z, where

 $-\epsilon < Z < \epsilon$ 

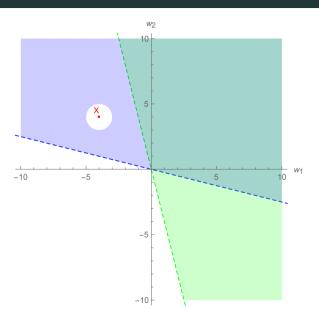


# New accuracy characterisation of Total Trust

- Suppose  $\pi$  does not Totally Trust p.
- Then there is some open set  $\mathcal{O}$  of gambles where Total Trust fails.
- Define:

$$\mathcal{O}^+ = \mathcal{O} \cap D_{\pi}, \quad \mathcal{O}^- = \mathcal{O} \cap D_{\pi}^c$$
 $D_p^* = (D_p \cup \mathcal{O}^+) \sim \mathcal{O}^-$ 

- You actually find the gambles in  $\mathcal{O}^+$  desirable, and those in  $\mathcal{O}^-$  not desirable.
- $D_p^*$  represents the opinions you would have if, at every possible world, you found the gambles in  $\mathcal{O}^+$  desirable and those in  $\mathcal{O}^-$  not desirable.



# New accuracy characterisation of Total Trust

- **Note**: At some possible worlds,  $D_p^*$  denotes an **incoherent** set of desirable gambles!
- But with INSERT NAME HERE we can measure its inaccuracy at all possible worlds!

#### **Theorem**

- 1. If  $\pi$  does not Totally Trust p, then there are measurable sets of gambles  $\mathcal{O}^+, \mathcal{O}^-$  such that  $\pi$  expects  $D_p^*$  to be strictly more accurate than  $D_p$  under every INSERT NAME HERE measure of inaccuracy.
- 2. If  $\pi$  Totally Trusts p, then for any measurable sets of gambles  $\mathcal{O}^+, \mathcal{O}^-, \pi$  expects  $D_p$  to be at least as accurate as  $D_p^*$  under every INSERT NAME HERE measure of inaccuracy.

## The New Argument

- Suppose  $\pi$  does not Totally Trust p.
- Then there is some (rigidly designated!) set of gambles  $\mathcal O$  such that you think you would be more accurate, under **every** INSERT NAME HERE measure of inaccuracy, if you made the same desirability judgements as  $\pi$  over  $\mathcal O$  at every possible world.
- So there is some set of gambles where you expect some rigidly designated credence function (e.g.  $\pi = (1/3, 1/3, 1/3)$ ) to be more accurate than your own credence function p.
- Alternatively: you expect that, if you determinately made the same judgements as  $\pi$  over  $\mathcal{O}$ , you would become more accurate than you are.

## Many open questions...

- Is it really bad to expect some possibly incoherent definite description to be more accurate than you, when you are *certain* to be coherent? I.e. is there any epistemic value to normative certainty?
- How do we determine which doxastic states we should compare your own with when evaluating you?
- What can we say about self-trust for imprecise doxastic states?