

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

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1 MASON'S GAIN FORMULA

2 BODE PLOT

2.1 Introduction

2.2 Example

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.1. Unit Step response of a linear time invariant (LTI) system is given by

$$y(t) = (1 - e^{-2t})u(t) \quad (4.1.1)$$

Assuming zero initial condition, the transfer function of the system is

Solution: We can convert this step response into s-domain using the Laplace Transform

$$Y(s) = \mathcal{L}(y(t)) \quad (4.1.2)$$

$$\text{where } \mathcal{L}(y(t)) = \int_{-\infty}^{\infty} y(t)e^{-st} dt \quad (4.1.3)$$

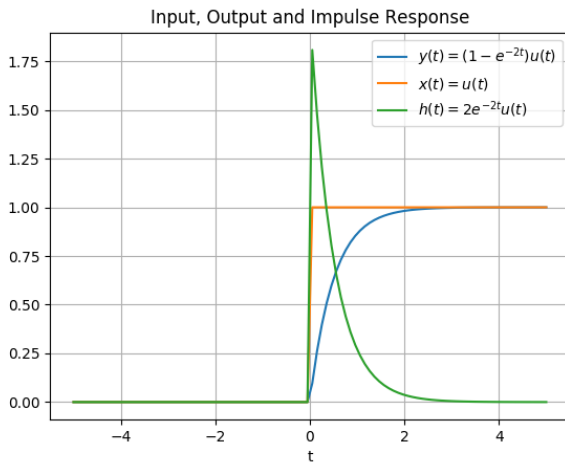
$$Y(s) = \mathcal{L}(y(t)) \quad (4.1.4)$$

$$\Rightarrow Y(s) = \mathcal{L}(u(t) - e^{-2t}u(t)) \quad (4.1.5)$$

$$\Rightarrow Y(s) = \mathcal{L}(u(t)) - \mathcal{L}(e^{-2t}u(t)) \quad (4.1.6)$$

$$\Rightarrow Y(s) = \frac{1}{s} - \frac{1}{s+2} \quad (4.1.7)$$

$$\Rightarrow Y(s) = \frac{2}{s(s+2)} \quad (4.1.8)$$



5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

6 NYQUIST PLOT

7 COMPENSATORS

8 PHASE MARGIN

$$X(s) = \frac{1}{s} \quad (4.1.9)$$

The Transfer Function $H(s)$ is given by

$$H(s) = \frac{Y(s)}{X(s)} \quad (4.1.10)$$

$$H(s) = \frac{2}{s+2} \quad (4.1.11)$$

Hence the Transfer Function $H(s)$ is $\frac{2}{s+2}$

4.2. Plotting the input, output and impulse response using Python **Solution:**

codes/EE18BTECH11021.py

4.3. Comment on the stability of the system from the obtained transfer function

Solution: The Transfer Function is

$$H(s) = \frac{2}{s+2} \quad (4.3.1)$$

which has only 1 pole at $s = -2$, which is on the left half of s-plane therefore the system is stable

We can see that the output function attains a steady state of $y = 1$ as t tends to infinity, this is the verification that the system is stable