Research talk

Towards trustworthy Bayesian networks: using choice functions as local models

Arthur Van Camp

Monday 15 May 2023

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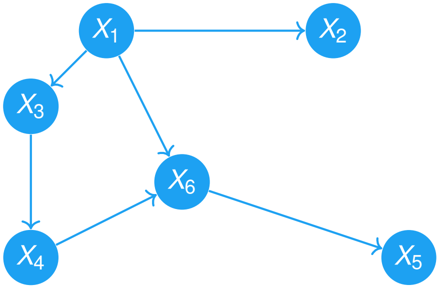
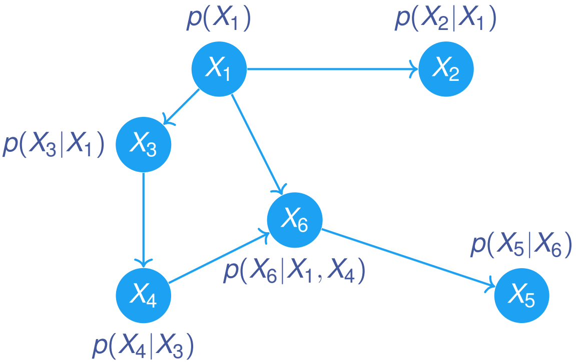
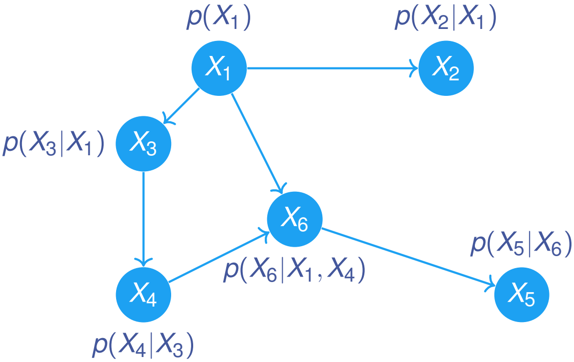
Bayesian networks



Bayesian networks

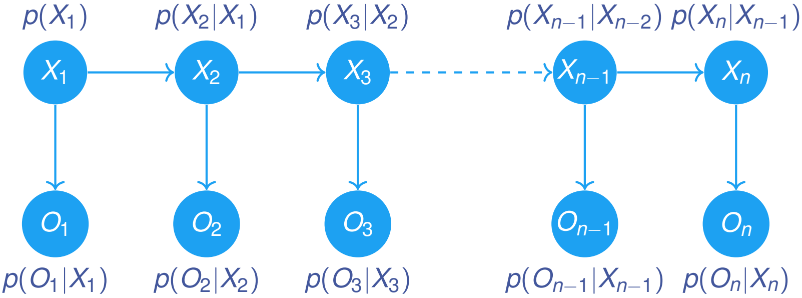


Bayesian networks



$$P(X_1, X_2, X_3, X_4, X_5, X_6) = P(X_1)P(X_2|X_1)P(X_3|X_1)P(X_4|X_3)P(X_5|X_6)P(X_6|X_1, X_4)$$

Bayesian networks



Specific Bayesian network: hidden Markov model

Towards trustworthy Bayesian networks:
using choice functions as local models

Choice functions

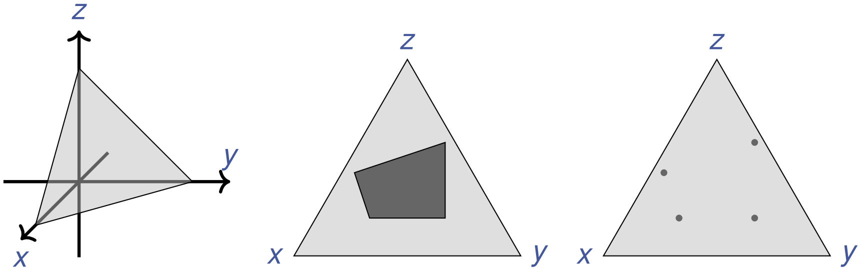
A choice function C maps any finite option set A to a subset $C(A)$:

$$C\colon \mathscr{Q} \to \mathscr{Q}\colon A \mapsto C(A) \subseteq A.$$

Typically, options f are gambles: real-valued functions of an uncertain variable X.

The non-chosen options $R(A) \coloneqq A \setminus C(A)$ are the rejected options.

Properties of choice functions



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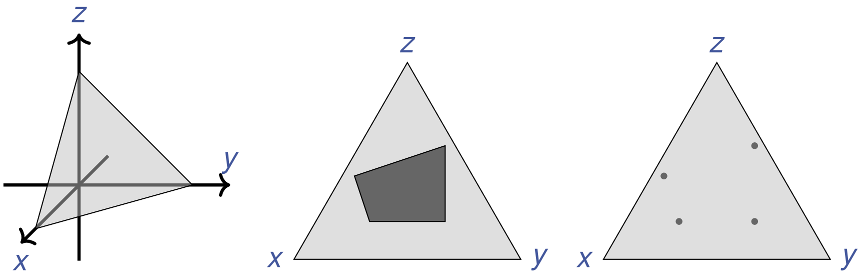
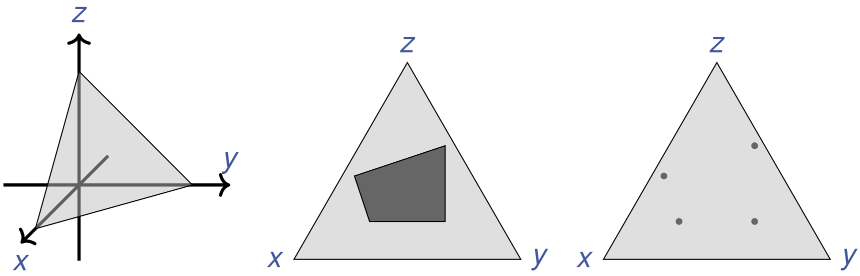


Choice functions give an operational meaning to non-convex sets of probabilities.

Properties of choice functions



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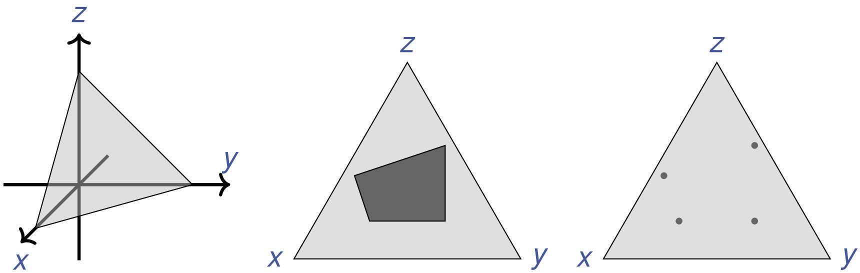

Choice functions give an operational meaning to non-convex sets of probabilities.

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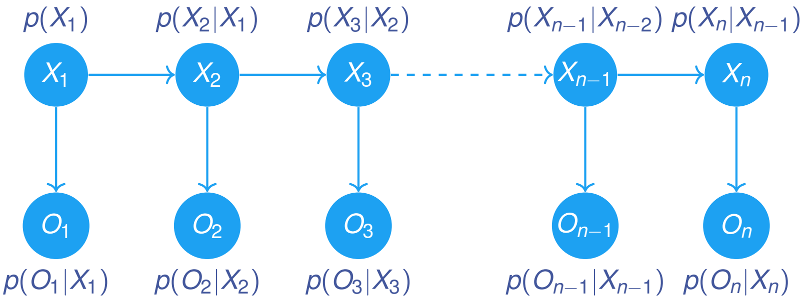
They yield a logic of desirability [work in progress: ‘OR’ statements].

Challenges

Combination into a joint

Doing inferences with the joint

Challenges: Combination into a joint



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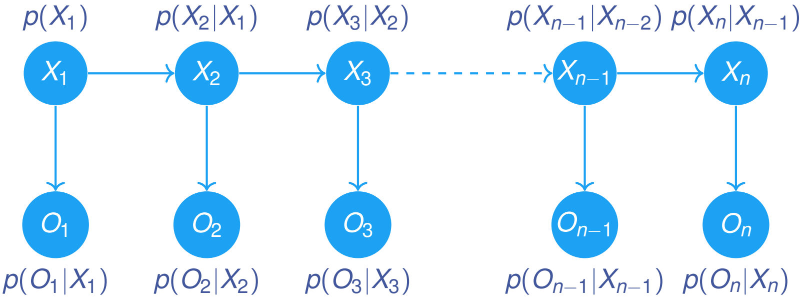

Combination of O_n with X_n: marginal extension (Bayes's rule):

$$p(X_n, O_n|X_{n-1}) = P(X_n|X_{n-1})P(O_n|X_n, X_{n-1}) = P(X_n|X_{n-1})P(O_n|X_n)$$

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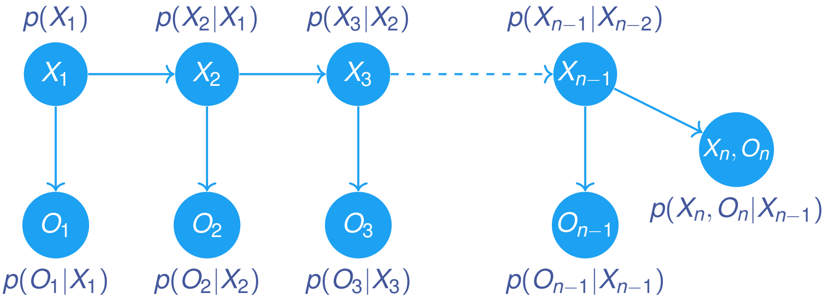

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For choice functions: Enrique Miranda & Arthur Van Camp, 'A Study of Jeffrey's Rule With Imprecise Probability Models', Accepted for ISIPTA 2023

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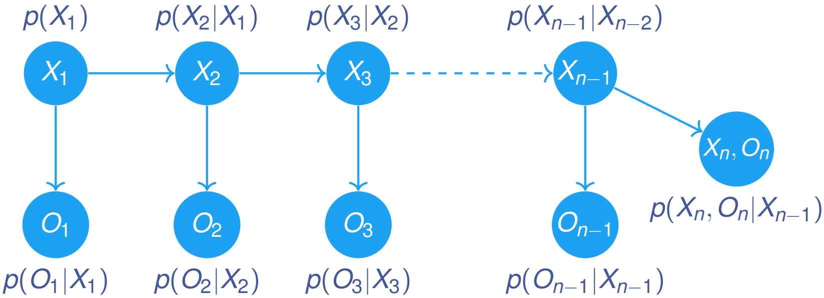

Combination X_n, O_n with O_{n-1}: use conditional independence

$$p(X_n, O_n, O_{n-1}|X_{n-1}) = p(X_n, O_n|X_{n-1})p(O_{n-1}|X_{n-1})$$

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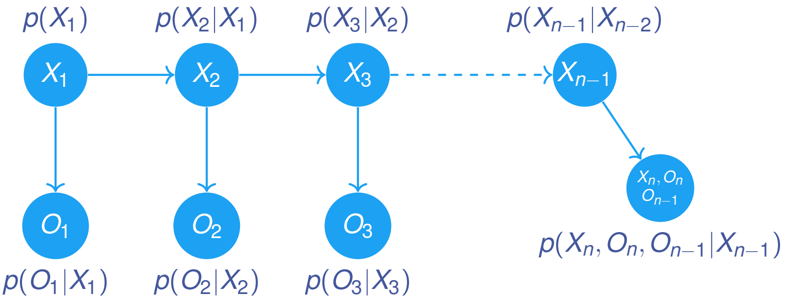

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For choice functions: Arthur Van Camp, Kevin Blackwell & Jason Konek, 'Independent Natural Extension for Choice Functions', IJAR 2023

Challenges: Doing inferences with the joint

Two operations:

Conditioning Arthur Van Camp & Enrique Miranda, ‘Modelling epistemic irrelevance with choice functions’, IJAR 2020

Marginalisation Arthur Van Camp, ‘Choice Functions as a Tool to Model Uncertainty’, PhD thesis 2018

Challenges: Doing inferences with the joint

These operations are instances of natural extension. Given information of the form

$$
\begin{aligned}
&R(A_1) \text{ is rejected from } A_1\\
&R(A_2) \text{ is rejected from } A_2\\
&\dots\\
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$$

is option f admissible within another option set A?

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$$

is option f admissible within another option set A?

Under some additional assumptions: linear programming.

Even convexity Arthur Van Camp & Teddy Seidenfeld, ‘Exposing some points of interest about non-exposed points of desirability’, IJAR 2022