Towards an Ontology of Roles and States

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Abstract

Facts (and, more generally, states of affairs) are plausibly individuated in an at least moderately coarse-grained way: the fact that this rose is red should not be distinguished from the fact that this rose fails to fail to be red. Something similar can be said for properties and relations. It is relatively easy to formulate principles that entail that the individuation of states of affairs, properties, and relations—in brief, intensional entities—is coarse-grained enough to conform to this idea. It is less easy to devise a complementary principle that imposes an *upper* bound on the coarse-grainedness of intensional entities. A promising approach to this problem is to formulate a principle according to which the *fundamental* entities (including fundamental properties and relations) are freely recombinable or 'logically independent'; but certain special provisions that have to be made with regard to relations tend to render such a principle unattractively complex. This paper attempts to find a simpler principle, based on a positionalistic conception of relations.

Keywords: states of affairs; properties; relations; positionalism; instantiation

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> Fruchtbaren Begriffsbildungen und Schlußweisen wollen wir, wo immer nur die geringste Aussicht sich bietet, sorgfältig nachspüren und sie pflegen, stützen und gebrauchsfähig machen.
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> – David Hilbert, 'Über das Unendliche'

> Die Wahrheit ist eben kein Kristall, den man in die Tasche stecken kann, sondern eine unendliche Flüssigkeit, in die man hineinfällt.
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> – Robert Musil, *Der Mann ohne Eigenschaften*

1 Introduction

Just as set theory is useful for mathematics and, at least indirectly, for any kind of quantitative science, so the theory of *intensional entities*—states of affairs, properties, and relations—is useful in semantics and various areas of philosophy.[1] But how are such entities to be individuated? If we help ourselves to a formal language whose formulas denote states of affairs, then we can say that, at least provisionally, no two states of affairs (or 'states' for short) are denoted by logically equivalent formulas. This is a reasonable first approximation if we think of obtaining states as ways the world *is*, rather than as ways in which the world might be represented.

Further, to make room for the plausible idea that Socrates' self-identity is distinct from Plato's (in symbols: $(\text{Socrates} = \text{Socrates}) \neq (\text{Plato} = \text{Plato})$), we might choose to weaken the mentioned principle into the claim that no two states are denoted by *analytically* equivalent formulas, where two formulas are 'analytically equivalent' iff (roughly) they are logically equivalent *and* contain free occurrences of the same atomic terms.[2] However, in saying that no two states are denoted by—in whatever sense—equivalent formulas, we are only imposing a *lower* bound on the coarse-grainedness of states. To have a full account of how states are individuated, we also have to specify a complementary *upper* bound. In other words, we need a 'fine-grainedness principle'.

Expanding on a proposal by Bacon (2020), Plate (2025: §5.4) has recently suggested a way in which such a principle might be formulated. The basic idea is that the *fundamental* intensional entities—as well as the non-intensional entities, or *particulars*—are 'logically independent' of each other. For example, suppose that P and Q are two fundamental properties. Then the self-identity of P (i.e., the state $P = P$) is distinct from the self-identity of Q. Very roughly, the principle that has this consequence is to the effect that a state s does not "necessitate" a state s^* unless a certain

[1] With regard to properties and relations, see, e.g., the articles collected in Fisher and Maurin (2024). While states of affairs may be less prominent in current analytic metaphysics, this should not be taken to mean that they are any less important. For instance, if properties and relations can serve as semantic values of predicates, then states of affairs may analogously serve as semantic values of declarative sentences (relative to a context).

[2] This notion of analytic equivalence traces back to Parry (1933); cf. also Parry (1989).

condition is satisfied; and this condition quantifies over formulas (or more generally terms) by which s and s^* can be respectively denoted. In particular, if the formulas in question contain only references to particulars or fundamental intensional entities, and if a few additional conditions are also satisfied, then the principle has it that s does not necessitate s^* *unless* the formula denoting s, possibly in conjunction with one or more "symmetry statements", *analytically entails* the formula denoting s^*.

Now, admittedly, all this is a mouthful, and it stands to become even more involved if the "additional conditions" are fully spelled out. In large part, as we will see, these complications arise from a traditional conception of relations that treats relations as polyadic analogues of properties without providing any deeper metaphysical analysis. This opens the prospect that, by adopting a more detailed conception, we can arrive at a simpler principle and, as a result, at a more satisfactory overall theory.

The paper falls into two parts. The *first* (Sections 2–9) sketches a theory of intensional entities along more-or-less traditional lines, while the second proposes a series of amendments. Section 2 begins by examining the concept of a relation and offers a general answer to the question of how this term may be usefully understood in a metaphysical context. This answer relies on a concept of *instantiation*, as expressed in 'x is an instantiation of y by $z_1, z_2, \ldots$, in this order'. Section 3 then introduces what I shall call *the Default View*: the thesis that this concept of instantiation is metaphysically primitive. Next, Sections 4–9 outline an ontology of intensional entities, complete with the fine-grainedness principle alluded to above. The *second* part (Sections 10–16) develops an alternative ontology, which largely results from abandoning the Default View in favor of a *positionalistic* analysis of the instantiation predicate and a corresponding conception of relations. The payoff is twofold: not only do we gain an explanation of why any converse of a fundamental relation is itself fundamental (Section 13), but we also become able to embrace a considerably more elegant fine-grainedness principle (Section 14). As indicated by the title, the proposed ontology is one "of roles and states". The relevant concept of a *role* can be seen as a generalization of the familiar concept of a property (on which more in Section 10.2). An assumption specifically concerning the individuation of roles is introduced in Section 16, and Section 17 concludes.

Before we go on, a word of caution: the consistency of the theory to be developed in this paper has not been formally established.[3] Instead, my goal here is simply to get the theory on the table and to motivate it philosophically. This way, if at some later point it can be shown that the theory is consistent (or *in*consistent, as the case may be), we will know that we have learnt something of interest.

2 What Is a Relation?

This question is something of a conceptual quagmire. Bertrand Russell, in his 1913 manuscript on the *Theory of Knowledge*, stipulated that

[3]Thanks to a referee for *Philosophical Studies* for raising this point.

> [a]n entity which *can* occur in a complex as "precedes" occurs in "*A* precedes *B*" will be called a *relation*. When it *does* occur in this way in a given complex, it will be called a "relating relation" in that complex. (p. 80)

Several decades later, David Lewis reserved the word 'relation' for "arbitrary classes of ordered pairs, triples ..." (1983: 344), where the ordered pairs and triples in question may have not only actual but also 'merely possible' entities as coordinates. Both of these stipulations are made on the background of contentious metaphysical assumptions: an ontology of 'complexes' in Russell's case, and an ontology of *possibilia* in Lewis's. But it is hardly plausible that our ordinary concept of relation should come with such heavy dialectical burdens. Can we say what a relation is—in the sense of answering not the question of *what it is to be* a relation, but rather that of how the term 'relation' is commonly understood in metaphysical contexts—without making contentious metaphysical commitments? And is there actually some *one* way in which that term is commonly understood in metaphysical contexts, as opposed to a variety of different ways?

The answer to this last question is clearly 'no'. To begin with, there is the *mathematical* conception, under which a relation is simply a set of tuples; its influence is clearly visible in Lewis's work. In addition, there is a conception—or family of conceptions—more recently influential in metaphysics, largely due to Williamson (1985) and Fine (2000), under which some or all relations are "neutral as regards sense", as Russell put it in his manuscript (p. 88). The first of these is quite sharply delineated, the second less so.

A *third* conception relies on the idea that relations (or at any rate *some* of them) are simply the 'semantic values' of transitive English verbs, such as 'loves' or 'admires'. But this is a somewhat naïve thought, since there will always be different viable ways of drawing up a semantics of natural language. (For example, let T be a semantic theory that assigns a certain entity X to the verb 'loves' as the latter's semantic value. With some trivial modifications, it will then be possible to transform T into an equally empirically adequate theory that instead assigns to 'loves' the *singleton* of X.[4]) There is little reason to think that, outside the context of a specific semantic theory, the phrase 'semantic value' has any meaning—any Fregean *Sinn*—that would determinately relate English transitive verbs to other entities.

A *fourth* conception, which I think holds the most promise, relies on the idea that at least some relations are such that some things 'bear' them to others. That a relation can be said to be borne by something *to* something is of course a deeply ingrained aspect of our usage of the term 'relation': already Quintilian speaks of a *relatione ad aliquid* (*Inst.* VIII.iv.21). To clarify what is here meant by 'to bear', we could say that, at least in various paradigmatic cases, a transitive English verb φ corresponds to an entity X in such a way that, for any two denoting noun phrases

[4] In a similar vein, Oliver (1996: 68) reminds us that "the assignment of semantic values is constrained by the goal of describing the truth-conditions of the sentences of the language, but this goal underdetermines that assignment. Anything that does the job will do".

N_1 and N_2, the following holds: the referent of N_1 *bears* X to the referent of N_2 iff the sentence $\ulcorner N_1\ \varphi\text{s}\ N_2 \urcorner$ is true.

If this is enough to fix the extension of 'to bear', then we could proceed to say that a relation is simply anything that is 'of the same kind' as those entities X that are in the relevant sense borne by one thing to another—though admittedly this talk of 'the same kind' introduces a certain degree of vagueness. It may also be objected that in this way we only manage (at best) to fix upon the *dyadic* relations. To overcome this latter limitation, we can switch from 'to bear' to a more flexible locution. Here I will use the verb 'to instantiate', as in, '$x_1, \ldots, x_n$, in this order, instantiate X'. To clarify what is meant by 'to instantiate', we can proceed in largely the same way as we have above in clarifying what was meant by 'to bear'.

I submit that, roughly along these lines, we can capture at least *one* way in which the term 'relation' may be usefully understood in a metaphysical context. From here we can go on to ask further questions. The most interesting and fruitful of these, at least for the metaphysician, may be that of whether 'to instantiate' may be amenable to *metaphysical analysis*: whether something informative can be said as to what it is for some entities $x_1, x_2, \ldots$ to instantiate an entity y.[5] In answering this question, neutrality with regard to our metaphysical commitments is no longer a desideratum.

Suppose we adopt a metaphysic that admits both obtaining and non-obtaining states. Whenever an entity x bears a relation R to an entity y, there will, on such a view, exist an obtaining state (or *fact*) of R's being borne by x to y, as well as a *non*-obtaining state of R's *not* being borne by x to y. Further, on the background of a conception of states as 'worldly' (and correspondingly coarse-grained), the state of R's being borne by x to y, if R is for instance the relation of *loving*, will be nothing else than the state that x loves y. Analogously for other cases.

Let us call the state of R's being borne by x to y—or in other words: the state that x bears R to y—the *instantiation* of R by x and y, in this order.[6] Equipped with this concept, together with that of obtainment, we may now turn around and (transitioning from conceptual to *metaphysical* analysis) say that for x to bear R to y is for there to be an obtaining instantiation of R by x and y. More generally, we may say that for some entities $y_1, y_2, \ldots$, to instantiate a given entity x is for there to be an obtaining instantiation of x by $y_1, y_2, \ldots$ (in this order). And one might further venture a definition of 'relation' under which something is a relation iff it has an instantiation (obtaining or not) by some entities $y_1, y_2, \ldots$ (in this order), provided that the length of the sequence $\langle y_1, y_2, \ldots \rangle$ is greater than 1.

Provided that one is happy to admit states into one's ontology, this should all seem more or less acceptable. However, it is very much a further question whether the primitive concept of instantiation expressed by 'is an instantiation of ... by ..., in this order' is also *metaphysically* primitive, or whether, armed with a sufficiently

[5]This question looms large in Dorr (2004), who considers and discards five different "systems of primitive predicates" that employ between them three different analyses of 'to bear' (§7). His aim is to show that "all relations are necessarily symmetric" (p. 158). For critical discussion, see MacBride (2015).

[6]To reduce clutter, the qualification 'in this order' will be occasionally omitted in the following.

expressive language, one could give an informative account of what it is for something to be an instantiation of something by some (other) things in a given order—in other words, whether one could give a metaphysical analysis.

3 The Default View

Presumably, the default view on this issue—if only because it is not immediately obvious how such an analysis would go—has it that the concept of instantiation is indeed metaphysically primitive. Let us call this *the Default View*. It might be worth pointing out right away that the Default View has relatively little to do with what Kit Fine, in his 'Neutral Relations, has called "the standard view on relations" (2000: 1). According to the latter, "we may meaningfully talk of a relation holding of several objects in a given order" (*ibid.*); but under our current understanding of the term 'relation' (as outlined in the previous section), *this* thesis is a mere truism.[7] By contrast, the Default View bears a certain affinity to what Barry Smith (1997, 2005) has denigratingly called 'fantology', meaning

> a doctrine to the effect that the key to the ontological structure of reality is captured syntactically in the 'F*a*' (or, in more sophisticated versions, in the 'R*ab*') of first-order logic, where 'F' stands for what is general in reality and '*a*' for what is particular. (2005: 153)

The Default View has at least one major shortcoming, already hinted at in the Introduction: it forces unattractive complications upon an otherwise plausible account of the individuation of states. Before we can say what those complications are, we will have to lay some conceptual groundwork, which will occupy us over the remainder of this and the next five sections. We begin with several definitions that directly or indirectly employ the concept of instantiation.

First, on the background of a metaphysic that admits both obtaining and non-obtaining states, we define the concept of an entity's *adicity*:

Definition 3.1. An ordinal α is an *adicity* of an entity x iff the following two conditions are satisfied:

(i) If $\alpha = 0$, then x is a state.

(ii) If $\alpha > 0$, then, for some α-sequence (i.e., sequence of length α) of entities $x_1, x_2, \ldots$, there exists an instantiation of x by $x_1, x_2, \ldots$, in this order.[8]

[7] Fine himself has "no quarrel with the standard view as such" (2000: 2) but argues, rather, that the standard view has to be *supplemented* by an alternative conception of relations. For some critical discussion of Fine's argument, see Liebesman (2014).

[8] Another way to put this, following Karp (1964), is as follows:

> If $0 < \alpha$, then, for some sequence $\langle x_\xi \mid \xi < \alpha \rangle$, there exists an instantiation of x by $x_0, \ldots, x_\xi, \ldots$, in this order.

I trust that the present notation does not cause any confusion. (Thanks to a referee for pressing me

As usual, following von Neumann (1923), we take an *ordinal* to be the set of its predecessors. Further, a *sequence* will be taken to be a function defined on an ordinal, *functions* will be conceived of as sets of ordered pairs, and—following Kuratowski (1921)—an *ordered pair* (a, b) will be taken to be the set $\{\{a\}, \{a, b\}\}$. In addition, the *length* of a given sequence will be the ordinal on which that sequence (*qua* function) is defined.

On the basis of the concept of an entity's adicity, we can next introduce the notions of property, attribute, and relation:[9]

Definition 3.2. Something is a *property* iff it has an adicity of 1.

Definition 3.3. Something is a *relation* iff it has an adicity of at least 2.

Definition 3.4. Something is an *attribute* iff it has an adicity of at least 1.

In other words, an entity x is a *property* iff there exists an entity y such that x has an instantiation by y; it is a *relation* iff, for some ordinal α greater than 1, there is an α-sequence of entities $y_1, y_2, \ldots$ such that x has an instantiation by $y_1, y_2, \ldots$, in this order; and it is an *attribute* iff it is either a property or a relation. Thus the present definition of 'relation' conforms to what has been said at the end of the previous section; and it is only natural to define the concept of a property in an analogous fashion. (Incidentally, since nothing that has been said so far rules out that an entity can have more than one adicity, we have not so far ruled out that there may be properties that are *also* relations.)

We will further need the concept of an f-*converse*, where f is a permutation on an ordinal, i.e., a bijective function that has an ordinal as both domain and range:

Definition 3.5. For any ordinal $\alpha > 0$, any permutation f on α, and any y: an entity x is an f-*converse* of y iff y is α-adic and, for any α-sequence of entities $z_0, z_1, \ldots$ and any state s, the following holds: s is an instantiation of x by $z_{f(0)}, z_{f(1)}, \ldots$, in this order, iff s is an instantiation of y by $z_0, z_1, \ldots$, in this order.

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on this.) Beyond merely notational differences, some readers may also prefer a 'modal' approach:

> If $\alpha > 0$, then it is *possibly* the case that, for some α-sequence of entities $x_1, x_2, \ldots$, there exists an instantiation of x by $x_1, x_2, \ldots$, in this order.

The reason why I would resist this suggestion is that I regard the relevant concept of metaphysical possibility as relatively murky. If an entity is such that it only *possibly* has an instantiation by some α-sequence of entities $x_1, x_2, \ldots$, then we can still call it 'possibly α-adic'; and it seems to me that this usage would be more transparent than the suggested alternative. A possible motivation for a modal definition of 'adicity' may lie in an intuition to the effect that properties and relations have their respective adicities not merely accidentally and would retain the latter even in counterfactual circumstances in which there is nothing to have an instantiation by. (Cf. Gilmore [2013: 221].) However, the attempt to do justice to this intuition by adopting a modal definition would render the concept not only more obscure but also more cumbersome to reason with.

[9]For precedent regarding the use of 'attribute', see Carnap (1942: 17). Regarding the distinction between property and relation, cf. also, e.g., Armstrong (1978: 75), who describes properties as "one-particulared or monadic universals" and relations as "two-, three, n-particulared, that is dyadic, triadic, n-adic universals". Armstrong does not, however, explain what he means by 'monadic', 'dyadic', etc.

In the simplest case, we have that $\alpha = 1$, f is the identity function on 1 (i.e., on $\{0\}$), and y is a property. It is easy to see that y is then an f-converse of itself. It is also clear that this fact generalizes to attributes of higher adicities: for any ordinal $\alpha > 0$, any α-adic attribute is an id_α-converse of itself, where id_α is the identity on α.

This last result may be rephrased by saying that every attribute is a *trivial* converse of itself, in the following sense:

Definition 3.6. An entity x is a *trivial converse* of an entity y iff, for some ordinal $\alpha > 0$, α is an adicity of y and x an id_α-converse of y.

Analogously, we define the notion of a *non*-trivial converse:

Definition 3.7. An entity x is a *non-trivial converse* of an entity y iff, for some ordinal $\alpha > 0$ and some permutation f on α that is distinct from id_α, α is an adicity of y and x is an f-converse of y.

And finally, the notion of a converse without qualification:

Definition 3.8. An entity x is a *converse* of an entity y iff, for some ordinal $\alpha > 0$ and some permutation f on α, α is an adicity of y and x is an f-converse of y.

In other words, a converse of a given entity is something that is either a trivial or a non-trivial converse of it. Obviously, every attribute is a trivial converse of itself; and whenever an entity is a converse of another, both entities are attributes. As may also be worth noting, nothing that has been said here rules out that an entity may be both a trivial *and* a non-trivial converse of one and the same attribute.

Indeed such cases are not unfamiliar. The dyadic relation of *adjacency*, for example, is plausibly not only a trivial but also a *non*-trivial converse of itself, since, plausibly, for any entities x and y, and for any state s, the following holds: s is an instantiation of adjacency by x and y, in this order, iff s is an instantiation of adjacency by y and x, in *this* order. In such cases, where an attribute is identical with one of its non-trivial converses, we may say that the attribute in question is *intensionally symmetric*.

4 The Individuation of Attributes

4.1 Properties and sets

As soon as one accepts the existence of *logically complex* attributes, there arises the question of how these things are individuated. Is *failing to fail to be red* the same property as *being red*? Is *failing to be red while being such that Socrates is self-identical* the same property as *failing to be red*? It is hard to be sure about the answers to these questions, but we can make educated guesses. On the approach that I shall adopt in the following, the individuation of states and attributes is *moderately coarse-grained*, so that, with regard to the two questions just raised, the answer to the first will be 'yes' while the answer to the second will be 'no': *failing to fail to be red* is the same property as *being red*, but *failing to be red while being such that Socrates is self-identical* is *not* the same as *failing to be red*.

These answers should not be too surprising. In the first place, a distinction between *failing to fail to be red* and *being red* would be a paradigmatic case of a 'distinction without a difference'. It may be granted that the *concept* of the failure of failing to be red is distinct from the concept of redness (and so the belief that a given rose fails to fail to be red may be distinguished, as it arguably should, from the belief that the rose is red); but the concept of redness is not the same as the *property* of being red. Second, for there to be such a thing as *failing to be red while being such that Socrates is self-identical*, the world intuitively has to be a richer place—in particular, a place in which there is not just redness but also Socrates—than it has to be in order for there to be such a thing as *failing to be red*.[10]

A critic might wonder whether it would not be best simply to reject the existence of logically complex attributes, and thereby spare ourselves the trouble of theorizing about the individuation of those entities. In reply, I would argue that, by reflecting on set theory and its applications, we can find good reason to accept the existence at least of logically complex *properties*. The argument proceeds in two steps. To begin with, we note that set theory plausibly deserves to be regarded as a source of real insight into mathematical questions. It allows us, for instance, to say something useful and informative as to what it means for there to be infinitely many Fs; and moreover, talk of sets presents itself as a natural medium for combinatorial reasoning (as may be seen from the frequent occurrence, in this kind of context, of the factor $\binom{n}{k}$, which gives the number of k-membered subsets of an n-membered set). These and other reasons suggest that we had better accept the existence of sets, or at least of (sufficiently numerous) entities capable of playing the same theoretical role.[11]

So much for the first step. The *second* step consists in the identification—or alternatively, the replacement—of sets with complex properties. This is motivated by the observation that the traditional Cantorian concept of a set is often thought to be shrouded in mystery: one would like to be told more as to what sort of thing a set *is* before one is happy to accept, e.g., that there is such a thing as an empty set. A proposal by Carnap (1947: §23) shows a way to dispel the mystery: we can conceive of sets as *disjunctive properties*, so that, e.g., the set $\{a, b\}$, for any a and b, is taken to be the property of *being identical with either a or b*. The special case of the *empty* set can be handled analogously by identifying $\emptyset$ with the uninstantiable property of *not being anything at all*.[12] Apart from helping to demystify the concept of a set,

[10] For related discussion, see, e.g., Plate (2016: §2.4).

[11] For relevant recent discussion concerning mathematical objects in general, see, e.g., Paseau and Baker (2023), Baron (2025).

[12] To see the analogy, one has to be prepared to countenance such a thing as the *empty disjunction*: the state that is the negation of the empty *con*junction. A conjunction of states obtains iff each of its conjuncts does; a *dis*junction iff at least one of its disjuncts does. Accordingly, the *empty* conjunction obtains no matter what, and the empty disjunction trivially *fails* to obtain. Now consider the property of *being either a or b*, and represent it as '$\lambda x\,((x = x) \wedge ((x = a) \vee (x = b)))$'. (The addition of the '$x = x$' should seem harmless, and is only needed to make sure that we still have a property when the disjunction that forms the second conjunct is empty.) Next, delete the two disjuncts, which yields

this proposal has the advantage of making for a cleaner ontological picture, in that it integrates the set-theoretic universe into the ontology of intensional entities. To be sure, a theorist might find the identification of sets with properties unpalatable and might therefore prefer to *replace* sets with the properties with which they are under the present proposal identified.[13] But either way, there would now seem to be at least *some* dialectical pressure to admit logically complex properties into our ontology. As a result it should appear worthwhile to investigate what a detailed account of their individuation might look like.

4.2 Attributes and their instantiations

It is a natural thought that states of affairs are zero-adic analogues of attributes: just as an attribute is a way for an individual thing to be or for some things to be related, so a state may be considered a way for the *world* to be. Accordingly, states should be just as coarse- or fine-grained as attributes. This thought can of course be turned around: attributes should be just as coarse- or fine-grained as states. And indeed, rather than to formulate principles that are directly concerned with the coarse- or fine-grainedness of *attributes*, it will be easier to formulate principles governing the individuation of *states* and to let the individuation of attributes fall out of the former via a separate 'bridge principle'.

While the formulation of principles governing the individuation of states requires some formal machinery that will be introduced further below, a useful bridge principle can be formulated already here. The basic idea, familiar at least since Ramsey (1925/1931: 35), is that no two attributes have all their instantiations in common. More precisely:

(A0′) For any ordinal α and any attributes A and B: if A is α-adic and the following holds—

> (∗) For any α-sequence of entities $x_1, x_2, \ldots$ and any state s: s is an instantiation of A by $x_1, x_2, \ldots$, in this order, iff s is an instantiation of B by $x_1, x_2, \ldots$, in this order.

—then B is identical with A.

'$\lambda x\,((x = x) \wedge \bot)$', where '$\bot$' represents the empty disjunction. What this new lambda-expression represents can be described as the property of 'being both self-identical and such that the empty disjunction obtains'. But since the empty disjunction trivially fails to obtain, this property is in an obvious sense uninstantiable. On a moderately coarse-grained conception of properties, it is thus identical with that of *not being anything at all*.

[13] Cf., e.g., Bealer (1982: ch. 5), van Inwagen (2023: 246ff.). The proposed conception of sets has been criticized by, e.g., Black (1971: 624) and Bigelow (1988: 105f.), but none of their objections strikes me as convincing. Bigelow, for instance, argues that "[t]here is no need for any single, 'disjunctive' property shared by" two entities (p. 106). However, if disjunctive properties can play the role of sets, then this should presumably incline us to welcome them into our ontology, precisely for the reasons outlined in the present paragraph.

With the help of a piece of terminology introduced in the previous section, this principle can be more succinctly stated as follows:[14]

(A0) Every attribute has at most one trivial converse.

A corollary of this principle is the thesis that, for any ordinal α and any permutation f on α, any α-adic attribute has at most one f-converse. For if it had two, then these two would be trivial converses of each other (as well as of themselves), thereby contradicting (A0).

As for the individuation of *states*, we will be able to formulate relevant principles only once we have put in place a formal language $\mathcal{L}$ that will allow us to represent the logical structure of intensional entities. This will be done in the next three sections. Following this, Section 8 will contribute some ontological stage-setting. The resulting system will in many ways resemble that of Plate (2025: §§3–4).[15] Meanwhile, readers mainly interested in the present paper's conception of *relations* (without the motivating backstory) are encouraged to skip ahead to Section 10.

5 The Syntax of $\mathcal{L}$

Given that we are conceiving of sets as disjunctive properties in the way indicated in Section 4.1, and given that there are infinitely large sets, our ontology will need to admit 'infinitarily disjunctive' properties. This in turn requires that our language $\mathcal{L}$ contain infinitely long formulas; and more generally, in order not to impose any arbitrary limitations on $\mathcal{L}$'s expressive strength, we should allow infinitely long argument lists as well as infinitely long lists of bound variables. (As will become clearer at the end of Section 8, the expressive strength of $\mathcal{L}$ directly determines the strength of the existential commitments of the ontology to be sketched in that section.)

Anything that is either a variable, a constant, a formula, or a lambda-expression will be called a *term* of $\mathcal{L}$. *Variables* come in two kinds, namely typed and untyped, and the type of a typed variable will be given by a positive ordinal, indicated by a superscript. (We assume that $\mathcal{L}$ has proper-class many variables of each type, as well as proper-class many untyped variables.) While variables will be written as italicized letters, constants will be written as unitalicized strings, such as 'Socrates'. The syntax of complex terms—i.e., of formulas and lambda-expressions—can be recursively specified as follows:

[14]Cf. Plate (2025: §3.3.3). By combining Definitions 3.4–3.6, it can be seen that a trivial converse of an α-adic attribute A (for $\alpha > 1$) is just an attribute B that satisfies clause $(*)$ of (A0′).

[15]There are, however, some deviations. First, the theory presented in the following contains no meaning postulate to the effect that the identity relation is fundamental, and the fine-grainedness principle developed in Section 9.2 below is slightly simpler than its counterpart in §5 of 'Ordinal Type Theory'. And second, although we will rely on the plausible assumption that any converse of a fundamental relation is again a fundamental relation, this assumption will not form any explicit part of the theory. (It *will*, however, be a theorem of the alternative theory developed in the second half of this paper.)

(i) For any term t: $\ulcorner\neg t\urcorner$ is a formula.

(ii) For any terms t_1 and t_2: $\ulcorner(t_1 = t_2)\urcorner$ is a formula.

(iii) For any (possibly empty) sequence of terms $t_1, t_2, \ldots$: $\ulcorner\&(t_1, t_2, \ldots)\urcorner$ is a formula.

(iv) For any (one or more) pairwise distinct variables $v_1, v_2, \ldots$ and any term t: $\ulcorner\exists v_1, v_2, \ldots\ t\urcorner$ is a formula and $\ulcorner\lambda v_1, v_2, \ldots\ t\urcorner$ is a lambda-expression.

(v) For any term t and any (one or more) terms $u_1, u_2, \ldots$: $\ulcorner(t)(u_1, u_2, \ldots)\urcorner$ is a formula.

(vi) Nothing else is a formula or lambda-expression.

This specification is extremely liberal, since even something like '$\neg\lambda x\, y$' counts as a well-formed formula and, moreover, formulas are allowed to occur not only at sentence-position. (This is why they are here counted as terms.) However, well-formedness does not guarantee semantic significance: it will be left to the semantics of $\mathcal{L}$, to be specified in the next section, to determine which terms have a denotation relative to a given interpretation and variable-assignment.

For any term t of $\mathcal{L}$, an occurrence of t will be said to stand *at predicate-position* iff it is the first occurrence of t in an occurrence of $\ulcorner(t)(u_1, u_2, \ldots)\urcorner$, for one or more terms $u_1, u_2, \ldots$; and a term will be said to occur *at sentence-position* iff it appears as an operand of either '$\neg$' or '&' or as the matrix of a term beginning with '$\exists$' or 'λ'. The distinction between *bound* and *free* variable-occurrences should be understood in the usual way, with '$\exists$' and 'λ' as the only variable-binding operators.

We will make use of several abbreviatory devices (apart from ellipses). First, where t and u are any terms, we will write $\ulcorner(t \neq u)\urcorner$ to abbreviate $\ulcorner\neg(t = u)\urcorner$. Second, for any sequence of terms $t_1, t_2, \ldots$ of a length greater than 1, we will write $\ulcorner(t_1 \wedge t_2 \wedge \ldots)\urcorner$ and $\ulcorner(t_1 \vee t_2 \vee \ldots)\urcorner$ to abbreviate $\ulcorner\&(t_1, t_2, \ldots)\urcorner$ and $\ulcorner\neg\&(\neg t_1, \neg t_2, \ldots)\urcorner$, respectively. And finally, for any variables $v_1, v_2, \ldots$ and any term t, we will write $\ulcorner\forall v_1, v_2, \ldots\ t\urcorner$ to abbreviate $\ulcorner\neg\exists v_1, v_2, \ldots\ \neg t\urcorner$. Parentheses will usually be omitted where this does not give rise to ambiguity. (In particular, we will omit parentheses around atomic terms at predicate-position.)

As noted above, $\mathcal{L}$ is intended to "allow us to represent the logical structure of intensional entities". By contrast, the ontological assumptions to be adopted in Section 8 will, like (A0) above, be formulated in *English*. This should not cause any confusion. $\mathcal{L}$ is not meant to be theorized in, but rather to serve as a tool for the representation of intensional entities. This tool will be useful not only for the ontology to be sketched in Section 8, but also for the definition of 'order of' to be given in Section 7 and the account of the individuation of states to be formulated in Section 9. In a nutshell, quantifying over terms of $\mathcal{L}$ will be a convenient way of talking about the ways an intensional entity might be 'constructed' from its 'constituents'.

6 The Semantics of $\mathcal{L}$

The semantics of $\mathcal{L}$ determines what terms denote what entities relative to a given interpretation and variable-assignment. Here an *interpretation* is any function defined on a set of constants, while a *variable-assignment* is any function defined on a set of variables. Several logico-metaphysical notions will be taken as primitive. Apart from the already-familiar concepts of instantiation, state, and obtainment, these are: (i) the concept of the *negation* of a state; (ii) the concept of the *identity* of an entity with itself or another entity;[16] (iii) the concept of the *conjunction* of a (possibly empty) set of states; and (iv) the concept of an attribute's *existential quantification*.[17]

To specify the semantics of $\mathcal{L}$, we now stipulate that, for any interpretation I and any variable-assignment g, the following holds:

(i) For any constant c and any x: c denotes x relative to I and g—or in short: c denotes$_{I,g}$ x—iff I maps c to x.

(ii) For any variable v and any x: v denotes$_{I,g}$ x iff g maps v to x and, if v is typed, x is of some order *less* than the type of v.[18]

(iii) For any term t and any x: $\ulcorner \neg t \urcorner$ denotes$_{I,g}$ x iff t denotes$_{I,g}$ a state of which x is the negation.

(iv) For any terms t_1 and t_2, and any x: $\ulcorner (t_1 = t_2) \urcorner$ denotes$_{I,g}$ x iff, for some entities y_1 and y_2: t_1 denotes$_{I,g}$ y_1, t_2 denotes$_{I,g}$ y_2, and x is the identity of y_1 and y_2.

(v) For any (possibly empty) sequence of terms $t_1, t_2, \ldots$ and any x: $\ulcorner \&(t_1, t_2, \ldots) \urcorner$ denotes$_{I,g}$ x iff there exists a set S of states such that: each t_i denotes$_{I,g}$ some member of S, each member of S is denoted$_{I,g}$ by some t_i, and x is the conjunction of S.

(vi) For any (one or more) pairwise distinct variables $v_1, v_2, \ldots$, any term t, and any x: $\ulcorner \exists v_1, v_2, \ldots\ t \urcorner$ denotes$_{I,g}$ x iff $\ulcorner \lambda v_1, v_2, \ldots\ t \urcorner$ denotes$_{I,g}$ an attribute of which x is the existential quantification.

(vii) For any term t, any ordinal $\alpha > 0$, any α-sequence of terms $u_1, u_2, \ldots$, and any x: $\ulcorner (t)(u_1, u_2, \ldots) \urcorner$ denotes$_{I,g}$ x iff t denotes$_{I,g}$ an attribute A such that, for

[16]Where x and y are any (not necessarily distinct) entities, the *identity of x and y* is a state that obtains iff x is numerically identical with y. (Cf. assumption (A4) in Section 8 below.)

[17]For precedent regarding this last concept, see, e.g., Chisholm (1976: 119), Bealer (1982: 50). Cf. also assumption (A8) in Section 8 below.

[18]The concept of an entity's order will be formally defined in the next section, based on the present semantics. (As a result, there will be a certain interdependence between semantics and definition; but this should not be taken to mean that the definition will be in a problematic way *circular*.)

some α-sequence of entities $x_1, x_2, \ldots$: each u_i denotes$_{I,g}$ the corresponding x_i, and x is an instantiation of A by $x_1, x_2, \ldots$, in this order.[19]

(viii) For any ordinal $\alpha > 0$, any α-sequence of pairwise distinct variables $v_1, v_2, \ldots$, any term t, and any x: $\ulcorner \lambda v_1, v_2, \ldots\ t \urcorner$ denotes$_{I,g}$ x iff x is an α-adic attribute such that, for any α-sequence of entities $x_1, x_2, \ldots$, and any state s: s is an instantiation of x by $x_1, x_2, \ldots$, in this order, iff, for some variable-assignment h that is just like g except for mapping each v_i to the corresponding x_i, s is denoted$_{I,h}$ by t.

With the semantics of $\mathcal{L}$ thus specified, the next order of business is to define the notion of *order* that is invoked in clause (ii).

7 The Assignment of Orders

An entity's *order* is simply an ordinal. The terminology goes back to the *Principia*; but in contrast to the latter, orders here start from zero.

To define the concept of an entity's order, we will employ that of a *fundamental* entity, which will be taken as primitive:[20]

Definition 7.1. An ordinal α is an *order of* an entity x (alternatively: x is of order α) iff α is the least ordinal β such that, for some interpretation I, variable-assignment g, and term t, the following three conditions are satisfied:

(i) t denotes$_{I,g}$ x.

(ii) For any atomic term u: if either $t = u$ or u has in t a free occurrence at predicate- or sentence-position, then u denotes$_{I,g}$ a fundamental entity.

(iii) For any variable v: if v has in t a bound occurrence at predicate- or sentence-position, then v is typed, and its type is less than or equal to β.

An easily verified consequence of this definition, which will become relevant below, is that any *fundamental* entity is of order zero (or 'zeroth-order'). Another important fact is that *not every* entity is zeroth-order. An example of a first-order property in the present sense would be $\lambda x\, \neg\exists y^1\, ((y^1 = x) \wedge y^1(y^1))$, i.e., the property of *not being identical with any zeroth-order entity that instantiates itself*.[21]

It may be worth emphasizing that orders are not cumulative: a zeroth-order entity is never also a fifth-order entity. However, as becomes clear from clause (ii)

[19] I say 'an instantiation' (and not '*the* instantiation') in order not to prejudge the question of whether there *is* an instantiation of the sort specified. However, by assumption (A9) of Section 8 below, there always exists at most *one* instantiation of a given attribute by a given sequence of entities.

[20] It might be suggested that the concept of a fundamental entity can be analyzed in terms of a primitive notion of *metaphysical grounding* (cf., e.g., Leuenberger [2020]) or in 'broadly logical' terms (e.g., along the lines suggested by my [2016] analysis of logical simplicity); but see Plate (MS[b]).

[21] Cf. Plate (2025: §5.1).

of the above semantics, there is still an element of cumulativity in the fact that a variable of type α (for $\alpha > 0$) 'ranges over' all entities of *any order less than* α—which is to say that, relative to any variable-assignment that maps the variable in question to such an entity, the variable will denote that entity.

8 An Ontology of States and Attributes

Now for some ontology. In addition to Section 4.2's (A0) ("Every attribute has at most one trivial converse"), we will take for granted the following eleven assumptions:

(A1) For any state s, there exists exactly one negation of s.

(A2) The negation of a state s is a state that obtains iff s itself does not obtain.

(A3) For any (not necessarily distinct) entities x and y, there exists exactly one identity of x and y.

(A4) The identity of any entities x and y is a state that obtains iff x is numerically identical with y.

(A5) For any set of states S, there exists exactly one conjunction of (the members of) S.

(A6) The conjunction of a set of states S is a state that obtains iff each member of S obtains.

(A7) For any attribute A, there exists exactly one existential quantification of A.

(A8) The existential quantification of an attribute A is a state that obtains iff A has at least one obtaining instantiation.

(A9) For any attribute A and any sequence of entities $x_1, x_2, \ldots$, there exists at most one instantiation of A by $x_1, x_2, \ldots$, in this order.

(A10) Any instantiation is a state.

(A11) Every attribute has at most one adicity.

A mildly interesting (as well as plausible) consequence of (A11) is that no attribute is a state. This follows because, by the definition of 'adicity' in Section 3 above, states are zero-adic, whereas, by the definition of 'attribute', any attribute has at least one adicity greater than zero. (A11) is not uncontroversial, since it may be held that there are 'multigrade' relations, i.e., relations with more than one adicity.[22] We will briefly return to this issue in Section 10.4.

[22]See, e.g., MacBride (2005: 571ff.). The term has been coined by Leonard and Goodman (1940: 50) for relations "without any fixed degree".

A further assumption is needed to ensure an abundance of attributes (as motivated by the remarks of Section 4.1 above):

(A12) For any interpretation I, variable-assignment g, term t, ordinal $\alpha > 0$, and any α-sequence of pairwise distinct variables $v_0, v_1, \ldots$: if t denotes$_{I,g}$ a state and, for each $i < \alpha$, the following two conditions are satisfied—

(i) v_i occurs free in t.

(ii) If v_i is identical with t or has in t a free occurrence at predicate- or sentence-position, then v_i is typed.

—then there exists at least one attribute A such that, for any α-sequence of entities $x_0, x_1, \ldots$ and any state s, the following holds: s is an instantiation of A by $x_0, x_1, \ldots$, in this order, iff, for some variable-assignment h that is just like g except for mapping each v_i to the corresponding x_i, s is denoted$_{I,h}$ by t.

It may be worth commenting very briefly on the two numbered clauses. Without clause (i), it would be possible to derive the unwelcome result that every state necessitates the existence of any entity whatsoever.[23] The purpose of the second numbered clause, meanwhile, is to prevent (A12) from giving rise to intensional antinomies, such as Russell's paradox of properties or the Russell–Myhill paradox.[24]

9 The Individuation of States

With the semantics of $\mathcal{L}$ and the above ontological assumptions in place, we are now in a position to tackle the task postponed in Section 4.2, which was to formulate principles governing the individuation of states. We will need two of them: a

[23]The relevant concept of necessitation will be formally introduced in Section 9.1 below. To see how the mentioned consequence would arise, let s be any state. Without clause (i), (A12) would then entail the existence of a property whose instantiation by any entity x was nothing but s itself. (This may be seen as follows. Let I be the empty interpretation, let g be a variable-assignment that maps 's' to s, let t be 's', let α be 1, and let v_0 be any variable other than 's'. Then apply the mutilated version of (A12).) Under the above semantics, this property would, relative to a variable-assignment that mapped 's' to s, be denoted by '$\lambda x\, s$'. Further, let x be any entity whatsoever. We then have, on the one hand, that $(\lambda x\, s)(x)$ would necessitate the existence of x; for if '$\lambda x\, s$' had a denotation relative to some variable-assignment, then—to use another concept to be introduced in Section 9.1—the formula '$(\lambda x\, s)(x)$' would *analytically entail* '$\exists y\, (y = x)$'. On the other hand, $(\lambda x\, s)(x)$ would be the instantiation of $\lambda x\, s$ by x, and would therefore be nothing other than s itself. It follows that s would necessitate the existence of x. But s was any state, and x was any entity. So any state would necessitate the existence of any entity. What makes this consequence unwelcome is that it would lead to the further result that, e.g., the existence of Socrates (i.e., the state $\exists x\, (x = \text{Socrates})$) and the existence of Plato necessitate each other. Under the coarse-grainedness principle to be adopted in the next section, this would mean that these 'two' states are one and the same, which is implausible on the face of it. (For related discussion, see, e.g., Dorr [2016: 57].)

[24]For details, see Plate (2025: §4.2).

'coarse-grainedness principle' that (as it were) imposes a lower bound on the coarse-grainedness of states, and a complementary *fine*-grainedness principle. We begin with the former.

9.1 The coarse-grainedness principle

As indicated at the beginning of Section 4.1, we would like a principle under which the property of *failing to fail to be red* comes out identical with *being red*; but at the same time the principle should *not* entail that *failing to be red while being such that Socrates is self-identical* is the same as *failing to be red*. A natural way to meet these desiderata relies on a concept of 'analytic entailment', applicable to terms of $\mathcal{L}$:

Definition 9.1. A term t *analytically entails* a term t' iff the following two conditions are satisfied:

(i) t denotes a state relative to some interpretation and variable-assignment.

(ii) For any interpretation I and variable-assignment g, the following holds: if t denotes$_{I,g}$ a state, then so does t'; and if t denotes$_{I,g}$ an *obtaining* state, then so does t'.

From the semantics of $\mathcal{L}$, it can be seen that, for any interpretation I and variable-assignment g, a term will not denote$_{I,g}$ anything at all if it contains a free occurrence of an atomic term that fails to have a denotation relative to I and g. Accordingly, a given term will analytically entail another *only* if any atomic term that occurs free in the latter does so also in the former. Thus the formula '$\neg\text{red}(x)$', for instance, does not analytically entail '$\neg\text{red}(x) \wedge (\text{Socrates} = \text{Socrates})$'. This will become relevant to the second desideratum mentioned above.

In the next step, we use this concept of analytic entailment to define a corresponding notion of necessitation applicable to states:

Definition 9.2. A state s *necessitates* a state s' iff there exist an interpretation I, a variable-assignment g, and terms t and t' such that the following three conditions are satisfied:

(i) t denotes$_{I,g}$ s.

(ii) t' denotes$_{I,g}$ s'.

(iii) t analytically entails t'.

With the help of this concept, we can now state the required principle:

(A13) No two states necessitate each other.

To apply this to the above example, let R be the property of *being red*, and let R' be the property of *failing to fail to be red*, whose existence (given that R exists) follows from (A12) in combination with (A1). Relative to a variable-assignment that maps 'R' to *being red*, this latter property can in $\mathcal{L}$ be denoted by '$\lambda x\, \neg\neg R(x)$'. We can observe straightaway that the formulas '$R(x)$' and '$(\lambda x\, \neg\neg R(x))(x)$' analytically entail each other. From this it can be seen that, for any x, the instantiation of R by x, if it exists, necessitates the instantiation of R' by x, and *vice versa*. By (A13), this means that, for any x and y: y is an instantiation of R by x iff y is an instantiation of R' by x. Hence, R' is a trivial converse of R. But since R is (like any attribute) a trivial converse of itself, it now follows by (A0) that R' and R are one and the same property, as desired.

9.2 The fine-grainedness principle

What (A13) provides is a necessary condition for the distinctness of states: a state s is distinct from a state s' *only* if s and s' do not necessitate each other. This imposes, as we have said, a lower bound on the coarse-grainedness of states. To impose a complementary *upper* bound, one might try to say that a state s is distinct from a state s' *whenever* s and s' do not necessitate each other. But this is just equivalent to the trivial claim that every state necessitates itself. Clearly, we need something more informative.

A more promising approach relies, not on the concept of necessitation, but on the more basic notion of analytic entailment. Suppose we have a variable-assignment g and two terms t and t' that both denote$_{\emptyset,g}$ a certain state s.[25] Might we not simply require that in each such case t should analytically entail t'? But this won't do, either; for suppose that t and t' are two untyped variables that g maps to s. Under the above semantics, t and t' will then both denote$_{\emptyset,g}$ s, and yet t will not analytically entail t'.

The obvious fix is to add a clause to the effect that g maps no two variables to the same entity. The resulting principle reads as follows:

(P) For any variable-assignment g, any terms t and t', and any state s: if t and t' both denote$_{\emptyset,g}$ s and g maps no two variables to the same entity, then t analytically entails t'.[26]

But unfortunately, this fails for a similar reason. Let t again be a variable, let v be some *other* variable, and let t' be the formula $\ulcorner \neg v \urcorner$. If g is the smallest variable-assignment that maps t to s and v to $\neg s$, then the antecedent of (P)'s main conditional will be satisfied; for under our semantics, t' will denote$_{\emptyset,g}$ $\neg\neg s$, which by (A13) is nothing other than s itself, so that t and t' will both denote$_{\emptyset,g}$ s. Yet t will again fail to entail t'.

At this point it becomes attractive to make use of the notion of fundamentality already employed in Section 7 above, and to insert a clause requiring that every entity in the range of g be fundamental. (The previous paragraph's argument will then not go through *unless* it is assumed that both s and $\neg s$ are fundamental.) This yields:

(P′) For any variable-assignment g, any terms t and t', and any state s: if the following three conditions are satisfied—

[25]For present purposes, we need not be concerned with non-empty interpretations.

[26]The 'analytically' will often be suppressed in the following.

(i) t and t' both denote$_{\varnothing,g}$ s.

(ii) Every entity in the range of g is fundamental.

(iii) No two variables are under g mapped to the same entity.

—then t analytically entails t'.

However, this is still only a start. (P′) is much too strong, and has to be weakened in no fewer than four ways. As we will see, two of these have to do with relations.

First, let x be some fundamental entity, let t and t' be (respectively) the formulas '$x = x$' and '$\exists y^1\,(y^1 = x)$', and let g be the smallest variable-assignment under which 'x' is mapped to x. Since x is fundamental, it is zeroth-order. From this, together with (A13), it can be seen that t', like t, denotes$_{\varnothing,g}$ the state $(x = x)$. For let h be a variable-assignment that maps 'x^1' to x. Then the self-identity of x is denoted$_{\varnothing,h}$ by '$x^1 = x^1$', while the state denoted$_{\varnothing,g}$ by t' can analogously be seen to be denoted$_{\varnothing,h}$ by '$\exists y^1\,(y^1 = x^1)$'; and since the formulas '$x^1 = x^1$' and '$\exists y^1\,(y^1 = x^1)$' analytically entail each other, it follows with (A13) that the state denoted$_{\varnothing,g}$ by t' is nothing other than $(x = x)$. The antecedent of (P′)'s main conditional is therefore satisfied; but since not every entity is zeroth-order, t does not analytically entail t'.

This problem can be solved naturally and straightforwardly by adding a further clause to the antecedent of (P′)'s main conditional, requiring that every variable in the domain of g should be of type 1. In the above example, the variable 'x' fails to meet this requirement, since it is untyped.

Second, let x be a fundamental property, let t and t' be (respectively) the formulas '$x^1 = x^1$' and '$x^1 = \lambda y\, x^1(y)$', and let g be the smallest variable-assignment that maps 'x^1' to x. Since x is fundamental, it is zeroth-order, and hence denoted$_{\varnothing,g}$ by 'x^1'. Further, since x is a fundamental (and hence zeroth-order) *property*, it is also denoted$_{\varnothing,g}$ by '$\lambda y\, x^1(y)$', as can be inferred from the semantics of lambda-expressions in combination with (A0). So the antecedent of (P′)'s main conditional is again satisfied; yet, as may be seen with the help of (A11), t does not entail t'.[27] Analogously, if x is a fundamental *state*, then the self-identity of x will be denoted$_{\varnothing,g}$ by both '$x^1 = x^1$' and '$x^1 = \&(x^1)$'; yet the first formula does not entail the second.

A natural solution of this problem would be to add to the antecedent of (P′)'s main conditional a clause to the effect that, for any variable v that occurs free in t and is under g mapped to an intensional entity, at least one free occurrence of v in t stands at predicate- or sentence-position.

[27]To see the relevance of (A11) ("Every attribute has at most one adicity"), note that the lambda-expression '$\lambda y\, x^1(y)$' will (relative to any variable-assignment) denote, if anything, a zeroth-order *property*. But from our ontology, we know that there are also, e.g., zeroth-order states and relations; and (A11) ensures that none of these is a property. (After all, states have an adicity of 0, while relations have one of at least 2. If any of these were a property, then that property would have at least two adicities.) Let then h be some variable-assignment that maps 'x^1' to a state or relation. Then t, i.e., '$x^1 = x^1$', will denote$_{\varnothing,h}$ the self-identity of that state or relation, but t', i.e., '$x^1 = \lambda y\, x^1(y)$', will not denote$_{\varnothing,h}$ anything at all, because '$\lambda y\, x^1(y)$' won't. It follows that t does not analytically entail t'.

Third, suppose that there exists—as for all we know there may—a fundamental dyadic relation R that is distinct from its non-trivial converse $\lambda x, y\, R(y, x)$.[28] Given that R is fundamental, its converse should very plausibly also count as fundamental; for presumably nature does not draw 'invidious distinctions' of the sort whereby, e.g., the relation *longer-than* is fundamental but *shorter-than* isn't.[29] Let now R' be the converse of R, let u and v be two variables of type 1, let t and t' be (respectively) the formulas $\ulcorner u = \lambda x, y\, u(x, y)\urcorner$ and $\ulcorner u = \lambda x, y\, v(y, x)\urcorner$, and let g be the smallest variable-assignment that maps u to R and v to R'. Given (A0), t will then denote$_{\varnothing,g}$ the self-identity of R; and the same goes for t', since $\ulcorner \lambda x, y\, v(y, x)\urcorner$ will denote$_{\varnothing,g}$ the converse of R', which is just R itself. But since there are pairs of dyadic relations that are not converses of each other (as can be seen from (A12) together with other assumptions of our ontology), t does not entail t'.

This problem can be solved fairly straightforwardly by strengthening clause (iii) of (P′): instead of requiring simply that g should map no two variables to the same entity, we can stipulate that g should map no two variables to the same entity *or to relations that are converses of each other*.[30]

Fourth, suppose that there exists a fundamental dyadic relation R that is *identical* with its own non-trivial converse. Let v be a variable of type 1, let t and t' be (respectively) the formulas $\ulcorner v = \lambda x, y\, v(x, y)\urcorner$ and $\ulcorner v = \lambda x, y\, v(y, x)\urcorner$, and let g be the smallest variable-assignment that maps v to R. Given that R is its own converse, t and t' will then both denote$_{\varnothing,g}$ the self-identity of R. But since—given (A12) and other assumptions of our ontology—not *every* zeroth-order dyadic relation is its own non-trivial converse, t does not entail t'.[31]

The solution to this last problem is somewhat more involved. Instead of adding yet another clause to the antecedent of (P′)'s main conditional, we have to weaken its consequent. To this end, let us first introduce three auxiliary notions:

Definition 9.3. A term is *true* relative to an interpretation I and a variable-assignment g—or true$_{I,g}$ for short—iff it denotes$_{I,g}$ an obtaining state.

Definition 9.4. A term t is *ontologically conservative* relative to a term t' iff any atomic term that occurs free in t also occurs free in t'.

Definition 9.5. A *symmetry statement* is a formula $\ulcorner u = \lambda v_0, v_1, \ldots\, u(v_{f(0)}, v_{f(1)}, \ldots)\urcorner$ where, for some ordinal $\alpha > 1$:

- u is an atomic term;

[28] Unless otherwise specified, by 'the converse' of a dyadic relation I will in the following mean its *non*-trivial converse.

[29] For remarks in this direction, see, e.g., Sider (2011: 219), Bacon (2020: 569f.), and Bacon and Dorr (2024: 145n.). Also cf. Plate (2025: §5.2). The present section is essentially based on §5.4 of that same paper, though following a very different order of presentation and leading to a slightly simpler principle.

[30] An analogous move is suggested by Bacon and Dorr (2024: 145n.) in discussing their principle of 'Fundamental Possibility'.

[31] A relevant example of a zeroth-order dyadic relation distinct from its non-trivial converse would be $\lambda x, y\, ((x = x) \wedge (y = \&()))$. (Here '$\&()$' refers to the empty conjunction.)

- $\langle v_0, v_1, \ldots \rangle$ is an α-sequence of pairwise distinct variables, all distinct from u; and
- f is a permutation on α.

A simple example of a symmetry statement is '$R = \lambda x, y\, R(y, x)$', which is ontologically conservative relative to (for instance) the formula '$R = R$'.

With these auxiliary concepts in place, we can now proceed to weaken the consequent of (P′)'s main conditional. Instead of the relatively demanding requirement that t should analytically entail t', we stipulate that a *conjunction* of t and zero or more symmetry statements—where each of these is required to be both true$_{\varnothing,g}$ and ontologically conservative relative to t—should analytically entail t'.[32] By combining this modification with the previous three changes to the antecedent, we arrive at:

(P″) For any variable-assignment g, any terms t and t', and any state s: if the following five conditions are satisfied—

(i) t and t' both denote$_{\varnothing,g}$ s.

(ii) Every entity in the range of g is fundamental.

(iii) No two variables are under g mapped to the same entity or to relations that are converses of each other.

(iv) Every variable in the domain of g is of type 1.

(v) For any variable v: if v occurs free in t and is under g mapped to an intensional entity, then at least one free occurrence of v in t stands at predicate- or sentence-position.

—then there are zero or more symmetry statements $s_1, s_2, \ldots$, each of which both true$_{\varnothing,g}$ and ontologically conservative relative to t, such that $\ulcorner t \wedge s_1 \wedge s_2 \wedge \ldots \urcorner$ analytically entails t'.

Evidently this is even more of a mouthful than the simpler but untenable (P′). It is not exactly plausible that our best account of the individuation of states and attributes should have to include a principle that is quite as inelegant as (P″)—at least not if, by suitable adjustments to our metaphysics, we can find some simpler principle that is able to do the same work. In the next five sections I will try to show that we can, provided that we abandon the Default View and stop thinking of relations as nothing more than polyadic analogues of properties. More specifically, the hope

[32] To see the need for the requirement that the symmetry statements in question should be true$_{\varnothing,g}$, let t be the formula '$R^1 \neq \lambda x, y\, R^1(y, x)$', let t' be the conjunction of t and '$\forall x, y\, R^1(x, y)$', and let g be the smallest variable-assignment that maps 'R^1' to a fundamental dyadic relation R that is distinct from its non-trivial converse. Then the conjunction of t and the *non*-true$_{\varnothing,g}$ symmetry statement '$R^1 = \lambda x, y\, R^1(y, x)$' will be a contradiction, and will hence entail t'. In this way our revised principle would fail to rule out that the fact that R is distinct from its converse is *the same state* as this fact's conjunction with the state $\forall x, y\, R(x, y)$. This would be unwelcome.

is that a more detailed conception of relations will enable us to dispense with any special provisions that (P″) has to make with regard to relations.[33]

10 Positionalism (Broadly Conceived)

The key difference that will distinguish the theory to be developed in the rest of this paper from the one discussed above is that the concept of instantiation will no longer be taken as metaphysically primitive (Section 10.4 below). The resulting view will be a form of what has, in the wake of Fine's 'Neutral Relations', become known as *positionalism*.

Positionalism's main defining feature lies in the fact that it conceives of relations as somehow associated with 'positions' or 'argument-places', considered not as *façons de parler* but as "entities in their own right" (Fine *op. cit.*, p. 16). One might thus characterize it as a view (or family of views) about the 'nature of relations'.[34] However, talk of the nature of relations should arguably be taken with a grain of salt. It suggests, to paraphrase Lewis (1986: 55f.), that we have "fixed once and for all" upon the things we call 'the relations', so that it is now up to us to *discover* what they are, with no room left for stipulative definition. As we will see, there is good reason to be skeptical about this way of looking at the issue, at least if the theory to be developed in the following—which may be best characterized as a view about the nature of *instantiation*—is correct. For if the concept of instantiation fails to be metaphysically primitive, then we should not expect its semantic grip to be so tight as to leave no room for stipulative definition (given that different analyses may be equally viable); and the same goes for the concept of relation.

10.1 Resultance

At the basis of the view to be developed in the following lie the two primitive concepts of *resultance* and *role*: we will say that states, such as Antony's loving Cleopatra, *result* from assignments of entities to *roles*. Here roles take the place of the 'argument-places' familiar from Fine's presentation of positionalism. Relatedly, where we speak of instantiations, Fine speaks of 'completions'. He describes the positionalist as adhering to a conception of completion under which

> the completion of a relation is the state that results from assigning certain objects to the argument-places of the relation; the completion of the neutral amatory relation under the assignment of Don José to *Lover*

[33]While there is more to be said about the truth-conduciveness of theoretical simplicity in metaphysics, a detailed consideration of this methodological issue would here lead too far afield. For recent discussion, see, e.g., Schaffer (2015), Brenner (2017), Sober (2022), Plate (MS[b]: §3.3).

[34]In Fine's presentation, it is moreover central to positionalism that it concerns the metaphysics of 'neutral' relations. ("Under this alternative conception, each neutral relation is taken to be endowed with a fixed number of argument-places or positions", as he writes on p. 10.) This aspect of the view will be elided in the following.

> and Carmen to *Beloved*, for example, is the state of Don José and Carmen standing in the relation of lover to beloved. [...] Completion [...] is relative to an assignment of objects to argument-places. (2000: 13)

The main task of the present section is to introduce a similarly positionalistic conception of completion (or instantiation). Unlike Fine's positionalist, however, we will *not* be concerned with 'neutral' relations but rather retain all the definitions laid down in Section 3 above, including that of 'relation'. In addition, we will keep on board nearly all of the ontological assumptions laid down in Sections 4.2, 8, and 9.1. In particular, we will retain the existence assumption (A12), which commits us, among other things, to the existence of infinitary relations.

Our central primitive concept of resultance will be understood as relating 'role assignments', on the one hand, to states of affairs on the other. To formalize the relevant notion of a role assignment, in a way that will allow us to accommodate infinitary relations, we have to begin by introducing the concept of a *generalized multiset*.

A *multiset* is commonly understood to be an ordered pair (A, m), where A is a set and m a function from A to the set of positive integers. An entity x is said to be a 'member' of a multiset (A, m) iff x is a member of A; and $m(x)$ is also called the 'multiplicity' of x in (A, m), or the number of times that x 'occurs' in that multiset. For present purposes, however, we will need a more general notion that allows m to be any function from A to a set of positive cardinals: thus we arrive at the concept of a *generalized* multiset. Intuitively put, a generalized multiset may contain each of its members any set-sized number of times. Curly brackets will be used to represent both sets and (generalized) multisets, depending on the context. For example, the multiset (A, m) with $A = \{0\}$ and $m = \{(0, 3)\}$ will be denoted by '$\{0, 0, 0\}$'.

The concept of a role assignment may now be defined as follows:

Definition 10.1. A *role assignment* is a non-empty generalized multiset of ordered pairs (r_i, x_i) such that each r_i is a role.

The reason why the entities that we are employing as role assignments cannot simply be *sets* of role–entity pairs (but have to be generalized *multi*sets of such pairs) stems from the fact that, motivated by considerations of ontological parsimony, we will allow that some relations have instantiations resulting from role assignments that pair *more than one* relatum with a single role. A fuller explanation will be given in Section 16 below.

10.2 Properties as roles

In using the term 'role' rather than 'position' or 'argument-place', I am following Sprigge (1970: 69f.) and Armstrong (1978: 94). Another option would have been to borrow a term from the scholastic tradition and to speak of 'relatives'.[35] But this would have been an awkward choice, since *properties* will here be treated as a special

[35]For an introduction to scholastic thinking about relational phenomena, see, e.g., Brower (2024).

case of roles, and to refer to a property such as redness as a 'relative' seems even less apt than to refer to it as a 'role'.

That properties will here be treated as roles calls for extra comment.[36] There are (at least) two ways in which the concept of a property may be generalized. The *first*, with which everyone is already familiar, is with respect to adicity: instead of requiring that the entities in question have an adicity of *exactly* 1, we only require that they have an adicity of *at least* 1. This gives us the notion of an attribute. The *second* way, which will be relevant here, proceeds in two steps. First, we reconceive property-instantiations as *resulting from assignments*. For example, where R is the property of *being red* and r is this rose, we reconceive the state $R(r)$—that this rose is red—as resulting from what might be called a 'property assignment' $\{(R, r)\}$. A *property assignment* is here simply the singleton of a pair (P, x), where P is a property and x some entity. In the second step, we generalize the notion of a property assignment by abandoning not only the requirement that any such assignment have only a single member, but also the requirement that it contain each of its members only once. Instead, we require that the assignment be a *generalized multiset*. We thereby make room for such things as might be called the roles of *Lover* and *Beloved*, with the state of John's loving Mary resulting from the generalized assignment $\{(\mathit{Lover}, \text{John}), (\mathit{Beloved}, \text{Mary})\}$; and we likewise make room for such things as a 'role of adjacency' (which may also be called *Next*), with the state of John's being next to Mary resulting from the assignment $\{(\mathit{Next}, \text{John}), (\mathit{Next}, \text{Mary})\}$. This gives us the concept of a *role*, as well as that of a role assignment.

It is easy to see that, even if every property is a role, the converse need not hold. For, by definition, every property is *monadic*. That is, for every property P, there is at least one entity x such that there exists an instantiation of P by x (cf. Definition 3.1). Under the previous paragraph's reconception of property-instantiations, this further means that, for every property P, there is at least one entity x such that some state results from the assignment $\{(P, x)\}$. Presumably, such roles as *Lover* and *Beloved* do not satisfy this condition; and if so, not every role is a property.

A theorist who prefers to avoid reference to role assignments might rephrase our resultance claims with the help of a more complex multigrade predicate, as in 'x results from assigning z_1 to y_1, z_2 to y_2, z_3 to y_3, ...'. However, since the dyadic 'results from' is in some ways more versatile than its multigrade counterpart, we will here almost exclusively be using the former.[37]

[36] Thanks here to a referee for *Erkenntnis*.

[37] As may be seen from the quotation at the beginning of the previous subsection, *something* like the present concept of resultance can also be found in Fine's presentation of positionalism in §3 of 'Neutral Relations'. Later in the paper, however, he instead employs a triadic concept of *occupation* (as in, 'x occupies α in s', where α is an argument-place and s a state). It is not altogether clear from Fine's exposition whether one of these notions—resultance or occupation—is supposed to be analyzable in terms of the other. In either case, the celebrated objection that he raises in footnote 10 of his paper (discussed in greater detail by Donnelly [2016]) presupposes that the positionalist operates with a concept of occupation. It is vital to this objection that the positionalist should be forced to characterize a certain state *Rabcd* as one in which the objects a, b, c, and d respectively occupy certain positions α, β, γ, and δ, and do so in such a way that a occupies *only* α, b occupies *only* β, and so on—

10.3 What relations could not be

How should a positionalist of the present (i.e., resultance-based) stripe understand talk of instantiation? In the above example, the state that this rose is red was understood both as the instantiation of *being red* by this rose and as resulting from the singleton of the ordered pair (R, r), where R is *being red* and r is this rose. Generalizing from this case, we arrive at a view of property-instantiation under which a given state s is an instantiation of a property P by an entity x iff s results from the singleton of the ordered pair (P, x). But what of *relational* instantiation: instantiation of some entity (which we will call a 'relation') by some entities $x_1, x_2, \ldots$, in this order, where the length of the sequence $\langle x_1, x_2, \ldots\rangle$ is greater than 1?

In §3 of 'Neutral Relations', Fine suggests that the positionalist might think of "each biased relation" as "an ordered pair $\langle R, O\rangle$ consisting of an unbiased [or 'neutral'] relation R and an ordering O of its argument-places" (p. 11). An 'ordering' of two roles r_1 and r_2 may in turn be identified with the ordered pair (r_1, r_2).[38] Thus, letting $\mathscr{A}$ be what Fine calls "the neutral amatory relation", with *Lover* and *Beloved* as its two associated roles, we have that the "biased" relation of *loving* is under this scheme identified with the ordered pair $(\mathscr{A}, (\mathit{Lover}, \mathit{Beloved}))$, which, under a common convention, is the ordered triple $(\mathscr{A}, \mathit{Lover}, \mathit{Beloved})$.

Considerations of ontological parsimony, however, suggest that we dispense with neutral relations and make do with roles alone. Our ordered triple then becomes the ordered pair $(\mathit{Lover}, \mathit{Beloved})$. Why not treat *this* as our relation of *loving*? A corresponding conception of relations can be easily obtained from our definitions of 'relation' and 'adicity' in Section 3 above, if only we adopt a suitable definition of the instantiation predicate. In particular, we could say:

(RI) For any α-sequence of entities $x_0, x_1, \ldots$ with $\alpha > 1$ and for any x: a state s is an *instantiation* of x by $x_0, x_1, \ldots$, in this order, iff x is an α-sequence of roles $r_0, r_1, \ldots$ such that s results from the role assignment $\{(r_i, x_i) \mid i < \alpha\}$.

Given our definitions of 'relation' and 'adicity', this has the consequence that an α-adic relation (for $\alpha > 1$) is just an α-sequence of roles $r_0, r_1, \ldots$ such that, for some

so that, if the relation R exhibits a cyclical symmetry, of a sort that renders *Rabcd* identical with *Rbcda*, the positionalist will not be able to accommodate this identity. For present purposes it is crucial to observe that this objection *has no force* against a positionalist who, instead of relying on a triadic occupation predicate, employs a notion of resultance by which role assignments are linked wholesale to states. This can be seen by simply noting that such a positionalist will be free to characterize the state *Rabcd* as one that results *both* from the assignment $\{(\alpha, a), (\beta, b), (\gamma, c), (\delta, d)\}$ *and* from the assignment $\{(\alpha, b), (\beta, c), (\gamma, d), (\delta, a)\}$. Fine's objection can therefore not be raised against the 'resultance-based' form of positionalism developed in the present paper. (See also Leo [2008: §2.4], who makes essentially the same point.)

Some other objections that have been raised against positionalist views have been discussed by Plate (forthcoming), albeit from the standpoint of a positionalist who operates with a triadic occupation predicate. For discussion of additional varieties of positionalism, see, e.g., Orilia (2011, 2014, forthcoming), Gilmore (2013), Dixon (2018), Litland (2022: §6), Bacon (2024: §12.4), Paolini Paoletti (2024: §§5.4f.).

[38] For conformity of notation, I here use parentheses where Fine uses angled brackets.

entities $x_0, x_1, \ldots$, there exists a state resulting from $\{(r_i, x_i) \mid i < \alpha\}$. However, there is a problem.

Let L be the dyadic relation of *loving*, and suppose that it has an instantiation by any given pair of people. (Any other non-symmetric relation would do as well.) Given (A5) and (A12), there then also exists a tetradic relation $\lambda x, y, z, w\,(L(x, y) \wedge L(z, w))$, whose instantiation by John, Mary, Dee, and Pat, in this order, is the state that John loves Mary and Dee loves Pat. Clearly, the order of these relata matters: by swapping, e.g., John with Mary, we obtain a different state (one in which Mary loves John), which is no longer an instantiation of that same relation by John, Mary, Dee, and Pat, in this order. Similarly for each of the other pairs. Further, under the previous paragraph's proposal, our tetradic relation—call it 'M'—is a sequence of roles $\langle r_1, r_2, r_3, r_4\rangle$. No problem so far.

Crucially, however, by what has just been said about the swapping of relata, our proposal requires that the roles $r_1, \ldots, r_4$ be pairwise distinct. (I delegate the argument to a footnote.[39]) A problem arises if we now consider the sequence $M' := \langle r_3, r_4, r_1, r_2\rangle$. Since the r_i are pairwise distinct, M' is distinct from M; and, given (RI), M' is a relation, too. In particular, just as a given state is an instantiation of M by John, Mary, Dee, and Pat, in this order, iff it results from $\{(r_1, \text{John}), (r_2, \text{Mary}), (r_3, \text{Dee}), (r_4, \text{Pat})\}$, so a state is an instantiation of M' by Dee, Pat, John, and Mary, in *this* order, iff it results from that very same role assignment. In other words:

(1) For any state s: s is an instantiation of M by John, Mary, Dee, Pat iff s results from $\{(r_1, \text{John}), (r_2, \text{Mary}), (r_3, \text{Dee}), (r_4, \text{Pat})\}$.

(2) For any state s: s is an instantiation of M' by Dee, Pat, John, Mary iff s results from $\{(r_1, \text{John}), (r_2, \text{Mary}), (r_3, \text{Dee}), (r_4, \text{Pat})\}$.

Meanwhile, given (A5)—which says that any set of states has *exactly one* conjunction—the instantiation of M by John, Mary, Dee, and Pat, in this order, is the same state as the instantiation of M by Dee, Pat, John, and Mary, in *this* order: each of the 'two' instantiations is just the conjunction of John's loving Mary and Dee's loving Pat. We thus have:

(3) For any state s: s is an instantiation of M by John, Mary, Dee, Pat iff s is an instantiation of M by Dee, Pat, John, Mary.

From (1)–(3), it now follows that a state is an instantiation of M' by Dee, Pat, John, and Mary (in this order) iff it is an instantiation of M by these same relata in the

[39]Suppose for *reductio* that, e.g., r_1 is identical with r_2. Then, given (RI), a state s is an instantiation of M by John, Mary, Dee, and Pat, in this order, iff s results from the assignment $\{(r_1, \text{John}), (r_2, \text{Mary}), (r_3, \text{Dee}), (r_4, \text{Pat})\}$; and a state s is an instantiation of M by Mary, John, Dee, and Pat, in *this* order, iff s results from the assignment $\{(r_1, \text{Mary}), (r_2, \text{John}), (r_3, \text{Dee}), (r_4, \text{Pat})\}$. Given that $r_1 = r_2$, these 'two' assignments are in fact one and the same. Hence any instantiation of M by John, Mary, Dee, and Pat, in this order, is also an instantiation of M by Mary, John, Dee, and Pat, in *this* order. But as we have seen, this is not the case. We can thus conclude that $r_1 \neq r_2$. By analogous reasoning it can be inferred that, for *any* distinct i and j in $\{1, \ldots, 4\}$, r_i is distinct from r_j.

same order; and of course the same holds for any other relata as well. Consequently, M' is a trivial converse of M. By (A0), M' is therefore identical with M, contradicting our earlier result.

10.4 Relations as sets of role sequences

What the above argument shows is that we have to give up *either* the identification of relations with sequences of roles *or* at least one of the relevant ontological assumptions. But those assumptions are rather plausible, or at least theoretically useful. It therefore seems preferable to amend the proposed account of relations. Fortunately, an alternative is close at hand: since the problem arises from the fact that some distinct role sequences always yield identical states (so that treating those sequences as relations will lead to a conflict with (A0)), we may instead want to reconceive relations as *equivalence classes* of such sequences.

As for the relevant sense of equivalence, we would have to say that two role sequences are equivalent iff they consist of the same roles (only differently ordered) and always 'yield' the same states. More formally:[40]

Definition 10.2. For any ordinal $\alpha > 0$, a role sequence σ of length α is *instantiation-equivalent* to a role sequence σ' (notation: $\sigma \cong \sigma'$) iff the following two conditions are satisfied:

(i) There exists a permutation f on α such that $\sigma' = \sigma \circ f$.

(ii) For any state s and any α-sequence of entities $x_0, x_1, \ldots$: s results from the role assignment $\{(\sigma(i), x_i) \mid i < \alpha\}$ iff s results from $\{(\sigma'(i), x_i) \mid i < \alpha\}$.

Two such instantiation-equivalent role sequences are $\langle r_1, r_2, r_3, r_4\rangle$ and $\langle r_3, r_4, r_1, r_2\rangle$ from the previous subsection's example.

Next, let us say that something is a *proto-relation* iff it is a non-empty equivalence class of role sequences of some length greater than 1:

Definition 10.3. An entity x is a *proto-relation* iff there exists a role-sequence σ, of some length greater than 1, such that $x = \{\sigma' \mid \sigma' \cong \sigma\}$.

Clearly, any proto-relation is a set. On this basis we can now define the concept of instantiation:

Definition 10.4. For any ordinal $\alpha > 0$, any entities x and y, and any α-sequence of entities $z_0, z_1, \ldots$: x is an *instantiation* of y by $z_0, z_1, \ldots$, in this order, iff one of the following two conditions is satisfied:

(a) $\alpha = 1$, y is a role, and x results from the role assignment $\{(y, z_0)\}$.

(b) $\alpha > 1$, y is a proto-relation, and there exists an α-sequence $\sigma \in y$ such that x results from the role assignment $\{(\sigma(i), z_i) \mid i < \alpha\}$.

[40] In the following, $\sigma \circ f$ is the function that is obtained by first applying f and then σ. To illustrate: if $f = \{(0,1), (1,2), (2,0)\}$ and $\sigma = \{(0, r_1), (1, r_2), (2, r_3)\}$, then $\sigma \circ f$ is the function $\{(0, r_2), (1, r_3), (2, r_1)\}$.

Let us take a moment to consider some of the consequences of these definitions. First, from clause (a) of Definition 10.4, combined with the definitions of 'adicity' and 'property' in Section 3, we can infer that every property is a role. Second, any α-adic relation (for any $\alpha > 1$) is a set whose members are α-sequences of roles. Many relations will be singletons, such as $\{\langle \mathit{Lover}, \mathit{Beloved} \rangle\}$; but the tetradic relation M from the previous subsection will be a pair set $\{\langle r_1, r_2, r_3, r_4 \rangle, \langle r_3, r_4, r_1, r_2 \rangle\}$.[41]

Third, recall that, under the Carnapian conception of sets adopted in Section 4.1, every set is a property. Consequently, since every relation is a set, it follows that every relation is a property, and hence a role. That every relation is a property further means that every relation has at least *two* adicities, which contradicts (A11). But fortunately, nothing important will be lost if we replace (A11) with:

(A11′) No role is a state.

Since every attribute is a role, this new assumption shares with (A11) the corollary that no attribute is a state.

Fourth, from clause (b) of Definition 10.4 (together with the definition of 'proto-relation'), it can be seen that every relation has exactly one adicity greater than 1. Since every relation is also a property but not (given (A11′)) a state, it further follows that every relation has *exactly* two adicities. That no relation has more than two adicities may be considered problematic by the friend of *multigrade* relations. In one of his objections against positionalism, Fine claims that "[t]here should, for example, be a relation of sup[p]orting that holds between any positive number of supporting objects $a_1, a_2, \ldots$ and a single supported object b just when $a_1, a_2, \ldots$ are collectively supporting b" (2000: 22). But on reflection, this is less than obvious. What there *should* be is an account of how there can be different states in which different numbers of objects perform analogous functions: e.g., a state in which a given piano is supported by three people and another state in which that same piano is supported by five people instead. But there is no good reason to think that such an account can only be given once we have made room for multigrade relations.

In the presence of Definition 10.4, talk of relations can be understood in terms of role sequences. However, this does not yet give us the more elegant fine-grainedness principle that we have been looking for. Before we come to that, we will first need a new formal language: one that allows us to represent the logical structure of states down to the level of roles. To have a name for it, let us call it '$\mathcal{L}^{\mathrm{p}}$'.

[41]A critic might suggest that, even if *non*-fundamental relations such as *brother-of* may be identified with sets of role sequences, such an identification will still be implausible in the case of *fundamental* relations. (Thanks here to a referee for *Philosophical Studies*.) It would be interesting to see this line developed in detail. At any rate, if it is felt that relations—or perhaps only *fundamental* relations—cannot be felicitously identified with sets of role sequences, then one could also reconceive the present proposal as a recommendation that relations be *replaced* with such sets. On this alternative construal, relations would be assigned to the realm of philosophical fiction, while the overall theory would still stand. Thus far, however, I am not convinced that the proposed identification is inadmissible. (The objection might be prompted by the thought that sets are not fundamental entities. However, the phrase 'fundamental relation' is certainly not synonymous with 'relation that is a fundamental entity', just as 'fundamental mistake' is not synonymous with 'mistake that is a fundamental entity'. We will return to the concept of a fundamental relation in Section 13.)

11 The Syntax of $\mathcal{L}^{\mathrm{p}}$

The syntax of $\mathcal{L}^{\mathrm{p}}$—the 'p' stands for 'positionalism'—is similar to that of $\mathcal{L}$, except for two major differences. First, instead of expressions of the form '$R(x_1, x_2, \dots)$', $\mathcal{L}^{\mathrm{p}}$ contains expressions of the form '$(r_1 x_1, r_2 x_2, \dots)$', which are also classified as formulas and serve to denote states by pairing names of roles with names of role bearers. For example, while in $\mathcal{L}$ we might write '$L(\mathrm{Dee}, \mathrm{Pat})$' to denote the state of Dee's loving Pat, in $\mathcal{L}^{\mathrm{p}}$ we would instead write '$(l\,\mathrm{Dee}, b\,\mathrm{Pat})$', with '$l$' and '$b$' standing for *Lover* and *Beloved*, respectively.[42]

Second, we now also have *rho-expressions*, of the form '$\rho x_1.x_2, x_3, \dots\ t$', which serve to denote individual roles. While roles that are *properties* can in $\mathcal{L}^{\mathrm{p}}$ still be denoted by lambda-expressions, this is not in general the case for the constituent roles of relations. For example, relative to a variable-assignment that maps 'l' to *Lover* and 'b' to *Beloved*, the lambda-expression

$$\lambda x, y\, ((lx, by) \wedge (ly, bx))$$

denotes in $\mathcal{L}^{\mathrm{p}}$ the dyadic relation of *loving each other*. Under the present view, this relation is a singleton $\{\langle r, r\rangle\}$, for a certain role r.[43] Further, on the supposition that r is not a property, this role cannot be denoted by a lambda-expression. With the help of rho-expressions, by contrast, it can be denoted by both '$\rho x.y\, ((lx, by) \wedge (ly, bx))$' and '$\rho y.x\, ((lx, by) \wedge (ly, bx))$'.

With this preamble, the syntax of the complex terms of $\mathcal{L}^{\mathrm{p}}$—that is, of formulas, lambda-expressions, and rho-expressions—can be specified as follows:

(i) For any term t: $\ulcorner \neg t \urcorner$ is a formula.

(ii) For any terms t and u: $\ulcorner (t = u) \urcorner$ is a formula.

(iii) For any (possibly empty) sequence of terms $t_1, t_2, \dots$: $\ulcorner \&(t_1, t_2, \dots) \urcorner$ is a formula.

(iv) For any (one or more) pairwise distinct variables $v_1, v_2, \dots$ and any term t: $\ulcorner \exists v_1, v_2, \dots\ t \urcorner$ is a formula and $\ulcorner \lambda v_1, v_2, \dots\ t \urcorner$ is a lambda-expression.

(v) For any ordinal $\alpha > 0$, any α-sequence of terms $t_0, t_1, \dots$, and any α-sequence of terms $u_0, u_1, \dots$: $\ulcorner (t_0 u_0, t_1 u_1, \dots) \urcorner$ is a formula.

(vi) For any (two or more) pairwise distinct variables $v_1, v_2, \dots$ and any term t: $\ulcorner \rho v_1.v_2, v_3, \dots\ t \urcorner$ is a rho-expression.[44]

(vii) Nothing else is a formula, lambda-expression, or rho-expression.

[42]For a broadly similar notation, see Castañeda (1975: 241f.).

[43]Cf. Section 16 below.

[44]The dot (instead of a comma) after the 'v_1' in '$v_1.v_2, v_3, \dots$' is meant to serve as a reminder that, even though the order of the *non*-first elements of the respective list of variables does not matter (as will become clearer in the next section), it *does* matter which variable comes first.

The mentioned syntactic differences between $\mathcal{L}$ and $\mathcal{L}^{\mathrm{p}}$ correspond to a different understanding of what it means for a term-occurrence to stand at predicate- or sentence-position. Thus, for any term t of $\mathcal{L}^{\mathrm{p}}$, an occurrence of t will be said to stand *at predicate-position* iff, for some term u, it is the first occurrence of t in an occurrence of a term pair $\ulcorner tu \urcorner$; and a term will be said to stand *at sentence-position* iff it appears as an operand of either '$\neg$' or '&' or as the matrix of a term starting with '$\exists$', 'λ', or 'ρ'. Finally, we adopt for $\mathcal{L}^{\mathrm{p}}$ the same abbreviatory devices that were introduced in Section 5 for expressions of $\mathcal{L}$.

12 The Semantics of $\mathcal{L}^{\mathrm{p}}$

With only two exceptions, the semantics of $\mathcal{L}^{\mathrm{p}}$ is exactly similar to that of $\mathcal{L}$. First, clause (vii) of the semantics of $\mathcal{L}$ (as specified in Section 6 above) has to be replaced with the following:

(vii) For any ordinal $\alpha > 0$, any α-sequence of terms $t_0, t_1, \ldots$, any α-sequence of terms $u_0, u_1, \ldots$, and any x: $\ulcorner (t_0 u_0, t_1 u_1, \ldots) \urcorner$ denotes$_{I,g}$ x iff there are an α-sequence of roles $r_0, r_1, \ldots$ and an α-sequence of entities $x_0, x_1, \ldots$ such that: each t_i denotes$_{I,g}$ the corresponding r_i, each u_i denotes$_{I,g}$ the corresponding x_i, and x results from the role assignment $\{(r_i, x_i) \mid i < \alpha\}$.

For the sake of illustration, suppose that the relation of *loving* is the set $\{\langle l, b \rangle\}$, for two roles l and b (short for, respectively, '*Lover*' and '*Beloved*'). Then, as has already been hinted at in the previous section, the state of Dee's loving Pat may, relative to a suitable interpretation and variable-assignment, be denoted by '$(l\,\mathrm{Dee}, b\,\mathrm{Pat})$'.

Second, we have to add a ninth clause to specify the semantics of rho-expressions. Here it is worth noting that not all roles are uniquely characterizable. For example, the tetradic relation M from the example of Section 10.3 can in $\mathcal{L}^{\mathrm{p}}$ be denoted by '$\lambda x, y, z, w\ ((lx, by) \wedge (lz, bw))$'. Under the present proposal, this relation is, as we have seen, a pair set $\{\langle r_1, r_2, r_3, r_4 \rangle, \langle r_3, r_4, r_1, r_2 \rangle\}$, for some pairwise distinct roles $r_1, \ldots, r_4$. However, there is nothing that would help us distinguish r_1 from r_3 or r_2 from r_4. Consequently, in drawing up a general semantics for rho-expressions, we cannot make any stipulation to the effect that such-and-such an expression should determinately denote such-and-such a role. The best we can do is to stipulate that (roughly put) *some* sequence $\sigma \in R$ is such that certain rho-expressions respectively denote the various element of σ.

More specifically, we can stipulate that any interpretation I and variable-assignment g meet the following condition:

(ix) For any ordinal $\alpha > 1$, any α-sequence of pairwise distinct variables $v_0, v_1, \ldots$, and any term t, the following two conditions are satisfied:

1. The rho-expression $\ulcorner \rho v_0 . v_1, v_2, \ldots\ t \urcorner$ has a denotation relative to I and

g *only* if the lambda-expression $\ulcorner\lambda v_0, v_1, \ldots\ t\urcorner$ denotes$_{I,g}$ a relation.[45]

2. If $\ulcorner\lambda v_0, v_1, \ldots\ t\urcorner$ denotes$_{I,g}$ a relation R, then there exists a sequence $\sigma \in R$ such that the following holds for each $i < \alpha$. Let $u_1, u_2, \ldots$ be pairwise distinct variables such that $\{u_1, u_2, \ldots\} = \{v_0, v_1, \ldots\} \setminus \{v_i\}$. Then, for any x: $\ulcorner\rho v_i.u_1, u_2, \ldots\ t\urcorner$ denotes$_{I,g}$ x iff $x = \sigma(i)$.

What (ix) essentially tells us is that, for any relation denoted by such-and-such a lambda-expression, the roles that 'make up' that relation will be denoted by certain rho-expressions whose matrices match that of the lambda-expression. For example, consider again the tetradic relation $\lambda x, y, z, w\,((lx, by) \wedge (lz, bw))$, which we said was identical with a set $\{\langle r_1, r_2, r_3, r_4\rangle, \langle r_3, r_4, r_1, r_2\rangle\}$. We can now write, e.g., '$\rho x.y, z, w\,((lx, by) \wedge (lz, bw))$' to denote either r_1 or r_3. While the semantics leaves indeterminate—as it has to—*which* of these two roles is denoted by that expression, it does entail that, whichever it is, the *other* role is denoted by '$\rho z.x, y, w\,((lx, by) \wedge (lz, bw))$'.

This completes the semantics of $\mathcal{L}^{\mathrm{p}}$. Along with almost all of the semantic clauses of Section 6, we can also retain the definition of 'order of' from Section 7, although it has to be kept in mind that the terms quantified over in that definition should now be taken to be terms of $\mathcal{L}^{\mathrm{p}}$ rather than of $\mathcal{L}$.

The present form of positionalism naturally goes hand in hand with a terminological shift with respect to 'intensional entity'. In particular, it should in the following be understood that an *intensional entity* is anything that is *either a role or a state* (rather than, as before, anything that is either an attribute or a state); and correspondingly, a *particular* should be understood to be anything that is neither a role nor a state. This shift is clearly warranted by the fact that we are now dealing with an ontology of roles and states rather than of *attributes* and states, and also by the fact that what is now called 'quantification into predicate-position' is quantification over *roles* and not exclusively attributes. As a result, the order-theoretic hierarchy looks markedly different from the way it did in Section 7. There, we had higher-order (i.e., non-zeroth-order) relations alongside higher-order properties and states. But since relations are now conceived of as sets, and since sets are conceived of as zeroth-order properties,[46] every relation is classified as zeroth-order. Instead of higher-order relations, we now have higher-order *relational roles*, such as $\rho x^1.y\,(x^1 y)$ and $\rho y.x^1\,(x^1 y)$.[47]

[45]Since a lambda-expression with two or more bound variables cannot denote anything *but* a relation, the present clause could be equivalently phrased as follows:

$\ulcorner\rho v_0.v_1, v_2, \ldots\ t\urcorner$ has a denotation relative to I and g only if $\ulcorner\lambda v_0, v_1, \ldots\ t\urcorner$ does.

[46]That every set is a zeroth-order property can be seen from the fact that, under the proposal adopted in Section 4.1 (and given that $\mathcal{L}^{\mathrm{p}}$, like $\mathcal{L}$, allows for infinitary disjunctions), any set can be denoted by either '$\lambda x\,\neg\exists y\,(y = x)$' or some lambda-expression of the form '$\lambda x\,((x = x_1) \vee (x = x_2) \vee \ldots)$'.

[47]By a 'relational role', I mean any role that is one of a relation's 'constituent roles', or in other words (using terminology to be introduced in the next section): a member of a relation's *role set*.

13 Fundamental Relations

As far as the ontological assumptions of Sections 4.2 and 8 are concerned, we have already seen that, given the above shift to resultance-based positionalism, (A11) has to be abandoned. But apart from this, the assumptions (A0)–(A10), as well as (A12) and (A13), can be left intact. It is now of interest to examine how the positionalistic turn affects the fine-grainedness principle (P″) of Section 9.2; but before we do that, it will be helpful to reflect for a moment on the concept of a *fundamental relation*.

Two principles that will be adopted in the next two sections will allow us to prove that no set is a fundamental entity. From this it will follow (given that relations are sets) that no relation is a fundamental entity. But even so, there is a sense in which we can reasonably speak of 'fundamental relations'; and indeed it is possible to provide a natural definition (or analysis) of this concept within the present setting. To this end, let us first introduce the concept of a relation's 'role set':

Definition 13.1. The *role set* of a relation R is the smallest set S such that $R \subseteq S^\alpha$, for some ordinal α.[48]

More intuitively put, the role set of a relation R is the set of all those roles of which R's members are sequences. For example, the role set of a relation $\{\langle r_1, r_2, r_3 \rangle\}$ is just the set $\{r_1, r_2, r_3\}$. On this basis, the concept of a fundamental relation may be analyzed as follows:[49]

Definition 13.2. A *fundamental relation* is a relation whose role set contains only *fundamental* roles (i.e., roles that are fundamental entities).

Thus, if the set $\{\langle r_1, r_2, r_3 \rangle\}$ is a relation, then it is a *fundamental* relation in the sense just defined iff each of r_1, r_2, and r_3 is a fundamental entity. (I trust that the disambiguation between the two senses of 'fundamental'—one being applicable only to relations, the other to everything else as well—can usually be left to context.)

We can now derive the following, highly plausible corollary:

Corollary 13.3. Any converse of a fundamental relation is a fundamental relation.

To see how this follows, let R be any fundamental α-adic relation, and let f be any permutation on α. With the help of Definitions 3.5 and 10.4, it can then be readily seen that the f-converse of R is just the set $R \circ f$, i.e., $\{\sigma \circ f \mid \sigma \in R\}$, a relation whose role set is the same as R's.

That the converses of a fundamental relation should again be fundamental is plausible even from the standpoint of the Default View. For even if the concept of instantiation is metaphysically primitive, one should not expect it to be the case that, for example, the relation *longer-than* is fundamental while its converse *shorter-than* isn't. (Cf. the relevant remark in Section 9.2.) However, the proponent of the Default View has no obvious explanation as to *why* the converses of fundamental relations

[48] Here S^α is the set of all functions from α to S, or in other words, the set of all α-sequences whose elements are members of S.

[49] Although the following is here labeled a 'definition', it does not seem inappropriate to count it as a metaphysical analysis of what it is to be a fundamental relation.

should again be fundamental: on the face of it, she will have to accept this as a brute fact. By contrast, as we have just seen, the present form of positionalism, coupled with the proposed analysis of the concept of a fundamental relation, *does* provide an explanation for that fact, without the need for any additional assumptions. This may be taken to lend additional support to the present view.

14 Fine-Grainedness Revisited

Let us now turn to the question of how to formulate a fine-grainedness principle within the present setting. Our hope in adopting a positionalistic conception of relations was that this would allow us to adopt a version of (P″) that dispenses with any special provisions with regard to relations. The principle that results from this simplification runs as follows:

(A14) For any variable-assignment g, any terms t and t', and any state s: if the following five conditions are satisfied—

(i) t and t' both denote$_{\varnothing,g}$ s.

(ii) Every entity in the range of g is fundamental.

(iii) No two variables are under g mapped to the same entity.

(iv) Every variable in the domain of g is of type 1.

(v) For any variable v: if v occurs free in t and is under g mapped to an intensional entity,[50] then at least one free occurrence of v in t stands at predicate- or sentence-position.

—then t analytically entails t'.

This differs from (P″) in two respects. First, clause (iii) has been shortened: it no longer contains the words, 'or to relations that are converses of each other'. That this particular simplification is possible can be seen by reflecting on the argument that has, in Section 9.2, led to the inclusion of this phrase in the original clause (iii). (Here it is relevant that in $\mathcal{L}^{\mathrm{p}}$ we no longer have terms of the form '$R(x_1, x_2, \dots)$', in which a name of a relation is applied to a list of arguments.)

The second difference consists in a radical shortening of the consequent, which no longer makes reference to symmetry statements. However, as welcome as this simplification may be, it invites an objection. For a critic might now put forward an argument largely analogous to the one that motivates the original complication in the consequent of the main conditional of (P″).

[50]This formulation presupposes that there are no 'improper' roles (as they might be called), i.e., roles r that are such that, for any entity x, *no* state results from any role assignment containing the pair (r, x). To take into account that, for all we know, there may be such roles, we would here have to replace 'intensional entity' with 'state or proper role' (cf. Plate [MS(a): §5.3]); but for present purposes I shall set this complication aside.

In particular, suppose that there exists a fundamental dyadic relation R that is identical with its converse, in such a way that, for two fundamental roles r_1 and r_2, R is the set $\{\langle r_1, r_2 \rangle, \langle r_2, r_1 \rangle\}$.[51] Next, let u and v be two variables of type 1, let t and t' be (respectively) the formulas $\ulcorner \lambda x, y\,(ux, vy) = \lambda x, y\,(ux, vy) \urcorner$ and $\ulcorner \lambda x, y\,(ux, vy) = \lambda x, y\,(uy, vx) \urcorner$, and let g be the smallest variable-assignment that maps u and v to, respectively, r_1 and r_2. Lastly, let s be the self-identity of R. Given that R is identical with its converse, we have that $\lambda x, y\,(r_1 x, r_2 y)$ is identical with $\lambda x, y\,(r_1 y, r_2 x)$, so that t and t' both denote$_{\emptyset,g}$ s; and the other four conditions in the antecedent of (A14)'s main conditional are also satisfied. Hence, with (A14), it follows that t analytically entails t'; but in fact t does *not* analytically entail t', since not every zeroth-order dyadic relation is its own (non-trivial) converse.[52] So either there is no relation R of the sort under consideration, or (A14) is false.

In response to this objection, we can choose between two routes. The *first* is to roll back the simplification just mentioned and to reintroduce reference to symmetry statements. (Due to the syntactic differences between $\mathcal{L}$ and $\mathcal{L}^{\mathrm{P}}$, the concept of a symmetry statement would in this case need to be suitably redefined.) The *second* option is to keep (A14) as is and accept the consequences. Let us briefly consider what this would mean.

With regard to the above argument, keeping (A14) as is would mean accepting that no fundamental relation is a pair set $\{\langle r_1, r_2 \rangle, \langle r_2, r_1 \rangle\}$ with $r_1 \neq r_2$. More generally, by a generalization of the above argument, it can be seen that a proponent of (A14) will have to accept:

Corollary 14.1. For any ordinal $\alpha > 1$, any permutation f on α, any fundamental α-adic relation R, and any sequence $\sigma \in R$: if R is its own f-converse, then $\sigma = \sigma \circ f$.

In other words, for any permutation f such that R is its own f-converse, any sequence in R will be fixed under f.

To be sure, an analogous move was already available in Section 9.2 above, for we could have simply refused to weaken the consequent of (P′)'s main conditional. This is essentially the path suggested by Bacon (2020), who accepts, as a consequence of his principle of 'Fundamental Independence', that "no fundamental relation is [intensionally] symmetric" (p. 570). This is also exactly the consequence that we would have been faced with, back in Section 9.2, if we had left the consequent of (P′)'s main conditional unchanged. However, even in the absence of immediately compelling counter-examples, the thesis that no fundamental relation is intensionally symmetric is certainly a bold claim: it rules out, for example, that the intensionally symmetric dyadic relation of *being a Planck length apart* is fundamental. By contrast, Corollary 14.1 is much less sweeping and seems, as a result, to be a much more

[51] A principle to be adopted in Section 16 actually rules out this specific type of relation, but let us ignore this for the sake of argument. A slightly more complex example that could be used to make the same point (without being ruled out by the mentioned principle) would be that of a fundamental triadic relation that, for three distinct roles r_1, r_2, and r_3, has the three sequences $\langle r_1, r_2, r_3 \rangle$, $\langle r_2, r_3, r_1 \rangle$, and $\langle r_3, r_1, r_2 \rangle$ as members.

[52] Cf. footnote 31 above.

reasonable claim to accept. In particular, it allows us to maintain that *being a Planck length apart* is fundamental, *as long as* the role set of this relation is a singleton (so that the relation itself is the set $\{\langle r, r\rangle\}$, for a certain fundamental role r).

For this reason, and because of the greater simplicity of (A14) as compared with the alternative, I consider the second option to be preferable to the first. If this is correct, we may say that the positionalistic turn affords a significantly simplified account of how states are individuated. Given the still-considerable complexity of (A14), this gain in simplicity may not seem overly exciting. But it is still progress, and a sign that we are moving in the right direction. In the next two sections, we will encounter two additional assumptions that will round out the present theory.

15 The Analyzability Assumption

The first of these assumptions states, in essence, that $\mathcal{L}^{\mathrm{P}}$'s expressive resources are sufficiently powerful to allow *every* entity to be 'analyzed' in terms of fundamental entities alone:[53]

(A15) For any entity x, there exist a variable-assignment g and a term t of $\mathcal{L}^{\mathrm{P}}$ such that: t denotes$_{\emptyset,g}$ x, and any entity in the range of g is fundamental.

This claim may at first blush look implausible when applied to ordinary material objects. Thus consider the following objection:

> My left thumb is surely neither a role nor a state, whereas the only complex terms of $\mathcal{L}^{\mathrm{P}}$ are ones that denote, if anything, roles and states. So the only kind of term that could denote my left thumb, relative to the empty interpretation and any variable-assignment, is a *variable*. By (A15) it then follows that my left thumb is fundamental, which seems absurd. Even worse, a straightforward generalization of the this argument shows that, if (A15) is correct, then *every* particular is fundamental.

Where the objection goes wrong, I suggest, is in assuming the falsity of the following disjunction: either there are no ordinary objects at all, or they are best identified with *events*.

To elaborate: the first option, i.e., *eliminativism* about ordinary objects, has been defended by various authors on a number of grounds (see Korman and Barker [2025: §2] for numerous references); and it may further be argued that *what there actually is* to play the theoretical role of ordinary objects—or at least the best existing candidates for this role—are events. Thus it may be argued that the objector's left thumb, if there is such a thing, will be best thought of as a complex jumble of molecular and sub-atomic processes, and that these processes may in turn be identified with vast conjunctions of facts (i.e., obtaining states) detailing, as it were, the changing spatiotemporal relationships among various particles. Obviously, here is not the

[53] A similar assumption can be found in Plate (2025: §5.3).

place to defend this idea in detail. At least the beginnings of such a defense may be found in Nolan (2012) and Sider (2013); and Whitehead's (1929/1978: §2.5) critique of "scientific materialism" is also relevant. But for now I will simply have to ask the reader to set aside whatever doubts about (A15) might arise from its application to material objects.[54]

What makes (A15) useful in the present context is the fact that it guarantees the applicability of (A14) to *any* state. It thereby allows us to prove, e.g.:

Proposition 15.1. No set is a fundamental entity.

For let x be any set, let s be the state $(x = x)$, and suppose for *reductio* that x is fundamental. By the identification of sets with properties proposed in Section 4.1, x will be either $\lambda x\, \neg\exists y\, (y \neq x)$ or a 'disjunctive' property $\lambda x\, ((x = x_1) \vee (x = x_2) \vee \dots)$. But both cases result in a contradiction,[55] and this completes the *reductio*: we can conclude that no set is a fundamental entity.

The usefulness of (A15) for establishing results like Proposition 15.1 constitutes the main reason for including *rho-expressions* in $\mathcal{L}^{\mathrm{p}}$. Had they not been included, any non-fundamental role that is not a property would have been a counter-example to the above assumption.

It now only remains to add one last principle, which specifically concerns the individuation of roles.

16 Equivalent Roles

Recall that, according to assumption (A0) (Section 4.2), every attribute has at most one trivial converse. This in effect renders the individuation of attributes parasitic upon that of their respective instantiations: the more coarse-grained the latter, the more coarse-grained the former. In thus constraining the individuation of attributes,

[54] Doubts about (A15) might also arise from its application to *non*-material objects, such as poems or string quartets, which, it may be argued, cannot be plausibly conceived of as states even if (some) material objects can. But this by itself does not mean that those immaterial objects are particulars. Some authors, such as Letts (2018), have identified musical works with properties, while others, such as Kania (2008) and Killin (2018), have defended a fictionalist stance.

[55] In the first case, let t and t' be, respectively, the formulas '$x^1 = \lambda y\, (x^1 y)$' and '$x^1 = \lambda x\, \neg\exists y\, (y \neq x)$', and let g be the smallest variable-assignment that maps 'x^1' to x. Since we are assuming that x is fundamental, it can then be seen that g, s, t, and t' jointly satisfy the five numbered clauses of (A14). So, from (A14), it now follows that t analytically entails t'. But, given that the empty set is not the only zeroth-order property, t does *not* analytically entail t'. So we have a contradiction.

In the remaining case, x is a disjunctive property $\lambda x\, ((x = x_1) \vee (x = x_2) \vee \dots)$. Here we can mount essentially the same argument, but we have to use (A15) in order to be able to infer that there exist terms $t_1, t_2, \dots$ and a variable-assignment g such that (i) each t_i denotes$_{\emptyset,g}$ the corresponding x_i and (ii) every entity in the range of g is fundamental. Without loss of generality, we may here suppose that g maps 'x^1' to x, that the untyped variable 'x' does not occur free in any of the t_i, that no two variables are under g mapped to the same entity, and that any variable in the domain of g is of type 1. With t as above and $t' = \ulcorner x^1 = \lambda x\, ((x = t_1) \vee (x = t_2) \vee \dots)\urcorner$, we then have that g, s, t, and t' jointly satisfy the five numbered clauses of (A14), and it can also be observed that t does not analytically entail t'. The result is another contradiction.

(A0) places a constraint also on the individuation of relational roles. For suppose you have a triadic relation R as well as a triadic relation R', and one day it turns out that R' is a trivial converse of R. If previously you had established that R is a set $\{\langle r_1, r_2, r_3\rangle\}$ while R' is a set $\{\langle r_1', r_2', r_3'\rangle\}$, then now you know, thanks to (A0), that $\langle r_1, r_2, r_3\rangle$ is identical with $\langle r_1', r_2', r_3'\rangle$, and you can accordingly conclude that $r_1 = r_1'$, $r_2 = r_2'$, and $r_3 = r_3'$.

But there is more to be said. Consider, for instance, the identity relation $\lambda x, y\,(x = y)$: under our identification of relations with equivalence classes of role sequences, there are—not necessarily distinct—roles r_1 and r_2 such that this relation is identical with the set $\{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$. At least *prima facie*, nothing about the identity relation forces us to regard r_1 and r_2 as distinct from each other; and it seems unlikely that nature would use two roles where one would suffice. If so, we should think of the identity relation as a singleton $\{\langle r, r\rangle\}$ rather than as a pair set. Generalizing from this case, it is further tempting to hypothesize that no two roles are (in a sense to be specified in a moment) *equivalent*.

To define the relevant notion of equivalence, we begin with the auxiliary concept of a role assignment's *realizing* a given role sequence:

Definition 16.1. For any ordinal α, a role assignment M *realizes* a role sequence σ of length α iff, for some entities $x_0, x_1, \ldots$: M is identical with the role assignment $\{(\sigma(i), x_i) \mid i < \alpha\}$, and there exists a state resulting from M.

A role sequence that is realized by some role assignment will also simply be called 'realized'. On this basis, we can now define our concept of equivalence:

Definition 16.2. A role r is *equivalent* to a role r' iff, for some ordinal $\alpha > 0$, some realized α-sequence σ, and some $i, j < \alpha$, the following three conditions are satisfied:

(i) $\sigma(i) = r$.

(ii) $\sigma(j) = r'$.

(iii) For some permutation f on α that merely transposes i and j: $\sigma \cong \sigma \circ f$.[56]

For example, if two sequences $\langle r_1, r_2\rangle$ and $\langle r_2, r_1\rangle$ are instantiation-equivalent, then the roles r_1 and r_2 will be equivalent to each other. With this concept in place, the principle alluded to above can be stated as follows:

(A16) No two roles are equivalent to each other.

This assumption, motivated by the above considerations of ontological parsimony, provides the reason why we have to think of role assignments as (generalized) multisets rather than sets. For let R be some intensionally symmetric dyadic relation $\{\langle r, r\rangle\}$ (for some role r), and let S be the triadic relation $\lambda x, y, z\,((rx, ry) \wedge (ry, rz) \wedge (rx, rz))$. With the help of (A5) and (A9), it can be seen that any two members of S's role set are equivalent. Consequently, by (A16), there is only one of them, so that S

[56]In the special case of $i = j$, the permutation on α that 'merely transposes' i and j should be taken to be the identity permutation on α.

turns out to be a singleton $\{\langle q,q,q\rangle\}$, for some role q. Next, let x and y be any two entities such that there exists an instantiation of R by x and y (in this order) as well as an instantiation by x and x and one by y and y. (In other words, let x and y be two entities such that each of the role assignments $\{(r,x),(r,y)\}$, $\{(r,x),(r,x)\}$, and $\{(r,y),(r,y)\}$ has a state resulting from it.) Then there will also exist an instantiation of S by x, x, and y (in this order); and this instantiation will typically be distinct from that of S by x, y, and y: for whereas the former necessitates (rx,rx), the latter instead necessitates (ry,ry).[57]

Now suppose that role assignments are sets rather than multisets—or, more carefully: that sets rather than multisets play in our theory the role of role assignments. Then the instantiation of S by x, x, and y, in this order, will result from the *set* $\{(q,x),(q,x),(q,y)\}$, which is identical with $\{(q,x),(q,y)\}$, while the instantiation of S by x, y, and y will result from the set $\{(q,x),(q,y),(q,y)\}$, which is *also* identical with $\{(q,x),(q,y)\}$. Thus both instantiations will result from the same role assignment, namely $\{(q,x),\ (q,y)\}$. From this it will further follow that both of them are instantiations of the relation $\{\langle q,q\rangle\}$ by x and y (in this order). But by (A9), any given relation has only at most *one* instantiation by a given sequence of entities; and so we will (typically) have a contradiction. The obvious way to avoid this problem is to think of role assignments as multisets rather than sets. For the multiset $\{(q,x),(q,x),(q,y)\}$ is *not* identical with the multiset $\{(q,x),(q,y),(q,y)\}$ if x is distinct from y.

A critic might worry that, with the addition of (A16) and the transition from $\mathcal{L}$ to $\mathcal{L}^{\mathrm{P}}$, our theory has become *more* complex than it would have been if we had rested content with simply treating relations as polyadic analogues of properties. Accordingly, one might wonder whether it would not be better to revert to the Default View, even if that means embracing (P″) instead of the simpler (A14) and having to accept as brute the fact that fundamental relations have fundamental converses. But I think the answer is fairly clearly 'no'. A more complex theory is not automatically a worse theory: there is a difference between the complexity that arises from the addition of epicycles and exceptions, and the complexity that simply comes from a more detailed picture of reality. To use a well-worn analogy, consider the phenomenon of mechanical watches. Typically, their hands rotate at more or less constant angular velocity in a clockwise direction. We *could* simply accept that "this is what watches do". Analogously, a friend of the Default View might co-opt Blanshard's dictum that "the business of a relation is to relate" (1984: 215), adding perhaps 'And that's all!'. But take one of those watches, and suppose you want to explain its behavior from first principles: you should then not feel embarrassed to posit springs and

[57] I say 'typically' because for some choices of R (and with r and S chosen accordingly, i.e., in such a way that $R=\{\langle r,r\rangle\}$ and $S=\lambda x,y,z\,((rx,ry)\wedge(ry,rz)\wedge(rx,rz))$), it turns out that the instantiations of S by—respectively—x,x,y and x,y,y both necessitate (rx,rx) and (ry,ry). An obvious example is the identity relation; and there are also non-reflexive relations that equally fit the bill. *Typically*, however, this will not be the case. Consider, e.g., the relation of *loving each other*: the conjunction of (i) Romeo and Juliet loving each other and (ii) Romeo and Romeo loving each other (i.e., Romeo's loving himself) does not necessitate that Juliet loves Juliet.

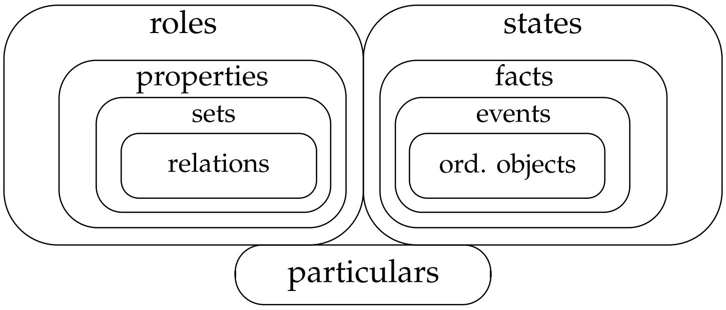

Figure 1: An ontology of roles and states (and particulars).

escapements. The situation is not so different for the metaphysician who tries to understand relational states.

17 Conclusion

We have arrived at an ontology of roles and states: on the one hand, relations are conceived of as sets, sets as properties, and properties as roles; and on the other hand, ordinary objects are (albeit tentatively) conceived of as events and events as facts, i.e., obtaining states (cf. Figure 1). This still leaves plenty of room for *particulars*, i.e., things that are neither roles nor states. These might be quarks and leptons, strings, monads, and/or spacetime points;[58] but the theory is officially neutral on whether there are any such things.

The positionalistic character of this ontology may not be evident from the 17 assumptions that form its backbone, of which only (A11$'$) and (A16) make explicit reference to roles. However, given the definition of the instantiation predicate put forward in Section 10.4, any talk of attributes within the context of this theory implicitly refers to either roles or sets of role sequences. At the same time it is worth emphasizing that the theory is not *merely* a form of positionalism, since it also contains several other not completely uncontroversial tenets, such as: that there are non-obtaining states and uninstantiated attributes; that there is a stratum of fundamental entities; that all particulars are fundamental; and that roles and states form a hierarchy of orders reminiscent of ramified type theory. I have here not (or at least not at any length) tried to defend these other aspects of the view.

In connection with this theory, there is a wealth of questions still unexplored. One could for instance wonder whether any states are fundamental entities, and under what conditions a relation might share its role set with another relation that is not a converse of it. (For example, let r be the sole member of the role set of the identity relation. Is the set $\{\langle r, r, r\rangle\}$ then also a relation?) More generally, one could try to investigate what might be called the 'Special Resultance Question': What does it take for a role assignment to be such that a state results from it? And of course, if

[58]Thanks here to an anonymous referee.

there is such a thing as the property of *being a role*, then one has to wonder whether it is a *fundamental* property; and the same may be asked about fundamentality itself.[59]

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