

Control Systems

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1 Feedback Circuits 1

Abstract—The objective of this manual is to introduce control system design at an elementary level.

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1 FEEDBACK CIRCUITS

1.0.1. Consider a Feedback Current Amplifier formed by cascading an Inverting Opamp μ with a MOSFET (NMOS). The output current is the Drain Current of the NMOS. Assume that Opamp has an input resistance R_{id} , an Open Circuit Voltage Gain μ , and an output resistance r_{o1}

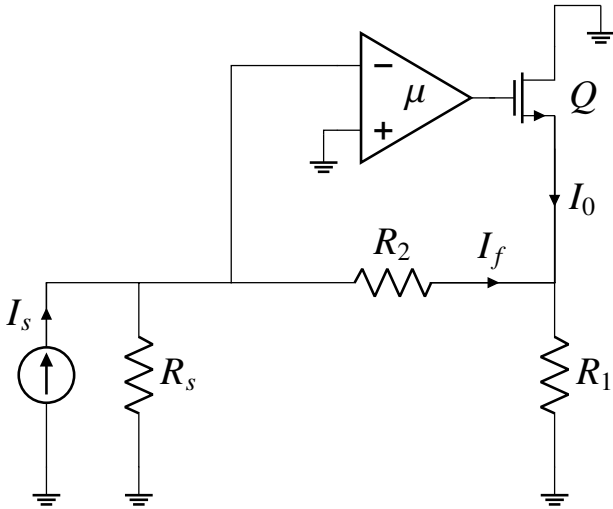


Fig. 1.0.1

1.0.2. Represent the Control System using a block diagram

Solution:

1.0.3. If loop gain is large, find approximate expression for closed loop gain T

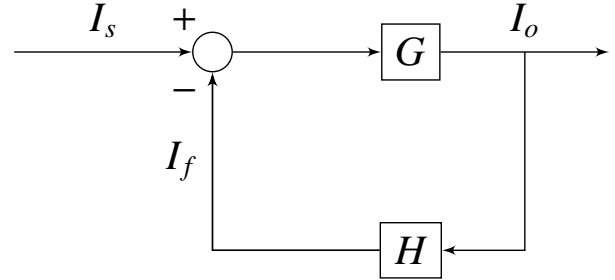


Fig. 1.0.2

Solution: Given,

$$GH \gg 1 \quad (1.0.3.1)$$

$$T = \frac{G}{1 + GH} \simeq \frac{1}{H} \quad (1.0.3.2)$$

$$H = \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2} \quad (1.0.3.3)$$

$$T \simeq \frac{1}{H} = -\left(1 + \frac{R_2}{R_1}\right) \quad (1.0.3.4)$$

1.0.4. Find the G Circuit i.e the Gain Circuit and approximate expressions for G , R_i , R_o

Solution: By replacing the Opamp with its equivalent model we can get the G circuit

$$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2) \quad (1.0.4.1)$$

$$V_i = I_i R_i \quad (1.0.4.2)$$

$$I_o = -\mu V_i \frac{1}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)} \quad (1.0.4.3)$$

$$G = \frac{I_o}{I_i} = -\mu \frac{R_i}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)} \quad (1.0.4.4)$$

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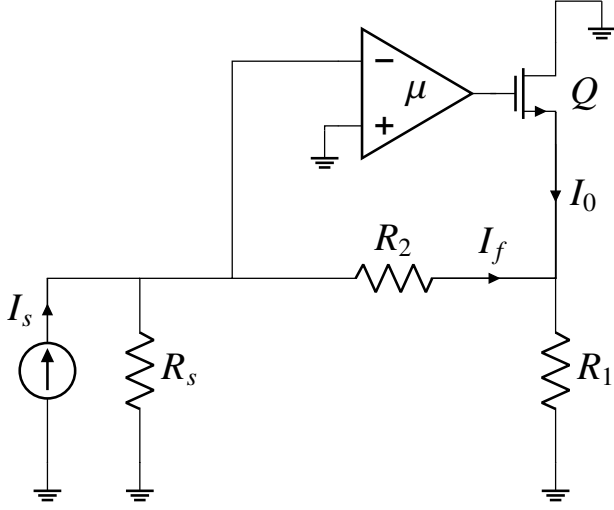


Fig. 1.0.4

We use the approximation

$$1/g_m \ll (R_1 \parallel R_2 \parallel r_{o2}) \quad (1.0.4.5)$$

This is because the $\frac{1}{g_m}$ is in order of few Ω s but, R_1 , R_2 and r_{o2} are in order of k Ω s

$$G = -\mu \frac{R_i}{R_1 \parallel R_2} \quad (1.0.4.6)$$

$$R_o = r_{o2} + (R_1 \parallel R_2) + (g_m r_{o2})(R_1 \parallel R_2) \quad (1.0.4.7)$$

$$\Rightarrow R_o \simeq g_m r_{o2} (R_1 \parallel R_2) \quad (1.0.4.8)$$

1.0.5. Give expressions for GH, T, R_{if} , R_{in} , R_{of} , R_{out}

Solution:

$$GH = \mu \frac{R_i}{\frac{1}{g_m} + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)} \frac{R_1}{R_1 + R_2} \quad (1.0.5.1)$$

Once again, using the approximation,

$$\Rightarrow GH \simeq \mu \frac{R_i}{R_1 \parallel R_2} \frac{R_1}{R_1 + R_2} = \mu \frac{R_i}{R_2} \quad (1.0.5.2)$$

For Input Resistance,

$$R_{if} = R_i / (1 + GH) \quad (1.0.5.3)$$

$$\Rightarrow \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{\mu}{R_2} \quad (1.0.5.4)$$

$$\Rightarrow R_{if} = R_i \parallel \frac{R_2}{\mu} \quad (1.0.5.5)$$

Substituting the value of R_i ,

$$R_{if} = R_s \parallel R_{id} \parallel (R_1 + R_2) \parallel \frac{R_2}{\mu} \quad (1.0.5.6)$$

$$R_{if} = R_s \parallel R_{in} \quad (1.0.5.7)$$

$$\Rightarrow R_{in} = R_{id} \parallel (R_1 + R_2) \parallel \frac{R_2}{\mu} \quad (1.0.5.8)$$

$$R_{in} \simeq \frac{R_2}{\mu} \quad (1.0.5.9)$$

For Output Resistance,

$$R_{of} = R_o(1 + GH) \simeq GHR_o \quad (1.0.5.10)$$

$$R_{of} \simeq \mu \left(\frac{R_i}{R_2} \right) (g_m r_{o2}) (R_1 \parallel R_2) \quad (1.0.5.11)$$

$$R_{out} = R_{of} = \mu \frac{R_i}{R_1 + R_2} (g_m r_{o2}) R_1 \quad (1.0.5.12)$$

1.0.6. Find numerical values for G, H, GH, T, R_i , R_{if} , R_{in} , R_o , R_{of} , R_{out} given the following values

Parameter	Value
μ	1000
R_s	∞
R_{id}	∞
r_{o1}	1k Ω
R_1	10k Ω
R_2	90k Ω
g_m	5mA/V
r_{o2}	20k Ω

TABLE 1.0.6

Solution: Using the given numerical values on the previously obtained equations, we obtain:

$$R_i = \infty \parallel \infty \parallel (10 + 90) = 100k\Omega \quad (1.0.6.1)$$

We can see that our approximation is valid

$$1/g_m = 0.2k\Omega \ll (10 \parallel 90 \parallel 20)k\Omega = 6.2k\Omega \quad (1.0.6.2)$$

$$G = -1000 \frac{100}{10 \parallel 90} = -11.11 \times 10^3 \quad (1.0.6.3)$$

$$H = -\frac{R_1}{R_1 + R_2} = -\frac{10}{10 + 90} = -0.1 \quad (1.0.6.4)$$

$$GH = 1111 \quad (1.0.6.5)$$

$$T \simeq \frac{1}{H} = -\frac{1}{0.1} = -10 \quad (1.0.6.6)$$

$$R_{in} = \frac{R_2}{\mu} = \frac{90k\Omega}{1000} = 90\Omega \quad (1.0.6.7)$$

$$R_o = g_m r_{o2}(R_1 || R_2) = 5 \times 20(10 || 90) = 900k\Omega \quad (1.0.6.8)$$

$$R_{out} = (1 + GH)R_o = 1112 \times 900 \simeq 1000M\Omega \quad (1.0.6.9)$$

Parameter	Value
R_i	100k Ω
$1/g_m$	200 Ω
G	-1.11×10^4
H	-0.1
GH	1111
T	-10
R_{in}	90 Ω
R_o	900k Ω
R_{out}	1000M Ω

TABLE 1.0.6

1.0.7. Verify the above calculations using a Python code.

Solution:

```
codes/ee18btech11021/ee18btech11021_calc.py
```