

# Multi-Head Attention

Comprehensive Demo and Analysis

Parallel attention heads with fused weight matrices, reshape/transpose operations, and output projection.

Random seed: 42  
Number of visualizations: 7

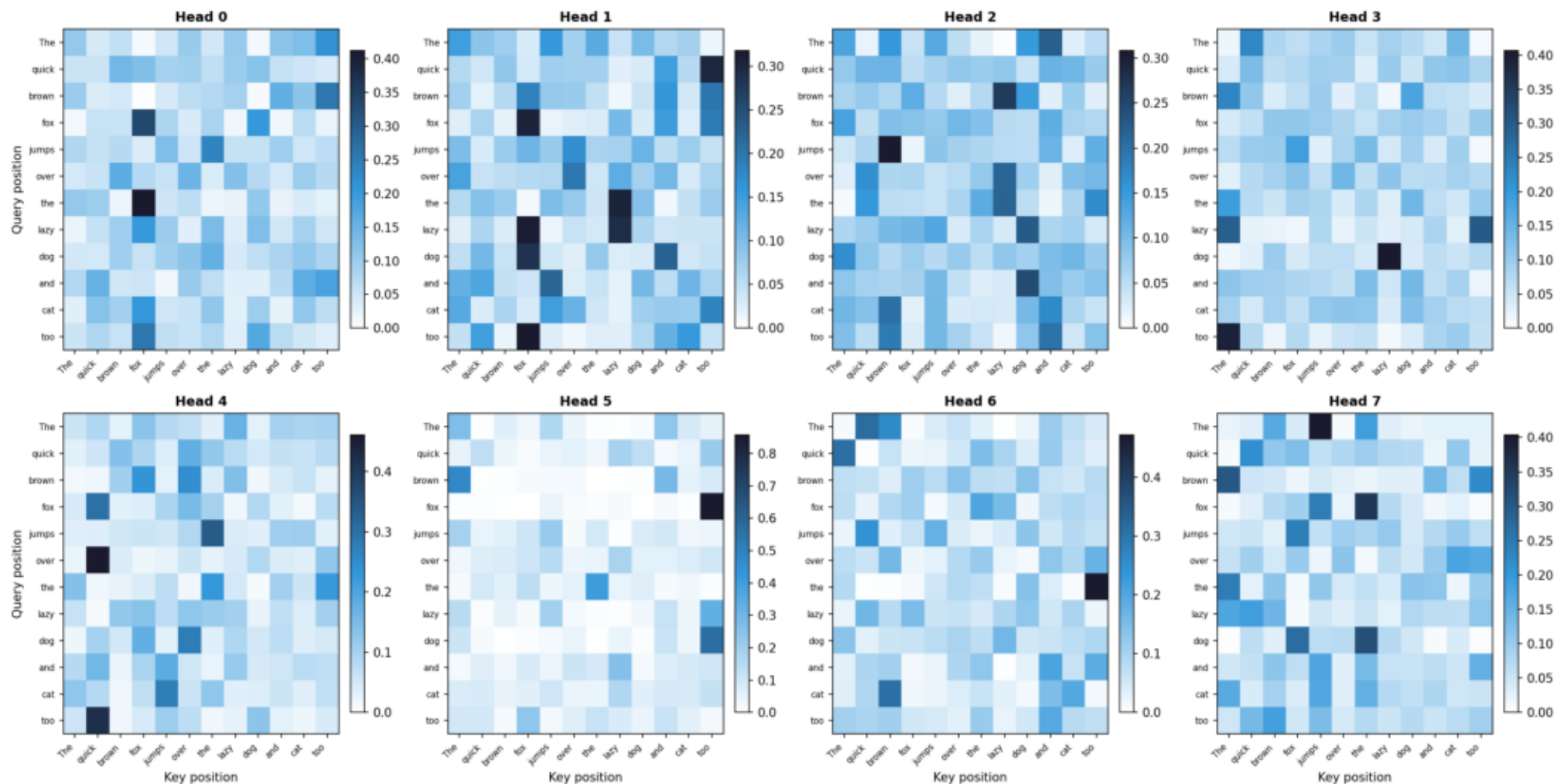
*Generated by demo.py*

## Summary of Findings

1. Attention Patterns: Each head learns distinct attention distributions, visible as different heatmap structures across 8 heads.
2. Multi-Head vs Single-Head: Multiple heads provide richer representational diversity. Standard deviation across heads reveals complementary patterns.
3. Head Diversity: Pairwise cosine similarity between head attention matrices confirms heads attend to different positions/relationships.
4. Causal Masking: Lower-triangular structure enforced correctly. Future positions receive zero attention weight. Row sums remain 1.0.
5. Memory Scaling: Attention matrix memory grows  $O(L^2)$ . At long sequences ( $L > 1024$ ), the attention matrix dominates total intermediate memory.
6. FLOPs Breakdown: Projection GEMMs dominate at short sequences. Attention core ( $QK^T + AV$ ) overtakes at longer sequences.
7. Single-Head Equivalence: MHA( $h=1$ ) matches SelfAttention: PASS  
Both forward and backward pass agree to machine precision.

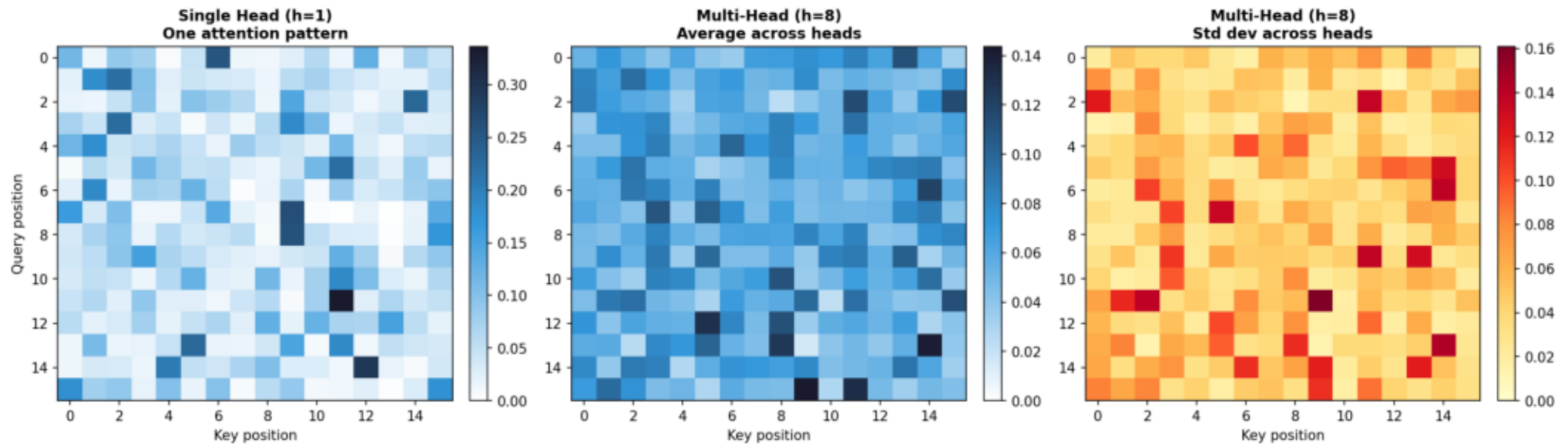
# Example 1: Per-Head Attention Heatmaps

Multi-Head Attention Patterns (8 Heads, d\_model=64)

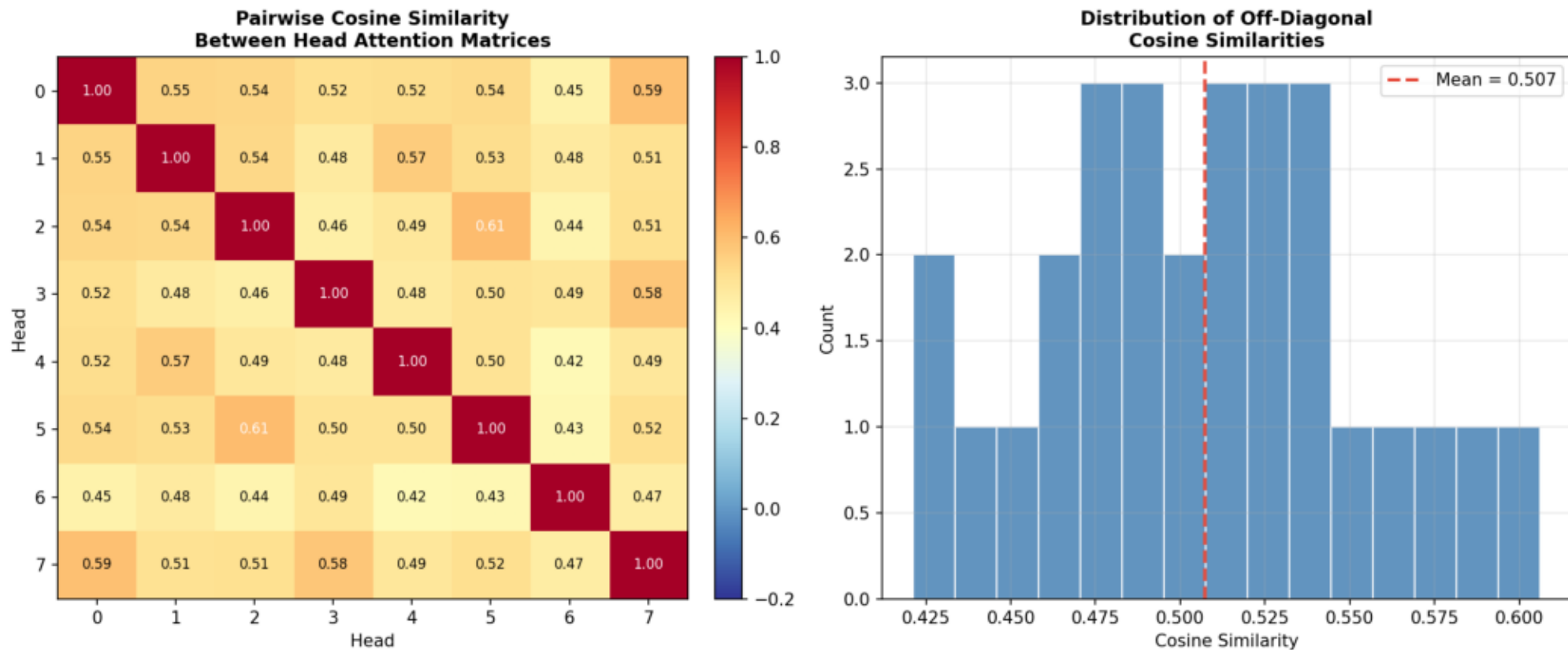


## Example 2: Multi-Head vs Single-Head Comparison

Single-Head vs Multi-Head: Attention Diversity

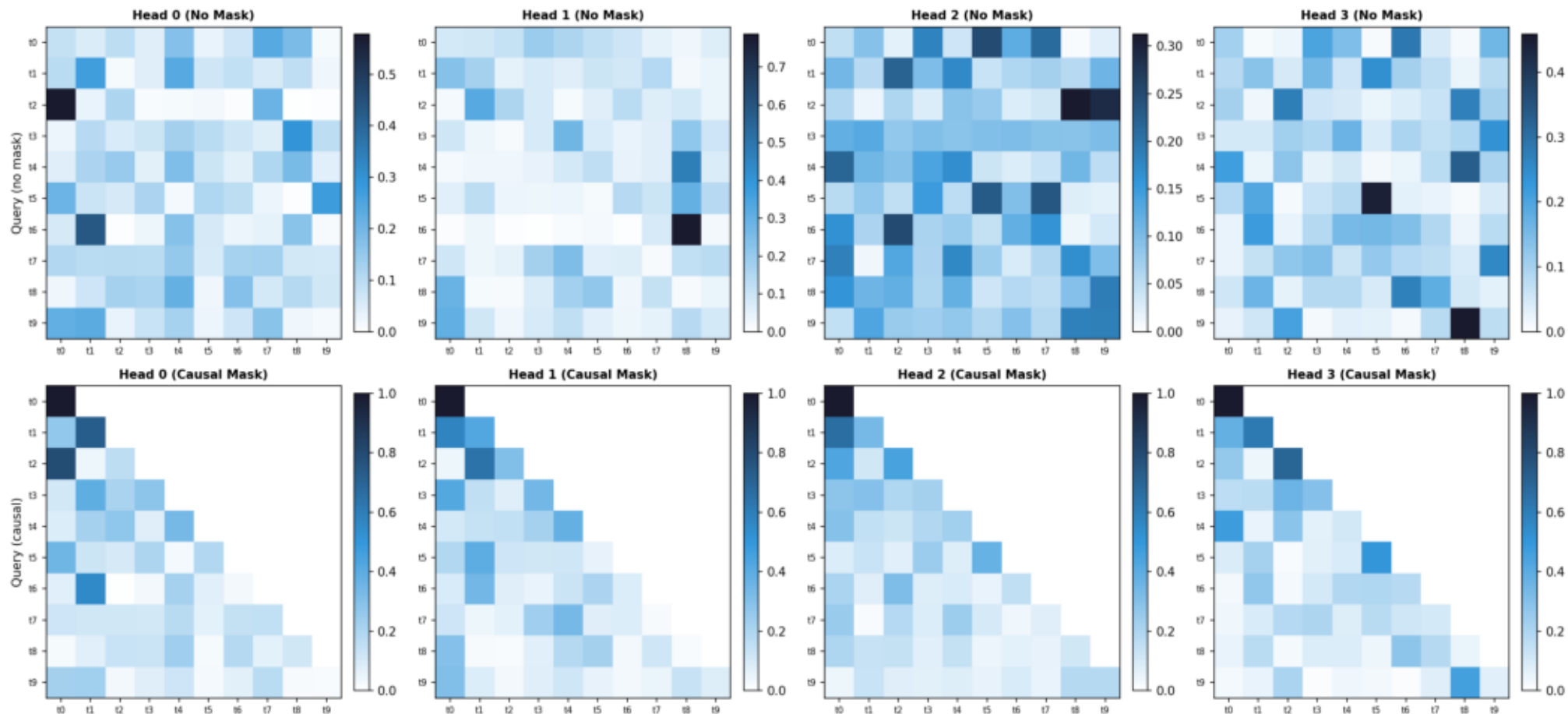


### Example 3: Head Diversity Analysis (Cosine Similarity)

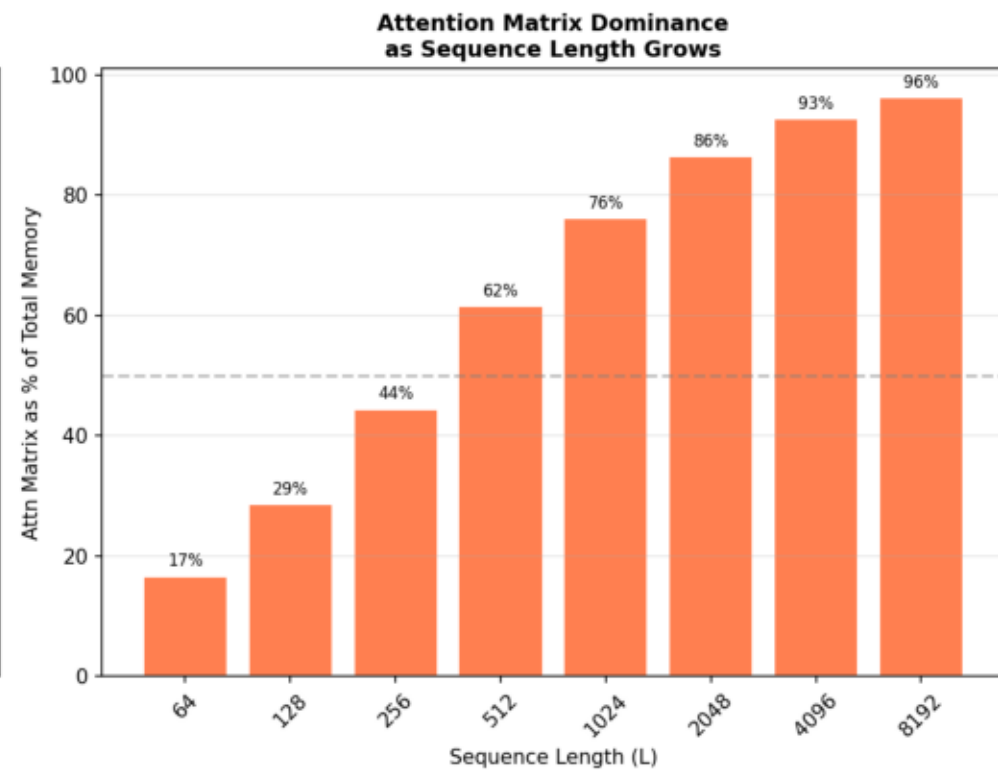
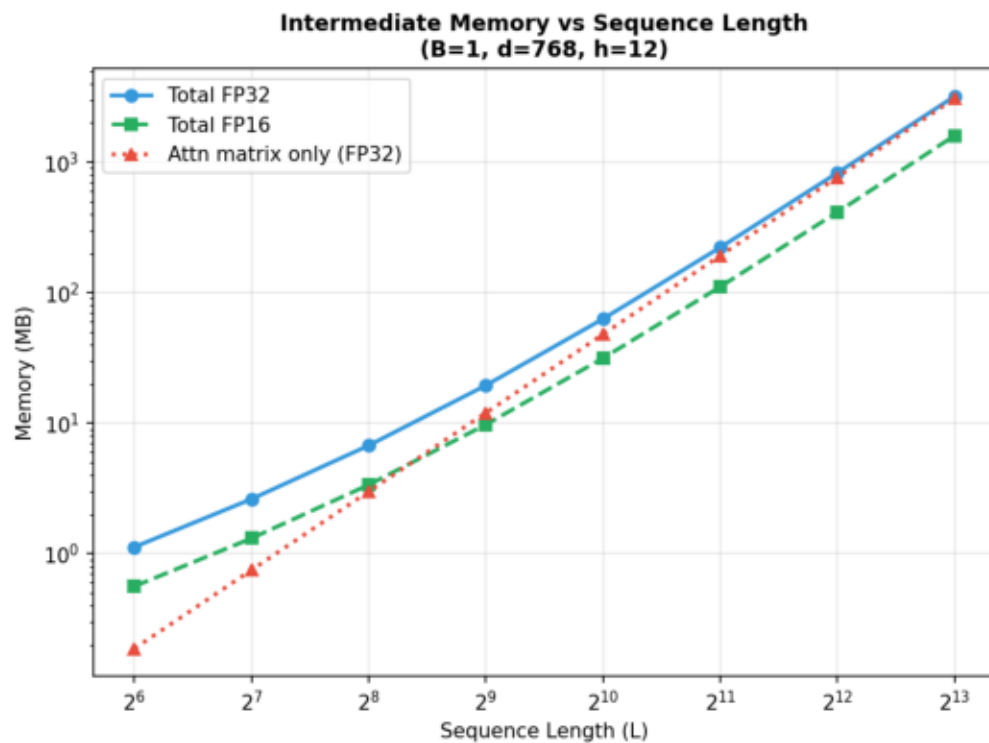


# Example 4: Causal Masking Visualization

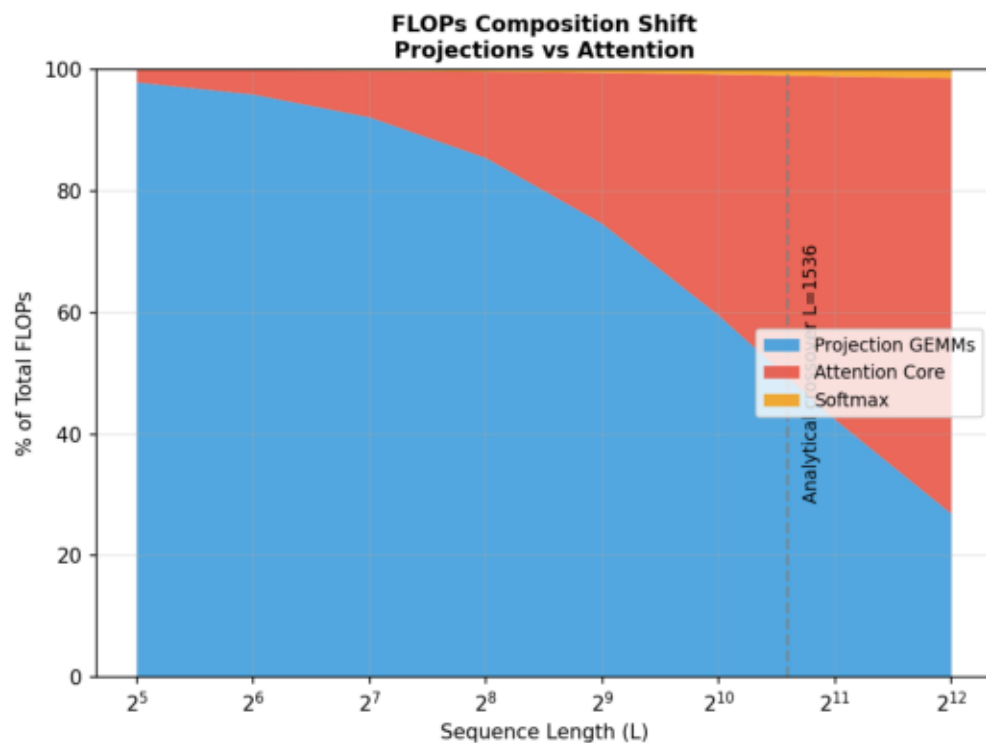
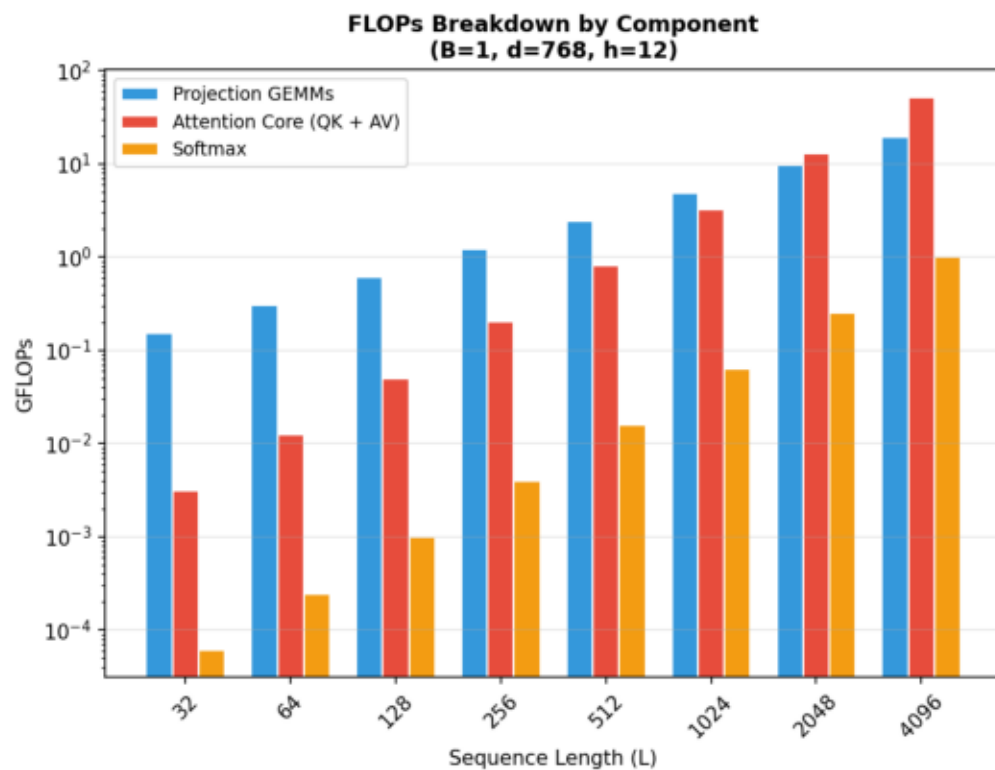
Effect of Causal Masking on Attention Patterns



## Example 5: Memory Scaling Analysis $O(L^2)$



## Example 6: FLOPs Breakdown by Component





## Example 7: Single-Head Equivalence Verification

