Purity and Perspicuity

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Abstract

Can we define a concept of *impurity* that—under plausible assumptions about the coarse-grainedness of properties—treats *being identical with Napoleon* and *being identical with redness* alike as impure? In this paper I suggest that we can, provided that we employ a notion of *perspicuity* (as applied to the terms of a certain formal language) that in turn rests on a concept of fundamentality. I take as my starting point an approach adopted by Rodriguez-Pereyra, under which a property is impure iff having it "consists in being related in a certain way to a certain object or objects in particular" (*Two Arguments for the Identity of Indiscernibles*, Oxford University Press, 2022, §1.8). If every property P is identical with that of *instantiating* P, then this approach has the trivializing consequence that every property is impure. Working in a framework recently proposed by Plate ('Ordinal Type Theory', *Inquiry* 68, 2025), I show that every property P is indeed identical with *instantiating* P. Hence, at least within this setting, there arises the need for an alternative approach. A straightforward way to eliminate the trivialization problem is to say that a property is impure iff it involves at least one *particular*; but this leads to an unequal treatment of *being identical with Napoleon* and *being identical with redness* (assuming that Napoleon is a particular). A perspicuity-based approach offers a way forward.

Keywords: metaphysical perspicuity; qualitativeness; properties; fundamentality

> If names are not rectified, speech will not accord with reality; when speech does not accord with reality, things will not be successfully accomplished.
>
> – Confucius

1 Introduction

In a recent book, Gonzalo Rodriguez-Pereyra (2022) draws a distinction between the "purely qualitative" and what he calls the "impurely qualitative" (or, more briefly, the "pure" and the "impure") properties as follows:

> [I]mpure properties are those such that having them consists in being related in a certain way to a certain object or objects in particular, *whether such object or objects are abstract or concrete*; pure properties, on the contrary, are those that are not impure: having them does not consist in being related to any object or objects in particular [...]. (§1.8; emphasis added)

As examples of impure properties, he lists *being identical with Napoleon*, *being a teacher of Aristotle*, and *orbiting the Earth*. So far, so familiar. But now consider: what about *instantiating redness*? On the face of it, having this property "consists in being related in a certain way to a certain object or objects in particular", namely in this case redness, a.k.a. the property of *being red*.[1] Apparently, then, *instantiating redness* is an impure property in Rodriguez-Pereyra's sense.

There is, however, a potential problem: Might *instantiating redness* be the same property as redness itself? A friend of Rodriguez-Pereyra's way of drawing the pure/ impure distinction may well be inclined to deny this, on the grounds that redness itself is *not* impure. But there is also another route that one might take, which deserves closer attention. For consider the following thesis:

> **Identity.** For any property P, the property of *instantiating* P is nothing else than P itself.

Any argument for this thesis is effectively an argument for the claim that, under Rodriguez-Pereyra's definition, *every* property is impure—which would suggest that that definition stands in need of amendment. So the question now becomes: Can it be argued that Identity holds?

As we will see in Section 3, the answer to this question is incontrovertibly 'yes', because I will there present just such an argument. What is more, that argument strikes me as sufficiently compelling that it will make sense to look for alternatives to Rodriguez-Pereyra's definition of 'impure'.[2] Two such alternatives (or three, depending on how one counts) will be examined in Section 4. The first of these amends Rodriguez-Pereyra's definition by simply restricting the class of relations quantified over in that definition, in such a way that *instantiation* relations are excluded. Unfortunately, this falls short of solving the problem unless one adopts some fairly broad

[1] In his footnote 7, Rodriguez-Pereyra stresses that the objects in question "could very well be properties".

[2] Rodriguez-Pereyra himself seems inclined to deny Identity, as becomes quite clear in the following passage:

> Indeed, even the property of *satisfying the predicable condition of being red* is relational and impure, and it is different from the property of *being red*. For when one predicates of something that it is red one predicates a non-relational condition of that object, not a condition that consists in being related to any universal, trope, class, or even the predicable condition one is predicating of it! (§1.8, original italics)

But as an *argument* for the denial of Identity, this certainly appears question-begging: a friend of Identity cannot be expected to grant that, "when one predicates of something that it is red", one does not ascribe to it a "condition that consists in being related to [...] the predicable condition" of *being red*.

conception of what is to count as an instantiation relation, and it is not immediately obvious how this is to be made precise. The *second* alternative restricts instead the class of *relata* quantified over, excluding anything that is a property, relation, or state of affairs. On this approach, *instantiating redness* is no longer classified as impure; but nor is that of *being identical with redness*. At least in some contexts, this is arguably an unwelcome result: there is something to be said for treating *being identical with redness* in the same way as *being identical with Napoleon*. To address this problem, Section 5 introduces an approach that relies on a notion of *perspicuity*, as applied to the terms of a certain formal language. The specific concept of perspicuity introduced in this section is quite straightforward and rather liberal, which renders it vulnerable to an objection raised in Section 6. Section 7 then introduces a more demanding concept, and Section 8 concludes.

The first order of business, however, is to settle on a suitable framework for theorizing about properties.

2 Two Frameworks

In the previous section I have spoken of 'instantiation relations' in the plural. The justification for this comes, in the first place, from the fact that there is instantiation not only of properties but also of relations. Thus—at least to a first approximation—we have a *dyadic* relation I_2 that holds between an entity x and a property P iff x instantiates P; a *triadic* relation I_3 that holds between some entities x and y and a dyadic relation R (in this order) iff x bears R to y; and so on. However, from a relation I_2 as just described it is but a short step to the admission of a property $\lambda x\, \neg I_2(x, x)$ of *non-self-instantiation*, and hence to Russell's paradox.

To avoid this problem, philosophers have often made use of the *Simple Theory of Types* (STT), which effectively excludes any given property or relation from its own 'range of application'. In one popular version, any bound variable would be required to carry a superscript such as 'e', '$\langle\rangle$', '$\langle e\rangle$', '$\langle e, \langle\rangle\rangle$', etc., which would indicate the respective variable's range. Instead of the above I_2 and I_3, we would have such relations as, for instance, $\lambda x^e y^{\langle e\rangle}.y^{\langle e\rangle}(x^e)$ and $\lambda x^e y^e z^{\langle e,e\rangle}.z^{\langle e,e\rangle}(x^e, y^e)$, which would themselves be assigned the types $\langle e, \langle e\rangle\rangle$ and $\langle e, e, \langle e, e\rangle\rangle$, respectively.[3] And since, instead of e, any other type would do just as well, we would in this way not only have exactly one dyadic, exactly one triadic (and so on) instantiation relation, but rather *infinitely many* dyadic (triadic, tetradic, ...) instantiation relations.

STT is pleasingly straightforward, but it is not without its defects. For example, suppose you want to do set theory with urelements, where the urelements are situated somewhere in STT's hierarchy. One of the first questions you would then be confronted with is: What type does the set-membership relation belong to? You might be happy to think of sets as 'individuals' (entities of type e), but if you wanted

[3] I here follow the notation used by Dorr (2016: §4). In English, $\lambda x^e y^{\langle e\rangle}.y^{\langle e\rangle}(x^e)$ can be described as a relation that holds between an 'individual' x and a 'property of individuals' y iff x instantiates y. Analogously for $\lambda x^e y^e z^{\langle e,e\rangle}.z^{\langle e,e\rangle}(x^e, y^e)$.

to allow for sets whose membership is drawn from *more than one* type, you'd be out of luck, because STT allows no such thing. If your set-membership relation is, e.g., of type $\langle e, e \rangle$, then the only sets you'll have are sets of individuals. Similarly, STT does not allow for a relation of *thinking-of* that can be borne by a given thinker to inhabitants of different types.

Another well-known point of criticism, perhaps even more damning, is that STT does nothing to block *Epimenidean* paradoxes (as discussed by, e.g., Prior [1961; 1971], Church [1976; 1993], Martino [2001]). For example, STT allows there to be a proposition—an entity of type $\langle\rangle$—to the effect that Prior's favorite proposition is false: we can symbolize it along the lines of '$\forall x^{\langle\rangle} (P^{\langle\langle\rangle\rangle}(x^{\langle\rangle}) \to \neg x^{\langle\rangle})$'. A contradiction arises once we suppose that this proposition *itself* is Prior's favorite proposition. In much the same way, STT is vulnerable to 'truth-teller versions' of these same paradoxes. Prior himself speaks in such cases of a "vicious self-dependence" (1971: 91), but provides no constructive way of ruling it out.

In my view, the only reasonable way to address these woes is to avoid committing oneself to an ontology that admits the offending propositions; and a reasonable way to do *that* is to join the 'ramifiers' in taking propositions—or states of affairs—to be arranged in a hierarchy. A recent proposal to this effect can be found in Plate (2025). In the context of this proposal, properties, relations, and states of affairs are collectively referred to as 'intensional entities' while everything else is a 'particular'. Moreover, and in contrast with STT, the formal language that is in this setting used for the representation of intensional entities contains both typed *and* untyped variables. The type of a typed variable is simply an ordinal number greater than 0, indicated by a superscript. Untyped variables 'range' over everything there is, whereas a variable of type α ranges over all entities of any order less than α; thus a variable of type ω will range over all entities of finite order.

At the base of the mentioned hierarchy lie the *zeroth-order* entities: particulars and fundamental intensional entities, along with all those intensional entities that can be denoted (in a certain formal language $\mathcal{L}$) by a formula or lambda-expression satisfying the following two conditions:

(i) Any constant or variable that has in it a free occurrence at predicate- or sentence-position denotes either a particular or a fundamental intensional entity.

(ii) It does not contain any bound occurrence of a typed variable at predicate- or sentence-position.

In the general case, an entity x is of order α just in case α is the least ordinal β such that x is denoted (relative to some interpretation I and variable-assignment g) by some *term*—i.e., some constant, variable, formula, or lambda-expression—that satisfies the following:[4]

[4]Cf. Plate *op. cit.*, p. 2369. For brevity's sake, I will in the following refer to this paper in the same way as to the theory it proposes, viz., as 'OTT'.

(i) Any constant or variable that is either identical with it or has in it a free occurrence at predicate- or sentence-position denotes (relative to I and g) either a particular or a fundamental intensional entity.

(ii) Any variable that has in it a bound occurrence at predicate- or sentence-position has a type that is less than or equal to β.

Since the lowest possible type that a variable can have is 1, this second condition will be satisfied for $\beta = 0$ only if the term in question contains *no* bound occurrence of a typed variable at predicate- or sentence-position.

The foregoing specifications provide the context for a hierarchically organized and plenitudinous ontology of intensional entities. To avoid the Epimenidean paradox mentioned above, the ontology has to avoid being committed to a state of affairs that could in the present setting be formalized as $\exists x\,(P(x) \wedge \neg x)$, where '$P$' stands for the property of *being Prior's favorite state of affairs*. To this end, the ontology in turn has to avoid commitment to a *property* that could be formalized as $\lambda x\,(P(x) \wedge \neg x)$; for if there *were* such a property, then there would (given one of the theory's general existence assumptions) also exist its 'existential quantification' $\exists x\,(P(x) \wedge \neg x)$. To see how the ontology avoids being committed to the existence of such a property, we have to turn to its *comprehension axiom*.

Roughly put, this axiom takes a variable or formula φ together with an interpretation, a variable-assignment, and a non-empty list of pairwise distinct variables, all of which are required to occur free in φ, and generates commitment to a corresponding property or relation: a *property* if the supplied list of variables is of length 1, and an α-adic *relation* otherwise (where α is the length of that list).[5] One of the clauses in the antecedent of the axiom's main conditional requires that, if any of those variables is either identical with φ or has in φ a free occurrence at predicate- or sentence-position, then it should be *typed*. So, for example, φ cannot just be an untyped variable: it has to be a *typed* variable, or of course a formula—and not just *any* formula, either. For suppose the supplied list of variables contains only 'x'; then '$(P(x) \wedge \neg x)$', for instance, won't do, because 'x' has here a free occurrence at sentence-position (preceded by '$\neg$'). What *would* do would, e.g., be the similar formula '$(P(x^1) \wedge \neg x^1)$', if instead of 'x' our list contained only the typed variable 'x^1'. And in this way the axiom generates commitment to a property $\lambda x^1\,(P(x^1) \wedge \neg x^1)$—which has instantiations only by zeroth-order entities—but not to a property $\lambda x\,(P(x) \wedge \neg x)$ and the attendant threat of paradox.

The previous four paragraphs constitute a (highly condensed and selective[6])

[5] Cf. OTT, p. 2366. An 'interpretation' is here a function (understood as a set of ordered pairs) that maps constants to entities, whereas a 'variable-assignment' is a function that maps variables to entities. Both interpretations and variable-assignments are *partial* functions; and the empty set can serve both as an interpretation and as a variable-assignment. Further, when we say that a given term t has such-and-such a denotation relative to a given variable-assignment g (leaving the interpretation unspecified), this should be understood as meaning that t has the specified denotation relative to g and the *empty* interpretation.

[6] 'Selective' mainly because I have left out OTT's account of how intensional entities are individu-

outline of *Ordinal Type Theory*, or 'OTT' for short. This is the framework on which I will for the most part be relying in the rest of this paper.

3 An Argument for Identity

As noted at the beginning of the previous section, STT admits multiple instantiation relations. For example, focusing only on the dyadic case, we have the relation $\lambda x^e y^{\langle e \rangle}.y^{\langle e \rangle}(x^e)$ corresponding to type e, the relations $\lambda x^{\langle\rangle} y^{\langle\langle\rangle\rangle}.y^{\langle\langle\rangle\rangle}(x^{\langle\rangle})$ and $\lambda x^{\langle e,e \rangle} y^{\langle\langle e,e \rangle\rangle}.y^{\langle\langle e,e \rangle\rangle}(x^{\langle e,e \rangle})$ corresponding (respectively) to types $\langle\rangle$ and $\langle e,e \rangle$, and so on. Something analogous holds for OTT: for each ordinal $\alpha > 0$, there exists a dyadic relation $\lambda x, y^\alpha\, y^\alpha(x)$.[7] But what exactly *are* these relations? More precisely, what does it mean for a given entity to bear one of these to some other entity?

To answer this, we have to turn to the semantics of lambda-expressions:[8]

(LE) For any ordinal $\alpha > 0$, any α-sequence of pairwise distinct variables $v_1, v_2, \ldots$, any term t, and any x: $\ulcorner \lambda v_1, v_2, \ldots\, t \urcorner$ denotes x relative to I and g iff x is an α-adic attribute such that, for any α-sequence of entities $y_1, y_2, \ldots$ and any state of affairs s, the following holds:

 ($*$) s is an instantiation of x by $y_1, y_2, \ldots$ (in this order) iff, for some variable-assignment h that is just like g except that it maps each v_i to the corresponding y_i, s is denoted by t relative to I and h.

Here an *attribute* is simply anything that is either a property or a relation. It bears noting that, in the context of OTT, attributes have instantiations *by definition*: something is said to be a property iff it has an instantiation (obtaining or not) by some single entity, and something is an α-adic relation (for $\alpha > 1$) iff, for at least one α-sequence of entities $x_1, x_2, \ldots$, it has an instantiation by $x_1, x_2, \ldots$, in this order. Instantiations, of course, are states of affairs (or henceforth simply 'states'); and it is a further thesis of OTT that no attribute is a state.[9]

Applying (LE) to, e.g., the lambda-expression '$\lambda x, y^1\, y^1(x)$', we can infer that the latter denotes an entity x relative to a given interpretation I and variable-assignment g iff x is a dyadic relation R such that, for any entities y and z and any state s, the following holds:

(†) s is an instantiation of R by y and z (in this order) iff, for some variable-assignment h that is just like g, except for mapping 'x' to y and 'y^1' to z, s is denoted by '$y^1(x)$' relative to I and h.

ated.

[7] In addition, there is an instantiation relation $\lambda x^\alpha, y^\beta\, y^\beta(x^\alpha)$ for any ordinals $\alpha, \beta > 0$; but let us set these aside for now. We will briefly return to the concept of an instantiation relation in the next section.

[8] Cf. OTT, p. 2368.

[9] Cf. OTT, p. 2364.

By the semantics of the language $\mathcal{L}$ that we are here operating with, for s to be denoted by '$y^1(x)$', relative to I and h as just specified, is simply to be *an instantiation of z by y*, with the proviso that z be zeroth-order. Thus (†) can be simplified to:

(‡) s is an instantiation of R by y and z (in this order) iff s is an instantiation of z by y, and z is zeroth-order.

We now have *almost* all we need to construct an argument for Identity, and at any rate enough to begin.

Let P be some zeroth-order property, such as (say) *being red*, and let P' be the property of *instantiating* P, taking the relevant instantiation relation to be $\lambda x, y^1\, y^1(x)$. What has to be shown is that P' is identical with P. Relative to a variable-assignment that maps 'P' to P, P' will be denoted by the following lambda-expression:

$$\lambda x\, (\lambda x, y^1\, y^1(x))(x, P).$$

Applying (LE) to the displayed expression, we can infer that a state s is an instantiation of P' by a given entity x iff s is an instantiation of $\lambda x, y^1\, y^1(x)$ by x and P, in this order. But in the previous paragraph we have seen that $\lambda x, y^1\, y^1(x)$—the relation R denoted by '$\lambda x, y^1\, y^1(x)$'—satisfies (‡). So, by (‡), we can now infer that a state s is an instantiation of P' by a given entity x iff s is an instantiation of P by x, and P is zeroth-order. Given that P *is* zeroth-order (by hypothesis), we can thus further conclude that, for any x, a state is an instantiation of P' by x iff it is an instantiation of P by x. And from *this*, by a principle about the individuation of attributes that forms an integral part of OTT (to be spelled out in a moment), we can finally infer that P' is identical with P.

The principle that has just now been alluded to is the following:

Uniqueness. Every attribute has at most one trivial converse.[10]

This needs some unpacking. An entity x is a *trivial converse* of an α-adic attribute A iff, for any α-sequence of entities $y_1, y_2, \ldots$ and any state s, the following holds:

s is an instantiation of x by $y_1, y_2, \ldots$, in this order, iff s is an instantiation of A by $y_1, y_2, \ldots$, in this order.

Since, as we have said, a state is an instantiation of P' by a given entity x iff it is an instantiation of P by x, we can now first of all conclude that P' is a trivial converse of P. But, like any attribute, P is already a trivial converse of *itself*. So, by Uniqueness, it follows that $P' = P$. We have thus reached the conclusion that the property of *instantiating* P is simply P itself. To be sure, P was any *zeroth*-order property. But this does not entail any loss of generality; for, by a completely parallel argument, one can show that any higher-order property Q is identical with the property of *instantiating* Q. (The only necessary adjustment would concern the choice of instantiation

[10]Cf. OTT, p. 2365.

relation.) Accordingly, Identity—or at least a suitable precisification of it—must be regarded as a theorem of OTT.[11] This suggests a very simple argument for Identity:

(P1) If OTT is correct, then Identity is true.

(P2) OTT is correct.

(C) Identity is true.

The argument is valid. Since we have established that (P1) holds, the only remotely viable way to resist it is to reject premise (P2) and, with it, OTT. But then the question arises what part, exactly, of OTT should be rejected.[12] I shall leave this question to the foes of Identity. Enough has been said, I think, to proceed on the assumption that Identity is indeed true. With this, let us now return to the question of how to draw the distinction between pure and impure properties.

4 Three Attempts at Defining 'Impure'

Recall that, under Rodriguez-Pereyra's definition, a property is impure iff having it "consists in being related in a certain way to a certain object or objects in particular". Somewhat more formally, this might be put as follows:

(I1) A property P is *impure* iff, for some ordinal $\alpha > 0$, some α-sequence of entities $y_1, y_2, \ldots$, and some $(\alpha+1)$-adic relation R, the following holds: P is identical with $\lambda x\, R(x, y_1, y_2, \ldots)$.

On the assumption that Identity is true, however, this has the consequence that *every* property is impure,[13] which means that (I1) is not a very useful definition. This makes it appear desirable to look for alternatives.

An initially promising way to amend (I1) is to add a clause that explicitly bans instantiation relations:

(I2) A property P is *impure* iff, for some ordinal $\alpha > 0$, some α-sequence of entities $y_1, y_2, \ldots$, and some $(\alpha+1)$-adic relation R, the following holds: P is identical with $\lambda x\, R(x, y_1, y_2, \ldots)$, and R is not an instantiation relation.

[11] The need for precisification arises from the fact that Identity speaks of "the property of *instantiating P*" without specifying the relevant instantiation relation. On the background of OTT, a suitable precisification may be given as follows:

> **Identity Precisified.** For any ordinal α and any property P of order α, the property $\lambda x\, R(x, P)$, where R is the relation $\lambda x, y^{\alpha+1}\, y^{\alpha+1}(x)$, is nothing else than P itself.

In the following, I shall be taking Identity to be understood in precisely this sense.

[12] For what it's worth, I myself am inclined to reject OTT in favor of a closely related view that incorporates a more sophisticated conception of relations. (Cf. Plate [MS(a),(b)].) But that is irrelevant here, because Identity is a theorem also of this latter view.

[13] Cf. footnote 11 above.

Here it should be noted that, if we are interpreting lambda-expressions in accordance with (LE) and individuating attributes by Uniqueness, then (I2) can be simplified. For if P is identical with $\lambda x\, R(x, y_1, y_2, \ldots)$, for some entities $y_1, y_2, \ldots$, then P will also be identical with $\lambda x\, S(x, y_1)$, where S is the relation $\lambda x, y\, R(x, y, y_2, y_3, \ldots)$.[14] So (I2) can be simplified to:

(I3) A property P is *impure* iff, for some entity y and some dyadic relation R, the following holds: P is identical with $\lambda x\, R(x, y)$, and R is not an instantiation relation.

In a similar fashion, we could also have simplified (I1).

The obvious problem of (I3)—and also of (I2), but for simplicity's sake I will focus on the former—is that the concept of an instantiation relation, at least as the term is understood here, is rather narrow. At the beginning of the previous section, I mentioned dyadic relations $\lambda x, y^\alpha\, y^\alpha(x)$, for $\alpha > 0$. Allowing the first bound variable (here 'x') to be typed as well, we can say that a dyadic relation R is an *instantiation relation* iff R is denoted (relative to any interpretation and variable-assignment) by a lambda-expression $\ulcorner \lambda u, v\, v(u) \urcorner$, for some (typed or untyped) variable u and some typed variable v distinct from u.

Unfortunately, in combination with this natural understanding of what it means for something to be a dyadic instantiation relation, (I3) yields, just like (I1), the unwelcome result that every property is impure. For let P be any zeroth-order property, and let R be, for example, the relation $\lambda x, y^1\, (P(x) \vee y^1(x))$, whose instantiation by an entity x and a zeroth-order property Q, in this order, is the disjunction of (i) P's instantiation by x and (ii) Q's instantiation by x. The instantiation of this relation by x and P, in this order, is accordingly the state $(P(x) \vee P(x))$, which is very plausibly the same state as $P(x)$, i.e., P's instantiation by x.[15] We can thus see that the properties $\lambda x\, R(x, P)$ and P are trivial converses of each other, which, by Uniqueness, means that P is identical with $\lambda x\, R(x, P)$; yet R is *not* an instantiation relation in the above sense.

One might try to address this problem by adopting a broader conception of what is to count as an instantiation relation. The trouble is that it is not easy to

[14] To see this, note that, given (LE), and given that $S = \lambda x, y\, R(x, y, y_2, y_3, \ldots)$, an instantiation of $\lambda x\, S(x, y_1)$ by an entity x is an instantiation of R by $x, y_1, y_2, \ldots$ (in this order), as well as *vice versa*. Hence, $\lambda x\, S(x, y_1)$ and $\lambda x\, R(x, y_1, y_2, \ldots)$ are trivial converses of each other.

[15] It would be a mistake, I think, to regard $(P(x) \vee P(x))$ as distinct from $P(x)$ on the ground that the former is 'disjunctive' or 'grounded in' the latter. One might say that Fred's *belief* that $P(x) \vee P(x)$ is a disjunctive belief, or that his knowledge that $P(x) \vee P(x)$ is grounded in his knowledge that $P(x)$ (whereas his knowledge that $P(x)$ is *not* grounded in his knowledge that $P(x) \vee P(x)$). But none of this is constitutes a compelling reason to think that the *state of affairs* $(P(x) \vee P(x))$ is distinct from $P(x)$. An example from Williamson may help to illustrate this:

> Imagine a seismologist discovering that Taiwan has so many earthquakes not because two tectonic plates converge nearby but instead because either two tectonic plates converge nearby or two tectonic plates converge nearby. (2024: 467)

A fully worked-out account of the individuation of states can be found in OTT, §3.3.2 and §5.4.

see how exactly this might go. It thus seems reasonable to try a different tack, and modify (I1) by restricting not the relevant class of relations but rather their *relata*:

(I4) A property P is *impure* iff, for some entity y and some dyadic relation R, the following holds: P is identical with $\lambda x\, R(x, y)$, and y is a *particular* (i.e., neither a state nor an attribute).

The reader might wonder why this restriction has to be so broad. For example, why exclude states? The answer is that anything that instantiates a property P will thereby also stand in a relation to some state. For instance, by being red, an object will stand in the relation $\lambda x, y^1\, ((x = x) \wedge y^1)$ to the fact that there are red things (assuming that this fact is zeroth-order). On a moderately coarse-grained way of individuating properties, any zeroth-order property P will thus be identical with $\lambda x\, R(x, \exists x\, P(x))$, where R is the relation $\lambda x, y^1\, (P(x) \wedge y^1)$; and analogously for any higher-order properties. So, if states were in (I4) not excluded from the range of admissible relata, then we would again be faced with the unwelcome result that every property whatsoever is impure. By a similar argument, it can be shown that relations have to be excluded, as well.

A critic might suggest that (I4) does not in fact overcome the problem that plagues the previous two approaches. For might we not say that, just by being red, an object will stand in the relation of *co-existence* to every single particular there is? On a suitably coarse-grained way of individuating properties, it should thus turn out that any property P is identical with $\lambda x\, R(x, a)$, where a is some particular and R is the relation $\lambda x, y\, (P(x) \wedge (y = y))$. So, even under (I4), every property whatsoever would turn out impure.

My reply is that the critic is here taking the coarse-grainedness of properties one step too far. It is certainly true that any red thing co-exists with every particular there is. But it would be quite implausible to regard a given object's being red as somehow requiring the existence of arbitrary particulars. Likewise, I consider it implausible to think that redness itself is identical with the property of *being red and such that Napoleon is self-identical*. If this is right, then (I4) does not in fact have the consequence that every property is impure.

Nonetheless, there is at least one respect in which (I4) is unsatisfactory. Let us first say that a property is *purely qualitative* iff it fails to be impure in the sense of (I4). The following thesis is highly plausible (as well as a theorem of OTT):

Inherited Purity. If a property P is purely qualitative, then the property of *being identical with P* is also purely qualitative.

Intuitively, a property is purely qualitative in the above sense iff it does not 'involve' any particulars.[16] Given that the property of *being identical with P* is analyzable in terms of P and the identity relation alone, and given that the identity relation itself does not seem to involve any particulars, it would appear that *being identical with P*

[16]For a definition of the relevant concept of involvement, see Plate (2022: 1304f.).

will involve a particular only if P does. Hence, if P is purely qualitative, then *being identical with P* should be purely qualitative, as well.[17]

With the concept of pure qualitativeness in place, it is natural to define a concept of *qualitative duplicate* as follows:

[17] In this footnote I provide a more formal argument for Inherited Purity, using two assumptions of OTT that have so far in this paper been left unmentioned. These are called the 'analyzability assumption' and the 'independence assumption' (OTT, §5.3 and §5.4). The former is to the effect that, for every entity x, there exists a term of $\mathcal{L}$—i.e., some constant, variable, formula, or lambda-expression—that denotes x relative to some variable-assignment whose range contains only particulars and *fundamental* intensional entities. By contrast, the independence assumption is more complex. One of its consequences (which I will here state without proof) is the thesis that, *if* two terms t and t' denote one and the same state relative to a variable-assignment g that satisfies the following four conditions—

(i) Any variable in the domain of g is of type 1.

(ii) Any intensional entity in the range of g is fundamental and distinct from the identity relation.

(iii) No two variables are under g mapped to the same entity or to relations that are converses of each other.

(iv) For any variable v: if v occurs free in t and is under g mapped to an intensional entity, then at least one free occurrence of v in t stands at predicate- or sentence-position.

—*then* every variable that occurs free in t' also occurs free in t. Call this 'the Lemma'. To see how Inherited Purity follows from all this, let P be any purely qualitative property, let P' be the property of *being identical with P*, and suppose for *reductio* that P' is impure in the sense of (I4). There then exist a particular y and a relation R such that P' is identical with $\lambda x\, R(x, y)$. By the analyzability assumption, there further exist terms π and ρ that respectively denote P and R relative to a variable-assignment g whose range contains only particulars and fundamental intensional entities. Let now t be the formula $\ulcorner(\lambda x\,(x = \pi) = \lambda x\,(x = \pi))\urcorner$. Without loss of generality, we may suppose that a certain variable v of type 1 is under g mapped to the aforementioned particular y and that, moreover, g and t jointly satisfy the above conditions (i)–(iv). In addition, let t' be the formula $\ulcorner(\lambda x\, \rho(x, v) = \lambda x\, \rho(x, v))\urcorner$. Relative to g, the two formulas t and t' then denote one and the same state, namely the self-identity of P'. So, by the Lemma, t contains a free occurrence of v. Since $t = \ulcorner(\lambda x\,(x = \pi) = \lambda x\,(x = \pi))\urcorner$, it follows that v occurs free in π. Now recall that, relative to g, π denotes P. Let χ be some variable that does *not* occur free in π, and let a be some entity by which P has an instantiation. (Such an entity exists because P is a property; cf. the previous section.) The formula $\ulcorner\pi(\chi)\urcorner$ then denotes a state—namely, P's instantiation by a—relative to a variable-assignment g' that is just like g except for mapping χ to a.

By our comprehension axiom (briefly discussed at the end of Section 2 above), there now exists a dyadic relation S such that, for any entities z and w and any state s: s is an instantiation of S by z and w, in this order, iff s is denoted by $\ulcorner\pi(\chi)\urcorner$ relative to a variable-assignment that is just like g' except for mapping χ to z and v to w. (Since v occurs free in π, this same variable also occurs free in $\ulcorner\pi(\chi)\urcorner$.) Next, consider the property $\lambda x\, S(x, y)$. For any x, a state s is an instantiation of this property by x iff s is an instantiation of S by x and y, in this order. By what has just been said, this means that a state s is an instantiation of $\lambda x\, S(x, y)$ by x iff s is denoted by $\ulcorner\pi(\chi)\urcorner$ relative to an assignment that is just like g' except for mapping χ to x and v to y. But an assignment that is just like g' except for mapping χ to x and v to y is nothing other than the assignment that is just like g except for mapping χ to x. So a state that is denoted by $\ulcorner\pi(\chi)\urcorner$ relative to an assignment that is just like g' except for mapping χ to x and v to y is simply P's instantiation by x. This means that P and $\lambda x\, S(x, y)$ are trivial converses of each other. So, by Uniqueness, they are identical—which means that P fails to be purely qualitative, contrary to hypothesis. This completes the *reductio*.

(D) For any entities x and y: x is a *qualitative duplicate* of y iff x is distinct from y and, for every purely qualitative intrinsic property P: x instantiates P iff y instantiates P.[18]

It is further plausible that the following holds:

Intrinsic Haecceities. For any x, the property of *being identical with* x is intrinsic.

Putting this together with Inherited Purity, we can infer that, for any purely qualitative property P, the property of *being identical with* P is both purely qualitative and intrinsic. Accordingly, if P is a purely qualitative property, then any qualitative duplicate of P will have to share with it the property of *being identical with* P while also being distinct from P. Since this cannot be done, it follows that no purely qualitative property has a qualitative duplicate.

Intuitively, however, there seems to be a sense of 'duplicate' in which the question of whether a given entity has a duplicate will not be settled in the negative simply by that entity's being a purely qualitative property. And we might also like to have a sense of 'impure' in which *being identical with redness* is just as impure as *being identical with Napoleon*, and for the same reason.[19] It should therefore be desirable to have an alternative to (I4).

5 Perspicuity: First Steps

What we want is a sense of 'impure' in which, for any property P, that of *being identical with* P is impure even if P itself is *not* impure. Now it might be reasonably hoped that this can be achieved by employing a concept of *perspicuity* that applies to the terms of a certain formal language—in particular, the language $\mathcal{L}$ of OTT, which has already been used above.

Where t is any term of $\mathcal{L}$, let us say (somewhat dramatically) that *t bears the mark of impurity* iff t contains a free occurrence of some term 'at argument-position', i.e., as an element of an argument-list. The lambda-expression '$\lambda x\,(x = \text{Sappho})$', for instance, bears the mark,[20] but the constant 'Sappho' doesn't. Life would be easy if we could say that a property is impure iff it is denoted, relative to some interpretation and variable-assignment, by a term that bears the mark of impurity! However, we can't. For one thing, we would get *false negatives*: terms that lack the mark of impurity while nonetheless denoting (relative to some interpretation and variable-assignment) a property that we would like to classify as impure. For

[18] To say that an entity x 'instantiates' a property P should here be taken to mean that there exists an obtaining instantiation of P by x. Regarding the concept of intrinsicality, see, e.g., Plate (2018).

[19] See Plate (2022: 1300f., 1312) and references therein. I am here assuming that Napoleon is a particular, though this might of course be doubted.

[20] In $\mathcal{L}$, we write '$(x = \text{Sappho})$' to abbreviate the formula '$\mathbf{I}(x, \text{Sappho})$'; '$\mathbf{I}$' functions as a 'logical' constant that denotes the identity relation relative to *every* interpretation and variable-assignment. Cf. OTT, pp. 2360, 2367.

example, relative to a variable-assignment that maps 'S' to *being identical with Sappho*, the variable 'S' denotes an impure property, and yet it lacks the mark of impurity. For another thing, we would also get *false positives*: terms that do bear the mark but fail to be impure in any sense that interests us here. Thus the lambda-expression '$\lambda x\, R(x, y)$' bears the mark but denotes, relative to a variable-assignment that maps 'y' to a zeroth-order property P and 'R' to the instantiation relation $\lambda x, y^1\, y^1(x)$, nothing else but P itself, regardless of whether P is impure.[21]

Enter perspicuity. At least as the term will be understood here, *concepts of perspicuity* promise to eliminate either the false positives, the false negative, or both. Like the concept of denotation for terms of $\mathcal{L}$, they should be understood as relative to interpretations and variable-assignments, meaning that, in general, a term can be meaningfully said to be perspicuous or imperspicuous only relative to some interpretation and variable-assignment. (For example, a variable v may be perspicuous relative to an assignment that maps v to a fundamental property, but not relative to an assignment that maps v to a highly complex non-fundamental property.) For brevity's sake, let us use the locution 'perspicuously denotable by ...' as an abbreviation of 'denoted by ... relative to an interpretation and variable-assignment relative to which ... is perspicuous'. We can then formulate the following three 'admissibility conditions' on concepts of perspicuity:

(A1) Every entity is perspicuously denotable by *some* term.

(A2) If every term by which a given property is perspicuously denotable bears the mark of impurity, then that property is impure.

(A3) If every term by which a given property is perspicuously denotable *lacks* the mark of impurity, then that property is *not* impure.

Equipped with an admissible concept of perspicuity that eliminates the false *positives*, we will be able to say that a property is impure iff it is perspicuously denotable by some term that bears the mark of impurity. The left-to-right direction will follow from (A3), while the right-to-left direction will follow from the absence of false positives. Analogously, equipped with an admissible concept of perspicuity that eliminates the false *negatives*, we will be able to say that a property *fails* to be impure iff it is perspicuously denotable by some term that *lacks* the mark.

While it may be tempting to devise a concept of perspicuity that eliminates the false negatives *as well as* the false positives, I will here pursue the less ambitious course of introducing a concept that only eliminates the false negatives. This concept will enable us to define the concept of an impure property as follows:

(I5) A property P is *impure* iff every term by which P is perspicuously denotable contains a free term-occurrence as an element of an argument-list.

Here the right-to-left direction will follow from (A2), while the left-to-right direction will follow from the absence of false negatives.

What concept of perspicuity might fill the bill? Conceivably, just the following:

[21] Cf. Section 3 above.

(P1) A term t is *perspicuous* relative to an interpretation I and a variable-assignment g iff every atomic term that occurs free in t denotes, relative to I and g, either a particular or a fundamental intensional entity.

This may seem simplistic, but let us see how far it gets us. First of all, we can note that, at least given OTT, the concept of perspicuity introduced by (P1) satisfies (A1), the requirement that every entity be perspicuously denotable. This is mainly because one of the assumptions of OTT—viz., the 'analyzability assumption', already mentioned in footnote 17 above—is to the effect that for every entity there exists a term that denotes that entity relative to (the empty interpretation and) some variable-assignment whose range contains only particulars and fundamental intensional entities. I leave the details to a footnote.[22]

As for (A2), this requirement is plausibly also met. It helps here that, given that (A1) is satisfied, there will be no property P such that the antecedent of (A2) is only *vacuously* satisfied for P: it will not be the case that every term by which P is perspicuously denotable bears the mark of impurity simply because there is no such term. Accordingly, if P is indeed such that every term by which it is perspicuously denotable bears the mark of impurity, then this should really tell us something about P; and plausibly, what it tells us is that P is impure.

Let us next consider the crucial question of whether (P1) allows for false negatives—the question, in other words, of whether any impure property is perspicuously denotable, in the sense corresponding to (P1), by a term that lacks the mark of impurity. At least *prima facie*, the answer to this question is plausibly 'no' (though see the next section). To be sure, a term t that is relative to a given interpretation I and variable-assignment g perspicuous in the sense of (P1) may exhibit any amount of 'redundant complexity' (such as double negations and repeated or tautological conjuncts) and in this way be intuitively imperspicuous. But it will *not* be imperspicuous in the sense of hiding the *non*-redundant complexity of intensional entities behind the syntactic simplicity of atomic terms. For example, if the variable 'S' denotes relative to I and g the property of *being identical with Sappho*, then 'S' will under (P1) not be perspicuous relative to I and g, because (i) 'S' trivially contains a free

[22] Let x be any entity. By the analyzability assumption, there then exist a term t and a variable-assignment g such that (i) t denotes x relative to the empty interpretation and g and (ii) the range of g contains only particulars and fundamental intensional entities. To see how this ensures that (A1) holds, we have to appeal to two (fairly standard) features of the semantics of $\mathcal{L}$. First, if a given term denotes an entity relative to a given variable-assignment (and the empty interpretation), then that term contains *no* free occurrence of any atomic term that does not have a denotation relative to that assignment. In particular, the term will not contain any constants other than '$\mathbf{I}$' (which denotes identity); any atomic term that occurs free in it will therefore be either '$\mathbf{I}$' or a variable. Second, relative to a given assignment, a variable will denote, if anything, the entity that it is mapped to under that assignment. Hence, given that the range of our assignment g contains only particulars and fundamental intensional entities, and given that t has a denotation relative to the empty interpretation and g, we can infer that any atomic term u that occurs free in t denotes, relative to g, either a particular or a fundamental intensional entity: if u is a variable, it will denote whatever it is mapped to under g; and if it is '$\mathbf{I}$', it will denote the identity relation, which in OTT is assumed to be fundamental. (Cf. OTT, p. 2377.) But this means, given (P1), that t is *perspicuous* relative to the empty interpretation and g. Accordingly, any entity whatsoever is perspicuously denotable, as required by (A1).

occurrence of 'S' and (ii) the property of *being identical with Sappho*, which is relative to I and g denoted by 'S', is provably non-fundamental.[23] This makes it seem plausible that (P1) is sufficient to eliminate false negatives: that no impure property will be perspicuously denoted, in the sense corresponding to (P1), by a term that lacks the mark of impurity. And if this is right, then we may further infer that (A3) is also satisfied. For let P be any property such that every term by which it is perspicuously denotable lacks the mark of impurity. Given that (A1) is satisfied, P will then be perspicuously denotable by *at least one* term that lacks the mark of impurity; and by the absence of false negatives, this means that P is impure, as required by (A3).

It seems, then, that the concept of perspicuity introduced by (P1) satisfies all three of the admissibility conditions laid down above; and thus the combination of (I5) and (P1) emerges as a viable alternative to the definitions of 'impure' canvassed in the previous section. In particular, it does not fall prey to the trivialization problem that plagues (I1), (I2), and (I3), and, unlike (I4), at least *prima facie*, it treats *being identical with redness* in the same way as it treats *being identical with Napoleon*.

Before we leave this section to discuss two objections, let us quickly note that any property that is impure in the sense of (I4) is also impure in the sense of (I5).[24] Provided that the converse does not hold, properties that are impure in the sense of (I4) might thus be justifiably called 'strongly impure'.

[23]The non-fundamentality of such properties as *being identical with Sappho* can be shown with the help of the analyzability assumption in combination with the 'independence assumption' referenced in footnote 17 above. (Cf. OTT, §5.5.2 as well as—for a more extensive argument in a slightly different setting—Plate [MS(b): §15].) The independence assumption also entails that instantiation relations are non-fundamental. Hence, if a variable-assignment g maps 'R' to, e.g., $\lambda x, y^1\, y^1(x)$, then the term '$\lambda x\, R(x,y)$', which contains a free occurrence of 'R', will under (P1) *not* be perspicuous relative to g. Of course, this is as it should be.

[24]This can be shown with the help of the analyzability assumption together with the 'Lemma' introduced in footnote 17 above. Suppose for *reductio* that there exists a property P that is impure in the sense of (I4) but not in the sense of (I5). Then, since P is impure in the sense of (I4), there exist a particular y and a dyadic relation R such that $P = \lambda x\, R(x,y)$. By the analyzability assumption, there further exist a term ρ and a variable-assignment g such that (i) g's range contains only particulars and fundamental intensional entities, and (ii) relative to g, R is denoted by ρ. Without loss of generality, we may suppose that g also maps a variable v of type 1 to y. Meanwhile, since P is *not* impure in the sense of (I5), we have that P is perspicuously denotable by a term μ that lacks the mark of impurity. From (P1) it can now be seen that we may, again without loss of generality, suppose that μ denotes P relative to g (and the empty interpretation). Further, since μ lacks the mark of impurity, it does not contain any free occurrence of v—or, for that matter, of any other variable that denotes, relative to g, a particular; for any such occurrence would have to stand at argument-position. (Given that μ has a denotation relative to g, any term that has in μ a free occurrence at predicate-position will, relative to g, denote an attribute, and any term that has in μ a free occurrence at sentence-position will, relative to g, denote a state.) Let now t and t' be, respectively, the formulas $\ulcorner(\mu = \mu)\urcorner$ and $\ulcorner(\lambda x\, \rho(x,v) = \lambda x\, \rho(x,v))\urcorner$. These two formulas denote one and the same state, namely the self-identity of P; and without loss of generality (once again), we may suppose that t and g jointly satisfy conditions (i)–(iv) of the Lemma. By the Lemma, it then follows that v, which occurs free in t', also occurs free in t. But since $t = \ulcorner(\mu = \mu)\urcorner$, this means that v occurs free in μ; yet we saw above that it doesn't. This completes the *reductio*.

6 Two Objections

We now come to two objections. The first concerns the overall approach and might occur to any casual observer, while the second is specifically concerned with (P1) and considerably less obvious, though also far more serious.

Probably the most striking difference between (I5) and its four predecessors is that it quantifies over terms of a formal language. This may seem to invite the objection that, since one's choice of language is often arbitrary, the question of what properties are impure is under (I5) made to depend on certain 'merely linguistic' facts. On closer inspection, however, this worry is unfounded. There are of course *some* features of $\mathcal{L}$ that are the result of arbitrary decisions. For instance, instead of a primitive conjunction operator, the language might have contained a primitive *dis*junction operator. But insofar as such factors have no influence on what properties (I5) classifies as impure, their arbitrariness is of no concern. It further bears noting that $\mathcal{L}$ is not just *any* language, but is rather closely bound up in the inner workings of OTT: it is employed in the comprehension axiom, the analyzability assumption relies on it, and so does the independence assumption referenced in footnote 17. In light of this, the choice of $\mathcal{L}$ as the formal language whose terms are quantified over in (I5) is anything *but* arbitrary.

A more serious objection arises from considerations about the coarse-grainedness of properties. As we have seen, the adequacy of (I5), when combined with (P1) as the relevant definition of 'perspicuous', rests on the ability of the corresponding concept of perspicuity to 'eliminate false negatives'. But now let P be any fundamental property, and consider the following lambda-expression:

$$\lambda x\, \exists y^1 \left((x = y^1) \wedge \forall z, w \left((z = y^1(w)) \leftrightarrow (z = P(w))\right)\right). \tag{1}$$

In English, the property that is denoted by this expression (relative to a variable-assignment that maps 'P' to P) might be referred to as 'the property of *being a zeroth-order entity that is a trivial converse of P*'. By Uniqueness (Section 3 above), every property has only one trivial converse, namely itself. Hence, for an entity to have the property in question, it will have to be identical with P; and by a plausible 'coarse-grainedness principle' that forms an integral part of OTT, it can be seen that the property in question is indeed nothing else than that of *being identical with P*.[25] Further, given that P is fundamental, (1) is under (P1) *perspicuous* relative to a variable-assignment that maps 'P' to P, and is therefore a term by which *being identical with P* is perspicuously denotable; yet it does not bear the mark of impurity. So, under the combination of (I5) and (P1), the property of *being identical with P* turns out *not* to be impure!

[25]The mentioned principle—which says that "[n]o two states necessitate each other" (OTT, p. 2364)—has in the first place the consequence that, for any entity x, a state s is an instantiation by x of *being a zeroth-order entity that is a trivial converse of P* iff s is an instantiation by x of *being identical with P*. This means that the 'two' properties just mentioned are trivial converses of each other; and this in turn means, by Uniqueness, that they are one and the same property.

For another example, let s be a fundamental state, and consider the property of *being identical with s*. Under the coarse-grainedness principle mentioned in the previous paragraph, the property of *being identical with s* will (relative to a variable-assignment that maps 's' to s) be denoted by

$$\lambda x\, \exists y^1 \left((x = y^1) \wedge \forall z, w^1 \left((z = (y^1 \wedge w^1)) \leftrightarrow (z = (s \wedge w^1)) \right) \right). \tag{2}$$

In English, this expression may be paraphrased as: 'the property of *being a zeroth-order entity y that is such that, for any zeroth-order entity w, something is a conjunction of y and w iff it is also a conjunction of s and w*'. Under the coarse-grainedness principle, this property turns out to be a trivial converse of *being identical with s*. By Uniqueness, it is therefore the same property. Further, given that s is fundamental, (2) is under (P1) *perspicuous* relative to a variable-assignment that maps 's' to s, and is therefore a term by which *being identical with s* is perspicuously denotable; yet it does not bear the mark of impurity. We thus have that *being identical with s* is not impure in the sense of (I5) when combined with (P1).

These results are disappointing. They suggest that, on the background of a (moderately) coarse-grained conception of properties, the combination of (I5) and (P1) does not give us what we were looking for, viz., a definition of 'impure' that reliably classifies *being identical with x* as impure, regardless of whether x is a particular, an attribute, or a state. The only way I see of repairing this defect is to adopt a more demanding concept of perspicuity.

7 Perspicuity Revisited

What is clear about the expressions (1) and (2) above is that they suffer from redundant complexity; and so it would be a natural idea to exploit this. However, it is not immediately clear how the relevant notion of 'redundant complexity' is to be made precise: are we dealing with a surplus number of variables, or parentheses, or what? Given that $\mathcal{L}$ allows for infinitely long expressions (OTT, p. 2357), counting *overall* numbers is a recipe for failure. A term may very well contain infinitely many variables and parentheses even without suffering from the redundancy exhibited by (1) and (2); and turning that term into one that *does* exhibit that kind of redundancy will then not necessarily increase those numbers. What we need is, rather, a 'local' approach that looks at individual term-occurrences.

Often, the term that results from replacing a term-occurrence o in a complex term t with a term t' has, relative to a given interpretation I and variable-assignment g, the same denotation as the original term t.[26] In such a case, we will say that o is relative to I and g *replaceable* by t'. Now, looking at (1) and (2), we can see that, in both cases, the second occurrence of 'y^1' is replaceable (relative to the respective variable-assignment) by an atomic term u—namely, 'P' in the case of (1), and 's' in

[26] We can think of an occurrence of an expression e in an expression E as an ordered triple (E, e, α), where α indicates the occurrence's starting point in E. For any expression E, the triple $(E, E, 0)$ is then E's occurrence 'in' itself. For related discussion and an alternative conception, see Wetzel (1993).

the case of (2)—in such a way that the resulting occurrence of u is *free* rather than bound. In particular, (1) can be transformed *salva denotatione* into

$$\lambda x\, \exists y^1 \left((x = P) \wedge \forall z, w \left((z = y^1(w)) \leftrightarrow (z = P(w))\right)\right), \tag{3}$$

and (2) into

$$\lambda x\, \exists y^1 \left((x = s) \wedge \forall z, w^1 \left((z = (y^1 \wedge w^1)) \leftrightarrow (z = (s \wedge w^1))\right)\right). \tag{4}$$

This observation relies on the same coarse-grainedness principle that is (together with Uniqueness) responsible for the fact that *being identical with P* and *being identical with s* are respectively denotable by (1) and (2). I would now like to propose that we make use of this observation for the purpose of defining a more demanding concept of perspicuity.

Intuitively, if a bound variable-occurrence is replaceable in the way the mentioned occurrences of 'y^1' are, then those variable-occurrences (along with the quantifier-occurrences that bind them) 'do no real work' but only contribute redundant complexity, thereby rendering the containing term imperspicuous. To rule out this kind of imperspicuity, we may adopt the following definition:

(P2) A term t is *perspicuous* relative to an interpretation I and a variable-assignment g iff the following two conditions are satisfied:

- (i) Every atomic term that occurs free in t denotes, relative to I and g, either a particular or a fundamental intensional entity.
- (ii) No bound variable-occurrence in t is relative to I and g replaceable by an atomic term u in such a way that the resulting occurrence of u is free.

Here the first clause repeats the right-hand side of (P1), while the second clause serves to rule out that any entity is perspicuously denotable by an expression such as (1) or (2). Obviously, this definition is tailor-made for the purposes of defining a concept of impurity: its express purpose is the elimination of the false negatives mentioned near the beginning of Section 5. But this is no defect. While one might think of (P2) as a mere second step (after (P1)) towards the grand goal of 'perfect perspicuity', our present project does not require that we go the rest of the way.

Does the concept of perspicuity introduced by (P2)—call it *2-perspicuity* for short—satisfy the requirement (A1) from Section 5 above? This would seem to be, at the very least, a reasonable conjecture; for it is hard to see why, in order to denote a given entity, a term should need to contain one or more bound variable-occurrences that are effectively replaceable by constants. Assuming that (A1) *is* satisfied, it is further plausible that (A2) is met, as well: here we can apply the same reasoning as already in the case of (P1). In addition, given that the concept of 2-perspicuity is by design more demanding than that introduced by (P1), the former should do a correspondingly better job of weeding out false negatives, in particular when it comes to expressions like (1) and (2) above. And of course, assuming that this concept *does* eliminate all false negatives (meaning that no impure property is 2-perspicuously denotable by a term that lacks the mark of impurity), (A3) will also be satisfied. Accordingly, I submit that the combination of (I5) and (P2) gives us what we wanted: a definition of 'impure' that reliably treats *being identical with x*—and likewise *being a lover of x*, etc.—as impure, regardless of what sort of entity x might be.[27]

8 Conclusion

The proposal we have now arrived at must be regarded as somewhat tentative: there is no guarantee that it achieves the stated aim. It *does*, however, seem sufficiently promising to merit further consideration. If nothing else, I hope in this paper to have shed some light on the difficulty of defining a satisfactory concept of impurity, and to have indicated a thus-far under-explored way of tackling it.[28]

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[27] This also holds, incidentally, if x is the property of *being red*. I therefore revoke the concession that I made to a potential critic in my (2022: 1312n.). Fairly obviously, one can love redness (or "the color red") only by standing in a certain relation—viz., *loving*—to redness. What a typical nominalistic paraphrase of 'loving the color red' yields is *not* a description of the property of *loving the color red* properly so called, but rather of something like the property of *loving red things*.

[28] An earlier (and radically different) version of this paper, referenced under the title 'Perspicuous Denotation' in my (2022), was written in the spring of 2021, with financial support from the Swiss National Science Foundation (project 100012_192200).

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