ARX & ARMAX Parameter Estimation

Real Time Learning in Intelligent Systems

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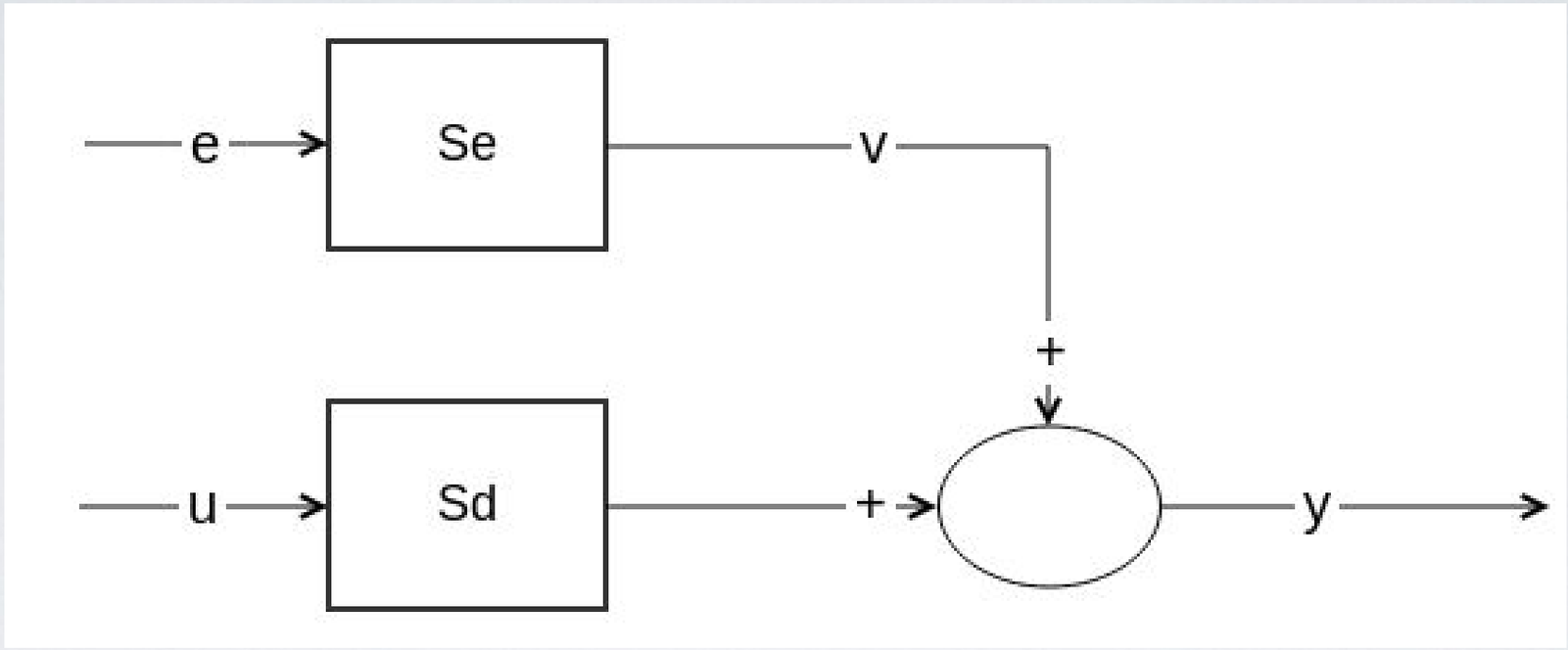
OBJECTIVES

- Recursive identification of two linear systems (ARX and ARMAX)
- Estimation of Parameters of the models

ARX Model



$$y_k = \frac{B(q)}{A(q)} u_k + \frac{1}{A(q)} e_k$$

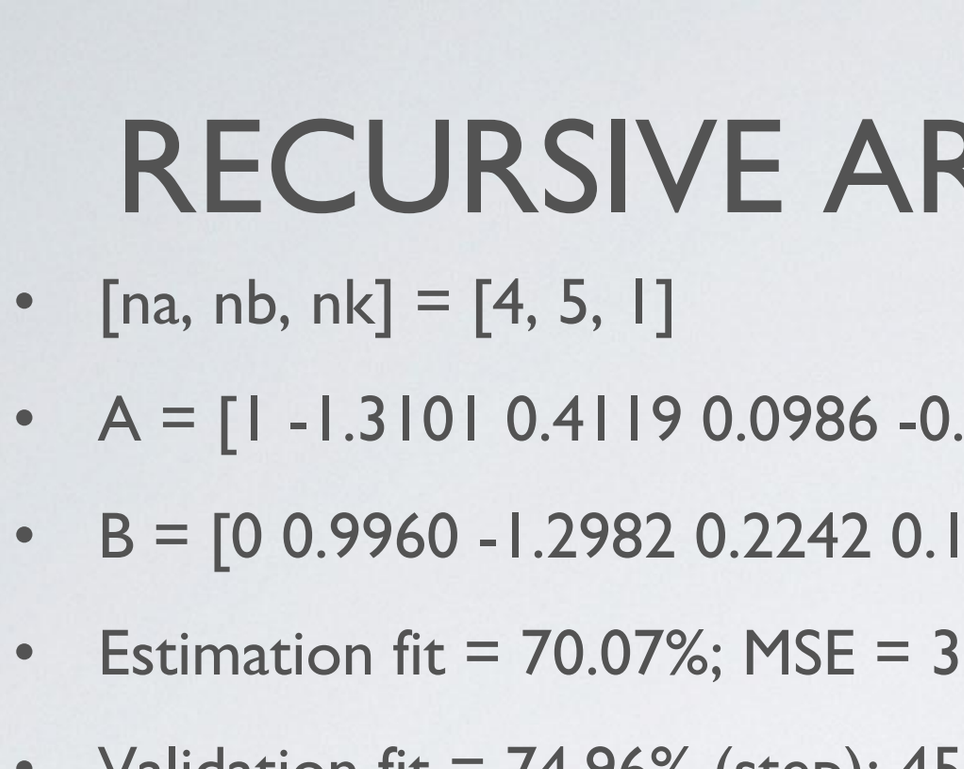
- Estimate order of the model (na, nb) and delay (nk)
- Estimate A(q) and B(q) from estimation data
- 2 estimation datasets of the same system

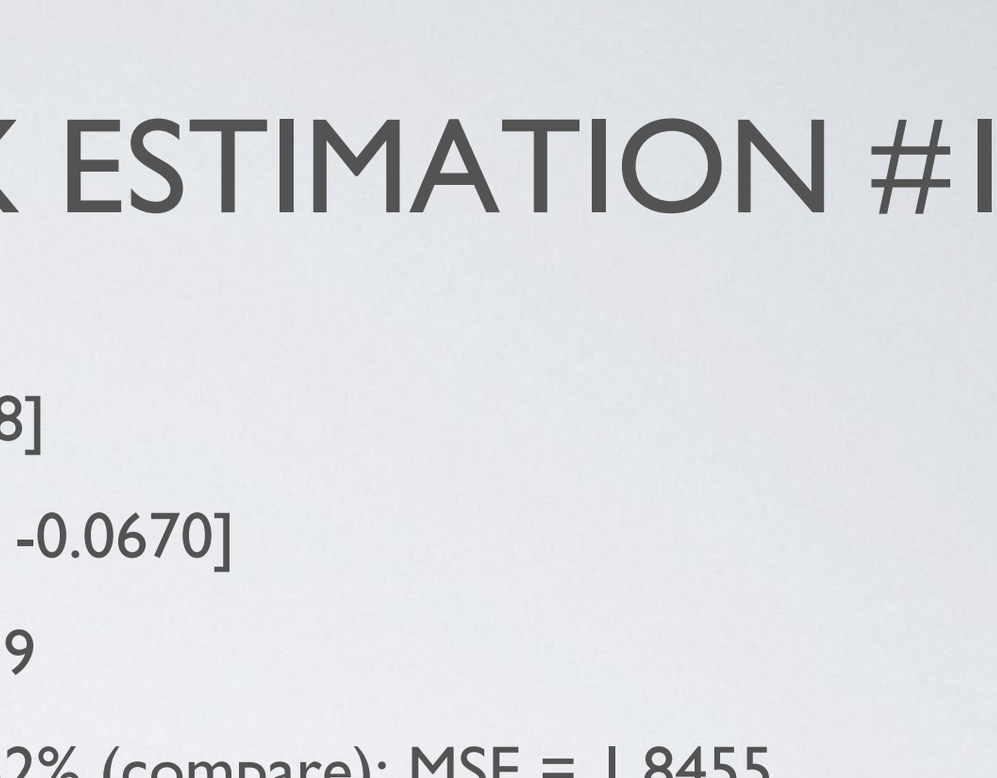
RECURSIVE ARX ESTIMATION

- Dataset division (70% estimation, 30% validation)
- Estimate model order from estimation and validation data (*selstruct* + *delayest*)
- Disturbance (ek) considered null (most probable value)
- Forgetting factor λ = 0.99; *recursiveARX* + *step* (Estimation)
- *Step* + *compare* (Validation)

RECURSIVE ARX ESTIMATION #1

- [na, nb, nk] = [4, 5, 1]
- A = [1 -1.3101 0.4119 0.0986 -0.0948]
- B = [0 0.9960 -1.2982 0.2242 0.1695 -0.0670]
- Estimation fit = 70.07%; MSE = 3.4019
- Validation fit = 74.96% (step); 45.5052% (compare); MSE = 1.8455





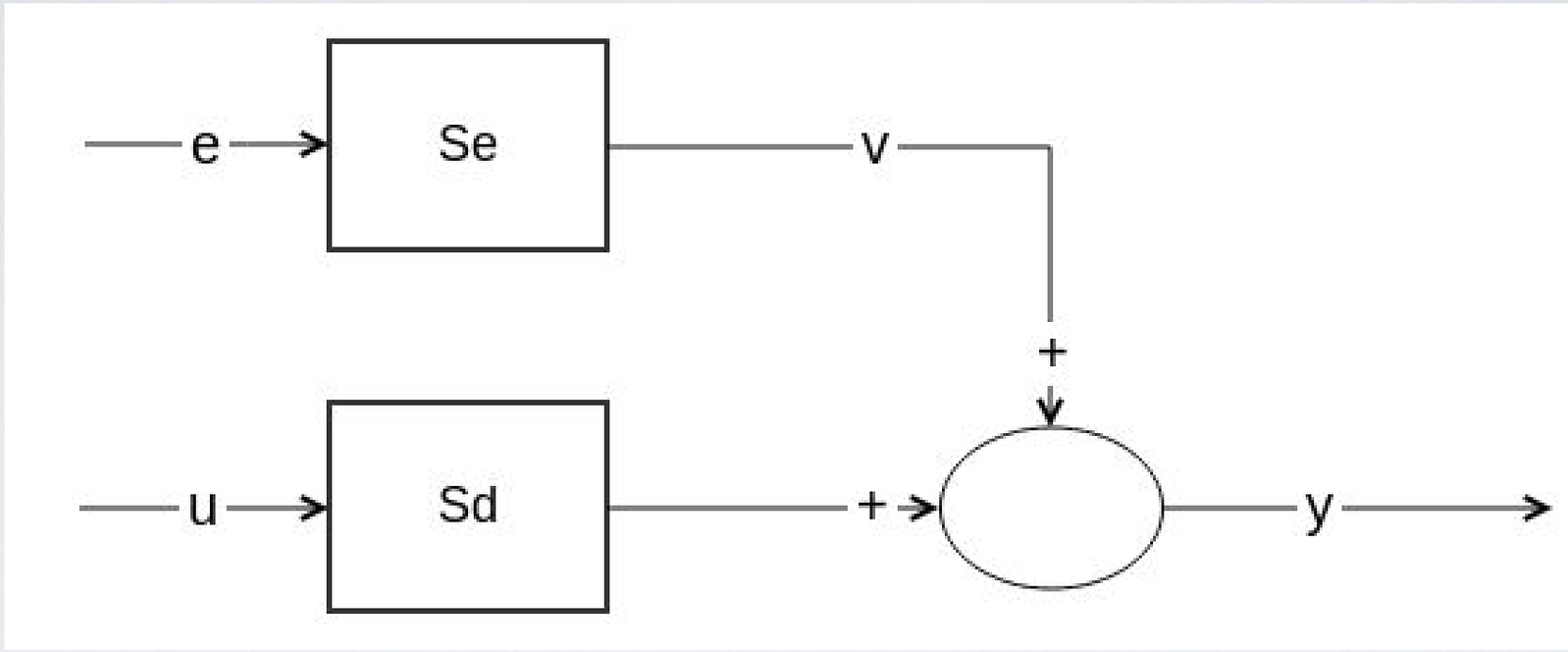
RECURSIVE ARX ESTIMATION #2

- [na, nb, nk] = [2, 3, 1]
- A = [1 -1.3059 0.4511]
- B = [0 0.9686 -1.2622 0.2578]
- Estimation fit = 65.70%; MSE = 2.5223
- Validation fit = 69.62% (step); 32.1336% (compare); MSE = 1.8668

RECURSIVE ARX COMPARISON

- ARX #1 validated with ARX #2 validation data
    - Validation fit = 65.23% (step); 30.7064% (compare)
    - MSE = 2.4457
- ARX #2 validated with ARX #1 validation data
    - Validation fit = 65.42% (step); 22.4408% (compare)
    - MSE = 1.8668

ARMAX MODEL



$$y_k = \frac{B(q)}{A(q)} u_k + \frac{C(q)}{A(q)} e_k$$

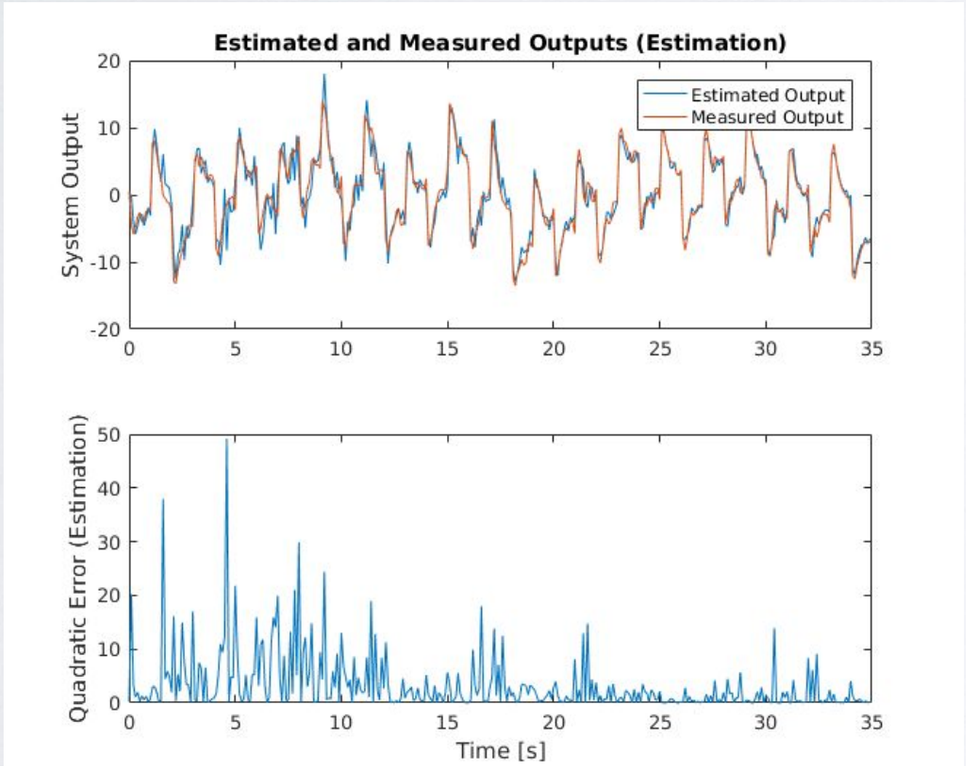
- Estimate order of the model (na, nb, nc) and delay (nk)
- Estimate A(q), B(q) and C(q) from estimation data

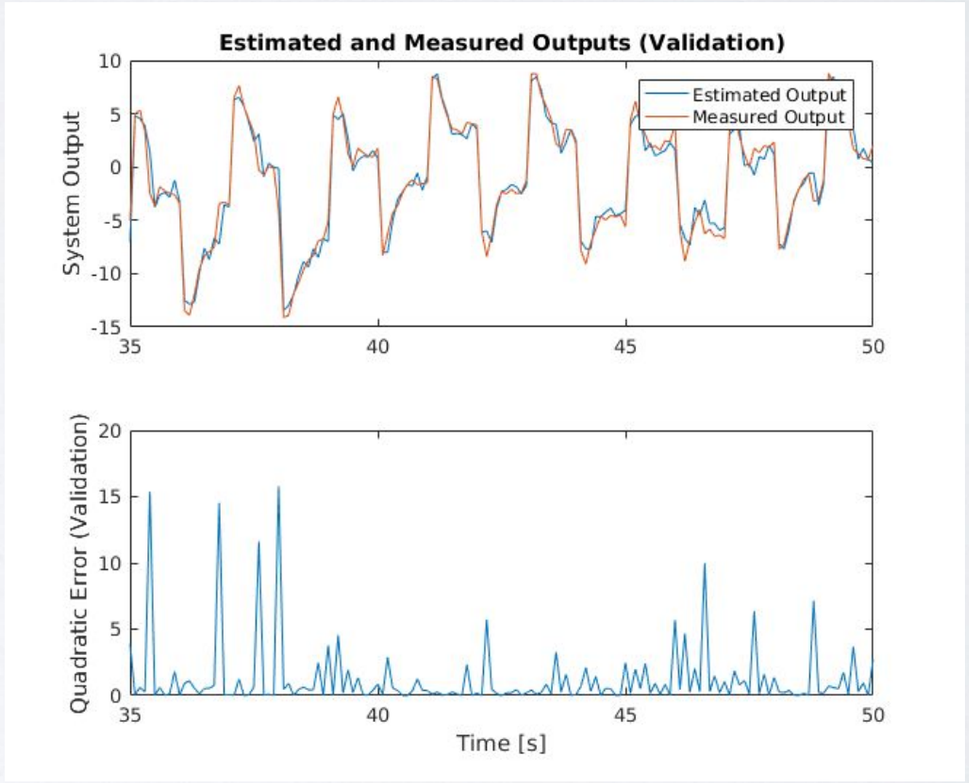
RECURSIVE ARMAX ESTIMATION

- Dataset division (70% estimation, 30% validation)
- Estimate model order with *armax* for all combinations of orders
- Disturbance (ek) considered null (most probable value)
- Forgetting factor λ = 0.99; *recursiveARMAX* + *step* (Estimation)
- *Step* + *compare* (Validation)

RECURSIVE ARMAX ESTIMATION

- [na, nb, nc, nk] = [7, 7, 10, 1]
- A = [1 -0.6594 -0.7420 -0.0457 0.6375 0.5912 -0.9068 0.1648]
- B = [0 0.9032 -0.6362 -0.9096 -0.0934 0.7044 0.7503 -0.8708]
- C = [1 0.3564 -0.7181 -0.5632 0.4154 0.7232 -0.1983 -0.1687 0.0836 0.0836 0.0139]
- Estimation fit = 68.25%; MSE = 3.2293
- Validation fit = 74.12% (step); 64.7406% (compare); MSE = 1.2521





CONCLUSION

- ARX #1, ARX #2 and ARMAX register similar values of fit
- ARX#1 presents best estimation and validation fit
- ARMAX lowest MSE in validation
- Fit in estimation and validation could be higher
  - Need to collect more data?
  - Low signal/noise ratio (signal and noise very similar)