Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

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1 Feedback Circuits

1.0.1. Consider a Feedback Current Amplifier formed by cascading an Inverting Opamp μ with a MOSFET (NMOS). The output current is the Drain Current of the NMOS. Assume that Opamp has an input resistance R_{id}, an Open Circuit Voltage Gain μ, and an output resistance r_{o1}

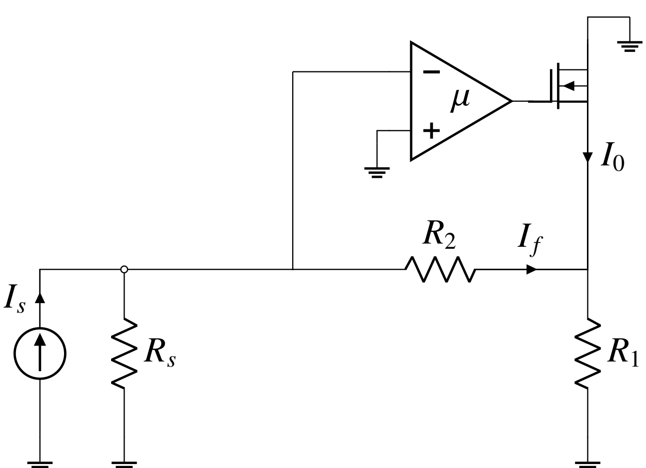

Fig. 1.0.1

1.0.2. If loop gain is large, find approximate expression for closed loop gain T
Solution: Given,

$$GH \gg 1 \tag{1.0.2.1}$$

$$T = \frac{G}{1+GH} \simeq \frac{1}{H} \tag{1.0.2.2}$$

$$H = \frac{I_f}{I_o} = -\frac{R_1}{R_1+R_2} \tag{1.0.2.3}$$

$$T \simeq \frac{1}{H} = -\left(1+\frac{R_2}{R_1}\right) \tag{1.0.2.4}$$

1.0.3. Find the G Circuit i.e the Gain Circuit and approximate expressions for G, R_i, R_o
Solution: By replacing the Opamp with its equivalent model we can get the G circuit

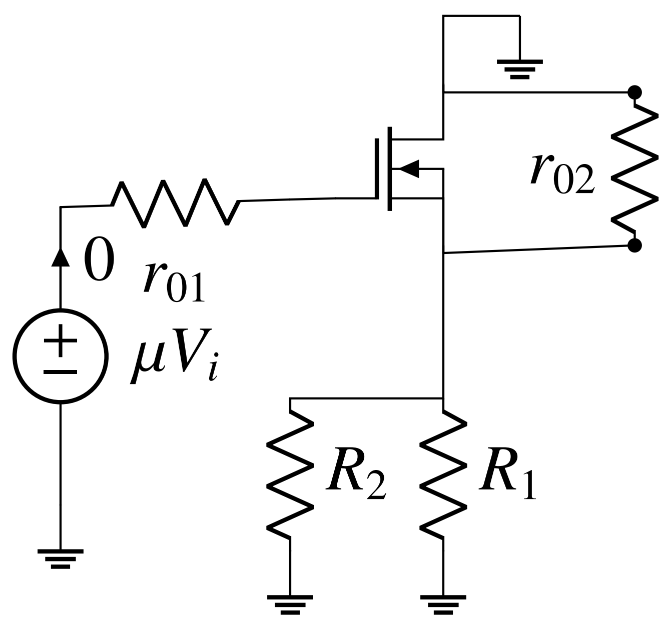

Fig. 1.0.3

$$R_i = R_s \| R_{id} \| (R_1+R_2) \tag{1.0.3.1}$$

$$V_i = I_i R_i \tag{1.0.3.2}$$

$$I_o = -\mu V_i \frac{1}{1/g_m + (R_1\|R_2\|r_{o2})}\frac{r_{o2}}{r_{o2}+(R_1\|R_2)} \tag{1.0.3.3}$$

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$$G = \frac{I_o}{I_i} = -\mu \frac{R_i}{1/g_m + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \tag{1.0.3.4}$$

We use the approximation

$$1/g_m \ll (R_1 \| R_2 \| r_{o2}) \tag{1.0.3.5}$$

This is because the $\frac{1}{g_m}$ is in order of few Ωs but, R_1, R_2 and r_{o2} are in order of kΩs

$$G = -\mu \frac{R_i}{R_1 \| R_2} \tag{1.0.3.6}$$

$$R_o = r_{o2} + (R_1 \| R_2) + (g_m r_{o2})(R_1 \| R_2) \tag{1.0.3.7}$$

$$\implies R_o \simeq g_m r_{o2} (R_1 \| R_2) \tag{1.0.3.8}$$

1.0.4. Give expressions for GH, T, R_{if}, R_{in}, R_{of}, R_{out}
Solution:

$$GH = \mu \frac{R_i}{\frac{1}{g_m} + (R_1 \| R_2 \| r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \| R_2)} \frac{R_1}{R_1 + R_2} \tag{1.0.4.1}$$

Once again, using the approximation,

$$\implies GH \simeq \mu \frac{R_i}{R_1 \| R_2} \frac{R_1}{R_1 + R_2} = \mu \frac{R_i}{R_2} \tag{1.0.4.2}$$

For Input Resistance,

$$R_{if} = R_i/(1 + GH) \tag{1.0.4.3}$$

$$\implies \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{\mu}{R_2} \tag{1.0.4.4}$$

$$\implies R_{if} = R_i \| \frac{R_2}{\mu} \tag{1.0.4.5}$$

Substituting the value of R_i,

$$R_{if} = R_s \| R_{id} \| (R_1 + R_2) \| \frac{R_2}{\mu} \tag{1.0.4.6}$$

$$R_{if} = R_s \| R_{in} \tag{1.0.4.7}$$

$$\implies R_{in} = R_{id} \| (R_1 + R_2) \| \frac{R_2}{\mu} \tag{1.0.4.8}$$

$$R_{in} \simeq \frac{R_2}{\mu} \tag{1.0.4.9}$$

For Output Resistance,

$$R_{of} = R_o(1 + GH) \simeq GHR_o \tag{1.0.4.10}$$

$$R_{of} \simeq \mu(\frac{R_i}{R_2})(g_m r_{o2})(R_1 \| R_2) \tag{1.0.4.11}$$

$$R_{out} = R_{of} = \mu \frac{R_i}{R_1 + R_2}(g_m r_{o2}) R_1 \tag{1.0.4.12}$$

1.0.5. Find numerical values for G, H, GH, T, R_i, R_{if}, R_{in}, R_o, R_{of}, R_{out} given the following values μ = 1000, R_s = inf,R$_{id}$ = inf,r$_{o1}$ = 1 kΩ, R_1 = 10 kΩ, R_2 = 90kΩ
Solution: Using the given numerical values on the previously obtained equations, we obtain:

$$R_i = \infty \| \infty \| (10 + 90) = 100k\Omega \tag{1.0.5.1}$$

We can see that our approximation is valid

$$1/g_m = 0.2k\Omega \ll (10 \| 90 \| 20)k\Omega = 6.2k\Omega \tag{1.0.5.2}$$

$$G = -1000 \frac{100}{10 \| 90} = -11.11 \times 10^3 \tag{1.0.5.3}$$

$$H = -\frac{R_1}{R_1 + R_2} = -\frac{10}{10 + 90} = -0.1 \tag{1.0.5.4}$$

$$GH = 1111 \tag{1.0.5.5}$$

$$T \simeq \frac{1}{H} = -\frac{1}{0.1} = -10 \tag{1.0.5.6}$$

$$R_{in} = \frac{R_2}{\mu} = \frac{90k\Omega}{1000} = 90\Omega \tag{1.0.5.7}$$

$$R_o = g_m r_{o2}(R_1 \| R_2) = 5 \times 20(10 \| 90) = 900k\Omega \tag{1.0.5.8}$$

$$R_{out} = (1 + GH)R_o = 1112 \times 900 \simeq 1000M\Omega \tag{1.0.5.9}$$