An Ontology of Roles and States

Jan Plate

August 28, 2025

Abstract

This paper develops an ontology of *roles* and states of affairs (or *states* for short), together with a conceptual framework that offers a novel conception of relations. In particular, relations are here conceived of not simply as polyadic universals but as *sets of role sequences*. The relation *shorter-than*, for instance, is identified with a singleton set $\{\langle \mathit{Shorter}, \mathit{Taller} \rangle\}$, for two roles *Shorter* and *Taller*; and an instantiation of it by Socrates and Simmias, in this order, is taken to be a state that 'results' from an assignment that pairs *Shorter* with Socrates and *Taller* with Simmias. This conception yields considerable explanatory benefits: it allows us to explain (i) how and why the adicities of relations are (at least in part) determined by their essential properties; (ii) why converse relations are 'on a par' with regard to fundamentality, (iii) why some relations *are* converses of others; and (iv) why certain specific relations, such as adjacency, are symmetric. The proposal moreover accommodates relations with 'complex' symmetries that have traditionally stymied (some) positionalistic conceptions. The overall ontology is highly generous with regard to roles and states, but quite austere otherwise: besides roles and states, no non-fundamental entities are admitted. (*Sets* are identified with certain complex properties.) The penultimate section examines the *Special Resultance Question*: Under what conditions is a given assignment of entities to roles such that a state results from it?

Keywords: properties; roles; relations; instantiation; ontology

1 Introduction

On a standard way of looking at relations, they are nothing but *polyadic analogues* of properties. Just as a property is instantiated by entities taken singly, so a *relation* is instantiated by two or more entities taken in sequence: in the case of a *dyadic* relation, the sequences are of length 2, in the case of a *triadic* relation, they are of length 3, and so on—and that is all there is to relations.

On the view to be developed in this paper, there *is* something more to relations: a kind of internal structure. The view has a precursor of sorts in a theory that Castañeda once ascribed to Plato. According to this latter theory, the sentence

(1) Socrates is shorter than Simmias

is "to be analyzed" along the following lines:

(2) Shortness (Socrates) Tallness (Simmias).

Here, as Castañeda goes on to explain, "the juxtaposition of the matrices 'Shortness ( )' and 'Tallness ( )' expresses the *with-respect-to* connection between the two participations in the Forms making up the relation shorterness" (1975: 241). The two Forms in question are referred to as 'Shortness' and 'Tallness', respectively, and the relation *shorter-than* (or "shorterness") is in Castañeda's view—or at any rate on the view he ascribes to Plato—simply the set of the two Forms just mentioned. In broadly similar fashion, the view to be developed below conceives of relations as set-theoretic constructions over a space of *roles*.[1]

More specifically, we will conceive of relations as non-empty sets of *sequences of roles*. For example, given suitable roles s and t (for '*Shorter*' and '*Taller*', respectively), the *shorter-than* relation will be taken to be the set $\{\langle s, t\rangle\}$, where $\langle s, t\rangle$ is the sequence that has the roles s and t as its first and second element. This conception is primarily motivated by its explanatory benefits; these will be presented in Section 2. It is further part of this view that (i) every property is a role and (ii) every set is a property. Putting all this together, we have that *every relation is a role*: since every relation is a set, every set a property, and every property a role. The overall ontological picture is marked by a stark dichotomy between roles and states of affairs. Since roles and states (the 'of affairs' will usually be dropped) are what the theory is mainly concerned with, I will refer to it as an *ontology of roles and states* (ORS).

Apart from its focus on roles and states and its novel conception of relations, ORS is characterized by the fact that it takes roles and states to be highly abundant, individuated in a moderately coarse-grained manner, and arranged in a transfinite hierarchy of 'orders', largely in accordance with Plate (2025). To make the present paper self-contained, some material from the former—mostly definitions—will need to be reproduced below. However, motivating commentary about the mentioned hierarchy, or about the 'fine-grainedness principle' adopted in Section 5.3, has here been largely omitted. Interested readers are referred to the paper just cited, as well as (MS(c)).[2]

The rest of this paper is organized as follows. Section 2 briefly elaborates on the mentioned conception of relations and motivates it by illustrating its explanatory potential with respect to four different kinds of *explanandum*. It also explains

[1] Orilia (2008; 2011; 2014; 2019; forthcoming) has developed a theory of *onto-thematic roles*, which are modeled on the *thematic roles* discussed by linguists. Cf. also Paolini Paoletti (2024: ch. 6). Role concepts can moreover be found in the field of conceptual modeling and adjacent areas; see, e.g., Bachman and Daya (1977), Steimann (2000), Mizoguchi et al. (2015).

[2] This latter paper also introduces the present conception of relations, but motivates it in a very different way, namely on the ground that it allows for a simpler fine-grainedness principle. This fact arguably constitutes a further reason—in addition to the explanatory benefits to be outlined in Section 2 below—to embrace that conception.

how ORS avoids a problem that Kit Fine (2000: 17n.) has raised for a form of *positionalism* that is superficially similar to the present view. Section 3 then introduces the theory's primitive concepts, along with a number of derived notions that will be useful in formulating the theory itself. Section 4 introduces the formal language $\mathcal{L}$ that will serve as a representational tool in some of ORS's most central assumptions. Those assumptions are finally laid out in Section 5. A large question left open by these assumptions is the following: What assignments of entities to roles have states 'resulting' from them? By way of a tentative answer, Section 6 introduces three additional principles, and Section 7 concludes.

2 Properties and Relations

2.1 The core idea

Consider the problem of *how to distinguish between properties and relations*. The usual approach, which we will follow here, is to characterize properties as monadic and relations as polyadic.[3] However, as for what exactly this *means*, there is surprisingly little consensus (as we will see in the next subsection). For the purposes of this paper, let us adopt the following definition:

Definition 2.1. An ordinal α is an *adicity* of an entity x iff the following two conditions are satisfied:

(i) If $\alpha = 0$, then x is a state.

(ii) If $\alpha > 0$, then there is an α-sequence (i.e., a sequence of length α) of entities $y_1, y_2, \ldots$ such that there exists an instantiation of x by $y_1, y_2, \ldots$, in this order.

Here an 'instantiation of x by $y_1, y_2, \ldots$, in this order' should be understood to be a state (not necessarily an *obtaining* state). Note that this definition does *not* rule out that an entity can have more than one adicity.

On the basis of the concept of adicity, we can also define those of property, relation, and attribute:

Definition 2.2. Something is a *property* iff it has an adicity of one.

Definition 2.3. Something is a *relation* iff it has an adicity of at least two.

Definition 2.4. Something is an *attribute* iff it is either a property or a relation.[4]

Thus defined, each of these concepts relies (via the definition of 'adicity') on that of instantiation. A natural view to take with regard to this latter concept—we might even call it the 'default view'—is that it is an unanalyzable primitive. In ORS, by contrast, the concept of instantiation receives a reductive analysis. That analysis falls

[3]See, e.g., Armstrong (1978: 75): "properties are one-particulared or monadic universals, while relations are two-, three-, n-particulared, that is dyadic, triadic, n-adic universals".

[4]For precedent, see, e.g., Carnap (1942: 17).

into two parts, one dealing with properties, the other with relations; and it mainly rests on three concepts: *role*, *role assignment*, and *resultance*. This last concept will here be taken as primitive, while the first two will be defined in Section 3.2 below.

For now, we can think of a role assignment as a multiset $\{(r_1, x_1), (r_2, x_2), \dots\}$, where each r_i is a role. *Properties* are treated as a special case of roles. In particular, we will say that properties are just those roles r that are such that, for some entity x, there exists a state that results from the assignment $\{(r, x)\}$; and for any such role r and entity x, a state that results from $\{(r, x)\}$ will be called an *instantiation* of r by x. For example, if M is the property of *being mortal*, then its instantiation by Socrates will be taken to be the state that results from $\{(M, \text{Socrates})\}$. In the case of *relations*, the matter is slightly more complicated. As has already been mentioned in the Introduction, we will treat relations as non-empty sets of role sequences. This will be a direct consequence of the above definitions of 'relation' and 'adicity' together with the to-be-proposed analysis of the concept of instantiation.

Under Definition 2.1 above, a relation R is α-adic (for $\alpha > 1$) iff it has an instantiation by some α-sequence of entities, which is to say: for some sequence $\langle x_1, x_2, \dots\rangle$ of length α, there exists an instantiation of R by $x_1, x_2, \dots$, in this order. Now, ostensibly, the question our analysis has to answer is simply this: what does it mean for a given state s to be such an instantiation? But note that we can frame our answer in such a way that it will *also* place a constraint on what it means for something to be an α-adic relation. For example, where $\langle x_1, x_2, \dots\rangle$ is a sequence of length α, we can (and will) say that a state s is an instantiation of R by $x_1, x_2, \dots$, in this order, iff R is a set that contains an α-sequence $\langle r_1, r_2, \dots\rangle$ such that s results from the role assignment $\{(r_1, x_1), (r_2, x_2), \dots\}$. Combined with Definition 2.1, this analysis entails that a relation will be α-adic *only* if it is a set containing an α-sequence of roles.

In the simplest case, a relation will contain only a single sequence of length 2. Thus *shorter-than* will, as already noted above, be identified with $\{\langle \mathit{Shorter}, \mathit{Taller}\rangle\}$, the relation of *loving* will be identified with $\{\langle \mathit{Lover}, \mathit{Beloved}\rangle\}$, its converse with $\{\langle \mathit{Beloved}, \mathit{Lover}\rangle\}$, and so on. Notice that these are all singletons. However, some relations—namely those that exhibit certain symmetry patterns—will be taken to contain more than one role sequence.[5]

Now, what motivates this rather unusual conception of relations? In the next four subsections, I will outline four explanatory benefits. They are not unique to this conception: Scott Dixon's *plural slot theory* (PST) provides them as well.[6] However, PST suffers from an arguably fatal problem that ORS avoids. This will be the topic of Section 2.6.

[5]A few examples of this will be briefly discussed in Section 2.6 below. Also cf. Plate (MS[c]: §10).

[6]Dixon develops PST in his (2018). In more recent work (2023, forthcoming(a),(b)), he defends a more traditional, 'directionalist' conception of relations.

2.2 Adicities

Cody Gilmore (2013) critically surveys ten different ways of defining the concept of adicity, before settling on an account that equates the adicity of an attribute (or 'universal') with its number of 'slots'. The definition offered above (Definition 2.1) is broadly similar to one of those he rejects:[7]

(D7) x specifies the adicy of y = df. (i) there is at least one x-tuple that saturates y, and (ii) anything that saturates y is an x-tuple.

To say that a given n-tuple $(a_1, \ldots, a_n)$ *saturates* a given "universal R" means here—at least "[l]oosely put"—that "*there is such a thing* as the proposition that $Ra_1, \ldots, a_n$, regardless of whether it's true" (*ibid.*, original emphasis). Admittedly, the similarity to Definition 2.1 may not be immediately evident. But suppose we delete the second numbered clause.[8] We then have:

(D7′) x specifies the adicy of y = df. there is at least one x-tuple that saturates y.

By renaming two of the variables and replacing the 'saturates' with its informal *definiens*, we can transform this into:

(D7″) n specifies the adicy of R = df. there is at least one n-tuple $(a_1, \ldots, a_n)$ such that there is such a thing as the proposition that $Ra_1, \ldots, a_n$, regardless of whether it's true.

Now the similarity to Definition 2.1—or, more precisely: to the *consequent of the second numbered clause* of that definition—should be obvious.

Gilmore's main objection to (D7), which might equally be leveled against Definition 2.1, is that it does not sit well with the following thesis: "for any *n*, if there is such a thing as the property of *being n-adic*, then that property is intrinsic" (p. 219). His label for this thesis is '(IA)', which is short for 'Intrinsicality of Adicy'. While he takes (IA) to be intuitively plausible, he also offers an argument for it. In particular, he claims that (IA) can be cited

> in conjunction with the fact that *universals are abstract objects* and the fact that *abstract objects have their intrinsic properties essentially*,[footnote omitted] as constituting the best explanation of the relatively uncontroversial fact that *universals have their adicies essentially*. (p. 221, original italics)

I agree with Gilmore that, *if* attributes (or universals) have their respective adicities essentially, then this would favor an account on which an attribute's having a certain adicity is an intrinsic feature of that attribute. However, it seems to me that it would

[7] *Op. cit.*, p. 218. I reproduce Gilmore's label for the definition, but omit two footnotes.

[8] Gilmore does not say why he considers this clause to be necessary. Together with the first, it effectively rules out that anything can have more than one 'adicy'; but it's unclear why this should need to be ruled out. (A note on terminology: while Gilmore uses 'adicy', I shall here stick with 'adicity'.)

not be implausible to accept instead a certain weaker, more cautious claim. Instead of the thesis that attributes have their respective adicities essentially, we might accept the following:

(E) What adicity (or adicit*ies*) a given attribute has is at least in part determined by its essential properties.

This weaker thesis is *not* in conflict with Definition 2.1. Supposing it to be true, we might now wonder how it is to be explained. One option would be to appeal to Gilmore's (IA); however, since (IA) conflicts with Definition 2.1, this option is not available to us. Fortunately, there is another option, which instead relies on the conception of relations sketched in the previous subsection.

Recall that an instantiation of a relation R by a given α-sequence of entities $x_1, x_2, \ldots$ is, under the above proposal, a state that results from the role assignment $\{(r_1, x_1), (r_2, x_2), \ldots\}$, where the sequence $\langle r_1, r_2, \ldots \rangle$ is a member of R. Combine this with Definition 2.1, and you have that a relation is α-adic only if it contains an α-sequence of roles. This much was already noted above. But on anyone's view, it is essential to a set what members it has, and essential to a sequence how long it is.[9] By a plausible inheritance principle, it now follows that it is essential to a set (and hence to a relation, if relations are sets) whether it contains a sequence of length α.[10] And from this, together with the foregoing, it can be inferred that the essential properties of a relation help determine whether it is α-adic. For example, if a given relation fails to contain a role sequence of length 2, then this will mean that it is *not* dyadic.

Thus, much like Gilmore's (IA) would help explain the (alleged) fact that attributes have their respective adicities essentially, the proposed conception of relations as sets of role sequences helps to explain (E).

2.3 Relational fundamentality

Consider next the following plausible claim about 'fundamental' relations:

(F) Converses of fundamental relations are themselves fundamental.

Intuitively, a relation and any of its converses are 'metaphysically on a par', as Bacon and Dorr (2024: 145n.) put it. Thus, if *shorter-than* is fundamental, then its converse *taller-than* should also be fundamental, in whatever sense a relation may be aptly said to be fundamental.[11]

Now it is one thing to accommodate (F) and another to *explain* it. If we flat-footedly took the fundamental relations to be just those fundamental entities that

[9] At least, this is so if sequences themselves are sets. Cf. Section 3.2 below.

[10] The principle I have in mind runs as follows: If it is essential to x to bear R to y, and y is essentially F, then it is essential to x to bear R to something that is F. Readers who, following Fine (1995), distinguish between 'mediate' and 'immediate' essence are invited to interpret my talk of essence as being concerned with *mediate or immediate* essence.

[11] For similar assessments, see, e.g., Sider (2011: 219), Bader (2020: 24).

are also relations,[12] then we could *accommodate* (F), but we would not yet have an explanation of why it should be true. By contrast, under the conception of relations sketched in Section 2.1, a slightly more intricate understanding of relational fundamentality suggests itself. In particular, it will be natural to regard a relation as a *fundamental* relation just in case *the roles from which it is constituted* are fundamental entities. (We will officially adopt a definition to this effect in Section 3.4 below.) Under this proposal, *shorter-than* will be a fundamental relation iff *Shorter* and *Taller* are fundamental entities; and similarly in other cases.

To see how this helps explain (F), we now merely have to note that, for any given relation R, any converse of R is constituted from exactly the same roles as R itself—a result that will be more formally established in Section 5.1.1 below, on the basis of a general principle about the individuation of attributes. Accordingly, we have here another example of the proposed conception's explanatory potential: the conception enables us to construct a natural account of relational fundamentality that (in combination with a general individuative principle) has the consequence that converse relations are equifundamental.

2.4 Converses

Let R be any non-symmetric dyadic relation (such as that of *loving*), and let $\breve{R}$ be its converse—or, more precisely, its *non-trivial converse*, to distinguish it from R itself. Under a definition to be adopted in the next section, the following is then the case:[13]

(C) For any entities x and y and any state s: s is an instantiation of R by x and y, in this order, iff s is an instantiation of $\breve{R}$ by y and x, in *this* order.

Even though this follows from the definition of 'converse' (or rather, 'non-trivial converse', but we will usually suppress the qualifier[14]), one might well think that

---

[12]This is the approach taken in Plate (2025).

[13]It is interesting to note that, as in the case of 'adicity', there seems to be no universally accepted definition of 'converse' in the metaphysical literature. In the present paper the concept is introduced on the basis of that of instantiation, but this has certainly not been the rule. For example, Dixon (forthcoming[b]) is willing to take the notion of conversehood as primitive, while Williamson (1985: 249) introduces it with the help of an infinitive construction ("for x to have one to y is for y to have the other to x"). Going much further back, we have Russell:

> A primitive proposition in regard to relations is that every relation has a converse, *i.e.* that, if R be any relation, there is a relation R' such that xRy is equivalent to $yR'x$ for all values of x and y. (1903: 25)

Since there can be co-extensive relations, and since one would typically like each dyadic relation to have only at most one converse, the 'equivalent' in this definition cannot very well be understood in the sense of *material* equivalence. So what is the alternative? In the present setting, the most natural approach is to understand it in terms of *identity*, applied to the states that are here respectively denoted (or otherwise represented; obviously only relative to an assignment) by the formulas 'xRy' and '$yR'x$'. This essentially yields the definition adopted in this paper.

[14]Concerning the locution 'non-trivial converse of R', I should caution that this is synonymous neither with 'converse of R that is distinct from R' nor with 'converse of R that is not a trivial

we have here something that calls out for an explanation. In particular, one might wonder what makes it so that R and $\breve{R}$—these distinct entities, on the face of it entirely separate from each other—jointly satisfy (C). Fine has drawn attention to essentially the same problem in motivating his acceptance of 'neutral relations': the "only plausible explanation", he says, relies on there being "a single underlying unbiased relation" (2000: 15).

Plausible or not, there *are* alternative explanations, at least one of which is available to the proponent of ORS.[15] For under the conception proposed here, R and $\breve{R}$, far from being "entirely separate", are constituted from exactly the same roles. More specifically, R is taken to be a set $\{\langle r_1, r_2\rangle\}$, for two roles r_1 and r_2; and $\breve{R}$ is then identified with $\{\langle r_2, r_1\rangle\}$.[16] From this identification, combined with the account of relational instantiation outlined in Section 2.1, we can infer that R and $\breve{R}$ jointly satisfy (C). For under that view, a state s is an instantiation of $\{\langle r_1, r_2\rangle\}$ by some entities x and y, in this order, iff s results from the role assignment $\{(r_1, x), (r_2, y)\}$, whereas, analogously, an instantiation of $\{\langle r_2, r_1\rangle\}$ by y and x, in *this* order, is a state that results from the role assignment $\{(r_2, y), (r_1, x)\}$. Since $\{(r_1, x), (r_2, y)\}$ and $\{(r_2, y), (r_1, x)\}$ are one and the same role assignment (just as $\{a, b\}$ is the same set as $\{b, a\}$ for any a and b), we have that any given state s is an instantiation of $\{\langle r_1, r_2\rangle\}$ by x and y, in this order, iff s is an instantiation of $\{\langle r_2, r_1\rangle\}$ by y and x, in *this* order. Hence, given the identification of R and $\breve{R}$ with, respectively, $\{\langle r_1, r_2\rangle\}$ and $\{\langle r_2, r_1\rangle\}$, it follows that R and $\breve{R}$ jointly satisfy (C), as required.

In this way, the to-be-explained fact receives its explanation not on the basis of an underlying 'neutral relation', but rather through a set of roles shared by R and $\breve{R}$.

---

converse of R'. A formal definition will be given in Section 3.4 below.

[15] For others, see Dixon (forthcoming[b]).

[16] Why could $\breve{R}$ not be a set $\{\langle r_3, r_4\rangle\}$ for two *further* roles r_3 and r_4, each distinct from r_1 and r_2? The answer is that, with $\{\langle r_2, r_1\rangle\}$ and $\{\langle r_3, r_4\rangle\}$, we would then have *two* converses of $\{\langle r_1, r_2\rangle\}$, and this is in effect ruled out by a general principle—already alluded to in the previous subsection—about the individuation of attributes. In the rest of this footnote, I will explain this in further detail.

The principle in question is the assumption (U2) (Section 5.1.1 below), according to which "no two attributes are trivial converses of each other". (The notion of a trivial converse will be introduced in Section 3.4 and crosscuts that of a *non*-trivial converse.) What this amounts to is that no two attributes A and B are such that, for some ordinal α, the following two conditions are satisfied:

(i) α is an adicity of both A and B.

(ii) For any α-sequence of entities $x_1, x_2, \dots$ and any state s: s is an instantiation of A by $x_1, x_2, \dots$, in this order, iff s is an instantiation of B by $x_1, x_2, \dots$, in this order.

Further, by the argument immediately following this footnote, the set $\{\langle r_2, r_1\rangle\}$ is a converse of R, given that $R = \{\langle r_1, r_2\rangle\}$. If now $\{\langle r_3, r_4\rangle\}$ were *also* a converse of R, then $\{\langle r_2, r_1\rangle\}$ and $\{\langle r_3, r_4\rangle\}$ would be trivial converses of *each other*: they would satisfy the two conditions just listed for $\alpha = 2$. But this would contradict (U2). So, given that $R = \{\langle r_1, r_2\rangle\}$, ORS practically dictates that we identify the converse of R with $\{\langle r_2, r_1\rangle\}$.

2.5 Symmetries

Finally, we can also explain why certain dyadic relations, such as *being next to*, are *symmetric*.[17] The relevant concept of a symmetric dyadic relation may here be defined in two steps as follows:

Definition 2.5. For any ordinal $\alpha > 0$ and any α-sequence of entities $x_1, x_2, \ldots$: an attribute A is *instantiated* by $x_1, x_2, \ldots$, in this order, iff there exists an obtaining instantiation of A by $x_1, x_2, \ldots$, in this order.

Definition 2.6. A dyadic relation R is *symmetric* iff, for any x and y: R is instantiated by x and y, in this order, iff R is instantiated by y and x, in *this* order.

For example, if all love were mutual, *loving* would be a symmetric relation. We can also define the stronger notion of an *intensionally* symmetric dyadic relation:[18]

Definition 2.7. A dyadic relation R is *intensionally symmetric* iff, for any entities x and y and any state s, the following holds: s is an instantiation of R by x and y, in this order, iff s is an instantiation of R by y and x, in *this* order.

Being next to is plausibly not just symmetric, but *intensionally* symmetric. Cleopatra's being next to Antony, for instance, is the same state as Antony's being next to Cleopatra; and similarly for any other relata.

Under the present view, any intensionally symmetric dyadic relation R is a set $\{\langle r, r\rangle\}$, for some role r.[19] For the sake of concreteness, let us say that *being next to* is the set $\{\langle Next, Next\rangle\}$. By combining this identification with what has been said in Section 2.1 about relational instantiation, we can now infer that any instantiation of *being next to* by some entities x and y, in this order, is a state that results from the role assignment $\{(Next, x), (Next, y)\}$, while any instantiation of the same relation by y and x, in *this* order, is a state that results from $\{(Next, y), (Next, x)\}$. But of course, $\{(Next, x), (Next, y)\}$ and $\{(Next, y), (Next, x)\}$ are one and the same role assignment. Hence, for any entities x and y and any state s, the following holds: s is an instantiation of *being next to* by x and y, in this order, iff s is an instantiation of *being next to* by y and x, in *this* order. In other words, *being next to* is intensionally symmetric.

I submit that this argument can stand as an explanation of the fact that *being next to* is intensionally symmetric, and thus also of the fact that it is symmetric.

[17]For ease of exposition, I will in this subsection focus only on dyadic relations. However, analogous remarks apply to higher-adic relations.

[18]For a similar concept (applicable to 'neutral relations'), see Fine (2000: 17).

[19]This follows from the assumption (U1) in Section 5.1.1 below, together with some definitions to be adopted in the next section. Notably, R could not be a singleton $\{\langle r_1, r_2\rangle\}$ for two distinct roles r_1 and r_2; for if it were, then, given that R is intensionally symmetric, the sequence $\langle r_1, r_2\rangle$ would be 'instantiation-equivalent' to $\langle r_2, r_1\rangle$ (cf. Definition 3.5 below), which would mean that $\{\langle r_1, r_2\rangle\}$ would not be a 'proto-relation' (Definition 3.6), and hence not a relation (Definition 3.7 in combination with Definitions 2.1 and 2.3 above). Nor could R be the pair set $\{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$, because r_1 and r_2 would be 'equivalent' to each other (Definition 3.10), which is directly ruled out by (U1). (Cf. the remark that immediately follows (U1) in Section 5.1.1.)

2.6 Pockets vs. roles

We have now seen the explanatory potential of ORS's conception of relations with respect to four different kinds of *explanandum*. As far as I am aware, the only extant theory that matches it in this regard is Dixon's (2018) *plural slot theory*, which (more than ORS) constitutes a form of what Fine (2000) has called *positionalism*.[20] However, as Dixon himself points out (p. 218n.), PST cannot accommodate relations that exhibit certain kinds of symmetry. The symmetries that PST *can* deal with are exactly those that can be described as a straightforward interchangeability of positions; a simple example of this is the relation of adjacency discussed in the previous subsection. On the other hand, among those symmetries that do *not* fall into this class (and which PST can accordingly not accommodate), we have, e.g., the *symmetry of the square*, which characterizes certain tetradic relations, such as the one whose instantiation by four entities x, y, z, and w (in this order) is the state that x and y are playing tug-of-war with z and w.[21]

ORS, by contrast, not only accommodates relations with this kind of symmetry but entails that there are infinitely many of them. To give just one example, it entails that there exists a relation $\lambda x, y, z, w\, ((x = y) \wedge (z = w))$, which exhibits exactly the same symmetry as the tug-of-war relation just mentioned.[22]

The crucial difference between PST and ORS lies in the fact that, in PST, instantiations of relations are characterized by specifying which slots (or 'pockets') are in them 'occupied' by what entities, whereas, in ORS, they are characterized by specifying what *role assignments* they result from.[23] To see how this matters, let R be the tetradic relation whose instantiation by x, y, z, and w, in this order, is the state that x loves y and z loves w. That PST cannot accommodate this relation may be seen from two considerations. First, R's symmetry—or rather, lack thereof—is such that no two of its four slots can be merged into a single 'pocket' without leading to the conflation of distinct states. And second, without such merging, R's instantiation by,

[20]It is tempting to apply this label also to ORS, but the matter is complicated by the fact that Fine introduces positionalism as a "conception of relations as neutral or unbiased" (p. 10), which does certainly not align with ORS. Whatever exactly a 'neutral relation' may be, it is *not* a relation in the sense of Definition 2.3, as long as 'adicity' is defined as in Definition 2.1.

[21]The example is due to MacBride (2007: 42f.). The 'square' in this case can be imagined as one that is formed by x, z, y, and w arranged in clockwise order. A clockwise rotation by 90 degrees (in which x takes z's place, z takes y's place, and so on) leads from the state that x and y are playing tug-of-war with z and w to the state that w and z are playing tug-of-war with x and y—which is plausibly just the same state as the former. For the original statement of the problem, see Fine (2000: 17n.); and see Donnelly (2016) for valuable related discussion.

[22]For the relevant existence assumption, see Section 5.2.1 below.

[23]This concept resultance essentially traces back to Fine's own exposition of positionalism. Fine describes the positionalist as someone who regards "the completion of a relation" as "the state that results from assigning certain objects to the argument-places of the relation" (2000: 13). The use of a triadic occupation predicate (as in, 'x occupies argument-place y in state z') does not appear until a few pages later, when Fine turns to presenting objections against positionalism. Cf. also Leo (forthcoming), who distinguishes between 'thick' (occupation-based) and 'thin' positionalism, the latter of which is based on a concept of substitution applied to states.

say, Dee, Pat, John, and Mary (in this order) will be incorrectly treated as distinct from R's instantiation by John, Mary, Dee, and Pat, in *this* order.[24] So there is a dilemma.

By contrast, under ORS, nothing hinders that one and the same state results from two or more role assignments. Accordingly, R's instantiation by Dee, Pat, John, and Mary, in this order, can result *both* from

$$\{(r_1, \text{Dee}), (r_2, \text{Pat}), (r_3, \text{John}), (r_4, \text{Mary})\}$$

and from

$$\{(r_1, \text{John}), (r_2, \text{Mary}), (r_3, \text{Dee}), (r_4, \text{Pat})\},$$

even if r_1, r_2, r_3, r_4 are pairwise distinct. In this way, ORS avoids the problem.[25]

That ORS provides the explanatory benefits outlined in this section, without running into the problem just now discussed, is in my view one of its main attractions. There are also others, which are largely unrelated to its treatment of relations. I will flag most of them as we go along.

3 A Conceptual Framework

The purpose of this section is to introduce the primitive metaphysical concepts from which ORS is constructed, as well as a battery of derived concepts that, together with some of those already defined in the previous section, will be useful in formulating the main part of the theory. Among these, the three most central concepts are those of *role*, *role assignment*, and *resultance*. On this basis, we will in Section 3.2 formulate an analysis (framed as a definition) of the concept of instantiation.[26] From this analysis, in conjunction with the definitions of 'property' and 'relation' stated in Section 2.1 above, it will be possible to infer that every property is a role and that every relation is a set of role sequences.

3.1 Primitive notions

ORS rests on nine primitive metaphysical (or 'logico-metaphysical') concepts, which are here expressed by the following locutions: 'is a state', 'obtains', 'is a rolehood of',

[24] I here assume that the state that Dee loves Pat and John loves Mary is not distinct from the fact that John loves Mary and Dee loves Pat. Cf. Sections 5.1.2 and 5.2.2 below.

[25] Cf. Leo (2008: §2.4). A critic might raise the following objection: "Granted that the displayed role assignments are such that the same state results from both: then this will surely be a remarkable fact that calls out for an explanation—and it is not clear what that explanation might look like." I disagree with the last part of this objection; but, for reasons of space, I will have to leave this issue for another occasion. (In a nutshell, I would suggest that the desired explanation can be found in the *essences* of the four roles in question. In order to make this precise, however, we would first have to construct a suitable account of essence.)

[26] The decision to formulate the proposed analysis as a definition stems from the fact that the latter is to *some* extent stipulative. There will be more to be said about this in footnote 31.

'is an identity of ... and ...', 'results from', 'is a negation of', 'is a conjunction of', 'is an existential quantification of', and 'is fundamental'. The conceptual relationships between the first eight of these are given by the following meaning postulates:

(M1) For any x: if x obtains, then x is a state.

(M2) For any x and y: if x is a rolehood of y, then x is a state.[27]

(M3) For any x, y, and z: if x is an identity of y and z, then x is a state that obtains iff y is numerically identical with z.

(M4) For any x and y: if x results from y, then x is a state and y a role assignment.

(M5) For any x and y: if x is a negation of y, then both x and y are states, and x obtains iff y does not.

(M6) For any ordinal α, entity x, and α-sequence of entities $y_0, y_1, \ldots$: if x is a conjunction of $y_0, y_1, \ldots$, then each y_i (for $i < \alpha$) is a state, and x is a state that obtains iff each y_i does.

(M7) For any x and y: if x is an existential quantification of y, then y is an attribute and x is a state that obtains iff y has at least one obtaining instantiation.

Applications of these postulates will usually be left implicit in the following.[28]

3.2 Roles and role assignments

On the basis of the primitive concepts expressed (respectively) by 'obtains' and 'is a rolehood of', the central concept of a *role* can be defined as follows:

Definition 3.1. An entity x is a *role* iff, for some y: y is a rolehood of x, and y obtains.

Since there is (by an assumption to be adopted in Section 5.2.2) only *one* rolehood of any given entity, one could equivalently say that something is a role iff its rolehood obtains.

[27] Informally, one could describe 'the rolehood' of a given entity as the state of affairs that that entity is a role (cf. Definition 3.1 below). In Section 5.2.2 we will adopt an assumption to the effect that, for any entity x, there exists exactly one rolehood of x.

[28] Since these postulates themselves employ logical vocabulary (such as 'and' and 'numerically identical'), a critic might worry that we are here introducing dubious 'metaphysical counterparts' of familiar *logical* notions. More particularly, the critic might suspect that we are engaging in needless duplication of conceptual functionality, for example when we introduce a concept expressed by 'is an identity of ... and ...' while we already have one expressed by 'is identical with'.

In reply, I would argue that the logical concepts and their metaphysical counterparts are more intimately linked than they might at first seem to be. For example, the familiar equation '$x = y$' may be understood as having the identity of x and y as its *truth-condition*. This is not to say that the logical concepts are somehow superfluous; after all we are still going to rely on them. But it *is* to say that their metaphysical counterparts should not be regarded as some out-of-the-blue addition. (Cf. Plate [MS(a): §2.4].)

To define the equally central concept of a role *assignment*, we have to start by generalizing the concept of a *multiset*. Intuitively, a multiset is a 'set-like object' that can contain its members 'more than once'. Formally, it is an ordered pair (A, m), where A is a set and m a function from A to the set of all positive integers. A given entity x is called a *member* of a multiset (A, m) iff x is a member of A. The integer $m(x)$ is also referred to as the *multiplicity* of x in (A, m). Now, to obtain the notion of a *generalized* multiset, we simply relax the requirement that multiplicities be finite, and instead allow *any positive cardinal* to be a multiplicity. This generalization is needed to allow our approach to accommodate certain infinitary relations.[29]

To represent specific (generalized) multisets, we will use curly brackets in the same way as with sets. For instance, we will write '$\{a, a, b\}$' to represent the multiset $(\{a, b\}, m)$, where m is the function on $\{a, b\}$ that maps a to 2 and b to 1.

With the concepts of role and generalized multiset in place, that of a role assignment can be defined as follows:

Definition 3.2. A *role assignment* is a non-empty generalized multiset of ordered pairs (r_i, x_i) such that each r_i is a role.

For a simple example, suppose that r is a role and x some arbitrary entity (possibly r itself): then $\{(r, x)\}$ is a role assignment, and so is $\{(r, x), (r, x)\}$. In the formal language $\mathcal{L}$ that will be introduced in the next section, a state that results from a role assignment $\{(r, x), (r, x)\}$ can be more succinctly denoted by the term '(rx, rx)'.

The following two concepts will also be useful:

Definition 3.3. An entity is an *intensional entity* iff it is either a role or a state.

Definition 3.4. An entity is a *particular* iff it is neither a role nor a state.

Thus, anything that is not an intensional entity is a particular, and *vice versa*.

Before we go on, we will need a little more terminology and notation. As usual, *sequences* will be conceived of as functions defined on ordinals. Functions, in turn, are sets of ordered pairs (which themselves are sets, following Kuratowski [1921]), and ordinals are the sets of their respective predecessors (following von Neumann [1923]). Thus, a sequence of length α has α itself as its domain; and for any such sequence σ, its ith element—counting from zero—is given by $\sigma(i)$. A 'permutation on' an ordinal α is a function that has α as both domain and range. And finally, for any such permutation f and any α-sequence σ, the function on α that maps each $i < \alpha$ to $\sigma(f(i))$ will, following common practice, be denoted by '$\sigma \circ f$'. (Informally, $\sigma \circ f$ is a 'reordering' of σ in accordance with f.)

[29]For example, our ontology will be committed to there being an ω-adic relation $\{\sigma\}$, where σ is an ω-sequence $\langle r, r, r, \ldots \rangle$, for some role r. An instantiation of this by $x, x, x, \ldots$ (for some entity x) will, under the next subsection's definition of 'instantiation', be a state that results from a generalized multiset $\{(r, x), (r, x), \ldots\}$, which contains the pair (r, x) with a multiplicity of $|\omega| = \aleph_0$. ("Why not simply use *sets* of role–entity pairs as your role assignments?" The answer to this is slightly more involved; see Section 5.1.2 below, in particular footnote 51.)

3.3 Instantiations

We are now well on our way to the definition (or analysis) of the concept of instantiation. An important consequence of this definition will be to the effect that relations are sets of *instantiation-equivalent* role sequences. The relevant concept of instantiation-equivalence can be defined as follows:

Definition 3.5. For any ordinal $\alpha > 0$, a role sequence σ of length α is *instantiation-equivalent* to a role sequence σ' (notation: $\sigma \cong \sigma'$) iff the following two conditions are satisfied:

(i) There exists a permutation f on α such that $\sigma' = \sigma \circ f$.

(ii) For any state s and any α-sequence of entities $x_0, x_1, \ldots$: s results from the role assignment $\{(\sigma(i), x_i) \mid i < \alpha\}$ iff s results from $\{(\sigma'(i), x_i) \mid i < \alpha\}$.

Here the first numbered clause ensures that two role sequences are instantiation-equivalent *only* if each can be transformed into the other by some reordering. So, for instance, the only sequences that might be instantiation-equivalent to $\langle \mathit{Lover}, \mathit{Beloved} \rangle$, as far as clause (i) is concerned, are $\langle \mathit{Lover}, \mathit{Beloved} \rangle$ itself and $\langle \mathit{Beloved}, \mathit{Lover} \rangle$. But the latter is ruled out by the *second* numbered clause, given that, e.g., the state that results from $\{(\mathit{Lover}, \text{Antony}), (\mathit{Beloved}, \text{Cleopatra})\}$ is distinct from the one that results from $\{(\mathit{Beloved}, \text{Antony}), (\mathit{Lover}, \text{Cleopatra})\}$.[30] This leaves $\langle \mathit{Lover}, \mathit{Beloved} \rangle$ as the *only* sequence that is instantiation-equivalent to it. Trivially, any role sequence is instantiation-equivalent to itself.

In the next step, we introduce the concept of a *proto-relation*. Simply put, a proto-relation is a non-empty set of instantiation-equivalent role sequences, of some length greater than 1. Slightly more formally:

Definition 3.6. An entity x is a *proto-relation* iff there exists a role sequence σ, of some length greater than 1, such that $x = \{\sigma' \mid \sigma' \cong \sigma\}$.

And on this basis, we can now define (or analyze) the concept of *instantiation*:

Definition 3.7. For any ordinal $\alpha > 0$, any entities x and y, and any α-sequence of entities $z_0, z_1, \ldots$: x is an *instantiation* of y by $z_0, z_1, \ldots$, in this order, iff the following two conditions are satisfied:

(i) If $\alpha = 1$, then y is a role and x results from the role assignment $\{(y, z_0)\}$.

(ii) If $\alpha > 1$, then y is a proto-relation and there exists an α-sequence $\sigma \in y$ such that x results from the role assignment $\{(\sigma(i), z_i) \mid i < \alpha\}$.

[30] Some theorists might prefer to prefix clause (ii) with a 'necessarily' on the grounds that, even in a world that contains only one entity (or rather, only one entity of which it could be meaningfully said that it loves or is loved), it should still turn out that $\langle \mathit{Beloved}, \mathit{Lover} \rangle$ fails to be instantiation-equivalent to $\langle \mathit{Lover}, \mathit{Beloved} \rangle$. However, I prefer the present non-modal formulation, because it is easier to work with and avoids awkward questions about the precise meaning of 'necessarily'.

By combining this with Section 2.1's definitions of 'adicity', 'property', and 'relation', it can be seen that a *property* is just a role r such that, for some entity x, there exists a state resulting from the role assignment $\{(r, x)\}$, while a *relation* is just a proto-relation that has an instantiation by some sequence of entities of a length greater than 1. So properties are roles, and relations are non-empty sets of role-sequences. This is the central result of this section.[31]

The following two concepts will be useful for reasoning about roles:

Definition 3.8. A role r is a *proper* role iff, for some entity x, there exists a state that results from a role assignment containing the pair (r, x).

Definition 3.9. A role r_1 is a *correlate* of a role r_2 iff, for some entities x and y, there exists a state that results from a role assignment containing the pairs (r_1, x) and (r_2, y).

For example, the roles *Lover* and *Beloved* are correlates of each other, given that, for some entities x and y, there exists a state resulting from the role assignment $\{(\mathit{Lover}, x), (\mathit{Beloved}, y)\}$.

Finally, for the purposes of formulating an assumption about the individuation of roles (in Section 5.1.1 below), we will need the following concept of equivalence:

Definition 3.10. A role r is *equivalent* to a role r' iff, for some ordinal $\alpha > 0$, some α-sequence σ of roles, and some $i, j < \alpha$, the following three conditions are satisfied:

(i) $\sigma(i) = r$ and $\sigma(j) = r'$.

(ii) $\sigma \cong \sigma \circ f$ for some permutation f on α that merely transposes i and j (or is the identity on α if $i = j$).

(iii) For some α-sequence of entities $x_0, x_1, \ldots$, there exists a state that results from the role assignment $\{(\sigma(i), x_i) \mid i < \alpha\}$.

Here the third clause is needed to avoid the result that any two roles whatsoever are classified as equivalent.[32] It is easy to verify that every proper role (in the sense of Definition 3.8) is equivalent to itself.

[31] It may be worthwhile at this point to reflect on the extent to which Definition 3.7 is stipulative. Instead of 'y is a proto-relation', clause (ii) might equally well (except for the needless complexity that would have resulted) have contained the words, 'y is the singleton of a proto-relation'. We would then have been able to infer that relations are *singletons* of proto-relations. Seeing this, a critic might be inspired to raise a complaint analogous to the objection that has been advanced by Benacerraf (1965) against the proposal to take the natural numbers to be sets. However, I am doubtful already about Benacerraf's original objection. To paraphrase a remark Lewis (1986: 55) made about our talk of properties: it is not as if, prior to adopting the set-theoretic conception of the natural numbers, we had already fixed, with perfect determinacy, on the things we call 'the natural numbers', so that all that remained for us to do was to *discover* what those things were. Rather, there was still room left for stipulation, for *deciding* what things we wanted to call 'the natural numbers'. I see little reason to think that our present situation with regard to 'the relations' is any different.

[32] Without the third clause, this unwanted result would plausibly come about as follows. First, let us note that, for any two roles r and r', there arguably exists at least one role q such that, for any entities x, y, and z, there exists *no* state resulting from the role assignment $\{(r, x), (r', y), (q, z)\}$. (For

3.4 Relational fundamentality and converses

We now turn to the concepts of a *fundamental relation* and a *converse*, both of which have already been encountered in Section 2. The first can be defined in two steps, beginning with the concept of a relation's *role set*:

Definition 3.11. The *role set* of a relation R is the smallest set S such that $R \subseteq S^\alpha$, for some ordinal α.

Here S^α is the set of all α-sequences whose elements are members of S. So, for instance, the role set of the relation $\{\langle s, t\rangle\}$ is just the set $\{s, t\}$. (Note that all the members of a role set are correlates of each other in the sense of Definition 3.9.) On this basis, the concept of a fundamental relation can be defined as follows:

Definition 3.12. A *fundamental relation* is a relation whose role set contains only fundamental roles.

Here a 'fundamental role' should be understood to be simply a fundamental entity that is also a role.

Finally, the concept of a *converse* can be defined in four steps:[33]

Definition 3.13. For any ordinal $\alpha > 0$, any permutation f on α, and any attribute A, an entity x is an *f-converse* of A iff A is α-adic and, for any α-sequence of entities $y_0, y_1, \ldots$ and any state s, the following holds: s is an instantiation of x by $y_{f(0)}, y_{f(1)}, \ldots$, in this order, iff s is an instantiation of A by $y_0, y_1, \ldots$, in this order.

Definition 3.14. An entity x is a *trivial converse* of an attribute A iff, for some ordinal $\alpha > 0$: α is an adicity of A, and x is an id_α-converse of A.[34]

Definition 3.15. An entity x is a *non-trivial converse* of an attribute A iff, for some ordinal $\alpha > 0$ and some permutation f on α that is distinct from id_α: α is an adicity of A, and x is an f-converse of A.

Definition 3.16. An entity x is a *converse* of an attribute A iff x is either a trivial or a non-trivial converse of A.

Usually, when we speak of 'the converse' of a dyadic relation, we mean its *non-trivial* converse. (That a dyadic relation only has a single non-trivial converse is a consequence of an assumption that will be adopted in Section 5.1.1 below.) It is worth emphasizing that the notions of a trivial and a non-trivial converse are not mutually exclusive. For example, a relation $\{\langle r, r\rangle\}$ (for some role r) will be both a trivial and a non-trivial converse of itself.

---

example, if r and r' are the roles of *Lover* and *Beloved*, then the role of *Perceiver* will presumably fill the bill, since it is implausible that, for some entities x, y, and z, there exists a state resulting from the assignment $\{(\mathit{Lover}, x), (\mathit{Beloved}, y), (\mathit{Perceiver}, z)\}$. For what would it mean for such a state to obtain?)

Let now r and r' be any two roles, and let q be as just specified. Then the role sequence $\langle r, r', q\rangle$ will be trivially instantiation-equivalent to $\langle r', r, q\rangle$—which means that, were it not for the third clause, the roles r and r' would be classified as equivalent to each other.

[33]The following definitions are (with negligible changes) taken from Plate (2025: §3.3.3).

[34]Here 'id_α' is the identity permutation on α, i.e., the function that maps each $i < \alpha$ to itself.

3.5 Sets as properties

Since the above definitions rely quite heavily on set theory, it is natural to ask how sets fit into the overall ontology of this paper. The answer is that sets will here be taken to be a special sort of property. In particular, the empty set will be identified with the uninstantiable property of *not being any entity* (in symbols: $\lambda x\, \neg\exists y\, (y = x)$); the singleton of an entity x will be taken to be the property of *being identical with* x; and the pair set $\{x, y\}$, for any x and y, will be identified with the property of *being identical with either* x *or* y. Similarly for larger and even infinite sets.[35] If it is objected—perhaps on linguistic grounds—that sets cannot admissibly be identified with properties, then the obvious fallback will be to *replace* sets with properties.[36] Either way, we will not need to worry about sets as forming some vast class of non-fundamental entities in *addition* to non-fundamental roles and states.

Having previously identified relations with proto-relations (and hence with sets), we can now conclude that relations are also *properties*, and hence roles. This has the consequence that every relation has at least two adicities: given that it is a property, one of its adicities is the ordinal 1, and given that it is a relation, another one of its adicities is some ordinal greater than 1. Further, since every relation is a proto-relation, and every proto-relation has *at most* one adicity greater than 1, it follows that every relation has exactly two adicities greater than 0. Under the reasonable assumption (to be officially adopted in Section 5.3) that no role is a state, we can even conclude that every relation has exactly two adicities, period. Now this result may seem odd for two reasons.

First, most of us are simply not used to thinking of a relation, such as *loving*, as something that is *also* a property. (For that matter, most of us are also not used to thinking of *loving* as a singleton set containing a role sequence!) But this does not amount to a strong objection. It could hardly be claimed, for instance, that *as a matter of Moorean certainty* the relation of *loving* is not also a property. While we may have the one or other pre-theoretic intuition as to what sort of thing a relation might be, such intuitions do not in general carry the same kind of weight as considerations revolving around theoretical virtues such as elegance and explanatory power. A philosopher committed to the idea that no relation is a property might preserve her commitment by rejecting either the proposed conception of relations as sets of role sequences or the identification (or replacement) of sets with properties. But to reject the former is to forego the associated explanatory benefits; and to reject the latter is to accept a needlessly inflated number of basic ontological categories (namely, sets in addition to roles and states) along with an increased ideological burden, which would now also have to include primitive concepts of sethood and set-membership.

[35] Cf. Carnap (1947: §23), Plate (2025: §3.1). *Pace* Black (1971: 624), the proposal does not "break down, of course, for infinite sets". Zermelo's well-ordering theorem, together with the syntactic specification of our formal language $\mathcal{L}$ in Section 4.1 below, guarantees that for any non-empty set there exists a disjunction $\ulcorner((x = v_0) \vee (x = v_1) \vee \ldots)\urcorner$ that (relative to a suitable variable-assignment) enumerates its members; and the assumption (E2) of Section 5.2.1 then entails that there exists a corresponding disjunctive property.

[36] Cf. Bealer (1982: ch. 5), van Inwagen (2023: ch. 6).

The second reason why one might find it odd that every relation should have exactly two adicities arises from the fact that it may seem plausible to think that some relations have more than one adicity greater than 1. Thus Fine writes that "[t]here should, for example, be a relation of sup[p]orting that holds between any positive number of supporting objects $a_1, a_2, \ldots$ and a single supported object b just when $a_1, a_2, \ldots$ are collectively supporting b" (2000: 22).[37] Similarly, it might be thought that there should be a relation of *colinearity* that has instantiations by any number (greater than 2) of points. In the present framework, there is no room for such relations. However, the claim that we should admit such relations can be defensibly rejected. By all appearances, we can make perfectly good metaphysical sense of a situation in which some objects support a further object, or some points are colinear, without having to invoke a variably polyadic relation. Thus, to describe a situation in which Pat, Dee, and John support Mary, we can say something to the effect that Pat, Dee, and John are placed underneath Mary and as a result experience a certain amount of compressive stress. And to describe a situation in which the points A, B, C, and D are colinear, we can say something to the effect that any three members of the set $\{A, B, C, D\}$ are colinear. So all that this latter example calls for is a *triadic* relation of colinearity.[38]

4 The Formal Language

At its core, ORS consists of 15 ontological assumptions, to be presented in the next section. Most of these will be simple and unsurprising, like the 'uniqueness claim' (U3) that no two states result from the same role assignment, or the assumption (O1) that no role is a state. While all of them will be stated in English, some—such as the comprehension axiom (E2) and the 'analyzability assumption' (O3)—will also rely on a certain formal language $\mathcal{L}$ that allows for the construction of complex names of roles and states. The purpose of the present section is to introduce that language.[39]

4.1 Syntax

By an *expression* of $\mathcal{L}$, we will mean a non-empty sequence each of whose elements is either

[37]Cf. also MacBride (2005: 571ff.), McKay (2006).

[38]It might be suggested that we can meet the Finean critic halfway by allowing there to be two roles of *Supporter* and *Supportee* such that, for any $n > 0$, the state of $a_1, \ldots, a_n$ supporting b results from the role assignment $\{(\mathit{Supporter}, a_1), \ldots, (\mathit{Supporter}, a_n), (\mathit{Supportee}, b)\}$. For each $n > 0$, there would then be an $(n+1)$-adic relation $\{\langle \underbrace{\mathit{Supporter}, \ldots, \mathit{Supporter}}_{n \text{ times}}, \mathit{Supportee}\rangle\}$. *Prima facie*, this is a very natural approach. However, as we will see in Section 6.2, it is ruled out by an attractively straightforward way of answering the Special Resultance Question, and so I do not think that we should adopt it.

[39]To a large extent, the material of this section is adapted from Plate (2025: §3.2, §§3.4–4.1). There are, however, significant differences. See also Plate (MS[c]: §§11–12).

(a) one of the ordinals $0, \ldots, 10$, or

(b) an ordered pair $(0, \alpha)$ or $(1, \alpha)$, for some ordinal α, or

(c) an ordered pair (α, β), for some ordinals $\alpha > 1$ and $\beta > 0$.

To represent these expressions in written form (so far as this is possible), we adopt the following five conventions:

1. If x is one of the ordinals $0, \ldots, 10$, then the expression $\langle x \rangle$ (i.e., the 1-sequence that has x as its only element) will be written as '$=$', '$\neg$', '℞', '&', '$\exists$', 'λ', 'ρ', ',', '.', '(', or ')', respectively.

2. If x is an ordered pair $(0, \alpha)$, for some ordinal α, then $\langle x \rangle$ will be written as an unitalicized string (such as 'Socrates') and be called a *constant*.

3. If x is an ordered pair $(1, \alpha)$ for some ordinal α, then $\langle x \rangle$ will be written as an italicized letter (such as 'x' or 'F'), possibly with subscript but *without* superscript, and will be called an *untyped variable*.

4. If x is an ordered pair (α, β) for some ordinals $\alpha > 1$ and $\beta > 0$, then $\langle x \rangle$ will be written as an italicized letter, possibly with subscript, that carries as superscript some representation of β (typically a numeral), and will be called a *typed variable of type β*.

5. Unabbreviated expressions of any length *greater* than 1 will be written by concatenating representations of expressions of length 1.

Some expressions of $\mathcal{L}$ will be called 'terms'. An *atomic* term is either a variable or a constant; and variables are further divided into typed and untyped, as indicated in conventions 3 and 4.

The *non*-atomic terms of $\mathcal{L}$ fall into three kinds: formulas, lambda-expressions, and rho-expressions. (The reason why formulas are here counted as 'terms' is that they serve as *names*, in particular of states.) Their syntax can be recursively specified as follows:

(i) For any terms t and u: $\ulcorner (t = u) \urcorner$ is a formula.

(ii) For any term t: the expressions $\ulcorner \neg t \urcorner$ and $\ulcorner ℞\, t \urcorner$ are formulas.[40]

(iii) For any (possibly empty) sequence of terms $t_1, t_2, \ldots$: $\ulcorner \&(t_1, t_2, \ldots) \urcorner$ is a formula.

[40]The unfamiliar symbol '℞' will here serve as a 'rolehood operator': for any entity x, the state ℞ x is the rolehood of x, which obtains iff x is a role. (Cf. Definition 3.1 above and clause (v) of the semantics given in Section 4.2 below.) By contrast, we do not have an analogous 'statehood operator', because none is really needed: with the help of the variadic conjunction operator '&', which will be introduced in the next clause, the thought that x is a state of some order less than 5 (say) can be expressed by the formula '$\exists y^5\, (x = \&(y^5))$'. More could be said about the reasons for not having a *perfectly general* way to express statehood, but this will have to be left for another paper.

(iv) For any (one or more) pairwise distinct variables $v_1, v_2, \ldots$ and any term t: $\ulcorner \exists v_1, v_2, \ldots\ t \urcorner$ is a formula, and $\ulcorner \lambda v_1, v_2, \ldots\ t \urcorner$ is a lambda-expression.

(v) For any ordinal $\alpha > 0$ and any α-sequences of terms $t_1, t_2, \ldots$ and $u_1, u_2, \ldots$: $\ulcorner (t_1 u_1, t_2 u_2, \ldots) \urcorner$ is a formula.

(vi) For any (two or more) pairwise distinct variables $v_1, v_2, \ldots$ and any term t: $\ulcorner \rho v_1 . v_2, v_3, \ldots\ t \urcorner$ is a rho-expression.

(vii) Nothing else is a formula, lambda-expression, or rho-expression.

This syntax is admittedly quite liberal: even 'nonsensical' expressions like '$\exists x\, \lambda y\, z$' count as terms. However, it is not the purpose of $\mathcal{L}$'s syntax to sort sense from nonsense; this task instead falls to the semantics, which is specified in the next subsection.

$\mathcal{L}$ provides exactly three variable-binding operators: '$\exists$', 'λ', and 'ρ'. Talk of 'free' and 'bound' variable-occurrences should be understood accordingly. In addition, we will say that a term occurs *at sentence-position* iff it occurs as an operand of either '$\neg$' or '&' or as the matrix of a term starting with '$\exists$', 'λ', or 'ρ', and that an occurrence of a term t stands *at predicate-position* iff it is the first occurrence of t in an occurrence of a term pair $\ulcorner tu \urcorner$ (for some term u).

It will be convenient to use several abbreviatory devices, apart from ellipses. '$\top$' and '$\bot$' will abbreviate '$\&()$' and '$\neg\&()$', respectively, and where t and u are any terms, we will write $\ulcorner (t \neq u) \urcorner$ to abbreviate $\ulcorner \neg(t = u) \urcorner$. Further, for any sequence of terms $t_1, t_2, \ldots$, of a length greater than 1, $\ulcorner (t_1 \wedge t_2 \wedge \ldots) \urcorner$ and $\ulcorner (t_1 \vee t_2 \vee \ldots) \urcorner$ will respectively abbreviate $\ulcorner \&(t_1, t_2, \ldots) \urcorner$ and $\ulcorner \neg\&(\neg t_1, \neg t_2, \ldots) \urcorner$. Finally, the outermost parentheses of any term $\ulcorner (tu) \urcorner$ will usually be omitted, provided that this does not create ambiguity. For example, instead of '$((Fx) \wedge (Gy))$', we will just write '$(Fx \wedge Gy)$'.

4.2 Semantics

To specify the semantics (or 'intended interpretation') of $\mathcal{L}$, we have to indicate which entity (if any) is denoted by a given term relative to a given interpretation and variable-assignment. Here an *interpretation* is a function that is defined on a set of constants, while a *variable-assignment* is a function defined on a set of variables. For brevity's sake, we will write 'denotes$_{I,g}$' instead of 'denotes relative to I and g'.

With these preliminaries, we can specify the semantics of $\mathcal{L}$ by stipulating that, for any interpretation I and any variable-assignment g, the following holds:

(i) For any constant c and any x: c denotes$_{I,g}$ x iff I maps c to x.

(ii) For any variable v and any x: v denotes$_{I,g}$ x iff g maps v to x and, if v is typed, then x is of some order *less* than the type of v.[41]

[41] The concept of an entity's *order* will be defined in the next subsection.

(iii) For any terms t and u, and any x: $\ulcorner(t = u)\urcorner$ denotes$_{I,g}$ x iff, for some entities y and z, t denotes$_{I,g}$ y, u denotes$_{I,g}$ z, and x is an identity of y and z.[42]

(iv) For any term t and any x: $\ulcorner\neg t\urcorner$ denotes$_{I,g}$ x iff t denotes$_{I,g}$ a state of which x is a negation.

(v) For any term t and any x: $\ulcorner\mathrm{R}\, t\urcorner$ denotes$_{I,g}$ x iff t denotes$_{I,g}$ an entity of which x is a rolehood.

(vi) For any ordinal α, any α-sequence of terms $t_1, t_2, \ldots$, and any x: $\ulcorner\&(t_1, t_2, \ldots)\urcorner$ denotes$_{I,g}$ x iff there exists an α-sequence of states $s_1, s_2, \ldots$ such that: each t_i denotes$_{I,g}$ the corresponding s_i, and x is a conjunction of $s_1, s_2, \ldots$.

(vii) For any (one or more) pairwise distinct variables $v_1, v_2, \ldots$, any term t, and any x: $\ulcorner\exists v_1, v_2, \ldots\ t\urcorner$ denotes$_{I,g}$ x iff $\ulcorner\lambda v_1, v_2, \ldots\ t\urcorner$ denotes$_{I,g}$ an attribute of which x is an existential quantification.

(viii) For any ordinal $\alpha > 0$, any α-sequence of terms $t_0, t_1, \ldots$, any α-sequence of terms $u_0, u_1, \ldots$, and any x: $\ulcorner(t_0 u_0, t_1 u_1, \ldots)\urcorner$ denotes$_{I,g}$ x iff there are an α-sequence of roles $r_0, r_1, \ldots$ and an α-sequence of entities $y_0, y_1, \ldots$ such that: each t_i denotes$_{I,g}$ the corresponding r_i, each u_i denotes$_{I,g}$ the corresponding y_i, and x results from the role assignment $\{(r_i, y_i) \mid i < \alpha\}$.

(ix) For any ordinal $\alpha > 0$, any α-sequence of pairwise distinct variables $v_1, v_2, \ldots$, any term t, and any x: $\ulcorner\lambda v_1, v_2, \ldots\ t\urcorner$ denotes$_{I,g}$ x iff x is an α-adic attribute such that, for any α-sequence of entities $y_1, y_2, \ldots$ and any state s: s is an instantiation of x by $y_1, y_2, \ldots$, in this order, iff, for some variable-assignment h that is just like g except for mapping each v_i to the corresponding y_i, s is denoted$_{I,h}$ by t.[43]

(x) For any ordinal $\alpha > 1$, any α-sequence of pairwise distinct variables $v_0, v_1, \ldots$, and any term t, the following two conditions are satisfied:

1. The rho-expression $\ulcorner\rho v_0.v_1, v_2, \ldots\ t\urcorner$ has a denotation relative to I and g *only* if the lambda-expression $\ulcorner\lambda v_0, v_1, \ldots\ t\urcorner$ denotes$_{I,g}$ a relation.
2. If $\ulcorner\lambda v_0, v_1, \ldots\ t\urcorner$ denotes$_{I,g}$ a relation R, then there exists a sequence $\sigma \in R$ such that the following holds for each $i < \alpha$:

   (∗) Let $u_1, u_2, \ldots$ be pairwise distinct variables such that $\{u_1, u_2, \ldots\} = \{v_0,\ v_1, \ldots\} \setminus \{v_i\}$. Then, for any x: $\ulcorner\rho v_i.u_1, u_2, \ldots\ t\urcorner$ denotes$_{I,g}$ x iff $x = \sigma(i)$.

[42] I say '*an* identity' (and not '*the* identity') in order not to build ontological presuppositions into the semantics of $\mathcal{L}$. However, by an assumption to be adopted in Section 5.2.2, there exists, for any entities y and z, only *one* identity of y and z. Similar remarks apply to the next five clauses.

[43] Given the assumption (U2) of Section 5.1.1 below (according to which no two attributes are trivial converses of each other), there will be at most one such attribute.

As may be seen from this last clause ($*$), the order of the u_i in a given rho-expression $\ulcorner \rho v.u_1, u_2, \ldots\ t \urcorner$ does not matter for its denotation. By contrast, it *does* matter which variable precedes that list (which is why it is separated from the rest by a dot rather than a comma). For example, suppose that the triadic relation of *giving* is the set $\{\langle G, g, r\rangle\}$, where G, g, and r are the roles of *Giver*, *Gift*, and *Recipient*. Then G may be denoted by both '$\rho x.y, z\,(Gx, gy, rz)$' and '$\rho x.z, y\,(Gx, gy, rz)$'. The immediate purpose of rho-expressions is to allow reference to the members of the role sets of non-fundamental relations. There will be more to be said about the need for them in Section 5.3 below.

4.3 The assignment of orders

From clause (ii) of the above semantics, it follows that a typed variable can only denote entities of some order less than the variable's type. Here an 'order' is simply an ordinal number. The criterion by which entities are assigned their respective orders is given by the following definition:[44]

Definition 4.1. For any ordinal α, an entity x is *αth-order* (alternatively: x is of order α) iff x is either a role or a state and α is the least ordinal β such that, for some interpretation I, variable-assignment g, and term t, the following three conditions are satisfied:

(i) t denotes$_{I,g}$ x.

(ii) For any atomic term u: if either $t = u$ or u has in t a free occurrence at predicate- or sentence-position, then u denotes$_{I,g}$ a fundamental entity.

(iii) For any variable v: if v has in t a bound occurrence at predicate- or sentence-position, then v is typed, and its type is less than or equal to β.

Note that *particulars*—entities that are neither roles nor states—fall outside of the order-theoretic hierarchy thus defined: they are not assigned any order at all.[45] By contrast, any fundamental role or state is classified as zeroth-order. This is because, under the previous subsection's semantics, anything at all can be denoted by an untyped variable. Thus, if x is a fundamental role or state, and if t is an untyped variable that is under g mapped to x, then any interpretation I will, together with g and t, satisfy the definition's three numbered clauses for $\beta = 0$.

[44]The ultimate purpose of this definition is to prevent the occurrence of 'vicious circles' by which the question of whether a given state obtains would lead back to itself, in the manner of an Epimenidean paradox. For more detailed discussion, see Plate (2025: §4).

[45]In this way, the present definition differs from Definition 16 in Plate (2025: §4.1). This change is motivated by a desire to leave statements as to how many particulars there are inexpressible in $\mathcal{L}$. If such statements *were* expressible in $\mathcal{L}$, they would under assumption (U4) (Section 5.1.2 below) be identical with $\top$ and in this sense 'logically necessary', which would be implausible. Another difference is that clause (ii) of the present definition speaks of 'a fundamental entity' instead of 'either a particular or a fundamental intensional entity'. This is due to the fact—which will become apparent in Section 5.3—that particulars are under the present view classified as fundamental.

Before we leave this section, let us quickly define two further concepts. In the next section, these will play an important role in the individuation of states:

Definition 4.2. A term t *analytically entails* a term t' iff the following two conditions are satisfied:

(i) t denotes a state relative to some interpretation and variable-assignment.[46]

(ii) For any interpretation I and variable-assignment g: if t denotes$_{I,g}$ a state, then so does t', and if t denotes$_{I,g}$ an *obtaining* state, then t' denotes$_{I,g}$ an obtaining state, as well.

Definition 4.3. A state s *$\mathcal{L}$-necessitates* a state s' iff there exist an interpretation I, a variable-assignment g, and terms t and t' of $\mathcal{L}$ such that: t denotes$_{I,g}$ s, t' denotes$_{I,g}$ s', and t analytically entails t'.

As might be expected, any state $\mathcal{L}$-necessitates itself. This can be seen from the fact that any state whatsoever can be denoted by an untyped variable, together with the fact that any such variable analytically entails itself.

5 Ontological Assumptions

The ontological assumptions of ORS can be divided into three groups, which will below be presented in their own respective subsections. We begin with four *uniqueness* claims (Section 5.1), followed by seven *existence* claims (Section 5.2) and, lastly, four somewhat miscellaneous assumptions.[47] The theory's mathematical bedrock, already made use of above, consists of ZFC with urelements. Some of the assumptions' consequences (and reasons for their adoption) will be pointed out along the way.

5.1 Uniqueness claims

The four *uniqueness* claims of ORS can be naturally divided into two pairs, the first two being concerned with the individuation of roles and the last two governing the individuation of states.

5.1.1 Roles

Our first assumption simply states that

(U1) No two roles are equivalent.

[46]The purpose of this clause is purely cosmetic. It serves to prevent terms that do not denote a state relative to *any* interpretation and variable-assignment (such as lambda-expressions) from being classified as 'analytically entailing' any term whatsoever.

[47]Most of the 'existence claims' will also contain assertions of uniqueness. For instance, the assumption (E5) will be to the effect that every state has *exactly* one negation.

Among other things, this means that, for any roles r_1 and r_2, the following holds: if the set $\{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$ is a relation, then $r_1 = r_2$.[48] (U1) can be motivated by considerations of ontological economy: why have two roles when one is enough? Relatedly, it can be motivated by explanatory considerations. For, while the equivalence of *two* roles with each other may call out for an explanation, the equivalence of a single role with *itself* does not.

Next, we have a broadly analogous principle that governs the individuation of attributes:

(U2) No two attributes are trivial converses of each other.

From Definition 3.14, it can be seen that every attribute is a trivial converse of itself. (U2) accordingly ensures that every attribute has itself as its *only* trivial converse. In this way, the individuation of attributes is effectively tied to that of states. For instance, if every state s is ts own double negation $\neg\neg s$, then (U2) will have the consequence that every property P is identical with *its* double negation $\lambda x\, \neg\neg(Px)$.[49] For suppose that every state is its own double negation. Then, for any state s and any x, the following holds: s is an instantiation of P by x iff s is an instantiation of $\lambda x\, \neg\neg(Px)$ by x. But this just means that $\lambda x\, \neg\neg(Px)$ is a trivial converse of P; and so, by (U2), the former is nothing else than P itself.[50]

We can further infer:

Proposition 5.1. Any converse of a relation R has the same role set as R.

For let R be any α-adic relation (for some $\alpha > 1$), and let R' be any converse of R. By the definition of 'converse', R' will then be an f-converse of R, for some permutation f on α. Indeed, given (U2), R' will be the *only* f-converse of R: for if there were two, they would be trivial converses of each other. Meanwhile, with the help of Definition 3.7 it can be seen that the set $\{\sigma \circ f \mid \sigma \in R\}$ is an f-converse of R. Hence we have that $R' = \{\sigma \circ f \mid \sigma \in R\}$; and so the role set of R' is the same as that of R, which was to be shown.

[48] To see this, let r_1 and r_2 be any roles such that $R := \{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$ is a relation. By the definitions of 'relation' and 'adicity', R then has an instantiation by some sequence of entities of a length greater than 1. Given our definition of 'instantiation' (and given that $R = \{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$), this means that:

(1) R is a proto-relation; and

(2) For some entities x_1 and x_2, there exists a state that results from $\{(r_1, x_1), (r_2, x_2)\}$.

From (1), together with the fact that $R = \{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$, we can infer by the definition of 'proto-relation' that the sequence $\langle r_1, r_2\rangle$ is instantiation-equivalent to $\langle r_2, r_1\rangle$. And from this, together with (2), it follows that r_1 is equivalent to r_2 (cf. Definition 3.10 above). From (U1) it can now be inferred that $r_1 = r_2$, as required.

[49] Needless to say, the concept of negation that applies to states is not the same as the one that applies to properties.

[50] Proponents of *higher-order metaphysics* speak in this connection of 'functionality'; cf. Dorr (2016: 100), Bacon and Dorr (2024: 121). For historical precedent, see Ramsey (1925/1931: 35)

5.1.2 States

It remains to introduce two uniqueness claims for states. The first of these should be uncontroversial:

(U3) No two states result from the same role assignment.

Given Definition 3.7, this has the welcome consequence that no attribute has more than one instantiation by any given entity or sequence of entities. Armed with (U1) and (U3), we can moreover give an argument—which I delegate to a footnote—for the conclusion that role assignments have to be thought of as (generalized) multisets rather than sets.[51]

Our final uniqueness claim captures the idea that states—and hence, given (U2), also roles—are at least *moderately* coarse-grained: 'moderately' because it should *not* be required that any two classically equivalent formulas, such as '(Sappho = Sappho)' and '(Socrates = Socrates)', denote, if anything, the same state. For, at least on the face of it, Sappho's self-identity does not require that Socrates exists, and nor does Socrates' self-identity require that Sappho exists. However, if two formulas are equivalent in the stronger sense of *analytically* entailing each other, then the principle will plausibly require that they denote (if anything) the same state. For example, consider the formulas '(Socrates = Socrates)' and '¬(Socrates ≠ Socrates)': these analytically entail each other, and there seems to be little point in drawing a distinction between the states they respectively denote. (Intuitively, a world in which Socrates fails to be self-distinct is no different from a world in

[51]Let R be the triadic relation whose instantiation by any entities x, y, and z, in this order, is the conjunction of the following three states: x and y love each other, x and z love each other, and y and z love each other. Since the relevant concept of conjunction is *unordered*, the order of R's relata does not matter: an instantiation of R by Dee, Pat, and John, in this or any other order, is simply the state that Dee, Pat, and John love each other. (Similarly, of course, for any other choice of relata.) By Definition 3.7, an instantiation of R by Dee, Pat, and John, in this order, will be a state that results from the assignment $\{(r_1, \text{Dee}), (r_2, \text{Pat}), (r_3, \text{John})\}$, for some not necessarily pairwise distinct roles r_1, r_2, r_3. But since the order of the relata does not matter, any two of these roles will be *equivalent* to each other. By (U1), this means that they are all one and the same role r, and our relation R turns out to be the singleton $\{\langle r, r, r\rangle\}$.

Let now s be R's instantiation by Dee, Dee, and Pat, and let s' be R's instantiation by Dee, Pat, and Pat. By Definition 3.7 again, s results from the assignment $\{(r, \text{Dee}), (r, \text{Dee}), (r, \text{Pat})\}$; and by what has been said above, it is the state that Dee and Dee, and Dee and Pat, love each other. (One could also call it the state that Dee loves both herself and Pat and is moreover loved by Pat.) Analogously, s' results from $\{(r, \text{Dee}), (r, \text{Pat}), (r, \text{Pat})\}$ and is the state that Pat and Pat, and Pat and Dee, love each other. Evidently, s and s' are distinct from each other: one of them might obtain while the other doesn't. By (U3), it then follows that they do *not* result from the same role assignment. But we also said that s results from $\{(r, \text{Dee}), (r, \text{Dee}), (r, \text{Pat})\}$ while s' results from $\{(r, \text{Dee}), (r, \text{Pat}), (r, \text{Pat})\}$. If now these assignments were *sets*, then they would be one and the same set, since they have the same members; and so s and s' would result from the same assignment, contradicting (U3). This shows that we have to think of role assignments as (generalized) *multi*sets: we have to allow them to contain a given role–entity pair 'more than once'. (As for the need for *generalized* multisets, see footnote 29 above.)

which Socrates is self-identical.) Similarly in other cases. Using the concept of $\mathcal{L}$-necessitation, we can express this thought as follows:[52]

(U4) No two states $\mathcal{L}$-necessitate each other.

Given that any state $\mathcal{L}$-necessitates its own double negation and *vice versa*, (U4) has the plausible consequence that any state is identical with its own double negation. And from this, as already mentioned, (U2) allows us to infer that any given property P is identical with $\lambda x \neg\neg(Px)$. Arguably, this is all as it should be. For instance, it seems intuitively plausible that the property of *failing to fail to be red* is nothing else than that of *being red*.[53]

A possibly more surprising consequence of (U4) is the following:

Proposition 5.2. For any roles r_1 and r_2: if r_1 is a correlate of r_2, then the state $(r_1 = r_1)$ is identical with $(r_2 = r_2)$.

In other words, the self-identity of any given role is *the very same state* as the self-identity of any of its correlates. This result stems from the fact that $\mathcal{L}$ contains rho-expressions, which allow any given (proper) role to be 'analyzed' in terms of its correlates. The proof is delegated to a footnote.[54] A critic might suspect that this result is merely a linguistic artifact, rooted in the particular way that $\mathcal{L}$ has been set up. This charge would be justified if the decision to include rho-expressions were arbitrary and unmotivated. In fact, however, rho-expressions are needed because of an assumption to be adopted later in this section—specifically, the 'analyzability assumption' (O3) (Section 5.3). We will return to this below.

5.2 Existence claims

The existence claims of ORS can, like its uniqueness claims, be divided into two groups. The first two posit roles, whereas the five remaining ones are concerned with various kinds of states.

[52]The concepts of analytic entailment and $\mathcal{L}$-necessitation have both been defined at the end of Section 4.3 above.

[53]For contrasting views, see, e.g., Zalta (1988), Menzel (1993).

[54]Let r_1 and r_2 be any two roles that are correlates of each other, such that the set $\{\langle r_1, r_2\rangle\}$ is a relation. (The argument will generalize straightforwardly to more complex cases.) Further, let g be a variable-assignment that maps 'r_1' and 'r_2' to, respectively, r_1 and r_2. Then r_1 and r_2 will be respectively denoted$_{\emptyset,g}$ by the rho-expressions '$\rho x.y\,(r_1 x, r_2 y)$' and '$\rho y.x\,(r_1 x, r_2 y)$'. Let now φ be the formula '$(\rho x.y\,(r_1 x, r_2 y) = \rho x.y\,(r_1 x, r_2 y))$', and let ψ be the formula '$(\rho y.x\,(r_1 x, r_2 y) = \rho y.x\,(r_1 x, r_2 y))$'. It can then be seen—since the variables that occur free in φ also occur free in ψ, and *vice versa*—that φ and ψ analytically entail each other. But φ denotes$_{\emptyset,g}$ the self-identity of r_1, while ψ denotes$_{\emptyset,g}$ the self-identity of r_2. Consequently, these states $\mathcal{L}$-necessitate each other, which means, by (U4), that they are one and the same state. So, for any roles r_1 and r_2 that are such that the set $\{\langle r_1, r_2\rangle\}$ is a relation, the state $(r_1 = r_1)$ is identical with $(r_2 = r_2)$. By a similar argument, the same can be shown for *any* roles that are correlates of each other.

5.2.1 Roles

At first blush, drawing a distinction between fundamental and non-fundamental entities does not require positing a property of fundamentality. However, if it is not nonsense (or otherwise semantically defective) to say of a given entity x that it is fundamental, then one may reasonably expect there to be a state of affairs to the effect that x is fundamental; and a natural way to think of such a state is to regard it as the instantiation by x of the property of *being a fundamental entity*. In this way, one is naturally led to the conclusion that there exists a property of fundamentality, after all.[55] Here we will assume not only that there is such a property, but also that it *itself* is fundamental:

(E1) There exists exactly one fundamental property F such that, for any x, there exists a state s that satisfies the following two conditions:

- (i) s is an instantiation of F by x.
- (ii) s obtains iff x is fundamental.

The symbol 'F' will in the following be used as a convenient name of the property in question. We can thus say that one of (E1)'s most obvious consequences is the thesis that the state FF—i.e., the instantiation of F by itself—obtains. In other words, fundamentality is fundamental. Clearly, this is a controversial thesis; I offer a defense in my (MS(b)).[56]

While (E1) generates ontological commitment to only a single attribute, our next assumption generates commitment to many:

(E2) For any interpretation I, variable-assignment g, term t, ordinal $\alpha > 0$, and any α-sequence of pairwise distinct variables $v_0, v_1, \ldots$: if t denotes$_{I,g}$ a state and, for each $i < \alpha$, the following two conditions are satisfied—

- (i) v_i occurs free in t.
- (ii) If v_i is identical with t or has in t a free occurrence at predicate- or sentence-position, then v_i is typed.

—then there exists at least one attribute A such that, for any α-sequence of entities $x_0, x_1, \ldots$ and any state s, the following holds: s is an instantiation of A by $x_0, x_1, \ldots$, in this order, iff, for some variable-assignment h that is just like g except for mapping each v_i to the corresponding x_i, s is denoted$_{I,h}$ by t.

[55] A skeptic about properties might here retort that, by a similar argument, we may be led to the unwelcome conclusion that there exists a property of non-self-instantiation. To this it can be replied that indeed there is such a property—only that, instead of a single, *perfectly general* property $\lambda x \neg(xx)$ (which would lead to Russell's paradox), we have an infinite series of non-self-instantiation properties $\lambda x^1 \neg(x^1x^1)$, $\lambda x^2 \neg(x^2x^2)$, and so on, which are limited in their respective 'ranges of application'.

[56] The question of whether fundamentality is fundamental has as its counterpart in a Lewisian setting the question of whether *perfect naturalness* is perfectly natural. This has been answered in the negative by Thompson (2016), whereas Sider (2011: §7.13), who operates with a more general notion of *structuralness*, gives a positive answer. Cf. also Wilson (forthcoming).

Given our earlier assumption (U2)—which rules out the existence of attributes with more than one trivial converse—the 'at least one' in the consequent of (E2)'s main conditional may also be read as 'exactly one'. The second numbered clause of the antecedent deserves special mention: it is responsible for the fact that (E2) does not, for example, commit us to there being a property $\lambda x \neg(xx)$, which would be a zeroth-order property of non-self-instantiation.[57]

Since $\mathcal{L}$ contains infinitely many variables and allows formulas to be infinitely long, many of those infinitely long formulas will contain free occurrences of infinitely many variables. If any such formulas manage to denote states (relative to some interpretations and variable-assignments),[58] then (E2) will accordingly generate ontological commitment to infinitary relations. I do not think that this is extravagant. Once one accepts that there are relations at all, it becomes difficult to see why all relations should be finitary. Now possibly it might be denied (as some have) that there *are* any relations. Here is not the place to discuss this question in detail. Pretty obviously, however, there are many things, related to each other in countless different ways, many of which are quite complex (cf. Blanshard [1984: 211f.]). It is just these ways of being related that we commonly talk about when we talk about relations; and so I take it that the question is not whether there are any, but how we might best conceive of them.

5.2.2 States

We now come to a handful of existence (and uniqueness) claims for states, corresponding to five of the nine primitive concepts introduced in Section 3.1 above:

(E3) For any entity x, there exists exactly one rolehood of x.[59]

(E4) For any entities x and y, there exists exactly one identity of x and y.

(E5) For any state x, there exists exactly one negation of x.

(E6) For any (possibly empty) set of states $\{s_1, s_2, \dots\}$, there exists exactly one conjunction of the s_i.

(E7) For any attribute A, there exists exactly one existential quantification of A.

In the presence of (E2), these assumptions help generate a plenitudinous ontology of roles. In defense of this plenitude, one can point to semantic considerations. We sometimes have occasion to make general claims as to what entities there are; an example would be the claim that F has at least one fundamental property. Now this claim can in $\mathcal{L}$ be expressed by the formula '$\exists x^1\, (x^1\mathsf{F} \wedge \mathsf{F}x^1)$', which, relative to an interpretation that maps 'F' to F, denotes the existential quantification of the

[57]See Plate (2025: §4.2) for related discussion.

[58]This is where the assumption (E3)–(E7) below become relevant.

[59]Recall that the 'rolehood of x' can be thought of as the state of affairs that x is a role.

higher-order property $\lambda x^1\,(x^1\mathsf{F} \wedge \mathsf{F}x^1)$. Naturally, if there were no such property, there would also be no existential quantification of it; and this would (implausibly) leave the mentioned claim without a truth-condition.[60]

5.3 Four other assumptions

Finally, we have four somewhat miscellaneous assumptions. These can be naturally grouped into two pairs: the first two create certain divisions, while the last two provide important work for F. The first is also the shortest:

(O1) No role is a state.

This is at least a reasonable default assumption, given that states and roles serve completely different theoretical functions.[61]

Next, we have an assumption that expresses a divide between fundamental and non-fundamental roles:

(O2) No fundamental role is a correlate of a non-fundamental role.

Intuitively, a non-fundamental role is either a property 'generated' by our comprehension axiom (E2) or a member of the role set of a relation so generated. But, plausibly, if it *is* such a member, then every *other* member of the role set in question will also be non-fundamental. It is thus hard to see how there might arise a relation whose role set contains both fundamental *and* non-fundamental roles.

The next assumption is considerably more controversial:

(O3) For any entity x, there exist a variable-assignment g and a term t such that: t denotes$_{\emptyset,g}$ x, and any entity in the range of g is fundamental.

This can be paraphrased as the claim that anything whatsoever is fully analyzable (within $\mathcal{L}$) in terms of fundamental entities.[62] Since complex terms of $\mathcal{L}$ denote nothing other than roles and states, (O3) has the consequence that any particular (i.e., anything that is *not* a role or state) is a fundamental entity. This consequence is *prima facie* counter-intuitive when applied to ordinary material objects such as tables and chairs, since one would usually not think of these things as fundamental, and would also not think of them as roles or states. This worry can be addressed in at least three ways.

First, it may be argued that ordinary material objects are not, in fact, particulars but rather *states* of a certain special sort. In particular, one might argue that they

[60]See Fine (2010: 587) for a similar argument. Another argument for the existence of complex properties, based on the idea that sets can be thought of as a special sort of property, can be found in Plate (2025: §6.1).

[61]For a possible way to motivate the idea that these two categories nonetheless overlap, see Dixon (2018: §6).

[62]Broadly similar principles include Sider's (2011: 116) 'Completeness' and Bacon's (2020: 566) 'Fundamental Completeness'.

are best thought of as events, and that events, in turn, are nothing but obtaining states (though, of course, not every obtaining state will count as an event).[63] Second, it might be argued that, strictly speaking, there are no ordinary material objects to begin with.[64] Third, one might defuse the worry by adding to $\mathcal{L}$ a term-forming operator that allows ordinary material objects to be denoted in terms of entities that *are* fundamental (such as, perhaps, their subatomic parts). On this third route, (O3) would no longer have the consequence that every particular is fundamental. But this result would come at the cost of our having to increase our burden of primitive notions, for example by a concept of parthood, in order to specify the semantics of that extra operator. This seems to be a good reason to favor one of the former options.

If (O3) is so controversial, why accept it? The answer lies in the value of what it allows us to prove. In particular, in combination with (O4) below, it allows us to show that every set (conceived of as a property, as indicated in Section 3.5) is a non-fundamental entity.[65] This is a valuable result: a good thing to be able to prove. Now, granted that (O3) can be motivated in some such way, another point worth noting is that (O3) in turn motivates the inclusion of rho-expressions in $\mathcal{L}$. For suppose there were none. Given (O1), there would then be no way to denote any role that is not a property, except by way of a variable or constant.[66] From (O3), it would then follow that any such role is fundamental. But this result would be wildly implausible. For example, the members of the role set of the dyadic relation $\lambda x, y\, ((x = \text{Sappho}) \vee (y = \text{Socrates}) \vee (x = y))$ are hardly properties (for what would it mean to instantiate either of them?), and they are certainly not fundamental entities. So each of them would constitute a counter-example to (O3)—and indeed, one of many. Hence the need for rho-expressions.

Our final assumption imposes an upper bound on the coarse-grainedness of states, complementary to (U4); one could thus call it a *fine-grainedness principle*. The task of formulating such a principle is less straightforward than one might think. If

[63]For arguments in this direction, see, e.g., Nolan (2012). Regarding the identification of events with obtaining states, see also, e.g., Taylor (1985: ch. 4), Tegtmeier (2000), Kaiserman (2017: §4). I here set aside discussion of a parallel worry about such things as texts and works of music, which might also be cited as counter-examples to the thesis that every particular is a fundamental entity. Arguably, such entities would be less plausibly classified as events than as *properties*—e.g., as types of inscription in the case of texts and as types of performance in the case of works of music. On the latter, see, e.g., Letts (2018). Cf. also Plate (MS[a]).

[64]This eliminativist stance has been defended by, e.g., Merricks (2001) and Benovsky (2018). Korman and Barker (2025: §2) provide a systematic overview of eliminativist (and generally 'anti-conservative') arguments. Here it might be worth emphasizing that an eliminativist view about ordinary material objects does not contradict my above assertion (at the end of Section 5.2.1) that "[p]retty obviously" there are many interrelated "things". For, by 'things', I meant nothing more specific than entities, which include such things as events and numbers.

[65]For the details, see **plateTO**.

[66](O1) is relevant here because it rules out that any role can be denoted by a *formula*, given that anything denoted by a formula is a state. (Meanwhile, anything denoted by a *lambda-expression* is a property, given that every relation is a property. Without rho-expressions, the only expressions by which non-property roles could be denoted would thus be constants and variables.)

we simply took the converse of (U4) and said that any states that fail to $\mathcal{L}$-necessitate each other are distinct, we would in effect be saying nothing more than that every state $\mathcal{L}$-necessitates itself, which is trivially true. To formulate a *non*-trivial fine-grainedness principle, we can instead impose a constraint on *what pairs of terms denote the same state*. The basic idea is that, for any terms t and t' and any variable-assignment g, the following holds: *if* t and t' denote (relative to g and the empty interpretation[67]) the same state, and if certain further conditions are satisfied, *then* t analytically entails t'. More fully:

(O4) For any variable-assignment g, any terms t and t', and any state s: if the following five conditions are satisfied—

(i) t and t' both denote$_{\varnothing,g}$ s.

(ii) Every entity in the range of g is fundamental.

(iii) No two variables are under g mapped to the same entity.

(iv) Every variable that is under g mapped to a role or state is of type 1.

(v) For any variable v: if v occurs free in t and is under g mapped to a state or proper role, then at least one free occurrence of v in t stands at predicate- or sentence-position.

—then t analytically entails t'.

In combination with (O2), this principle implies that the fundamental entities are 'independent' of each other, in the following sense: for any two fundamental entities x and y (except for roles that are correlates of each other), the state $(x = x)$ is distinct from $(y = y)$.[68,69]

[67]The reason why we are here not considering non-empty interpretations is simply that anything that can be denoted by a constant can also be denoted by a variable. ("Then why has $\mathcal{L}$ been set up in such a way as to contain constants in the first place?" The reason is largely cosmetic: we would like to be able to write something like '$\exists x\,(x = \text{Socrates})$', without having to count something like '$\exists \text{Socrates}\,(\text{Socrates} = x)$' as a well-formed formula.)

[68]This follows from a more general result established, with the help of (O2), in Plate (MS[b]: §2.5).

[69]A slightly different version of (O4) can be found in Plate (MS[c]: §14); the differences concern clauses (iv) and (v). The (MS(c)) version of clause (iv) refers, quite generally, to variables in the domain of g, rather than to variables that are under g mapped to roles or states. The reason for the present narrower formulation lies in the fact that particulars do here not count as zeroth-order entities and are accordingly not denotable by variables of type 1.

As for clause (v), the (MS(c)) version refers to intensional entities rather than, again more narrowly, to states and *proper* roles. To see how the present formulation is motivated, we first have to note that, for any variable v and any variable-assignment g that maps v to an improper role, the following holds:

(i) Any term that contains a free occurrence of v at predicate-position does not denote$_{\varnothing,g}$ anything. (This can be readily seen from the semantics of $\mathcal{L}$.)

(ii) Given (O1), any term that contains a free occurrence of v at *sentence*-position does not denote$_{\varnothing,g}$ anything, either.

Much more could be said about the consequences of (O4). However, my aim in the rest of this paper is not to examine the consequences of ORS, but rather to fill a lacuna.

6 The Special Resultance Question

6.1 What it is and how it arises

Throughout this paper, I have been at pains to avoid presupposing that every property has an instantiation (obtaining or not) by *every* entity. Analogously, I have avoided presupposing that every α-adic relation (for $\alpha > 1$) has an instantiation by every α-sequence of entities. Thus, when describing a dyadic relation R and its converse $\breve{R}$ in Section 2.4 above, I did *not* say:

> For any entities x and y: the instantiation of R by x and y, in this order, is identical with the instantiation of $\breve{R}$ by y and x, in *this* order,

but rather:

> For any entities x and y and any state s: s is an instantiation of R by x and y, in this order, iff s is an instantiation of $\breve{R}$ by y and x, in *this* order.

This second formulation allows that, for some x and y, R simply does not *have* an instantiation by x and y. This caution was in part due to the presence in $\mathcal{L}$ of typed variables and the consequent existence, in the present ontology, of such properties as $\lambda x^1\, x^1$, which have instantiations only by *some* entities. (For instance, $\lambda x^1\, x^1$ has instantiations only by zeroth-order states.[70])

But even apart from such cases, it is often an open question whether a given property has an instantiation by a given entity, or whether a given α-adic relation has an instantiation by a given α-sequence of entities. For example, one can legitimately wonder whether there exists an instantiation of *being green* by the number 9.[71] And in ORS, where we are dealing not just with attributes but with roles more generally, we can ask a correspondingly more general question: Under what conditions does

Consider now the demand imposed by clause (v) of (O4). If the antecedent of this clause's main conditional had 'an intensional entity' instead of 'a state or proper role', then, in those cases in which some variable that occurs free in t is under g mapped to an improper role, the condition would in effect entail that t lacks a denotation relative to g. This would in turn mean that, for any term t that contains a free occurrence of a variable that is under g mapped to an improper role, conditions (i) and (v) *are not both satisfied*. Thus, if we wanted to know whether the self-identity of a given improper role was the same state as $\top$, (O4) would not give us the expected negative answer. This would be unwelcome.

[70]By Definition 3.7, combined with the meaning postulate (M4) (Section 3.1 above), any instantiation of $\lambda x^1\, x^1$ by an entity x is a state. Further, by clauses (ii) and (ix) in Section 4.2, any given state s is an instantiation of $\lambda x^1\, x^1$ by an entity x iff x is identical with s as well as zeroth-order. Consequently, any instantiation of $\lambda x^1\, x^1$ is an instantiation by a zeroth-order state.

[71]See Magidor (2013: ch. 4) for related discussion.

a role assignment have a state resulting from it? With a nod to van Inwagen (1990), one might call this the 'Special Resultance Question' (SRQ). Despite the name, it is of course a rather general question.

The following is an important sub-question of SRQ:

(Q) Under what conditions are two roles correlates of each other?[72]

Plausibly, not just any old roles are correlates of each other! We may safely assume that *Lover* and *Beloved* are correlates, but *Lover* and *Perceiver* seem to be an exceedingly likely case of non-correlates. We can imagine two people being related as lover and beloved, but what could it possibly mean for them to be related as lover and *perceiver*?

Facts as to what role assignments have states resulting from them presumably observe certain regularities. For example, if there is a state resulting from the assignment $\{(\mathit{Lover}, \text{Desdemona}), (\mathit{Beloved}, \text{Othello})\}$, then there should also exist a state that results from $\{(\mathit{Lover}, \text{Desdemona}), (\mathit{Beloved}, \text{Cassio})\}$ (whether or not the latter obtains); and similarly in other cases. But what are those regularities, exactly? In the *Simple Theory of Types*, the analogous question is typically understood to have a straightforward answer: there exists a state of affairs (or 'proposition') resulting from the application of an attribute A to a sequence of entities $x_1, x_2, \ldots$ just in case A is of a type $\langle \tau_1, \tau_2, \ldots \rangle$ such that each τ_i is the type of the corresponding x_i.[73] It would be desirable if we could formulate a similarly elegant principle, or set of principles, that would help to answer the Special Resultance Question.

6.2 How it might be answered

The proposal I would like to put forward in the rest of this section falls into two unequal parts, the first concerned with *fundamental* roles, the second with non-fundamental ones.

Regarding the former, it is at least a natural working hypothesis that the fundamental properties and relations—i.e., those properties that are fundamental entities and those relations whose role sets contain only fundamental entities—are 'transcendental' in the sense of having instantiations by *any* entities (or appropriately long

[72]In other words: under what conditions are two roles r_1 and r_2 such that, for some entities x and y, there exists a state resulting from a role assignment that contains both (r_1, x) and (r_2, y)? (Cf. Definition 3.9.)

[73]As Church writes, using a slightly different notation for his types:

> In the interpretation of the theory it is intended that the subscript shall indicate the type of the variable or constant, o being the type of propositions, ι the type of individuals, and $(\alpha\beta)$ the type of functions of one variable for which the range of the independent variable comprises the type β and the range of the dependent variable is contained in the type α. (1940: 56f.)

For present purposes, the important point to note here is that the functions in question are *total* functions.

sequences of entities), regardless of their ontological category. In terms of states resulting from role assignments, this can be phrased as follows:

(R1) For any ordinal α and any α-sequence of fundamental roles $r_0, r_1, \ldots$: if there exists a state that results from $\{(r_i, x_i) \mid i < \alpha\}$ for *some* entities $x_0, x_1, \ldots$, then, for *every* α-sequence of entities $y_0, y_1, \ldots$, there exists a state that results from $\{(r_i, y_i) \mid i < \alpha\}$.

In other words, every fundamental property has an instantiation by any entity whatsoever; and every fundamental α-adic *relation* has an instantiation by any α-adic sequence of entities. This is a rather sweeping assumption, but not, I think, implausible.

Let us now turn to *non*-fundamental roles. The following definition will be useful:

Definition 6.1. An entity x is *assignable* to a given role r iff there exists a state that results from some role assignment containing the pair (r, x).

Our assumption (O3) tells us that any non-fundamental role is analyzable in terms of fundamental entities. Now, such an analysis—i.e., a complex term of $\mathcal{L}$ by which the role in question can be denoted—will already tell us, sometimes in combination with (R1), what entities are assignable to it. For example, if the role in question is denoted by '$\rho x^1.y\,(x^1 y)$', then we can infer that at least *some* of the entities assignable to it are zeroth-order properties.[74] And similarly, if a role is denoted by '$\rho x.y\,((lx, by) \wedge (ly, bx))$' relative to a variable-assignment that maps 'l' to *Lover* and 'b' to *Beloved*, then, assuming that *Lover* and *Beloved* are fundamental, we can infer from (R1) that every entity whatsoever is assignable to it.

However, we are still faced with some open questions—in particular, (Q). As far as *fundamental* roles are concerned, this question will here be left open, and perhaps it is just a brute fact which fundamental roles are correlates of each other. But we can say something more about *non*-fundamental roles.

Consider the properties of *being Sappho* and *being Socrates*, i.e., $\lambda x\,(x = \text{Sappho})$ and $\lambda x\,(x = \text{Socrates})$. Might it not be that these are the very same non-fundamental roles that form the role set of $\lambda x, y\,((x = \text{Sappho}) \vee (y = \text{Socrates}))$? On grounds of ontological parsimony, some may be inclined to answer in the affirmative: in particular, it may seem natural to identify $\lambda x\,(x = \text{Sappho})$ with $\rho x.y\,((x = \text{Sappho}) \vee (y = \text{Socrates}))$, i.e., the 'Sappho-associated' member of that relation's role set.[75] On reflection, however, this move appears dubious. First of all, one would have to wonder why those two properties do not instead form the role set of the *different* relation $\lambda x, y\,((x = \text{Sappho}) \wedge (y = \text{Socrates}))$. And, more importantly, the proposed identification is in tension with (O4). To see this, suppose for the moment

[74] One might be forgiven for thinking that the *only* entities assignable to this role are zeroth-order properties. However, nothing we have said so far rules out that this same role is identical with, for example, $\rho x.y\,(x = y)$, to which *every* entity is assignable (as may be seen from (E4) above in combination with the semantics of rho-expressions).

[75] See Bacon (2024: §12.4) for related discussion in a slightly different setting.

that Sappho and Socrates are particulars (and hence, given (O3), fundamental), and let g be a variable-assignment that maps 's_1' and 's_2' to Sappho and Socrates, respectively. Under the proposed identification, the formulas '$\exists y\,(y = \lambda x\,(x = s_1))$' and '$\exists z\,(z = \rho x.y\,((x = s_1) \vee (y = s_2))$' will then denote$_{\emptyset,g}$ one and the same state, viz., the existential quantification of the property of *being identical with* r, where r is the role in question (namely, *being Sappho*). Suppose that they do. Applying (O4), we can now infer that the first formula analytically entails the second. But clearly, it doesn't—since the second but not the first contains a free occurrence of 's_2'. This should throw at least *some* cold water onto the idea that *being Sappho* and *being Socrates* might be correlates.

Even so, (O4) does not in general offer a decisive verdict on this kind of question. Thus, instead of Sappho and Socrates, consider the states $\top$ and $\bot$. With the help of (O4), both of these states, as well as the properties $\lambda x\,(x = \top)$ and $\lambda x\,(x = \bot)$, can be shown to be non-fundamental.[76] Analogously to above, we can now ask whether the properties $\lambda x\,(x = \top)$ and $\lambda x\,(x = \bot)$ might form the role set of the relation $\lambda x, y\,((x = \top) \vee (y = \bot))$. And on *this* question, (O4) is simply silent. We are thus still faced with a lacuna.

As far as I can see, the most principled stance on this issue is to say that a non-fundamental property does not belong to *any* relation's role set:

(R2) For any non-fundamental property P and any relation R: P is not a member of the role set of R.

So, for example, $\lambda x\,(x = \top)$ is not a member of the role set of $\lambda x, y\,((x = \top) \vee (y = \bot))$, and $\lambda x\,(x = \text{Sappho})$ is not a member of the role set of $\lambda x, y\,((x = \text{Sappho}) \vee (y = \text{Socrates}))$—or of any other relation.

Apart from resolving the above issue in a principled manner, (R2) has a welcome sanitizing effect on our ontology, in that it straightforwardly rules out such apparently spurious relations as $\{\langle s, s\rangle\}$, where s is the property of *being Socrates*. If there were such a relation, we would be faced with the vexing question of what it could possibly mean for an entity to bear $\{\langle s, s\rangle\}$ to itself. Would it mean to be identical with Socrates 'twice over'?

Supposing that (R2) is correct, it will be natural to adopt a similarly restrictive principle with regard to non-fundamental roles that are *not* properties:

(R3) For any non-fundamental role r and any relations R and R': if r is a member of the role sets of both R and R', then R and R' are converses of each other.

For example, consider the identity relation $\lambda x, y\,(x = y)$, which, on the present view, is a set $\{\langle r, r\rangle\}$ for a certain non-fundamental role r. One of the things that (R3) rules out is that the set $\{\langle r, r, r\rangle\}$ is a relation, too; and so it also rules out that, for any entities x, y, and z, there exists a state that results from the role assignment $\{(r, x), (r, y), (r, z)\}$. On the face of it, this is not an implausible result.

[76] Cf. the non-fundamentality results in Plate (2025: §5.5.2), which are based on a similar principle.

A noteworthy feature of (R2) and (R3) is that, with the help of our earlier assumptions (O2) and (O4), these two principles generalize to *all* roles, including fundamental ones. To see this, suppose that a certain fundamental role r is a property, and let g be a variable-assignment that maps 'r^1' to r. The self-identity of r will then be denoted$_{\varnothing,g}$ not only by '$(r^1 = r^1)$' but also by '$(r^1 = \lambda x\,(r^1 x))$'. Suppose further that r is not just a property but also a member of a relation's role set. To take the simplest case, suppose that that relation is the set $\{\langle r, r\rangle\}$. We then have that r is denoted$_{\varnothing,g}$ not only by '$\lambda x\,(r^1 x)$' but also by '$\rho x.y\,(r^1 x, r^1 y)$'. But from this it follows that the self-identity of r is also denoted$_{\varnothing,g}$ by '$(r^1 = \rho x.y\,(r^1 x, r^1 y))$'. So, by (O4), '$(r^1 = \lambda x\,(r^1 x))$' analytically entails this latter formula. But it doesn't, because, as can be seen with the help of (R2), there exists a variable-assignment relative to which '$(r^1 = \lambda x\,(r^1 x))$' denotes a state while '$(r^1 = \rho x.y\,(r^1 x, r^1 y))$' denotes nothing at all.[77] So we have a contradiction; and, given (O2), we get a similar contradiction if we suppose r to be a member of any other relation's role set. But r was *any* fundamental role. We can thus infer the following

Corollary 6.2. No property is a member of any relation's role set.

Moreover, by a completely parallel argument, we can also generalize (R3), which yields

Corollary 6.3. No role is a member of the role sets of two relations that are not converses of each other.

In other words, any two relations with overlapping role sets are converses of each other. If we wish to hold on to both (R2) and (R3) while avoiding these two corollaries, then we might do so by an appropriate weakening of (O4). It is difficult to see, however, how this could be done without making this latter assumption more complex than it is already.

The above principles (R1)–(R3) recommend themselves mainly by their simplicity. In time, some other way of answering the Special Resultance Question may come to seem preferable; but for now I think they can stand as a reasonable set of working assumptions.

7 Conclusion

My principal goal in this paper has been the presentation and partial defense of a theory of intensional entities, specifically an ontology of roles and states. In the previous section, I have moreover suggested a way of expanding the theory by offering an answer to the Special Resultance Question. There is much work left to do, both

[77] For example, let h be a variable-assignment that maps 'r^1' to $\lambda x\,(x = \top)$. Given that $\lambda x\,(x = \top)$ is non-fundamental, it follows from (R2) that this property is not a member of any relation's role set. *A fortiori*, the set $\{\langle T, T\rangle\}$, with $T = \lambda x\,(x = \top)$, is not a relation. This means, in the first place, that the lambda-expression '$\lambda x, y\,(r^1 x, r^1 y)$' does not denote$_{\varnothing,h}$ anything. But then, by clauses (iii) and (x.1) of the semantics of $\mathcal{L}$ (Section 4.2 above), the formula '$(r^1 = \rho x.y\,(r^1 x, r^1 y))$' does not denote$_{\varnothing,h}$ anything, either.

in comparing ORS with rivals and in examining its consequences. Relatedly, it is at present an open question whether this theory, with or without the additions suggested in the previous section, is free from contradictions. But even so, there seems to me good reason to think that *something* like ORS is probably correct. In support of this, let me conclude by briefly enumerating what I see as its main attractions:

1. Its conception of relations provides considerable explanatory benefits.[78]
2. It is sufficiently generous, ontologically speaking, to integrate the set-theoretic universe (interpreted as a class of properties[79]), along with higher-order roles and states.
3. It is not constrained by the rigidity of simple type theory: e.g., it admits self-instantiating properties.
4. At the same time it is attractively austere, in that the only non-fundamental entities it admits are roles and states.
5. It is also *ideologically* austere, in that it employs neither a primitive concept of essence nor any primitive modal or mereological notions.
6. It does justice to the intuitively plausible idea that properties and states (and relations) are at least moderately coarse-grained.
7. It contains, with (O4), a non-trivial fine-grainedness principle that also helps to narrow down the extension of 'fundamental'.

Provided that ORS is consistent, these points should make it seem a promising contender in the arena of available theories.[80]

References

Armstrong, David M. (1978). *A Theory of Universals*. Cambridge University Press.

Bachman, Charles W. and Manilal Daya (1977). The Role Concept in Data Models. *Proceedings of the Third International Conference on Very Large Data Bases*. Vol. 3, pp. 464–76.

Bacon, Andrew (2020). Logical Combinatorialism. *The Philosophical Review* 129, pp. 537–89. DOI: 10.1215/00318108-8540944.

— (2024). *A Philosophical Introduction to Higher-Order Logics*. New York: Routledge.

[78]See Section 2 above.

[79]See Section 3.5 above. The terms 'class' and 'universe' are here used only in an informal sense.

[80]Many of the ideas presented in this paper were developed during a time when I enjoyed financial support from the Swiss National Science Foundation (project 100012_192200, directed by Kevin Mulligan) and institutional support from the University of Italian Switzerland. I am grateful to both, and especially to Mulligan.

Bacon, Andrew and Cian Dorr (2024). Classicism. In: P. Fritz and N. K. Jones (eds.), *Higher-Order Metaphysics*. Oxford: Oxford University Press. Pp. 109–90.

Bader, Ralf M. (2020). Fundamentality and Non-Symmetric Relations. In: D. Glick, G. Darby, and A. Marmodoro (eds.), *The Foundation of Reality: Fundamentality, Space, and Time*. Oxford: Oxford University Press. Pp. 15–45.

Bealer, George (1982). *Quality and Concept*. Oxford University Press.

Benacerraf, Paul (1965). What Numbers Could Not Be. *The Philosophical Review* 74, pp. 47–73.

Benovsky, Jiri (2018). *Eliminativism, Objects, and Persons*. New York: Routledge.

Black, Max (1971). The Elusiveness of Sets. *The Review of Metaphysics* 24, pp. 614–36.

Blanshard, Brand (1984). Bradley on Relations. In: A. Manser and G. Stock (eds.), *The Philosophy of F. H. Bradley*. Oxford: Oxford University Press. Pp. 211–26.

Carnap, Rudolf (1942). *Introduction to Semantics*. Harvard University Press.

— (1947). *Meaning and Necessity: A Study in Semantics and Modal Logic*. Chicago: University of Chicago Press.

Castañeda, Hector-Neri (1975). Relations and the Identity of Propositions. *Philosophical Studies* 28, pp. 237–44.

Church, Alonzo (1940). A Formulation of the Simple Theory of Types. *The Journal of Symbolic Logic* 5, pp. 56–68.

Dixon, T. Scott (2018). Plural Slot Theory. *Oxford Studies in Metaphysics* 11, pp. 193–223. DOI: 10.1093/oso/9780198828198.003.0006.

— (2023). Symmetric Relations. *Philosophical Studies* 180, pp. 3615–39.

— (forthcoming[a]). Directionalism and Relations of Arbitrary Symmetry. *Dialectica* 76, pp. 35–74. DOI: 10.48106/dial.v76.i2.02.

— (forthcoming[b]). How to Be a Postmodal Directionalist. Forthcoming in *Philosophical Studies*.

Donnelly, Maureen (2016). Positionalism Revisited. In: A. Marmodoro and D. Yates (eds.), *The Metaphysics of Relations*. Oxford University Press. Pp. 80–99.

Dorr, Cian (2016). To Be F Is to Be G. *Philosophical Perspectives* 30, pp. 39–134. DOI: 10.1111/phpe.12079.

Fine, Kit (1995). Senses of Essence. In: W. Sinnott-Armstrong (ed.), *Modality, Morality, and Belief*. Cambridge: Cambridge University Press. Pp. 53–73.

— (2000). Neutral Relations. *The Philosophical Review* 109, pp. 1–33. DOI: 10.1215/00318108-109-1-1.

— (2010). Towards a Theory of Part. *The Journal of Philosophy* 107, pp. 559–89. DOI: 10.5840/jphil20101071139.

Gilmore, Cody S. (2013). Slots in Universals. *Oxford Studies in Metaphysics* 8, pp. 187–233. DOI: 10.1093/acprof:oso/9780199682904.003.0005.

Kaiserman, Alex (2017). Causes and Counterparts. *Australasian Journal of Philosophy* 95, pp. 17–28.

Korman, Daniel Z. and Jonathan Barker (2025). Ordinary Objects. In: E. N. Zalta and U. Nodelman (eds.), *The Stanford Encyclopedia of Philosophy*. Spring 2025. Metaphysics Research Lab, Stanford University.

Kuratowski, Kazimierz (1921). Sur la notion de l'ordre dans la Théorie des Ensembles. *Fundamenta Mathematica* 2, pp. 161–71.
Leo, Joop (2008). Modelling Relations. *Journal of Philosophical Logic* 37, pp. 353–85.
— (forthcoming). On Reconciling Positionalism and Antipositionalism. *Dialectica* 76, pp. 75–104. DOI: 10.48106/dial.v76.i2.03.
Letts, Philip (2018). The Property Theory of Musical Works. *Journal of Aesthetics and Art Criticism* 76, pp. 57–69. DOI: 10.1111/jaac.12435.
Lewis, David K. (1986). *On the Plurality of Worlds*. Blackwell.
MacBride, Fraser (2005). The Particular–Universal Distinction: A Dogma of Metaphysics? *Mind* 114, pp. 565–614. DOI: 10.1093/mind/fzi565.
— (2007). Neutral Relations Revisited. *Dialectica* 61, pp. 25–56.
Magidor, Ofra (2013). *Category Mistakes*. Oxford University Press.
McKay, Thomas J. (2006). *Plural Predication*. Oxford University Press.
Menzel, Christopher (1993). The Proper Treatment of Predication in Fine-Grained Intensional Logic. *Philosophical Perspectives* 7, pp. 61–87.
Merricks, Trenton (2001). *Objects and Persons*. Oxford University Press.
Mizoguchi, Riichiro, Antony Galton, Yoshinobu Kitamura, and Kouji Kozaki (2015). Families of Roles: A New Theory of Occurrent-Dependent Roles. *Applied Ontology* 10, pp. 367–99.
Nolan, Daniel (2012). Categories and Ontological Dependence. *The Monist* 94, pp. 277–301. DOI: 10.5840/monist201194214.
Orilia, Francesco (2008). The Problem of Order in Relational States of Affairs: A Leibnizian View. In: R. Egidi and G. Bonino (eds.), *Fostering the Ontological Turn: Essays on Gustav Bergmann*. Frankfurt am Main: Ontos. Pp. 161–86.
— (2011). Relational Order and Onto-Thematic Roles. *Metaphysica* 12, pp. 1–18. DOI: 10.1007/s12133-010-0072-0.
— (2014). Positions, Ordering Relations and O-Roles. *Dialectica* 68, pp. 283–303. DOI: 10.1111/1746-8361.12058.
— (2019). Relations, O-Roles, and Applied Ontology. *Philosophical Inquiries* 7, pp. 115–31.
— (forthcoming). Converse Relations and the Sparse–Abundant Distinction. *Dialectica* 76, pp. 135–63. DOI: 10.48106/dial.v76.i2.05.
Paolini Paoletti, Michele (2024). *The Ontology of Relations*. New York: Routledge.
Plate, Jan (2025). Ordinal Type Theory. *Inquiry* 68, pp. 2344–2400. DOI: 10.1080/0020174X.2023.2278031.
— (MS[a]). All Structure Is Logical Structure. Unpublished manuscript.
— (MS[b]). Fundamentality in an Ontology of Roles and States. Unpublished manuscript.
— (MS[c]). Towards an Ontology of Roles and States. Unpublished manuscript.
Ramsey, Frank P. (1925/1931). The Foundations of Mathematics. In: R. B. Braithwaite (ed.), *The Foundations of Mathematics and Other Logical Essays*. London: Routledge & Kegan Paul. Pp. 1–61.
Russell, Bertrand (1903). *The Principles of Mathematics*. London: Allen & Unwin.
Sider, Theodore (2011). *Writing the Book of the World*. Oxford University Press.

Steimann, Friedrich (2000). On the Representation of Roles in Object-Oriented and Conceptual Modeling. *Data & Knowledge Engineering* 35, pp. 83–106.
Taylor, Barry M. (1985). *Modes of Occurrence: Verbs, Adverbs and Events*. Blackwell.
Tegtmeier, Erwin (2000). Events as Facts. *Poznań Studies in the Philosophy of the Sciences and the Humanities* 76, pp. 219–28.
Thompson, Naomi (2016). Is Naturalness Natural? *American Philosophical Quarterly* 53, pp. 381–95.
Van Inwagen, Peter (1990). *Material Beings*. Cornell University Press.
— (2023). *Being: A Study in Ontology*. Oxford University Press.
Von Neumann, John (1923). Zur Einführung der transfiniten Zahlen. *Acta Scientiarum Mathematicarum (Szeged)* 1, pp. 199–208.
Williamson, Timothy (1985). Converse Relations. *The Philosophical Review* 94, pp. 249–62. DOI: 10.1215/00318108-109-1-1.
Wilson, Jessica M. (forthcoming). *The Fundamentality First Approach to Metaphysical Structure*. Forthcoming in *Australasian Philosophical Review*.
Zalta, Edward N. (1988). *Intensional Logic and the Metaphysics of Intentionality*. MIT Press.