

# Fundamentality in an ontology of roles and states

Jan Plate

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## Abstract

I defend the thesis that *fundamentality is fundamental*—that the property of *being a fundamental entity* is itself a fundamental entity—by arguing against several rival approaches. The broader theoretical context is provided by what I call an *ontology of roles and states* (ORS), where the concept of a *role* is to be understood as a generalization of the concept of a property, while a *state* is just a state of affairs. In this context, the concept of fundamentality is invoked by a *fine-grainedness principle* that governs the individuation of states, and indirectly also of roles. This principle in turn forms the basis for an argument from considerations of theoretical simplicity against an otherwise attractive proposal to equate fundamentality with ‘logical simplicity’, understood along the lines of Plate (2016). In addition, it is argued that fundamentality should not be analyzed in terms of such notions as grounding, building, or structuralness.

**Keywords:** fundamentality; states of affairs; roles; ontology; metaphysical structure

## 1 Introduction

The past two decades have seen a profusion of work on fundamentality. An interesting characteristic of much of this literature is its ‘ecumenical’ character, by which I mean its eschewal of any particular ontological framework.<sup>1</sup> This has the advantage that any results obtained following this approach can claim to transcend the boundaries between individual ontological theories. But arguably there are also *disadvantages*. Most obviously, by abstaining from the commitments of a particular theory, the ecumenical metaphysician will inevitably rob herself of the chance to generate results that are only available to those willing to make those commitments.

The approach taken in this paper is the opposite of ecumenical: it is a discussion of the property of *being a fundamental entity*, using the resources of *a specific metaphysical theory*, and taking the term ‘fundamental’ in the sense that it has within the context of that theory. This may render the following discussion ‘parochial’ insofar

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<sup>1</sup>Cf., e.g., Wilson (2020: 283).

as its results do not easily carry over to other frameworks. However, I consider the theory in question to be sufficiently well motivated to be of interest to the general metaphysician, and especially to anyone in search of a suitable framework within which to theorize about intensional entities. If this is right, then results obtained on the basis of this theory should be of corresponding philosophical interest.

For reasons that will become clearer soon, I refer to the theory in question as an *ontology of roles and states* (ORS). Section 2 introduces those aspects of ORS that will be relevant for the subsequent discussion. Meanwhile, the thesis to be defended in this paper can be stated as follows:

(FF) The property of *being a fundamental entity* is itself a fundamental entity.

In other words, fundamentality is fundamental. But why believe that there is any such thing as a property of *being a fundamental entity*?

At first blush it may well seem that we can meaningfully distinguish between fundamental and non-fundamental entities without also having to think that there is a *property* of fundamentality. Nonetheless, a property of fundamentality is a perfectly sensible thing to have in one's ontology. In support of this, I offer the following argument. (Readers who are already convinced should feel free to skip ahead to the last paragraph of this section.) We proceed in four steps. First, taking the term 'fundamental' in the sense that it has in the context of ORS, it is not nonsense to say of a given entity  $x$  that it is fundamental. There should then be a state of affairs, obtaining or not, to the effect that  $x$  is fundamental.<sup>2</sup>

Second step: suppose we wish to construct an ontology of states of affairs and properties that admits *logically complex* properties.<sup>3</sup> A natural way to do so is to adopt a suitable comprehension axiom. The basic idea is that open sentences correspond to properties. Very roughly, where  $g$  is a variable-assignment and  $\varphi$  a formula (of a suitable formal language) containing a free occurrence of a variable  $v$  and denoting some state of affairs relative to  $g$ , we assume that there exists a property  $P$  such that, for any  $x$  and any state of affairs  $s$ :  $s$  is an instantiation of  $P$  by  $x$  iff  $s$  is denoted by  $\varphi$  relative to a variable-assignment that is just like  $g$  except for mapping  $v$  to  $x$ .<sup>4</sup>

Third step: since our ontology is not just one of properties but also of states of affairs (or henceforth simply 'states'), and since we wish to admit into this ontology a state to the effect that a given entity is *fundamental*, our formal language will need to contain some device for expressing fundamentality. That device might

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<sup>2</sup>Proponents of 'higher-order metaphysics'—on which see, e.g., Skiba (2021)—generally prefer to use the term 'proposition', but I shall use 'state of affairs' (or simply 'state') instead.

<sup>3</sup>One reason for doing so is that one might want to admit *existentially quantified* states of affairs, many of which are naturally viewed as requiring the existence of logically complex properties. Thus the state of affairs *that there is a brown cow* is naturally regarded as the 'existential quantification' of the complex property of *being a brown cow*. Cf., e.g., Fine (2010: 587). Another reason for admitting complex properties is that these can play the role of sets; see Section 2.1 below.

<sup>4</sup>This is rough in part because, depending on one's choice of formal language, certain provisions have to be put in place to avoid Russell's paradox; but we can leave this issue aside for now.

be a dedicated predicate ‘ $F$ ’, or a syncategorematic sentence-forming operator, or perhaps simply a variable that can stand in predicate-position and be mapped to the property of *being a fundamental entity*. In any event, the state that a given entity  $x$  is fundamental will be denoted, relative to some variable-assignment  $g$ , by some formula  $\varphi$  containing a free occurrence of a variable that is under  $g$  mapped to  $x$ ; and typically, for any entity  $y$  and any variable-assignment  $h$  that is just like  $g$  except for mapping that variable to  $y$ ,  $\varphi$  will relative to  $h$  denote, if anything, the state that  $y$  is fundamental.<sup>5</sup>

Fourth step: by applying the aforementioned comprehension axiom to the formula  $\varphi$  that has just been specified, one can derive a commitment to a property  $P$  such that, for any  $y$  and any state  $s$ :  $s$  is an instantiation of  $P$  by  $y$  iff  $s$  is the state that  $y$  is fundamental. I submit that such a property may be justifiably called ‘the property of *being a fundamental entity*’.<sup>6</sup>

Unfortunately, as far as I can see, there is no similarly straightforward argument for (FF).<sup>7</sup> What I will do instead is to argue against a number of salient rival views. In particular, Section 3 considers the question of whether fundamentality might be analyzed in broadly logical terms and asks, more particularly, whether fundamentality may be equated with *logical simplicity*. The discussion there hinges on the idea that metaphysicians should *ceteris paribus* favor simpler theories over more complex ones, and also puts forward a way of justifying this idea. Section 4 then asks whether the concept of fundamentality might be best expressed by a syncategorematic operator; and Sections 5–7 assess the prospects of analyzing fundamentality in terms of (respectively) grounding, building, and structuralness. Section 8 concludes.

## 2 An ontology of roles and states

### 2.1 Overview

ORS has grown out of a theory of that has recently been proposed by Plate (2025) under the name ‘ordinal type theory’ (or OTT for short). OTT itself is characterized by the fact that it takes all entities to be arranged in a transfinite hierarchy in which

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<sup>5</sup>I say ‘typically’ because there are conceivable exceptions. For example, someone might choose to express the fundamentality of  $x$  by something like ‘ $F(x) \wedge F(y)$ ’, where ‘ $x$ ’ and ‘ $y$ ’ are both mapped to  $x$ . This complication need not detain us here.

<sup>6</sup>A shorter argument for the existence of such a property is suggested by the remarks of footnote 3. In particular, since it is not nonsense to say that there is at least one fundamental entity, there should exist a state of affairs to this effect; and a natural way to think of this state is to think of it as the existential quantification of *being a fundamental entity*.

<sup>7</sup>A few arguments for theses that exhibit at least some affinity to (FF), due to Fine (2001) and Wilson (2014), respectively, have been critically discussed by Bennett (2017: §5.10). More recently, Wilson (forthcoming) has defended the thesis that “[w]hat makes it the case that some goings-on at a world  $w$  are fundamental at  $w$  is metaphysically primitive”. This also exhibits a certain affinity to (FF). Wilson defends her thesis by arguing that it satisfies certain desiderata no less well than certain other views. For reasons of space, it will not be possible to discuss her arguments in this paper.

each entity is assigned an ordinal number as its ‘order’, starting from zero. At the base of this hierarchy lie the particulars—those entities that are neither states, nor properties, nor relations—as well as the *fundamental* states, properties, and relations. All of these entities, and others besides, are classified as ‘zeroth-order’. The term ‘attribute’ is used to refer to anything that is a property or a relation.

Under ORS, this picture changes in several ways. Most notably, whereas OTT takes as primitive the concept of something’s being an *instantiation* of some entity by some one or more entities in a given order, ORS instead takes as primitive, first, the concept of a *role* and, second, that of a state’s *resulting* from a given ‘role assignment’.<sup>8</sup> For example, the state of Cleopatra’s loving Antony is in ORS conceived of as the state that results from a role assignment that maps the role of *Lover* to Cleopatra and that of *Beloved* to Antony.<sup>9</sup> Formally, a role assignment is a non-empty *generalized multiset* of ordered pairs  $(r_i, x_i)$ , with each  $r_i$  being a role. While a multiset is ordinarily thought of as an ordered pair  $(A, m)$ , where  $A$  is a set and  $m$  a function from  $A$  to the set of positive integers, a *generalized multiset* is given by a set and a function that has as its range a set of positive *cardinals*. This allows the framework to handle infinitary relations.

In the special case in which, for some entity  $x$ , there exists a state resulting from the role assignment  $\{(r, x)\}$ , the role  $r$  will also be called a *property*, and the state that results from  $\{(r, x)\}$  will be called the *instantiation* of  $r$  by  $x$ . Thus, properties are in ORS treated as a special kind of role. The property of *being mortal*, for instance, is identified with a role  $M$  such that the fact (i.e., obtaining state) that Socrates is mortal results from the role assignment  $\{(M, \text{Socrates})\}$ .

The treatment of *relations* is more complex, in that these are conceived of as sets of *role sequences*. For example, the relation of *loving* would be the set  $\{\langle \text{Lover}, \text{Beloved} \rangle\}$ , its converse would be  $\{\langle \text{Beloved}, \text{Lover} \rangle\}$ , the *taller-than* relation would be  $\{\langle \text{Taller}, \text{Shorter} \rangle\}$ , and so on. Notice that these are all singletons. *Non-singleton* sets are needed for relations that exhibit certain symmetry patterns. Thus suppose that  $R$  is a triadic relation that is identical with its ‘converses’  $\lambda x, y, z R(y, z, x)$  and  $\lambda x, y, z R(z, x, y)$ , but not with  $\lambda x, y, z R(y, x, z)$ . Then  $R$  will in ORS be conceived of as a set that contains not only the sequence  $\langle r_1, r_2, r_3 \rangle$  (for some pairwise distinct roles  $r_1, r_2, r_3$ ), but also the sequences  $\langle r_2, r_3, r_1 \rangle$  and  $\langle r_3, r_1, r_2 \rangle$ . In general, an *instantiation* of a relation  $R$  by some entities  $x_1, x_2, \dots$  is simply the state that results from  $\{(r_1, x_1), (r_2, x_2), \dots\}$ , where  $\langle r_1, r_2, \dots \rangle$  is one of the sequences in  $R$ .

As usual, we think of sequences as functions defined on ordinals, and of functions as sets of ordered pairs. Further, as already in OTT, sets are in ORS conceived of as *properties*, in such a way that any non-empty set  $\{x_1, x_2, \dots\}$  is identified with the property of *being identical with  $x_1$  or with  $x_2$  or  $\dots$* , while the empty set is identi-

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<sup>8</sup>Slightly more accurately, the concept of a role is defined on the basis of that of an entity’s *rolehood*: an entity is a role iff its rolehood obtains. But this nuance will not matter in what follows.

<sup>9</sup>Cf. Fine (2000: 12). Using a term coined by Fine, ORS can be aptly described as a ‘positionalistic’ theory. Fine himself is opposed to positionalism. For responses to his objections, see, e.g., Plate (forthcoming: §5; **plateAT**).

fied with the uninstantiable property  $\lambda x \neg \exists y (x = y)$ .<sup>10</sup> Accordingly, since relations are conceived of as sets, this makes them properties, and hence roles. The reconception of relations as sets of role sequences is chiefly motivated by explanatory considerations. Among other things, it allows for a straightforward explanation as to why it is that the converses of fundamental relations are again fundamental. For we may naturally regard a relation as a *fundamental* relation iff its ‘constituent roles’, i.e., those roles that are the elements of the sequences of which it is a set, are fundamental entities. Since a converse of a relation is simply the result of reordering those sequences, fundamentality is preserved across converses.<sup>11</sup>

The conceptual changes described above correspond to a change in the underlying formal language in which states are represented. For example, while the state of Cleopatra’s loving Antony can in OTT, as in first-order logic, be represented by the formula ‘ $L(\text{Cleopatra}, \text{Antony})$ ’, in ORS this state would instead be represented by, say, ‘ $(l \text{ Cleopatra}, b \text{ Antony})$ ’, where ‘ $l$ ’ and ‘ $b$ ’ stand for the roles of *Lover* and *Beloved*, respectively.

In ORS, anything that is neither a state nor a role is called a *particular*, whereas in OTT this term is reserved for those entities that are neither states nor attributes. The way in which entities that are neither states nor roles are treated in ORS differs in two respects from the way in which entities that are neither states nor attributes are treated in OTT. First, those things that are neither states nor roles are in ORS classified as fundamental entities. Second, these entities are treated as falling *outside* of the order-theoretic hierarchy, in the sense that they are not assigned any order.<sup>12</sup>

In some ways, ORS is an extraordinarily lean ontology. It admits no Aristotelian substances, no ‘mere aggregates’, nor even any pluralities. Instead, there are only roles and states and fundamental particulars (such as, possibly, quarks and leptons). With respect to those roles and states, however, the ontology is highly generous. Thus it recognizes not only negations of states and existential quantifications of attributes, but also infinitary conjunctions of states, as well as attributes that are ‘abstracted’ from such states by way of lambda-abstraction. In the rest of this section, I will present some details of this theory that will become relevant later on.

## 2.2 The formal language

Some of ORS’s assumptions rely on a formal language  $\mathcal{L}$  as a tool for the representation of roles and states. The present subsection gives an overview of  $\mathcal{L}$ ’s main features. There are two main innovations. First, the format of predication familiar from first-order logic (with predicates followed by argument lists) is replaced with a different format, in which a predication consists of a list of term pairs, with the first

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<sup>10</sup>Cf. Carnap (1947: §23), van Inwagen (2023: 246ff.).

<sup>11</sup>Since the phrase ‘fundamental relation’ is here not understood in the sense of ‘relation that is a fundamental *entity*’, we have as a result that the word ‘fundamentality’ is correspondingly ambiguous. But this is nothing new: a fundamental mistake and a fundamental belief are not fundamental entities, either.

<sup>12</sup>Here I am following Plate (MS[a]: §5.3) rather than my more transitional (MS(b)).

term of each pair denoting a role.<sup>13</sup> Second, in addition to formulas and lambda-expressions, there are now also *rho-expressions*, which serve to denote roles that are not properties and can as such not be denoted by lambda-expressions.

In general, a term of  $\mathcal{L}$ —henceforth simply ‘term’—has or lacks a denotation only relative to an interpretation and a variable-assignment. Here an *interpretation* is just a function defined on a set of constants (represented as unitalicized strings, such as ‘Cleopatra’ and ‘Socrates’), while a *variable-assignment* is a function defined on a set of variables. We will write ‘denotes<sub>*I,g*</sub>’ as shorthand for ‘denotes relative to *I* and *g*’. Each *atomic* term is either a constant or a variable, and variables can be *typed* or *untyped*. The type of a typed variable is a positive ordinal, indicated by a superscript: ‘*x*<sup>1</sup>’ is of type 1, ‘*x*<sup>2</sup>’ is of type 2, and so on. Untyped variables range over everything whatsoever, whereas a variable of type  $\alpha$  ranges over all and only those entities that are of some order *less* than  $\alpha$ . Thus a typed variable *v* denotes an entity *x* relative to a given variable-assignment *g* just in case two conditions are met: *v* is under *g* mapped to *x*, and *x* has an order less than *v*’s type.

Let us now briefly go over the three kinds of *complex* term: formulas, lambda-expressions, and rho-expressions.<sup>14</sup>

1. *Formulas* are used to denote states and come in six varieties: (i)  $\lceil (t = u) \rceil$  for some terms *t* and *u*, (ii)  $\lceil \neg t \rceil$  and (iii)  $\lceil \mathbb{R} t \rceil$  for some term *t*, (iv)  $\lceil (t_1 u_1, t_2 u_2, \dots) \rceil$  for one or more terms *t*<sub>1</sub>, *t*<sub>2</sub>, ... and *u*<sub>1</sub>, *u*<sub>2</sub>, ..., (v)  $\lceil \&(t_1, t_2, \dots) \rceil$  for *zero* or more terms *t*<sub>1</sub>, *t*<sub>2</sub>, ..., and (vi)  $\lceil \exists v_1, v_2, \dots t \rceil$  for one or more pairwise distinct variables *v*<sub>1</sub>, *v*<sub>2</sub>, ... and some term *t*. In the case of (iii) and (iv), the semantics of these formulas calls for special comment. First, where *I* and *g* are an interpretation and variable-assignment relative to which *t* denotes an entity *x*, the formula  $\lceil \mathbb{R} t \rceil$  denotes<sub>*I,g*</sub> the ‘rolehood’ of *x*, or in other words, the state of affairs that *x* is a role. Second, a formula  $\lceil (t_1 u_1, t_2 u_2, \dots) \rceil$  denotes<sub>*I,g*</sub> an entity *x* iff there exist roles *r*<sub>1</sub>, *r*<sub>2</sub>, ... and entities *x*<sub>1</sub>, *x*<sub>2</sub>, ... such that:

- (i) Each *t*<sub>*i*</sub> denotes<sub>*I,g*</sub> the corresponding *r*<sub>*i*</sub>;
- (ii) Each *u*<sub>*i*</sub> denotes<sub>*I,g*</sub> the corresponding *x*<sub>*i*</sub>; and
- (iii) *x* is a state that results from the role assignment  $\{(r_1, x_1), (r_2, x_2), \dots\}$ .

The semantics for the other kinds of formula is largely as one would expect. For example, if *t* and *u* respectively denote<sub>*I,g*</sub> entities *x* and *y*, then  $\lceil (t = u) \rceil$  denotes<sub>*I,g*</sub> a state that obtains iff *x* is identical with *y*.

2. A *lambda-expression* is an expression  $\lceil \lambda v_1, v_2, \dots t \rceil$  for one or more pairwise distinct variables *v*<sub>1</sub>, *v*<sub>2</sub>, ... and some term *t*. These serve to denote properties, including properties that are relations. For example, relative to a variable-assignment that maps ‘*l*’ and ‘*b*’ to, respectively, the roles of *Lover* and *Beloved*, the lambda-expression ‘ $\lambda x, y (lx, by)$ ’ denotes the relation of *loving*.

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<sup>13</sup>An example of this would be the formula ‘(*l* Cleopatra, *b* Antony)’ already encountered above.

<sup>14</sup>That formulas are counted as ‘terms’ is historically unusual, but is here justified by the fact that they serve as names of states.

3. A *rho-expression* is an expression  $\lceil \rho v_1.v_2.v_3.\dots t \rceil$  for two or more pairwise distinct variables  $v_1, v_2, \dots$  and some term  $t$ . As noted above, rho-expressions will be used to denote those roles that are not properties. For example, relative to a suitable variable-assignment, the rho-expression  $\lceil \rho x.y(lx, by) \rceil$  denotes *Lover* while  $\lceil \rho y.x(lx, by) \rceil$  denotes *Beloved*.

The only variable-binding devices of  $\mathcal{L}$  are  $\lceil \exists \rceil$ ,  $\lceil \lambda \rceil$ , and  $\lceil \rho \rceil$ ; talk of free and bound variable-occurrences should be understood accordingly. A term occurs *at sentence-position* iff it either occurs as an operand of  $\lceil \neg \rceil$  or  $\lceil \& \rceil$  or as the matrix of a term starting with  $\lceil \exists \rceil$ ,  $\lceil \lambda \rceil$ , or  $\lceil \rho \rceil$ . By contrast, an occurrence of a term  $t$  will be said to stand *at predicate-position* iff it is the first occurrence of  $t$  in an occurrence of a term pair  $\lceil tu \rceil$ , for some term  $u$ .

Finally, we will make use of a few abbreviatory devices. The formula  $\lceil \&() \rceil$  will often be written as  $\lceil \top \rceil$ , the formula  $\lceil \neg \&() \rceil$  as  $\lceil \perp \rceil$ , and  $\lceil \&(t_1, t_2, \dots) \rceil$  as  $\lceil (t_1 \wedge t_2 \wedge \dots) \rceil$ . The outermost parentheses of a formula  $\lceil (tu) \rceil$  will usually be omitted, as long as this does not create ambiguity.

### 2.3 Roles

Let us say that a role assignment is *live* iff there exists a state, obtaining or not, that results from it. The role assignment  $\{(Lover, Antony), (Beloved, Cleopatra)\}$  is plausibly live, even if Antony does not love Cleopatra. By contrast, the assignment  $\{(Beloved, Antony), (Perceiver, Cleopatra)\}$  is, at least on the face of it, *not* live, as it is deeply unclear what it would mean for a state resulting from this assignment to obtain. A minor quirk of ORS lies in the fact that it does not rule out the existence of ‘improper’ roles: roles  $r$  that are such that, for any  $x$ , no live role assignment contains the pair  $(r, x)$ . Roles that are *not* improper in this sense will naturally be called *proper* roles:

**Definition 2.1.** An entity  $x$  is a *proper role* iff  $x$  is a role and, for some entity  $y$ , some live role assignment contains the pair  $(x, y)$ .

Another useful concept is that of ‘correlateship’ among roles:

**Definition 2.2.** An entity  $x$  is a *correlate* of an entity  $y$  iff  $x$  and  $y$  are roles and, for some entities  $z$  and  $w$ , there exists a live role assignment containing the pairs  $(x, z)$  and  $(y, w)$ .

Clearly, any proper role is a correlate of itself. A typical example of two correlate roles are those of *Lover* and *Beloved*, whereas those of *Beloved* and *Perceiver* are plausibly *not* correlates of each other.

With regard to correlate roles, ORS contains the following assumption:

(CR) No fundamental role is a correlate of a non-fundamental role.

I take this thesis to be intuitively plausible. It is hard to see how a relation could turn out to be such that only *some* of its constituent roles are fundamental while others aren’t. For instance, if *Lover* is fundamental, then surely its correlate *Beloved* should be fundamental, too. (CR) will become relevant in Section 2.5 below.

## 2.4 The coarse-grainedness principle

We now turn to ORS's account of the individuation of roles and states. The individuation of *roles* is tied to that of states; for the purposes of this paper, we can set this topic aside. (For details, see Plate [MS(b): §4.2, §16; MS(a): §5.1.1].) With regard to the individuation of *states*, ORS generally follows the standard 'intensionalistic' approach that treats any two logically equivalent formulas as co-denotational. However, an exception is made for those formulas—and terms more generally—that do not contain free occurrences of the same constants and variables. For example, while '(Socrates = Socrates)' is classically equivalent to '(Sappho = Sappho)', it seems implausible to think that Socrates' self-identity is the same state as Sappho's. Intuitively, only the former requires the existence of Socrates, and only the latter requires the existence of Sappho. An account of the individuation of states should therefore allow these two formulas to denote *distinct* states.

The individuation of states is in ORS governed by two principles that impose a lower and an upper bound, respectively, on the coarse-grainedness of states. The first of these will here be called a *coarse-grainedness principle*, the second a *fine-grainedness principle*. Both rely on a concept of *analytic entailment* applicable to terms of  $\mathcal{L}$ :

**Definition 2.3.** A term  $t$  *analytically entails* a term  $t'$  iff the following two conditions are satisfied:

- (i)  $t$  denotes a state relative to some interpretation and variable-assignment.
- (ii) For any interpretation  $I$  and variable-assignment  $g$ : if  $t$  denotes $_{I,g}$  a state, then so does  $t'$ ; and if  $t$  denotes $_{I,g}$  an *obtaining* state, then so does  $t'$ .

To illustrate the import of this definition, let us return to the above example: the formula '(Socrates = Socrates)' does not analytically entail '(Sappho = Sappho)', because, relative to an interpretation that is defined only on 'Socrates' but not on 'Sappho', '(Socrates = Socrates)' denotes a state while '(Sappho = Sappho)' denotes nothing at all.

Next, we can define a concept of  $\mathcal{L}$ -necessitation:

**Definition 2.4.** A state  $s$   $\mathcal{L}$ -necessitates a state  $s'$  iff there exist an interpretation  $I$ , a variable-assignment  $g$ , and terms  $t$  and  $t'$  such that: (i)  $t$  denotes $_{I,g}$   $s$ , (ii)  $t'$  denotes $_{I,g}$   $s'$ , and (iii)  $t$  analytically entails  $t'$ .

It is easy to see that any given state  $\mathcal{L}$ -necessitates itself. For any given state can be denoted (relative to a suitable variable-assignment) by an untyped variable  $v$ ; and any variable analytically entails itself. Note, however, that from this definition it does *not* follow that, e.g., Socrates' self-identity does not  $\mathcal{L}$ -necessitate Sappho's. For all that has been said so far, the formulas '(Socrates = Socrates)' and '(Sappho = Sappho)' may yet denote the same state, even relative to an interpretation that maps 'Socrates' and 'Sappho' to different people.

With the concept of  $\mathcal{L}$ -necessitation in place, our coarse-grainedness principle can be formulated as follows:



(CG) No two states  $\mathcal{L}$ -necessitate each other.

In other words, if a state  $s$   $\mathcal{L}$ -necessitates a state  $s'$  and *vice versa*, then  $s = s'$ . Given that the formulas ' $M$  Socrates' and ' $\neg\neg(M$  Socrates)' analytically entail each other, one of the things that can be inferred from (CG) is that Socrates' being mortal is the same state as its double negation. By contrast, given that ' $(\text{Socrates} = \text{Socrates})$ ' and ' $(\text{Sappho} = \text{Sappho})$ ' do *not* analytically entail each other, it cannot be inferred from (CG) that these formulas denote the same state. Of course, this is as it should be.

Now it might be suggested that we can expand (CG) into a full-fledged account of state individuation by simply turning the conditional into a biconditional:

(CG') For any states  $s$  and  $s'$ :  $s = s'$  iff  $s$  and  $s'$   $\mathcal{L}$ -necessitate each other.

But this suggestion overlooks that the left-to-right direction of (CG') is *trivial*. This follows straightforwardly from the fact, noted above, that every state  $\mathcal{L}$ -necessitates itself. As a result, (CG') is equivalent to (CG), and we are still without a (non-trivial) upper bound on the coarse-grainedness of states. The biconditional format of (CG') should not deceive us into thinking that our work is done.

## 2.5 The fine-grainedness principle

To formulate an adequate (and in particular non-trivial) fine-grainedness principle, we can employ semantic ascent. In a nutshell, our principle is to the effect that, *if* two terms  $t$  and  $t'$  denote the same state, and if certain further conditions are satisfied, *then*  $t$  analytically entails  $t'$ . For reasons of space, I shall here omit any motivating discussion of those further conditions.<sup>15</sup> The principle itself reads as follows:

(FG) For any terms  $t$  and  $t'$ , any interpretation  $I$ , and any variable-assignment  $g$ : if the following five conditions are satisfied—

- (i)  $t$  and  $t'$  denote <sub>$I, g$</sub>  the same state.
- (ii) Every entity in the range of  $I \cup g$  is fundamental.
- (iii) No two atomic terms are under  $I \cup g$  mapped to the same entity.
- (iv) Every atomic term that is under  $I \cup g$  mapped to a role or state is a variable of type 1.
- (v) For any atomic term  $u$ : if  $u$  occurs free in  $t$  and is under  $I \cup g$  mapped to a state or proper role, then at least one free occurrence of  $u$  in  $t$  stands at predicate- or sentence-position.

—then  $t$  analytically entails  $t'$ .

To apply this to our above example, let us suppose for the moment that Socrates and Sappho are fundamental particulars. Then, from the fact that ' $(\text{Socrates} = \text{Socrates})$ '

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<sup>15</sup>Interested readers can find such discussion in my (MS(b): §9.2, §14); cf. also Plate (2025: §5.4).

does not analytically entail  $'(Sappho = Sappho)'$ , (FG) allows us to infer, fairly straightforwardly, that Socrates' self-identity is not the same state as Sappho's.<sup>16</sup>

For the purposes of this paper (and especially the next section), it is crucial to note that (FG) has consequences not only for the individuation of states, but also for the question of *what entities are fundamental*. For example, it can be readily inferred that the state  $\top$ —i.e.,  $\&()$ —is *not* fundamental. For, from the supposition that it is, it follows by (FG) that the formula  $'(x^1 = \&(x^1))'$  analytically entails  $'(x^1 = \&())'$ , which is patently not the case.<sup>17</sup>

To get a fuller picture of (FG)'s import, it may be helpful to derive a theorem. To this end, let us first introduce a concept of 'purity':<sup>18</sup>

**Definition 2.5.** An entity is *pure* iff it is denoted $_{\emptyset, \emptyset}$  by some term.

More intuitively put, an entity is pure iff it can be represented by using the expressive resources of  $\mathcal{L}$  alone, i.e., by a term that contains no free occurrence of any variable or constant. With the help of the above assumptions (CR) and (CG), (FG) then allows us to prove the following:

**Proposition 2.6.** Let  $A$  and  $B$  be any *pure* attributes, let  $x_1, x_2, \dots$  and  $y_1, y_2, \dots$  be some (not necessarily pairwise distinct) fundamental entities such that there exist an instantiation of  $A$  by  $x_1, x_2, \dots$  and an instantiation of  $B$  by  $y_1, y_2, \dots$ , and suppose that at least one of the following two conditions is satisfied:

- (a) At least one of the  $x_i$  has no correlate among the  $y_j$ .<sup>19</sup>
- (b) At least one of the  $y_i$  has no correlate among the  $x_j$ .

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<sup>16</sup>In particular, let  $t$  be  $'(Socrates = Socrates)'$ , let  $t'$  be  $'(Sappho = Sappho)'$ , and let  $I$  be the smallest interpretation that maps 'Socrates' and 'Sappho' to Socrates and Sappho, respectively. In addition, let  $g$  be the empty set. Clauses (ii)–(v) will then be satisfied; and if Socrates' self-identity *were* the same state as Sappho's, then clause (i) would be satisfied, too. By (FG), it would therefore follow that  $t$  analytically entails  $t'$ . But  $t$  does *not* analytically entail  $t'$ .

<sup>17</sup>Cf. Plate (2025: §5.5.2). More slowly: suppose for *reductio* that  $\top$  is fundamental. It then follows that  $\top$  is zeroth-order, so that the state  $(\top = \top)$  will be denoted by both  $'(x^1 = \&(x^1))'$  and  $'(x^1 = \&())'$  relative to the empty interpretation and any variable-assignment that maps  $'x^1'$  to  $\top$ . (The formula  $'\&(x^1)'$  is, as it were, the conjunction that has  $'x^1'$  as its only conjunct.) Hence, if we let  $I$  be the empty interpretation and  $g$  the smallest variable-assignment that maps  $'x^1'$  to  $\top$ , and further let  $t$  and  $t'$  be, respectively, the formulas  $'(x^1 = \&(x^1))'$  and  $'(x^1 = \&())'$ , then clause (i) is satisfied; and so are (ii)–(v). From (FG), it then follows that  $t$  analytically entails  $t'$ , which, as noted in the text, is not the case. This completes the *reductio*.

<sup>18</sup>For precedent, see Bacon (2020: 546f.). Bacon investigates a closely related principle under the name *Quantified Logical Necessity*. In fact the two principles can be regarded as analogous to each other, although there are also some quite significant differences. For example, Bacon's principle specifically concerns the question of *when a given state is identical with  $\top$* , rather than the individuation of states more generally.

<sup>19</sup>The relevant sense of 'correlate' is given by Definition 2.2 above. As may be inferred from that definition, only proper roles have correlates. As a result, the present condition can also be put as follows: At least one of the  $x_i$  is either not a role at all or is a (possibly improper) role that has no correlate among the  $y_j$ .

Then the instantiation of  $A$  by  $x_1, x_2, \dots$  is distinct from the instantiation of  $B$  by  $y_1, y_2, \dots$  (in this order).

I delegate the proof sketch to a footnote.<sup>20</sup> Proposition 2.6 roughly captures, but also

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<sup>20</sup>We focus on the case in which  $A$  and  $B$  are pure *properties*, and let  $x$  and  $y$  be two fundamental entities such that: there exists an instantiation of  $A$  by  $x$  as well as an instantiation of  $B$  by  $y$ , and  $x$  is not a correlate of  $y$ . (It will be easy to see how the argument generalizes.) Our goal is to find terms  $t$  and  $t'$  and a variable-assignment  $g$  such that, first,  $t$  and  $t'$  denote $_{\emptyset, g}$ , respectively, the instantiation of  $A$  by  $x$  and the instantiation of  $B$  by  $y$  and, second,  $t, \emptyset$  (the empty set, playing here the part of the empty interpretation), and  $g$  jointly satisfy clauses (ii)–(v) of (FG). Clause (v) complicates things; it is the reason why we have to use (CR).

Since  $A$  and  $B$  are pure, they are respectively denoted, relative to any interpretation and variable-assignment, by some terms that contain no free occurrence of any variable or constant. Let  $u$  and  $u'$  be such terms, so that  $u$  denotes $_{\emptyset, \emptyset} A$  and  $u'$  denotes $_{\emptyset, \emptyset} B$ . Further, let  $v$  and  $v'$  be two variables such that the following holds:

- $v$  is untyped if  $x$  is a particular, and of type 1 otherwise.
- $v'$  is untyped if  $y$  is a particular, and of type 1 otherwise.

In addition, let  $t'$  be the formula  $\ulcorner u'v' \urcorner$ . Relative to any variable-assignment that maps  $v'$  to  $y$ ,  $t'$  will then denote the instantiation of  $B$  by  $y$ , which is to say: the state that results from the role assignment  $\{(B, y)\}$ . (Cf. Sections 2.1 and 2.2 above.) We have to distinguish two cases.

*Case 1.* If  $x$  is either a particular or a state, a property, or improper role, then we can proceed as follows. First, let  $g$  be the smallest variable-assignment that maps  $v$  to  $x$  and  $v'$  to  $y$ . Together with (CG) and the fundamentality of  $x$ , the semantics of formulas then ensures that, if  $x$  is a state, it is also denoted $_{\emptyset, g}$  by  $\ulcorner \&(v) \urcorner$ . Similarly, the semantics of lambda-expressions, together with the fundamentality of  $x$  and an assumption about the individuation of roles, ensures that, if  $x$  is a property, then it is also denoted $_{\emptyset, g}$  by  $\ulcorner \lambda z v z \urcorner$ . (For the semantics of lambda-expressions and details about the individuation of roles, I refer the reader to my (MS(b): §4.2, §6) and (MS(a): §4.2, §5.1.1).)

Next, if  $x$  is a particular or an improper role, let  $t$  be  $\ulcorner uv \urcorner$ ; if  $x$  is a state, let  $t$  be  $\ulcorner u\&(v) \urcorner$ ; and if  $x$  is a property, let  $t$  be  $\ulcorner u\lambda z v z \urcorner$ . By what has been said in the previous paragraph, it can be seen that  $t$  then denotes $_{\emptyset, g}$  the instantiation of  $A$  by  $x$ . Meanwhile, by what has been said two paragraphs ago,  $t'$  denotes $_{\emptyset, g}$  the instantiation of  $B$  by  $y$ . In addition,  $t, \emptyset$ , and  $g$  jointly satisfy clauses (ii)–(v) of (FG). Now suppose for *reductio* that the instantiation of  $A$  by  $x$  is identical with the instantiation of  $B$  by  $y$ . Then clause (i) is also satisfied, so that, by (FG), we have that  $t$  analytically entails  $t'$ . But in fact,  $t$  does *not* analytically entail  $t'$ , since  $t'$  contains a free occurrence of a variable—viz.,  $v'$ —that does not occur free in  $t$ . This completes the *reductio*; it follows that the instantiation of  $A$  by  $x$  is distinct from that of  $B$  by  $y$ .

*Case 2.* It remains to consider the case in which  $x$  is a proper role but not a property. There then exists, for some ordinal  $\alpha > 0$ , an  $\alpha$ -sequence (i.e., a sequence of length  $\alpha$ ) of roles  $r_1, r_2, \dots$  such that, for some  $(\alpha + 1)$ -sequence of entities  $x_0, x_1, x_2, \dots$ , there is a state resulting from the role assignment  $\{(x, x_0), (r_1, x_1), (r_2, x_2), \dots\}$ . These roles are *correlates* of  $x$ . Since  $x$  itself is fundamental, it follows by (CR) that each  $r_i$  is fundamental, too, and hence zeroth-order. Moreover, since  $x$  is by hypothesis not a correlate of  $y$ , we have that none of the  $r_i$  is identical with  $y$ . Let  $u_1, u_2, \dots$  be an  $\alpha$ -sequence of variables, each of type 1 and distinct from  $v'$ , and let  $g$  be the smallest variable-assignment that maps  $v$  to  $x$ ,  $v'$  to  $y$ , and each  $u_i$  to the corresponding  $r_i$ . Since each  $r_i$  is zeroth-order, we have that each  $u_i$  denotes $_{\emptyset, g}$  the corresponding  $r_i$ . In addition, of course,  $v$  denotes $_{\emptyset, g} x$ . The semantics of rho-expressions (on which see my (MS(b): §12) and (MS(a): §4.2)) then ensures that  $x$  is also denoted $_{\emptyset, g}$  by  $\ulcorner \rho v_0.v_1.v_2.\dots (vv_0, u_1v_1, u_2v_2, \dots) \urcorner$ , for some pairwise distinct variables  $v_0, v_1, \dots$ , all of which are distinct from both  $v$  and each of the  $u_i$ .

Let now  $t$  be the formula  $\ulcorner u\tilde{\zeta} \urcorner$ , where  $\tilde{\zeta}$  is the rho-expression just mentioned. Since  $x, y$ , and each of the  $r_i$  is fundamental, clause (ii) of (FG) is satisfied. Without loss of generality, we may moreover assume that clause (iii) is satisfied, too. (While we have stipulated that each  $u_i$  be distinct from  $v'$ ,

adds an important qualification to, the thought that any state expressible in terms of *one* set of fundamental entities is distinct from any state expressible in terms of some *other* set of fundamental entities. The qualification is that it is not enough for the two sets to be merely distinct: rather, at least one of the two has to have a member that *has no correlate* among the members of the other.

## 2.6 The analyzability assumption

Finally, we come to what may be the most controversial among ORS's assumptions. Intuitively put, it is the thesis that  $\mathcal{L}$  is expressive enough to represent the metaphysical structure (down to the *fundamentalia*) of any entity:

- (A) For any entity  $x$ , there exist a term  $t$ , an interpretation  $I$ , and a variable-assignment  $g$  such that: (i)  $t$  denotes $_{I,g}$   $x$ , and (ii) any entity in the range of  $I \cup g$  is fundamental.

This assumption is useful in part because it allows us to show, with the help of (FG), that every set is a non-fundamental entity.<sup>21</sup> However, the relative austerity of  $\mathcal{L}$ —which contains, apart from formulas and atomic terms, only lambda- and rho-expressions—makes it a correspondingly strong claim. For instance, (A) has the consequence that any given particular is fundamental, since particulars can in  $\mathcal{L}$  only be denoted by atomic terms. To avoid this, one might augment  $\mathcal{L}$  by adding a way to construct complex terms capable of denoting particulars. For present purposes, however, we can set this issue aside, since those more controversial consequences of (A) will not be of interest in what follows. Let us instead focus on the main task of this paper, which is to defend (FF).

## 3 Might fundamentality be analyzed in broadly logical terms?

If, instead of taking our concept of fundamentality as primitive, we can analyze it in terms of some pre-existing 'broadly logical' concepts, then we should probably do so—or that is what considerations of ideological parsimony seem to suggest.<sup>22</sup> Such an analysis of the *concept* of fundamentality will typically suggest an analogous metaphysical analysis of the *property* of fundamentality, just as an analysis of, e.g., the concept of a vixen in terms of those of fox and female will suggest an analogous analysis of the property of vixenhood that makes reference to the properties of *being*

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this does not create a problem, since none of the  $r_i$  is identical with  $y$ .) So is clause (iv). And since both  $v$  and each of the  $u_i$  stands in  $t$  at predicate-position, clause (v) is satisfied, as well. The rest of the proof proceeds as before.

<sup>21</sup>See Plate (MS[b]: §15).

<sup>22</sup>I here use the qualifier 'broadly' to give myself license to include certain logico-metaphysical concepts, such as those of role and state.

*female* and *being a fox*. Given (FG), this property can then be expected *not* to be fundamental, contrary to (FF).

In the present section, I will discuss a proposal according to which fundamentality is nothing other than *logical simplicity*. I will first give an account of logical simplicity, then formulate the proposal, and finally (in Section 3.3) argue that it should be rejected.

### 3.1 Logical simplicity

Intuitively, an entity is *logically simple* iff it is not ‘constructed’ by logical operations, such as conjunction and negation, from simpler entities. However, the devil is in the details. To see the problems with this intuitive explication, it will be helpful to make it more precise, which can be done in two steps. We first stipulate that every *particular*—i.e., every entity that is neither a role nor a state—is logically simple, and that a *state* is logically simple (or ‘simple’ for short) iff it is neither the negation nor the conjunction of any other states, and is also not the existential quantification of any property or relation. In the second step, we expand this account to roles:

- (S) A role  $r$  is simple iff, for every state  $s$ , the following holds: if  $s$  results from a role assignment that, for some entity  $x$ , contains the pair  $(r, x)$ , then  $s$  is simple.

In this way, the simplicity of simple roles is made to result from the simplicity of simple states. Notice that, under (S), every *improper* role is classified as simple. Another consequence is that, if every state is simple, then so is every role; and likewise, if every state *fails* to be simple, then no proper role is simple.

Unfortunately, this straightforward proposal won’t quite do if we adopt an at least moderately coarse-grained conception of states, such as the one outlined in the previous section. For under that conception, every state is the negation of its own negation, so that, under the proposal now under discussion, *every* state—and consequently, given (S), also every proper role—fails to be simple. If we then further equate fundamentality with logical simplicity, we are faced with the consequence that no state or proper role is fundamental. This would be unwelcome, and would certainly not sit well with our assumption (A). For there are presumably many states, such as Cleopatra’s loving Antony, that cannot be analyzed in terms of particulars and improper roles alone.<sup>23</sup>

What we need, then, is an account of logical simplicity that does not collapse when combined with a coarse-grained conception of states. Such an account has been proposed by Plate (2016: §3), and with a bit of work it can be adapted to the present setting. Its basic idea is that an attribute is simple iff, for any lambda-expression  $L$  that denotes that attribute, one can find a ‘reduction’ of  $L$  that simplifies *salva denotatione* to a lambda-expression of a particularly simple form, such

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<sup>23</sup>More formally put: there are many states that are not denoted by a term of  $\mathcal{L}$  relative to some interpretation  $I$  and variable-assignment  $g$  such that the range of  $I \cup g$  contains only particulars or improper roles.

as  $\lambda x F(x)$  or  $\lambda x, y G(x, y)$ , in which an atomic predicate is applied to a list of bound variables. Here a reduction of  $L$  is essentially a lambda-expression that results from  $L$  by replacing occurrences of atomic terms with codenotational other terms. The property of *being a vixen*, for example, is under this account plausibly *not* simple, because it can—in the formal language used by Plate—be denoted by  $\lambda x (\text{female}(x) \wedge \text{fox}(x))$ , and it is difficult to see how this lambda-expression could have a reduction that simplifies to  $\lambda x \text{vixen}(x)$ .<sup>24</sup>

Adapting this account to the present setting requires a few changes. First, we will obviously be using a different formal language. For example, there is in  $\mathcal{L}$  no lambda-expression  $\lambda x, y G(x, y)$ ; the closest counterpart would be something like  $\lambda x, y (Qx, Ry)$ , with  $'Q'$  and  $'R'$  standing for roles. Second, in order to address the question of what *states* are fundamental, we have to generalize the account in such a way that it is also applicable to states, rather than only to roles. Third, we have to make the account applicable to roles that are not properties. And fourth, given that relations in ORS are sets, which (as noted in Section 2.6 above) are non-fundamental entities, we should *not* count an entity as simple if every lambda-expression by which it can be denoted has a reduction that simplifies to something like  $\lambda x, y (Qx, Ry)$ . For such an entity will still be a relation, and should therefore count as non-fundamental, and hence also as non-*simple* if we are to equate fundamentality with simplicity.

With these points in mind, let us now move towards a suitable definition of 'logically simple'. To begin with, the central concept of reduction can be defined exactly as in Plate (*op. cit.*, p. 24). The following is merely a stylistic variant:

**Definition 3.1.** For any interpretation  $I$  and variable-assignment  $g$ , a term  $t$  is a *reduction* of a term  $t'$  relative to  $I$  and  $g$  iff  $t$  is the result of replacing in  $t'$  zero or more free occurrences of atomic terms with (relative to  $I$  and  $g$ ) codenotational other terms, in such a way that no free variable-occurrence in any of the replacing terms is captured.

For example, if  $I$  is the empty set and  $g$  a variable-assignment that maps  $'V'$ ,  $'F'$ , and  $'G'$  to vixenhood, femaleness, and foxhood, respectively, then  $\lambda x (Fx \wedge Gx)$  is relative to  $I$  and  $g$  a reduction of  $'V'$ ; and likewise,  $(\lambda x (Fx \wedge Gx)x)$  is a reduction of  $'(Vx)'$ .<sup>25</sup>

Let us further say that two terms are *analytically equivalent* iff they analytically entail each other in the sense of Definition 2.3. On this basis, we can finally produce the required definition:

**Definition 3.2.** An entity  $x$  is *logically simple* iff, for any term  $t$ , interpretation  $I$ , and variable-assignment  $g$ , the following holds: if  $t$  denotes <sub>$I, g$</sub>   $x$ , then there

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<sup>24</sup>Essentially, the problem is how to get rid of the conjunction sign. It *could* be done if foxhood and femaleness were somehow both analyzable in terms of vixenhood and broadly logical notions (and nothing besides), but of course this is quite implausible.

<sup>25</sup>Note here that  $(\lambda x (Fx \wedge Gx)x)$  is syntactically unambiguous. In particular, it cannot be parsed as  $(\lambda x ((Fx \wedge Gx)x))$ , since this would be a lone lambda-expression enclosed in parentheses, and there is no such term in  $\mathcal{L}$ .

exist an interpretation  $I' \supseteq I$  and a variable-assignment  $g' \supseteq g$  that satisfy the following two conditions:

- (i) If  $x$  is a state, then, for some atomic term  $u$ ,  $t$  has relative to  $I'$  and  $g'$  a reduction that is analytically equivalent to  $u$ .
- (ii) If  $x$  is a proper role, then at least one of the following two conditions is met:
  - (a) For some atomic term  $u$  and variable  $v$ :  $t$  has relative to  $I'$  and  $g'$  a reduction  $\ulcorner \lambda v \varphi \urcorner$ , where  $\varphi$  is analytically equivalent to  $\ulcorner uv \urcorner$ .
  - (b) For some ordinal  $\alpha > 1$ , some  $\alpha$ -sequence of atomic terms  $u_1, u_2, \dots$ , and some  $\alpha$ -sequence of pairwise distinct variables  $v_1, v_2, \dots$ :  $t$  has relative to  $I'$  and  $g'$  a reduction  $\ulcorner \rho v_1.v_2, \dots \varphi \urcorner$ , where  $\varphi$  is analytically equivalent to  $\ulcorner (u_1 v_1, u_2 v_2, \dots) \urcorner$ .

Here the first of the two clauses (i) and (ii) deals with states, while the second concerns proper roles; and *its* two subclauses deal, respectively, with properties and proper roles that are *not* properties. As already in the case of Plate (*op. cit.*), the quantification over supersets of  $I$  and  $g$  is motivated by considerations of extensional adequacy: if  $I$  is defined only on ‘vixen’ and  $g$  is empty, then ‘vixen’ will not have a reduction relative to  $I$  and  $g$  that would reveal vixenhood to be a conjunction of foxhood and femaleness.

Since the clauses (i) and (ii) are vacuously satisfied by any particular and any improper role, it can be seen that, under Definition 3.2, any particular and any improper role is classified as logically simple, just as under the previous proposal. With regard to *states*, by contrast, the present definition is considerably more demanding.

### 3.2 How ORS has to be modified if fundamentality is equated with logical simplicity

Let us now suppose that, in the context of ORS, fundamentality is to be equated with logical simplicity in the sense just defined. Unfortunately, this does not turn out to be entirely smooth sailing. On the background of a sufficiently coarse-grained conception of properties, the above definition of ‘logically simple’ does not differentiate between a property and its negation. In other words, if a given property  $P$  is simple, then so is  $\lambda x \neg(Px)$ .<sup>26</sup> This creates a problem for the idea that fundamentality may in the context of ORS be equated with logical simplicity; for (FG) entails that, for any fundamental property  $P$ , the negation of  $P$  is *non-fundamental*.<sup>27</sup>

Accordingly, if fundamentality is to be equated with logical simplicity, then (FG) has to be given up. In order to have nonetheless an account of state individuation

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<sup>26</sup>See the argument in Plate (2016: 28), which carries over to the present setting with minor adjustments.

<sup>27</sup>Cf. Plate (2025: §5.5.2). The basic shape of the argument has already been alluded to in Section 2.5 above, in connection with the non-fundamentality of  $\top$ .

that preserves the spirit of (FG), we can replace both (FG) and (A) with a *single* principle according to which *some set* of fundamental (i.e., simple) entities satisfies suitably modified versions of (A) and (FG):

- (F) Some set  $S$  of fundamental entities satisfies the following two conditions:
- (1) For any entity  $x$ , there exist a term  $t$ , an interpretation  $I$ , and a variable-assignment  $g$  such that: (i)  $t$  denotes <sub>$I, g$</sub>   $x$ , and (ii) any entity in the range of  $I \cup g$  is a member of  $S$ .
  - (2) For any terms  $t$  and  $t'$ , any interpretation  $I$ , and any variable-assignment  $g$ : if the following five conditions are satisfied—
    - (i)  $t$  and  $t'$  denote <sub>$I, g$</sub>  the same state.
    - (ii) Every entity in the range of  $I \cup g$  is a member of  $S$ .
    - (iii) No two atomic terms are under  $I \cup g$  mapped to the same entity.
    - (iv) Every atomic term that is under  $I \cup g$  mapped to a role or state is a variable of type 1.
    - (v) For any atomic term  $u$ : if  $u$  occurs free in  $t$  and is under  $I \cup g$  mapped to a state or proper role, then at least one free occurrence of  $u$  in  $t$  stands at predicate- or sentence-position.
 —then  $t$  analytically entails  $t'$ .

Here conditions (1) and (2) differ from (A) and (FG) only insofar as the predicate ‘is fundamental’ has in both cases been replaced by ‘is a member of  $S$ ’. The need to include condition (1) stems from the fact that (2) is trivially satisfied for  $S = \emptyset$ , so that, without (1), (F) would be trivially true.<sup>28</sup> Due to the inclusion of this first condition, (A) itself is no longer needed: its work is now done by (F).

By replacing (FG) and (A) with (F), we avoid the result that the negation of a fundamental property is non-fundamental. This solves the above problem, leaving us free to equate fundamentality with logical simplicity and thereby “reduce our burden of primitive notions”, as Lewis (1986: 154) puts it. What is not to like?

### 3.3 Why not to equate fundamentality with logical simplicity

To my mind, the most compelling reason to reject the above proposal lies in the complexity it adds to ORS. While (F) by itself is only *slightly* more complicated than (A) and (FG) taken together, it is clearly *significantly* more complex when combined with Definitions 3.1 and 3.2. It would be nice if this added complexity could be

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<sup>28</sup>To see this, suppose that clauses (2.i) and (2.ii) are both satisfied. With  $S = \emptyset$ , (2.ii) entails that the range of  $I \cup g$  is empty. But if the range of  $I \cup g$  is empty, then no term that has a denotation <sub>$I, g$</sub>  will contain a free occurrence of a variable or constant. Hence, given that (2.i) is satisfied, so that  $t$  and  $t'$  denote <sub>$I, g$</sub>  the same state (so that, *a fortiori*, each of them denotes <sub>$I, g$</sub>  *something*), neither  $t$  nor  $t'$  contains a free occurrence of a variable or constant. But in that case, the consequent of (2)’s main conditional follows, as can be seen by consulting Definition 2.3. Hence (2) is satisfied whenever  $S = \emptyset$ .



### 3 Might fundamentality be analyzed in broadly logical terms?

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avoided; and the obvious way to do so is to take the concept of fundamentality as primitive.<sup>29</sup>

Admittedly, this may not sway the friend of the above proposal. Under what conditions, she may ask, should we prefer a simpler theory over a more complex one? Is it not only if and when the discovery of the simpler theory comes as a surprise—a surprise resulting from the fact that, on the assumption that the more complex theory was correct, it was not to be expected that some simpler theory would have a comparable rate of predictive success? (I delegate a more detailed explanation to a footnote.<sup>30</sup>) But in the present case there is no surprise, because the

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<sup>29</sup>For recent discussions of simplicity as a criterion of theory choice in metaphysics, see, e.g., Schaffer (2015), Brenner (2017), Da Vee (2020), Sober (2022). Another potential problem for the above proposal may seem to arise from the observation that the fundamental entities, taken together, are often held to form a *minimal* basis of everything there is (cf. Tahko [2023: §1.3]). This constraint is plausibly violated if, e.g., the negations of fundamental properties are themselves fundamental. However, this constraint is not set in stone. It is hardly absurd to allow *some* redundancy among the fundamental entities. For example, Sider (2020: 121) suggests that the mereological concepts of parthood and overlap are both of them fundamental. Transposed into the present setting, this would mean that the *relations* of parthood and overlap, despite being analyzable in terms of each other, are both of them fundamental in the sense of having fundamental constituent roles.

<sup>30</sup>One situation in which we can be justified in penalizing complex theories arises when we are confronted with potentially misleading patterns in ‘noisy’ data. (See, e.g., Forster and Sober [1994], Claeskens and Hjort [2008].) The rationale hinted at in the text is largely orthogonal to this concern. To explain that rationale, it will be helpful to distinguish two kinds of complexity:

1. A theory’s *syntactic* complexity is the length of its most concise statement (in any language). As usual, a theory’s primitive vocabulary may include ‘observation terms’; and if it employs *defined* terms, the relevant definitions should be included in the theory. (Note that metaphysical analyses do *not* count as definitions.)
2. What I shall call the *material* complexity of a theory *T* is the syntactic complexity that a *by-and-large complete* theory of the phenomena that *T* is concerned with—where a theory’s degree of completeness is to be assessed, very roughly, by looking at what proportion of relevant facts about the phenomena in question can be logically derived from it—may be reasonably expected to have if *T* were true. (Finocchiaro [2021: 620] speaks in a similar vein of a theory’s ‘objective simplicity’.) For example, if a theory *T* of particle physics posits ten fundamental types of particle, and we suppose *T* to be true, then, even if *T* itself says *nothing whatsoever* about those particles, we can reasonably expect that a by-and-large complete theory of the phenomena that *T* is concerned with will contain a number of statements as to how the particles of each type behave; and the material complexity of *T* will be correspondingly greater than that of a theory that posits only one fundamental type of particle.

This is admittedly *quite* rough, but it is enough to go on with. The kind of simplicity at issue in the text, as well as in the rest of this footnote, should be understood to be material rather than syntactic simplicity.

So why are we justified in preferring simpler theories over more complex ones? The basic idea is quite straightforward. Under a more complex theory *T*, the world is a more complicated place than under a simpler theory *T\**; and so one would expect this additional complexity to show up somewhere in the data, provided that the latter are of sufficient quality. But if *T\** enjoys essentially the same predictive success as *T* in the same areas as *T*, then there is no such signal. In this case, then, a researcher who works under the assumption that *T* is correct has put herself into a position where one of the expectations she is justified (on the strength of that assumption) to have turns out to be mistaken. This serves to disconfirm her theory. By contrast, someone who works with a

simplification essentially consists in the ‘trick’ of putting a new primitive (viz., ‘fundamental’) in place of the complex notion of logical simplicity. Cheap tricks deserve no rewards, and certainly do not justify the introduction of a primitive concept!

This objection makes a few important points. Clearly, the sort of simplification that results from simply throwing out a definition and taking the *definiendum* as primitive does not, in general, result in a better theory. Nor do I wish to contest the objector’s claim about the conditions under which a simpler theory deserves preference over a more complex one. Instead, my reply is that those conditions *are actually satisfied in the present case*. In particular, there *is* a surprise, and the ‘trick’ decried by the objector is not as cheap as all that. For consider how the replacement of a complex concept with a novel primitive would usually play out: We would start with a certain theory that, in at least one of its assumptions, makes use of a certain predicate *F*. Then we would replace each occurrence of this predicate with the new primitive *G*. In doing so, we would typically lose the information encoded in the definition of *F*; and consequently, we would have to restore that information by adding a suitable assumption. For example, if in our original theory fundamentality was defined as logical simplicity, and we then moved to a theory in which that definition was abandoned and the concept of fundamentality taken as primitive, we would typically have to add an assumption to the effect that an entity is fundamental iff it is logically simple. In thereby restoring the information lost by abandoning the definition, we would undo that gain in simplicity.

But this is not how things play out in the present case. For suppose that, instead of (A) and (FG), our original theory contains (F) together with a definition under which an entity is fundamental just in case it is logically simple. To arrive at ORS, we will then have to replace (F) with (A) and (FG) and get rid of the mentioned definition, thus turning ‘fundamental’ into a primitive. Crucially, however, there is here no need to add an assumption to the effect that any *x* is fundamental iff *x* is logically simple; for (FG) already imposes a very strong constraint on what entities are fundamental. For example, it is one of the consequences of (FG) that, for any two fundamental properties *P* and *Q*, their ‘conjunction’, i.e.,  $\lambda x (Px \wedge Qx)$ , is *non-fundamental*.<sup>31</sup> This is the surprise, and the reason why the gain in simplicity achieved by taking fundamentality as primitive is not merely the result of a cheap

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simpler theory need *not* be surprised to find that there exists a more complex theory with a similar or greater rate of predictive success in the same areas, since greater complexity tends to provide greater flexibility in accommodating the data. (Cf. Huemer [2009: §II.4].)

To be sure, there are cases in which a researcher would *not* have been justified in having the sort of expectation just described. For example, if *T* posits ten fundamental types of particle and *in addition* claims that the instances of these ten types all behave in exactly the same way, then it will be no surprise to learn of a simpler theory that works just as well: in particular, a theory that posits only a single fundamental type of particle instead of *T*’s ten. But it seems tolerably clear that in this scenario the prior probability of *T* is much lower than that of its simpler rival. (Cf. Jansson and Tallant [2017].)

<sup>31</sup>This can be shown, once again, by an argument that is analogous to the one given in Section 2.5 above for the conclusion that  $\top$  is non-fundamental. But I should note that my point here is *not* that (FG) guarantees that an entity is fundamental iff it is logically simple—this is a commitment that can be easily given up—but rather that (FG) guarantees the *sparseness* of the fundamental entities.

trick. Rather, it marks a shift to a genuinely more elegant theory.<sup>32</sup>

A line of reply still open to the friend of the above proposal is to insist that the greater simplicity of a theory that takes fundamentality as primitive is trumped by the greater *ideological parsimony* of a theory that equates fundamentality with logical simplicity. It is not very clear, however, how this reply might be justified. In the absence of a compelling argument for the claim that ideological parsimony *does* trump simplicity, it appears to be a reasonable default position that, over and above the extent to which a theory's ideological parsimony contributes to its simplicity, there is no special reason to regard ideological parsimony as truth-conducive. (Not implausibly, the same might be said about *ontological* parsimony.)

We have now considered what—at least to my mind—seems to be the only promising way of analyzing fundamentality in broadly logical terms, viz., to equate it with logical simplicity. The existence of still further ways to analyze fundamentality in broadly logical terms has not thereby been ruled out; but it is certainly not obvious what they might be.

## 4 Should we go syncategorematic?

Even if the concept of fundamentality cannot be analyzed in broadly logical terms, it might be thought that the concept could perhaps *itself* be treated as broadly logical. More specifically, it might be suggested that we add to  $\mathcal{L}$  a new term-forming operator ' $\Delta$ ' with the following semantics:

For any interpretation  $I$ , variable-assignment  $g$ , term  $t$ , and entity  $x$ : if  $t$  denotes <sub>$I, g$</sub>   $x$ , then ' $\Delta t$ ' denotes <sub>$I, g$</sub>  the *fundamentality of  $x$* ,

where 'the fundamentality of  $x$ ' is a technical expression for what might otherwise be called 'the state of affairs that  $x$  is fundamental'. Let *syncategorematicism* be the view that results from ORS when we: (i) add to  $\mathcal{L}$ 's basic vocabulary the operator

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<sup>32</sup>A reader might still wonder whether this gain in elegance has any epistemic value—especially since it may be unclear how the considerations of footnote 30 above carry over to the present case. The main source of apparent disanalogy seems to lie in the fact that, in the case discussed there, the researcher in question has to pay close attention to the *rates of predictive success* of the theories under consideration. For, typically, predictive success is not a prominent factor in comparisons of metaphysical theories.

However, even a metaphysical theory will conform or fail to conform with empirical data, and there is such a thing as *coverage*: we often have at least a rough sense of how much a given metaphysical theory achieves, of the range of topics that it has something to say about, and whether it suffers from any obvious lacunae. And in general we have little reason to prefer a simpler metaphysical theory over a more complex one if the latter has much greater coverage and fits the empirical data at least equally well. Here, then, is the analogy: just like a scientist working under the assumption that a certain complex theory is correct should (normally) be surprised to find that a significantly simpler theory enjoys an at least similar rate of predictive success, so a metaphysician working under the assumption that a certain complex theory is correct should be surprised to find that there is a significantly simpler theory that, without lapsing into absurdity or empirical falsehood, has at least roughly equal coverage.

‘ $\exists$ ’, with the semantics just outlined, and (ii) adopt a definition of the predicate ‘is fundamental’ under which an entity  $x$  is fundamental just in case  $\exists x$  obtains.<sup>33</sup> The label ‘syncategorematicism’ is suggested by the fact that ascriptions of fundamentality are under this scheme effected through a *syncategorematic* operator (namely, ‘ $\exists$ ’), rather than through some constant that *denotes* the property of *being a fundamental entity*.

With  $\mathcal{L}$  modified in the way just described, it follows from (FG) that the property of fundamentality, which can now be denoted by ‘ $\lambda x \exists x$ ’, fails to be fundamental. The argument is analogous to the proof of the non-fundamentality of  $\top$ .<sup>34</sup> And of course, in just the same way it can also be shown that *non-fundamentality*, i.e.,  $\lambda x \neg \exists x$ , is non-fundamental. The view should thus appeal to theorists with a taste for symmetry. What is not to like?

Maybe the following. Given (CG), which says that no two states  $\mathcal{L}$ -necessitate each other, syncategorematicism has the consequence that any fact as to how many fundamental entities there are is identical with  $\top$ .<sup>35</sup> How much of a problem is this?

It is true that, already under ORS, states are individuated in a relatively coarse-grained manner. For example,  $\top$  is identical with the fact (i.e., obtaining state) that  $\top = \top$ , and also with the fact that self-identity is self-identical. Thus the formulas ‘ $\top$ ’, ‘ $(\top = \top)$ ’, and ‘ $\lambda x (x = x) \lambda x (x = x)$ ’ all denote the same fact. That the *truths*—the true propositions or ‘objects of belief’—expressed by these formulas are (as one may argue) pairwise distinct does not mean that they cannot all correspond to the same *fact*, in the sense of having the same fact as their common ‘truth condition’.<sup>36</sup> Nor is every truth that corresponds in this way to  $\top$  what one would typically call a ‘trivial’ truth. One of them, for instance, is the proposition that there are infinitely many entities, which follows from the set-theoretic axiom of Infinity. Let us call this proposition ‘ $p_\infty$ ’. Since  $p_\infty$  is expressible by a formula of  $\mathcal{L}$  that contains no free occurrence of a constant or variable, the fact to which  $p_\infty$  corresponds is nothing other than  $\top$ . This identification should not strike us as absurd. It would be a mistake to think that the *epistemic* distinction between trivial and non-trivial truths has to be neatly mirrored by a *metaphysical* division among facts.<sup>37</sup>

<sup>33</sup>A related proposal would be to introduce a special quantifier ‘ $\exists^*$ ’ that ranges over all and only the fundamental things. The fundamentality predicate could then be introduced by saying that an entity  $x$  is fundamental iff  $\exists^* y (y = x)$ . Cf. McDaniel (2017: 199).

<sup>34</sup>Cf. Section 2.5 above.

<sup>35</sup>*Proof sketch.* Let  $s$  be some obtaining state (or ‘fact’) as to how many fundamental entities there are. For example, supposing that there are at least two,  $s$  might be the fact that there are at least two fundamental entities. (Another possibility would be to let  $s$  be the fact that there are *exactly* two fundamental entities; but let us here go with the former option.) In our modified formal language,  $s$  will then be denoted, relative to any interpretation and variable-assignment, by the formula ‘ $\exists x, y ((x \neq y) \wedge \exists x \wedge \exists y)$ ’. Now notice that this formula and ‘ $\top$ ’ analytically entail each other. This means that  $s$  and  $\top$   $\mathcal{L}$ -necessitate each other, so that, by (CG), they are one and the same fact.

<sup>36</sup>A critic might wonder what sort of thing a proposition could possibly be, if not a state. According to a currently prominent view, however, propositions are *act-types*, which arguably makes them properties. Cf. Hodgson (2021) and references therein.

<sup>37</sup>For related recent discussion, see Williamson (2022; 2024: ch. 5).

But now consider claims as to how many *fundamental* entities there are. Plausibly, there are not too many of them to form a set. If so, there is a set of them, whose size is given by a certain cardinal  $\kappa$ . It is also natural to think that the fact that there are exactly  $\kappa$ -many fundamental entities is in a certain sense ‘contingent’: whatever  $\kappa$  may be, one would think that there could surely have been  $(\kappa + 1)$ -many fundamental entities instead. If this is right, then the fact that there are exactly  $\kappa$ -many fundamental entities should certainly *not* be identified with  $\top$ . Working backwards, we can then conclude that fundamentality should not be expressed by some operator of  $\mathcal{L}$  that, like ‘=’ and ‘ $\neg$ ’, has a fixed semantics.

Unfortunately, a critic might now point out that, if the foregoing argument works at all, it seems to generate a dilemma for the adherent of ORS. For on the one hand, if there could have been more fundamental entities than there in fact are, one would think that there could likewise have been more *logically simple states* (or logically simple properties, or logically simple roles with more than one correlate, etc.) than there in fact are. But on the other hand, logical simplicity—or something like it—is apparently ‘pure’ in the sense of Definition 2.5 above.<sup>38</sup> Hence, assuming that there are exactly  $\lambda$ -many logically simple states (for some cardinal  $\lambda$ ), the fact that this is so will also be pure: it will be denoted $_{\emptyset, \emptyset}$  by some formula of  $\mathcal{L}$  that contains no free occurrence of any constant or variable. With (CG), it then follows that this fact is nothing else than  $\top$ , which seems unwelcome in the light of the above argument.

What lesson should we draw from this dilemma? Some may take it to show that (CG) should be rejected, but I am inclined to draw a different lesson: our modal intuitions are best taken with a grain of salt, at least when it comes to basic metaphysics.<sup>39</sup> From this perspective, the force of the above argument, based as it is on the intuition that facts about the number of fundamental entities are contingent, ap-

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<sup>38</sup>The ‘or something like it’ is here supposed to indicate that we are sliding over a complication. In particular, instead of a *single* property of logical simplicity, there is only a class of properties that might be said to approximate, to varying degrees, the ideal of being *the* property of logical simplicity. The reason for this lies in our definition of ‘logically simple’ (i.e., Definition 3.2 above), which invokes the concept of denotation by a term of  $\mathcal{L}$ . If there were such a thing as *the* denotation relation for terms of  $\mathcal{L}$ , then this relation would be denotable by a term of  $\mathcal{L}$ , which would mean that the semantics of  $\mathcal{L}$  could be formulated in  $\mathcal{L}$  itself. But this cannot be, as Tarski famously showed. So there is no such thing as *the* denotation relation for terms of  $\mathcal{L}$ . Accordingly, there can also not be any such thing as *the* property of being *logically simple*.

<sup>39</sup>See, e.g., Williamson (2002, 2013) and Dasgupta (2016) for related discussion. Indeed the best way forward might be to renounce talk of metaphysical necessity altogether and instead simply speak of identity with  $\top$ , so as to avoid misleading connotations. But even so, some theorists may wish to insist that  $\top$  should not be identified with a fact as to how many logically simple states there are. Such theorists might be tempted to buy into the fiction of *possible universes*. They might adopt a slightly more complex semantics of  $\mathcal{L}$  that involves relativization not only to interpretations and variable-assignments but also to universes (as domains of quantification), and they might redefine ‘analytically entails’ accordingly. Obviously, the commitment to possible universes would be highly problematic. A less adventurous option would be to ‘simulate’ those universes with the help of set theory; but I am not convinced that the philosophical payoff would be worth the considerable complications that this would involve.

pears certainly much diminished. Still, I do not think that the argument is worthless. When we are asked why, for instance, the notion of *electronhood* should not be treated as broadly logical, thoughts about the apparent contingency of electron-related matters are among the first that spring to mind, and it is difficult to imagine a better basis on which to argue that electronhood should not be regarded as broadly logical. Even if our modal intuitions cannot always have the final say, it may be wise not to flout them unnecessarily.

So much for syncategorematicism. In the next three sections, I will briefly discuss reductive accounts of fundamentality in terms of distinctively metaphysical notions, viz., grounding, building, and structuralness. Of these, the concept of *ground* (or grounding) has been by far the most influential, and this is where we'll start.

## 5 Should we resort to ground?

Grounding is said in many ways. One of the principal distinctions to be drawn in this area concerns the question of what kind of thing is supposed to be grounded: a fact, a true proposition, or just any old entity. We can accordingly speak of 'fact-grounding', 'proposition-grounding', and 'entity-grounding'. Using the first of these notions, we might give an account of fundamentality along the following lines:

(F1) An entity  $x$  is *fundamental* iff  $x$  is a constituent of some ungrounded fact.<sup>40</sup>

But what is a 'constituent'? In the context of ORS, this concept might (to a first approximation) be defined by saying that an entity is a constituent of another iff some term denoting the latter has a free occurrence in a term denoting the first. More precisely:

(C) An entity  $x$  is a *constituent* of an entity  $y$  iff there exist an interpretation  $I$ , a variable-assignment  $g$ , and terms  $t$  and  $t'$  that satisfy the following two conditions:

- (i)  $t$  occurs free in  $t'$  and is distinct from  $t'$ .
- (ii)  $t$  and  $t'$  respectively denote <sub>$I, g$</sub>   $x$  and  $y$ .

Given (CG) (Section 2.4 above), the concept thus defined is extremely liberal: since any state  $s$  is identical with its own double negation  $\neg\neg s$ , it follows from (C) that, for any state  $s$ , the negation of  $s$  is a constituent of  $s$ . A proponent of (F1) may therefore wish to adopt a more fine-grained conception of states, so as to avoid the result that no state is fundamental.

Alternatively, one could use a concept of *entity-grounding* and follow Schaffer (2009: 373) in simply saying that:

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<sup>40</sup>Proponents of (F1) include Rosen (2010: 112), Litland (2017: 284), Shumener (2020: 2075). The right-to-left direction (a.k.a. 'Purity'; cf. Sider [2011: §7.2]) has moreover been endorsed by, e.g., Fine (2001: 20), deRosset (2013, 2023), Lo (2022). Critical voices include Barker (2022), Correia (2023). (In personal communication, Lo has indicated that he now also takes a more critical stance towards (F1).)

(F2) An entity  $x$  is *fundamental* iff nothing grounds  $x$ .

In Schaffer's view, grounding is "an unanalyzable but needed notion—it is *the primitive structuring conception of metaphysics*" (p. 364; original emphasis).

A friend of a ground-theoretic approach might propose to adopt either (F1) or (F2) as her reductive account of fundamentality. In addition, she may either propose to add to  $\mathcal{L}$  a grounding operator ' $\prec$ ' (equipped with a suitable semantics) or directly postulate the existence of a dyadic *grounding relation*. And she may argue that the resulting theory is attractive because (i) we need a concept of grounding in any case and (ii) it is unclear how one might give an account of grounding on the basis of a primitive concept of fundamentality.<sup>41</sup> What is not to like?

A first point to note is that (F1) looks rather doubtful on closer inspection. To adapt a famous example from J. L. Austin (1956: 11n.), suppose that the archangel Gabriel owns a donkey named 'Dave', and that one day he conceives a dislike for Dave. Suppose further that the relation of dislike is a *fundamental relation*, in the sense that its two constituent roles (*Disliker* and *Dislikee*) are fundamental entities, and that not only is there nothing that has caused Gabriel to dislike Dave, but there is not even anything *in virtue of which* he dislikes him, either—he just does.<sup>42</sup> Now, the fact that Gabriel dislikes Dave has at least the following two constituents: Gabriel and Dave. Since there is nothing in virtue of which this fact obtains, it is an ungrounded fact. So, according to (F1), both Gabriel and Dave are fundamental entities. But surely they aren't.<sup>43</sup>

For a less fanciful example, suppose that there exists a fundamental relation of temporal or causal precedence that holds among events.<sup>44</sup> The supposition is respectable, but we should not therefore want to say that events are fundamental entities. It thus seems reasonable to conclude that (F1) should be rejected. The upshot is that we had better rely on (F2) instead of (F1), and correspondingly employ a concept of *entity-grounding*, if we are to analyze fundamentality in terms of ground.

The second point to note is that the ground-theoretically modified version of ORS, formulated using entity- rather than fact- or proposition-grounding, leaves open many questions as to what grounds what. For consider: as before, our fine-grainedness principle (FG) gives us valuable guidance as to what entities are non-fundamental and hence, when combined with (F2), as to what entities are grounded. For example, we can tell in this way that the states  $\top$  and  $\perp$  are both grounded. (Recall from Section 2.2 that ' $\perp$ ' abbreviates ' $\neg \&()$ '.) But then they must presumably be grounded in *something*; and what could that be?

<sup>41</sup>Thus Schaffer (2016) in a critical discussion of Wilson (2014). For Wilson's reply, see her (2016).

<sup>42</sup>Some might object that the relation of dislike cannot plausibly be supposed to be fundamental. But presumably it is not absurd to think that there might be *some* fundamental relation in which archangels can stand to their donkeys. If so, the example can be easily adjusted.

<sup>43</sup>If the friend of (F1) insists that Gabriel and Dave should both be recognized as fundamental, then the argument can be repaired by replacing Dave with some entity, such as the state  $\top$ , whose non-fundamentality can be derived from (FG).

<sup>44</sup>The theoretical framework I have in mind here is that of *causal set theory*. See Wüthrich (2024) for a recent and philosophically oriented introduction.

Here the perhaps most plausible thing to say is that  $\top$  and  $\perp$  are *zero-grounded*, to use a term coined by Fine (2012: 47f.) in connection with proposition-grounding.<sup>45</sup> This move requires that we employ, instead of a grounding relation between individual entities, one that takes as its first relatum *sets* (or pluralities) of entities. We can then, for example, express the idea that  $\top$  and  $\perp$  are zero-grounded by saying that these states are grounded in the empty set (or the empty plurality).<sup>46</sup> What is still missing, however, is a principled reason as to *why*  $\top$  and  $\perp$  should be grounded in the empty set. By what principled reason can it be ruled out, e.g., that  $\perp$  is grounded in  $\top$  or *vice versa*? Here (FG) gives us no guidance at all; and as long as the concept of entity-grounding is taken as primitive, it is difficult to see where such reasons might come from.<sup>47</sup>

In a nutshell, then, our situation is this: with (FG) and (F2), we have a combination of principles that tells us that various entities fail to be ungrounded, without also telling us what those entities are grounded *in*, or whether they are zero-grounded. To avoid this quandary, a grounding theorist may be moved to abandon the orthodox idea that grounding should be taken as primitive and start to entertain a reductive account. With regard to fact- and proposition-grounding, a number of such accounts have already been put forward.<sup>48</sup> In the present context, we might turn to the concept of constituency, as tentatively defined in (C) above, and say that an entity  $x$  *grounds* an entity  $y$  just in case  $x$  is a constituent of  $y$ . To avoid the consequence that states are grounded by their own negations, we might further adopt a more fine-grained conception of states. (Never mind, for now, whether such a move would be *ad hoc*.) If we then combine the resulting account of entity-grounding with (F2), we get as an immediate consequence that an entity is fundamental iff it has no constituents. But now there are two further points to consider.

First, (FG) still tells us that  $\top$  is non-fundamental. So, by (F2),  $\top$  has at least one constituent; but on the face of it, it doesn't. That is one problem.

Second, by combining (F2) with the proposal to equate entity-grounding with constituency (as defined in (C)), we have adopted a view that faces essentially the same objection as the proposal discussed in Section 3.3 above. Namely, the view is unattractively complex when compared to a theory that takes the notion of fundamentality as primitive. Given that (FG) already guarantees the sparseness of the fundamental entities, we do not *also* need a reductive account that achieves the same result by effectively equating fundamentality with the lack of constituents. The pro-

<sup>45</sup>For a recent book-length study, see Kappes (2023).

<sup>46</sup>I am here sliding over the fact that instantiation by pluralities is not unproblematic. Cf. McGee and Rayo (2000).

<sup>47</sup>For a more complex example, consider the non-fundamental relation  $\lambda x, y ((x = x) \wedge (y = y))$ , whose instantiation by any entities  $x$  and  $y$  is the conjunction of the states  $(x = x)$  and  $(y = y)$ . Under ORS, this relation is a singleton set  $\{\langle r, r \rangle\}$  for some role  $r$  that can be denoted by  $\rho_{x,y}((x = x) \wedge (y = y))'$ . Like the relation itself, this role is not fundamental. But what is it grounded in? There seems to be no obvious path to an answer.

<sup>48</sup>See, e.g., Wilsch (2015, 2016), Poggiollesi (2016, 2021), Litland (2017), Correia and Skiles (2019), Correia (2021), Haderlie and Litland (forthcoming).



ponent of a ground-theoretic reframing of ORS thus finds herself confronted with a dilemma: either take the relevant notion of ground as primitive and be saddled with a host of open questions as to what grounds what, or embrace a reductive account and justify the choice of a needlessly complex theory.

To be sure, a staunch defender of primitive ground will not be fazed by the first horn of this dilemma. (“So what if there are a few more open questions?”) It is worth considering, however, how *unnecessary* many of those open questions appear to be.<sup>49</sup> If we operate with a primitive notion of fundamentality, instead of trying to analyze fundamentality in terms of ground, we won’t have to wonder what non-fundamental entities such as  $\top$  and  $\perp$  might be grounded in, and nor will we need to make sense of ‘zero-grounding’. This seems to be a considerable advantage.

## 6 Should we appeal to building relations?

According to an account of “absolute fundamentality” proposed by Karen Bennett, an entity  $x$  is fundamental iff nothing stands to  $x$  in any *building relation* (2017: 106). Here a building relation is one that satisfies the following three conditions:<sup>50</sup>

- (i) It is antisymmetric and irreflexive.
- (ii) It is ‘necessitating’, roughly in the sense that “builders necessitate what they build”.
- (iii) It is ‘generative’, in the sense that “[b]uilt entities exist or obtain because that which builds them does”.

This account of fundamentality has some initial plausibility, but it faces questions analogous to those encountered in the previous section. In particular, the states  $\top$  and  $\perp$ , among many other entities (both states and roles), are not fundamental; but what entities are they built from, and by what building relations?

It might be suggested that we embrace a notion of *zero-building* and say, e.g., that  $\top$  is built from the empty set by a certain relation that relates any given set of states to their conjunction. This would require a deviation from Bennett’s original vision, which instead relies on the concept of a building relation as a dyadic relation between the individual ‘builders’ and the things they build. My main worry, however, is that, even if a building-based account of fundamentality can be fully worked out, *part* of this account will be an account of *what it takes for something to be a building relation*, so that the overall account of fundamentality will be correspondingly more complicated. This extra complexity will in turn be inherited by any theory that relies on such an account of fundamentality. Accordingly, the building-based

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<sup>49</sup>Wilson (2018) speaks in this connection quite fittingly of ‘spandrel questions’. While these questions may not be objectionable *per se*, they do point to an unjustified surplus of ‘material complexity’ (as I have called it in footnote 30) of the theory from which they arise.

<sup>50</sup>See *op. cit.*, p. 32.

account faces essentially the same difficulty that already afflicts two of the proposals discussed above, on which fundamentality amounts to either logical simplicity or the lack of constituents.

## 7 Should we invoke structuralness?

In *Writing the Book of the World*, Ted Sider proposes a highly distinctive approach to fundamentality that employs a syncategorematic sentence-forming operator ‘ $\mathcal{S}$ ’ (for ‘structural’), capable of being combined with expressions “of any grammatical category” (p. 92). He is led to this approach in part by the thought that it is not enough to ask merely which *predicates* “carve at the joints”. Instead, we must also ask what *quantifiers* carve at the joints; and indeed we can even raise an analogous question about *linguistic categories*, such as that of a sentence.<sup>51</sup>

From this perspective, the approach taken by ORS must seem entirely inadequate. Yes, we can say that such-and-such roles and states are fundamental. But should we not also be able to say that the operators of  $\mathcal{L}$ , such as ‘&’ and ‘ $\neg$ ’, or the concepts expressed by them, are privileged in much the same way? Yet since those operators or concepts are fairly clearly not *ontologically* fundamental (they aren’t *fundamental entities* in the way a quark or electron might be), the concept of fundamentality operative in ORS is apparently of no help in this regard. A proponent of Sider’s approach might therefore suggest an alternative concept: that of *structuralness*, expressed by an operator that can attach to ‘&’ and ‘ $\neg$ ’ just as easily as to the name of a state or role.

In reply, I would argue that, even without Sider’s operator, we can articulate a metaphysically relevant sense in which the operators of  $\mathcal{L}$  may be said to be privileged. In a first step, we can (if ORS is correct) say that  $\mathcal{L}$  satisfies the following condition:

- (P) The assumptions (CG), (FG), and (A)—each of which can be understood as a claim about  $\mathcal{L}$ —are all true.

In other words,  $\mathcal{L}$ ’s expressive resources are sufficient for the purposes of our account of the individuation of states, and also sufficient to allow every entity to be represented in terms of *fundamentalia*. This is not something that can be said of *every* language! But of course it is not as if, whenever a given language satisfies (P), its operators could be said to be ‘privileged’. After all, starting from a language that satisfies (P), one can easily construct another language with various further operators that do no interesting work, and the language will still satisfy (P). However, if a given language not only satisfies (P) but is also relatively *parsimonious* in its number of operators, then (I submit) we *may* reasonably claim that its operators are metaphysically privileged. And that is the case with  $\mathcal{L}$ .<sup>52</sup>

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<sup>51</sup>See *op. cit.*, §11.8. In his more recent book (2020), Sider employs a primitive concept of fundamentality that applies to *concepts*.

<sup>52</sup>I say ‘relatively parsimonious’ instead of ‘maximally parsimonious’ because it may be argued that

When I say that the operators of  $\mathcal{L}$  are ‘metaphysically privileged’ in being the operators of a relatively parsimonious language that satisfies (P), this should not be taken to mean that no other operators are similarly privileged. For example, we could equally well have a language in which ‘&’ is replaced by an infinitary disjunction operator, or in which ‘ $\exists$ ’ is replaced by a universal quantifier. From the perspective of ORS, the choice between these languages is simply a matter of taste. By contrast, Sider’s approach entails that there is an objective fact of the matter as to whether  $\mathcal{S}(\wedge)$  or  $\mathcal{S}(\vee)$ , and also as to whether  $\mathcal{S}(\exists)$  or  $\mathcal{S}(\forall)$ . At least *prima facie*, this renders Sider’s approach implausible.<sup>53</sup> The conception of metaphysical privilege sketched in the previous paragraph does not have this problem.

This completes my reply to Sider’s approach. Let us now very briefly consider how ORS fares with regard to a challenge that has recently been raised by Dasgupta (2018), mainly against Sider. What Dasgupta essentially points out is that, if the notion of joint-carvingness is taken as primitive, then it remains a mystery why we should strive to theorize in joint-carving terms. This takes direct aim at Sider’s assertion that “[a] good theory isn’t merely likely to be *true*. Its ideology is also likely to carve at the joints” (2011: 12, original emphasis). The focus of Dasgupta’s attack lies on the word ‘good’. He in effect asks: A joint-carving ideology may be good *for us*, but is it also *objectively* good? A reader might wonder whether Dasgupta’s challenge does not also arise for ORS, given the latter’s reliance on a primitive concept of fundamentality.

My reply is almost embarrassingly simple: ORS in no way implies that theories are better when they are formulated in terms of fundamental entities. This is sufficient to show that Dasgupta’s challenge does not arise for the present view.

## 8 Conclusion

We have examined five salient approaches to the concept of fundamentality operative in ORS. In each case we have found reason to be dissatisfied. So, at least for the time being, it seems that we can defensibly retain (FF)—the claim that fundamentality itself is fundamental—as a reasonable working hypothesis.

A critic might point out that the same can be said of the following rival thesis:

(NF) The property of *not being a fundamental entity* is fundamental.

And they would be right. However, the disagreement between (FF) and (NF) strikes me as relatively uninteresting. In the first place, there seems to be nothing that

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a language that is much like  $\mathcal{L}$ , but which instead of the latter’s conjunction and negation operators contains an infinitary version of the Sheffer stroke, will be even more parsimonious. One might try to resist this argument on the grounds of the more complicated inferential behavior of the infinitary Sheffer stroke, but for present purposes we can leave this issue aside.

<sup>53</sup>As Sider himself notes, “[y]ou don’t need to be a logical positivist to feel that metaphysics has gone off the rails if it leads to questions like whether universal or existential quantification is more fundamental” (2020: 189f.).

could possibly decide the issue, and in the second place, nothing important seems to hinge on it. If an oracle were to reveal that (NF) rather than (FF) is correct, then no part of ORS would need to be rewritten, except of course for the claim that fundamentality is fundamental. It would still be the case, for instance, that the negation of a fundamental property is non-fundamental.

The claim that non-fundamentality is fundamental may sound counter-intuitive, in a way that also threatens (FF). For as we go over certain paradigmatically non-fundamental entities, such as the fact that Sappho's birth precedes Socrates', or the property of *being a female fox*, it may strike us that each of them is non-fundamental *in virtue* of being in some way 'constructed' from some other things. And if this is what makes something non-fundamental, should then not fundamentality be analyzable as the property of *not* being constructed in any such way?

The reasoning is seductive, but can be resisted at (at least) two points. First, as I have tried to stress by using such examples as  $\top$  and  $\perp$ , it is often far from clear what other things a given non-fundamental entity should be taken to be constructed from. And second, as I have also tried to make clear, we do not *need* a reductive analysis of fundamentality, given that the non-fundamentality of those paradigmatically non-fundamental entities already falls out of our account of how states are individuated. To paraphrase Lewis (1983: 352), an *account* of fundamentality need not be an *analysis*!

I would like to close with two brief remarks. First, a critic might raise the following worry about the role fundamentality plays in ORS: 'What if there is no fundamental level?' My reply is that this is no objection. ORS presupposes that it makes sense to distinguish between the fundamental and the non-fundamental, and takes fundamentality itself to be one of the fundamental things. To ask, 'What if there is no fundamental level?' is thus no more of an objection than to ask, 'What if your theory is wrong?'. A theorist who insists that it should at least be *possible* for there to be no fundamental level may be led to this view by a conception that takes fundamentality to be analyzable in terms of some priority relation such as grounding. In imagining a scenario in which this relation is non-well-founded, such a theorist may take herself to be imagining a scenario in which there is no fundamental level. But as I have argued, we should resist the urge to understand fundamentality in terms of either grounding or building.

Second, the above defense of the thesis that fundamentality is fundamental has not been 'ecumenical'. That was unavoidable, for we could not otherwise have made use of the principles fueling this paper's argument.<sup>54</sup> Generally speaking, results obtained by relying on a specific theory may appear 'local' and 'parochial', in the sense of having relevance only within a given theory or framework. But once one takes seriously the idea that the theory in question is *true*, there will seem to be nothing

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<sup>54</sup>This is not to say that a very similar argument could not have been constructed on the basis of ordinal type theory rather than ORS. I have here chosen the latter only because it allows for a slightly more elegant fine-grainedness principle, and because it is arguably (and not unrelatedly) a better theory.

‘local’ about those results, provided that the theory itself is general enough.<sup>55</sup>

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