

A note on the propagation of quantized vortex rings through a quantum turbulence tangle: Energy transport or dissipation?

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We investigate the role of vortex ring emission in the decay of quantum turbulence. Through geometric arguments and high resolution numerical simulations we study the effectiveness of vortex rings emission as a potential energy dissipation mechanism. We show the typical mean free path estimate for the distance a vortex ring propagates before reabsorption should be considered as a mean free path before reconnection. Furthermore, we provide evidence, supported by numerical simulations, to indicate that upon reconnection with a vortex line, a vortex ring will, on average, retain approximately 75% of its energy. This leads to the much greater estimates for the distance energy can be transported by a vortex ring through a quantum turbulence tangle. Indeed, using typical experimental measurements, we conclude that a large proportion of the energy of the vortex ring will be dissipated at the boundaries. Ultimately, this provides evidence supporting the notion that vortex ring emission in quantum turbulence could be an effective energy dissipation mechanism, particularly for random unstructured tangle.

INTRODUCTION

The study of vortex rings is an old one, dating back at least to the 1850s¹⁸; despite a long history they still provide a very active area of fluid dynamics research^{9,21}. One system which provides the ideal playground to investigate the dynamics of vortex rings are those of quantum fluids such as superfluid helium, and atomic Bose-Einstein condensates. The reasons are two-fold, firstly quantized vortices are stable topological defects, with a fixed circulation. Secondly, particularly in superfluid helium the vortex core size is typically orders of magnitude smaller than the radius of the vortex ring⁷. This is in stark contrast to vortex rings in a classical fluid where both viscous spreading and instabilities due to their (relatively) thick cores severely constrain the lifetime of even an isolated ring. Another active field of research in quantum fluids is that of quantum turbulence⁵. Here vortex rings offer an ideal experimental tool to both generate^{6,22} and probe²² quantum turbulence. Recently a great deal of focus has turned to the role that vortex rings may play in the decay of quantum turbulence^{10,17}.

Indeed both Kerr⁸ and Kursa *et al.*¹⁴ noted that reconnections between almost anti-parallel vortices lead to the emission of a cascade of vortex rings, which could provide an alternative energy dissipation mechanism other than a Kelvin wave cascade^{12,15}. Numerical evidence suggests that reconnections between anti-parallel vortex reconnections dominate in tangles that contain no large-scale vortex bundles⁴, while further numerical studies by Kondaurova and Nemirovskii¹⁰ have highlighted the importance of vortex ring emission in the decay of the (random, unstructured) Vinen tangle.

In contrast, for the case of the much studied quasi-classical picture of quantum turbulence (at 0 K), vortex ring emission is not expected to be as important¹³. Here energy is injected at large scales (scales greater than the typical vortex separation) and energy is proposed to be transferred to smaller scales through a Richardson cascade of polarized vortex bundles¹. This process is assumed to occur until energy reaches scales of the inter-vortex distance. The energy transfer mechanism at this scale is still somewhat unknown^{11,16}, but vortex reconnections are thought to play an essential role. The general consensus is that energy eventually gets transferred to Kelvin waves that propagate along individual quantized vortex lines, nonlinearly interacting with each other until energy is dissipated at the phonon scale as heat.

It is typical (and appealing) to apply simple geometrical arguments to investigate the importance of vortex ring emission in the decay of quantum turbulence, see for example arguments concerning the ‘opaqueness’ of a tangle in¹⁴. For example if we have a tangle of quantized vortices of total length Λ , confined within a volume V then we define the vortex line density as $L = \Lambda/V$. If a single vortex ring is created (through vortex reconnection for example) with a radius r within the tangle, then the probability per unit time that the ring will meet and hence reconnect with a vortex line is $2rLv_{\text{ring}}$, where v_{ring} is the velocity of the vortex ring. If one neglects the influence of other vortices

and only considers the self-induced motion of the vortex ring, which is perhaps justified for small rings (where the curvature is large), then the speed of a quantized vortex ring is given by

$$v_{\text{ring}} = \frac{\Gamma}{4\pi r} [\ln(8r/a) - \alpha], \quad (1)$$

where a is the core radius ($a \approx 10^{-8}$ cm for superfluid ^4He), Γ is the quantum of circulation ($\Gamma = 9.97 \times 10^{-4}$ cm/s² for superfluid ^4He) and α depends on the assumed vortex core structure⁷. This leads to a prediction for the mean free path, $\langle D \rangle$ of a vortex ring of radius r in a tangle of vortex line density L as

$$\langle D \rangle = \frac{1}{2rL}. \quad (2)$$

The mean free path $\langle D \rangle$ is used as the expected distance for a vortex ring to propagate before being reabsorbed back into the vortex tangle. In principle, one could use Eq. (2) to estimate the importance of vortex ring emission with regards to energy dissipation. One of the main goals of this paper is to investigate the validity of estimate (2) and to further understand the behaviour of vortex rings propagating through a quantum turbulence tangle.

Indeed, there are a number of reasons one may question the validity of Eq. (2) as a prediction for the mean distance that a vortex ring will propagate before transferring energy back into the system. Firstly, nonlinear interactions between the vortex ring and vortex lines may substantially change the dynamics of the ring; for example, an approaching ring could be deflected around vortex lines before reconnection. Secondly, a reconnection event may leave a substantial part of the ring free to propagate further. Moreover even if the vortex ring is fully absorbed back into the tangle, one could imagine that large amplitude Kelvin waves would also be generated. Such perturbations along a vortex line could trigger self-reconnections and additional vortex ring emissions, as have been observed in numerical simulations¹⁹.

In order to probe vortex ring propagation in quantum turbulence we perform a suite of numerical simulations using the vortex filament method, which is described in the next section.

NUMERICAL TECHNIQUES

To simulate superfluid turbulence, we utilize the vortex filament model of Schwarz²⁰ where quantized vortex lines are discretized by one-dimensional space curves $\mathbf{s} = \mathbf{s}(\xi, t)$ where ξ is arc length and t is time. At zero temperature, where mutual friction effects are absent and in the regime where there is no external superfluid flow, the dynamics of the quantized vortex lines are determined by the Biot-Savart Law

$$\frac{d\mathbf{s}}{dt} = \frac{\Gamma}{4\pi} \oint_{\mathcal{L}} \frac{(\mathbf{r} - \mathbf{s})}{|\mathbf{r} - \mathbf{s}|^3} \times d\mathbf{r}. \quad (3)$$

Here, we use parameters that correspond to pure superfluid ^4He : circulation $\Gamma = 9.97 \times 10^{-4}$ cm²/s and a vortex core radius $a = 1 \times 10^{-8}$ cm, but our results can be generalized to turbulence in low temperature $^3\text{He-B}$.

The line integral in Eq. (3) extends over the entire vortex configuration \mathcal{L} , which is discretized into a large number of points \mathbf{s}_i where $i = 1, \dots, N$. The Biot-Savart law (3) contains a singularity when $\mathbf{r} = \mathbf{s}$, which we regularize in a standard way by considering the local and non-local contributions to the integral separately. Consequently, if we denote the position of the i^{th} discretization point as \mathbf{s}_i along the vortex line, then Eq. (3) becomes

$$\frac{d\mathbf{s}_i}{dt} = \frac{\Gamma}{4\pi} \ln \left(\frac{\sqrt{\ell_i \ell_{i+1}}}{a_0} \right) \mathbf{s}'_i \times \mathbf{s}''_i + \frac{\Gamma}{4\pi} \oint_{\mathcal{L}'} \frac{(\mathbf{r} - \mathbf{s}_i)}{|\mathbf{r} - \mathbf{s}_i|^3} \times d\mathbf{r}. \quad (4)$$

Here ℓ_i and ℓ_{i+1} are the arc lengths of the curve between points \mathbf{s}_{i-1} and \mathbf{s}_i and between \mathbf{s}_i and \mathbf{s}_{i+1} respectively, and \mathcal{L}' represents the remaining (nonlocal) vortex tangle.

The precise details of the techniques on how we discretize the vortex lines into a variable number of points \mathbf{s}_i where, $i = 1, \dots, N$ held at minimum separation distance of $\Delta\xi/2$ are described in³, whilst information on how we implement the artificial vortex reconnections are found in². All spatial derivatives are calculated using a fourth-order finite difference scheme, and time-stepping is achieved with a third-order Runge-Kutta method. Finally, all numerical simulations are performed in a periodic cube with sides of length $\mathcal{D} = 1$ cm.

VORTEX RING PROPAGATION THROUGH A TANGLE

To begin, we wish to investigate the validity of the mean free path estimate of Eq (2). We expect that the distribution of the vortex ring propagation distance before reconnection to be exponentially distributed. Therefore, given the prior estimate of the mean free path of the vortex ring Eq. (2), we expect that the probability of the vortex ring to propagate a distance d within the tangle to be

$$P(d) = \frac{1}{\langle D(r) \rangle} \exp \left(-\frac{d}{\langle D(r) \rangle} \right). \quad (5)$$

Using the exponential distribution (5), one can reformulate the problem to consider the probability of a vortex ring of radius r propagating a distance d without undergoing a reconnection:

$$P(r) = 2rL \exp(-2drL), \quad (6)$$

where we have substituted the formula for the mean free path in Eq. (2). To check the probability distribution (6) we create a numerical experiment where a variety of vortex rings of differing radii are ‘fired’ through a vortex tangle of fixed width d .

We begin by creating a random tangle at the centre of the numerical box. To do this we initialize the numerical simulation with a large number of randomly oriented vortex rings, uniformly distributed in a small strip in the xy -plane of the numerical box around the origin with a width in z is 1×10^{-1} cm. The system is time evolved using Eq. (4) for a sufficient period so that a random tangle is created as seen in Fig. 1. The tangle contains a total vortex line length of $\Lambda = 35.4$ cm inside a volume of $V = 0.6$ cm \times 1 cm \times 1 cm, hence the width of the tangle in z is now $d = 0.6$ cm. Subsequently, the respective vortex line density of the tangle is $L = \Lambda/V = 5.88 \times 10^1$ cm $^{-2}$.

In order to simulate the large number of vortices required to produce sufficient statistics, we take a numerical resolution of $\Delta\xi = 5 \times 10^{-3}$ cm and a timestep of $\Delta t = 1 \times 10^{-4}$ s.

This vortex tangle configuration displayed in Fig. 1 is used as the initial condition for another series of numerical simulations where we inject a single vortex ring of a given radius r , randomly positioned in the xy -plane at the edge of the box $z = -\mathcal{D}/2$, oriented such that the vortex ring propagates in the direction towards the vortex tangle. In total we perform 600 numerical simulations for three different radii of vortex rings $r = 1.8 \times 10^{-2}$, 3.6×10^{-2} , and 5.4×10^{-2} cm. Each ring is tracked and if the initial ring reaches the opposite end of the box at $x = \mathcal{D}/2$ without undergoing vortex reconnection then the ring is deemed to have propagated successfully through the vortex tangle.

Results

In Fig. 2 we compare the numerically obtained probabilities to those predicted by Eq. (6), shown by the solid black curve. We observe that the probability distribution of Eq. (6) vastly over estimates the likelihood of a successful vortex ring transition through the vortex tangle.

Note, however that the estimate of the probability (6) neglects the temporal dynamics of the vortex tangle as the vortex ring propagates. Therefore, one must take into account the gradual diffusion of the vortex tangle in space and the subsequently reduction in its vortex line density L as the vortex ring transverses the box.

Based on the speed of the vortex ring given in Eq. (1), we can calculate the expected transition time for the ring to transverse the numerical box: $t^*(r) = 1/v_{\text{ring}}$. Now, during the vortex ring propagation time t^* , the vortex tangle will have diffused a distance $\delta = (t^*\Gamma)^{1/2}$ at either ends. Then, taking into account that the transition time of the ring depends on its radius, we can compute the distance the vortex tangle will diffuse in terms of the initial radius of the vortex ring:

$$\delta(r) = \left[\frac{4\pi r}{\ln(8r/a) - 1/2} \right]^{1/2}, \quad (7)$$

where we have assumed a hollow vortex core for the ring $\alpha = 1/2$. Adjusting Eq. (6) for the slowly varying vortex line density and vortex tangle width, now both a function of the vortex ring radius r , we get a new estimate for the probability of a vortex ring surviving the propagation through the tangle

$$P(r) = \frac{2rdL}{d + 2\delta(r)} \exp(2drL), \quad (8)$$

where d and L are the initial width and vortex line density respectively. In fact, prediction (8) should be a lower bound as we have presumed that the vortex tangle has diffused a distance δ either side before the vortex ring is ‘injected’.

The new estimate for the probability, Eq. (8) is shown in Fig. 2 by the black dashed curve. We observe that the numerical data shows good agreement to the new prediction and is initially slightly above the prediction as expected. For last data point, for vortex rings of radius $r = 5.4 \times 10^{-2}$ cm, the lower than expected probability is possibly due to the fact that the time taken (for largest radius rings) to move a distance equal to the inter-vortex spacing $\ell/v_{\text{ring}} \simeq 5$ s is comparable to timescale of dynamics at the scale of the inter-vortex spacing, $\tau_\ell \sim \ell^2/\Gamma \simeq 17$ s. Hence the fact that the tangle is not static becomes more important for larger radius rings.

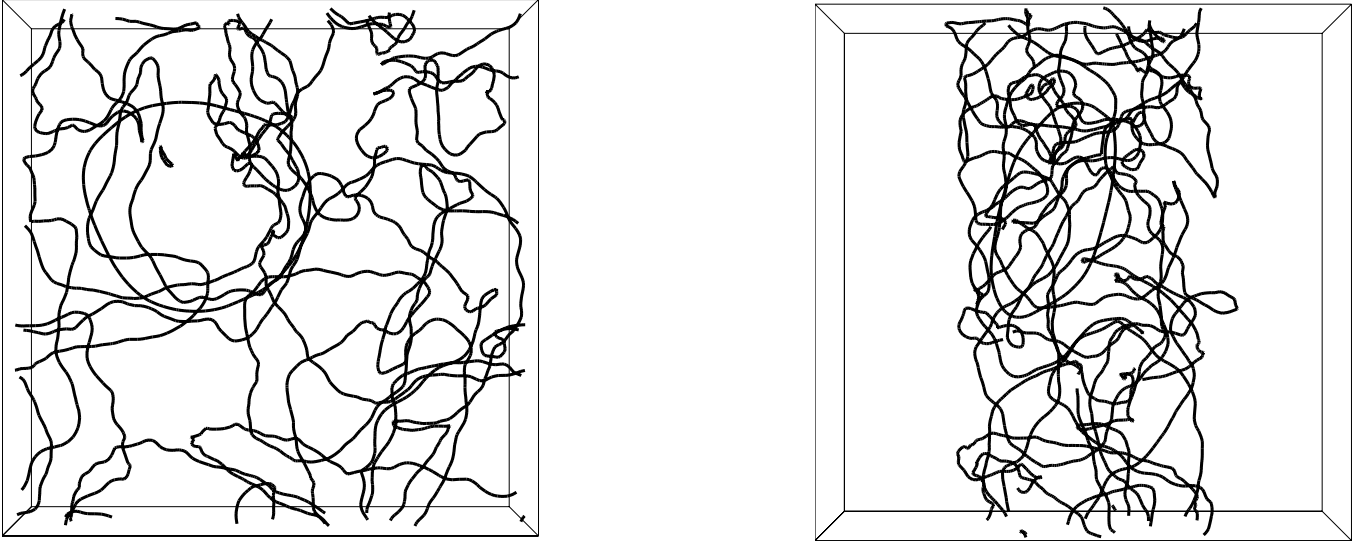


FIG. 1. Image of the vortex tangle created at the center of the numerical box across the xy -plane. View of the tangle along the z -plane (left) and along the x -plane (right).

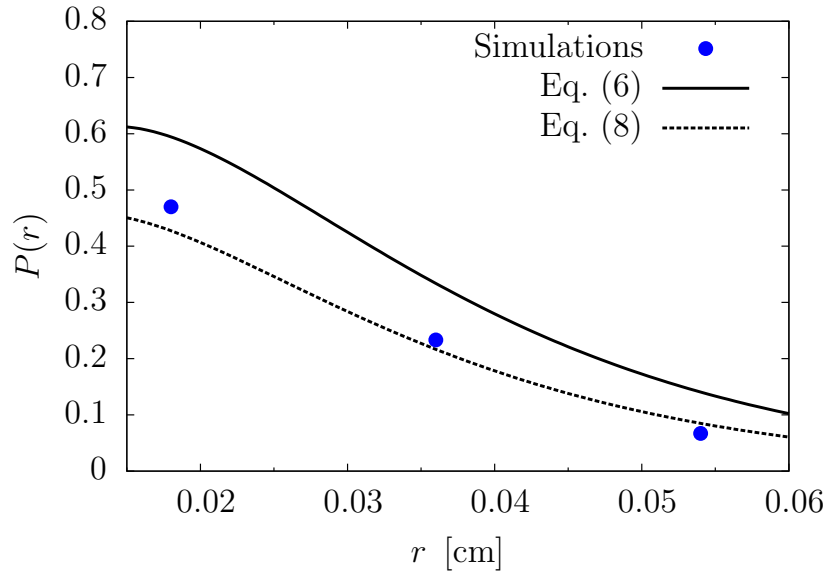


FIG. 2. Probability of a vortex ring of radius r transversing the vortex tangle without experiencing a vortex reconnection. The numerical data (blue circles) are plotted for three vortex ring radii of $r = 1.8 \times 10^{-1}$, 3.6×10^{-1} and 5.4×10^{-1} cm. The solid black curve is the theoretical prediction of Eq. (6) and the dashed black curve of prediction (8).

The good agreement with the theory indicates that our hypothesis on the exponential distribution of the propagation distance of a vortex ring before a vortex reconnection and the mean free path estimate of Eq. (2) are correct. However,

we do note that many rings also propagate through the system undergoing at least one reconnection, which are not included in the data plotted in Fig. 2. The reconnected rings still appear as coherent objects and this does lead us to question whether $\langle D \rangle = 1/2rL$ should be used to measure the amount of energy that vortex rings can dissipate. In the next section we turn our attention to the reconnection of a single quantized vortex ring with a straight vortex line to estimate how much energy can be dissipated by vortex ring emission.

VORTEX RING-LINE RECONNECTION

A geometric picture

Consider a single vortex ring of radius r propagating towards a vortex line, as depicted in Fig. 3. We define the impact factor, q as the distance between the centre of the vortex ring and the axis of the vortex line. In such a setup the ring will propagate towards the vortex line and reconnect, if we assume very a crude ‘cut and paste’ procedure, as depicted in Fig. 3, then we can estimate the circumference, C , of the post-reconnection vortex ring as

$$C(q) = 2(r^2 - q^2)^{1/2} + 2r \cos^{-1}\left(\frac{q}{r}\right), \quad (9)$$

where the impact factor $q \in [-r, r]$.

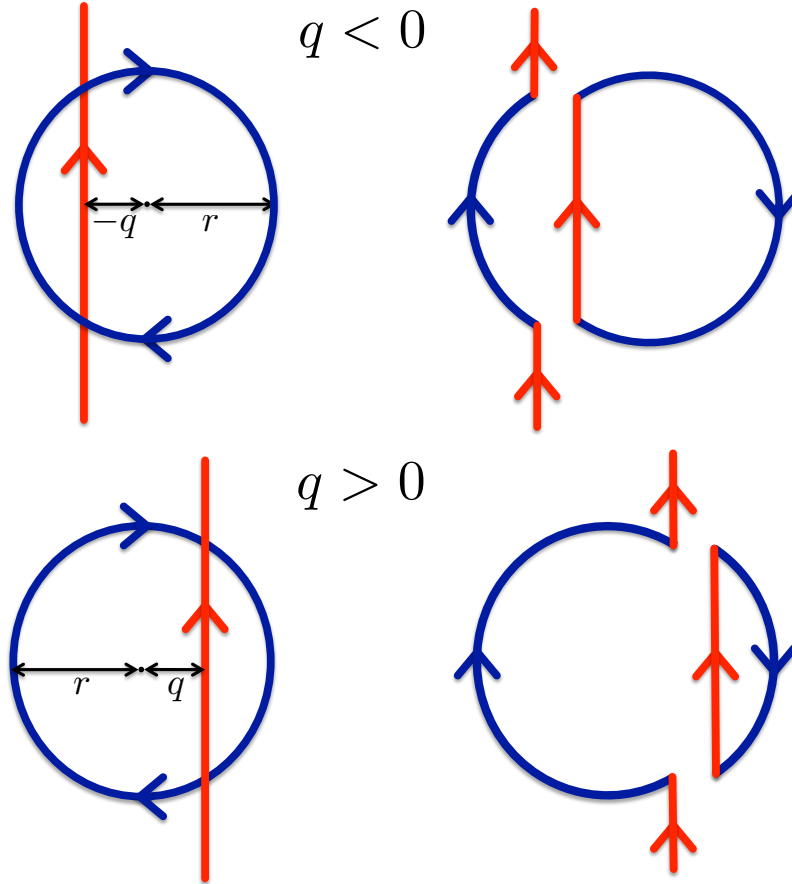


FIG. 3. Schematic to show head on reconnection between a vortex ring (blue) and a vortex line (red) depends on the orientation.

If we further assume that the impact factors are uniformly distributed, then we can compute the expected value of the post-reconnection vortex ring circumference by integrating $C(q)$ over $q \in [-r, r]$ and dividing by $2r$:

$$\langle C \rangle = \frac{1}{2r} \int_{-r}^r C(q) dq = \frac{3\pi r}{2}. \quad (10)$$

Hence, from Eq. (10), that the expected radius of the vortex ring after reconnection is simply

$$\langle r_{\text{post}} \rangle = \frac{\langle C \rangle}{2\pi} = \frac{3r}{4}. \quad (11)$$

We remark that this geometric picture can provide a plausible explanation for an interesting feature noticed in the recent experimental study of Walmsley *et al.* ²³, where they measured the time of flight of charged vortex rings propagating through a quantized vortex tangle. They discovered that some vortex rings were found to propagate through the system faster than one would expect based on their initial radius. The explanation of Walmsley *et al.* was that reconnections could produce quantized vortex rings with a very small radius, orders of magnitude smaller than the initial injected ring, resulting in a faster velocities as indicated by Eq. (1). In our simple geometric picture, such tiny vortex rings are created when the impact factor is large $q \simeq r$.

To test our prediction of Eq. (10), and hence Eq. (11), we perform a set of numerical simulations that consider a vortex-line reconnection across a range of impact factors $q \in [-r, r]$. Numerically, we take a point separation of $\Delta\xi = 1 \times 10^{-3}$ cm and use a timestep of $\Delta t = 2.5 \times 10^{-5}$ s. The initial vortex ring has a radius of $r = 1/4\pi \simeq 8.0 \times 10^{-2}$ cm (initially discretized into 500 vortex segments) with a straight vortex located in the centre of the box. We vary the initial position of the vortex ring in order to cover a range of impact parameters q . After reconnection we allow the vortex ring to propagate until it reaches the edge of the numerical domain before computing its post-reconnection circumference. Fig. 4 shows the initial and final states of the the vortex-line configuration for a particular simulation where $q = 0$.

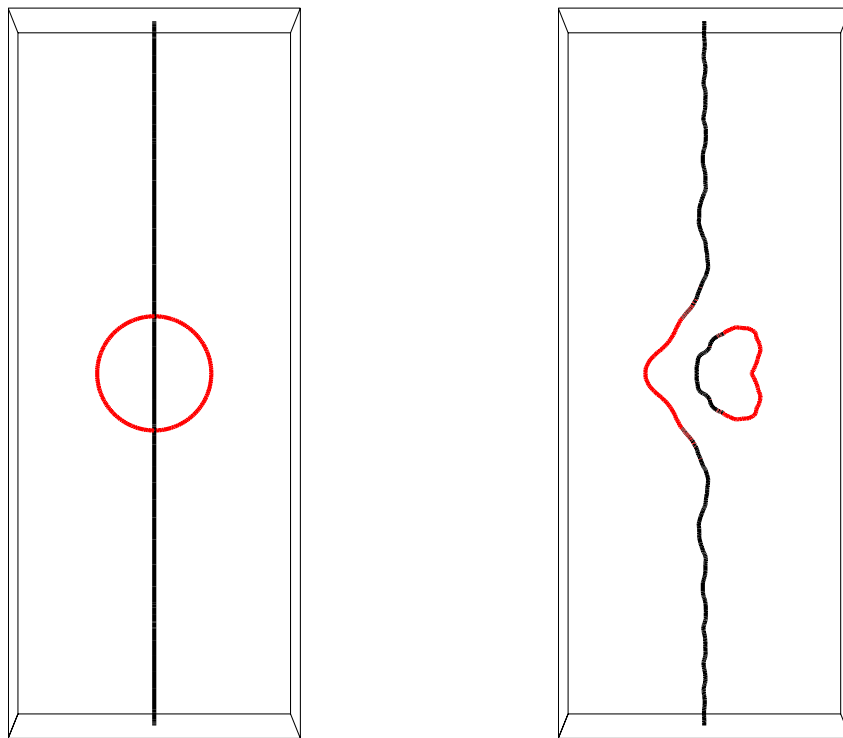


FIG. 4. Initial (left) and final (right) state of vortex-line reconnection between a vortex ring (red) and vortex line (black) with $q = 0$. Notice the exchange of vortex line segments between the ring and the line post-reconnection.

Results

In Fig. 5 we present the numerically obtained circumferences and compare to the theoretical prediction given by Eq. (10). We observe very good agreement between the numerical data and the theory. There is a slight discrepancy at large q which is possibly due to the lack of resolution for the particularly small size of the vortex rings. By numerically integrating the data, we get an estimate for the mean circumference post reconnection to be $\langle C_{\text{num}} \rangle = 3.6 \times 10^{-1}$ cm

which is remarkably close to the prediction of Eq. (11) which for the initial vortex ring radius used in the simulations is $\langle C \rangle = 3r/4 = 3.75 \times 10^{-1}$ cm.

Hence, we can conclude that on average, quantized vortex rings are very robust structures that will preserve a majority of their vortex line length whilst undergoing a reconnection with a straight vortex line perpendicular to its propagation direction. Therefore, they have the potential to endure multiple sequential vortex reconnections and still transport a substantial fraction of their original energy to the boundaries where it can be dissipated. This will be investigated further in the next subsection.

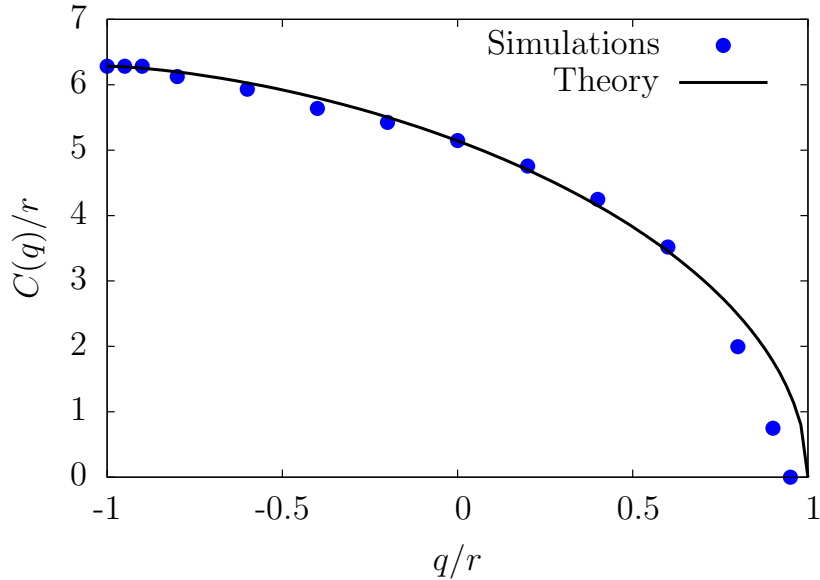


FIG. 5. Normalized circumference $C(q)/q$ against normalized impact factor q/r for a post-reconnection vortex ring. Blue circles indicate numerical data of the mean circumference of the post-reconnection vortex ring. Solid black curve is the theoretical estimate from Eq. 10.

Vortex ring radii in a random tangle

Before we go on to investigate whether vortex ring emission could be an effective energy dissipation mechanism, it is important to understand at what scales vortex rings are likely to be produced in a quantized vortex tangle. Kozik and Svistunov¹³ suggested that in side a vortex tangle, the expected scale in which vortex rings will be generated through self-reconnection of the vortex lines will be of the order

$$r^* \sim \ell / [\ln(\ell/a)]^{1/2}, \quad (12)$$

where $\ell = L^{-1/2}$ is the inter-vortex scale, and a is the vortex core radius.

To check at which scale vortex rings are generate inside a vortex tangle, we perform an additional numerical simulation. This time, we set an initial condition of 50 vortex rings of radius $r = 0.08$ cm, randomly distributed and orientated in a sphere of radius 1×10^{-1} cm located in the center of an open numerical box. The resulting vortex line density of the initial condition inside the sphere is $L = 5.97 \times 10^3 \text{ cm}^{-2}$. We evolve the system for a short period of time until the tangle has randomized as shown in Fig. 6. We note that during this short evolution, we observe very little spatial diffusion of the sphere in general, with only the ejection of a few small vortex rings.

Then by considering only vortex rings of radii smaller than the initial vortex rings, we produce the kernel density estimate of the probability density distribution (PDF) of the radii of the vortex rings inside the vortex tangle in Fig. 7. We observe a very skewed distribution of vortex ring radius that has a mean value of $\langle r \rangle = 1.4 \times 10^{-2}$ cm. However, as indicated by the vertical black dashed line, the estimate of Eq. (12) which for this tangle is $r^* = 3.45 \times 10^{-3}$ cm ($\ell = L^{-1/2} = 1.29 \times 10^{-3}$ cm), is remarkably close to that of the modal vortex ring radius of $\langle r \rangle_{\text{mode}} = 3.34 \times 10^{-3}$ cm.

Therefore, our numerical simulation provides evidence that the estimate of Eq. (12) is a reliable prediction for the scale at which vortex rings are produced inside a vortex tangle.

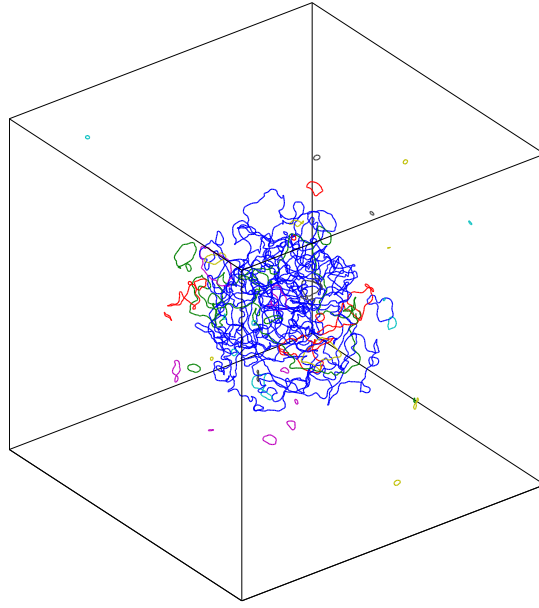


FIG. 6. Snapshot of the randomized tangle. Individual vortex rings are displayed in different colours.

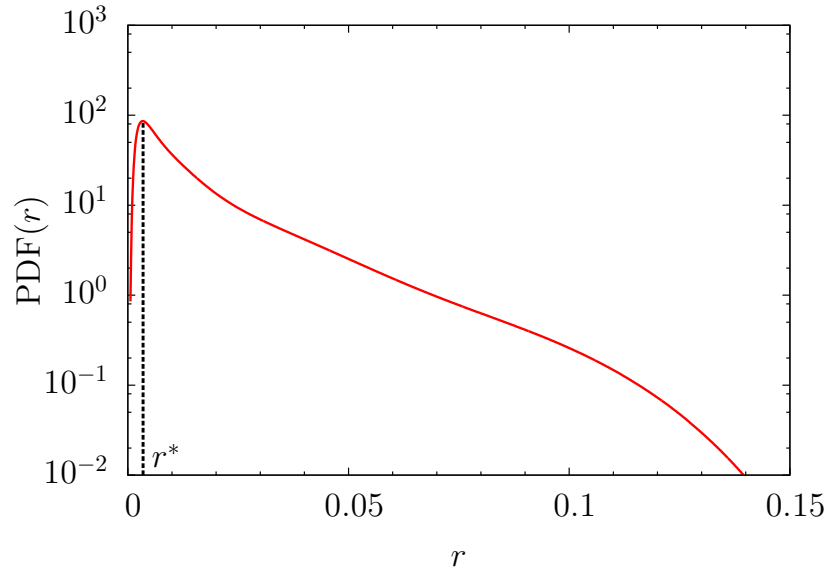


FIG. 7. Kernel density estimation of the probability distribution of vortex ring radius in the random tangle presented in Fig. 6. The black dashed line indicates Kozik and Svistunov's¹³ predicted radius $r^* \sim \ell / [\ln(\ell/a)]^{1/2} = 3.45 \times 10^{-3}$ cm.

Energy transport or energy dissipation?

Given that the energy of a vortex ring is proportional to its radius $E \propto r$, we can use these numerical results to estimate the potential energy dissipation of a vortex ring emission. Using formula (2) as a measure of the expected distance a vortex ring will travel before undergoing a reconnection and assuming that upon reconnection the radius of the vortex ring shrinks, on average by a factor of 3/4, Eq. (11), we expect that after n reconnections, a vortex ring will travel on average a distance

$$\langle D \rangle = \frac{1}{2Lr} \sum_{k=0}^n \left(\frac{4}{3} \right)^k. \quad (13)$$

Alternatively, one may also ask what is the expected number of reconnections the vortex ring will experience when traveling a distance $\langle D \rangle$? Which by rearranging the previous formula is

$$n(\langle D \rangle) = \left\lfloor \frac{\ln(1 + 2LrD/3)}{\ln(4/3)} \right\rfloor - 1. \quad (14)$$

Indeed, using formulae (13) and (14) we can estimate what is the expected amount of energy to be dissipated at the boundaries of a typical experiment by a vortex ring. As the energy of a vortex ring is $E \propto r$, we know that after n reconnections the vortex ring will have an expected energy of $E_n = (3/4)^n E_0$, where E_0 is the initial energy of the ring.

Assuming for simplicity that we are in a one-dimensional box and that a vortex ring is equally likely to be produced anywhere along the domain and orientated in either direction, we can compute the expected energy of a vortex ring hitting either of the boundaries to be

$$\langle E \rangle = \frac{E_0}{\mathcal{D}} \int_0^{\mathcal{D}/2} \left(\frac{3}{4} \right)^{n(x+\mathcal{D}/2)} + \left(\frac{3}{4} \right)^{n(\mathcal{D}/2-x)} dx. \quad (15)$$

Taking typical values of a quantum vortex tangle in an experimental configuration, such as those of the Manchester experimental setup²², where the experimental cell is a cube with sides of length $\mathcal{D} = 4.5$ cm with a vortex line density of $L \simeq 1 \times 10^3$ cm (appropriate for the ultra-quantum regime). Then supposing that the typical vortex ring is created with a radius given by the prediction in Eq. (12): $r^* \simeq 2 \times 10^{-3}$ cm, we expect that on average the energy still remaining in the vortex ring when it hits a boundary will be $\langle E \rangle \simeq 0.48 E_0$, i.e. roughly 50% of the energy that was contained in the initial vortex ring will be dissipated at the walls of the container. It is clear from these estimates that energy dissipation due to vortex ring emission may be an important factor when considering superfluid turbulence at scales smaller than the inter-vortex spacing. That said, work is still required to understand how much energy can be transferred to vortex rings as a whole in quantum turbulence. As is indicated by the works of Kerr⁸ and Kursa *et al.*¹⁴, vortex ring formation through reconnections may be optimized in tangles that contain many large angle vortex reconnections, where the vortices are close to being anti-parallel. These types of reconnections are particularly dominant in the random Vinen and counterflow tangles as opposed to quasi-Kolmogorov tangles as noted by Baggaley *et al.*⁴.

CONCLUSIONS

To conclude we have investigated the propagation of small vortex rings through a tangle of quantized vortex lines, with the aim of gaining a better understanding of the role of vortex ring emission in the decay of quantum turbulence. We have showed that the mean free path estimate for a vortex ring $\langle D \rangle = 1/2rL$ agrees well with direct numerical simulations and that the free path is exponentially distributed. We emphasise, that the mean free path should only be viewed as an estimate for the distance before a reconnection and not as a typical distance for reabsorption. Through a combination of geometrical arguments and high-resolution numerical simulations we further showed that after a reconnection with a straight vortex line, the vortex ring remains a coherent ring, post-reconnection, with approximately 75% of its original energy. Using this and data from a recent experiment²² we compute estimates indicating that vortex ring emission could lead to approximately half of the vortex ring energy being dissipation at the boundaries.

Finally, we note that much of our analysis is based upon a vortex ring reconnection head on with a straight vortex line. Therefore, further analysis is required to understand the dynamics the vortex rings colliding with tilted vortex lines and other vortex rings.

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