Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Controllers

1.1. Write the general expression for the trasfer function of a PID controller.
Solution: The Transfer Function of the PID Controller is

$$K_p\left(1 + T_d s + \frac{1}{T_i s}\right) \tag{1.1.1}$$

1.2. Write the general expression for the trasfer function of a PD controller.
Solution: PD Controllers are special cases of PID Controllers in which only Proportional and Derivative Controls are used.
The Transfer Function of PD Controller is

$$K_p(1 + T_d s) \tag{1.2.1}$$

1.3. Write the general expression for the transfer function of a PI controller.
Solution: PD Controllers are special cases of PID Controllers in which only Proportional and Integral Controls are used.
The Transfer Function of PI Controller is

$$K_p\left(1 + \frac{1}{T_i s}\right) \tag{1.3.1}$$

1.4. For a unity Feedback system

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)} \tag{1.4.1}$$

Design a PD Controller with $K_v = 2$ and Phase Margin 30°
Solution: PD Controller is cascaded with the given G(s). The Transfer Function of the PD Controller is $K_p(1 + T_d s)$

$$G_c(s) = \frac{K_p(1 + T_d s)K}{s(s+2)(s+4)(s+6)} \tag{1.4.2}$$

$$K_v = \lim_{s \to 0} sG_1(s) = 2 \tag{1.4.3}$$

If we choose $K_p = 1$

$$\implies K = 96 \tag{1.4.4}$$

For Phase Margin 30°, at Gain Crossover Frequency w

$$\tan^{-1}(T_d\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)\tan^{-1}\left(\frac{\omega}{6}\right) = -60 \tag{1.4.5}$$

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{T_d^2 w^2 + 1}}{w\sqrt{(w^2+4)(w^2+16)(w^2+36)}} = 1 \tag{1.4.6}$$

By Hit and Trial, one of the best combinations is

$$w = 4 \tag{1.4.7}$$

$$T_d = 1.884 \tag{1.4.8}$$

We get a Phase Margin of 30.31°

1.5. Verify using a Python Plot
Solution:

```
codes/EE18BTECH11021_3.py
```

1.6. Design a PI Controller with $K_v = \infty$ and Phase

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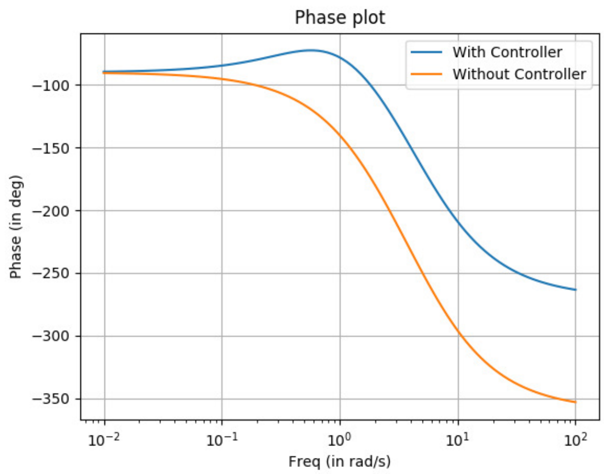
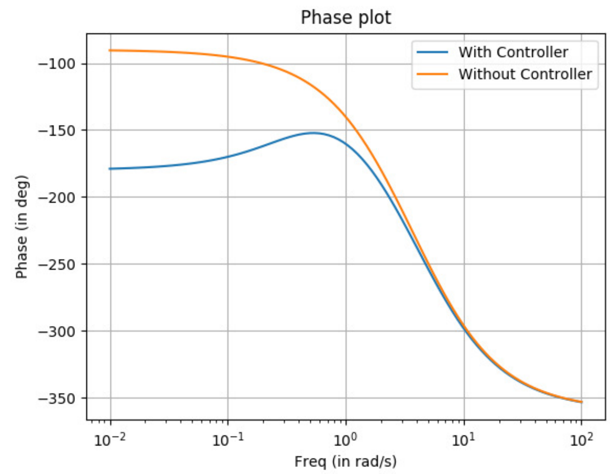




Margin 30°

Solution: PI Controller is cascaded with the given G(s). The Transfer Function of the PI Controller is $K_p\left(1+\frac{1}{T_i s}\right)$

$$G_1(s) = \frac{K_p\left(1+\frac{1}{T_i s}\right)K}{s(s+2)(s+4)(s+6)} \tag{1.6.1}$$

Choose $K_pK = 96$ This can be written as

$$G_1(s) = \frac{96(T_i s+1)}{T_i s^2(s+2)(s+4)(s+6)} \tag{1.6.2}$$

For Phase Margin 30°, at Gain Crossover Frequency w

$$\tan^{-1}(T_i\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)\tan^{-1}\left(\frac{\omega}{6}\right) = 30 \tag{1.6.3}$$

$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{T_i^2w^2+1}}{T_i^2w^2\sqrt{(w^2+4)(w^2+16)(w^2+36)}} = 1 \tag{1.6.4}$$

By Hit and Trial, one of the best combinations is

$$w = 0.75 \tag{1.6.5}$$

$$T_i = 2.713 \tag{1.6.6}$$

We get a Phase Margin of 25.53°

1.7. Verify using a Python Plot

Solution:

```
codes/EE18BTECH11021_4.py
```

1.8. Design a PID Controller with $K_v = \infty$ and Phase Margin 30°

Solution: PID Controller is cascaded with the given G(s). The Transfer Function of the PD Controller is $K_p\left(1+T_d s+\frac{1}{T_i s}\right)$

$$G_1(s) = \frac{K_p\left(1+T_d s+\frac{1}{T_i s}\right)K}{s(s+2)(s+4)(s+6)} \tag{1.8.1}$$

Choose $K_pK = 96$ This can be written as

$$G_1(s) = \frac{96(T_iT_d s^2+T_i s+1)}{T_i s^2(s+2)(s+4)(s+6)} \tag{1.8.2}$$

For Phase Margin 30°, at Gain Crossover Frequency w

$$\tan^{-1}\left(\frac{T_i\omega}{1-TiT_dw^2}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)\tan^{-1}\left(\frac{\omega}{6}\right) = 30 \tag{1.8.3}$$

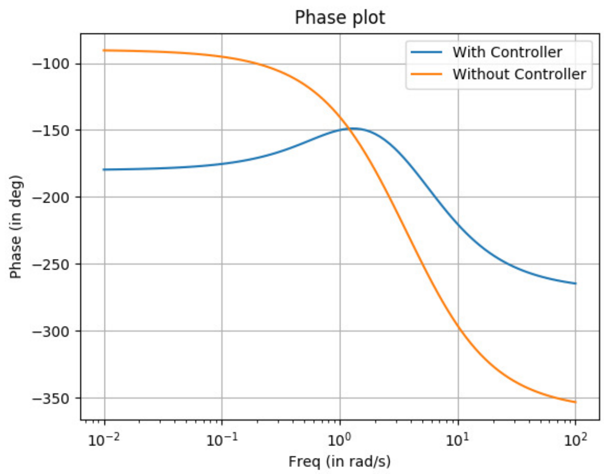
$$\left|G_1(\jmath\omega)\right| = \frac{96\sqrt{(1-TiT_dw^2)^2+T_i^2}}{T_i^2w^2\sqrt{(w^2+4)(w^2+16)(w^2+36)}} = 1 \tag{1.8.4}$$

By Hit and Trial, one of the best combinations is

$$w = 1 \tag{1.8.5}$$

$$T_i = 1.738 \tag{1.8.6}$$

$$T_d = 0.4 \tag{1.8.7}$$



We get a Phase Margin of 30°

1.9. Verify using a Python Plot

Solution:

```
codes/EE18BTECH11021_5.py
```