Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

1 Feedback Circuits

1.0.1. Consider a Feedback Current Amplifier formed by cascading an Inverting Opamp μ with a MOSFET (NMOS). The output current is the Drain Current of the NMOS. Assume that Opamp has an input resistance R_{id}, an Open Circuit Voltage Gain μ, and an output resistance r_{o1}

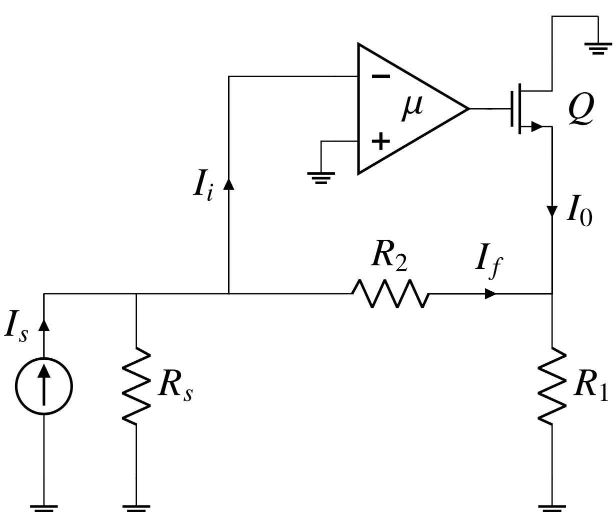

Fig. 1.0.1: Complete Circuit

1.0.2. Represent the given circuit using a Small Signal Equivalent Model.
Solution:

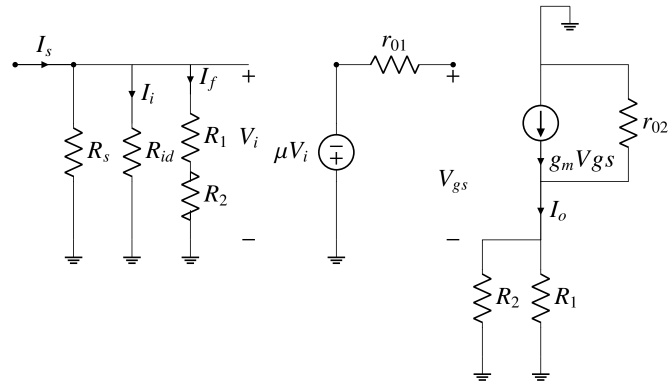

Fig. 1.0.2: Small Signal Model

1.0.3. Represent the Control System using a block diagram
Solution:

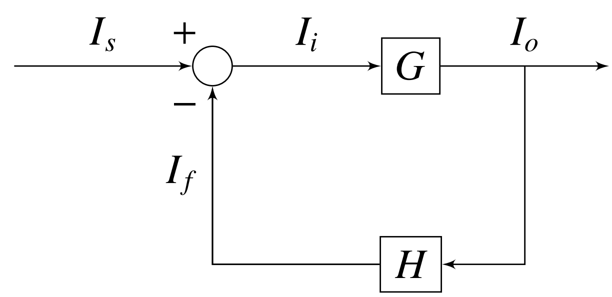

Fig. 1.0.3: Block Diagram

1.0.4. Find approximate expressions for G, R_i, R_o
Solution:

$$R_i = R_s \| R_{id} \| (R_1 + R_2) \tag{1.0.4.1}$$

$$V_i = I_i R_i \tag{1.0.4.2}$$

$$I_o = -\mu V_i \frac{1}{1/g_m + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \tag{1.0.4.3}$$

$$G = \frac{I_o}{I_i} = -\mu \frac{R_i}{1/g_m + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \tag{1.0.4.4}$$

We use the approximation

$$1/g_m \ll (R_1 \| R_2 \| r_{o2}) \tag{1.0.4.5}$$

This is because the $\frac{1}{g_m}$ is in order of few Ωs

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but, R_1, R_2 and r_{o2} are in order of kΩs

$$G = -\mu \frac{R_i}{R_1 \| R_2} \tag{1.0.4.6}$$

$$R_o = r_{o2} + (R_1 \| R_2) + (g_m r_{o2})(R_1 \| R_2) \tag{1.0.4.7}$$

$$\implies R_o \simeq g_m r_{o2} \left(R_1 \| R_2\right) \tag{1.0.4.8}$$

1.0.5. Find expression for Loop Gain H
Solution:

$$H = \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2} \tag{1.0.5.1}$$

1.0.6. If loop gain is large, find approximate expression for closed loop gain T
Solution: Given,

$$GH \gg 1 \tag{1.0.6.1}$$

$$T = \frac{G}{1 + GH} \simeq \frac{1}{H} \tag{1.0.6.2}$$

$$T \simeq \frac{1}{H} = -\left(1 + \frac{R_2}{R_1}\right) \tag{1.0.6.3}$$

1.0.7. Give expressions for GH, T, R_{if}, R_{in}, R_{of}, R_{out}
Solution:

$$GH = \mu \frac{R_i}{\frac{1}{g_m} + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \frac{R_1}{R_1 + R_2} \tag{1.0.7.1}$$

Once again, using the approximation,

$$\implies GH \simeq \mu \frac{R_i}{R_1 \| R_2} \frac{R_1}{R_1 + R_2} = \mu \frac{R_i}{R_2} \tag{1.0.7.2}$$

For Input Resistance,

$$R_{if} = R_i / (1 + GH) \tag{1.0.7.3}$$

$$\implies \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{\mu}{R_2} \tag{1.0.7.4}$$

$$\implies R_{if} = R_i \| \frac{R_2}{\mu} \tag{1.0.7.5}$$

Substituting the value of R_i,

$$R_{if} = R_s \| R_{id} \| (R_1 + R_2) \| \frac{R_2}{\mu} \tag{1.0.7.6}$$

$$R_{if} = R_s \| R_{in} \tag{1.0.7.7}$$

$$\implies R_{in} = R_{id} \| (R_1 + R_2) \| \frac{R_2}{\mu} \tag{1.0.7.8}$$

$$R_{in} \simeq \frac{R_2}{\mu} \tag{1.0.7.9}$$

For Output Resistance,

$$R_{of} = R_o(1 + GH) \simeq GHR_o \tag{1.0.7.10}$$

$$R_{of} \simeq \mu(\frac{R_i}{R_2})(g_m r_{o2})(R_1 \| R_2) \tag{1.0.7.11}$$

$$R_{out} = R_{of} = \mu \frac{R_i}{R_1 + R_2}(g_m r_{o2}) R_1 \tag{1.0.7.12}$$

1.0.8. Given the following values

Parameter	**Value**
μ	1000
R_s	∞
R_{id}	∞
r_{o1}	$1k\Omega$
R_1	$10k\Omega$
R_2	$90k\Omega$
g_m	$5mA/V$
r_{o2}	$20k\Omega$

TABLE 1.0.8

Find numerical value of R_i and use it to find the value of G
Solution: Using the given numerical values on the previously obtained equations, we obtain:

$$R_i = \infty \| \infty \| (10 + 90) = 100k\Omega \tag{1.0.8.1}$$

$$G = -1000 \frac{100}{10 \| 90} = -11.11 \times 10^3 \tag{1.0.8.2}$$

1.0.9. Check the validity of the approximation that we use to neglect $1/g_m$
Solution:

$$1/g_m = 0.2k\Omega \ll (10 \| 90 \| 20)k\Omega = 6.2k\Omega \tag{1.0.9.1}$$

Hence, we can see that our approximation is valid

1.0.10. Find the value of feedback gain H and open loop gain GH

Solution:

$$H = -\frac{R_1}{R_1 + R_2} = -\frac{10}{10 + 90} = -0.1 \quad (1.0.10.1)$$

$$GH = 1111 \gg 1 \quad (1.0.10.2)$$

1.0.11. Find the approximate value of closed loop gain T

Solution:

$$T \simeq \frac{1}{H} = -\frac{1}{0.1} = -10 \quad (1.0.11.1)$$

1.0.12. Find the values of R_{in} and R_{out}

Solution:

$$R_{in} = \frac{R_2}{\mu} = \frac{90k\Omega}{1000} = 90\Omega \quad (1.0.12.1)$$

$$R_o = g_m r_{o2}(R_1 \| R_2) = 5 \times 20(10\|90) = 900k\Omega \quad (1.0.12.2)$$

$$R_{out} = (1 + GH)R_o = 1112 \times 900 \simeq 1000M\Omega \quad (1.0.12.3)$$

Parameter	**Value**
R_i	$100k\Omega$
$1/g_m$	200Ω
G	-1.11×10^4
H	-0.1
GH	1111
T	-10
R_{in}	90Ω
R_o	$900k\Omega$
R_{out}	$1000M\Omega$

TABLE 1.0.12

1.0.13. Verify the above calculations using a Python code.

Solution:

```
codes/ee18btech11021/ee18btech11021_calc.
    py
```