# Process Modelling with Neuro-Fuzzy Systems

University of Coimbra
Doctoral Program in Information Science and Technology
Real Time Learning in Intelligent Systems

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Joaquim Pedro Bento Gonçalves Pratas Leitão

# OBJECTIVES

- Sugeno-type Neuro-Fuzzy System (NFS) to model the dynamics of a process with intertia.
- Map system's input space to output space by a series of *IF-THEN* rules

# MODELLED SYSTEM

- $G(s) = \frac{2}{s^3 + 5\, s^2 + 6.75\, s + 2.25}$

- Third-order system, with memory of 3 instants in both the inputs and outputs and inertia

- $$y(k) = f(y(k-1), y(k-2), y(k-3), \\ u(k-1), u(k-2), u(k-3))$$

# METHODOLOGY

1. Learning Stage
   - Data Collection
   - Fuzzy Rules Initialisation
   - Fuzzy Rules Optimisation
2. Assessment Stage

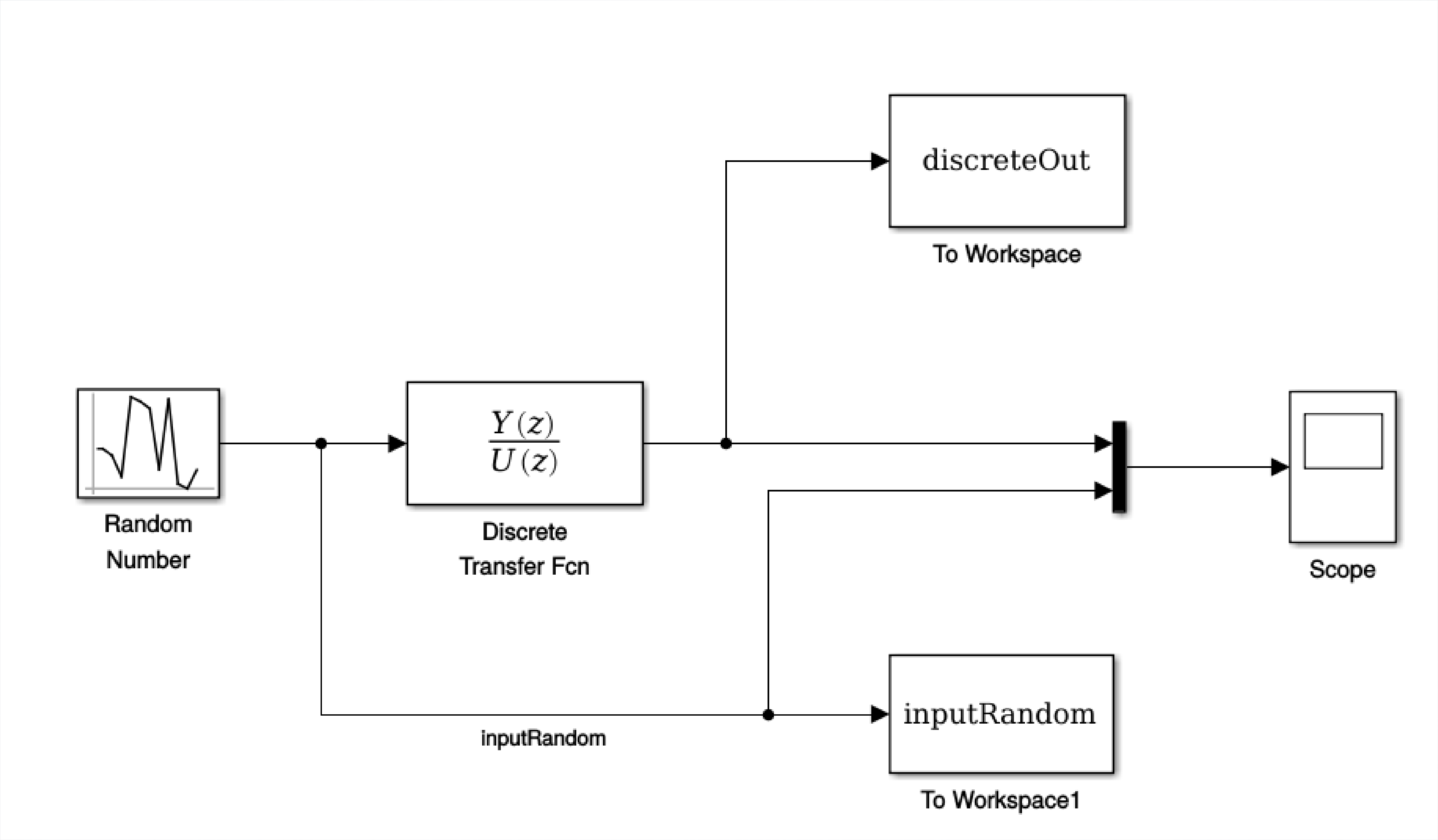
# DATA COLLECTION

- Generate random input sequence and compute the system's output
- *Simulink* environment
- Sampling interval less than the inverse of the smaller pole of the system ($\frac{1}{3}$ was used)
- Discretization of the transfer function using *c2dm* function

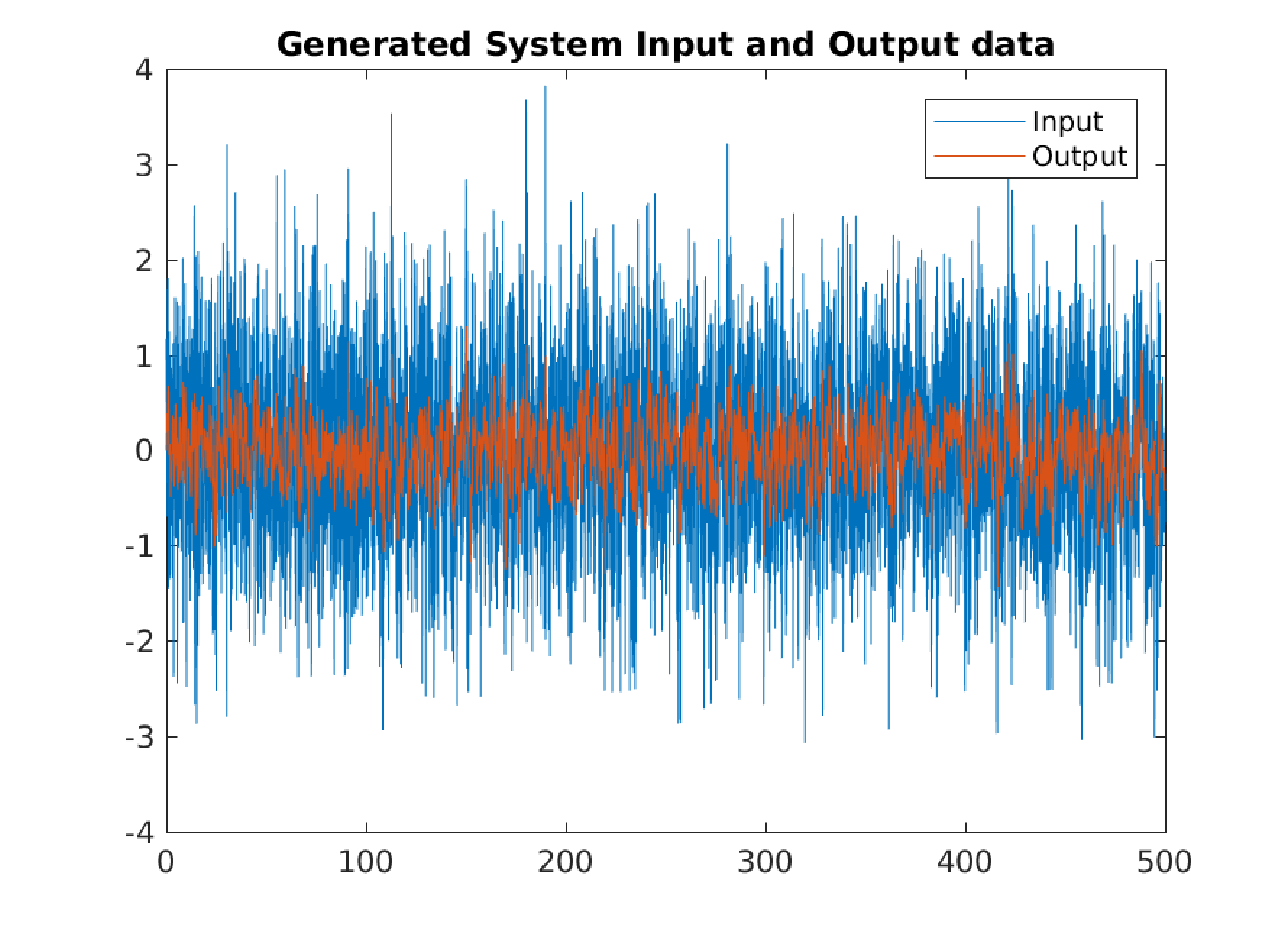
# DATA COLLECTION

- $G(z) = \dfrac{0.1079z^2 + 0.1413z + 0.009003}{z^3 - 0.8794z^2 + 0.1766z - 0.006738}$
- Total Simulation time of 500 instants
- 70-30 split of generated data

# DATA COLLECTION



# DATA COLLECTION



# FUZZY RULES INITIALISATION

- Fuzzy Inference Systems (FIS)
- Initial estimations for the fuzzy rules are produced as a result of a clustering process on training data (*genfis* function)
    - Subtractive Clustering
    - Fuzzy C-Means Clustering

# FUZZY RULES INITIALISATION

Input Membership Functions - Subtractive Clustering.

| **Input Number** | **Membership Function Type** | **Parameters** |
|---|---|---|
| in1 | Gaussian Curve (gaussmf) | $\sigma = [0.1095; 0.1095; 0.1095]$<br>$C = [-0.0221; 0.1294; -0.1326]$ |
| in2 | Gaussian Curve (gaussmf) | $\sigma = [0.1095; 0.1095; 0.1095]$<br>$C = [-0.0242; 0.1236; -0.1426]$ |
| in3 | Gaussian Curve (gaussmf) | $\sigma = [0.1095; 0.1095; 0.1095]$<br>$C = [-0.0276; 0.1155; -0.1513]$ |
| in4 | Gaussian Curve (gaussmf) | $\sigma = [1.2188; 1.2188; 1.2188]$<br>$C = [-0.056; -0.0963; 0.2091]$ |
| in5 | Gaussian Curve (gaussmf) | $\sigma = [1.2188; 1.2188; 1.2188]$<br>$C = [0.1841; -0.2404; -0.0825]$ |
| in6 | Gaussian Curve (gaussmf) | $\sigma = [1.2188; 1.2188; 1.2188]$<br>$C = [0.0674; -0.021; -0.0921]$ |

# FUZZY RULES INITIALISATION

Input Membership Functions - Fuzzy C-Means Clustering.

| Input Number | Membership Function Type | Parameters |
|---|---|---|
| in1 | Gaussian Curve (gaussmf) | $\sigma = [0.0636; 0.0638; 0.0636]$ <br> $C = [-0.0086; -0.0071; -0.0108]$ |
| in2 | Gaussian Curve (gaussmf) | $\sigma = [0.0637; 0.0637; 0.0636]$ <br> $C = [-0.0098; -0.0069; -0.0099]$ |
| in3 | Gaussian Curve (gaussmf) | $\sigma = [0.0637; 0.0637; 0.0635]$ <br> $C = [-0.0101; -0.0069; -0.0097]$ |
| in4 | Gaussian Curve (gaussmf) | $\sigma = [0.5497; 0.5851; 0.5863]$ <br> $C = [-0.0334; -0.4924; 0.4953]$ |
| in5 | Gaussian Curve (gaussmf) | $\sigma = [0.5592; 0.5909; 0.5669]$ <br> $C = [-0.2425; 0.5136; -0.3109]$ |
| in6 | Gaussian Curve (gaussmf) | $\sigma = [0.6152; 0.5591; 0.5746]$ <br> $C = [0.6764; -0.2763; -0.4373]$ |

# FUZZY RULES OPTIMISATION

- ANFIS structure to optimise both FIS
    - ➢ Backpropagation Optimisation
    - ➢ Hybrid Optimisation
- *MATLAB's anfis* function with 200 epochs
- Performance assessed with test data and Root Mean Squared Error (RMSE) metric

# FUZZY RULES OPTIMISATION

- Subtractive Clustering + Backpropagation ☑
- Fuzzy C-Means Clustering + Backpropagation ☑
- Subtractive Clustering + Hybrid ☒
- Fuzzy C-Means Clustering + Hybrid ☒

# FUZZY RULES OPTIMISATION

Final Input Membership Functions - Subtractive Clustering and Hybrid Optimisation Method.

| **Input Number** | **Membership Function Type** | **Parameters** |
|---|---|---|
| in1 | Gaussian Curve (gaussmf) | $\sigma = [0.2036; 6.5753e-05; 0.0723]$ <br> $C = [-0.0207; 0.2147; -0.1315]$ |
| in2 | Gaussian Curve (gaussmf) | $\sigma = [0.2036; 0.0641; 0.0702]$ <br> $C = [-0.0219; 0.1512; -0.1613]$ |
| in3 | Gaussian Curve (gaussmf) | $\sigma = [0.2032; 0.0692; -2.2305e-05]$ <br> $C = [-0.0237; 0.1266; -0.2394]$ |
| in4 | Gaussian Curve (gaussmf) | $\sigma = [1.2305; 1.2134; 1.2130]$ <br> $C = [-0.0553; -0.0965; 0.2101]$ |
| in5 | Gaussian Curve (gaussmf) | $\sigma = [1.2313; 1.2134; 1.2135]$ <br> $C = [0.1815; -0.2415; -0.0827]$ |
| in6 | Gaussian Curve (gaussmf) | $\sigma = [1.2294; 1.2135; 1.2138]$ <br> $C = [0.0644; -0.0212; -0.0924]$ |

# FUZZY RULES OPTIMISATION

Final Input Membership Functions - Fuzzy C-Means Clustering and Hybrid Optimisation Method.

| **Input Number** | **Membership Function Type** | **Parameters** |
| --- | --- | --- |
| in1 | Gaussian Curve (gaussmf) | $\sigma = [-2.3364e-04; 0.0964; -0.0021]$<br>$C = [-0.0305; -0.0128; 0.0204]$ |
| in2 | Gaussian Curve (gaussmf) | $\sigma = [0.0145; 0.0966; 2.1969e-04]$<br>$C = [-0.0058; -0.0133; -0.0145]$ |
| in3 | Gaussian Curve (gaussmf) | $\sigma = [-1.5273e-04; 0.0969; 0.0059]$<br>$C = [-0.0029; -0.014; -0.0093]$ |
| in4 | Gaussian Curve (gaussmf) | $\sigma = [0.5424; 0.6011; 0.5833]$<br>$C = [-0.0328; -0.4880; 0.4956]$ |
| in5 | Gaussian Curve (gaussmf) | $\sigma = [0.5521; 0.6047; 0.5634]$<br>$C = [-0.242; 0.5106; -0.3113]$ |
| in6 | Gaussian Curve (gaussmf) | $\sigma = [0.6123; 0.5757; 0.5718]$<br>$C = [0.6761; -0.2744; -0.4362]$ |

# FUZZY RULES OPTIMISATION

Final results of the optimisation process.

| Cluster Technique | Optimisation Type | Minimum Training Error | Test Error | AND Method | OR Method | Number Rules | Order |
|---|---|---|---|---|---|---|---|
| Subtractive | Backpropagation | 9.1984e-17 | 0.0053 | prod | probor | 3 | 1 |
| Subtractive | Hybrid | 1.5467e-06 | 8.9969e-04 | prod | probor | 3 | 1 |
| Fuzzy C-Means | Backpropagation | 2.1481e-16 | 0.0044 | prod | probor | 3 | 1 |
| Fuzzy C-Means | Hybrid | 1.5467e-06 | 1.5086e-06 | prod | probor | 3 | 1 |

- Lack of generalisation capacity for Backpropagation method
- Errors with Hybrid method remain more stable during training and testing
    - ➢ Suggest a more general (and more desirable) model



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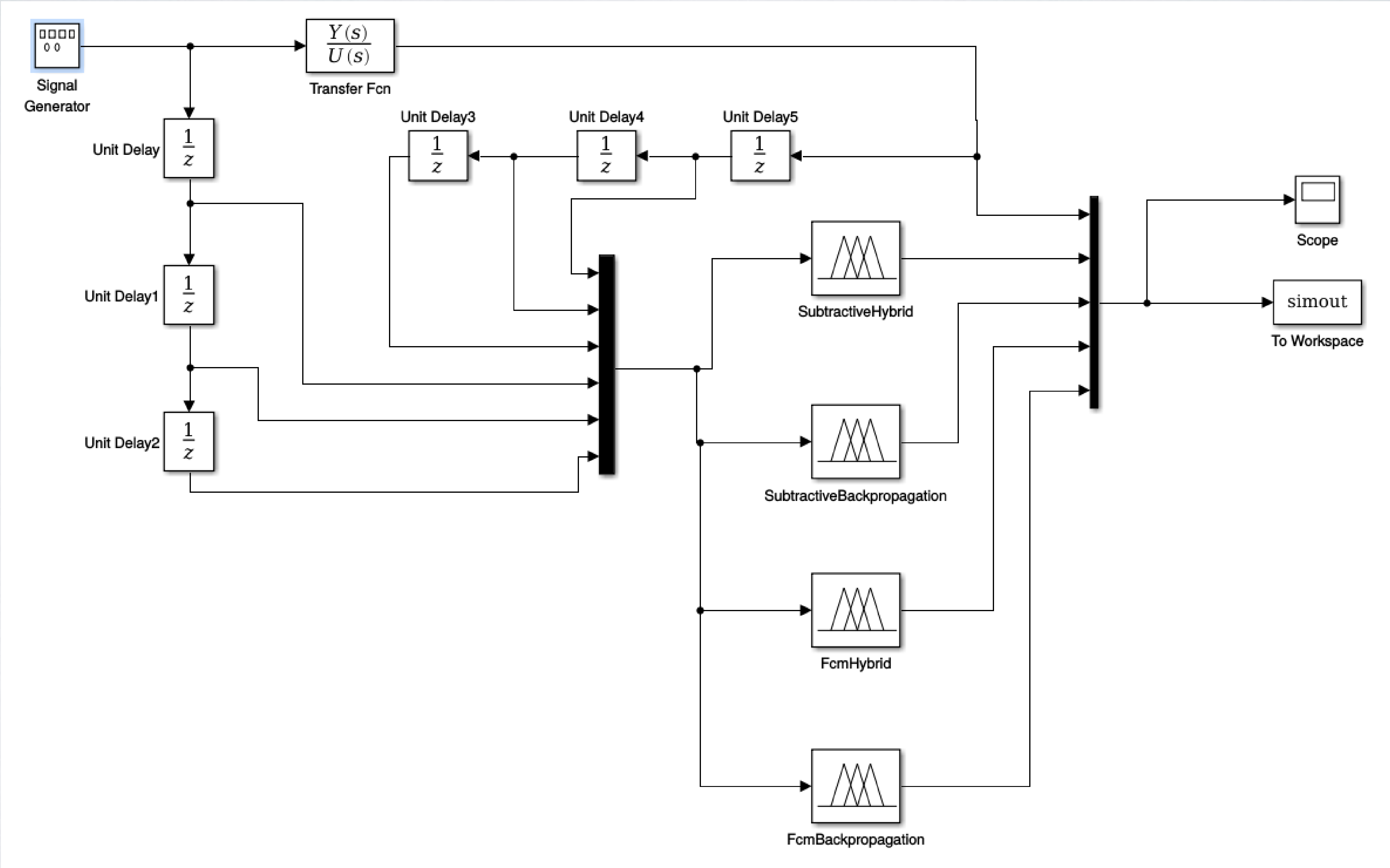
# ASSESSMENT



Signal Generator
$\frac{Y(s)}{U(s)}$
Transfer Fcn
Unit Delay
Unit Delay1
Unit Delay2
Unit Delay3
Unit Delay4
Unit Delay5
$\frac{1}{z}$
SubtractiveHybrid
SubtractiveBackpropagation
FcmHybrid
FcmBackpropagation
Scope
simout
To Workspace

# ASSESSMENT – SIGNAL GENERATOR

- Sawtooth
- Sine
- Square

➢ Amplitude 1; Units in Hz

➢ Sampling interval set to value defined during data collection

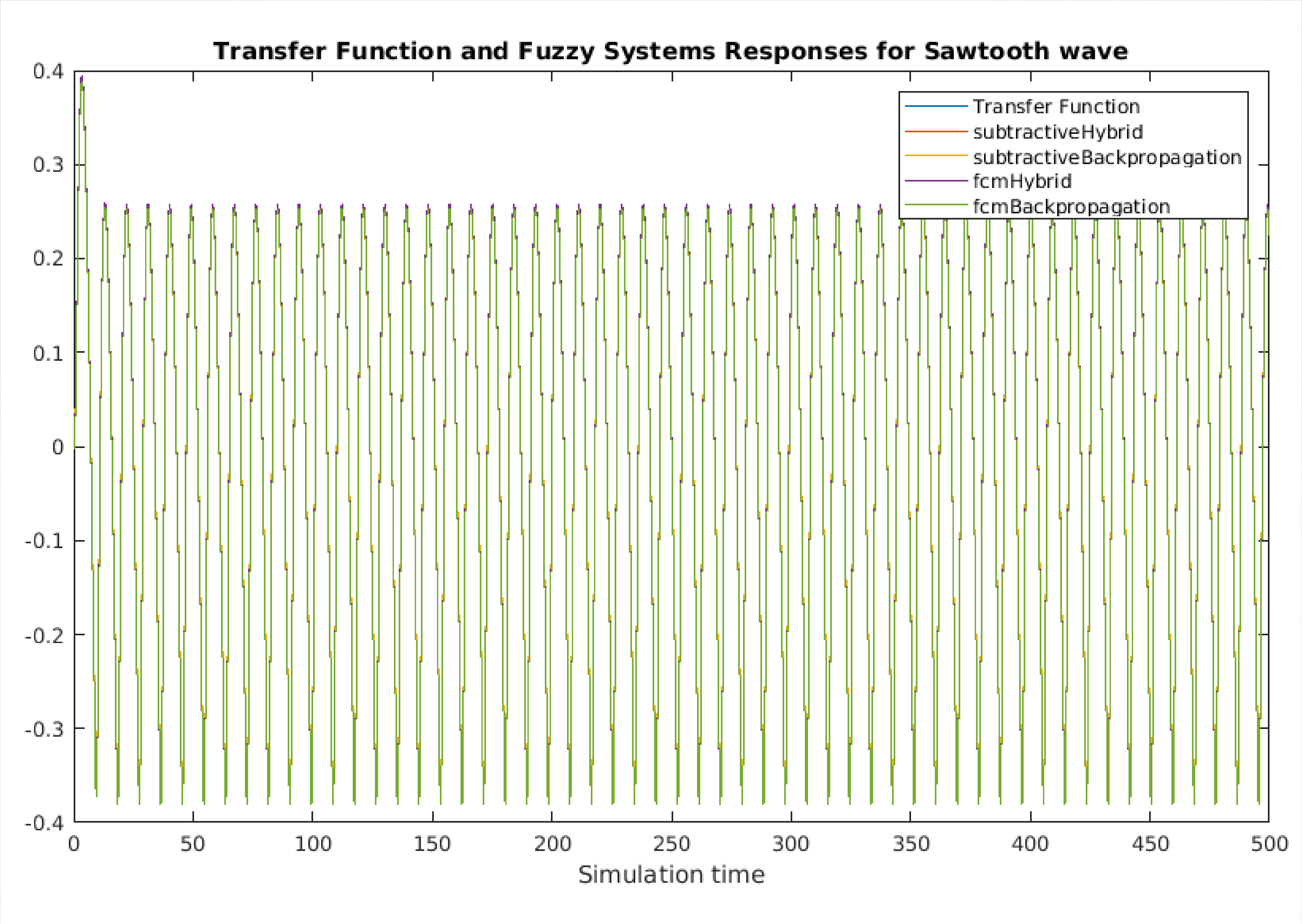
# ASSESSMENT

| Wave Type | Cluster Technique | Optimisation Type | RMSE |
|---|---|---|---|
| Sawtooth | Subtractive | Hybrid | 3.0734e-04 |
| Sawtooth | Subtractive | Backpropagation | 0.0051 |
| Sawtooth | Fuzzy C-Means | Hybrid | 3.0734e-04 |
| Sawtooth | Fuzzy C-Means | Backpropagation | 0.0031 |
| Sine | Subtractive | Hybrid | 8.4137e-04 |
| Sine | Subtractive | Backpropagation | 0.0056 |
| Sine | Fuzzy C-Means | Hybrid | 5.7272e-05 |
| Sine | Fuzzy C-Means | Backpropagation | 0.0041 |
| Square | Subtractive | Hybrid | 3.4867e-04 |
| Square | Subtractive | Backpropagation | 0.0059 |
| Square | Fuzzy C-Means | Hybrid | 3.4867e-04 |
| Square | Fuzzy C-Means | Backpropagation | 0.005 |

# ASSESSMENT

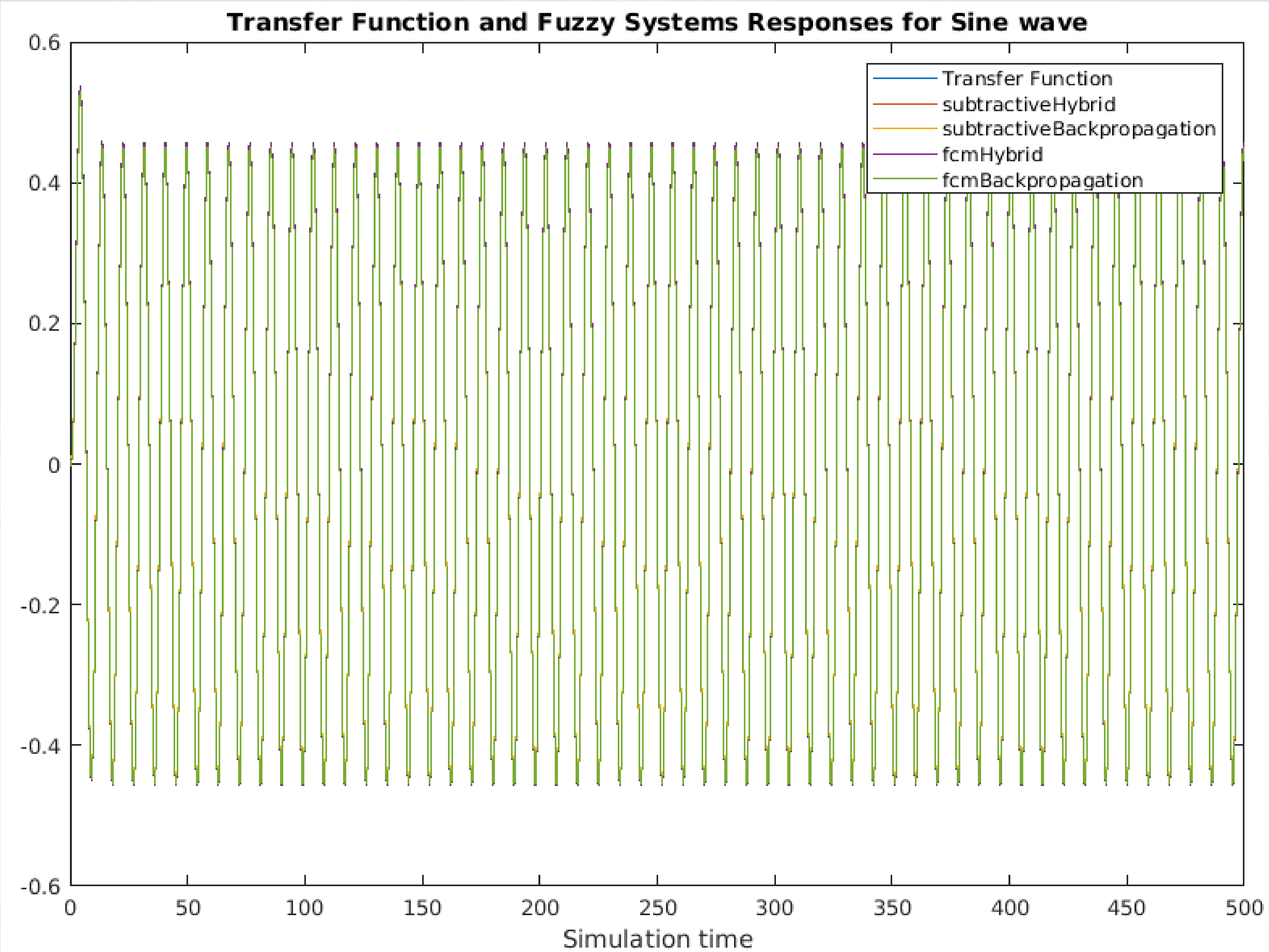
- Low RMSE registered for the different wave types
- NFSs can accurately reproduce the dynamics of the system being modelled

# ASSESSMENT

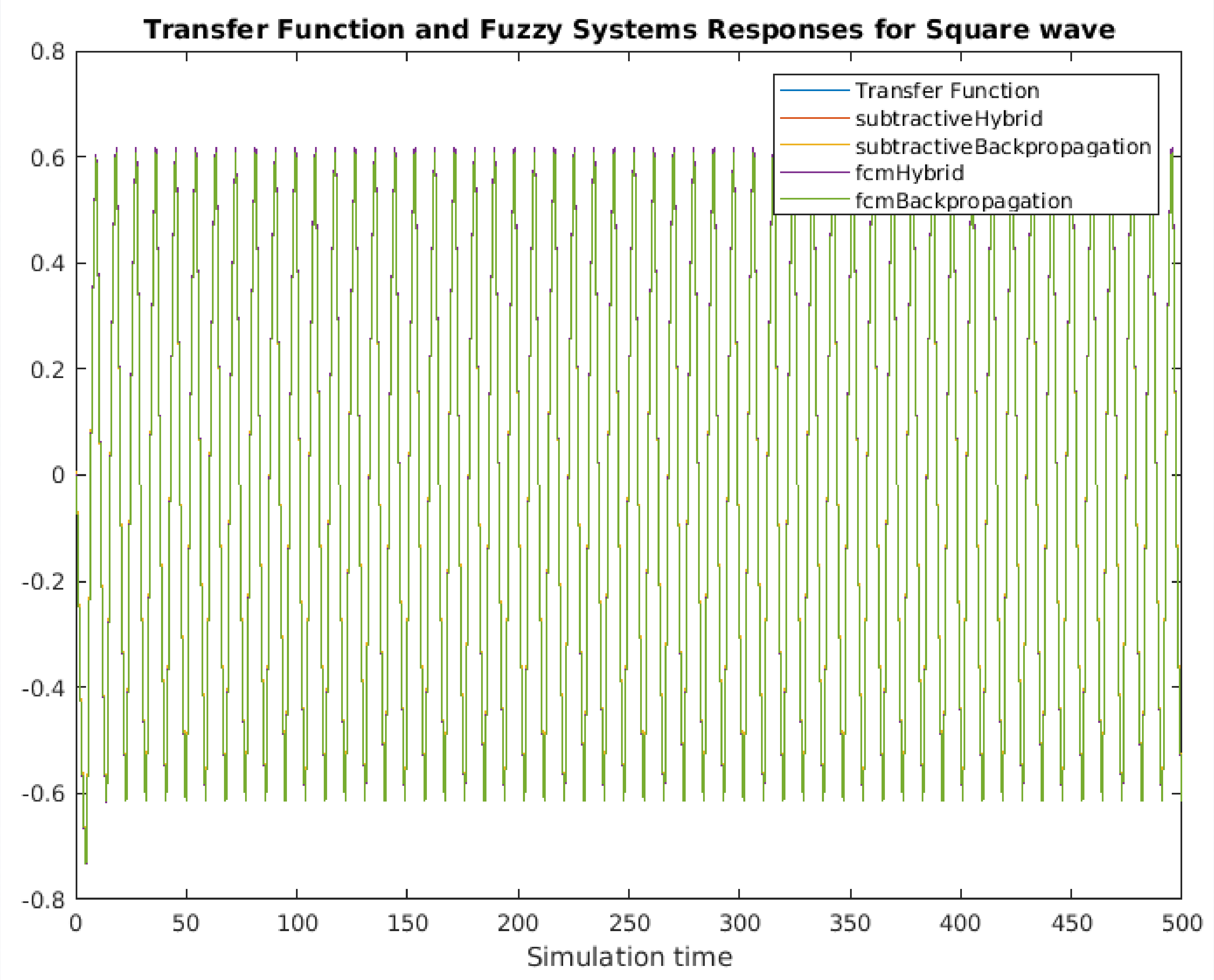


Transfer Function and Fuzzy Systems Responses for Sawtooth wave
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
50
100
150
200
250
300
350
400
450
500
Simulation time
Transfer Function
subtractiveHybrid
subtractiveBackpropagation
fcmHybrid
fcmBackpropagation

# ASSESSMENT


Transfer Function and Fuzzy Systems Responses for Sine wave
Transfer Function
subtractiveHybrid
subtractiveBackpropagation
fcmHybrid
fcmBackpropagation
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
50
100
150
200
250
300
350
400
450
500
Simulation time

# ASSESSMENT


Transfer Function and Fuzzy Systems Responses for Square wave
Transfer Function
subtractiveHybrid
subtractiveBackpropagation
fcmHybrid
fcmBackpropagation
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
50
100
150
200
250
300
350
400
450
500
Simulation time

# CONCLUSIONS

- Neuro-Fuzzy Systems were projected and developed to model the dynamics of a given process
- Outputs very similar to the real system, for the different NFSs trained
- Hybrid optimisation produced more general NFSs, with smaller RMSEs in test data for different input signals