Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```


*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

1 Mason's Gain Formula

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Damping

3.2 Example

4 Routh Hurwitz Criterion

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.1. Unit Step response of a linear time invariant (LTI) system is given by

$$y(t) = (1 - e^{-2t})u(t) \tag{4.1.1}$$

Assuming zero initial condition, the transfer function of the system is
Solution: We can convert this step response into s-domain using the Laplace Transform

$$Y(s) = \mathcal{L}(y(t)) \tag{4.1.2}$$

$$where \quad \mathcal{L}(y(t)) = \int_{-\infty}^{\infty} y(t)e^{-st}dt \tag{4.1.3}$$

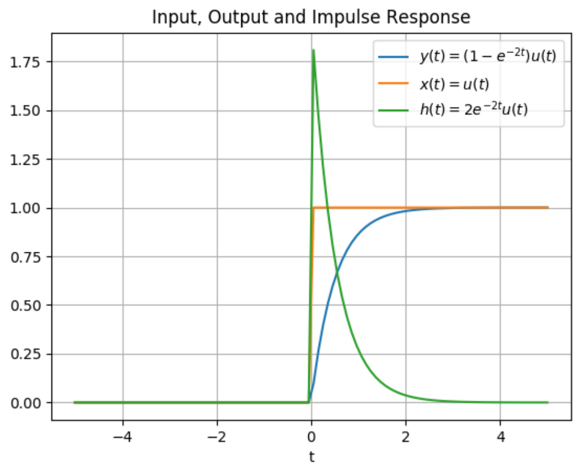
$$Y(s) = \mathcal{L}(y(t)) \tag{4.1.4}$$

$$\implies Y(s) = \mathcal{L}(u(t) - e^{-2t}u(t)) \tag{4.1.5}$$

$$\implies Y(s) = \mathcal{L}(u(t)) - \mathcal{L}(e^{-2t}u(t)) \tag{4.1.6}$$

$$\implies Y(s) = \frac{1}{s} - \frac{1}{s+2} \tag{4.1.7}$$

$$\implies Y(s) = \frac{2}{s(s+2)} \tag{4.1.8}$$



$$X(s) = \frac{1}{s} \tag{4.1.9}$$

The Transfer Function $H(s)$ is given by

$$H(s) = \frac{Y(s)}{X(s)} \tag{4.1.10}$$

$$H(s) = \frac{2}{s+2} \tag{4.1.11}$$

Hence the Transfer Function $H(s)$ is $\frac{2}{s+2}$

4.2. Plotting the input, output and impulse response using Python **Solution:**

codes/EE18BTECH11021.py

4.3. Comment on the stabilty of the system from the obtained transfer function
Solution: The Transfer Function is

$$H(s) = \frac{2}{s+2} \tag{4.3.1}$$

which has only 1 pole at s = -2, which is on the left half of s-plane therefore the system is stable
We can see that the output function attains a steady state of y = 1 as t tends to infinity, this is the verification that the system is stable

5 State-Space Model

5.1 Controllability and Observabiity

5.2 Second Order System

6 Nyquist Plot

7 Compensators

7.1. What is the purpose of a phase lead compensator?
Solution:
- Phase lead compensators are used to produce an output having a phase lead when an input is applied
- The major application is to help improve the Phase Margin (P.M) of the system.

7.2. Through an example show how the compensator in Problem 7.1 can be used in a control system
Consider a control system with the Transfer Function

$$G(s) = \frac{1}{s(3s+1)} \tag{7.2.1}$$

$$G(\jmath\omega) = \frac{1}{(j\omega)(3j\omega+1)} \tag{7.2.2}$$

$$\implies \left|G(\jmath\omega)\right| = \frac{1}{\omega\left(\sqrt{9\omega^2+1}\right)} \tag{7.2.3}$$

$$\angle G(\jmath\omega) = -\tan^{-1}(3\omega) - 90^\circ \tag{7.2.4}$$

At Gain Crossover,

$$\left|G(\jmath\omega)\right| = 1 \tag{7.2.5}$$

$$\implies \frac{1}{\omega\left(\sqrt{9\omega^2+1}\right)} = 1 \tag{7.2.6}$$

$$\implies \omega_{gc} = 0.531 \tag{7.2.7}$$

$$\angle G(\jmath\omega) = -147.88^\circ \implies PM = 32.12^\circ \tag{7.2.8}$$

Phase Margin is only 32.12°which is below the acceptable range of Phase Margin i.e 45°.

Now we will apply Lead Compensator

$$D(s) = \frac{3(s + \frac{1}{3T})}{(s + \frac{1}{T})} \tag{7.2.9}$$

Choose T = 1 for this compensator
We have,

$$D(s) = \frac{3(s + \frac{1}{3})}{(s + 1)} \tag{7.2.10}$$

By cascading the Compensator and the Open Loop Transfer Function, we get

$$D(s)G(s) = \frac{1}{s(3s + 1)} \frac{3(s + \frac{1}{3})}{(s + 1)} \tag{7.2.11}$$

$$Let G_1(s) = D(s)G(s) \tag{7.2.12}$$

$$\implies G_1(s) = \frac{1}{s(s + 1)} \tag{7.2.13}$$

Calculating Phase Margin for cascaded Transfer Function

$$G_1(s) = \frac{1}{s(s + 1)} \tag{7.2.14}$$

$$G_1(\jmath\omega) = \frac{1}{(j\omega)(j\omega + 1)} \tag{7.2.15}$$

$$\implies \left|G_1(\jmath\omega)\right| = \frac{1}{\omega\left(\sqrt{\omega^2 + 1}\right)} \tag{7.2.16}$$

$$\angle G_1(\jmath\omega) = -\tan^{-1}(\omega) - 90° \tag{7.2.17}$$

At Gain Crossover,

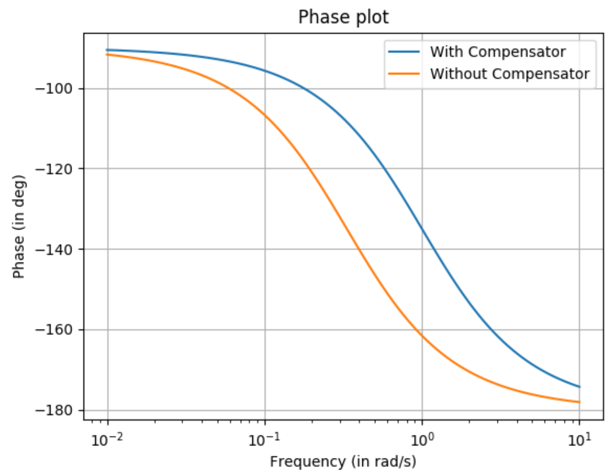
$$\left|G_1(\jmath\omega)\right| = 1 \tag{7.2.18}$$

$$\implies \frac{1}{\omega\left(\sqrt{\omega^2 + 1}\right)} = 1 \tag{7.2.19}$$

$$\implies \omega_{gc} = 0.786 \tag{7.2.20}$$

$$\angle G(\jmath\omega) = -128.167° \implies PM = 51.83° \tag{7.2.21}$$

As we can see, the Phase Margin has gone



above 45°which is the acceptable range of Phase Margin.
Therefore, through this example we have shown that a Lead Compensator is helpful in improving the Phase Margin of the System.

7.3. Verify the above improvement in Phase Margin with the help of a Python Code **Solution:**

codes/EE18BTECH11021_2.py

8 Phase Margin