Roles, States, and Fundamentality

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Abstract

This paper develops and defends a positionalistic ontology of intensional entities, expanding on a proposal ('Towards an Ontology of Roles and States', unpublished) that is in turn partially based on my recent 'Ordinal Type Theory' (forthcoming in this journal). Three central primitive concepts of the theory proposed here are those of *rolehood*, *resultance*, and *fundamentality*. Whereas traditionally a state of affairs—or 'state' for short—is often thought of as the result of 'applying' a property or relation to a given entity or sequence of entities, the present theory takes states to *result* from 'role assignments', which in turn are generalized multisets of ordered pairs (r_i, x_i), with each r_i being a *role*; hence the significance, within this theory, of the concepts of rolehood and resultance. Meanwhile, the concept of *fundamentality* is employed, among other things, in an account of the individuation of states. A main thesis of the paper is the claim that fundamentality is a fundamental property. This thesis is defended by arguing against several rival approaches. The penultimate section draws attention to the 'Special Resultance Question': under what conditions is a role assignment such that a state results from it? A tentative answer is provided that invokes a fundamental role of *connectedness*.

Keywords: states of affairs; properties; positionalism; fundamentality

1 Introduction

My principal aim in this paper is to develop and defend an ontology of intensional entities that is 'positionalistic' insofar as it conceives of relations not merely as polyadic analogues of properties but as set-theoretic constructions over a space of *roles*. This ontology has something of a precursor in a theory that, some decades ago, Hector-Neri Castañeda has ascribed to Plato. On this view, the sentence

(1) Socrates is shorter than Simmias

is "to be analyzed" by something like the following:

(2) Shortness (Socrates) Tallness (Simmias).

Here, as Castañeda goes on to explain, "the juxtaposition of the matrices 'Shortness ( )' and 'Tallness ( )' expresses the *with-respect-to* connection between the two participations in the Forms making up the relation shorterness" (1975: 241). The two Forms in question are referred to as 'Shortness' and 'Tallness', respectively, and the relation *shorter-than* (or "shorterness") is in Castañeda's view—or at any rate on the view he ascribes to Plato—simply the set of the two Forms just mentioned.

In characterizing the theory to be developed in this paper as 'positionalistic', I have in effect called it a form of *positionalism*. This latter term has gained currency in the wake of Kit Fine's seminal 'Neutral Relations' (2000), where it is used for a certain conception of 'neutral' relations, i.e., of relations of which it makes no sense to say that they apply to their relata in a given order. More specifically, Fine introduces positionalism as a conception under which

> each neutral relation is taken to be endowed with a fixed number of argument-places or positions. But by this, I do not merely mean that the relation is of fixed degree—binary, ternary, or what have you. I mean that there are *specific entities* that are the argument-places of the relation. Thus, in the case of the neutral amatory relation, there will be two such entities, the argument-places *Lover* and *Beloved*, that in some primitive sense *belong* to the relation. (2000: 10)

Contrary to what this quotation might lead a reader to expect from a view that bills itself as a form of positionalism, the theory to be developed in this paper is *not* a view about neutral relations. In fact it does not even accommodate the idea that there *are* such things. Nonetheless I think that the label is apt, in the same way in which it is (arguably) apt when applied to the view that Castañeda has ascribed to Plato. That is, it is a form of positionalism in the sense that it tries to understand relations in terms of some more basic entities that, in a certain vague and abstract sense, 'stand between' (Fine speaks of 'mediation') a relation's relata and the relational states that are its instantiations. For Castañeda, these more basic entities are Platonic Forms; for Fine's positionalist, they are 'argument-places'. Here I will be following Sprigge (1970: 69f.), Armstrong (1978: 94), and Orilia (2011) in speaking instead of 'roles'.[1]

To eliminate any undue suspense: the view to be proposed below follows Plate (MS) in conceiving of relations as non-empty sets of *sequences of roles*. For example, given suitable roles s and t (for '*Shorter*' and '*Taller*', respectively), the *shorter-than* relation is the set $\{\langle s, t\rangle\}$. Another part of the view is the thesis that *properties* form a special case of roles; and moreover, *sets* are taken to be properties.[2] Putting all this together, we arrive at the thesis that every relation is a role: since every relation is

[1]In a series of papers, Francesco Orilia (e.g., 2008; 2011; 2014; 2019; 2022) has developed a theory of *onto-thematic roles*, which are modeled on the thematic roles discussed by linguists. Other recent proponents of positionalism include Gilmore (2013) and Dixon (2018), though Dixon has more recently (2022) embraced a non-positionalist view.

[2]The identification of sets with properties is somewhat optional, in that one could also understand the present theory as one in which sets are *replaced* with properties (of a certain special kind). Cf. footnote 11 below. On this reading, relations are directly identified with properties.

a set, every set a property, and every property a role. As it is also part of the view that no *state of affairs*—or 'state' for short—is a role, the overall ontological picture is one marked by a grand dichotomy between roles and states. Accordingly, to have a convenient name for this view, we might call it an *ontology of roles and states*, or 'ORS' for short. In the context of this theory, anything that is either a role or a state is called an *intensional entity*, while anything else is called a *particular*. It should be kept in mind, however, that ORS is neutral on the question of whether there *are* any particulars. In general it has little to say about them, except that each particular is a fundamental entity.

Apart from its inherent positionalism and the mentioned dichotomy between roles and states, ORS is characterized by the fact that it takes roles and states to be arranged in a hierarchy of 'ordinal types'. This hierarchy is largely borrowed from Plate (forthcoming) along with, among other things, two principles that are taken to govern the individuation of states. Both of these commitments—ordinal type theory and the principles of individuation—bring with them a reliance on a concept of *fundamentality*; and it is *this* aspect of the theory that will be the main subject of discussion once the theory itself is on the table. More specifically, I will be defending the thesis that the property of *being a fundamental entity* is itself fundamental. I will do so by arguing, among other things, that the relevant concept of fundamentality should *not* be analyzed in terms of grounding. Considerations of theoretical simplicity will play a central part in this discussion.

The rest of this paper is organized as follows. Sections 2–5 are dedicated to the exposition of ORS. Section 2 introduces the theory's primitive concepts and briefly motivates the mentioned conception of relations as sets of role sequences. Section 3 provides further details and formal machinery. Section 4 introduces the formal language $\mathcal{L}$ that serves as a representational tool in some of ORS's most central assumptions, which in turn are laid out in Section 5. Once all this is in place, we proceed to a discussion of several noteworthy aspects of the theory. By far the most space will be taken up by Section 6, which defends ORS's treatment of fundamentality, and in particular the thesis that fundamentality is a fundamental property. Next, Section 7 discusses and defends ORS's treatment of the concepts of rolehood and statehood. Section 8 rounds out the discussion by drawing attention to the *Special Resultance Question*—roughly: what assignments of entities to roles have states 'resulting' from them?—and provides a tentative answer in the form of four additional principles. Section 9 concludes.

2 Primitive Notions

ORS is formulated using the following nine primitive concepts: *state of affairs*, *obtainment*, *rolehood*, *identity*, *resultance*, *negation*, *conjunction*, *existential quantification*, and *fundamentality*. Let us quickly go over them one by one:

1. *State of affairs*. We can think of a state of affairs (or simply 'state') as a way for the world to be. As will become clearer in Section 5, states will here be taken

to be individuated in a moderately coarse-grained manner.

2. *Obtainment*. States can obtain or fail to obtain. Non-obtaining states can seem mysterious but are needed for a straightforward and philosophically satisfying treatment of negation, among other things. For example, the state that *this rose is not black* is naturally understood as the negation of the *non-obtaining* state that this rose is black. If this latter state did not exist, there would be nothing for the former to be a negation *of*.[3]

3. *Rolehood*. This is the first really novel concept on this list. The *rolehood* of a given entity x is a special kind of state, which may be informally described as the state that x is a role. The central concept of a *role* is defined on the basis of those of rolehood and obtainment: something is a role iff its rolehood obtains.

4. *Identity*. Contrary to what might be expected, this is also a (relatively) unfamiliar concept. The *identity* of two entities x and y is a state that may be informally described as the state that x and y are one and the same entity. The familiar relational concept expressed by 'is identical with' may be defined with the help of this concept as follows: an entity x is identical with an entity y iff the identity of x and y obtains.

5. *Resultance*. We invoke the concept of resultance when we say that a given state 'results' from such-and-such an assignment of entities to roles. (The notion of a *role assignment* will be formally introduced in the next section.)

6. *Negation*. We invoke the concept of negation when we say that a given state is the negation of another. Another concept of negation is expressed by the word 'not' when we say that something is not the case. On the present view, this latter concept can be understood in terms of the former: when we say that something is not the case, we can be understood as saying (barring reservations about the existence of states) that a given state fails to obtain; and this in turn may be understood as saying that the *negation* of that state obtains.

7. *Conjunction*. We invoke this concept when we say that a given state is the conjunction of a set of states. The concept expressed by the word 'and' when we say that such-and-such *and* such-and-such are both the case may be understood on the basis of it: given any set of states $\{s_1, s_2, \ldots\}$, it is the case that

[3]The existence of non-obtaining states is admittedly a controversial issue. S. Barker and Jago (2012) take a different approach that eschews non-obtaining states and instead invokes two kinds of instantiation: 'normal' instantiation and *anti*-instantiation. At least in the context of ORS, however, non-obtaining states are essential for the ontology of logically complex attributes. For example, in a world in which something is green but nothing is red, there should nonetheless exist a property of *being either red or green*. One reason for this is that this latter property is needed in order for there to be the fact that *something* is either red or green. There will be more to be said in defense of such a 'plenitudinous' ontology of states (and attributes) at the end of Section 5.2 below.

s_1 obtains *and* that s_2 obtains *and* that s_3 obtains, and so on, iff the conjunction of that set obtains.[4]

8. *Existential quantification*. The existential quantification of an attribute (i.e., of a property or relation) is a state that obtains iff there exists an obtaining instantiation of that attribute. Or this is how one might try to explain this concept on the basis of those respectively expressed by 'there exists', 'obtains', 'instantiation of', and 'attribute'. Taken as primitive, this concept may instead serve to explain the meaning of 'there exists': there exists an F iff the existential quantification of the property denoted by 'F' obtains.[5]

9. *Fundamentality*. Using a popular architectural metaphor, one could say that an entity is fundamental iff it is 'unbuilt', but here the concept will be taken as primitive. It may be worth noting that it is *not* part of the present understanding that the fundamental things exist in some privileged sense or to

[4]Accordingly, the concept of conjunction that is in this paper expressed by the logical 'and' presupposes, in a certain sense, the concept of set. It is important not to read too much into this; for example, nothing that has been said here entails that a person who does not have the concept of set cannot be a competent reasoner. However, we do enter a certain regress when we ask how the concept of a set of states $s_1, s_2, \ldots$ is to be analyzed. On the usual approach, that concept can be analyzed as that of a set x such that, for any y: y is a member of x iff $y = s_1$ or $y = s_2$ or (and so on). In the present setting, since we are identifying sets with properties, the same concept can also be analyzed as that of a certain disjunctive property. But in either case, the relevant concept of disjunction is not taken as primitive but is analyzed in terms of conjunction and negation; and this means that we have to invoke the concept of a *further* set of states, distinct from the original. This further concept in turn requires analysis, and so on *ad infinitum*. To avoid this regress, one could switch from the dyadic concept of conjunction to a *variadic* concept that takes one plus any set-sized number of arguments. Instead of 'x is a conjunction of $\{s_1, s_2, s_3\}$', one could then say something along the lines of, 'x is a conjunction of s_1, s_2, s_3'. When dealing with *infinitary* conjunctions, however, the use of such a concept would require a language in which predicates can take infinitely many arguments. Since English is no such language, and because of the greater versatility of the dyadic concept, I will here stick with the latter; but the reader should certainly feel free to imagine this paper written in a more powerful language.

As the reader may have noticed, there also looms a second regress (or threat of circularity), since the phrase 'the conjunction of that set obtains' is typically formalized as an existentially quantified conjunction: some s is such that (i) s is a conjunction (or *the* conjunction) of $\{s_1, s_2, \ldots\}$, *and* (ii) s obtains. To avoid this second regress, we will need a language in which states can be named by complex terms. In such a language, the proposed definition of the logical 'and' may be put along the following lines: it is the case that s_1 obtains *and* that s_2 obtains *and* that s_3 obtains, and so on, iff $\&(s_1, s_2, s_3, \ldots)$ obtains.

[5]Analogously for quantifications with more than one variable. For example, there exist an x and a y such that $x \neq y$ iff the existential quantification of the relation $\lambda x, y\,(x \neq y)$ obtains. However, since in defining the concept of relation we will be relying on quantificational resources (see Definitions 2.1 and 2.3 below), there arises a regress similar to the one described in the first paragraph of the previous footnote. Fortunately, the present form of positionalism suggests a solution: one could switch from the dyadic concept of existential quantification to a *variadic* concept that applies to sequences of arguments of any positive set-sized length. Instead of 'x is an existential quantification of $\{\langle s, t\rangle\}$', one could then say something along the lines of, 'x is an existential quantification of s, t (in this order)'. When dealing with *infinitary* relations, however, the use of such a concept would require a language in which predicates can take infinitely long lists of arguments. Since English is no such language, and because of the greater versatility of the dyadic concept, I will here stick with the latter.

some stronger degree than everything else. Nor will I be assuming that fundamental things are somehow easier to refer to than non-fundamental things.

Conspicuously absent from this list—although it does get mentioned in connection with the concept of existential quantification—is the concept of *instantiation*, by which I mean the concept of something's being an instantiation *of* a given attribute *by* such-and-such entities (in a given order). The motivation for the positionalistic aspect of ORS derives mainly from the consequences of taking this concept as primitive while defining that of *relation* in terms of it. Let me explain.

To begin with, here is how one may define the four concepts of *adicity*, *property*, *relation*, and *attribute*, taking the concept of instantiation, at least for now, as primitive:

Definition 2.1. An ordinal α is an *adicity* of an entity x iff the following two conditions are satisfied:

(i) If $\alpha = 0$, then x is a state.

(ii) If $\alpha > 0$, then there is an α-sequence (i.e., a sequence of length α) of entities $y_1, y_2, \ldots$ such that there exists an instantiation of x by $y_1, y_2, \ldots$, in this order.

Definition 2.2. Something is a *property* iff it has an adicity of one.

Definition 2.3. Something is a *relation* iff it has an adicity of at least two.

Definition 2.4. Something is an *attribute* iff it is either a property or a relation.

Here the definition of 'relation' relies indirectly (via that of 'adicity') on the concept of instantiation. Having adopted these definitions, and having (if only provisionally) taken the concept of instantiation as primitive, how should we understand talk of 'fundamental relations'? Presumably a relation will have to be counted as fundamental just in case it is a *fundamental entity*.

But now suppose that the dyadic relation *shorter-than* is a fundamental relation. It is then plausibly the case that its converse, the relation *taller-than*, is also fundamental; for why should nature favor one of these over the other? It would be desirable, however, if we had some *explanation* as to why it is that the converse of a fundamental relation is again fundamental, rather than just an argument for the conclusion that this is how things are. Yet as long as the concept of instantiation (on the basis of which that of relation has just been defined) is taken as primitive, we seem to have no good way of telling a deeper story. Partly for this reason, it is my view that the concept of instantiation should *not* be taken as primitive. (Another reason, which is set out in Plate (MS: §9), lies in the fact that a primitive concept of instantiation will lead to awkward complications in that part of the theory that deals with the individuation of states.)

In the context of ORS, the concept of instantiation is not taken as primitive but rather defined—or analyzed—in terms of the primitive concept of *resultance*. This will be the main topic of the next section. As we will see, the to-be-proposed definition of 'instantiation', combined with the above definition of 'relation', has the consequence that relations are sets of role sequences. To return to the example used in the Introduction, the relation *shorter-than* will under this conception be nothing else than the set $\{\langle s, t\rangle\}$, where $\langle s, t\rangle$ is a sequence that has the roles s and t as its first and second element. On the background of this conception of relations, the concept of a *fundamental* relation can now be naturally understood in such a way that a given entity is a fundamental relation iff it is a relation all of whose constituent roles are fundamental entities. (In this sense, a fundamental relation is *not* a relation that is also a fundamental entity. Indeed, *no* relation is a fundamental entity, given that no set is.) This gives us the wherewithal to provide a straightforward explanation as to why the converse of a fundamental relation is again a fundamental relation. For consider once more the relation *shorter-than*, and suppose it to be fundamental in the sense just explained. Then, given that *shorter-than* is the set $\{\langle s, t\rangle\}$, the two roles s and t will be fundamental entities; and the *converse* of *shorter-than*, which is the relation *taller-than*, will under that same conception be the set $\{\langle t, s\rangle\}$.[6] And now it is easy to see that *this latter relation, too, will be a fundamental relation*, because, as has just been said, the roles s and t are fundamental entities. Generalizing from this case, we can infer that fundamental relations have fundamental converses, and we can also see that this fact will no longer have to be regarded as brute. (The result that fundamental relations have fundamental converses will be more formally established at the end of the next section.)

Admittedly, the question of why fundamental relations have fundamental converses is not exactly a long-standing philosophical problem. But I think that it is nonetheless a problem worth solving, and it is a point in favor of ORS that it solves it. Admittedly also, the conception of relations as sets of role sequences can take some getting used to. But the same may be said of other conceptions. For example, neither Lewis's (1983; 1986) identification of relations with sets of tuples of *possibilia*, nor Fine's proposal to think of relations as coming in "two kinds, one neutral and the other biased" (2000: 1n.), is significantly less revisionary than the positionalistic conception outlined above.[7]

After this preview and brief sketch of a defense of ORS's conception of relations, let us now look more closely at the concepts of resultance and instantiation.

3 Resultance and Instantiation

In order properly to introduce the primitive concept of resultance, we have to start with the notion of a *generalized multiset*. Intuitively, a multiset is a 'set-like object' that can contain its members 'more than once'. Formally, it is an ordered pair (A, m), where A is a set and m a function from A to the set of positive integers. A given entity x is said to be a *member* of a multiset (A, m) iff x is a member of A; and the

[6] This will become clearer once we have defined the concept of a (non-trivial) converse, which will be done in the next section.

[7] For critical discussion of Fine's 'antipositionalist' conception of relations, see, e.g., MacBride (2007) and Gaskin and Hill (2012).

number $m(x)$ is also called the *multiplicity* of x in (A, m), or the number of times that x 'occurs' in the latter. To arrive at the notion of a *generalized* multiset, we have to relax the requirement that m be a function to the set of positive integers: we will instead allow m to be any function that assigns to each member of A a positive—and possibly infinite—cardinal. (Thus, a generalized multiset is an ordered pair (A, m), where A is a set and m a function from A to some set of positive cardinals.) This generalization is needed in order to allow our approach to accommodate infinitary relations.

To represent specific (generalized) multisets, we will use curly brackets in the same way as with sets. For instance, we will write '$\{a, a, b\}$' to represent the multiset $(\{a, b\}, m)$, where m is the function on $\{a, b\}$ that maps a to 2 and b to 1. In addition, we will usually speak as if sets were a special case of multisets—so that, e.g., the set $\{a, b\}$ will also be said to be a multiset—even though in fact the reverse is the case, since multisets are ordered pairs and ordered pairs (which will here be conceived of *à la* Kuratowski [1921]) are sets.

With the help of the concept of a generalized multiset, that of a *role assignment* can be defined as follows:

Definition 3.1. A *role assignment* is a non-empty generalized multiset of ordered pairs (r_i, x_i) such that each r_i is a role.

For a simple example, suppose that r is a role and x some other entity (or even r itself): then $\{(r, x)\}$ is a role assignment, and so is $\{(r, x), (r, x)\}$. As has already been mentioned in the previous section, the concept of *resultance* relates role assignments to states: we can use it to say that such-and-such a state results from such-and-such a role assignment. In the formal language $\mathcal{L}$ that will be introduced in the next section, a state that results from a role assignment $\{(r, x), (r, x)\}$ can be more succinctly denoted by the term '$[rx, rx]$'.[8]

Before we go on, we will need a little more terminology. As usual, *sequences* will be taken to be functions defined on ordinals. Thus the sequence $\langle a, b\rangle$ (for any

[8] A reader worried about 'Bradley's regress' or about the 'problem of the unity of the proposition' may wonder just how a state that results from a given role assignment is supposed to be 'held together'. Will there not need to be some special relation that ties an entity x to a role r so as to form the state that, as we say, 'results' from the role assignment $\{(r, x)\}$? The answer is 'no'; and incidentally, we are also not going to assume that there exists a general 'resultance relation'. What we *will* be committed to, via an assumption to be adopted in Section 5.2, is the existence of such relations as $\lambda x, y^1, z\,(x = [y^1 z])$ and $\lambda x, y^1_1, y^1_2, z_1, z_2\,(x = [y^1_1 z_1, y^1_2 z_2])$. These might be referred to as *special* resultance relations. But it would be a mistake, in my view, to think that some relation (or other metaphysical gadget) is needed to 'hold a state together'. The thought that there has to be some such relation may be motivated by the idea that there has to be something that accounts for the difference between, on the one hand, a state that results from a role assignment $\{(r, x)\}$ and, on the other, the 'mere aggregate' of the role r and the entity x. But this idea strikes me as doubly mistaken. First, mainly for reasons of ideological parsimony (cf., e.g., Sider [2013]), I reject the presupposition that there are such things as 'mere aggregates'. Second, even if there were such things, I see no reason why, in order to specify how the state differs from the aggregate, it would not be enough to say that the state but not the aggregate results from the mentioned role assignment. See Eklund (2019) for related discussion.

a and b) is the function that is defined on the ordinal 2—which in turn is the set $\{0, 1\}$—and maps 0 to a and 1 to b.[9] Given that functions are sets of ordered pairs, it follows that the sequence $\langle a, b\rangle$ is nothing else than the set $\{(0, a), (1, b)\}$. Further, by a 'permutation on an ordinal', we will mean a function that has a certain ordinal as both domain and range. Accordingly, if f is a permutation on an ordinal α and σ is an α-sequence, then there will also be such a thing as the function on α that maps each $i < \alpha$ to $\sigma(f(i))$. Following common notational practice, this latter function—which can be viewed as a 'reordering' of σ in accordance with f—will be denoted by '$\sigma \circ f$'.

With this terminology in place, we can next define the concept of *instantiation-equivalence*, which is applicable to role sequences:[10]

Definition 3.2. For any ordinal $\alpha > 0$, a role sequence σ of length α is *instantiation-equivalent* to a role sequence σ' (notation: $\sigma \approx \sigma'$) iff the following two conditions are satisfied:

(i) There exists a permutation f on α such that $\sigma = \sigma' \circ f$.

(ii) For any state s and any α-sequence of entities $x_0, x_1, \ldots$: s results from the role assignment $\{(\sigma(i), x_i) \mid i < \alpha\}$ iff s results from $\{(\sigma'(i), x_i) \mid i < \alpha\}$.

From the first numbered clause it can be seen that, for any given role sequence σ of length α, there are only at most as many role sequences instantiation-equivalent to σ as there are permutations on α. Trivially, any role sequence is instantiation-equivalent to itself.

Next, we introduce the term 'proto-relation', which simply refers to any non-empty set of instantiation-equivalent role sequences of length greater than one:

Definition 3.3. An entity x is a *proto-relation* iff there exists a role sequence σ, of some length greater than 1, such that $x = \{\sigma' \mid \sigma' \approx \sigma\}$.

On this basis, we can now define (or analyze) the central concept of *instantiation*:

Definition 3.4. For any ordinal $\alpha > 0$, any entities x and y, and any α-sequence of entities $z_0, z_1, \ldots$: x is an *instantiation* of y by $z_0, z_1, \ldots$, in this order, iff one of the following two conditions is satisfied:

[9] In identifying ordinals with the sets of their predecessors, I follow the standard approach due to von Neumann (1923/1967). This approach is here accepted in the same pragmatic spirit as Kuratowski's set-theoretic construal of ordered pairs. (For precedent and elaboration, see Quine's [1960: §53f.] remarks on 'ordered pair' and 'natural number'. Lewis's [1986: 55f.] remarks on 'property' are also relevant.) I take this to disarm the sort of argument that has been influentially raised by Benacerraf (1965) against set-theoretic construals of the natural numbers, the point being that it has never been determinately fixed just what sort of entity we refer to when we speak of 'the ordinals', 'the natural numbers', etc. If an objector insists, perhaps on linguistic grounds, that ordinals cannot admissibly be identified with sets, we can offer an alternative reading of the present theory, under which ordinals are *replaced* with sets. Similarly in other cases, such as the identification of relations with sets or that of sets with properties (which will be adopted later in this section).

[10] By a 'role sequence', I simply mean a sequence of roles.

(a) $\alpha = 1$, y is a role, and x results from the role assignment $\{(y, z_0)\}$.

(b) $\alpha > 1$, y is a proto-relation, and there exists an α-sequence $\sigma \in y$ such that x results from the role assignment $\{(\sigma(i), z_i) \mid i < \alpha\}$.

By combining this with the definitions of 'adicity', 'property', and 'relation' that have already been stated in the previous section, it can be seen that relations have now been 'reduced' to sets and properties to roles. For recall that, under the above definitions, properties are just entities that have instantiations by individual things, while relations are entities that have instantiations by two or more things in a given order. Hence, given Definition 3.4, a *property* is just a role r such that, for some x, there exists a state resulting from the role assignment $\{(r, x)\}$; and a *relation* is just a proto-relation that has an instantiation.

But we can go further. Following a proposal by Carnap (1947: §23), we can reduce sets to properties. Under this proposal, the empty set is identified with the uninstantiable property $\lambda x\, \neg\exists y\, (y = x)$, the singleton of an entity x with the property of *being identical with* x, the pair set $\{x, y\}$, for any x and y, with the property of *being identical with either* x *or* y, and so on.[11] Having previously identified relations with sets, we can now conclude that relations are also *properties*—and hence roles. This has the consequence that every relation has exactly two adicities: given that it is a property, one of its adicities is the ordinal 1, and given that it is a relation, it also has as one of its adicities some ordinal greater than 1. While this may initially seem odd, it is really no cause for concern. In practice, when we are dealing with a relation and treating it as such (considering for instance some instantiation of it by some two or more entities), we can simply ignore that it is also a property.[12]

[11] A philosopher who has held an opposing view, arguing for a strict divide between sets and properties, is Linnebo (2006). Linnebo remarks that properties "have complements whereas (ordinary) sets don't" (p. 157). (For Linnebo's more recent views on the nature of sets, see his [2018: ch. 12].) The claim that sets "don't have complements" is of course correct if read as expressing the idea that the complements of sets *are not again sets*, since, for any set S, there are too many things outside of S to form a set. But this does not rule out an identification of sets with properties: for nothing prevents us from allowing that the complement of a set is some other property that is *not* a set.

Linnebo further argues that "properties and sets have completely different essential properties" (p. 160). In doing so he relies on the premise that "properties are just nominalized concepts" (*ibid.*). I reject this premise. Even taking to heart (as one should) the Lewisian point that we have some leeway with regard to what kind of thing we choose to refer to when using the term 'property' (cf. footnote 9 above), I would *not* agree to apply that term to a "nominalization of a concept", for the simple reason that I doubt that there are such things. It may also be noted that the proposal to treat properties as "nominalizations of concepts" leaves wide open the question of what a *concept* is to begin with. Rather than to regard the ontology of properties and relations as somehow parasitic on that of concepts, it may be more adequate to think of the latter as forming merely a smallish part of the former. (This will become clearer in footnote 53 below.)

[12] Against the idea that every relation has exactly two adicities, it might be objected that we should allow for relations that have *multiple* adicities. For example, Fine writes that "[t]here should be a relation of sup[p]orting that holds between any positive number of supporting objects $a_1, a_2, \ldots$ and a single supported object b just when $a_1, a_2, \ldots$ are collectively supporting b" (2000: 22). But it is hardly obvious that there should be any such relation. In the first place, it is debatable that we

In connection with properties and other roles, the following three concepts will be occasionally useful:

Definition 3.5. An entity x *instantiates* a property P iff there exists an obtaining state that results from the role assignment $\{(P, x)\}$.

Definition 3.6. An entity x *fills* a role r iff there exists an obtaining state that results from a role assignment containing the pair (r, x).

Definition 3.7. An entity x *fits* a role r (alternatively: r suits x) iff there exists a state that results from a role assignment containing the pair (r, x).

These concepts are successively more general. Thus anything that instantiates a given property also 'fills' it (but the converse need not hold), and anything that fills a role also fits it, but not necessarily *vice versa*. An interesting question to consider is whether there are any fundamental roles that suit only *some* entities, rather than every entity or none. We will briefly return to this in Section 8.

It is now time to state formal definitions of a few concepts that have already been invoked in the previous section, in particular those of a *fundamental relation* and a *converse*. The first of these can be defined in two steps, beginning with the concept of a relation's *role set*:

Definition 3.8. The *role set* of a relation R is the smallest set S such that $R \subseteq S^\alpha$, for some ordinal α.

Here the set S^α is the set of all α-sequences whose elements are members of S. So, for instance, the role set of the relation $\{\langle s, t\rangle\}$ is just the set $\{s, t\}$. On this basis, the concept of a fundamental relation can be defined as follows:

Definition 3.9. A *fundamental relation* is a relation whose role set contains only fundamental roles.

Next, to define the notion of converse, we proceed in four steps:[13]

Definition 3.10. For any ordinal $\alpha > 0$, any permutation f on α, and any y: an entity x is an *f-converse* of y iff y is α-adic and, for any α-sequence of entities $z_0, z_1, \ldots$ and

need such a relation when it comes to constructing an adequate semantics of natural language. (Cf., e.g., Brisson [2003: §6].) In the second place, even from a purely metaphysical point of view it is far from obvious that a situation in which an object b is supported by $a_1, a_2, \ldots$ cannot simply be understood as a situation in which each of $a_1, a_2, \ldots$ contributes in a certain way to the event of b's being held aloft. If this is denied, a purely positionalistic treatment is also available. For instead of positing a multigrade relation, one could equally well hypothesize that there are two roles r_1 and r_2 (for '*Supporter*' and '*Supportee*', respectively) such that the state of b's being supported by $a_1, a_2, \ldots$ results from the role assignment $\{(r_1, a_1), (r_1, a_2), (r_1, a_3), \ldots, (r_2, b)\}$.

There is also the related issue of whether we should admit relations (and properties) that have instantiations by *pluralities* of entities. The thesis that there are such relations has been popular among analytic philosophers since the 1990s. (See, e.g., McKay [2006] and references therein.) It has been less widely noted that this thesis leads to problematic—indeed paradoxical—consequences as soon as relations are understood to have instantiations. See McGee and Rayo (2000) and Pruss and Rasmussen (2015).

[13] The following definitions are (with negligible changes) taken from Plate (forthcoming: §3.3.3).

any state s, the following holds: s is an instantiation of x by $z_{f(0)}, z_{f(1)}, \ldots$, in this order, iff s is an instantiation of y by $z_0, z_1, \ldots$, in this order.

Definition 3.11. An entity x is a *trivial converse* of an entity y iff, for some ordinal $\alpha > 0$: α is an adicity of y, and x is an id_α-converse of y.[14]

Definition 3.12. An entity x is a *non-trivial converse* of an entity y iff, for some ordinal $\alpha > 0$ and some permutation f on α that is distinct from id_α: α is an adicity of y, and x is an f-converse of y.

Definition 3.13. An entity x is a *converse* of an entity y iff x is either a trivial or a non-trivial converse of y.

Usually, when we speak of 'the converse' of a dyadic relation, we will mean its *non-trivial* converse. It is worth emphasizing that the notions of a trivial and a non-trivial converse are not mutually exclusive. For example, a relation $\{\langle r, r\rangle\}$ will be both a trivial and a non-trivial converse of itself. It also bears noting that *any* attribute is a trivial converse of itself.

A plausible principle that will be officially adopted in Section 5.1 is to the effect that any attribute is its own and *only* trivial converse. From this principle, taken together with the above definitions, it can be inferred that, for any fundamental relation R, any converse of R is again a fundamental relation.[15]

4 The Formal Language

At its core, ORS consists of nineteen 'ontological assumptions' that will be listed in the next section. While those assumptions will be formulated in English, some of them—such as the comprehension axiom (E2) and the 'analyzability assumption' (O2)—also rely on a certain formal language $\mathcal{L}$ that provides names of roles and states. The purpose of the present section is to introduce that language. It is worth noting that $\mathcal{L}$ will here *not* be used for the formalization of proofs. Any reasoning in this paper will be done in English (using classical logic), so that there will be no need to lay down axioms or inference rules for reasoning in $\mathcal{L}$. Instead, we only have to specify its syntax and semantics.[16]

[14] Here 'id_α' is the identity permutation on α, i.e., the function that maps each $i < \alpha$ to itself.

[15] To see this, let R be any α-adic relation (for some $\alpha > 1$), and let R' be any converse of R. R' will then be an f-converse of R, for some permutation f on α. Given the mentioned principle, R' will be the *only* f-converse of R; for if there were two, they would be trivial converses of each other, in contradiction of the principle. But from Definition 3.4 it can be seen that the set $R \circ f$, i.e., $\{\sigma \circ f \mid \sigma \in R\}$, is an f-converse of R. Hence we have that $R' = R \circ f$; and so the role set of R' is the same as R's. It follows that, if R is a fundamental relation, then so is R'.

[16] To a large extent, the material of the rest of this section is adapted from Plate (forthcoming: §3.2, §§3.4–4.1). There are, however, significant differences. See also Plate (MS: §§11–12).

4.1 Syntax

We have to begin by distinguishing between the expressions of $\mathcal{L}$ themselves and the ways they are represented on paper. What will here be called an *expression* of $\mathcal{L}$ is a sequence of certain pure sets, of possibly infinite length. Specifically, a given sequence is an expression of $\mathcal{L}$ iff each of its elements is either

(a) one of the ordinals $0, \ldots, 12$, or

(b) an ordered pair $(0, \alpha)$ or $(1, \alpha)$, for some ordinal α, or

(c) an ordered pair (α, β), for some ordinals $\alpha > 1$ and $\beta > 0$.

To represent these sequences in written form (so far as this is possible), we adopt the following conventions:

(C1) If x is one of the ordinals $0, \ldots, 12$, then the expression $\langle x \rangle$ (a sequence of length one, with x as its only element) will be written as '$=$', '$\neg$', '$\mathbb{R}$', '&', '$\exists$', 'λ', 'ρ', ',', '.', '(', ')', '[', or ']', respectivedly.

(C2) If x is an ordered pair $(0, \alpha)$, for some ordinal α, then $\langle x \rangle$ will be written as an unitalicized string (such as 'Socrates') and be called a *constant*.

(C3) If x is an ordered pair $(1, \alpha)$ for some ordinal α, then $\langle x \rangle$ will be written as an italicized letter (such as 'x'), possibly with subscript but *without* superscript, and will be called an *untyped variable*.

(C4) If x is an ordered pair (α, β) for some ordinals $\alpha > 1$ and $\beta > 0$, then $\langle x \rangle$ will be written as an italicized letter, possibly with subscript, that carries as superscript a numeral (or other expression) representing β, and will be called a *typed variable*.

(C5) Except for the use of abbreviatory devices (such as ellipses), expressions of any length greater than one will be written by concatenating representations of expressions of length one.

A special subclass of the expressions of $\mathcal{L}$ will be called 'terms', of which we distinguish two general kinds: atomic and non-atomic (or 'complex'). An *atomic* term is either a variable or a constant; and variables are further divided into typed and untyped, as indicated in (C3) and (C4). For any typed variable $\langle(\alpha, \beta)\rangle$, the ordinal β will be called the *type* of that variable.

The *complex* terms of $\mathcal{L}$ fall into three kinds: formulas, lambda-expressions, and rho-expressions (which might also be called 'relational role abstractors'). Their syntax can be recursively specified as follows:

(i) For any terms t and u: $\ulcorner(t = u)\urcorner$ is a formula.

(ii) For any term t: the expressions $\ulcorner\neg t\urcorner$ and $\ulcorner\mathbb{R}\, t\urcorner$ are formulas.

(iii) For any (possibly empty) sequence of terms $t_1, t_2, \ldots$: $\ulcorner \&(t_1, t_2, \ldots) \urcorner$ is a formula.

(iv) For any (one or more) pairwise distinct variables $v_1, v_2, \ldots$ and any term t: $\ulcorner \exists v_1, v_2, \ldots\ t \urcorner$ is a formula and $\ulcorner \lambda v_1, v_2, \ldots\ t \urcorner$ a lambda-expression.

(v) For any ordinal $\alpha > 0$ and any α-sequences of terms $t_1, t_2, \ldots$ and $u_1, u_2, \ldots$: $\ulcorner [t_1 u_1, t_2 u_2, \ldots] \urcorner$ is a formula.

(vi) For any (two or more) pairwise distinct variables $v_1, v_2, \ldots$ and any term t: $\ulcorner \rho v_1 . v_2, v_3, \ldots\ t \urcorner$ is a rho-expression.

(vii) Nothing else is a formula, lambda-expression, or rho-expression.

This syntax is admittedly quite liberal: even 'nonsensical' expressions like '$\neg \lambda x\, y$' and '$\exists z\, \lambda x\, p$' count as terms. But this is harmless, and it is at any rate not the purpose of $\mathcal{L}$'s syntax to sort sense from nonsense. This will be left to the semantics.

$\mathcal{L}$ provides exactly three variable-binding operators: '$\exists$', 'λ', and 'ρ'. Talk of 'free' and 'bound' variable-occurrences should be understood accordingly.[17] A further distinction will be needed to describe the *place* (or places) at which a term occurs. The overall idea is that an occurrence of a term t will be said to stand at sentence-position (predicate-position) iff, in order for the embedding term to have a denotation, t would need to denote a state (role). To put this in purely syntactic terms, we will say that a term-occurrence stands *at sentence-position* iff it is immediately preceded by an occurrence of '$\neg$', '$\&($', or $\ulcorner \&(t_1, t_2, \ldots, \urcorner$ (for one or more terms $t_1, t_2, \ldots$) or by an occurrence of either $\ulcorner \exists v_1, v_2, \ldots \urcorner$ or $\ulcorner \lambda v_1, v_2, \ldots \urcorner$ (for one or more variables $v_1, v_2, \ldots$) or $\ulcorner \rho v_1 . v_2, v_3, \ldots \urcorner$ (for two or more variables $v_1, v_2, \ldots$). And we will say that an occurrence o of a term t stands *at predicate-position* iff, for some term u, o is the first occurrence of t in an occurrence of $\ulcorner tu \urcorner$ that appears as an element of a comma-delimited list of term-pairs enclosed in square brackets (as in, $\ulcorner [\ldots, tu, \ldots] \urcorner$).

It will be convenient to use several abbreviatory devices apart from ellipses. '$\top$' and '$\bot$' will abbreviate '$\&()$' and '$\neg\&()$', respectively. Where t and u are any terms, we will write $\ulcorner (t \neq u) \urcorner$ to abbreviate $\ulcorner \neg(t = u) \urcorner$. Further, for any sequence of terms $t_1, t_2, \ldots$, of a length greater than one, $\ulcorner (t_1 \wedge t_2 \wedge \ldots) \urcorner$ and $\ulcorner (t_1 \vee t_2 \vee \ldots) \urcorner$ will respectively abbreviate $\ulcorner \&(t_1, t_2, \ldots) \urcorner$ and $\ulcorner \neg\&(\neg t_1, \neg t_2, \ldots) \urcorner$, while $\ulcorner (t_1 \leftrightarrow t_2 \leftrightarrow \ldots) \urcorner$ will abbreviate $\ulcorner \neg\&(\neg\&(t_1, t_2, \ldots), \neg\&(\neg t_1, \neg t_2, \ldots)) \urcorner$. Finally, for any variables $v_1, v_2, \ldots$ and any term t, we will write $\ulcorner \forall v_1, v_2, \ldots\ t \urcorner$ to abbreviate the formula $\ulcorner \neg\exists v_1, v_2, \ldots\ \neg t \urcorner$. Parentheses will usually be omitted where this is not likely to cause confusion.

[17] For a way of formally defining these notions, see Plate (2018: 853). In this connection, an *occurrence* of an expression e of length α in an expression f may be taken to be an ordered triple (e, f, β), for some ordinal β such that $e(i) = f(\beta + i)$ for each $i < \alpha$.

4.2 Semantics

To specify the semantics of $\mathcal{L}$, we have to specify which entities are denoted by which terms relative to a given interpretation and variable-assignment. Here an *interpretation* is a function that is defined on a set of constants.[18] Analogously, a *variable-assignment* is a function defined on a set of variables. For brevity's sake, we will write 'denotes$_{I,g}$' instead of 'denotes relative to I and g'.

The semantics of $\mathcal{L}$ may now be specified by stipulating that, for any interpretation I and any variable-assignment g, the following holds:

(i) For any constant c and any x: c denotes$_{I,g}$ x iff I maps c to x.

(ii) For any variable v and any x: v denotes$_{I,g}$ x iff g maps v to x and, if v is typed, then x is of some order *less* than the type of v.[19]

(iii) For any terms t and u, and any x: $\ulcorner(t = u)\urcorner$ denotes$_{I,g}$ x iff, for some entities y and z, t denotes$_{I,g}$ y, u denotes$_{I,g}$ z, and x is an identity of y and z.[20]

(iv) For any term t and any x: $\ulcorner\neg t\urcorner$ denotes$_{I,g}$ x iff t denotes$_{I,g}$ a state of which x is a negation.

(v) For any term t and any x: $\ulcorner\mathrm{R}\, t\urcorner$ denotes$_{I,g}$ x iff t denotes$_{I,g}$ an entity of which x is a rolehood.

(vi) For any (possibly empty) sequence of terms $t_1, t_2, \ldots$ and any x: $\ulcorner\&(t_1, t_2, \ldots)\urcorner$ denotes$_{I,g}$ x iff there exists a set S of states such that: each t_i denotes$_{I,g}$ some member of S, each member of S is denoted$_{I,g}$ by some t_i, and x is a conjunction of S.

(vii) For any (one or more) pairwise distinct variables $v_1, v_2, \ldots$, any term t, and any x: $\ulcorner\exists v_1, v_2, \ldots\ t\urcorner$ denotes$_{I,g}$ x iff $\ulcorner\lambda v_1, v_2, \ldots\ t\urcorner$ denotes$_{I,g}$ an attribute of which x is an existential quantification.

(viii) For any ordinal $\alpha > 0$, any α-sequence of terms $t_0, t_1, \ldots$, any α-sequence of terms $u_0, u_1, \ldots$, and any x: $\ulcorner[t_0 u_0, t_1 u_1, \ldots]\urcorner$ denotes$_{I,g}$ x iff there are an α-sequence of roles $r_0, r_1, \ldots$ and an α-sequence of entities $y_0, y_1, \ldots$ such that: each t_i denotes$_{I,g}$ the corresponding r_i, each u_i denotes$_{I,g}$ the corresponding y_i, and x results from the role assignment $\{(r_i, y_i) \mid i < \alpha\}$.

(ix) For any ordinal $\alpha > 0$, any α-sequence of pairwise distinct variables $v_1, v_2, \ldots$, any term t, and any x: $\ulcorner\lambda v_1, v_2, \ldots\ t\urcorner$ denotes$_{I,g}$ x iff x is an α-adic attribute such that, for any α-sequence of entities $y_1, y_2, \ldots$ and any state s: s is an

[18] Since $\mathcal{L}$ has proper-class many constants, there is no such thing as *the* set of constants.

[19] The concept of an entity's *order* will be defined in the next subsection.

[20] I say '*an* identity' (and not '*the* identity') in order not to build ontological presuppositions into the semantics of $\mathcal{L}$. However, in Section 5.1 we will adopt an assumption according to which, for any y and z, there exists at most one identity of y and z. Analogous remarks apply to the next five clauses.

instantiation of x by $y_1, y_2, \dots$, in this order, iff, for some variable-assignment h that is just like g except for mapping each v_i to the corresponding y_i, s is denoted$_{I,h}$ by t.[21]

(x) For any ordinal $\alpha > 1$, any α-sequence of pairwise distinct variables $v_0, v_1, \dots$, any term t, and any relation R:

(∗) There is a sequence $\sigma \in R$ such that, for each $i < \alpha$, any term τ, and any y: τ denotes$_{I,g}$ y iff $y = \sigma(i)$ and, for some variables $u_1, u_2, \dots$ with $\{u_1, u_2, \dots\} = \{v_0, v_1, \dots\} \setminus \{v_i\}$: τ is identical with $\ulcorner \rho v_i.\, u_1, u_2, \dots\ t \urcorner$

iff R is denoted$_{I,g}$ by $\ulcorner \lambda v_0, v_1, \dots\ t \urcorner$.

This last clause specifies the semantics of rho-expressions. (To help with the parsing: (∗) is the left-hand side of the main biconditional of (x).) As may be seen from the right-hand side of (∗), the order of the u_i in a given rho-expression $\ulcorner \rho v.\, u_1, u_2, \dots\ t \urcorner$ does not matter, but it *does* matter which variable precedes that list. This is the reason why a dot (rather than a comma) is used to separate that variable from the list of the u_i. For example, suppose that the triadic relation of *giving* is the set $\{\langle G, g, r \rangle\}$, where G, g, and r are (respectively) the roles of *Giver*, *Gift*, and *Recipient*. Then G may be denoted both by '$\rho x.\, y, z\, [Gx, gy, rz]$' *and* by '$\rho x.\, z, y\, [Gx, gy, rz]$'. The immediate purpose of rho-expressions is to allow reference to the members of the role sets of non-fundamental relations. This will become relevant in connection with the 'analyzability assumption' of Section 5.3.[22]

We next have to turn to the concept of an entity's 'order', which has already been made use of in clause (ii) above.

4.3 The Assignment of Orders

The following example may help to illustrate some aspects of how the above semantics works and also give a hint as to how the order-theoretic hierarchy that characterizes the present ontology is motivated.

Let F be the property of *being Frege's favorite state*. From clause (ii) of the above semantics, it can be seen that a typed variable can only denote entities of some order less than the variable's type. More particularly, the variable 'x^1' will denote an entity x relative to an interpretation I and a variable-assignment g only if g maps 'x^1' to x and x is zeroth-order. From this, together with clauses (iv), (vi), and (viii), it follows that the formula '$[Fx^1] \wedge \neg x^1$' (which without the use of abbreviatory devices is written as '$\&([Fx^1], \neg x^1)$') will have a denotation relative to a given interpretation and variable-assignment *only* if the latter maps 'x^1' to a zeroth-order state. And from *this*, together with (ix), it follows that any property denoted by '$\lambda x^1\, ([Fx^1] \wedge \neg x^1)$' will have instantiations only by zeroth-order states. So the state $\exists x^1\, ([Fx^1] \wedge$

[21] Given the assumption (U2) of Section 5.1 below (according to which no attribute has any trivial converses other than itself), there will be at most one such attribute.

[22] See Plate (MS: §12) for more extensive commentary on (x).

$\neg x^1$), which obtains iff $\lambda x^1\,([Fx^1] \wedge \neg x^1)$ has an obtaining instantiation, will fail to obtain unless some zeroth-order state instantiates $\lambda x^1\,([Fx^1] \wedge \neg x^1)$—which in turn requires that some zeroth-order state instantiate F. Hence, if it happens that the state $\exists x^1\,([Fx^1] \wedge \neg x^1)$ is the *only* thing to instantiate F, then this state will fail to obtain unless it is zeroth-order. (And it had better *not* be zeroth-order, or else we would be faced with an Epimenidean paradox!)

The relevant notion of order can be defined as follows:[23]

Definition 4.1. For any ordinal α, an entity x is *αth-order* (alternatively: x is of order α) iff x is either a role or a state and α is the least ordinal β such that, for some interpretation I, variable-assignment g, and term t, the following three conditions are satisfied:

(i) t denotes$_{I,g}$ x.

(ii) Any atomic term that is either identical with t or has in t a free occurrence at predicate- or sentence-position denotes$_{I,g}$ a fundamental entity.

(iii) For any variable v: if v has in t a bound occurrence at predicate- or sentence-position, then v is typed, and its type is less than or equal to β.

It is easy to see that, under this definition, any fundamental role or state is classified as zeroth-order.[24] Likewise, for any x, the states $\text{Ʀ}\,x$ and $(x = x)$, the property $\lambda y\,(y = x)$, and the relational role $\rho x.\,y\,(x = y)$ are all zeroth-order. In general, to show that a given entity's order does not exceed a certain ordinal α, it is enough to produce a suitable term t that denotes that entity (relative to some interpretation I and variable-assignment g)—in particular, a term that satisfies the clauses (ii) and (iii) of the above definition for $\beta = \alpha$.

5 Ontological Assumptions

Let us say that something is an *intensional entity* iff it is either a role or a state; and let us say that a *particular* is anything that is neither a role nor a state. ORS can be very broadly described as offering a 'three-category ontology', in the sense that it treats each thing as falling into one of three very general categories—those of roles, states, and particulars—of which the first two are assumed to be non-empty. While there may certainly be distinctions to be drawn between different kinds of

[23]Cf. Plate (forthcoming: §4.1; MS: §8), but note the following difference: the present definition rules out that any particular (i.e., anything that is neither a role nor a state) has an order. Thus, particulars *fall outside* of the order-theoretic hierarchy as defined here. The reason for this has to do with the matters discussed in Section 7 below and, more specifically, with the desideratum that statements as to how many particulars there are should not turn out to be either necessarily true or necessarily false. (Another relevant difference lies in the fact that, as already foreshadowed in the Introduction, particulars are in the present paper classified as fundamental. This will become apparent in Section 5.3.)

[24]Cf. Plate (forthcoming: Corollary 3).

particulars that are every bit as significant as that between roles and states, the only distinctive positive claim that ORS makes about particulars is that each of them is a *fundamental* entity. This claim (which, as far as ORS is concerned, may be only vacuously true) follows from the 'analyzability assumption' (O2) that will be formulated in Section 5.3. All the other assumptions are primarily concerned with roles and states.

5.1 Uniqueness Claims

We begin our list with nine assumptions that will be labeled '(U*n*)', because each of them entails a characteristic uniqueness claim. The first concerns the individuation of roles and relies on the following concept of equivalence:

Definition 5.1. An entity x is *equivalent* to a role r iff, for some ordinal $\alpha > 0$, some α-sequence σ of roles, and some $i, j < \alpha$, the following four conditions are satisfied:

(i) $\sigma(i) = x$.

(ii) $\sigma(j) = r$.

(iii) For some permutation f on α that merely transposes i and j,[25] σ is instantiation-equivalent to $\sigma \circ f$.

(iv) For some α-sequence of entities $x_0, x_1, \ldots$, there exists a state that results from the role assignment $\{(\sigma(i), x_i) \mid i < \alpha\}$.

Here the fourth numbered clause is needed to avoid classifying as 'equivalent' any two roles that, intuitively put, have nothing to do with each other, such as *Lover* and *Teacher*. Another effect of this clause is that it is an open question whether every role is equivalent to itself.[26] By contrast, it is trivially the case that nothing *other* than a role is equivalent to a role. Our first assumption goes one step further:

(U1) For any role r: nothing other than r is equivalent to r.

An easy consequence of (U1) is that no relation is a pair set $\{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$, for two *distinct* roles r_1 and r_2.[27] As a result, any dyadic relation that is 'strictly symmetric',

[25]In the case of $i = j$, a permutation that 'merely transposes' i and j is simply the identity permutation.

[26]A role that fails to be equivalent to itself in the sense just defined is a role that does not 'suit' (in the sense of Definition 3.7) any entity. Such a role may also be called an *improper* role. A peculiarity of improper roles is that they are denotable neither by lambda- nor by rho-expressions; for the semantics of both requires (via the definitions of 'property' and 'relation') that the denoted roles be *proper* (i.e., not improper). For more on improper roles, see footnote 37 below.

[27]Suppose to the contrary that, for two distinct roles r_1 and r_2, the pair set $R := \{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$ is a relation. Then, by the definition of 'relation', R has an adicity greater than one. By the definition of 'adicity', it further follows that R has an instantiation by some sequence of entities of a length greater than one. Given the definition of 'instantiation' (and given that $R = \{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$), this means

in the sense of being its own non-trivial converse, is a singleton $\{\langle r, r\rangle\}$, for some role r. A salient example is the identity relation $\lambda x, y\, (x = y)$.

Our next assumption concerns the individuation of attributes:

(U2) For any attribute A: nothing other than A is a trivial converse of A.

Given that every attribute is a trivial converse of itself, (U2) ensures that every attribute has *exactly one* trivial converse, namely itself.[28]

The next seven assumptions all concern the individuation of states. We begin by defining a concept of *analytic entailment*:

Definition 5.2. A term t *analytically entails* a term t' iff the following two conditions are satisfied:

(i) t denotes a state relative to some interpretation and variable-assignment.

(ii) For any interpretation I and variable-assignment g: if t denotes$_{I,g}$ a state, then so does t', and if t denotes an *obtaining* state, then so does t'.

With the help of this concept, we can next define the following:

Definition 5.3. A state s *$\mathcal{L}$-necessitates* a state s' iff there exist an interpretation I, a variable-assignment g, and terms t and t' of $\mathcal{L}$ such that: (i) t denotes$_{I,g}$ s, (ii) t' denotes$_{I,g}$ s', and (iii) t analytically entails t'.

For example, given that the terms 's' and '$\neg\neg s$' analytically entail each other, we have that any state s $\mathcal{L}$-necessitates its double negation $\neg\neg s$, and *vice versa*. Trivially, any state $\mathcal{L}$-necessitates itself. The concept of $\mathcal{L}$-necessitation now allows us to formulate an assumption that, as it were, imposes a lower bound on the coarse-grainedness of states:

(U3) For any state s: no state other than s both $\mathcal{L}$-necessitates and is $\mathcal{L}$-necessitated by s.

More concisely put, no two states $\mathcal{L}$-necessitate each other. Together with (U2), this assumption also imposes a constraint on the individuation of *attributes* (and hence of roles).[29]

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that:

(1) R is a proto-relation; and

(2) For some entities x_1 and x_2, there exists a state that results from $\{(r_1, x_1), (r_2, x_2)\}$.

From (1), together with the fact that $R = \{\langle r_1, r_2\rangle, \langle r_2, r_1\rangle\}$, we can infer by the definition of 'proto-relation' that the sequence $\langle r_1, r_2\rangle$ is instantiation-equivalent to $\langle r_2, r_1\rangle$. And from this, together with (2), it follows that r_1 is equivalent to r_2 in the sense of Definition 5.1. From (U1) it can now be inferred that $r_1 = r_2$, but by hypothesis they are distinct: a contradiction.

[28]For slightly more extensive discussion, see Plate (forthcoming: §3.3.3).

[29]For example, since any state $\mathcal{L}$-necessitates its double negation and *vice versa*, it follows by (U3) that any state is identical with its double negation. Hence, for any property P, *its* 'double negation'—i.e., the property $\lambda x\, \neg\neg[Px]$—is a trivial converse of P. By (U2), we thus have that $\lambda x\, \neg\neg[Px]$ is identical with P. In this way, (U2) ensures that attributes are no less coarse-grained than their instantiations.

Our fourth assumption entails a uniqueness claim for the results of role assignments:

(U4) For any entity x and any role assignment S: if x results from S, then the following holds:

(i) x is a state.

(ii) Nothing other than x results from S.

This assumption has the welcome consequence that no attribute has more than one instantiation by any given entity or sequence of entities. Together with (U1) and (U3), it is moreover responsible for the fact that role assignments—the things from which states are said to 'result'—have to be thought of as (generalized) multisets rather than sets.[30]

We next adopt more or less analogous assumptions for rolehood, identity, and negation:

(U5) For any entities x and y: if x is a rolehood of y, then the following holds:

(i) x is a state.

(ii) Nothing other than x is a rolehood of y.

(U6) For any entities x, y, and z: if x is an identity of y and z, then the following holds:

(i) x is a state.

(ii) Nothing other than x is an identity of y and z.

(U7) For any entities x and y: if x is a negation of y, then the following holds:

(i) x is a state.

(ii) y is a state.

(iii) Nothing other than x is a negation of y.

Here the second numbered clause reflects the plausible idea that only states have negations (in the present sense of 'negation'). Similar assumptions suggest themselves for conjunction and existential quantification:

(U8) For any entities x and y: if x is a conjunction of (the members of) y,[31] then the following holds:

(i) x is a state.

[30]See Plate (MS: §16).

[31]The parenthetical 'the members of' is added for purely idiomatic reasons: it seems more natural to say that something is 'the conjunction of the members of S' (where S is a set of states) than to say that x is the conjunction of S.

(ii) y is a set of states.

(iii) Nothing other than x is a conjunction of (the members of) y.

(U9) For any entities x and y: if x is an existential quantification of y, then the following holds:

(i) x is a state.

(ii) y is an attribute.

(iii) Nothing other than x is an existential quantification of y.

None of these last six assumptions should be particularly surprising or controversial. But now we come to *existence* claims.

5.2 Existence Claims

We begin with an assumption that introduces a role (and more particularly, a property) of fundamentality, which is itself claimed to be fundamental:

(E1) There exists a fundamental property F such that, for any x, there is an instantiation of F by x, which obtains iff x is fundamental.

A straightforward consequence of this is the thesis that the state $[\mathsf{FF}]$—i.e., the instantiation of F by itself—obtains; or in other words, that F instantiates itself.[32] Clearly, this is a controversial assumption. It is controversial enough that it does not seem inappropriate that a large part of this paper (viz., Section 6 below) should be dedicated to its defense.

Since (E1) is mainly concerned with just a single property, it is a very specific assumption. Our next assumption is far more general:

[32]To readers familiar with the literature on metaphysical grounding, the claim that there is a fundamental property of fundamentality will be reminiscent of the following passage from Jessica Wilson:

> [T]he fundamental should not be metaphysically characterized in negative terms—or indeed, in *any* other terms. The fundamental is, well, *fundamental*: entities in a fundamental base play a role analogous to axioms in a theory—they are basic, they are 'all God had to do, or create'. (2014: 560)

A broadly similar sentiment can be found in Fine's (2001: 25) remarks on "the absolute notion of fundamental reality", which he claims to be not "in need of a relational underpinning". (Also cf. his [2005: §2; 2006], as well as Barnes [2012: 876f.].) A crucial difference between Fine's approach to fundamentality and the present treatment lies in the fact that his notion of fundamental reality applies to facts or sentences rather than entities in general. Thus he likens fundamental reality to a container of *facts* rather than simply of things. For relevant critical discussion, see Lipman (2018). The present notion of fundamentality, by contrast, applies to entities in general.

The question of whether fundamentality is fundamental has as its counterpart in a Lewisian setting the question of whether *perfect naturalness* is perfectly natural. This latter question has been answered in the negative by Thompson (2016), whereas Sider (2011: §7.13), who operates with a more general notion of *structuralness*, has given a positive answer. Relevant critical discussion of Sider's view may be found, e.g., in Torza (2017). Also see Section 6.4.2 below.

(E2) For any interpretation I, variable-assignment g, term t, ordinal $\alpha > 0$, and any α-sequence of pairwise distinct variables $v_0, v_1, \ldots$: if t denotes$_{I,g}$ a state and, for each $i < \alpha$, the following two conditions are satisfied—

(i) v_i occurs free in t.

(ii) If v_i is identical with t or has in t a free occurrence at predicate- or sentence-position, then v_i is typed.

—then there exists at least one attribute A such that, for any α-sequence of entities $x_0, x_1, \ldots$ and any state s, the following holds: s is an instantiation of A by $x_0, x_1, \ldots$, in this order, iff, for some variable-assignment h that is just like g except for mapping each v_i to the corresponding x_i, s is denoted$_{I,h}$ by t.

Given (U2)—which rules out the existence of attributes with more than one trivial converse—the 'at least one' in the consequent of (E2)'s main conditional may also be read as 'exactly one'. The second numbered clause of (E2) deserves special attention: it is responsible for the fact that (E2) does not, for example, commit us to there being (for any property P) a state $\exists x\,([Px] \wedge \neg x)$, whose existence would lead to intensional paradox if it were the only entity instantiating P.[33]

Five additional existence claims are complementary to (U5)–(U9):

(E3) For any entity x, there is at least one rolehood of x.

(E4) For any entities x and y, there is at least one identity of x and y.

(E5) For any state x, there is at least one negation of x.

(E6) For any set of states S, there is at least one conjunction of (the members of) S.

(E7) For any attribute A, there is at least one existential quantification of A.

Compared with (E1), these further assumptions should seem fairly innocuous. This is not to say that they are uncontroversial. A reader attracted to nominalism or some form of the 'Eleatic principle' will be strongly inclined to reject all of them, and unfortunately I do here not have the space for detailed justifications. However, a quick defense of a similarly plenitudinous system may be found in §4.2 and §6.1 of Plate (forthcoming). Very roughly, the starting point is the idea that we have excellent reason to believe in sets (or at any rate in things that *can play the theoretical role* of sets), and that it is philosophically attractive to integrate the universe of sets into the ontology of intensional entities, which in turn calls for a plenitudinous ontology of properties. Further, a natural way of constructing such an ontology makes use of a comprehension axiom; but for such an axiom—which would be (E2) in the present case—to yield a plenitudinous ontology of properties, we need a sufficiently generous ontology of states that those properties can be 'abstracted from'. Hence the need for assumptions like (E3)–(E7).

[33]Cf. the example discussed at the beginning of Section 4.3. Like (U2) and (U3), (E2) is essentially taken from Plate (forthcoming: §3).

In addition, there are also semantic considerations, which are especially relevant with regard to the question of why one should believe in the existence of *higher-order* intensional entities, and also with regard to the prior question of why one should believe that there are any such things as states or properties (or relations) at all. For example, even if there is not strictly speaking anything referred to by the English noun phrase 'the sky', we do express *a fact* when we say (in a typical context) 'The sky is blue', in a way in which we do *not* express a fact—or any other state—when we say 'Mome raths outgrabe'. I take this to be an absolutely indispensable distinction, and a philosopher would only seem to be passing the buck if she were to try to account for this difference by suggesting that 'The sky is blue' but not 'Mome raths outgrabe' is translatable into a certain privileged metalanguage. To see the relevance of this thought for the existence of higher-order intensional entities, one only has to apply it to the observation that we sometimes (as in the context of ORS) make general claims as to what roles or states there are—claims that, if they were to be translated into $\mathcal{L}$, would involve quantification into predicate- or sentence-position.

5.3 Three Other Assumptions

The remaining three assumptions are not classifiable as existence claims and do also not entail any salient uniqueness claims. I will label them '(O*n*)'; the 'O' stands for 'other'. The first of these simply expresses a dichotomy between states and roles:

(O1) No state is a role.

This is at least a plausible default assumption, as it is hard to see what plausible principle (or set of principles) would entail that there is some entity that is *both* a state and a role.

Our next assumption may be called an 'analyzability principle', since it can be paraphrased as the claim that anything whatsoever is 'fully analyzable' in terms of fundamental entities:[34]

(O2) For any entity x, there exist a variable-assignment g and a term t such that: t denotes$_{\emptyset,g}$ x, and any entity in the range of g is fundamental.

Since complex terms of $\mathcal{L}$ exclusively denote roles and states, (O2) has the consequence that any particular (i.e., anything that is *not* a role or state) is a fundamental entity. While this consequence may be counter-intuitive when one considers ordinary material objects such as tables and chairs, it can arguably be made (more) palatable by thinking of those objects as *events*, and of events as obtaining states;[35] for under this conception, those objects will no longer be particulars. Alternatively, if the conception of ordinary objects as events should prove unacceptable, one could

[34] Broadly similar principles include Sider's (2011: 116) 'Completeness' and Bacon's (2020: 566) 'Fundamental Completeness'.

[35] Cf., e.g., Nolan (2012).

also take an *eliminativist* stance and deny that, strictly speaking, there are any ordinary objects to begin with.[36] Finally, one could add to $\mathcal{L}$ a term-forming mereological fusion operator. This would allow one to admit particulars that are fully analyzable in terms of their fundamental parts, and it would no longer follow from (O2) that every particular is fundamental. The obvious downside of this last proposal, however, is that it would increase our "burden of primitive notions" (to use Lewis's memorable phrase) by the addition of a mereological concept.[37]

Complementary to Section 5.1's (U3), our final assumption imposes an *upper* bound on the coarse-grainedness of states. It is not easy to formulate such a principle without ending up with a claim that is either trivial or implausible. For instance, if we simply took the converse of (U3) and said that any states that fail to $\mathcal{L}$-necessitate each other are distinct, we would in effect say nothing more than that every state $\mathcal{L}$-necessitates itself, which is trivially true. To formulate a *non*-trivial principle, it will be convenient to talk about terms by which states can be denoted, and to impose a constraint on what pairs of terms can denote the same state. The basic idea is that, for any terms t and t' and any variable-assignment g, the following holds: *if* t and t' denote (relative to g and the empty interpretation[38]) the same state, and if certain further conditions are satisfied, *then* t analytically entails t'. More fully:

(O3) For any variable-assignment g, any terms t and t', and any state s: if the following five conditions are satisfied—

(i) t and t' both denote$_{\varnothing,g}$ s.

(ii) Any entity in the range of g is fundamental.

(iii) No two variables are under g mapped to the same entity.

(iv) Any variable that is under g mapped to an intensional entity (i.e., to a role or state) is of type one.

[36]See, e.g., van Inwagen (1990), Merricks (2001), Benovsky (2018). One could of course also adopt the 'disjunctive' view that either ordinary objects are events or there aren't any. Similarly for such things as poems, novels, and works of music, which form another *prima facie* non-empty class of entities that are commonly treated as non-fundamental particulars. Thus it may be held that *either* those entities are in fact properties—types of inscriptions or performances, say—*or* there aren't any such things as poems, novels, etc. Fictionalism about works of music (as understood in the Western classical tradition) has been defended by Kania (2008) and Killin (2018).

[37]For relevant discussion of the theoretical virtue of simplicity, see Section 6.1 below. Incidentally, in nearly the same way in which it can be shown to follow from (O2) that every particular is fundamental, it can also be shown to follow that every 'improper' role—i.e., every role that fails to suit anything (cf. footnote 26)—is fundamental. For in the first place, from the semantics of lambda- and rho-expressions, it can be inferred that no improper role is denotable by a lambda- or rho-expression; and in the second place, from (O1) we can infer that no role (and *a fortiori* no improper role) is denotable by a formula. Consequently, no improper role is denotable by a complex term of $\mathcal{L}$—which means, given (O2), that every improper role is fundamental. (Whether there *are* any improper roles is of course a further question.)

[38]Since anything that can be denoted by a constant can also be denoted by a variable, there is no need for present purposes to consider any terms that contain constants, and hence no need to consider any non-empty interpretations. This allows for a slightly simpler formulation of the principle in question.

(v) For any variable v: if v occurs free in t and is under g mapped to an intensional entity, then at least one free occurrence of v in t stands at predicate- or sentence-position.

—then t analytically entails t'.

For the motivation behind clauses (ii)–(v) and some of the consequences of this assumption, I have to refer the reader to Plate (MS: §9.2, §14)). A principle that is in some important ways similar has been discussed by Bacon (2020) under the label *Quantified Logical Necessity*.[39] For present purposes, we can refer to (O3) as an 'independence assumption', since it effectively asserts that the fundamental entities are independent of each other, in the sense that, for any fundamental entities x and y—with a potential exception to be noted in a moment—the existence of x is distinct from that of y.

To state the exception that has just been alluded to, we first have to introduce a concept of 'correlateship' among roles:

Definition 5.4. A role r is a *correlate* of a role r' iff, for some entities x and y, there exists a state that results from a role assignment containing the pairs (r, x) and (r', y).

For example, given that there exists a state that results from the role assignment $\{(\textit{Lover}, \text{Anthony}), (\textit{Beloved}, \text{Cleopatra})\}$, we can first infer that the roles *Lover* and *Beloved* are correlates of each other. Further, those two roles can also be denoted (relative to a variable-assignment that maps 'l' to *Lover* and 'b' to *Beloved*) by the rho-expressions '$\rho x.\, y\, [lx, by]$' and '$\rho y.\, x\, [lx, by]$', respectively. Since each of those two roles is thus denotable by a term in which a name of the respectively other role occurs free, it can now be inferred that the fact that *Lover* is self-identical and the fact that *Beloved* is self-identical $\mathcal{L}$-necessitate each other.[40] By (U3), these 'two' facts are therefore one and the same. Since this result would also obtain if *Lover* and *Beloved* were fundamental, we have here a potential exception to the mutual independence of fundamental entities.[41]

This completes my exposition of ORS, or at least its 'purely metaphysical' part. One important part that has so far gone unmentioned is the theory's mathematical bedrock, which consists of ZFC with urelements. Rules of inference and logical axioms have also been left implicit (although it *has* been mentioned that we would be using classical logic). Perhaps the three most controversial aspects of ORS can be found in its conception of relations, its plenitudinous character, and its treatment of fundamentality. The first two have already been defended (however briefly). It is time to turn to the third.

[39] Cf. also Plate (forthcoming: §5.4). A slight weakening of (O3) will be proposed in Section 8 below.

[40] Let g be a variable-assignment that maps 'l' and 'b' to (respectively) *Lover* and *Beloved*, let φ be the formula '$\rho x.\, y\, [lx, by] = \rho x.\, y\, [lx, by]$', and let ψ be the formula '$\rho y.\, x\, [lx, by] = \rho y.\, x\, [lx, by]$'. It can then be seen (since the variables that occur free in φ also occur free in ψ, and *vice versa*) that φ and ψ analytically entail each other. But φ denotes$_{\emptyset,g}$ the self-identity of *Lover*, while ψ denotes$_{\emptyset,g}$ the self-identity of *Beloved*. Hence these two states necessitate each other, as required.

[41] Cf. Plate (MS: §16).

6 Approaches to Fundamentality

Our assumption (E1) (Section 5.2) asserts, in effect, that there exists a fundamental property of fundamentality. We can think of this assumption as the outcome of three decisions:

1. For the purposes of theorizing about fundamentality, should we try to make do with pre-existing ('broadly logical') concepts, or should we help ourselves to a new primitive? Here we took the second route.

2. Given that we help ourselves to a new primitive, should this be a concept of fundamentality itself (or at least of something very closely related, such as *non*-fundamentality), or should it be a concept of something significantly different, such as grounding? Here we took the first path.[42]

3. Should we regard this primitive concept as picking out a fundamental or rather a *non*-fundamental attribute (where an attribute is fundamental '*qua* attribute' iff it is either a fundamental property or a fundamental relation in the sense of Definition 3.9)? We went with the first choice.

In Figure 1, this set of decisions is represented as 'Option C'. To be sure, not every proponent of Option C has to embrace (E1). For example, someone might hold that not fundamentality itself but *non*-fundamentality ('derivativeness') is a fundamental property. But arguably, the more interesting disagreements are to be found among proponents of *distinct* Options, and it is these 'inter-Optional' disagreements that will be at issue in the rest of this section. Primarily I will be concerned with defending Option C against the other three, and will do so by arguing against them. (Further, rather than to try to rule out every possible view that falls under each of those other Options, I will focus on those that I take to be either the most promising or the most prominent.) In addition, in Section 6.3 I will discuss a potential objection that might be raised against the specific form of Option C that is represented by ORS.

6.1 Against Option A

Considerations of ideological parsimony clearly favor Option A. After all, if we can do without introducing a new primitive, then it would be in the interest of ideological parsimony that we do so. According to the proposal that I would like to discuss in the rest of this subsection, fundamentality can be equated with *logical simplicity*, which in turn may be analyzed (following a proposal by Plate [2016: §3]) in a way that is analogous to a certain analysis of *mereo*logical simplicity.

[42]Nothing important will hinge on how exactly the phrase 'significantly different' is understood here.

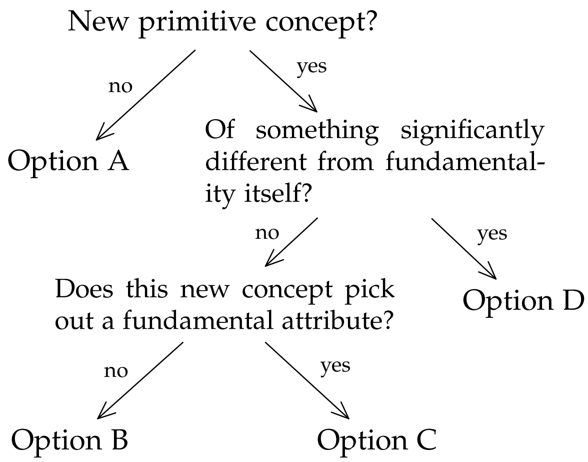

Figure 1: Some broad options for theorizing about fundamentality.

6.1.1 Logical simplicity

The analysis in question relies on a concept of 'reduction': for any interpretation I and variable-assignment g, a term t' (of $\mathcal{L}$) is a *reduction* of a term t relative to I and g iff t' is the result of replacing in t zero or more free occurrences of atomic terms with (relative to I and g) codenotational other terms, in such a way that no free variable-occurrence in any of the replacing terms 'becomes bound'. For an over-simple example, let I be any interpretation that maps 'Cicero' and 'Tully' to the same person, and let g be any variable-assignment. Then the term 'Cicero = Cicero' is a reduction of 'Cicero = Tully' relative to I and g (as well as as *vice versa*).

Let us further say that two terms of $\mathcal{L}$ are *analytically equivalent* iff they analytically entail each other (in the sense of Definition 5.2). We can then formulate the following analysis of logical simplicity:[43]

Definition 6.1. An entity x is *logically simple* iff, for any term t and variable-assignment g, the following holds: if t denotes$_{\varnothing,g}$ x, then there exists a variable-assignment $g' \supseteq g$ that satisfies the following two conditions:

(i) If x is a state, then, for some atomic term u, t has (relative to $\varnothing$ and g') a reduction that is analytically equivalent to u.

(ii) If x is a role, then, for some atomic terms $u_1, u_2, \ldots$ and variables $v_1, v_2, \ldots$, t has (relative to $\varnothing$ and g') a reduction $\ulcorner \lambda v_1\, \varphi \urcorner$ or $\ulcorner \rho v_1 . v_2, \ldots\, \varphi \urcorner$, where φ is analytically equivalent to $\ulcorner [u_1 v_1, u_2 v_2, \ldots] \urcorner$.

Note that any particular (i.e., anything that is neither a state nor a role) is trivially classified as logically simple.

[43]Here I am adapting my 2016 proposal to the present setting. The original proposal comes in two parts: one dealing with attributes (*op. cit.*: 25) and the other with states (34f.). Those two parts correspond to the two numbered clauses of the present definition.

An important point to note about the concept of logical simplicity, as it has just been defined, is that, on the background of a sufficiently coarse-grained conception of intensional entities (such as the one embraced here), that concept does not differentiate between a property and the 'negation' of that property. Thus, if a given property P is logically simple, then so is $\lambda x\, \neg[Px]$, and *vice versa*.[44] This creates a slight problem for the idea that fundamentality may in the context of ORS be equated with logical simplicity; for (O3) entails that, for any fundamental property P, the negation of P is *non*-fundamental.[45] Accordingly, if everything that is logically simple is to count as fundamental, then (O3) has to be weakened. Such a weakening could be effected by replacing both (O2) and (O3)—i.e., the two assumptions of ORS that make direct reference to fundamental entities—with a single principle according to which *some set* of fundamental entities satisfies suitably modified versions of (O2) and (O3):

(R) Some set S of fundamental entities satisfies the following two conditions:

(1) For any entity x, there exist a variable-assignment g and a term t such that: t denotes$_{\emptyset,g}$ x, and any entity in the range of g is a member of S.

(2) For any variable-assignment g, any terms t and t', and any state s: if the following five conditions are satisfied—

(i) t and t' both denote$_{\emptyset,g}$ s.

(ii) Any entity in the range of g is a member of S.

(iii) No two variables are under g mapped to the same entity.

(iv) Any variable that is under g mapped to an intensional entity (i.e., to a role or state) is of type one.

(v) For any variable v: if v occurs free in t and is under g mapped to an intensional entity, then at least one free occurrence of v in t stands at predicate- or sentence-position.

—then t analytically entails t'.

Here condition (2) differs from (O3) only in its second numbered clause, and (1) corresponds in a similar way to (O2). Condition (1) is needed to prevent (R) from

[44]See the argument in Plate (2016: 28), which carries over to the present setting with minor adjustments.

[45]See Plate (forthcoming: §5.5.2). It might be thought that there is also another problem. The fundamental things, taken together, are often held to provide a *minimal* basis for all there is (in some suitable sense of 'basis'; cf., e.g., Tahko [2023: §1.3]). For example, Lewis takes the "sparse" or "perfectly natural" properties to be such that "there are only just enough of them to characterise things completely and without redundancy" (1986: 60). This constraint is plausibly violated if we classify the negations of fundamental properties as fundamental, too. Arguably, however, it is not set in stone that there cannot be any redundancy among the fundamental entities. It is not clear that the non-redundancy constraint does any useful theoretical work, and if it only gets in the way of a promising approach to thinking about fundamentality, then there is little reason not to drop it. (Cf. also Sider [2020: 121].)

being a trivial truth: without it, the single remaining condition would be (2), which is trivially satisfied for $S = \emptyset$.[46]

And this seems to solve our problem. By simply replacing (O2) and (O3) with the single (though more complex) assumption (R), we can modify ORS in such a way as to allow us to equate fundamentality with logical simplicity, thereby reducing our burden of primitive notions. What is not to like?

6.1.2 Why not to equate fundamentality with logical simplicity

As far as I can see, the most compelling reason to reject the proposal under discussion lies in the added complexity it brings to ORS. Thus consider the thesis (R). In the form in which it has here been stated, this thesis is admittedly only *slightly* more complicated than the two principles it is designed to replace—namely (O2) and (O3)—taken together. But clearly, (R) becomes *significantly* more complex once fundamentality is taken to be logical simplicity in the sense of Definition 6.1. What is more, the same goes for the sort of principle to be discussed in Section 8 below, insofar as those principles rely on our concept of fundamentality. It would be nice if this added complexity could be avoided; and the obvious way to do so is to take the concept of fundamentality as primitive.

To be sure, this may not sway the friend of Option A. Under what conditions, she may ask, should we prefer a simpler theory over a more complex one? Is it not only if and when the discovery of the simpler theory comes as a surprise—a surprise resulting from the fact that, on the assumption that the more complex theory was correct, it was not to be expected that some simpler theory would have a comparable rate of predictive success?[47] But in the present case there is no surprise,

[46] With $S = \emptyset$, condition (2.ii) entails that the range of g is empty. But if the range of g is empty, then any term that denotes$_{\emptyset,g}$ something will contain no free occurrence of any variable or constant. Hence, for any terms t and t' and any state s: *either* condition (2.i) fails to be satisfied, *or* both t and t' denote$_{\emptyset,g}$ s, and neither of them contains a free occurrence of a variable or constant. In the first case, the antecedent of (2)'s main conditional fails to be satisfied. In the second case, the consequent of (2)'s main conditional *is* satisfied. In either case, (2) is satisfied.

[47] For recent discussions of simplicity as a criterion for theory choice in metaphysics, see, e.g., Schaffer (2015), Bradley (2018), Da Vee (2020), Sober (2022). Andrew Brenner has suggested that "for the most part, there are no good arguments for simplicity's being truth conducive in *any* contexts" (2017: 2688) and that our belief that simplicity *is* truth-conducive has to be accepted as "an epistemological bedrock, without which we would likely be stuck with a great deal of skepticism regarding, e.g., induction, other minds, and the external world more generally" (2688n.). This would be sobering if true.

One situation in which we can be justified in penalizing complex theories (or models) arises when we are confronted with 'noisy' data that contain potentially misleading patterns. (See, e.g., Claeskens and Hjort [2008].) The rationale suggested in the main text is largely orthogonal to this concern. To explain that rationale in a little more detail, it will be helpful to begin by distinguishing two kinds of complexity. A theory's *syntactic* complexity may be understood as the length of its most concise statement (in any language); and if the theory uses *defined* terms, then the relevant definitions have to be included. (As usual, a theory's primitive vocabulary may include 'observation terms'.) By contrast, what I will call the *theoretical* complexity of a theory T is the syntactic complexity that a *by and large complete* theory of the phenomena that T is concerned with—where a theory's 'degree of because the simplification essentially consists in the cheap trick of putting a new primitive (viz., 'fundamental') in place of the complex notion of logical simplicity. Cheap tricks deserve no rewards, and certainly don't justify the introduction of a primitive concept of fundamentality.

My reply is that the 'trick' in question is not as cheap as all that, and that there *is* a surprise, after all. For consider how the replacement of a complex concept with a novel primitive would usually play out: We start with a certain theory that, in at least one of its assumptions, makes use of a certain predicate F that is either itself complex or has a complex definition. Then we replace each occurrence of that predicate with our new primitive, G. In doing so, we typically lose the information encoded in the definition of F; and so, if the latter was to the effect that, for any

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completeness' is to be assessed, very roughly, by looking at what proportion of relevant facts (about the phenomena in question) can be derived from it—may be reasonably expected to have if T were true. (Finocchiaro [2021: 620] speaks in a similar vein of a theory's 'objective simplicity', which he defines as "something like simplicity with regard to the picture of the world provided by the theory".) For example, if a certain theory T posits ten fundamental types of particle, and we suppose T to be true, then, even if T itself says *nothing whatsoever* about those particles, we should presumably expect that a by and large complete theory of the phenomena that T is concerned with—which are, let us suppose, those of particle physics—will contain ten separate statements as to how the particles of each type behave; and the theoretical complexity of T will be correspondingly greater than that of a theory that posits only one fundamental type of particle. This is admittedly *quite* rough, but it is enough to go on with. The kind of simplicity at issue in the main text, as well as in the rest of this footnote, should be understood to be theoretical (as opposed to merely syntactic) simplicity.

So why are we justified in preferring simpler theories over more complex ones? The basic idea is quite straightforward. Suppose Vicky is a researcher who has been assuming that a certain theory T is correct. One day it turns out that there is a rival theory T^* that enjoys essentially the same measure of predictive success as T in the same areas as T, but is substantially simpler. (See Schaffer (2015: 648) for a similar example.) Normally at least, this should be a surprise. After all, under T, the syntactic complexity of a by and large complete theory of the phenomena T is concerned with may be reasonably expected to be substantially greater than under T^*; it would, e.g., have more statements corresponding to a greater number of fundamental kinds of particle. Since, in this sense, the world is a more complicated place under T than under T^*, one would expect this additional complexity to show up somewhere in the data (provided that the latter is of sufficient quality). But by hypothesis, it doesn't, for we said that T^* enjoys essentially the same measure of predictive success as T in the same areas as T. So, working with the assumption that T is correct, Vicky has put herself into a position where one of the expectations she was justified (on the strength of that assumption) to have turns out to be mistaken. This serves to disconfirm her theory. By contrast, someone working with a simpler theory need *not* be surprised to find that there exists a more complex theory with a similar or greater rate of predictive success in the same areas; for, as is well-known, with greater complexity comes greater flexibility in accommodating the data. (This last part of the argument should be reminiscent of the similarly probabilistic argument discussed by Huemer [2009: §II.4], which rests on the thought that a simpler theory often "makes more specific predictions" than a more complex one; but the *overall* argument is markedly different.)

Of course, there are certain special cases in which Vicky would *not* have been justified in having the described expectation. For example, suppose her original theory T posits ten fundamental types of particle and *in addition* claims that the instances of these ten types all behave in exactly the same way. In this case it would have been no surprise at all to learn of a simpler theory that works just as well: in particular, a theory that posits only a single fundamental type of particle *in lieu* of T's ten. But it seems clear that in this scenario the prior probability of T would have been considerably lower than that of its simpler rival. (Cf. Jansson and Tallant [2017].)

x, Fx iff φ (for some formula φ in which 'x' occurs free), then we typically have to restore that information by adding an assumption to the effect that, for any x, Gx iff φ. Thus, in the case under discussion, we would add an assumption to the effect that, for any x, x is fundamental iff x is logically simple in the sense of Definition 6.1.

But this is not how things play out in the present case. For suppose that, instead of (O2) and (O3), our original theory contains (R) together with a definition under which an entity is fundamental just in case it is logically simple. To arrive at the theory presented in Sections 2–5 above (i.e., ORS), we will then have to replace (R) with (O2) and (O3) and get rid of the mentioned definition, thus turning 'fundamental' into a primitive. Crucially, however, there is here no need to add an assumption to the effect that any x is fundamental iff x is logically simple; for (O3) already takes care of that. For example, it is one of the consequences of (O3) that, for any two fundamental properties P and Q, their 'conjunction', i.e., $\lambda x\,([Px] \wedge [Qx])$, is *non*-fundamental.[48] This is the surprise: given (O3), we *do not need* the information provided by the definition that equates fundamentality with logical simplicity. So there is a clear sense in which the proponent of the 'original' theory—i.e., the one that contains (R) instead of (O2) and (O3) and equates fundamentality with logical simplicity—misses out on an opportunity to adopt a simpler, more elegant theory; and achieving this gain in elegance would not be merely a cheap trick. Meanwhile, a proposal to simplify that theory in a different way, by simply discarding condition (2) from (R), would not do, because that would leave us without an upper bound on the coarse-grainedness of intensional entities, and consequently with a rather glaring lacuna.[49]

A line of reply still open to the friend of Option A is to insist that the greater elegance or simplicity of the present theory (which takes fundamentality as primitive) is trumped by the greater ideological parsimony of a theory that equates fundamentality with logical simplicity, where a theory's ideological parsimony is measured by

[48] Cf., again, Plate (forthcoming: §5.5.2). My point here is not that (O3) guarantees that an entity is fundamental *iff* it is logically simple, but rather that (O3) guarantees the *sparseness* of the fundamental entities.

[49] Even though, as I have argued, the simplification that consists in taking fundamentality as primitive and replacing (R) with (O2) and (O3) should not be regarded as a 'cheap trick', it might still be wondered whether the resulting gain in simplicity has any epistemic value—especially since it may be unclear how the considerations of footnote 47 carry over to the present case. I do think, however, that they do carry over. The main source of apparent disanalogy lies in the fact that, in the above footnote, the researcher in question ('Vicky') had to pay close attention to the *rates of predictive success* of the theories under consideration. (A theory that is not very good at predicting things should not usually be preferred over one that is, even if it is simpler.) Typically, predictive success is not a very prominent factor in comparisons of metaphysical theories. But there is such a thing as *coverage*: we can have at least a rough sense of how much a given metaphysical theory achieves, how many questions it settles, how many it leaves open, and whether it suffers from any obvious lacunae. We have little reason, in general, to prefer a simpler metaphysical theory over a more complicated one if the latter has significantly greater coverage. And now the analogy should be clear: If Vicky is a metaphysician working under the assumption that a certain complex theory T is correct, then she should be surprised to find that there is a significantly simpler theory that, without lapsing into absurdity, has at least roughly equal coverage.

the number of primitive notions employed in it. It would be interesting to see this reply developed in detail. Here I can only note that the burden of proof seems to lie with its proponents. In the absence of a compelling argument for the claim that ideological parsimony *does* trump theoretical simplicity, it appears to be a reasonable default position to consider the former but a special case (or 'aspect') of the latter, and accordingly to suppose that, over and above the extent to which it contributes to a theory's simplicity, there is no special reason to regard ideological parsimony as truth-conducive.

6.2 Against Option B

6.2.1 Preliminaries

Option B agrees with ORS that, for the purposes of theorizing about fundamentality, we should help ourselves to a dedicated primitive concept that is either one of fundamentality itself or of something very closely related (such as non-fundamentality). The disagreement between ORS and Option B concerns only the question of whether that new primitive should be taken to pick out a fundamental attribute.

In trying to adjudicate this issue, it may be best to start by setting aside a certain class of views that fall under Option B, on the grounds that these are not interestingly different from views falling under Option C. Thus suppose our theorist takes as primitive the concept of fundamentality, but it is not *this* concept that she takes to pick out a fundamental attribute, but rather that of *non*-fundamentality. I do not have much to say against such an approach. While our theorist technically disagrees with (or at least deviates from) Option C, this disagreement or deviation is not of much philosophical interest. Her position could be criticized by pointing out that it would be more natural for her, given that she regards the property of non-fundamentality as fundamental, to take the concept of *non*-fundamentality (rather than that of fundamentality) as primitive. This should seem an innocuous move to make; and if she makes it, she will thereby have landed in the camp of Option C. So the initial disagreement, if it can be called that, does in this case not seem very substantial at all.

Another relatively uninteresting way to deviate from Option C is to adopt a 'quietist' view that is simply neutral between Options B and C, in not taking any stance at all as to whether the primitive concept in question (whichever it may be) picks out a fundamental attribute. Such a view would of course be safer than either of those Options, but I think that in adopting it we would also be following a less fruitful and ultimately less illuminating course than if we were to adopt a more definite stance. Let us therefore set this view aside, as well.

6.2.2 Fundamentality treated as 'syncategorematic'

The form of Option B that I would like to focus on in the next few paragraphs is one under which *neither* the property of fundamentality, nor that of *non*-fundamentality, nor any other attribute that is fully analyzable in terms of fundamentality alone, is fundamental. To render ascriptions of fundamentality formally expressible, the approach in question adds to $\mathcal{L}$'s primitive vocabulary a new sentence-forming operator '$\mathrel{\text{Ⅎ}}$', whose semantics is such that, relative to any interpretation and any variable-assignment that maps 'x' to an entity x, the formula '$\text{Ⅎ}\, x$' denotes what may be called 'the state of affairs that x is fundamental' or, more succinctly, the *fundamentality of* x. For the sake of definiteness, let us call *syncategorematicalism* the view that results from ORS when we: (i) drop (E1), (ii) add to $\mathcal{L}$'s basic vocabulary the symbol 'Ⅎ', with the semantics just outlined, and (iii) adopt a definition of the monadic predicate 'is fundamental' under which an entity x is fundamental just in case $\text{Ⅎ}\, x$ obtains.[50]

With $\mathcal{L}$ modified in the way just described, it follows from (O3) that the property of fundamentality, which can now be denoted by '$\lambda x\ \text{Ⅎ}\, x$', fails to be fundamental. The argument is analogous to the one by which it can be shown in ORS that, e.g., the states $\top$ and $\bot$ are non-fundamental.[51] And in just the same way it can be shown that the property of *non*-fundamentality, i.e., $\lambda x \neg \text{Ⅎ}\, x$, is non-fundamental, too. The view should thus be appealing to theorists with a taste for symmetry. What is not to like?

Maybe the following. Given (U3) (which says that no two states $\mathcal{L}$-necessitate each other), syncategorematicalism has the consequence that any fact as to how many fundamental entities there are is identical with the 'empty conjunction' $\top$.[52]

How much of a problem is this? It is true that, already under ORS, states are individuated in a somewhat coarse-grained manner. For example, $\top$ is identical with the fact that $\top$ is self-identical, and it is also identical with the fact that no zeroth-order state both obtains and fails to obtain. Thus the formulas '$\top$', '$\top = \top$', and '$\neg\exists x^1\,(x^1 \wedge \neg x^1)$' all denote the same fact. That these formulas express distinct *truths*—distinct true propositions or 'objects of belief'—does not mean that they cannot all correspond to the same *fact*.[53] Nor is every truth corresponding to

[50]An alternative way to be a syncategorematicalist would be to introduce a special quantifier '$\exists^*$' that is supposed to range over all and only the fundamental things. The fundamentality predicate could then be introduced by saying that an entity x is *fundamental* iff it is the case that $\exists^* y\,(y = x)$. A closely related though slightly more complex account has been suggested by McDaniel (2017: 199): "If something exists and is in the domain of a perfectly natural quantifier, it has the highest degree of being: it *fundamentally* exists".

[51]Cf. Plate (forthcoming: §5.5.2).

[52]Let s be some fact (i.e., an obtaining state) as to how many fundamental entities there are. For example, supposing that there are at least two, s might be the fact that there are at least two fundamental entities. In our modified formal language, s can then be denoted (relative to any interpretation and variable-assignment) by the formula '$\exists x, y\,((x \neq y) \wedge \text{Ⅎ}\, x \wedge \text{Ⅎ}\, y)$'. Let t be this formula, let t' be '$\top$' (which can also be written as '$\&()$'), and notice that t and t' analytically entail each other. This means that s and $\top$ necessitate each other, so that, by (U3), they are one and the same fact.

[53]Propositions should not be confused with states. As the term is used here, a proposition is a special kind of concept, where a concept is, in the monadic case, just "a way of thinking of something" (Evans 1982: 21). A way of thinking of something may in turn, at least to a first approximation, be taken to be a relation between thinkers and things thought of: in order for me to think of an object x in a certain way, there will have to be a certain relation holding between myself and x. To be sure, if we wish to accommodate propositions involving *semantically defective* concepts, such as Pegasus or Batman, then this simple conception is no longer adequate. Instead, we might think

$\top$ what one would typically call a 'trivial' truth. One of them is the proposition—which follows from the set-theoretic axiom of Infinity—that there are infinitely many things. Let us call this proposition 'p'. Since p is expressible in 'broadly logical' terms (more precisely: by a formula of $\mathcal{L}$ that contains no free occurrence of a constant or variable), the fact to which p corresponds is nothing other than $\top$. This identification should not strike us as absurd. It would be a mistake to think that the *epistemic* distinction between trivial and non-trivial truths has to be neatly mirrored by a *metaphysical* division among facts.

But now consider claims as to how many *fundamental* things there are. I know of no good argument that would show that the number of fundamental things exceeds every cardinality. Plausibly, there is just a set of them. If so, the number of fundamental things is given by a certain cardinal κ. It is also plausible to think that the fact that there are exactly κ-many fundamental things is in a certain sense 'contingent': there could have been more, and (at least as long as $\kappa > 1$) there could have been fewer.[54] Supposing, then, that facts about the number of fundamental entities are contingent, the fact that there are κ-many fundamental entities should certainly *not* be identified with $\top$. Accordingly, syncategorematicalism should be rejected.

6.2.3 A complication

In this connection it should be noted that the thesis that "facts about the number of fundamental entities are contingent" does not hold in full generality: an exception has to be made for the claim that there exists at least *one* fundamental entity. To see this, we have to begin with the following theorem:

Proposition 6.2. If there is *no* fundamental entity, then $\top$ and $\perp$ are the only two states; but if there is at least one fundamental entity that is not a state, then there are proper-class many zeroth-order facts.[55]

From this, together with (O3), it is easy to infer the following biconditional:

Corollary 6.3. There exists at least one fundamental entity iff there are more than two zeroth-order states.[56]

of monadic concepts as relations between thinkers and *mental representations*: for me to 'think of Pegasus' in a certain way, there has to be a certain relation holding between myself and my mental Pegasus-representation. But either way, monadic concepts would appear to be dyadic relations, and propositions would analogously appear to be properties. On the present view, that makes them roles; and from (O1) ("No state is a role") it then follows that no proposition is a state.

[54]How exactly should these modal claims be understood? One option would be to spell them out in terms of 'possible worlds', which in turn may be conceived of as states. For present purposes, however, it will be enough to rely on a purely intuitive grasp, leaving the question of analysis for some later time.

[55]This result is nearly identical to Proposition 2 in Plate (forthcoming: §5.5.3) and can be obtained in the present setting by a very similar argument, relying on (U3), (O2), and (O3). (Note that, in the context of ORS, the identity relation is classified as *non*-fundamental.)

[56]To prove the left-to-right direction, suppose that there is at least one fundamental entity, and let x be some such entity. Then either x is a fundamental *state* or it isn't. Suppose it is. Then, since $\top$ and

Meanwhile, it may be thought that the fact (if it is a fact) that there are more than two zeroth-order states can in $\mathcal{L}$ be denoted by the formula

$$\exists x^1, y^1, z^1 \left((x^1 \neq y^1) \ \wedge \ (x^1 \neq z^1) \ \wedge \ (y^1 \neq z^1) \ \wedge \right. \\ \left. (x^1 = \&(x^1)) \ \wedge \ (y^1 = \&(y^1)) \ \wedge \ (z^1 = \&(z^1)) \right),$$

where the three conjuncts on the second line serve to express the statehood of the entities x^1, y^1, and z^1, respectively. Whether or not it is a fact, let s be the state expressed by the displayed formula.

Now, if s is indeed a fact, then it is identical with $\top$; otherwise with $\bot$. This much follows from (U3). But if it is a contingent matter whether there exists a fundamental entity, then, given Corollary 6.3, it should also be contingent whether there are more than two zeroth-order states, and hence whether s obtains. And by what has been said at the end of Section 6.2.2, this would seem to *rule out* that s may be identified with either $\top$ or $\bot$. We would thus have a contradiction. Therefore it cannot be granted that it is contingent whether there exists a fundamental entity.

I accept this result. However, I still wish to maintain that any claim about the number of fundamental entities that is *stronger* than 'There exists at least one fundamental entity' should be regarded as contingent; and this will be sufficient for the purposes of our above argument against syncategorematicalism. Making an exception for the claim just mentioned should not seem *ad hoc* in the context of ORS, given that ORS contains the assumption that the property of fundamentality itself is fundamental, which has that claim as a consequence.

Although I take the above argument (from Corollary 6.3 onwards) to be sound, it contains at least one problematic point, namely the suggestion that "the fact (if it is a fact) that there are more than two zeroth-order states can in $\mathcal{L}$ be denoted" by the displayed formula. This will become clearer in the next subsection.

6.3 Against Option C

The theorist who casts her lot with Option C helps herself to a novel concept, either of fundamentality or of something closely related, and takes this concept to pick out a fundamental attribute. In the case of ORS, this concept is of course that of fundamentality itself. In this section I will briefly discuss a potential objection against ORS. If it succeeds, then this may also go some way towards undermining Option C in general.

The objection revolves around properties of the form $\lambda x \, \exists y^\alpha \, (x = y^\alpha)$ for $\alpha > 0$, but here it will suffice to consider only $\lambda x \, \exists y^1 \, (x = y^1)$. From the semantics of lambda-expressions, it can be seen that this property is instantiated by all and only zeroth-order entities. It thus appears that, for all intents and purposes, this property

$\bot$ are zeroth-order but neither of them is fundamental—as can be shown with the help of (O3)—it follows that x, $\top$, and $\bot$ are three zeroth-order states; and hence there are more than two such states. Next, suppose that x is *not* a state: in this case we can apply the second conjunct of Proposition 6.2. Finally, the right-to-left direction follows from the first conjunct of that Proposition.

can be referred to as 'the property of *being a zeroth-order entity*'. But now consider also the definition of 'order of' stated in Section 4.3. From that definition we learn that an entity x is zeroth-order iff zero is the least ordinal that is such that, for some interpretation I and variable-assignment g, there exists a term t satisfying the following three conditions:

(i) t denotes$_{I,g}$ x.

(ii) Any atomic term that is either identical with t or has in t a free occurrence at predicate- or sentence-position denotes$_{I,g}$ a *fundamental* entity.

(iii) Any variable that has in t a bound occurrence at predicate- or sentence-position is of type zero.

Since there are in $\mathcal{L}$ no variables of type zero, this can be rephrased by saying that an entity x is zeroth-order iff it is denoted, relative to some interpretation I and variable-assignment g, by some term that satisfies condition (ii) while containing *no* bound variable-occurrence at predicate- or sentence-position.

But this slight simplification does not (the objection continues) change the fact that the property of *being a zeroth-order entity*—or 'zeroth-orderhood' for short—must be analyzable in terms of fundamentality, as can be seen from condition (ii). And under the assumption that fundamentality itself is fundamental (as ORS claims), it can be shown with the help of (O3) that zeroth-orderhood is *distinct* from $\lambda x\, \exists y^1\, (x = y^1)$.[57] So this property can*not* be referred to as 'the property of *being a zeroth-order entity*', which plainly contradicts what we have said above. This may be taken to show that there is something fundamentally wrong with ORS, or at least with the claim that fundamentality is fundamental.

Thus far the objection. Its Achilles heel lies (arguably) in its justification of the claim that $\lambda x\, \exists y^1\, (x = y^1)$ can "for all intents and purposes" be referred to as 'the property of *being a zeroth-order entity*'. While it is true that $\lambda x\, \exists y^1\, (x = y^1)$ is instantiated by all and only zeroth-order entities, this does not guarantee that it can be felicitously referred to in any specific way in English, and ORS makes no prescriptions on this matter. It is thus open to a proponent of ORS to regard the argument as showing merely that $\lambda x\, \exists y^1\, (x = y^1)$ can only for *some* intents and purposes be referred to as 'the property of *being a zeroth-order entity*'. Once this way out is seen, the objection is effectively defused.

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[57] That zeroth-orderhood is 'analyzable in terms of fundamentality' should here be taken to mean that there exist a variable-assignment g and a term u such that u denotes$_{\emptyset,g}$ zeroth-orderhood while containing a free occurrence of a variable that is under g mapped to fundamentality. To prove the claim in question, let g and u be some such variable-assignment and term, let L be the lambda-expression '$\lambda x\, \exists y^1\, (x = y^1)$', and suppose for *reductio* that zeroth-orderhood is identical with $\lambda x\, \exists y^1\, (x = y^1)$. Further, let s be the self-identity of zeroth-orderhood, let t be the formula $\ulcorner L = L \urcorner$, and let t' be the formula $\ulcorner u = u \urcorner$. Then the first numbered clause of (O3) will be satisfied, since both t and t' denote$_{\emptyset,g}$ s; and without loss of generality we may assume that the other four clauses are satisfied, as well. It then follows from (O3) that $\ulcorner L = L \urcorner$ analytically entails $\ulcorner u = u \urcorner$. But since u but not L contains a free occurrence of a variable, we have that $\ulcorner L = L \urcorner$ but not $\ulcorner u = u \urcorner$ has a denotation relative to the empty variable-assignment. Consequently, $\ulcorner L = L \urcorner$ does *not* analytically entail $\ulcorner u = u \urcorner$, and so we have a contradiction. This completes the *reductio*.

6.4 Against Option D

Finally, we come to Option D, which involves helping oneself—for the purpose of theorizing about fundamentality—to a new primitive concept of something "significantly different from fundamentality itself". The currently most prominent concept (or family of concepts) that has been employed in this role is without doubt that of *metaphysical grounding*. This will be discussed first. In Section 6.4.2, I will then turn to the 'structural' approach defended by Sider (2011).

6.4.1 Grounding

Grounding is said in many ways. One of the principal distinctions to be drawn in this area concerns the question of what kind of thing is supposed to be grounded: a fact, a (true) proposition, or just any old entity? We can accordingly speak of 'fact-grounding', 'proposition-grounding', and 'entity-grounding'. Using the first of these notions, we might give an account of fundamentality along the following lines:

(F1) An entity x is *fundamental* iff x is a constituent of some ungrounded fact (i.e., of a fact that is not grounded by any plurality of facts).[58]

But what is a 'constituent'? In the context of ORS, this concept might (to a very first approximation) be explicated as follows:

(C) An entity x is a *constituent* of an entity y iff there exist a variable-assignment g and terms t and t' of $\mathcal{L}$ that satisfy the following two conditions:

(i) t occurs free in t'.

(ii) t and t' respectively denote$_{\emptyset,g}$ x and y.

Given (U3), the concept thus defined is extremely liberal: since any state s is under (U3) identical with its own double negation $\neg\neg s$, it follows from (C) that, for any state s, the negation of s (in symbols, $\neg s$) is a constituent of s. A proponent of (F1) may therefore wish to adopt a more fine-grained conception of intensional entities. Alternatively, we could use a concept of *entity*-grounding and follow Schaffer (2009: 373) in simply saying that:

(F2) An entity x is *fundamental* iff nothing grounds x.

[58]Cf. Fine (2001: 20): "If a given constituent C occurs in a true basic factual proposition then it must be a fundamental element of reality". A true proposition is 'basic' in Fine's sense just in case it is "not grounded in other propositions" (p. 17). Other proponents of (F1) include deRosset (2013, 2023), Litland (2017, 2023), Shumener (2020), Lo (2022); critical voices include Dasgupta (2016: 411), J. Barker (2022), Correia (2023). In personal communication, Lo has indicated that he now also takes a more critical stance towards (F1). *Prima facie*, of course, criticisms of (F1) will not make much sense unless it is understood as a substantive thesis. But even where (F1), or something like it, is taken as a stipulative definition (as in the case of Litland [2017: 284]), a prior intuitive grasp of the term 'fundamental' exerts its pull, so that it becomes possible, at least in principle, to criticize (F1) by casting doubt on its extensional adequacy. (In fact, this is exactly how I *will* criticize it two paragraphs further down.)

On this second approach, we no longer need a notion of constituency.

In order to reframe ORS along ground-theoretic lines, we might adopt either (F1) or (F2) as our definition of 'fundamental' and replace Section 5.2's (E1) with an assumption to the effect that there exists a fundamental relation of grounding.[59] A grounding theorist may argue that the resulting theory is superior to the original version of ORS, because (i) we need a concept of grounding in any case and (ii) it is unclear how one might give an account of grounding on the basis of a primitive concept of fundamentality.[60] What, then, is not to like?

A first point to note is that (F1), despite the support it enjoys in the literature, looks rather doubtful on closer inspection. To adapt a famous example from J. L. Austin (1956: 11n.), consider the archangel Gabriel, and suppose that he owns a goat named 'Sue'.[61] Suppose further that one day he takes a liking to that goat, and that the relation of *liking* is a *fundamental* relation in the sense of Definition 3.9. (Readers who find this hard to accept should feel free to substitute a relation they take to be fundamental.) In our story, nothing has caused Gabriel to like his goat; and more importantly, as we may also suppose, there is nothing *in virtue of which* he likes it, either—he just does. Now, the fact that Gabriel likes Sue has at least the following two constituents: Gabriel and Sue. (We can ignore for now the two roles of *Liker* and *Likee*.) Since there is nothing in virtue of which this fact obtains, it is an ungrounded fact. So, if we take (F1) as our definition of 'fundamental', then the goat turns out to be a fundamental entity—which seems absurd.[62] I conclude that (F1) should be rejected. The upshot is that we had better rely on (F2) instead of (F1), and correspondingly employ a notion of *entity*-grounding. (This is of course not to say that any concept of fact- or proposition-grounding must now be discarded. It only means that we won't be employing any such concept in our account of fundamentality.)

We next have to note that the grounding-theoretically modified version of ORS (formulated using entity- rather than fact- or proposition-grounding) leaves open many questions as to what grounds what. For consider: as before, our assumption (O3) gives us valuable guidance with respect to the question of what entities are or aren't fundamental, and hence with respect to what entities are or aren't ungrounded. For example, it tells us that the states $\top$ and $\bot$ are *not* ungrounded. But if these states are not ungrounded, then presumably they must be grounded in

[59]There are also more complex ground-theoretic definitions of 'fundamental'. For example, Raven (2016) proposes to equate fundamentality with a kind of 'ineliminability' that is in turn spelled out in terms of fact-grounding and constituency. As far as I am aware, however, any such proposal—Raven's included—takes the right-hand side of (F1) to express a condition that is *sufficient* for fundamentality; and that is enough to render it vulnerable to the next paragraph's objection against (F1).

[60]Thus Schaffer (2016) in a critical discussion of Wilson (2014), in which he charges that "it is not at all obvious that using absolute fundamentality as a primitive will allow one to say everything one wants to say in terms of relative fundamentality, or in the even stronger linking terms of the grounding connection" (158). For Wilson's reply, see her (2016).

[61]Admittedly, I have altered Austin's example in a few ways.

[62]If the friend of (F1) insists that the goat is fundamental, let us modify the argument by replacing the goat with some entity, such as the state $\top$, whose non-fundamentality can be proved by (O3).

something; and what could that be?

Perhaps the most plausible thing to say in response is that $\top$ and $\bot$ are *zero-grounded*, to use a term coined by Fine (2012: 47f.) in connection with proposition-grounding.[63] This move requires that we employ, instead of a grounding relation between individual entities, one that takes as its first relatum *sets* (or pluralities) of entities. With this conceptual shift behind us, we can, for example, express the idea that $\top$ and $\bot$ are zero-grounded by saying that these states are grounded in the empty set (or the empty plurality).[64] So far, so good. What is still missing, however, is a principled reason as to *why* $\top$ and $\bot$ should be grounded in the empty set, as opposed to simply being ungrounded. Here (O3) gives us no guidance at all; and as long as the concept of ground is taken as primitive, it is difficult to see where such reasons might come from. And of course, this epistemological malaise does not only affect the question of what grounds $\top$ and $\bot$. For example, consider the non-fundamental relation $\lambda x, y\,((x = x) \wedge (y = y))$, whose instantiation by any entities x and y is the conjunction of the states $(x = x)$ and $(y = y)$. Is this relation grounded in that of identity, or rather in the property of *being self-identical*, or in both? There seems to be no obvious path to an answer.

In a nutshell, then, our situation is this: with (F2) and (O3), we have a combination of principles that tells us that various entities fail to be ungrounded, without also telling us *what* those entities are grounded in. Faced with this awkwardness, a grounding theorist may be moved to abandon the orthodox idea—promoted by, e.g., Schaffer (2009) and Rosen (2010)—that grounding should be taken as primitive and start to entertain a reductive account. With regard to fact- and proposition-grounding, a number of such accounts have already been put forward.[65] In the present context, we might turn to the concept of constituency, as tentatively defined in (C) above, and say that an entity x *grounds* an entity y just in case x is a constituent of y. To avoid the consequence that states are grounded by their own negations, we might further adopt a more fine-grained conception of states, which can be achieved by strengthening the concept of analytic entailment. (Never mind, for now, whether such a move would be *ad hoc*.) If we then combine the resulting account of entity-grounding with (F2), we get as an immediate consequence the result that an entity is fundamental iff it has no constituents.[66] But now there are two further points to make.

First, we should keep in mind that (O3) still tells us that $\top$ is non-fundamental. Yet presumably we do not wish to say that this state has constituents; and so, under the account now under consideration, $\top$ turns out to be fundamental, after all: we have reached a contradiction. So we have to either give up (O3) or adopt a different

[63] For a recent book-length study, see Kappes (2023).

[64] I am here sliding over the fact that instantiation by pluralities is a rather problematic affair; see footnote 12 above.

[65] See, e.g., Wilsch (2015, 2016), Poggiolesi (2016, 2021), Litland (2017), Correia and Skiles (2019), Correia (2021), Haderlie and Litland (forthcoming).

[66] Cf. Bennett (2017: 103): "to be fundamental is to be unbuilt".

account of entity-grounding.

Second, by combining (F2) with the proposal to equate entity-grounding with constituency (as defined in (C)), we have left Option D behind and have effectively adopted a view that falls under Option A. But, as we have already seen in Section 6.1, Option A should arguably be rejected: the resulting version of ORS is unattractively complex when compared to a theory that simply takes the notion of fundamentality as primitive. Likewise, by analyzing our concept of fundamentality in terms of grounding (be it fact-, proposition-, or entity-grounding) and then *further* spelling out the relevant concept of grounding in terms of whatever other notions, we substantially complicate not only our account of the individuation of states, but also the concept of order that underlies our order-theoretic hierarchy. The proponent of a ground-theoretic reframing of ORS thus finds herself confronted with a dilemma: either take the relevant notion of ground as primitive and be saddled with a host of open questions as to what grounds what, or embrace a reductive account and justify the choice of a needlessly complex theory.

To be sure, a staunch defender of a primitive concept of grounding will not be fazed by the first horn of this dilemma. ("So what if there are a few more open questions?") It is worth considering, however, how *unnecessary* many of these open questions appear to be, and how much easier life becomes once they are eliminated. If we operate with a primitive notion of fundamentality, instead of trying to analyze fundamentality in terms of grounding, we won't have to wonder what non-fundamental entities such as $\top$ and $\lambda x, y\,((x = x) \wedge (y = y))$ might possibly be grounded in, and nor will we need to make sense of 'zero-grounding'. At least to my mind, this is an advantage that is hard to pass up. If our grounding theorist still wishes to theorize in terms of a primitive concept of grounding, then she should certainly feel free to do so, and she should also feel free to define a concept of fundamentality in terms of it. But enough has been said to make us skeptical of the idea that a concept of grounding (whether primitive or not) should be employed in the foundation of ORS.[67]

6.4.2 Structure

In his *Writing the Book of the World*, Ted Sider has put forward a highly distinctive approach to fundamentality that arguably also falls under Option D. Sider rejects the

[67] Jon Litland has recently suggested that the logical operations of conjunction and disjunction (as well as identity and universal quantification) may be defined in terms of fact-grounding. If this can be sustained, then it may be argued that there is a place for grounding in the foundation of ORS, after all. For instance, Litland proposes to define *conjunction* as the unique dyadic operation R that is such that, for any propositions p and q (or *states* in our terminology), the proposition Rpq is grounded in p, q (in symbols: $p, q \ll Rpq$) while the negation of Rpq is grounded in each of $\neg p$ and $\neg q$ taken separately (2023: 864). Circularity is avoided through the use of a 'real definition operator' that can be flanked on its right-hand side by a *list* of sentences. All this forms part of an ambitious ongoing research program, which it is here not possible to do justice to. For now it may suffice to note that it is not easy to see, given that we have rejected (F1), how the concept of a fundamental entity might be defined on the basis of Litland's notion of fact-grounding.

use of a fundamentality predicate altogether and opts instead for a syncategorematic sentence-forming operator '$\mathscr{S}$' (for 'structural'). This operator can combine with expressions "of any grammical category" (p. 92) from any (interpreted) language, including English and the formal language of predicate logic. Sider is led to this approach in part by the thought that it is not enough to ask merely which *predicates* 'carve at the joints', since an analogous question also arises about quantifiers and other expressions of which a nominalist would be even more reluctant to say that they correspond to anything 'out there'.[68] From this perspective, the approach taken by ORS must seem quite inadequate: Yes, we can say that such-and-such roles are fundamental, but should we not also say that the operators of $\mathcal{L}$ (such as 'λ', 'ρ', or the square brackets) are privileged in exactly the same sense—a sense that may be provisionally captured by saying that all these things somehow 'carve at the joints'? But since those operators—expressions of $\mathcal{L}$—are obviously not fundamental *entities*, our fundamentality predicate is ostensibly of no help in this regard, and neither is the constant '$\mathbf{F}$' by which we have been trying to refer to the alleged property of fundamentality.

This objection strikes me as misguided. It can be granted that the various operators of $\mathcal{L}$ are in *some* sense privileged, but there is little reason to think that this sense is the same as the sense in which fundamental entities might be said to be privileged. To make this clearer, here is *one* way in which the operators of $\mathcal{L}$ may be said to be privileged (at least if ORS is correct):

> For each operator O of $\mathcal{L}$, the following holds:
>
> ($*$) The 'analyzability assumption' (O2) would not be true if O were missing from $\mathcal{L}$.

For example, if the operator 'ρ' were missing from $\mathcal{L}$, then any given role that is not a property could in $\mathcal{L}$ only be denoted by a variable or constant. By (O2), it would then follow that every such role is fundamental;[69] but since *it is not the case* that every such role is fundamental, this would mean that (O2) is false.

Let us say that an operator is *$\mathcal{L}$-privileged* iff it is privileged in the way just adumbrated. As indicated, each operator of $\mathcal{L}$ is $\mathcal{L}$-privileged. Similarly, if $\mathcal{L}'$ is a language exactly like $\mathcal{L}$ except for employing a *disjunction* operator instead of $\mathcal{L}$'s conjunction operator, then each operator of $\mathcal{L}'$ will be $\mathcal{L}'$-privileged. We can also imagine a 'redundant' language $\mathcal{L}''$ only *some* of whose operators are $\mathcal{L}''$-privileged: for example, a language that contains *both* a disjunction and a conjunction operator.

[68] By what has just been said about the 'grammar' of '$\mathscr{S}$', Sider's ascriptions of structuralness (i.e., sentences that have '$\mathscr{S}$' as their main operator) have to be thought of as formulated in languages that are obtained from pre-existing languages by adding '$\mathscr{S}$'. This raises the question of how one might hope to construct a formal semantics of such an 'augmented' language without relying on the semantic values of the expressions in question, but I won't press the issue here. In more recent work, Sider (2020: §1.8) has employed a primitive concept of fundamentality that applies to *concepts*. (But what sort of thing is a concept? In footnote 53 above, I suggest that it is a special kind of role.)

[69] While (O2) does not explicitly mention $\mathcal{L}$, it is to be understood in such a way that the terms it quantifies over are terms of $\mathcal{L}$.

And in addition we can imagine an expressively impoverished language for which (O2) does not even hold in the first place: a language for which it is not the case that for every entity x there exist a variable-assignment g and a term t (of that language) such that t denotes$_{\emptyset,g}$ x and every entity in the range of g is fundamental. If ORS is correct, then $\mathcal{L}$ may be said to be privileged over many other languages in that it is *not* impoverished in this sense.

A critic might object that being $\mathcal{L}$-privileged is itself not a very privileged way of being privileged. Instead it is only one among many, as illustrated by the case of the language $\mathcal{L}'$ just mentioned. My reply has two parts. First: To a large extent, this is as it should be. In the same way in which it would be odd if nature were to privilege the relation *longer-than* over its converse *shorter-than*, it would also be odd if a conjunction operator were in some objective, absolute sense privileged over a *dis*junction operator. By contrast, Sider's approach entails that there is an objective fact of the matter as to whether $\mathscr{S}(\wedge)$ or $\mathscr{S}(\vee)$, and this renders it *prima facie* implausible. Second: For many languages, we can still identify an objective, absolute sense in which the operators of our present language $\mathcal{L}$ are of greater interest to the metaphysician, and better to theorize with, than those of that other language. For one thing, the operators of $\mathcal{L}$ are relatively few in number. This makes them easier to remember than those of a rival language that has, say, a dedicated α-adic conjunction operator for each ordinal α. For another thing, the operators of $\mathcal{L}$ are in an important sense 'pure': as long as we do not also use constants or free variables, the states we can express with them either hold or fail to hold 'no matter what'. For a simple example, the state $\exists x, y\,(x \neq y)$ obtains regardless of how many people there are currently in this room. This is another reason why the operators of $\mathcal{L}$ should be of interest to the metaphysician (whose concern, after all, lies with *general* matters), or at least of greater interest than they would have been otherwise.[70]

That we can point to these ways in which the operators of $\mathcal{L}$ are privileged over those of many (though by no means all) other languages, rather than having to rely on a primitive operator of 'structuralness', means that the proponent of ORS has no great difficulty replying to the kind of challenge raised by Dasgupta (2018); and this arguably gives us another reason to prefer the present approach over Sider's. To elaborate: what Dasgupta essentially points out is that, if the notion of joint-carvingness is taken as primitive, then it remains mysterious why we should strive

[70]For a similar point and related criticism of Sider's view, see Busse (2020: 110n.). There is also yet another sense in which, under the assumption that ORS is correct, the operators of $\mathcal{L}$ may be said to be privileged over those of many other languages. For let us say that a state s *absolutely necessitates* a state s' iff s is identical with the conjunction of s and s'. If ORS is correct, then $\mathcal{L}$-necessitation coincides with absolute necessitation: a state s $\mathcal{L}$-necessitates a state s' iff s absolutely necessitates s'. (The left-to-right direction follows from (U3), while the right-to-left direction is trivial.) This condition would plausibly *not* hold if $\mathcal{L}$ were to contain, e.g., a term-forming operator 'ι' that would allow the fact that Scott is Scott to be denoted by '$\iota x\,[Wx, wy] = \iota x\,[Wx, wy]$' relative to a variable-assignment that maps 'y' to *Waverley* while mapping 'W' and 'w' to the roles of *Writer* and *Work* (respectively). For then the fact that Scott is Scott would (given a natural semantics of the 'ι' operator) $\mathcal{L}$-necessitate the self-identity of *Waverley*; whereas, plausibly, the self-identity of *Waverley* is not *absolutely* necessitated by that of Scott.

to theorize in terms of joint-carving notions. But the advantages of $\mathcal{L}$ that have been identified in the previous paragraph do not rely on any such notion, and this means that Dasgupta's challenge does not arise for them.

A reader might wonder whether Dasgupta's challenge does not still arise for our concept of fundamentality, which after all *is* primitive. But this, too, has a straightforward answer. To begin with, it is no part of the present view that theorists should strive to theorize in terms of fundamental roles and states. Nonetheless, I think that the present concept of fundamentality should be *of interest* to the metaphysician, and it is not hard to see why. After all, in the context of ORS, that concept is given real work to do, both in the individuation of states and in the construction of the order-theoretic hierarchy. This should be enough to motivate an interest in the question of which roles and states, if any, are the fundamental ones.[71]

6.5 Taking Stock

None of the above arguments against the four Options amounts to a fatal objection. Still, the argument against Option A, which is based on considerations of theoretical simplicity, seems to me reasonably strong. The argument against Option D rests on a variety of considerations, depending on whether fundamentality is to be spelled out in terms of grounding or structuralness. These seem to me fairly compelling as well; and unlike friends of Option A, proponents of Option D cannot point to the greater ideological parsimony of their respective proposals. The argument against Option B, meanwhile, strikes me as a good deal weaker than those against Options A and D. Its central premise is the thesis that facts about the number of fundamental entities—or at least those that are stronger than 'There is at least one fundamental entity'—are 'contingent'; and although the relevant notion of contingency deserves to be examined more closely (and theorists of a certain rationalist stripe might demur[72]), the intuitive pull of this thesis seems hard to deny. Finally, the argument against Option C is surely the weakest of the four.

Consequently, if no other objection is forthcoming, then Option C would appear to be the most promising approach to the treatment of fundamentality, at least given the other assumptions of ORS (such as, most notably, (O3)). So we have here, to

[71]A Dasgupta-style anti-realist might reply that there could be a "different community" that is not so much interested in the individuation of roles and states as it is in their *grindividuation*, which concerns the *gridentity* conditions of those things. That may be; but unless more is said about gridentity, it remains unclear why anyone would be interested in grindividuation, whereas the interest of questions concerning identity and distinctness should be beyond dispute. (It is surely no accident that it matters to most of us whether we have two hands or only one!) The anti-realist may retort that, apart from the things that are of interest, there are also those that are of *grinterest*. But that would be a red herring (or perhaps an attempt to remove the goal post), for what we care about are of course the things that are of interest, not those that are of grinterest. For Sider's own reply to Dasgupta's challenge, see his (2022). Another, much briefer reply can be found in Gómez Sánchez (2023: 101n.), whose account of naturalness relies (much like ORS) on a primitive notion of fundamentality.

[72]On metaphysical rationalism, see, e.g., Lin (2012), Dasgupta (2016). For a very different kind of skepticism regarding contingency claims, see Heil (2015: §IV), Kimpton-Nye (forthcoming).

some extent, a vindication of the thought that fundamentality is fundamental.[73] I say 'to some extent' because Option C is of course also compatible with the idea that *non*-fundamentality is fundamental; for instead of the concept of fundamentality, we could just as well have taken the concept of non-fundamentality (or 'derivativeness') as primitive. But since this difference does not affect the overall shape of the theory, I will continue, if only for the sake of definiteness, to take the concept of fundamentality as primitive and to regard it as picking out a fundamental property.

Since this is already a long paper, the remaining sections will be kept relatively brief.

7 Rolehood and Statehood

Two concepts of ORS that are arguably even more central than that of fundamentality are those of *role* and *state*. These are treated very differently from that of fundamentality. To begin with, ORS does not assume that there even *is* a generally applicable property (let alone a *fundamental* property) of statehood. Instead we have a series of 'statehood properties' that cover a successively greater range of cases. Thus (E2) yields a property $\lambda x\, \exists y^1\, (x = \&(y^1))$ of *being a zeroth-order state*, a property $\lambda x\, \exists y^2\, (x = \&(y^2))$ of *being a first-order state*, and so on; and none of these is fundamental, as can be shown with the help of (O3).[74] The treatment of rolehood is different again. While we have posited no *fundamental* property of rolehood, we at least have something that may be called 'the property of *being a role*', namely $\lambda x\, \mathbb{R}\, x$. What motivates this variety?

To start with rolehood, notice first that the term '$\lambda x\, \mathbb{R}\, x$' contains no free occurrence of any variable or constant. Hence, unlike true statements about the number of fundamental things, which can in $\mathcal{L}$ only be expressed with the help of some variable or constant referring to the property of fundamentality (or some related property), true statements in $\mathcal{L}$ as to how many *roles* there are (i.e., statements such as '$\exists x, y\, (\mathbb{R}\, x \wedge \mathbb{R}\, y \wedge (x \neq y))$') all denote, by (U3), the same state as '$\top$', namely the empty conjunction. I take this to be unobjectionable, because there is arguably nothing contingent about how many roles there are. One way to convince oneself of this is to recognize that there is nothing contingent about there being proper-class many ordinals, and that there is nothing contingent, either, about the fact that each ordinal is a set (and hence a property, and hence a role).

[73] A critic might be tempted to raise an objection that starts with something like the following question: 'What if there is no fundamental level?'. But it is difficult to see how this is to be fleshed out. According to the proponent of ORS (and, in particular, (E1)), there is at least one fundamental entity: fundamentality itself. And what else would deserve to be called 'the fundamental level' if not the set of all fundamental entities (assuming that they do indeed form a set)? Someone who asks the proponent of ORS, 'What if there is no fundamental level?' is thus essentially asking, 'What if your view is wrong?'. But this is no objection at all.

[74] Cf. once again Plate (forthcoming: §5.5.2). A worry might be raised as to whether it is in fact appropriate to refer to $\lambda x\, \exists y^1\, (x = \&(y^1))$ as 'the property of *being a zeroth-order state*' (cf. Section 6.3 above). However, having flagged this worry, I will set it aside in the following.

But why not extend the same treatment to the concept of statehood and introduce, for example, a syncategorematic operator '\$' that would allow us to write '$\lambda x\ \$\, x$' to denote a general property of *being a state*?

If there were such a property, then it would indeed be appropriate for it to be denotable in $\mathcal{L}$ by a term such as '$\lambda x\ \$\, x$' that contains, like '$\lambda x\ \mathbb{R}\, x$', no free occurrence of a variable or constant. But the problem is that in this case there would also be a property of *being a particular* (i.e., the property of *being neither a state nor a role*), which would be denotable by '$\lambda x\, (\neg \mathbb{R}\, x \wedge \neg \$\, x)$'. By (U3), any fact as to how many particulars there are would then be identical with $\top$, which would run counter to the plausible idea that at least some such facts should be treated as contingent. To be sure, it may be held that *some* particulars—such as God—exist necessarily; and from this it would follow that some statements as to how many particulars there are (such as the claim that there is at least one) express necessary truths. But this does little to undermine the idea that many other such statements express contingent truths. To argue against this latter thesis, it would have to be made plausible *either* that there could not be any more particulars than a certain cardinal *or* that, necessarily, there are proper-class many particulars; and the prospects for either option seem slim.[75]

An objector might point out that the above argument shows at most that $\mathcal{L}$ should not allow us to express *both* that something is a role and that something is a state. It thus leaves open that $\mathcal{L}$ contains a statehood but no rolehood operator; whereas we have here taken the opposite route. What if we hadn't? Suppose we had an operator '\$' that allowed us to express the idea that a given entity x is a state by writing '$\$\, x$', but *no* operator '$\mathbb{R}$' to express in analogous fashion the thought that x is a role. Since it is not nonsense to say of a given entity that it is a role, there should presumably still exist, for any given entity x, a state to the effect that x is a role; or at least there should be a way of approximating this goal via an infinite series consisting of

the state that x is a zeroth-order role,
the state that x is a first-order role,
the state that x is a second-order role,

and so on. The simplest way to ensure that one's ontology admits, for any x, a state to the effect that x is a role is of course to posit a property of rolehood. But it would not be plausible to do so without equipping $\mathcal{L}$ with the means to denote such a property by a 'purely logical' term: i.e., by a term that contains no free occurrence of a variable or constant. For, just as the fact that there are at least two zeroth-order states is in $\mathcal{L}$ denoted by the purely logical formula

$$\exists x^1, y^1\, \big((x^1 \neq y^1) \wedge (x^1 = \&(x^1)) \wedge (y^1 = \&(y^1))\big)$$

[75] Timothy Williamson (2013) has famously argued that "necessarily everything is necessarily something". If this is combined with the B axiom ($p \to \Box\Diamond p$) and the plausible assumptions (i) that necessarily every particular is necessarily a particular and (ii) that for any cardinal κ there could have existed at least κ-many particulars, then one can derive the (*prima facie* implausible) conclusion that there are proper-class many particulars. One way to resist this conclusion is to reject the B axiom; see Bacon (2018: §5) for discussion.

(so that this fact is, given (U3), identical with $\top$), so also should the fact that there are at least two zeroth-order *roles* be denoted by some purely logical formula. It would be extremely odd if rolehood were to be treated in a way that deviates in this respect from our treatment of statehood.

At this point it may be suggested that, rather than to take the concept of rolehood as primitive, we can define it in terms of resultance. Thus it might be proposed that a 'role assignment' is just any generalized multiset of ordered pairs, and that an entity x is a *role* iff, for some y, the pair (x, y) is a member of some role assignment from which there results a state. In $\mathcal{L}$, the idea that a given entity x is a role in this sense may be approximately expressed by an infinitary disjunction of the form

$$\begin{aligned}\exists y, z\,(z = [xy]) \vee \exists y, z, u_1^1, v_1\,(z = [xy, u_1^1 v_1]) \vee \\ \exists y, z, u_1^1, u_2^1, v_1, v_2\,(z = [xy, u_1^1 v_1, u_2^1 v_2]) \vee \\ \cdots \\ \vee \exists y, z, u_1^2, v_1\,(z = [xy, u_1^2 v_1]) \ \vee\ \ldots\,.\end{aligned}$$

But this approach strikes me as altogether unattractive, because it effectively turns any approximate ascription of rolehood into an ascription of a highly disjunctive and extrinsic property, whereas, at least intuitively, there is nothing disjunctive or extrinsic about something's being a role. Accordingly, I think that we should reject the approach currently under consideration. It is preferable to have in $\mathcal{L}$ an atomic operator '$\mathbb{R}$' that allows for straightforward ascriptions of rolehood while leaving ascriptions of statehood to be approximated by such formulas as '$\exists y^1\,(x = \&(y^1))$', '$\exists y^2\,(x = \&(y^2))$', etc.[76]

8 The 'Special Resultance Question'

We now come to the final question to be discussed in this paper: Under what circumstances does a role assignment have a state resulting from it? One might call this the 'Special Resultance Question'.

Facts as to what role assignments have states resulting from them presumably observe certain regularities. For example, if there is a state resulting from the assignment $\{(\mathit{Lover}, \text{Edward}), (\mathit{Beloved}, \text{Rose})\}$, then there should also exist a state that results from $\{(\mathit{Lover}, \text{Edward}), (\mathit{Beloved}, \text{Flora})\}$ (whether or not the latter obtains); and similarly in other cases. But what are those regularities, exactly? In the Simple

[76]Given that ascriptions of statehood are only approximately expressible, the same goes for ascriptions of particularity: when we say that a given entity x is a particular, we in effect say that x is neither a role nor a zeroth-order state, nor a first-order state, nor . . . . In a formal semantic treatment of this assertion, the semantic value that would have to be assigned to it would thus be a disjunction of a certain set of states. More precisely, the set in question would contain the state $\neg\,\mathbb{R}\,x$ as well as, for each positive ordinal α less than a certain ordinal β, the state $\neg\exists y^\alpha\,(x = \&(y^\alpha))$. To be sure, when we say that x is a particular, we do not mean to leave open that x might be a state of order β or higher. The best (and possibly only) way for the semanticist to do justice to this thought may be to take our 'domain of discourse' to be restricted to particulars and to intensional entities of orders less than β.

Theory of Types, the analogous question is typically understood as having a straightforward answer: there exists a state of affairs (or 'proposition') resulting from the application of an attribute A to a sequence of entities $x_1, x_2, \ldots$ just in case A is of a type $\langle \tau_1, \tau_2, \ldots \rangle$ such that each τ_i is the type of the corresponding x_i.[77] It would be desirable if we could formulate some similarly elegant principle for ORS, at least as a working hypothesis.

The proposal I would like to put forward in the rest of this section falls into two parts, the first concerned with *non*-fundamental roles, the second with fundamental ones. To begin with the former, let P be the non-fundamental property of *being Alonzo Church*, and consider the question of whether there exists a state that results from the role assignment $\{(P,x),(P,y)\}$, for any x and y. At least *prima facie*, it seems reasonable to answer in the negative: all the instantiations of P that anyone would need or have reason to believe in are instantiations by single entities. This stance can be more generally expressed by the following principle:

(R1) For any non-fundamental property P and any relation R: P is not a member of the role set of R.[78]

To apply this to the above example: if some state resulted from the role assignment $\{(P,x),(P,y)\}$, then there would exist a relation $\{\langle P,P\rangle\}$ that has $\{P\}$ as its role set. But this is precisely what (R1) rules out. Further, if (R1) is correct, then it stands to reason that we may adopt a similarly restrictive principle to handle the case of non-fundamental roles that are *not* properties:

(R2) For any non-fundamental role r and any relations R and R': if r is a member of the role sets of both R and R', then, for some ordinal $\alpha > 1$ and some permutation f on α: $R' = \{\sigma \circ f \mid \sigma \in R\}$.

For example, consider the identity relation $\lambda x, y\,(x = y)$, which, on the present view, is a set $\{\langle i,i\rangle\}$ for a certain non-fundamental role i.[79] One of the things that (R2) rules out is that the set $\{\langle i,i,i\rangle\}$ is a relation, too; and so it also rules out that, for any entities x, y, and z, there exists a state that results from the role assignment $\{(i,x),(i,y),(i,z)\}$.

The second part of my proposal concerns *fundamental* roles. It combines two ideas, both of which are somewhat speculative. The first idea is to the effect that, if some pairwise distinct fundamental roles $r_1, r_2, \ldots$ are *correlates* of each other (in the sense of Definition 5.4; an example would be the roles of *Lover* and *Beloved*) and no other role is a correlate of any of these, then, for *any* entities $x_1, x_2, \ldots$, there exists a

[77] See, e.g., Church (1940: 56f.). Where we speak of attributes, Church speaks of 'propositional functions'. Crucially, these are intended to be *total* functions. Thus, a propositional function of type $\langle e\rangle$, or $(o\iota)$ in Church's notation, is supposed to be a function that maps *each* entity of type e to some proposition or other. For critical discussion of simple type theory as applied to intensional entities, see Plate (forthcoming: §2).

[78] The concept of a relation's role set is defined in Definition 3.8 above.

[79] To see that i is non-fundamental, note first that this role can be denoted by '$\rho x.\,y\,(x = y)$'. That i is non-fundamental can then be shown with the help of (O3).

state resulting from the role assignment $\{(r_1, x_1), (r_2, x_2), \dots\}$. The simplest case to which this idea can be applied is that of a fundamental *property*. Thus consider the property F of fundamentality. Under the present proposal there exists, for any entity x whatsoever, a state resulting from the role assignment $\{(\mathsf{F}, x)\}$, which may in $\mathcal{L}$ be denoted by '$[\mathsf{F}x]$'. This is presumably as it should be: if there were an entity y such that there existed no state $[\mathsf{F}y]$, then it would be in an important sense *meaningless* to ask whether that entity is fundamental; and that would be very odd.

Turning now to the second idea, let us begin by noting that, if two fundamental roles are correlates of each other, then this will be in a certain sense an 'extrinsic' matter, since their being correlates is a matter of there being some state that results from a role assignment of a certain sort. It is not implausible to suspect, however, that this extrinsic relatedness between the two roles can be traced back to their standing in some more intimate, *intrinsic* relation to each other. To have a name for this latter relation, let us call it *connectedness*. Plausibly, connectedness not just symmetric but *strictly* symmetric: for any roles r_1 and r_2, the state of r_1's being connected to r_2 is just the same state as r_2's being connected to r_1. But if so, then, by our assumption (U1) (Section 5.1), the dyadic connectedness relation will be a singleton $\{\langle \mathsf{C}, \mathsf{C} \rangle\}$, for some role C.

Putting these two ideas together, we arrive at the following:

(R3) For any ordinal $\alpha > 0$ and any α-sequences of entities $x_1, x_2, \dots$ and $y_1, y_2, \dots$: the state $[\mathsf{C}x_1, \mathsf{C}x_2, \dots]$ obtains iff each x_i is a fundamental role and there exists a state resulting from the role assignment $\{(x_1, y_1), (x_2, y_2), \dots\}$.

Before we turn to applications, let us note that, through its use of the schema '$[\mathsf{C}x_1, \mathsf{C}x_2, \dots]$', (R3) presupposes the following auxiliary principle:

(R4) For any non-empty sequence of entities $x_1, x_2, \dots$, there exists a state that results from the role assignment $\{(\mathsf{C}, x_1), (\mathsf{C}, x_2), \dots\}$.

Accordingly, C is in fact a *property*; and moreover, for each cardinal $\kappa > 1$, there exists a κ-adic[80] 'connectedness relation' in the form of a singleton $\{\langle \mathsf{C}, \dots, \mathsf{C} \rangle\}$ whose sole member is a sequence of length κ that maps each ordinal $i < \kappa$ to C.

From (R4) and the right-to-left direction of (R3), we can infer the following corollary:

(C) For any cardinal $\kappa > 0$, the state $[\underbrace{\mathsf{CC}, \dots, \mathsf{CC}}_{\kappa\text{-many}}]$ obtains.

By applying the left-to-right direction of (R3) to (C), it can further be inferred that C is a *fundamental* role. We can thus observe a pleasing symmetry: given that F is a property, it follows from the right-to-left direction of (R3) that the state $[\mathsf{CF}]$ obtains; and given that C is fundamental, it follows from (E1) that the state $[\mathsf{FC}]$ obtains as well.

[80] As is standard, cardinals are here taken to be 'initial' ordinals: each cardinal is the least member of a maximal set of equinumerous ordinals.

With (R1)–(R4), we have an answer to the Special Resultance Question that, at least for the time being, recommends itself by its relative simplicity. However, there is a wrinkle: (R4) is incompatible with (O3). This can be seen by observing that, if (R4) is true, then, relative to a variable-assignment that maps 'C^1' to $\mathbf{C}$, this latter role will be denoted both by '$\lambda x\,[C^1 x]$' and by '$\rho x.\,y\,[C^1 x, C^1 y]$', so that the state ($\mathbf{C} = \mathbf{C}$) will be denoted both by '$C^1 = \lambda x\,[C^1 x]$' and by '$C^1 = \rho x.\,y\,[C^1 x, C^1 y]$'; and the former does not analytically entail the latter.[81] Hence, if we wish to adopt (R3) and (R4), then (O3) has to be weakened. The apparently most straightforward and least disruptive way to do so is to modify the fifth numbered clause of (O3). Thus, instead of—

(v) For any variable v: if v occurs free in t and is under g mapped to an intensional entity, then at least one free occurrence of v in t stands at predicate- or sentence-position.

—we may put the following:

(v′) For any term u: if every variable that occurs free in u also occurs free in t, and t' analytically entails $\ulcorner u = u \urcorner$, then so does t.

In the above example, t would be the formula '$C^1 = \lambda x\,[C^1 x]$', while t' would be '$C^1 = \rho x.\,y\,[C^1 x, C^1 y]$'. To see how (v′) solves the problem, let u be the term '$\rho x.\,y\,[C^1 x, C^1 y]$', and note that t' then analytically entails $\ulcorner u = u \urcorner$ while t doesn't.

Admittedly, replacing (v) with (v′) makes (O3) somewhat harder to work with, since it is usually less easy to check whether a given pair of terms t and t' satisfies (v′) than it is to check whether that same pair satisfies (v). But this may be the price we have to pay for an answer to the Special Resultance Question.

9 Conclusion

The main goal of this paper has been the presentation and defense of an at least somewhat comprehensive theory of intensional entities. More particularly, in Sections 6 and 7 I have defended ORS's treatment of fundamentality, rolehood, and statehood; and in Section 8 I have suggested a way of expanding the theory by offering an answer to the Special Resultance Question. Throughout all this, only two fundamental entities have been posited, namely $\mathbf{F}$ and $\mathbf{C}$, the roles (and properties) of fundamentality and connectedness. Arguably there are others, which may undergird various more specific phenomena of our *Lebenswelt*. At any rate, what has been proposed here is, apart from being yet another theory about intensional entities, just as much a framework in which further questions can be formulated, and parts of our world described.

Unless a theory about intensional entities is either trivial or inconsistent, it is notoriously difficult to know for sure whether it is correct. But if something like

[81] That this is so can be seen with the help of (R1).

the theory presented in this paper is found to be at least defensible, then we should arguably not just let matters rest there. If possible, we should try to *use* that theory to arrive at answers to further questions and see (without falling into dogmatism) where that leads us. This way, we can at least gain a better understanding of the available options, and, with some luck, the answers we arrive at may even unlock answers to still further questions: an exciting prospect.[82]

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