An Accuracy Argument for Self-Trust

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Seld-Doubt and Self-Trust

Self-Doubt

Two kinds of self-doubt

1. **Alethic self-doubt**: doubting that my beliefs are accurate.
2. **Normative**: doubting that my beliefs are rational.

I will focus on alethic self-doubt here.

Rational Self-Doubt

It seems rational to doubt the accuracy of my own beliefs.

- **Plenty of evidence** that I have been wrong, and that my peers are wrong.
- **Preface-like cases**: I'm confident that some of my beliefs about biology are *false* (e.g. "Mammals don't lay eggs").
- **Cartesian Circle**: No non-circular way to rule out the possibility that our beliefs are thoroughly inaccurate.

Irrational Self-Doubt

Some cases of extreme self-doubt seem irrational.

E.g. believing a Moorean sentence:
"It's raining, but it's not the case that I believe that it's raining".
$\mathrm{Bel}(A \wedge \neg\mathrm{Bel}(A))$

Questions

- **Why** are certain kinds of self-doubt irrational? Equivalently: why is some amount of self-trust rationally required?
- **How much** may we rationally doubt ourselves? Equivalently: what is the minimum amount of self-trust that is rationally required?
- What about **graded doxastic states**? E.g. being *very confident* in “It’s raining, and I’m *very confident* that it’s not raining” seems nearly as bad as believing a Moorean sentence.

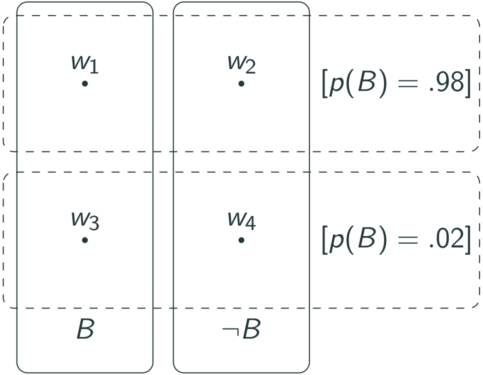
Goal: Use **accuracy** to answer these questions.

Notation

- $\mathcal{W} = \{w_1, ..., w_n\}$ finite set of *possible worlds*.
- Greek letters π, γ denote *rigidly designated credence functions*, i.e. vectors in $\mathbb{R}^n$.
- Latin letters p, q denote *definite descriptions of credence functions*.
  - p is a function from possible worlds to credence functions. So $p(w_i)$ is a credence function for every $w_i \in \mathcal{W}$.
  - Can think of them as vector-valued random variables.
  - Abuse notation: p_i instead of $p(w_i)$.
- If ϕ is a property of credence functions, $[\phi(p)]$ is the proposition $\{w_i : \phi(p_i)\}$

Example

- $p =$ My radiologist's credence function.
- $B =$ I have a broken bone.



$$p_1 = p_2 = (.97, .01, .01, .01) \quad p_3 = p_4 = (.01, .01, .01, .97)$$

A self-trust requirement

- Let p be a definite description of your credence function.
- Let π be your actual credence function (i.e. $\pi = p_i$ where w_i is the actual world)

Total Trust

You Totally Trust yourself iff:

$$\mathbb{E}_\pi(X|[\mathbb{E}_p(X) \geq r]) \geq r \tag{1}$$

whenever $X : \mathcal{W} \to \mathbb{R}$, $r \in \mathbb{R}$, and the above conditional expectation is defined.

I want to argue that rational agents Totally Trust themselves.

Consequences of Total Trust

Coherence + Total Trust $\implies$ you cannot be maximally confident in A as well as in $[p(A) \leq \text{low}]$. Because:

$$\pi(A \wedge [p(A) \leq \text{low}]) = 1 \tag{2}$$

$$\iff \frac{\pi(A \wedge [p(A) \leq \text{low}])}{\pi([p(A) \leq \text{low}])} = 1 \tag{3}$$

$$\iff \pi(A|[p(R) \leq \text{low}]) = 1 > \text{low} \tag{4}$$

which violates Total Trust.

More generally: Coherence + Total Trust $\implies$ you cannot be very highly confident of both A and $[p(A) \leq \text{low}]$.

Total Trust rules out high confidence in Moore-like sentences.

An Accuracy Argument for Total Trust

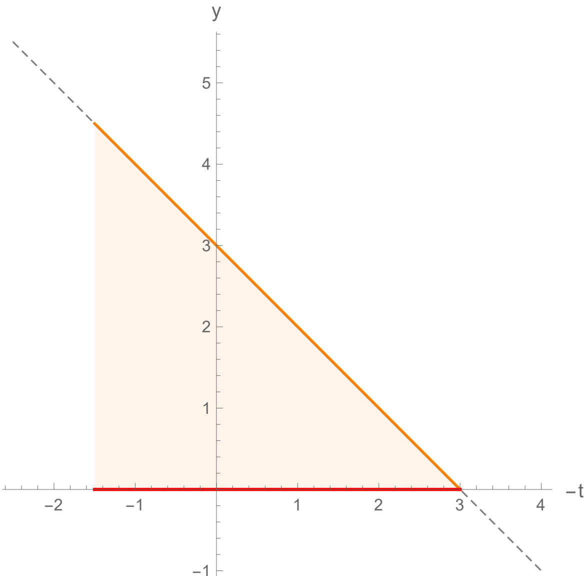
Generalised Strictly Proper Scores

How inaccurate is expectation $\mathbb{E}_\pi(X)$ when X has value $X(w_i) = x_i$?

- Interpret $\mathbb{E}_\pi(X)$ as a *unique fair price* for gamble X.
- $(X - t)$ is desirable whenever $t < \mathbb{E}_\pi(X)$
- "Add up" the losses resulting from these desirability judgements when w_1 is the case.

Example

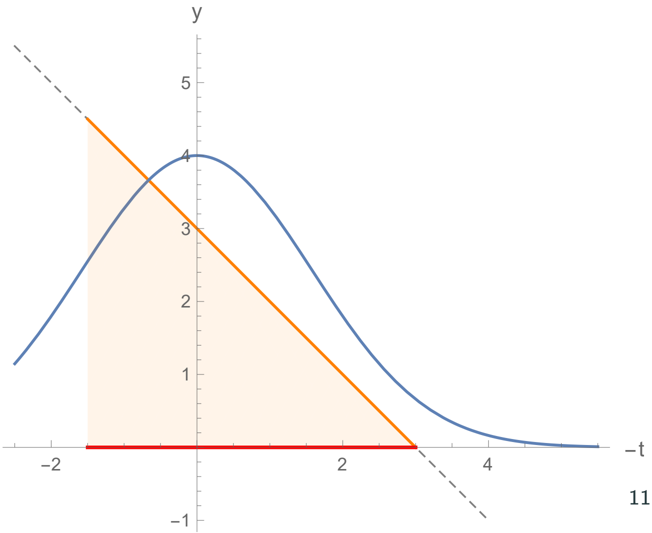
- $\mathcal{W} = \{w_1, w_2\}$.
- $X = (-3, 6)$.
- $\pi = (1/2, 1/2)$, so $\mathbb{E}_\pi(X) = 1.5$.
- Say w_1 is the case, so $X = -3$



Example

$$S(\pi(X), x_i) = \int_{x_i}^{\pi(X)} -(x_i - t)\lambda(dt)$$

Different λ yield different GSP measures of inaccuracy.



An Accuracy Argument for Total Trust

Theorem (Dorst et al. 2012, Th.3.2)

π Totally Trusts p iff for *every* GSP measure of inaccuracy, π expects p to be at least as accurate as π.

- Suppose I don't Totally Trust myself, i.e. π does not Totally Trust p.
- Then there is a rigidly designated credence function π (e.g. $(1/3, 1/3, 1/3)$) that I think is more accurate than p under GSP measure S.
- I expect that I would be more accurate, as measured by S, by having credence function π at all possible worlds!

Problem

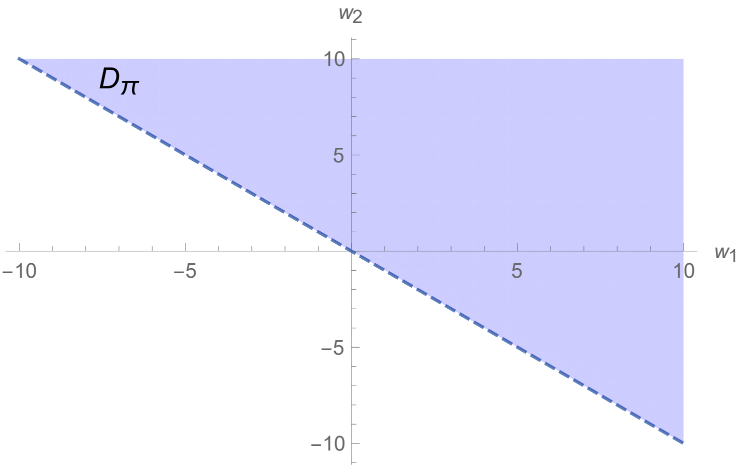
- The theorem shows that, if I don't Totally Trust myself, then I expect some rigidly designated credence function to be more accurate than myself under **some** GSP measure of accuracy.
- But why should we care about **that** measure?

Maybe more abt why this is a problem...

Improving the Argument

From credences to desirability judgements

The desirability judgements induced by a coherent credence function π via its expectation $\mathbb{E}_\pi$, interpreted as unique fair price, are **extremely structured**.



From credences to desirability judgements

Represent opinions via **sets of desirable gambles** for more expressive power.

Subtle Point: We want to show that **rational** agents Totally Trust themselves.

- Rational agents have *coherent* and (for this talk) *precise* doxastic states.
- So we need to show that agents with *coherent*, *precise* doxastic states should trust themselves.
- Your beliefs are still representable by a coherent credence function π.
- Added expressive power lets us consider ways your beliefs **could be** that don't correspond to any coherent credence function.

From credences to desirability judgements

We can express Total Trust in desirability terms.

- Let p be a definite description of your credence function. $D_p = \{X : p(X) > 0\}$ is a definite description of a set of gambles.
- Let π be your actual credence function, rigidly designated. So $D_\pi = \{X : \pi(X) > 0\}$ is a rigidly designated set of gambles.

Total Trust

π Totally Trusts p iff:

$$X \in D_{\pi(\cdot|[X \in D_p])} \tag{5}$$

whenever $X : \mathcal{W} \to \mathbb{R}$, $r \in \mathbb{R}$, and the above conditional expectation is defined.

From GSP to INSERT NAME HERE measures of inaccuracy

We can use [INSERT NAME HERE] to measure the inaccuracy of an arbitrary set of desirable gambles at a world.

- For every X which you find desirable, you get a penalty if X is not actually desirable.
- For every X which you don't find desirable, you get a penalty if X is actually desirable.

$$S(\pi, w_i) = \int_{D_{w_i} \sim D_\pi} x_i \, d\mu - \int_{D_\pi \sim D_{w_i}} x_i \, d\mu \tag{6}$$

GSP vs INSERT NAME HERE

GSP:

- The single value $\mathbb{E}_\pi(X)$ determines the desirability of *all gambles* in form $(X - t)$.
- These judgements jointly determine the inaccuracy of the expectation value $\mathbb{E}_\pi(X)$.
- **Structural assumption**: desirable gambles are a half-space through the origin.

INSERT NAME HERE

- Each desirability judgement *contributes individually* to your total score.
- **No structural assumptions** on desirability judgements.

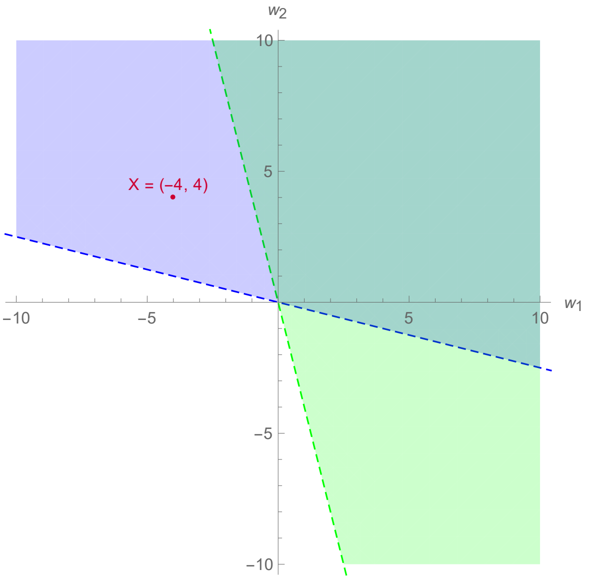
A useful fact

Fact 1

If Total Trust fails on some gamble, then it fails on some open set of gambles.

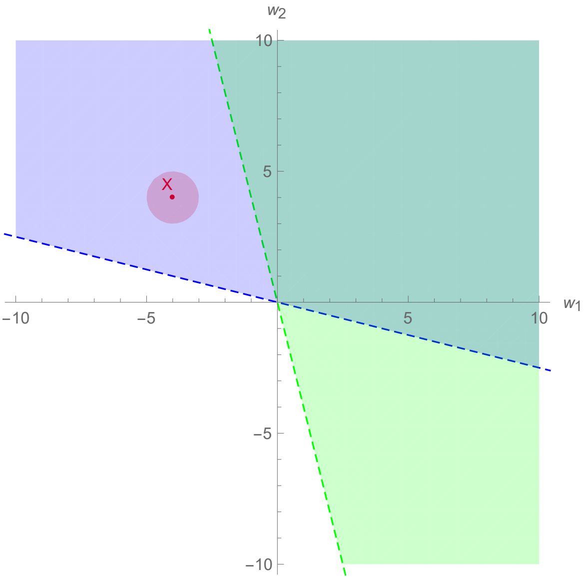
Example

- $\mathcal{W} = \{w_1, w_2\}$
- w_2 is the actual world.
- $p_1 = (.2, .8)$
- $p_2 = \pi = (.8, .2)$
- $X = (-4, 4)$



Example

- $[X \in D_p] = \{w_1\}$.
- $\pi(X|\{w_1\}) = -4$.
- So $X \notin D_{\pi(\cdot|\{w_1\})}$.
- Similarly for $X + Z$, where $-\epsilon < Z < \epsilon$



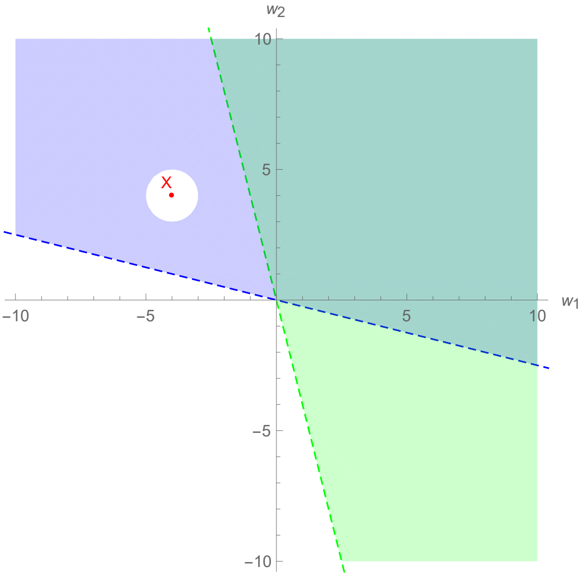
New accuracy characterisation of Total Trust

- Suppose π does not Totally Trust p.
- Then there is some open set $\mathcal{O}$ of gambles where Total Trust fails.
- Define:

$$\mathcal{O}^+ = \mathcal{O} \cap D_\pi, \quad \mathcal{O}^- = \mathcal{O} \cap D_\pi^c$$
$$D_p^* = (D_p \cup \mathcal{O}^+) \sim \mathcal{O}^-$$

- You actually find the gambles in $\mathcal{O}^+$ desirable, and those in $\mathcal{O}^-$ not desirable.
- D_p^* represents the opinions you would have if, at every possible world, you found the gambles in $\mathcal{O}^+$ desirable and those in $\mathcal{O}^-$ not desirable.

Example



New accuracy characterisation of Total Trust

- **Note**: At some possible worlds, D_p^* denotes an **incoherent** set of desirable gambles!
- But with INSERT NAME HERE we can measure its inaccuracy at all possible worlds!

Theorem

1. If π does not Totally Trust p, then there are measurable sets of gambles $\mathcal{O}^+, \mathcal{O}^-$ such that π expects D_p^* to be strictly more accurate than D_p under **every** INSERT NAME HERE measure of inaccuracy.
2. If π Totally Trusts p, then for any measurable sets of gambles $\mathcal{O}^+, \mathcal{O}^-$, π expects D_p to be at least as accurate as D_p^* under **every** INSERT NAME HERE measure of inaccuracy.

The New Argument

- Suppose π does not Totally Trust *p*.
- Then there is some (rigidly designated!) set of gambles $\mathcal{O}$ such that you think you would be more accurate, under **every** INSERT NAME HERE measure of inaccuracy, if you made the same desirability judgements as π over $\mathcal{O}$ at every possible world.
- So there is some set of gambles where you expect some rigidly designated credence function (e.g. $\pi = (1/3, 1/3, 1/3)$) to be more accurate than your own credence function *p*.
- Alternatively: you expect that, if you *determinately* made the same judgements as π over $\mathcal{O}$, you would become more accurate than you are.

Many open questions...